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Learning about analysts ☆

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Abstract

We examine an analyst with career concerns making cheap talk recommendations to a sequence of traders, each of whom possesses private information concerning the analyst's ability. The recommendations of the analyst influence asset prices that are then used to evaluate the analyst. An endogeneity problem thus arises. In particular, if the reputation of the analyst is sufficiently high then an incompetent but strategic analyst is able to momentarily hide her type. An equilibrium in which the market eventually learns the analyst type always exists. However, under some conditions, an equilibrium also exists in which the incompetent analyst is able to hide her type forever.

JEL classification: D82; D83; G12; G14

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1. Introduction

A key task of financial analysts is to make recommendations to investors. As analyst ability is not directly observable, reputation – measured by rankings such as Institutional Investor's

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All-American Research Team – is a major factor affecting an analyst's compensation.[1] However, since a stock's true value is unknown, evaluating the quality of an analyst's advice is a complicated task. Indeed, analyst forecasts influence prices which are later on used to evaluate analyst ability. This suggests that an incompetent but *strategic* analyst might be able to bias her recommendations in order to appear better than she really is. Can incompetent analysts hide their type and, if so, how? What are the implications for market liquidity and asset prices' convergence to the true asset values?

To address these questions, we propose a stylized model in the spirit of Glosten and Milgrom (1985) but in which traders obtain information through the recommendations of a financial analyst (she). The analyst is either good (G) or bad (B). The revenue of the analyst in a period is an increasing function of the public belief that she is type G, which we refer to as the reputation of the analyst. The (fixed) asset value is either high or low and each period the analyst receives a binary piece of information concerning the realized asset value. The G analyst receives more precise information than the B analyst. After receiving information, the analyst makes a cheap-talk recommendation, either "buy" or "sell". The G analyst is truthful, but the B analyst makes recommendations with a view to maximize the expected discounted sum of her future revenues. A new trader (he) arrives each period. The trader is either a liquidity trader (i.e. trades randomly), or a profit-maximizing speculator. Each speculator privately observes a noisy signal of the analyst type. Trade takes place in a competitive dealer market, but market makers observe the recommendations with a lag.[2] Reputation is updated at the end of each period based on the observation of the latest recommendation and trade order.

We start by showing that if the reputation of the analyst is sufficiently high then the analyst can momentarily hide her type. Increasing reputation simultaneously reduces the importance of speculators' private signals about the analyst and increases the impact of the current recommendation on a speculator's valuation. Hence, when the reputation of the analyst is sufficiently high information incorporated in the latest recommendation trumps a speculator's private signal about analyst ability: speculators buy with probability 1 following a buy recommendation and sell with probability 1 following a sell recommendation, preventing the market from learning about the analyst by observing the order flow.

We then show that the only possible way B can hide her type indefinitely is if her recommendations are informative asymptotically. The intuition is as follows. As G is better informed than B, a speculator whose private signal provides unfavorable information about the analyst type will see his valuation drawn back toward the prior expectation of the asset value. The longer the history the greater the reversion toward the mean. By that logic, when the number of past recommendations is very large, either B's recommendations are sufficiently informative so as to curb the reversion to the mean indicated above or speculators start trading based on their private

1 According to Michaely and Womack (2005), "At most brokerage firms, analyst compensation is based on two major factors. The first is the analyst's perceived (external) reputation. The annual Institutional Investor All-American Research Teams poll is perhaps the most significant external influence driving analyst compensation. All-American rankings are based on a questionnaire asking over 750 money managers and institutions to rank analysts in several categories: stock picking, earnings estimates, written reports, and overall service." Leone and Wu (2007) and Fang and Yasuda (2014) find that reputation as measured by Institutional Investor's ranking of analysts seems to be driven by skill. Groysberg et al. (2011) show empirically that analyst compensation is positively correlated to "All-Star" recognition and being named a top stock picker by the Wall Street Journal.

2 This reflects the fact that recommendations are often first disclosed to client investors before they are publicized (Michaely and Womack, 2005). In Rüdiger and Vigier (2017) we investigate under what circumstances traders are better informed than market makers.

signals: when the price is high, speculators whose private signals of the analyst type are favorable choose to buy the asset; those whose private signals are unfavorable choose to sell. Thus either B's recommendations are informative asymptotically or the market eventually learns the analyst type. This implies in turn that, in equilibrium, either the market learns the analyst type or the market learns the true asset value.[3] Our second main result goes on to establish that –in spite of the occurrence of informational cascades of the kind previously described– an equilibrium in which the market learns the analyst type *always* exists.

Under certain conditions, another equilibrium exists, in which the B analyst succeeds at hiding her type forever. The conditions required are that (i) B possesses enough private information about the asset and that (ii) no speculator be too well informed about the analyst type. In the latter equilibrium B mimics G's frequencies of buy and sell recommendations. At the same time the B analyst ensures that her recommendations are sufficiently informative so as to induce the speculators to trade based on the recommendations rather than on their private signals of the analyst type. Consequently neither the order flow nor the pattern of recommendations made convey any information about the analyst. In an extension we show that in fact when the asset value is not fixed but follows a known Markov process then, even if she is completely uninformed, the B analyst can hide her type indefinitely. In that setting, uncertainty about possible value changes never decreases over time, allowing the B analyst to make recommendations that are never trumped by speculators' private signals of the analyst type.

The related literature is discussed below. The model is presented in Section 2. Section 3 analyzes an example illustrating the basic workings of the model. Section 4 contains our main results. The implications of our model for market liquidity and price discovery are examined in Section 5, where the example of Section 3 is simulated computationally. Changing asset values are examined in Section 6. Section 7 concludes.

Related literature To the best of our knowledge the present paper is the first to analyze a sequential trading model in the spirit of Glosten and Milgrom (1985) but where traders' information proceeds from the recommendations of an analyst with unknown ability and reputational concerns. Specifically, in our model the market (a) learns about the asset value from the information of an analyst and (b) learns about the ability of the analyst from the order flow, given that speculators have private information about the analyst's ability. The learning mechanism (a) is along the lines of Glosten and Milgrom (1985) (that is, essentially, the Bikhchandani et al. (1992) statistical herding model with an endogenous price). Compared to Glosten and Milgrom's setting, the novel feature is the learning mechanism (b) concerning the ability of the analyst. The market can only learn about the analyst's ability if the order flow is affected by the speculators' signals about analyst ability. In turn, learning about analyst ability impacts learning about the asset value, by enabling the market to assess the quality of the information supplied by the analyst.

Our paper bridges two strands of literature. The first strand examines the (lack of) aggregation of private information in sequential trade settings (Avery and Zemsky, 1998; Lee, 1998; Chari and Kehoe, 2004; Decamps and Lovo, 2006; Cipriani and Guarino, 2008; Dasgupta and Prat, 2008; Park and Sabourian, 2011). The central difference between these papers and ours is that instead of being exogenous, in our paper traders' information about the asset is provided by a financial analyst (with unknown ability) who may act strategically. The second strand of litera-

[3] By "learning‘" we mean holding beliefs coming arbitrarily close to the realization of the relevant random variable.

ture connected to our work explores the behavior of financial analysts motivated by reputational concerns (Scharfstein and Stein, 1990; Benabou and Laroque, 1992; Trueman, 1994; Ottaviani and Sørensen, 2006). The main difference between these papers and ours is the feedback channel through which the market learns about the analyst. In these papers the analyst forecasts asset values which are then observed (with noise). By contrast, in our model the asset value is never observed, and the main feedback channel is the order flow, which is endogenous. This difference is key since trade orders can be influenced, whereas fundamental asset values cannot. The next paragraphs discuss these papers in greater details.

The literature on informational cascades in financial markets builds on Bikhchandani et al. (1992) and Smith and Sørensen (2000). To the best of our knowledge Avery and Zemsky (1998) were first to note that informational cascades are precluded in the framework of Glosten and Milgrom (1985), the reason being that price adjustment by competitive market makers provides incentives to traders and thereby prevents information getting trapped. The authors showed that herd behavior is however possible; the necessary and sufficient conditions for herding to occur are established by Park and Sabourian (2011). Various papers show how natural modifications of the baseline framework could be reconciled with the occurrence of informational cascades.[4] Lee (1998) introduces transaction costs; Chari and Kehoe (2004) relax the assumption that traders move in a prespecified order; Decamps and Lovo (2006) consider traders and market makers who differ in their risk aversion; in Cipriani and Guarino (2008) traders' utility derived from the asset differs from that of the market makers; Dasgupta and Prat (2008) consider traders who care both about reputation and about trading profits. In all of the aforementioned papers, informational cascades about the asset value are possible. In our setting, the informational cascades are about the analyst type instead of being about the asset value. Yet the strategic behavior of the B analyst hinders price discovery since, by hiding her type, the B analyst prevents the market from efficiently evaluating information contained (or not) in the recommendations.

The literature on financial analysts with reputational concerns analyzes how such concerns may lead a strategic analyst to misrepresent her private information. Scharfstein and Stein (1990), Trueman (1994) and Ottaviani and Sørensen (2006) all develop the idea that in order to appear good, analysts strategically shade their forecasts toward the prior mean, thereby reducing the informativeness of the recommendations made; in Benabou and Laroque (1992) the analyst cares about reputation but also engages in insider trading, creating an incentive for the analyst to mislead the market in order to make more trading profits. As mentioned above, in all these papers, feedback concerning analyst type takes a very different form than in our setting. Instead of learning about the analyst by observing the true asset values, in our setting the market is forced to learn about the analyst's type based on endogenous variables, namely traders' response to the recommendations. However, when the reputation of the analyst is sufficiently high, traders' response to the recommendations are uninformative about the analyst type. An informational cascade then occurs (with respect to the analyst type).

[4] The baseline framework has the following features: (i) the asset value is time-invariant, (ii) prices are set by competitive market makers, (iii) trade is sequential, (iv) at most one unit of the asset is traded each period, (v) a trader is either a liquidity trader or a speculator, (vi) each trader is given one opportunity to trade at a predetermined time, (vii) there are no transaction costs.

2. Model

An analyst (she) makes cheap-talk recommendations to a sequence of traders (he) trading a risky asset in a competitive dealer market. The risky asset has value V: $V = 1$ with probability $\frac{1}{2}$ and $V = 0$ with probability $\frac{1}{2}$. Time is discrete and indexed by $t = 1, 2, \ldots$.

Analyst The analyst is either good or bad. Her type $\theta \in \{G, B\}$ is private information; ρ_t indicates the beginning-of-period-t public belief that $\theta = G$. We refer to ρ_t as the analyst reputation and assume that $\rho_1 \in (0, 1)$. Each period the analyst observes $x_t \in \{0, 1\}$; conditional on θ and V, the sequence $\{x_t\}_{t\in\mathbb{N}}$ is independent and identically distributed according to

$$\mathbb{P}(x_t = V \mid V, \theta) = q_\theta, \quad \text{where } 1 > q_G > q_B \geq \frac{1}{2}.$$

The G analyst is therefore better informed than B, and B may be completely uninformed.

Recommendations The recommendation of the analyst in period t is denoted

$$r_t \in \{\text{buyrec}, \text{sellrec}\},$$

with buyrec/sellrec standing for buy/sell recommendation. We assume that G makes truthful recommendations, i.e., conditional on $\theta = G$, then $r_t =$ buyrec if and only if $x_t = 1$. This assumption enables us to focus the analysis on B's attempt to manipulate the market in order to appear to be type G. We discuss in Section 4 what would change if this assumption were relaxed.

Payoffs In each period t, the B analyst maximizes the expectation of the discounted payoff U_t given by

$$U_t := u(\rho_{t+1}) + \delta u(\rho_{t+2}) + \delta^2 u(\rho_{t+3}) + \ldots,$$

where $u(\cdot)$ is a strictly increasing function. These payoffs could for instance represent revenues derived from selling advice on a secondary market, where an analyst known to be type G would obtain the price $u(1)$. Alternatively, the analyst could be employed on a contract that specified wage as a function of reputation.

Traders Following Glosten and Milgrom (1985) a new trader is drawn i.i.d. each period. With probability $\pi \in (0, 1)$ the trader is a speculator, and with probability $1 - \pi$ he is a liquidity trader. Speculators trade to maximize profits, while liquidity traders trade at random, independently of all other random variables. To save notation, we assume that liquidity traders buy, sell and abstain from trading the asset with probability $1/3$ each. The trade order in period t is denoted

$$y_t \in \{\text{buy}, \text{sell}, \text{abstain}\}.$$

Each speculator is endowed with private information concerning the analyst type. For simplicity, this information takes the form of a binary signal $s_t \in \{g, b\}$ drawn i.i.d. across speculators and satisfying

$$\mathbb{P}(s_t = g|\theta) = \gamma_\theta, \quad \text{where } \gamma_G > \gamma_B.$$

Thus $s_t = g$ (respectively $s_t = b$) represents favorable (resp. unfavorable) information about the analyst type. We refer to speculators having observed the signal realization g (respectively b) as type-g (resp. type-b) speculators.

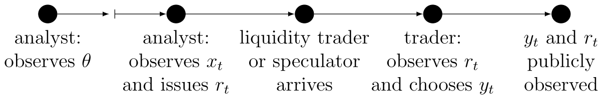

Fig. 1. Timeline.

Financial market Trade takes place in a competitive dealer market. The public information at the beginning of period t is denoted h_t, and consists of all past recommendations and trade orders, that is, $h_1 = \emptyset$ and

$$h_t := \{r_1, ..., r_{t-1}; y_1, ..., y_{t-1}\}, \quad \forall t > 1.$$

The current recommendation, r_t, is initially observed only by trader t. This assumption is crucial; it reflects the fact that recommendations are first disclosed to client investors before they are publicized (see Michaely and Womack (2005)).[5] The ask price p_t^a and bid price p_t^b are therefore given by

$$p_t^a = \mathbb{E}[V|h_t, y_t = \text{buy}];$$
$$p_t^b = \mathbb{E}[V|h_t, y_t = \text{sell}].$$

That is, competitive market makers price the asset at its expected value.

Strategies and equilibrium The timeline is summarized in Fig. 1. First, the analyst observes her type, θ. The remaining timeline, to the right of the broken arrow, is for an arbitrary period t. Within a given period, the analyst first observes x_t and then issues her recommendation, r_t. A new trader is then drawn at random (probability π for speculators and $1-\pi$ for liquidity traders). The trader observes the current recommendation, and chooses his trade order (to maximize profits if he is a speculator, or uniformly at random if he is a liquidity trader). Finally, the trade order and the current recommendation are publicly observed, and reputation is updated.

A (behavior) strategy of the B analyst specifies the probabilities of making recommendations $r_t = \text{buyrec}$ and $r_t = \text{sellrec}$ for all tuples $(h_t, x_1, \ldots, x_t)$. A strategy of speculator t specifies, for all triples (h_t, r_t, s_t), the probabilities of buying, selling, and abstaining from trading the asset. At the end of each period, the analyst's reputation is updated using Bayes' rule. The equilibrium concept is perfect Bayesian equilibrium.

2.1. Notation and terminology

Various expectations of the asset value (henceforth referred to as valuations) play a central role throughout the analysis. The expected asset value based only on the public history is denoted by v_t, that is,

$$v_t := \mathbb{E}[V|h_t].$$

As usual in the literature, we slightly abuse terminology and refer to v_t as the price. Speculator t's valuation of the asset is denoted by $v_t(r, s)$, that is,[6]

[5] In Rüdiger and Vigier (2017), we show that for a range of the cost of acquiring information, the unique equilibrium is such that speculators become informed with probability 1 whereas market makers choose to remain uninformed.

[6] If trader t is a liquidity trader then his valuation is irrelevant.

$$v_t(r,s) := \mathbb{E}[V|h_t, r_t = r, s_t = s].$$

We will sometimes say that speculators *screen the analyst* when speculators' trade orders depend on their private signals of the analyst, and that *screening breaks down* if instead a speculator's trade order is independent of s_t. Lastly, the expected asset value conditional both on the history and the type of the analyst will often be useful; we therefore define

$$v_t^\theta := \mathbb{E}[V|h_t, \theta].$$

In particular, $v_t = \rho_t v_t^G + (1-\rho_t)v_t^B$.

Since the model is symmetric with respect to buy and sell recommendations, the difference n_t between the total numbers of buy and sell recommendations having occurred before time t will be useful in the analysis:

$$n_t := \sum_{k<t} \left(\mathbf{1}_{\{r_k=\text{buyrec}\}} - \mathbf{1}_{\{r_k=\text{sellrec}\}}\right),$$

where $\mathbf{1}_X$ denotes the indicator function of X. We refer to n_t as the net recommendation count. Some results will be stated for n_t positive only in order to shorten the exposition. Lastly, to shorten notation the probability of a buy recommendation in the current period conditional on the analyst being type θ will be denoted R_t^θ, that is, $R_t^\theta := \mathbb{P}(r_t = \text{buyrec} \mid h_t, \theta) = 1 - \mathbb{P}(r_t = \text{sellrec} \mid h_t, \theta)$.

3. Example: uninformed bad analyst

In this section we illustrate the basic workings of the model by way of a simple example. Specifically, we set in this section

$$q_B = \frac{1}{2}; \tag{A1}$$

$$\gamma_B = 0 \text{ and } \gamma_G \in (0,1); \tag{A2}$$

$$u(\rho_t) = \rho_t \text{ and } \delta = 0. \tag{A3}$$

Assumption (A1) implies that the B analyst is completely uninformed about the realization of the asset value. Assumption (A2) implies that $s_t = g$ perfectly reveals $\theta = G$, while $s_t = b$ is imperfectly informative. Assumption (A3) implies $U_t = \rho_{t+1}$, i.e. each period the objective of the B analyst is to maximize the expectation of her reputation one period ahead. The proofs of this section are in Appendix A.

We first examine the ordering of speculators' equilibrium valuations and show that, provided the net recommendation count is at least two ($n_t \geq 2$), then the valuation of speculator t is highest when $(r_t, s_t) = (\text{buyrec}, g)$ and lowest when $(r_t, s_t) = (\text{sellrec}, b)$. Furthermore, $v_t(\text{buyrec}, g)$ is strictly above the ask price and $v_t(\text{sellrec}, b)$ is strictly below the bid price.

Lemma 1. *In any equilibrium, given any history satisfying $n_t \geq 2$:*

(i) $v_t(\textit{buyrec}, g) = \max_{(r,s)}\{v_t(r,s)\}$;
(ii) $v_t(\textit{sellrec}, b) = \min_{(r,s)}\{v_t(r,s)\}$;
(iii) $v_t(\textit{buyrec}, g) > p_t^a$ *and* $v_t(\textit{sellrec}, b) < p_t^b$.

The logic behind parts (i) and (ii) of the lemma is as follows. When $n_t \geq 2$ then, holding fixed the current recommendation, the valuation of type-b speculators is strictly less than the valuation of type-g speculators, that is, $v_t(r, b) < v_t(r, g)$ irrespective of r. The idea is straightforward: as B is uninformed, unfavorable information about the analyst draws valuations back toward the prior expectation of the asset. Parts (i) and (ii) of the lemma then follow from the remark that buy (respectively sell) recommendations tend to push the valuations upwards (resp. downwards).[7] To understand the final part of the lemma just note that, due to the presence of liquidity traders, speculators with the most extreme valuations of the asset must be making strictly positive profits.

We next present this section's first main result, showing that if the reputation of the analyst is sufficiently high then in equilibrium the market temporarily stops learning anything about the type θ of the analyst. In what follows, let $\hat{v}_t(r_t, s_t)$ indicate speculator t's valuation of the asset under the belief that the B analyst "mimics" type G, that is, given the belief that $R_t^B = R_t^G$. Define also

$$\hat{p}^a(\rho_t, n_t) := \frac{\pi R_t^G}{\pi R_t^G + \frac{1-\pi}{3}}\Big(\rho_t v_t^G(\text{buyrec}) + (1-\rho_t)\frac{1}{2}\Big) + \frac{\frac{1-\pi}{3}}{\pi R_t^G + \frac{1-\pi}{3}}\Big(\rho_t v_t^G + (1-\rho_t)\frac{1}{2}\Big) \tag{1}$$

and

$$\hat{p}^b(\rho_t, n_t) := \frac{\pi (1-R_t^G)}{\pi(1-R_t^G) + \frac{1-\pi}{3}}\Big(\rho_t v_t^G(\text{sellrec}) + (1-\rho_t)\frac{1}{2}\Big) + \frac{\frac{1-\pi}{3}}{\pi(1-R_t^G) + \frac{1-\pi}{3}}\Big(\rho_t v_t^G + (1-\rho_t)\frac{1}{2}\Big). \tag{2}$$

The function $\hat{p}^a(\rho_t, n_t)$ captures the ask price of market makers computed under the assumptions that (i) $R_t^B = R_t^G$ and (ii) speculator t chooses to buy the asset if and only if $r_t = \text{buyrec}$. Similarly, the function $\hat{p}^b(\rho_t, n_t)$ captures the bid price under the assumptions that (i) $R_t^B = R_t^G$ and (ii) speculator t chooses to sell the asset if and only if $r_t = \text{sellrec}$. We can now define, for $n_t \geq 2$,

$$\underline{\rho}(n_t) := \min\big\{\rho_t > 0 : \hat{v}_t(\text{buyrec}, b) \geq \hat{p}^a(\rho_t, n_t) \text{ and } \hat{v}_t(\text{sellrec}, g) \leq \hat{p}^b(\rho_t, n_t)\big\}. \tag{3}$$

We show in Appendix A that $\underline{\rho}(n_t)$ is well-defined, with $\underline{\rho}(n_t) \in (0, 1)$.[8] The interpretation is the following: $\underline{\rho}(n_t)$ is the minimum reputation such that, when $R_t^B = R_t^G$, and the asset is priced at $\hat{p}^a$ and $\hat{p}^b$, then buying the asset if and only if $r_t = \text{buyrec}$ and selling the asset if and only if $r_t = \text{sellrec}$ comprises an optimal strategy of speculator t.

Observe that by construction if an equilibrium and history $\tilde{h}_t$ exist such that $\rho_t > \underline{\rho}(n_t)$, then an equilibrium exists such that, given $\tilde{h}_t$: (i) $R_t^B = R_t^G$, (ii) the asset is priced at $\hat{p}^a$ and $\hat{p}^b$, and (iii) speculator t buys (respectively sells) the asset if and only if $r_t = \text{buyrec}$ (resp.

[7] We say "tend to" because the recommendation conveys information about the analyst type, opening up for the possibility that $v_t(\text{buyrec}, b) < v_t(\text{sellrec}, b)$. We rule out this possibility in the proof.

[8] See Lemma 5 in Appendix A. Note that $\underline{\rho}(n_t)$ is defined independently of any equilibrium.

$r_t = \text{sellrec}$).[9] In this equilibrium, given $\tilde{h}_t$, neither the recommendation made in period t nor the trade order reveals any information about the analyst type; thus $\rho_{t+1} = \rho_t$ with probability one conditional on $\tilde{h}_t$. The next proposition shows that in fact, given *any* equilibrium, if a history induces $\rho_t > \underline{\rho}(n_t)$ then $\rho_{t+1} = \rho_t$ with probability one conditional on this history.

Proposition 1. *Let $\underline{\rho}(n_t)$ be defined by (3). Consider an arbitrary equilibrium.*

(i) *Fix a history h_t such that $\rho_t > \underline{\rho}(n_t)$, and $n_t \geq 2$. Then $\rho_{t+1} = \rho_t$ with probability one.*
(ii) *Conversely, fix a history h_t such that $\rho_{t+1} = \rho_t$ with probability one, and $n_t \geq 2$. Then $\rho_t \geq \underline{\rho}(n_t)$.*

To understand Proposition 1, notice that increasing analyst reputation simultaneously reduces the importance of speculators' private signals about the analyst type *and* increases the impact of the current recommendation on a speculator's valuation. Hence, when the reputation of the analyst is sufficiently high speculators ignore their private signals about the analyst type and make trading decisions solely based on the current recommendation. Mimicking the G analyst then enables B to completely hide her type, thereby inducing an informational cascade with respect to analyst type.

We next inquire whether informational cascades of the kind described above can last indefinitely. Note to start with that in any equilibrium and given any history, $\mathbb{P}(r_t = r_{t+1} = \cdots = r_{t+T} = \text{buyrec}|h_t) > 0$, and so

$$\mathbb{P}(n_{t+T} = n_t + T|h_t) > 0, \quad \text{for all } T > 0. \tag{4}$$

Can an equilibrium and history h_t exist such that $\mathbb{P}(\rho_{t+T} = \rho_t|h_t) = 1$ for arbitrary $T > 0$? If they existed then, combining (4) with part (ii) of Proposition 1 would imply $\rho_t \geq \underline{\rho}(n_t + T)$, for all $T > 0$. Therefore, if an equilibrium-history pair exists such that $\mathbb{P}(\rho_{t+T} = \rho_t|h_t) = 1$ for arbitrary $T > 0$ then the function $\underline{\rho}(\cdot)$ must be bounded above by $\rho_t < 1$. Our next result shows however that $\lim_{n_t \to +\infty} \underline{\rho}(n_t) = 1$.

Proposition 2. *Let $\underline{\rho}(n_t)$ be defined by (3). Then $\lim_{n_t \to +\infty} \underline{\rho}(n_t) = 1$. In particular, given any equilibrium and history h_t, there exists $T < \infty$ such that $\mathbb{P}(\rho_{t+T} = \rho_t|h_t) < 1$.*

We conclude from Proposition 2 that informational cascades of the kind described in Proposition 1 cannot last indefinitely. When the net recommendation count becomes sufficiently large relative to the analyst's reputation, the market again starts accumulating information about the type θ of the analyst. The mechanism is as follows. As long as analyst reputation is strictly positive, increasing the net recommendation count simultaneously reduces the impact of the current recommendation on a speculator's valuation *and* enhances the importance of beliefs concerning the analyst type. Intuitively, since B is uninformed, if the analyst is in fact type B then mispricing must be substantial, allowing speculators to make profits by trading *against* historical trends (that is, selling if the price is high and buying if the price is low). When the net recommendation

[9] Given $R_t^B = R_t^G$, $p_t^a = \hat{p}^a(\rho_t, n_t)$, and $p_t^b = \hat{p}^b(\rho_t, n_t)$, then ignoring s_t is optimal for speculator t. On the other hand, if speculator t ignores s_t and reputation is updated based on the belief that $R_t^B = R_t^G$, then the analyst is guaranteed $\rho_{t+1} = \rho_t$ and, therefore, is indifferent between choosing $r_t = \text{buyrec}$ or choosing $r_t = \text{sellrec}$. Lastly, given $R_t^B = R_t^G$ and speculator t buying (respectively selling) the asset if and only if $r_t = \text{buyrec}$ (resp. $r_t = \text{sellrec}$) then the ask price (resp. bid price) is given by $\hat{p}^a(\rho_t, n_t)$ (resp. $\hat{p}^b(\rho_t, n_t)$).

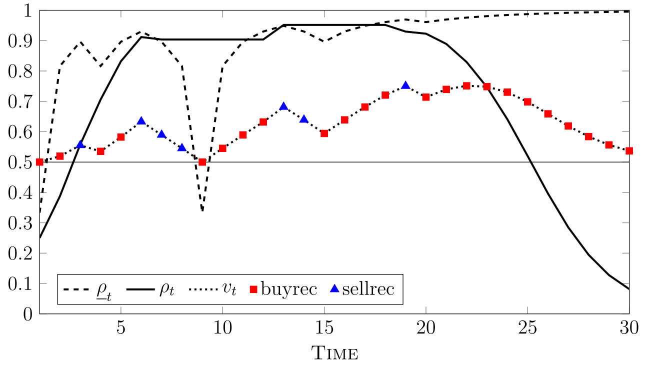

Fig. 2. Computational example. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

count is sufficiently large speculators thus ignore the current recommendation and make trading decisions solely based on their private signals of the analyst type. The former mechanism in turn enables the market to gradually learn about θ, yielding this section's last result.

Proposition 3. *In any equilibrium the market learns the analyst type, that is, as $t \to \infty$, with probability 1, $\rho_t \to 0$ conditional on $\theta = B$ and $\rho_t \to 1$ conditional on $\theta = G$.*

As we shall see in the next section, the current example's assumption that the B analyst has no information about V is crucial for the result in Proposition 3 to hold. When instead B is informed, certain conditions enable the recommendations made to never be trumped by the speculators' private signals of the analyst type. Whenever these conditions hold, equilibria in which the analyst indefinitely hides her type then exist alongside equilibria in which the market learns θ.

We illustrate in Fig. 2 the workings of the model by simulating the model computationally over the course of 30 periods.[10] In the simulation, the analyst is type B; the parameters chosen are $q_G = 0.55$, $\gamma_G = 0.7$, $\pi = 0.3$, and $\rho_1 = 0.25$. The solid curve in the figure represents the reputation of the analyst; the dashed curve shows the evolution of the screening threshold $\underline{\rho}(n_t)$. The periods in which screening breaks down thus correspond to the periods in which the solid curve lies above the dashed curve. The dotted curve depicts the price v_t, with red squares representing $r_t =$ buyrec and blue triangles representing $r_t =$ sellrec.

Since $\rho_1 < \underline{\rho}(n_1)$, initially speculators screen the analyst. At first, the analyst experiences a streak of luck: for the first 5 periods, trade orders are in line with the recommendations and therefore the reputation goes up. By period 6, the market attaches 91% probability to the analyst being type G ($\rho_6 = 0.91$). At this point, the impact of the recommendations on the price is substantial: 4 buy and 1 sell recommendations give $v_6 = 0.63$. Next period, $\rho_7 > \underline{\rho}(n_7)$, speculators' screening of the analyst breaks down and reputation momentarily stops evolving. By period 12, a string of buy recommendations pushes the screening threshold above the reputation, allowing once again the market to evaluate the analyst's ability. However, another lucky event enables the analyst to increase her reputation, triggering a new breakdown of screening starting

[10] We explain the simulation in Appendix D (available online). The code of the simulation is available on the authors' websites.

in period 13 and lasting until period 17. At this point the analyst runs out of luck: a prolonged sequence of trade orders goes counter to the recommendations made and reputation therefore decreases. From period 20 on, the analyst only makes buy recommendations, but at this point her reputation has dropped so much that she has a limited impact on the price. Eventually, her buy recommendations become insufficient to move the price away from the prior.

4. General results

In this section we present the paper's main results. Throughout this section we will say that the market *learns the analyst type* (respectively *learns the true asset value*) if, as $t \to \infty$, with probability 1, $\rho_t \to 0$ conditional on $\theta = B$ (resp. $v_t \to 0$ conditional on $V = 0$) and $\rho_t \to 1$ conditional on $\theta = G$ (resp. $v_t \to 1$ conditional on $V = 1$). The proofs of this section are in Appendix B.

Can the B analyst manipulate the market in order to hide her type and what are the implications for price discovery (i.e. for the convergence of the price to the true asset value)? We start with a positive result, in the sense that if the B analyst successfully hides her type then it must be that the market learns the true asset value.

Theorem 1. *In any equilibrium, either the market learns the true asset value or the market learns the analyst type.*

The logic of the theorem is the following. In order to successfully hide her type, B needs screening to break down, that is, B requires speculators to follow the recommendations irrespective of their type. In other words, to hide her type, B needs to make recommendations worth listening to. But in that case the recommendations of both types of the analyst are informative, and so as $t \to \infty$ the price must converge to the true asset value.

We provide in the next paragraph some details of the proof of Theorem 1 which help to shed light on the rest of the analysis. By virtue of the law of large numbers, conditional on $\theta = G$ the frequency of buy recommendations must converge either to q_G or to $1 - q_G$. Hence, either B makes buy recommendations with frequency q_G or $1 - q_G$, or B is discovered. We will say that "B mimics G" in the former case. Next suppose that B does indeed mimic G and makes buy recommendations with frequency q_G. Then since $q_G > \frac{1}{2}$ the net recommendation count, n_t, must tend to infinity as $t \to \infty$. The valuation conditional on $\theta = G$ therefore converges to 1. Imagine now that B's recommendations lost their informativeness in the limit as $t \to \infty$ and that the valuation conditional on $\theta = B$ therefore did not converge to 1. Then, choosing t sufficiently large, the valuation $v_t(r, b)$ of a type-b speculator sharply reverts toward the prior. In other words, as $t \to \infty$ type-b speculators trade against historical trends, selling the asset when the price is high and buying the asset when the price is low. But then speculators of types g and b trade in opposite directions, implying efficient screening in the limit as $t \to \infty$.[11] Hence, either B's recommendations remain informative forever or the market eventually learns the analyst type.

The next definition formalizes the idea of B's recommendations losing informativeness in the limit as $t \to \infty$.

[11] This generalizes our observation in the example of Section 3. There, valuations conditional on $\theta = B$ were equal to $\frac{1}{2}$.

Definition 1. Say that B's strategy is asymptotically uninformative if for any outcome[12]

$$\mathbb{P}(r_t = \text{buyrec}|V = 1, h_t, B) - \mathbb{P}(r_t = \text{buyrec}|V = 0, h_t, B) \underset{t\to\infty}{\to} 0.$$

Say that an equilibrium is class U if in the equilibrium considered B's strategy is asymptotically uninformative.

We can now state the following result.

Proposition 4. *A class-U equilibrium exists for all parameter values.*

The existence of class-U equilibria is easy to show. For all values of q_B, an equilibrium can always be constructed in which B's recommendations are independent of V and where, for any history h_t, B recommends buy/sell according to the probabilities of an equilibrium corresponding to $q_B = \frac{1}{2}$.[13] The following corollary is a consequence of Theorem 1.

Corollary 1. *In all class-U equilibria the market learns the analyst type.*

Note that $q_B = \frac{1}{2}$ implies that all equilibria are class-U equilibria. So Proposition 3 of the previous Section immediately follows from Corollary 1.

Combining Proposition 4 and Corollary 1 shows existence of equilibria such that the market learns the analyst type. We next inquire: can an equilibrium exist in which the B analyst succeeds at hiding her type forever, that is, in which the market does not learn the analyst type? The next theorem shows that as long as (i) the B analyst possesses some private information about the asset and (ii) no speculator is too well informed about the analyst type, then an equilibrium exists in which the market learns nothing about θ.

Theorem 2. *Suppose $q_B > \frac{1}{2}$ and $\gamma_G \in (0, 1)$. There exists $\eta > 0$ such that, if $\gamma_B > \gamma_G - \eta$, then an equilibrium exists in which $\mathbb{P}(\rho_t = \rho_1) = 1$ irrespective of t.*

The existence of an equilibrium in which B successfully hides her type is, in our opinion, a relatively surprising result. Here is why. We argued earlier in this section that if B's recommendations lost their informativeness in the limit as $t \to \infty$ then the valuations of type-b speculators would eventually sharply revert toward the prior, and induce type-b speculators to trade against historical trends. Suppose now for the sake of argument that instead of losing their informativeness, B's recommendations became maximally informative, i.e. suppose B stopped lying. In that case, since $q_G > q_B$, then B would reveal herself statistically over the course of time through her recommendations. What the theorem demonstrates is the possibility for B to strike a balance between garbling her recommendations sufficiently so as to avoid revealing herself statistically, while at the same time making the recommendations sufficiently informative so as to ensure that they are never trumped by the speculators' private signals of the analyst type.

Applying Theorem 1 to any equilibrium satisfying the condition stated in Theorem 2 shows that the market must learn the true asset value. The caveat however is that, to appear like she is type G, the B analyst artificially autocorrelates the recommendations she makes. By garbling her

12 That is, for any outcome in the underlying probability space.

13 Existence of an equilibrium is assured by Lemma 11 in Appendix E (available online).

private information this way, B also slows down the process of price discovery. We will return to this important remark in the simulations of the next section.

We close this section with a brief discussion of the assumption that the G analyst makes truthful recommendations. Note first that if G were strategic then Theorem 1 would not be true. For a counterexample, consider a pure babbling equilibrium in which neither type of the analyst ever makes recommendations that are correlated with V. A pure babbling equilibrium is easily constructed in which the market neither learns the true asset value nor the analyst type.[14] In contrast, Theorem 2 remains true when G is strategic. The logic is straightforward. If the market accumulates no information about the analyst type when beliefs are such that G makes truthful recommendations then making truthful recommendations each period is an optimal strategy of the G analyst.

5. Market liquidity and price discovery

In this section we examine the model's implications regarding market liquidity and price discovery, and propose two benchmarks for comparative purposes. We first analyze the evolution of reputation, and then show how reputation affects market learning about asset values.

We focus in the simulations of this section on the example analyzed in Section 3, given by $q_B = \frac{1}{2}$, $\gamma_B = 0$, $u(\rho_t) = \rho_t$ and $\delta = 0$. The other parameters are chosen as follows: $\rho_1 = 0.4$, $q_G = 0.7$, $\gamma_G = 0.7$, and $\pi = 0.3$. All simulation results in this section show average values over 30000 simulations.[15]

Two factors deter learning in our model: (i) the breakdown of screening by speculators and (ii) B's strategic attempt to appear like she is type G. To gain insights into their respective importance, we compare the model of this paper (henceforth referred to as the baseline model) to two benchmarks, described below.

The no-breakdown model In this model, the trade order y_t is unobserved. Public learning about the analyst type occurs through i.i.d. draws from probability distributions matching those of the trade order in the baseline model under efficient screening.[16] Thus, screening is constant in this model and breakdowns never occur. Notice that B's optimal strategy in this case is to mimic G in all periods, that is, to make recommendations satisfying $\mathbb{P}(r_t = \text{buyrec} \mid h_t, \theta = B) = \mathbb{P}(r_t = \text{buyrec} \mid h_t, \theta = G)$.

The non-strategic model In this model the B analyst behaves non-strategically, truthfully recommending $r_t = x_t$ in all periods. Since B's signal x_t is uninformative and x_t is i.i.d., then x_t is uniformly distributed. As a consequence, B sends out a buy/sell recommendation with equal probability each period.

[14] A pure babbling equilibrium can also be constructed in which although the market does not learn V, the market learns θ.

[15] The code for the simulations is available at the authors' websites. In Appendix D (available online) we describe in detail the simulation algorithm.

[16] I.e. the trade order probability distribution in the baseline model when, fixing r_t, the support of speculator t's strategy conditional on $s_t = b$ does not intersect the support of his strategy conditional on $s_t = g$.

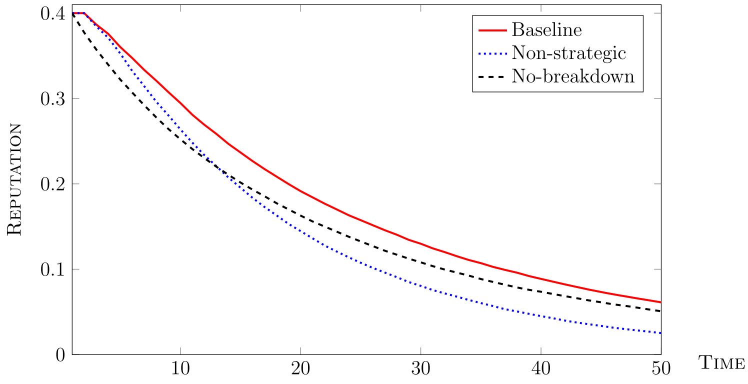

Fig. 3. Reputation of B analyst.

5.1. Reputation

We compare in Fig. 3 the evolution of the analyst's reputation in the baseline model to that obtained in the benchmarks. The simulations of the figure are for $\theta = B$. The solid curve corresponds to the baseline model, the dashed curve to the no-breakdown model and the dotted curve to the non-strategic model. The solid curve lies above the other curves, reflecting slower learning in the baseline model relative to the benchmarks. This is unsurprising since each benchmark was constructed so as to switch off one of the factors impeding learning in the baseline model.

Screening breaks down at $t = 1$ in both the baseline model and the non-strategic model, giving $\rho_2 = \rho_1$. By contrast, $\rho_2 < \rho_1$ in the no-breakdown model since the market in that case accumulates each period information about the analyst. Initially, learning is therefore fastest in the no-breakdown model. As time passes and reputation decreases, screening breakdowns gradually cease to occur in the baseline model (see Proposition 1).[17] The dashed and solid curves therefore converge towards one another. In sharp contrast the solid and dotted curves diverge. Intuitively, the greater the number of past recommendations the more conspicuous the absence of autocorrelation in recommendations made by the non-strategic B analyst. The dashed and dotted curves therefore cross at $t = 13$, after which point reputation is lowest in the non-strategic model.

5.2. Spread

The spread, defined as $p_t^a - p_t^b$, provides a widely used measure of market liquidity. We now examine how the reputation affects the spread over time. There are two closely related ways of interpreting the spread. One may view the spread as a measure of (minus) the expected utility of a liquidity trader, and one may view the spread as a measure of adverse selection facing market makers. In either case, it provides a measure of the market's information about the asset value at any given point in time.

We show in Fig. 4 the evolution of the spread.[18] The first remark, which captures a central feature, is that the spread increases at first and decreases eventually. This pattern can be traced back

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[17] In the baseline model screening breaks down on average 34% of the time in the first 5 periods, and on average 10% of the time over the first 20 periods.

[18] In the figure the average is taken as ρ_1 times the average spread conditional on $\theta = G$ plus $1 - \rho_1$ times the average spread conditional on $\theta = B$.

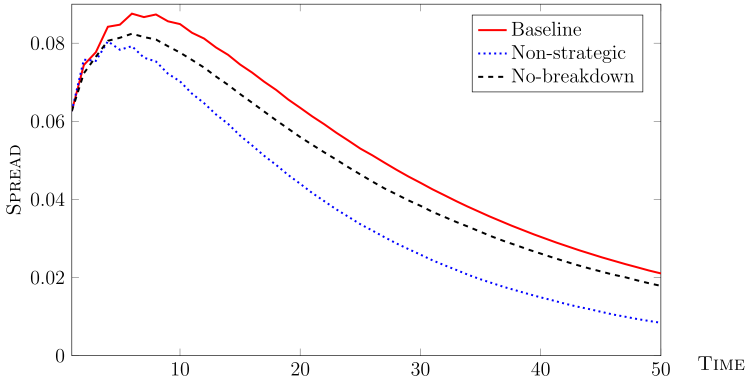

Fig. 4. Spread.

to the structure of asymmetric information between speculators and market makers in our model. In each period, a speculator possesses private information about the analyst type (in the form of the signal s_t) and private information about the asset (the latest recommendation). The greater the number of past recommendations the more important s_t, and therefore the spread grows initially, as the accumulation of recommendations makes the speculators' private signals of the analyst type more valuable. This is unlike traditional sequential trade models, where more information always tends to decrease spreads. However, by virtue of Corollary 1, the market eventually learns the analyst type. Asymmetric information between speculators and market makers therefore disappears as t tends to infinity.

Our second observation is that the difference between spreads in the baseline and non-strategic models remains significant even past $t = 50$. This remark reflects two things. First, as we emphasized in the previous subsection learning about the analyst type is much quicker in the non-strategic model than in the baseline model. Second, the non-strategic model implies $\mathbb{P}(r_t = \text{buyrec} \mid h_t, \theta = B) = \frac{1}{2}$. The average net recommendation count conditional on $\theta = B$ therefore remains close to 0 irrespective of t, which in turns lowers the importance of speculators' private information about the analyst type. Combining the two effects creates a substantial difference between adverse selection in the baseline and non-strategic models.

5.3. *Squared price error*

We now discuss the pattern of the squared price error, defined as $(v_t - V)^2$ and traditionally used as a measure of (inverse) price discovery.[19]

We show in Fig. 5 the evolution of the squared price error. First, notice that the squared price error does not increase initially in the same manner as the spread. Recall that the spread increases initially as the accumulation of recommendations implies that the speculator's private signal s_t becomes more important, and adverse selection therefore drives up the spread. To see why the squared price error is not subject to the same effect, we first remark that the squared price error combines information contained in quotes with information contained in trade orders. Consider

[19] Since in the simulations $q_B = \frac{1}{2}$, that is to say B is uninformed, the squared price error is taken to be $(v_t - 1/2)^2$ whenever $\theta = B$. In Fig. 5 the average is taken as ρ_1 times the average squared price error conditional on $\theta = G$ plus $1 - \rho_1$ times squared price error conditional on $\theta = B$.

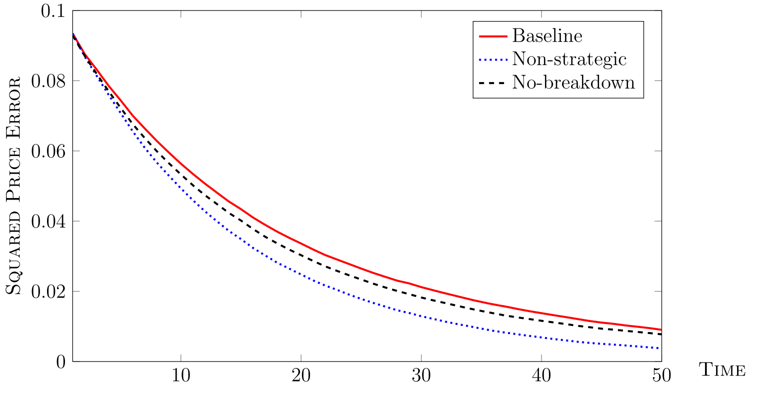

Fig. 5. Squared price error.

now what happens when the accumulation of recommendations causes the spread to increase through the aforementioned effect. In isolation, the higher spread implies that quotes are farther away from the expected asset value, affecting price discovery negatively (increase the squared price error). However, trade orders also become more informative, as the speculator now has better information. The latter effect affects price discovery positively (decrease the squared price error). The combined effect is that the squared price error decreases over time, and therefore we do not observe an initial 'hump' in squared price error in the same manner that we observed an initial hump in the spread.

Second, observe the substantial difference of squared price error in the baseline model and in the non-strategic model. Comparatively, the no-breakdown model is far closer to the baseline model. This is line with the observations from Subsection 5.1, where the same effect was identified for the spread. These observations suggest that in the baseline model the greatest impediment to learning the true asset value is B's strategic behavior.

Lastly, we relate our results to the literature on the speed of learning in observational learning models. Vives (1993, 1995) shows that price adjustment in financial markets may be slow, but that this process is made faster by market makers who, by setting competitive prices, provide incentives for traders to use and therefore reveal their private information. However, Vives (2010) remarks that herding may lead market makers to update prices only very slowly, due to uncertainty about the informativeness of the trade orders. Our model differs from the models considered in the literature on the speed of observational learning in that the quality of the traders' information is uncertain and endogenous, via the strategic incentives of the B analyst. Screening breakdowns may slow down learning in the same manner that herding may slow down learning in the aforementioned models.

6. Changing fundamentals

In this section we extend the model by allowing the underlying asset value V_t to change over time. We saw through Proposition 3 that in the baseline model, for $q_B = \frac{1}{2}$, the market learns the analyst type in any equilibrium. We now show that with changing asset values even a completely uninformed B analyst can hide her type indefinitely. The proofs of this section are in Appendix C.

In what follows we place ourselves in the setting of the example of Section 3, that is, $q_B = \frac{1}{2}$, $\gamma_B = 0$ and $\gamma_G \in (0, 1)$, $u(\rho_t) = \rho_t$ and $\delta = 0$. However, we assume that $\{V_t\}_{t=1}^{\infty}$ follows a commonly known Markov process with transition matrix

	0	1
0	$1 - \frac{z}{2}$	$\frac{z}{2}$
1	$\frac{z}{2}$	$1 - \frac{z}{2}$

where $z \in [0, 1]$. The idea is as follows. Each period, a new draw is made with probability z; each new draw is such that V_t equals 0 or 1 with probability $\frac{1}{2}$ each. With probability $1 - z$ this period's value equals last period's value. The total probability that the value in a period equals the value in the previous period is thus equal to $1 - \frac{z}{2}$. The baseline model corresponds to $z = 0$. We can now state this section's main result.

Proposition 5. *For all $z > 0$, there exists $\underline{\rho} < 1$ such that, in equilibrium, if $\rho_t > \underline{\rho}$ then $\mathbb{P}(\rho_s = \rho_t) = 1$ for all $s > t$.*

We saw in Section 3 that with a fixed asset value informational cascades of the kind uncovered in Proposition 1 had to be transitory. The argument relied on the observation that increasing the net recommendation count simultaneously reduced the impact of the current recommendation on a speculator's valuation and raised the importance of beliefs concerning the analyst's true type. When the net recommendation count became sufficiently large speculators thus started ignoring the current recommendation and made trading decisions solely based on their private signals of the analyst type. The logic of Proposition 5 is straightforward: a changing asset value ensures that the marginal information incorporated in the recommendations of the G analyst never vanishes, even as n_t tends to infinity. Once reputation is sufficiently high mimicking G then allows B to hide her type infinitely long, in a way similar to the mechanism highlighted in connection to Proposition 1.

Interestingly, since our model exhibits twofold uncertainty (with respect to the asset value and with respect to analyst ability) here changing fundamentals facilitate the occurrence of informational cascades with respect to analyst ability. These insights naturally complement those of Moscarini et al. (1998) concerning the fragility of informational cascades in a changing environment:[20] In our setting, if the type of the analyst evolved instead of the asset value then speculators' screening of the analyst would not cease, since the screening threshold $\underline{\rho}(n_t)$ from Proposition 1 would then never be reached.[21]

7. Concluding remarks

There is ample evidence that analyst recommendations influence stock prices, and high-reputation analysts have greater impact than low-reputation analysts (Stickel, 1995; Womack, 1996; Leone and Wu, 2007; Loh and Stulz, 2011; Fang and Yasuda, 2014). The question remains as to whether fund managers and investors actually learn analyst ability over time. If price movements are effectively dictated by star analysts then evaluating their true ability becomes a complicated task. Fang and Yasuda (2014) for instance document that being elected

[20] In Moscarini et al. (1998) the state of the world changes stochastically over time. The resulting information depreciation implies that, in their setting, only temporary informational cascades can arise.

[21] We thank an associate editor for this insightful remark.

All-American analyst predicts future performance of recommendations, even controlling for the likelihood of being elected. In our model, fundamental asset values are never observed. Traders' private information about the analyst is backed out from the order flow. An endogeneity problem thus arises. An equilibrium in which the market learns the analyst type always exists. However, under certain conditions, an equilibrium exists in which the incompetent analyst succeeds at hiding her type indefinitely: each trader ignores his own private information and 'herds' on the analyst's recommendations. Moreover, the incompetent analyst may be able to momentarily hide her type even in the equilibria where the market eventually learns the analyst type. These results are broadly consistent with empirical evidence. For example, Brown et al. (2013) find that mutual funds often herd in the direction of analyst recommendation changes, and that the stocks later tend to underperform.[22] In a similar vein, several papers document 'self-fulfilling prophecies' in connection to analyst recommendations publicized in the mass media (Sant and Zaman, 1996; Keasler and McNeil, 2010).[23]

Appendix A. Proofs of Section 3

The following notation and terminology will help the exposition of the proofs. Define

$$v_t^\theta(r) := \mathbb{E}[V|h_t, r_t = r, \theta].$$

Let $\rho_t(r, s)$ denote the probability assigned by speculator t to $\theta = G$. Assumptions (A1)–(A2) imply

$$v_t^B(\text{buyrec}) = v_t^B(\text{sellrec}) = v_t^B = \frac{1}{2}, \tag{5}$$

$$\rho_t(\text{buyrec}, g) = \rho_t(\text{sellrec}, g) = 1, \tag{6}$$

$$\rho_t(\text{buyrec}, b) = \frac{\rho_t(1-\gamma_G)R_t^G}{\rho_t(1-\gamma_G)R_t^G + (1-\rho_t)R_t^B}, \tag{7}$$

$$\rho_t(\text{sellrec}, b) = \frac{\rho_t(1-\gamma_G)(1-R_t^G)}{\rho_t(1-\gamma_G)(1-R_t^G) + (1-\rho_t)(1-R_t^B)}. \tag{8}$$

Lemma 2. *In equilibrium, if $R_t^B > R_t^G$ then either $v_t(\text{buyrec}, b) = p_t^b$, or $v_t(\text{buyrec}, g) > p_t^a$ implies $v_t(\text{buyrec}, b) \geq p_t^a$.*

Proof. Consider an arbitrary equilibrium and fix a history h_t. To shorten notation, henceforth all probabilities and expectations will be understood conditional on h_t. For all $(y, r) \in \{\text{buy, sell, abstain}\} \times \{\text{buyrec, sellrec}\}$, let $L(y|r)$ denote the likelihood ratio of trade order y given recommendation r, that is,

$$L(y|r) := \frac{\mathbb{P}(y_t = y|r_t = r, \theta = G)}{\mathbb{P}(y_t = y|r_t = r, \theta = B)}.$$

[22] As stated in the paper, "...we find clear evidence that mutual funds herd on analyst recommendation changes, and that this trading impacts stock prices in a manner consistent with overreaction by funds to the consensus signal provided by analysts".

[23] Sant and Zaman focus on stocks mentioned in Business Week's 'Inside Wall Street' column. This column mainly includes quotes from analysts, but also from investment company managers and other financial market agents. Keasler and McNeil examine the recommendations of Jim Cramer in the CNBC show 'Mad Money'.

Notice that for all r,[24]

$$\mathbb{E}_{y_t}[L(y_t|r)|r, B] = 1. \tag{9}$$

Define $f(x) := \frac{\rho_t x}{\rho_t x + (1-\rho_t)}$, noting that $f(\cdot)$ is increasing and concave. Moreover, applying Bayes' rule,

$$\mathbb{E}[\rho_{t+1}|r_t = \text{buyrec}, \theta = B] = \mathbb{E}_{y_t}\Big[f\Big(\frac{R_t^G}{R_t^B} L(y_t|\text{buyrec})\Big)\Big|r_t = \text{buyrec}, \theta = B\Big] \tag{10}$$

and

$$\mathbb{E}[\rho_{t+1}|r_t = \text{sellrec}, \theta = B] = \mathbb{E}_{y_t}\Big[f\Big(\frac{1-R_t^G}{1-R_t^B} L(y_t|\text{sellrec})\Big)\Big|r_t = \text{sellrec}, \theta = B\Big]. \tag{11}$$

Assume henceforth that in the equilibrium considered $v_t(\text{buyrec}, b) \neq p_t^b$, $R_t^B > R_t^G$ and $v_t(\text{buyrec}, g) > p_t^a$. We aim to show that $v_t(\text{buyrec}, b) \geq p_t^a$. Suppose instead that $v_t(\text{buyrec}, b) < p_t^a$. Then $v_t(\text{buyrec}, g) > p_t^a > v_t(\text{buyrec}, b)$, that is, conditional on a buy recommendation, type-g (respectively type-b) speculators buy the asset with probability 1 (resp. 0). As $v_t(\text{buyrec}, b) \neq p_t^b$, conditioning on $r_t = \text{buyrec}$ and $\theta = B$ the random variable $L(y_t|\text{buyrec})$ has support

$$\left\{\frac{\frac{1-\pi}{3} + \pi(1-\gamma_G)}{\frac{1-\pi}{3} + \pi}, 1, \frac{\frac{1-\pi}{3} + \pi\gamma_G}{\frac{1-\pi}{3}}\right\}$$

and satisfies $\mathbb{P}(L(y_t|\text{buyrec}) = 1|r_t = \text{buyrec}, B) = \frac{1-\pi}{3}$. Now, conditioning instead on $r_t = \text{sellrec}$ and $\theta = B$, either $L(y_t|\text{sellrec})$ is distributed as above or $L(y_t|\text{sellrec})$ is equal to 1 with probability 1.[25] Combined with (9) the previous remarks imply that the distribution of $L(y_t|\text{buyrec})$ conditional on $r_t = \text{buyrec}$ and $\theta = B$ is a mean-preserving spread of the distribution of $L(y_t|\text{sellrec})$ conditional on $r_t = \text{sellrec}$ and $\theta = B$; call this remark R1.

Next,

$$\begin{aligned}\mathbb{E}[\rho_{t+1}|r_t = \text{buyrec}, \theta = B] &= \mathbb{E}\Big[f\Big(\frac{R_t^G}{R_t^B} L(y_t|\text{buyrec})\Big)\Big|r_t = \text{buyrec}, \theta = B\Big] \\ &< \mathbb{E}\Big[f\Big(\frac{1-R_t^G}{1-R_t^B} L(y_t|\text{buyrec})\Big)\Big|r_t = \text{buyrec}, \theta = B\Big] \\ &\leq \mathbb{E}\Big[f\Big(\frac{1-R_t^G}{1-R_t^B} L(y_t|\text{sellrec})\Big)\Big|r_t = \text{sellrec}, \theta = B\Big] \\ &= \mathbb{E}[\rho_{t+1}|r_t = \text{sellrec}, \theta = B].\end{aligned}$$

The first equality is obtained from (10); the first inequality is due to $f(\cdot)$ being increasing and $R_t^B > R_t^G$; the second inequality follows from remark R1 and concavity of $f(\cdot)$; the last equality is obtained from (11).

[24] We write $\mathbb{E}_{y_t}$ to indicate that the expectation is with respect to y_t.

[25] We ignore for simplicity the knife-edge cases in which, conditional on a sell recommendation, one type of speculator is indifferent between two trade orders. These cases lead to the same conclusion since the pattern of trade induced by a buy recommendation reveals the maximum possible amount of information about θ.

The highlighted sequence of inequalities gives $\mathbb{E}[\rho_{t+1}|r_t = \text{sellrec}, \theta = B] > \mathbb{E}[\rho_{t+1}|r_t = \text{buyrec}, \theta = B]$, that is, the expected payoff of the B analyst is strictly greater for a sell recommendation than it is for a buy recommendation. Equilibrium consistency therefore requires $R_t^B = 0$, contradicting the initial assumption that $R_t^B > R_t^G$. Hence, either $v_t(\text{buyrec}, b) = p_t^b$, or $R_t^B > R_t^G$ and $v_t(\text{buyrec}, g) > p_t^a$ imply $v_t(\text{buyrec}, b) \geq p_t^a$. □

Proof of Lemma 1. First notice that since (i) $q_G < 1$ and (ii) liquidity traders uniformly randomize over actions, then in any equilibrium and given any history, $\rho_t \in (0, 1)$. Consider an arbitrary equilibrium and fix a history h_t with $n_t \geq 2$. Because the net recommendation count is at least two,

$$v_t^G(\text{buyrec}) > v_t^G(\text{sellrec}) > \frac{1}{2}. \tag{12}$$

Furthermore, as $v_t^B(r) = \frac{1}{2}$ irrespective of r,

$$v_t(r, s) = \rho_t(r, s)v_t^G(r) + \big(1 - \rho_t(r, s)\big)v_t^B(r) = \rho_t(r, s)v_t^G(r) + \big(1 - \rho_t(r, s)\big)\frac{1}{2} \tag{13}$$

for all $(r, s) \in \{\text{buyrec}, \text{sellrec}\} \times \{g, b\}$. As $\rho_t(r, g) = 1$ irrespective of r, (12) and (13) give $v_t(r, g) = v_t^G(r)$ and $v_t(\text{buyrec}, g) = \max_{(r,s)}\{v_t(r, s)\}$, establishing part (i) of the lemma.

We next show $p_t^a > v_t > p_t^b$. We have $v_t(\text{buyrec}, g) > v_t$. Thus $p_t^a > v_t$. Suppose for a contradiction that $p_t^b = v_t$. By virtue of the law of total expectation,

$$v_t = \mathbb{P}(y_t = \text{sell}|h_t)p_t^b + \mathbb{P}(y_t = \text{abstain}|h_t)\mathbb{E}[V|h_t, y_t = \text{abstain}] + \mathbb{P}(y_t = \text{buy}|h_t)p_t^a.$$

Therefore, as $p_t^a > v_t$, we must have $\mathbb{E}[V|h_t, y_t = \text{abstain}] < v_t$. Observe that if only liquidity traders abstained we would have $\mathbb{E}[V|h_t, y_t = \text{abstain}] = v_t$. So speculator t must abstain in some contingency (r, s) such that $v_t(r, s) < p_t^b$. This is evidently impossible in equilibrium. Hence $p_t^b < v_t$.

We can now show part (ii) of the lemma. Using (12) and (13) gives

$$\begin{aligned} v_t(\text{sellrec}, b) &= \rho_t(\text{sellrec}, b)v_t^G(\text{sellrec}) + \big(1 - \rho_t(\text{sellrec}, b)\big)\frac{1}{2} \leq v_t^G(\text{sellrec}) \\ &= v_t(\text{sellrec}, g). \end{aligned}$$

Hence, by part (i) of the lemma,

$$v_t(\text{sellrec}, b) = \min_{(r,s)}\{v_t(r, s)\} \Leftrightarrow v_t(\text{sellrec}, b) \leq v_t(\text{buyrec}, b). \tag{14}$$

Suppose $v_t(\text{sellrec}, b) > v_t(\text{buyrec}, b)$ (and therefore $v_t(\text{buyrec}, b) = \min_{(r,s)}\{v_t(r, s)\}$). The combination of (12) and (13) then gives $\rho_t(\text{sellrec}, b) > \rho_t(\text{buyrec}, b)$ which, using (7)–(8), implies $R_t^B > R_t^G$. On the other hand, since we established that $p_t^b < v_t$, $v_t(\text{buyrec}, b) = \min_{(r,s)}\{v_t(r, s)\}$ implies $v_t(\text{buyrec}, b) < p_t^b$. Gathering the previous remarks gives (a) $R_t^B > R_t^G$ on the one hand and (b) $v_t(\text{buyrec}, b) < p_t^b < p_t^a < v_t(\text{buyrec}, g)$ on the other. We saw in Lemma 2 that (a) and (b) cannot hold at once. Hence $v_t(\text{sellrec}, b) \leq v_t(\text{buyrec}, b)$ and, by (14), $v_t(\text{sellrec}, b) = \min_{(r,s)}\{v_t(r, s)\}$, establishing part (ii) of the lemma. □

Lemma 3. *Let $\rho_t \in (0, 1)$. In equilibrium, $R_t^B < R_t^G$ implies*

$$\min\{v_t(\textit{buyrec}, b), v_t(\textit{buyrec}, g)\} \leq p_t^a.$$

Proof. Consider an arbitrary equilibrium and fix a history h_t with $\rho_t \in (0,1)$. To shorten notation, all expectations will be understood conditional on h_t.

Suppose $R_t^B < R_t^G$ and $\min\{v_t(\text{buyrec}, b), v_t(\text{buyrec}, g)\} > p_t^a$. The latter inequality implies that there is no screening following a buy recommendation. Hence following a buy recommendation the only public information about θ comes from the recommendation itself. As $R_t^B < R_t^G$, we obtain $\mathbb{E}[\rho_{t+1}|r_t = \text{buyrec}, \theta = B] > \rho_t$. However, conditional on $\theta = B$, ρ_t is a super-martingale. Hence $\mathbb{E}[\rho_{t+1}|r_t = \text{sellrec}, \theta = B] < \rho_t$. Equilibrium consistency thus requires $R_t^B = 1$, contradicting the assumption that $R_t^B < R_t^G$. □

Lemma 4. *Let $\rho_t \in (0,1)$. In equilibrium, $R_t^B > R_t^G$ implies*

$$\max\{v_t(\textit{sellrec}, b), v_t(\textit{sellrec}, g)\} \geq p_t^b.$$

Proof. Lemma 4 is the mirror image of Lemma 3. Their proofs are almost identical. We do not repeat the details. □

Lemma 5. *Fix $n_t \geq 2$. Let $\hat{v}_t(r,s)$ denote speculator t's valuation of the asset given the belief that $R_t^B = R_t^G$. Then*

$$\left\{\rho_t > 0 : \hat{v}_t(\textit{buyrec}, b) \geq \hat{p}^a(\rho_t, n_t) \textit{ and } \hat{v}_t(\textit{sellrec}, g) \leq \hat{p}^b(\rho_t, n_t)\right\} = \left\{\rho_t : \rho_t \geq \underline{\rho}(n_t)\right\},$$

with $\hat{p}^a(\rho_t, n_t)$, $\hat{p}^b(\rho_t, n_t)$ and $\underline{\rho}(n_t)$ defined by (1), (2) and (3), respectively. Moreover, $\underline{\rho}(n_t) \in (0,1)$.

Proof. The lemma follows from noting that (i) $\hat{v}_t(\text{buyrec}, b) - \hat{p}^a(\rho_t, n_t)$ is concave in ρ_t, strictly greater than 0 at $\rho_t = 1$ and equal to 0 at $\rho_t = 0$, and (ii) $\hat{p}^b(\rho_t, n_t) - \hat{v}_t(\text{sellrec}, g)$ is linear in ρ_t and strictly greater than 0 at $\rho_t = 1$. □

Proof of Proposition 1. We prove first part (ii) of the proposition. Consider an arbitrary equilibrium, and a history h_t such that $n_t \geq 2$ and $\mathbb{P}(\rho_{t+1} = \rho_t | h_t) = 1$. Then $R_t^B = R_t^G$, otherwise the market would learn about θ from observing r_t at the beginning of period $t+1$. Similarly, speculator t's trade order cannot reveal anything about s_t. We know by part (iii) of Lemma 1 that speculator t buys with probability 1 conditional on $(r_t, s_t) = (\text{buyrec}, g)$ and sells with probability 1 conditional on $(r_t, s_t) = (\text{sellrec}, b)$. So speculator t must buy with probability 1 conditional on $r_t = \text{buyrec}$ and sell with probability 1 conditional on $r_t = \text{sellrec}$. The previous remarks in turn imply $p_t^a = \hat{p}^a(\rho_t, n_t)$ and $p_t^b = \hat{p}^b(\rho_t, n_t)$.[26] Now, buying the asset is optimal for speculator t conditional on $(r_t, s_t) = (\text{buyrec}, b)$, giving $\hat{v}_t(\text{buyrec}, b) \geq \hat{p}^a(\rho_t, n_t)$. Similarly, selling the asset is optimal conditional on $(r_t, s_t) = (\text{sellrec}, g)$, giving $\hat{v}_t(\text{sellrec}, g) \leq \hat{p}^b(\rho_t, n_t)$. Hence $\rho_t \in \left\{\rho_t > 0 : \hat{v}_t(\text{buyrec}, b) \geq \hat{p}^a(\rho_t, n_t) \text{ and } \hat{v}_t(\text{sellrec}, g) \leq \hat{p}^b(\rho_t, n_t)\right\}$. Lemma 5 concludes the proof of part (ii) of the proposition.

We now prove part (i) of the proposition. Consider an arbitrary equilibrium, and a history h_t such that $n_t \geq 2$ and $\rho_t > \underline{\rho}(n_t)$. First, we claim that $R_t^B = R_t^G$. Suppose for a contradiction that $R_t^B < R_t^G$. Then,

[26] Recall, the function $\hat{p}^a(\rho_t, n_t)$ captures the ask price of market makers computed under the assumptions that (i) $R_t^B = R_t^G$ and (ii) speculator t chooses to buy the asset if and only if $r_t = \text{buyrec}$. Similarly, the function $\hat{p}^b(\rho_t, n_t)$ captures the bid price under the assumptions that (i) $R_t^B = R_t^G$ and (ii) speculator t chooses to sell the asset if and only if $r_t = \text{sellrec}$.

$$\begin{cases} v_t(\text{buyrec}, g) = \hat{v}_t(\text{buyrec}, g) \\ v_t(\text{buyrec}, b) > \hat{v}_t(\text{buyrec}, b) \\ v_t(\text{sellrec}, g) = \hat{v}_t(\text{sellrec}, g) \\ v_t(\text{sellrec}, b) < \hat{v}_t(\text{sellrec}, b). \end{cases}$$

Now, observe first that $\rho_t > \underline{\rho}(n_t)$ combined with Lemma 1 yields $v_t(\text{sellrec}, b) < v_t(\text{sellrec}, g) = \hat{v}_t(\text{sellrec}, g) < \hat{p}^b(\rho_t, n_t) < v_t < p_t^a$. Hence in the equilibrium considered speculator t never buys the asset conditional on $r_t = \text{sellrec}$. Second, $\rho_t > \underline{\rho}(n_t)$ implies that $\hat{v}_t(\text{buyrec}, b)$ is strictly greater than the weighted average of $\hat{v}_t(\text{buyrec}, g)$ and v_t.[27] Since $v_t(\text{buyrec}, b) > \hat{v}_t(\text{buyrec}, b)$, we therefore obtain $v_t(\text{buyrec}, b) > p_t^a$. Thus, by Lemma 1,

$$\min\{v_t(\text{buyrec}, b), v_t(\text{buyrec}, g)\} > p_t^a. \tag{15}$$

But (15) contradicts Lemma 3.

Similarly, suppose for a contradiction that $R_t^B > R_t^G$. Then,

$$\begin{cases} v_t(\text{buyrec}, g) = \hat{v}_t(\text{buyrec}, g) \\ v_t(\text{buyrec}, b) < \hat{v}_t(\text{buyrec}, b) \\ v_t(\text{sellrec}, g) = \hat{v}_t(\text{sellrec}, g) \\ v_t(\text{sellrec}, b) > \hat{v}_t(\text{sellrec}, b). \end{cases}$$

Moreover, Lemma 2 gives $v_t(\text{buyrec}, b) \geq p_t^b$, so neither $v_t(\text{buyrec}, b)$ nor $v_t(\text{buyrec}, g)$ affect the bid price. Now, $\rho_t > \underline{\rho}(n_t)$ implies that $\hat{v}_t(\text{sellrec}, g)$ is strictly less than the weighted average of $\hat{v}_t(\text{sellrec}, b)$ and v_t.[28] Since $v_t(\text{sellrec}, b) > \hat{v}_t(\text{sellrec}, b)$, we therefore obtain $v_t(\text{sellrec}, g) < p_t^b$. Thus, by Lemma 1,

$$\max\{v_t(\text{sellrec}, b), v_t(\text{sellrec}, g)\} < p_t^b. \tag{16}$$

But (16) contradicts Lemma 4. The previous steps establish the claim that $R_t^B = R_t^G$.

Finally, repeating the remarks made above, $\rho_t > \underline{\rho}(n_t)$ implies (i) $\hat{v}_t(\text{buyrec}, b)$ is strictly greater than the weighted average of $\hat{v}_t(\text{buyrec}, g)$ and v_t, and (ii) $\hat{v}_t(\text{sellrec}, g)$ is strictly less than the weighted average of $\hat{v}_t(\text{sellrec}, b)$ and v_t. Therefore $\hat{v}_t(\text{buyrec}, b) > p_t^a$ and $\hat{v}_t(\text{sellrec}, g) < p_t^b$, giving $\mathbb{P}(\rho_{t+1} = \rho_t | h_t) = 1$ and concluding the proof of part (i) of the proposition. □

Proof of Proposition 2. Fix $\rho_t < 1$. Then, choosing n_t sufficiently large:

$$v_t(\text{sellrec}, g) = v_t^G(\text{sellrec}) > \rho_t v_t^G + (1 - \rho_t)\frac{1}{2} = v_t \geq p_t^b > v_t(\text{sellrec}, b). \tag{17}$$

For the first inequality notice that $\rho_t < 1$ and $\lim_{n_t \to +\infty} v_t^G = \lim_{n_t \to +\infty} v_t^G(\text{sellrec}) = 1$; the second inequality follows from standard arguments, and the third inequality is by part (iii) of Lemma 1.

On the other hand, by (3), $\rho_t \geq \underline{\rho}(n_t)$ implies

$$v_t(\text{sellrec}, g) = \hat{v}_t(\text{sellrec}, g) \leq \hat{p}^b(\rho_t, n_t) \leq v_t.$$

27 With weights $\pi \rho_t \gamma_G R_t^G$ and $\frac{1-\pi}{3}$, respectively.

28 With weights $\pi\big(\rho_t(1-\gamma_G) + (1-\rho_t)\big)(1 - R_t^G)$ and $\frac{1-\pi}{3}$, respectively.

Hence (17) implies $\rho_t < \underline{\rho}(n_t)$, for all n_t sufficiently large. Since $\rho_t < 1$ was chosen arbitrarily, we obtain $\lim_{n_t \to +\infty} \underline{\rho}(n_t) = 1$. □

Proof of Proposition 3. The proposition follows from Corollary 1 once we note that for $q_B = \frac{1}{2}$ all equilibria are class-U equilibria. □

Appendix B. Proofs of Section 4

We make use throughout this appendix of the additional notation and terminology defined at the start of Appendix A. We will say with a slight abuse of terminology that a strategy of the B analyst is a non-degenerate mixed strategy if $\mathbb{P}(r_t = r|h_t, \theta = B) \in (0, 1)$ for all public histories h_t.[29]

Lemma 6. *Any equilibrium strategy of the B analyst is a non-degenerate mixed strategy.*

Proof. Consider a strategy of the B analyst such that for some h_t and r, $\mathbb{P}(r_t = r|h_t, \theta = B) = 0$. Then applying Bayes' rule, given history h_t, the recommendation $r_t = r$ induces $\rho_{t+1} = 1$ whereas $r_t \neq r$ induces $\rho_{t+1} < 1$ (recall, $q_G < 1$, therefore each period the G analyst makes both recommendations with positive probabilities). Hence, the strategy considered cannot be an equilibrium strategy. □

Lemma 7. *For any non-degenerate mixed strategy of the B analyst,* $\mathbb{P}(\lim_{t\to\infty} \rho_t = 1|\theta = B) = 0$ *and* $\mathbb{P}(\lim_{t\to\infty} \rho_t = 0|\theta = G) = 0$. *Similarly,* $\mathbb{P}(\lim_{t\to\infty} v_t = 1|V = 0) = 0$ *and* $\mathbb{P}(\lim_{t\to\infty} v_t = 0|V = 1) = 0$.

Proof. We prove here the first part of the lemma; the proof of the second part is analogous. Consider an arbitrary non-degenerate mixed strategy of the B analyst. Reputation, being a bounded martingale, converges with probability 1. Let $\rho := \lim_{t\to\infty} \rho_t$. Then

$$\mathbb{E}\Big[\frac{\rho_{t+1}}{1-\rho_{t+1}}\Big|B\Big] = \mathbb{E}\Big[\sum_{r_{t+1},y_{t+1}} \mathbb{P}(r_{t+1}, y_{t+1}|h_t, B) \cdot \frac{\rho_t \mathbb{P}(r_{t+1}, y_{t+1}|h_t, G)}{(1-\rho_t)\mathbb{P}(r_{t+1}, y_{t+1}|h_t, B)}\Big|B\Big]$$
$$= \mathbb{E}\Big[\frac{\rho_t}{1-\rho_t}\sum_{r_{t+1},y_{t+1}} \mathbb{P}(r_{t+1}, y_{t+1}|h_t, G)\Big|B\Big]$$
$$= \mathbb{E}\Big[\frac{\rho_t}{1-\rho_t}\Big|B\Big].$$

Hence, $\mathbb{E}\Big[\frac{\rho_t}{1-\rho_t}\Big|B\Big] = \frac{\rho_1}{1-\rho_1}$. Fatou's Lemma then gives $\mathbb{E}\Big[\frac{\rho}{1-\rho}\Big|B\Big] \leq \frac{\rho_1}{1-\rho_1}$. This shows $\mathbb{P}(\rho = 1|B) = 0$. Similar derivations show $\mathbb{P}(\rho = 0|G) = 0$. □

Proof of Theorem 1. Consider an arbitrary equilibrium. We will show that, with probability 1, either

[29] We are slightly abusing terminology since a strategy of the B analyst could also depend on what she observed privately up to period t, namely $x_1, \dots, x_t$.

$$\lim_{t\to\infty} v_t = V, \tag{18}$$

or

$$\rho := \lim_{t\to\infty} \rho_t \in \{0, 1\}. \tag{19}$$

In case (19) holds, applying Lemma 7 gives $\rho = 0$ conditional on $\theta = B$ and $\rho = 1$ conditional on $\theta = G$. So showing that at least one of (18) and (19) has to hold is sufficient to prove the theorem.

For any history h_t, let m_t denote the number of buy recommendations up to period t. By virtue of the law of large numbers:

$$\lim_{t\to\infty} \frac{m_t}{t} \in \{q_G, 1 - q_G\} \tag{20}$$

with probability 1 conditional on $\theta = G$. Hence, either property (20) holds as well with probability 1 conditional on $\theta = B$ or whenever (20) does not hold then applying Bayes' rule gives $\lim_{t\to\infty} \rho_t = 0$ (in which case (19) evidently holds). We therefore restrict attention in the rest of the proof to the set of outcomes such that (20) holds.

Suppose for concreteness that $\lim_{t\to\infty} \frac{m_t}{t} = q_G$ (the other case is symmetric), so that $\lim_{t\to\infty} v_t^G = 1$. If $\lim_{t\to\infty} v_t^B = 1$ then $\lim_{t\to\infty} v_t = 1$. On the other hand, Lemma 7 gives $\mathbb{P}(\lim_{t\to\infty} v_t = 1 | V = 0) = 0$. So (18) holds whenever $\lim_{t\to\infty} v_t^B = 1$. Consider next the case $\lim_{t\to\infty} v_t^B = \ell < 1$. Pick $\varepsilon \in (0, \frac{1-\ell}{2})$. Then choosing t large enough ensures

$$\ell - \varepsilon < v_t^B(r) < \ell + \varepsilon < 1 - \varepsilon < v_t^G(r), \quad \forall r \in \{\text{buyrec}, \text{sellrec}\}. \tag{21}$$

Suppose by way of contradiction that (19) fails to hold, i.e. $\rho \in (0, 1)$. Then, for any $\eta > 0$,

$$\left|\mathbb{P}(r_t = \text{buyrec}|h_t, B) - \mathbb{P}(r_t = \text{buyrec}|h_t, G)\right| \le \eta$$

for all sufficiently large t. So by choosing η sufficiently small we obtain $k > 0$ such that, for all sufficiently large t,[30]

$$\begin{cases} \rho_t(r, g) > \rho + k, & \forall r \in \{\text{buyrec}, \text{sellrec}\}; \\ \rho_t(r, b) < \rho - k, & \forall r \in \{\text{buyrec}, \text{sellrec}\}. \end{cases} \tag{22}$$

Now, a speculator's valuation can be written as

$$v_t(r, s) = v_t^B(r) + \rho_t(r, s)\big(v_t^G(r) - v_t^B(r)\big). \tag{23}$$

Thus, combining (21), (22) and (23) gives

$$v_t(r, b) < \ell + \varepsilon + (\rho - k)(1 - \ell + \varepsilon), \quad \forall r \in \{\text{buyrec}, \text{sellrec}\}, \tag{24}$$

and

$$v_t(r, g) > \ell - \varepsilon + (\rho + k)(1 - \ell - 2\varepsilon), \quad \forall r \in \{\text{buyrec}, \text{sellrec}\}, \tag{25}$$

for all sufficiently large t. Choosing ε small enough, (24) and (25) in turn imply

$$\min_r\{v_t(r, g)\} - \max_r\{v_t(r, b)\} > k(1 - \ell) \tag{26}$$

30 Recall, $\gamma_G > \gamma_G$.

for all sufficiently large t. Furthermore, by choosing ε and η sufficiently small, both $v_t(\text{buyrec}, g) - v_t(\text{sellrec}, g)$ and $v_t(\text{buyrec}, b) - v_t(\text{sellrec}, b)$ can be made arbitrarily close to 0. The latter remark combined to (26) establishes $v_t(r, b) < p_t^b$ for all $r \in \{\text{buyrec}, \text{sellrec}\}$ and $v_t(r, g) > p_t^a$ for all $r \in \{\text{buyrec}, \text{sellrec}\}$, that is, type-$g$ speculators buy irrespective of r_t while type-b speculators sell irrespective of r_t. It follows from these observations that the set of outcomes in which $\ell < 1$ and $\rho \in (0, 1)$ both hold is a subset of the event in which speculators' screening of the analyst is efficient eventually. Therefore, the set of outcomes in which $\ell < 1$ and $\rho \in (0, 1)$ both hold has probability 0. This concludes the proof of the theorem. □

Definition 2. Say that B mimics G if for all h_t,

$$\mathbb{P}(r_t = \text{buyrec}|h_t, B) = \mathbb{P}(r_t = \text{buyrec}|h_t, G) \tag{27}$$

and at least one of the following conditions holds:

$$\mathbb{P}(r_t = \text{buyrec}|h_t, x_t = 1, B) = 1, \tag{28}$$

$$\mathbb{P}(r_t = \text{sellrec}|h_t, x_t = 0, B) = 1. \tag{29}$$

Lemma 8. *A mimicking strategy exists, and can be chosen moreover so as to satisfy*

$$\sup_{h_t} \mathbb{P}(r_t = \textit{buyrec}|x_t = 0, h_t, B) \leq 1 - \frac{1 - q_G}{q_B} \tag{30}$$

and

$$\sup_{h_t} \mathbb{P}(r_t = \textit{sellrec}|x_t = 1, h_t, B) \leq 1 - \frac{1 - q_G}{q_B}. \tag{31}$$

Proof. Our objective will be to construct a mimicking strategy recursively, starting with $t = 1$. Let

$$\mathbb{P}(r_1 = \text{buyrec}|h_1, x_1 = 1, B) = 1 = \mathbb{P}(r_1 = \text{sellrec}|h_1, x_1 = 0, B). \tag{32}$$

As $v_1 = \frac{1}{2}$, then (32) implies (27) (with h_1 substituting h_t). That (32) implies (28) and (29) is immediate.

Next suppose a strategy can be constructed such that, up to period $T - 1 \geq 1$, for all h_t with $t \leq T - 1$, equation (27) and at least one of (28) and (29) are satisfied. Now fix a history h_T; we will consider 3 cases separately.

Case 1: if

$$v_T^B q_B + (1 - v_T^B)(1 - q_B) = v_T^G q_G + (1 - v_T^G)(1 - q_G),$$

let

$$\mathbb{P}(r_T = \text{buyrec}|h_T, x_T = 1, B) = 1 = \mathbb{P}(r_T = \text{sellrec}|h_T, x_T = 0, B).$$

Case 2: if instead

$$v_T^B q_B + (1 - v_T^B)(1 - q_B) < v_T^G q_G + (1 - v_T^G)(1 - q_G), \tag{33}$$

let

$$\mathbb{P}(r_T = \text{buyrec}|h_T, x_T = 1, B) = 1,$$

and

$$\mathbb{P}(r_T = \text{buyrec}|h_T, x_T = 0, B) = z(h_T),$$

with $z(h_T)$ solving

$$v_T^B[q_B + (1-q_B)z] + (1-v_T^B)[(1-q_B) + q_B z] = v_T^G q_G + (1-v_T^G)(1-q_G). \tag{34}$$

The right-hand side of (34) is bounded above by q_G, and $q_G < 1$; the left-hand side evaluated at $z = 1$ is equal to 1; lastly, the left-hand side evaluated at $z = 0$ is strictly smaller than the right-hand side (by (33)). So $z(h_T)$ exists (and is unique, moreover).

Case 3: if

$$v_T^B q_B + (1-v_T^B)(1-q_B) > v_T^G q_G + (1-v_T^G)(1-q_G), \tag{35}$$

let

$$\mathbb{P}(r_T = \text{sellrec}|h_T, x_T = 0, B) = 1,$$

and

$$\mathbb{P}(r_T = \text{sellrec}|h_T, x_T = 1, B) = z(h_T),$$

with $z(h_T)$ solving

$$v_T^B[q_B z + (1-q_B)] + (1-v_T^B)[(1-q_B)z + q_B] = v_T^G(1-q_G) + (1-v_T^G)q_G. \tag{36}$$

The right-hand side of (36) is bounded above by q_G, and $q_G < 1$; the left-hand side evaluated at $z = 1$ is equal to 1; lastly, the left-hand side evaluated at $z = 0$ is strictly smaller than the right-hand side (by (35)). So $z(h_T)$ exists (and is unique, moreover).

It is now a simple matter to check that, irrespective of which of the cases above holds, then given h_T, equation (27) and at least one of (28) and (29) are satisfied. As h_T was chosen arbitrarily, a strategy can therefore be constructed such that for all h_t with $t \leq T$, equation (27) and at least one of (28) and (29) are satisfied. But then a mimicking strategy exists, by induction.

We show next that the mimicking strategy constructed above satisfies (30). In Cases 1 and 3, $\mathbb{P}(r_t = \text{buyrec}|x_t = 0, h_t, B) = 0$, so (30) trivially holds. In Case 2, $\mathbb{P}(r_t = \text{buyrec}|x_t = 0, h_t, B) = z(h_t)$, with $z(h_t)$ solving (34). So we need to show $z(h_t) \leq 1 - \frac{1-q_G}{q_B}$. Notice that the left-hand side of (34) is increasing in z and, as $q_B \geq \frac{1}{2}$, is also increasing in v_t^B. The right-hand side of (34) is bounded above by q_G. Since $v_t^B \geq 0$, the solution to

$$(1-q_B) + q_B z = q_G$$

is thus an upper bound for $z(h_t)$. This gives $z(h_t) \leq 1 - \frac{1-q_G}{q_B}$. The mimicking strategy constructed above therefore satisfies (30). That it satisfies (31) follows by symmetry. □

Define in the remaining of the appendix, for any non-degenerate mixed strategy of the B analyst,

$$L_{\theta,t} := \frac{\mathbb{P}(h_t|V=0,\theta)}{\mathbb{P}(h_t|V=1,\theta)}$$

and

$$\phi_{\theta,t}^r := \frac{\mathbb{P}(r_t = r|V=0, h_t, \theta)}{\mathbb{P}(r_t = r|V=1, h_t, \theta)}.$$

Lemma 9. *For any non-degenerate mixed strategy of the B analyst and any history h_t such that $v_t^G(sellrec) \neq v_t^B(sellrec)$ and $v_t^G(buyrec) \neq v_t^B(buyrec)$:*

$$\frac{v_t^B(buyrec) - v_t^B(sellrec)}{v_t^G(sellrec) - v_t^B(sellrec)} = \frac{(1+\phi_{G,t}^{sell} L_{G,t})(\phi_{B,t}^{sell} - \phi_{B,t}^{buy})}{(1+\phi_{B,t}^{buy} L_{B,t})(\phi_{B,t}^{sell} - \phi_{G,t}^{sell} \frac{L_{G,t}}{L_{B,t}})}; \tag{37}$$

$$\frac{v_t^B(buyrec) - v_t^B(sellrec)}{v_t^G(buyrec) - v_t^B(buyrec)} = \frac{(1+\phi_{G,t}^{buy} L_{G,t})(\phi_{B,t}^{sell} - \phi_{B,t}^{buy})}{(1+\phi_{B,t}^{sell} L_{B,t})(\phi_{B,t}^{buy} - \phi_{G,t}^{buy} \frac{L_{G,t}}{L_{B,t}})}; \tag{38}$$

$$\frac{v_t^G(buyrec) - v_t^G(sellrec)}{v_t^G(buyrec) - v_t^B(buyrec)} = \frac{(1+\phi_{B,t}^{buy} L_{B,t})(\phi_{G,t}^{sell} - \phi_{G,t}^{buy})}{(1+\phi_{G,t}^{sell} L_{G,t})(\phi_{B,t}^{buy} \frac{L_{B,t}}{L_{G,t}} - \phi_{G,t}^{buy})}; \tag{39}$$

$$\frac{v_t^G(buyrec) - v_t^B(buyrec)}{v_t^G(sellrec) - v_t^B(sellrec)} = \frac{(1+\phi_{G,t}^{sell} L_{G,t})(1+\phi_{B,t}^{sell} L_{B,t})(\phi_{B,t}^{buy} - \phi_{G,t}^{buy} \frac{L_{G,t}}{L_{B,t}})}{(1+\phi_{G,t}^{buy} L_{G,t})(1+\phi_{B,t}^{buy} L_{B,t})(\phi_{B,t}^{sell} - \phi_{G,t}^{sell} \frac{L_{G,t}}{L_{B,t}})}. \tag{40}$$

Proof. Applying Bayes' rule,

$$v_t^\theta(r) = \frac{1}{1+\phi_{\theta,t}^r L_{\theta,t}}.$$

Tedious but straightforward algebra then yields (37)–(40). □

Lemma 10. *If B mimics G, (30)–(31) hold, and $q_B > \frac{1}{2}$ then*

$$\inf_{h_t} |\phi_{B,t}^{sell} - \phi_{B,t}^{buy}| > 0. \tag{41}$$

Furthermore, if $\lim_{t\to\infty} n_t = +\infty$, then

$$\lim_{t\to\infty} L_{B,t} = 0. \tag{42}$$

Proof. Assume $q_B > \frac{1}{2}$, B mimics G and (30)–(31) hold. Fix a history h_t. Either (28) holds, or (29) does. We consider below the two possibilities.

If (28) holds then $\mathbb{P}(r_t = \text{buyrec}|x_t = 0, h_t, B) = z(h_t)$, with $z(h_t)$ solving (34). Hence,

$$\begin{aligned}\mathbb{P}(r_t = \text{buyrec}|V=1, h_t, B) &- \mathbb{P}(r_t = \text{buyrec}|V=0, h_t, B)\\ &= q_B + (1-q_B)z(h_t) - [(1-q_B) + q_B z(h_t)]\\ &= (2q_B - 1)(1 - z(h_t))\\ &\geq \frac{(2q_B-1)(1-q_G)}{q_B},\end{aligned}$$

where we used (30) to obtain the final inequality.

If instead (29) holds then $\mathbb{P}(r_T = \text{sellrec}|h_T, x_T = 1, B) = z(h_T)$, with $z(h_t)$ solving (36). Hence,

$$\begin{aligned}\mathbb{P}(r_t = \text{buyrec}|V=1, h_t, B) &- \mathbb{P}(r_t = \text{buyrec}|V=0, h_t, B)\\ &= q_B(1 - z(h_t)) - (1-q_B)(1-z(h_t))\\ &= (2q_B - 1)(1 - z(h_t))\end{aligned}$$

$$\geq \frac{(2q_B - 1)(1 - q_G)}{q_B},$$

where we used (31) to obtain the final inequality.

As h_t was chosen arbitrarily, we find

$$\inf_{h_t} \Big\{ \mathbb{P}(r_t = \text{buyrec} | V = 1, h_t, B) - \mathbb{P}(r_t = \text{buyrec} | V = 0, h_t, B) \Big\}$$
$$\geq \frac{(2q_B - 1)(1 - q_G)}{q_B}. \tag{43}$$

Since $q_B > \frac{1}{2}$ and $q_G < 1$, (43) implies (41).

For the second part of the lemma, write

$$\begin{aligned} L_{B,t} &= \frac{\mathbb{P}(h_t | V = 0, B)}{\mathbb{P}(h_t | V = 1, B)} \\ &= \prod_{\tau < t} \frac{\mathbb{P}(r_\tau, y_\tau | h_\tau, V = 0, B)}{\mathbb{P}(r_\tau, y_\tau | h_\tau, V = 1, B)} \\ &= \left(\prod_{\tau < t} \frac{\mathbb{P}(r_\tau | h_\tau, V = 0, B)}{\mathbb{P}(r_\tau | h_\tau, V = 1, B)} \right) \cdot \left(\prod_{\tau < t} \frac{\mathbb{P}(y_\tau | r_\tau, h_\tau, V = 0, B)}{\mathbb{P}(y_\tau | r_\tau, h_\tau, V = 1, B)} \right). \end{aligned}$$

Now y_τ depends on (r_τ, s_τ, h_τ) only and s_τ is independent of V. Therefore $\mathbb{P}(y_\tau | r_\tau, h_\tau, V = 0, B) = \mathbb{P}(y_\tau | r_\tau, h_\tau, V = 1, B)$, and so the second bracketed term is equal to 1. If moreover $\lim_{t \to \infty} n_t = +\infty$ then, by virtue of (43), the first bracketed term goes to zero as $t \to \infty$. □

Proof of Theorem 2: Suppose B mimics G (we are not saying at this point that such a strategy is optimal for B). Note to start with that if we were able to show that, for any t and any history h_t,

$$\min_s v_t(\text{buyrec}, s) > p_t^a; \tag{44}$$

$$\max_s v_t(\text{sellrec}, s) < p_t^b, \tag{45}$$

then we would deduce (by induction) (a) $\mathbb{P}(\rho_t = \rho_1) = 1$ for all t and h_t, and also (b) mimicking G is an optimal strategy of the B analyst. So to prove the theorem we just need to show that (44)–(45) can be made to hold for all t and all h_t under the premises that B mimics G and $\rho_t = \rho_1$ irrespective of t and h_t. Now consider $\epsilon > 0$. As long as $\gamma_G \in (0, 1)$, we can choose $\eta > 0$ such that $\gamma_B > \gamma_G - \eta$ implies

$$\max_{r,s} |\rho_1(r, s) - \rho_1| < \varepsilon. \tag{46}$$

Our goal in the rest of the proof will be to show that if

(i) B mimics G,
(ii) (30)–(31) hold,
(iii) (46) holds,
(iv) $\rho_t = \rho_1$ for all t and h_t,

then (44)–(45) hold for all t and h_t as long as we choose ε sufficiently small. By virtue of the previous remarks, this will prove the theorem.

Assume $q_B > \frac{1}{2}$, $\gamma_G \in (0,1)$ and (i)-(iv) listed above. Below, we focus on histories for which $v_t^G(\text{buyrec}) > v_t^B(\text{buyrec})$ and $v_t^G(\text{sellrec}) > v_t^B(\text{sellrec})$ (the other cases are analogous). In this case (44)–(45) hold if we can show the following inequalities:

$$(\rho_1 - \epsilon)v_t^G(\text{buyrec}) + (1-\rho_1+\epsilon)v_t^B(\text{buyrec}) > p_t^a; \tag{47}$$

$$(\rho_1 + \epsilon)v_t^G(\text{sellrec}) + (1-\rho_1-\epsilon)v_t^B(\text{sellrec}) < p_t^b. \tag{48}$$

Let $\alpha_t := \mathbb{P}(r_t = \text{buyrec}|h_t)$ and $\beta_t := \mathbb{P}(r_t = \text{sellrec}|h_t)$. The prices appearing in (47) and (48) can then be written as

$$p_t^a = \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}[\rho_1 v_t^G(\text{buyrec}) + (1-\rho_1)v_t^B(\text{buyrec})] + \left(1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}\right)v_t;$$

$$p_t^b = \frac{\pi\beta_t}{\pi\beta_t + \frac{1-\pi}{3}}[\rho_1 v_t^G(\text{sellrec}) + (1-\rho_1)v_t^B(\text{sellrec})] + \left(1 - \frac{\pi\beta_t}{\pi\beta_t + \frac{1-\pi}{3}}\right)v_t.$$

Define $v_t(r) := \rho_1 v_t^G(r) + (1-\rho_1)v_t^B(r)$. Then $v_t = \alpha_t v_t(\text{buyrec}) + (1-\alpha_t)v_t(\text{sellrec})$. Some straightforward algebra now shows that we can write (47)–(48) as

$$\frac{v_t(\text{buyrec}) - v_t(\text{sellrec})}{v_t^G(\text{buyrec}) - v_t^B(\text{buyrec})} > \left(\frac{1}{1-\alpha_t}\right)\left(\frac{\epsilon}{1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}}\right); \tag{49}$$

$$\frac{v_t(\text{buyrec}) - v_t(\text{sellrec})}{v_t^G(\text{sellrec}) - v_t^B(\text{sellrec})} > \left(\frac{1}{\alpha_t}\right)\left(\frac{\epsilon}{1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}}\right). \tag{50}$$

Define

$$\Delta_t^c := \frac{v_t^G(\text{buyrec}) - v_t^G(\text{sellrec})}{v_t^G(\text{buyrec}) - v_t^B(\text{buyrec})};$$

$$\Delta_t^d := \frac{v_t^B(\text{buyrec}) - v_t^B(\text{sellrec})}{v_t^G(\text{buyrec}) - v_t^B(\text{buyrec})}.$$

Then (49) becomes

$$\rho_1\Delta_t^c + (1-\rho_1)\Delta_t^d > \left(\frac{1}{1-\alpha_t}\right)\left(\frac{\epsilon}{1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}}\right). \tag{51}$$

Note first that $v_t^G(\text{buyrec}) > v_t^G(\text{sellrec})$, $v_t^B(\text{buyrec}) > v_t^B(\text{sellrec})$ and, moreover, we are focusing on histories such that $v_t^G(\text{buyrec}) > v_t^B(\text{buyrec})$. Thus $\Delta_t^c > 0$ and $\Delta_t^d > 0$ for all relevant histories. Furthermore, Lemma 10 shows that $\lim_{n_t\to+\infty} L_{B,t} = 0$ and repeating arguments used in the proof of Theorem 1 establishes $\lim_{n_t\to+\infty} L_{G,t} = \lim_{n_t\to+\infty} \frac{L_{G,t}}{L_{B,t}} = 0$. Hence, applying Lemmata 9 and 10,

$$\lim_{n_t\to+\infty} \Delta_t^d = \lim_{n_t\to+\infty}\left(\frac{\phi_{B,t}^{\text{sell}} - \phi_{B,t}^{\text{buy}}}{\phi_{B,t}^{\text{buy}}}\right) > 0.$$

For all relevant histories, the left-hand side of (51) is therefore bounded below by a strictly positive number. As α_t is bounded away from 0 and 1, choosing ε sufficiently small assures that (51) holds. Repeating the steps above starting from (50) instead of (49) shows in a similar manner that (50) holds as long as we choose ε sufficiently small. □

Appendix C. Proofs of Section 6

Proof of Proposition 5. The method of proof is similar to the method used to prove Theorem 2. Suppose B mimics G (we are not saying at this point that such a strategy is optimal for B). Note to start with that if we were able to show that, for any t and any history h_t,

$$\min_s v_t(\text{buyrec}, s) > p_t^a; \tag{52}$$

$$\max_s v_t(\text{sellrec}, s) < p_t^b, \tag{53}$$

then we would deduce (by induction) (a) $\mathbb{P}(\rho_t = \rho_1) = 1$ for all t and h_t, and also (b) mimicking G is an optimal strategy of the B analyst. So to prove the proposition we just need to show that (52)–(53) can be made to hold for all t and all h_t under the premises that B mimics G and $\rho_t = \rho_1$ irrespective of t and h_t. Now consider $\epsilon > 0$. Choosing $\overline{\rho} < 1$ sufficiently large, $\rho_1 > \overline{\rho}$ implies

$$\max_{r,s} |\rho_1(r, s) - \rho_1| < \varepsilon. \tag{54}$$

Our goal in the rest of the proof will be to show that if

(i) B mimics G,
(ii) (54) holds,
(iii) $\rho_t = \rho_1$ for all t and h_t,

then (52)–(53) hold for all t and h_t as long as we choose ε sufficiently small. By virtue of the previous remarks, this will prove the proposition.

Assume (i)-(iii) listed above. Below, we focus on histories for which $v_t^G(\text{sellrec}) > 1/2$ (the other cases are analogous). In this case (52)–(53) hold if we can show the following inequalities:

$$(\rho_1 - \epsilon)v_t^G(\text{buyrec}) + (1 - \rho_1 + \epsilon)\frac{1}{2} > p_t^a; \tag{55}$$

$$(\rho_1 + \epsilon)v_t^G(\text{sellrec}) + (1 - \rho_1 - \epsilon)\frac{1}{2} < p_t^b. \tag{56}$$

Let $\alpha_t := \mathbb{P}(r_t = \text{buyrec}|h_t)$ and $\beta_t := \mathbb{P}(r_t = \text{sellrec}|h_t)$. The prices appearing in (55) and (56) can then be written as

$$p_t^a = \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}[\rho_1 v_t^G(\text{buyrec}) + (1-\rho_1)\frac{1}{2}] + \left(1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}\right)v_t;$$

$$p_t^b = \frac{\pi\beta_t}{\pi\beta_t + \frac{1-\pi}{3}}[\rho_1 v_t^G(\text{sellrec}) + (1-\rho_1)\frac{1}{2}] + \left(1 - \frac{\pi\beta_t}{\pi\beta_t + \frac{1-\pi}{3}}\right)v_t.$$

Define $v_t(r) := \rho_1 v_t^G(r) + (1 - \rho_1)\frac{1}{2}$. Then $v_t = \alpha_t v_t(\text{buyrec}) + (1 - \alpha_t)v_t(\text{sellrec})$. Some straightforward algebra now shows that we can write (55)–(56) as

$$\frac{\rho_1\big(v_t^G(\text{buyrec}) - v_t^G(\text{sellrec})\big)}{v_t^G(\text{buyrec}) - \frac{1}{2}} > \left(\frac{1}{1-\alpha_t}\right)\left(\frac{\epsilon}{1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}}\right); \tag{57}$$

$$\frac{\rho_1\big(v_t^G(\text{buyrec}) - v_t^G(\text{sellrec})\big)}{v_t^G(\text{sellrec}) - \frac{1}{2}} > \left(\frac{1}{\alpha_t}\right)\left(\frac{\epsilon}{1 - \frac{\pi\alpha_t}{\pi\alpha_t + \frac{1-\pi}{3}}}\right). \tag{58}$$

Next, denote $\overline{v}_t^G(r)$ the valuation conditional on $\theta = G$, $r_t = r$ and $V_t = V_{t-1}$. We can then write

$$v_t^G(\text{buyrec}) - v_t^G(\text{sellrec}) = (1-z)[\overline{v}_t^G(\text{buyrec}) - \overline{v}_t^G(\text{sellrec})] + z[v_1^G(\text{buyrec}) - v_1^G(\text{sellrec})].$$

This gives $v_t^G(\text{buyrec}) - v_t^G > z[v_1^G(\text{buyrec}) - v_1^G(\text{sellrec})] > 0$. The left-hand sides of (57) and (58) are therefore bounded below by a strictly positive number. As α_t is bounded away from 0 and 1, choosing ε sufficiently small assures that (57)–(58) hold. □

Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2019.01.001.

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