Optimists and Pessimists in (In)Complete Markets

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Abstract

We study the effects of market incompleteness on speculation, investor survival, and asset pricing moments, when investors disagree about the likelihood of jumps and have recursive preferences. We consider two models. In a model with jumps in aggregate consumption, incompleteness barely matters, since the consumption claim resembles an insurance product against jump risk and effectively reproduces approximate spanning. In a long-run risk model with jumps in the long-run growth rate, market incompleteness affects speculation, and investor survival. Jump and diffusive risks are more balanced regarding their importance and, therefore, the consumption claim cannot reproduce approximate spanning.

Keywords: Market (in)completeness, heterogeneous beliefs, jumps in the long-run growth rate, jumps in aggregate consumption, recursive preferences

JEL: D51, D52, G12

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I. Introduction

The probability of rare disasters is by definition hard to estimate. Therefore, investors likely disagree about it, which creates an incentive to trade to engage in speculation. In addition, investors cannot insure against disasters. For example, Bates (2008) shows that the insurance market against "large" risks suffers from substantial under-capitalization, i.e., there are not enough insurers willing to offer, e.g., deep-out-of-the-money put options so that the market potentially can be incomplete. Therefore, we ask in this paper: If two investors trade with each other because they disagree about the likelihood of jumps, under what circumstances does market incompleteness or, more precisely, spanning incompleteness matter?

Dieckmann (2011) argues that in a model with two log utility investors who disagree about the likelihood of jumps in aggregate consumption, the "risk premium increases upon market completion, and the increase can be as large as 30%" (p. 460), i.e., it increases to 1.06% from 0.82%. Nevertheless, looking at an absolute difference of 0.24%, market incompleteness does not seem to matter that much. We investigate this issue in the context of two models: (1) a model with jumps in aggregate consumption, and (2) a long-run risk model with jumps in the long-run growth rate. In each model, there are two investors with recursive preferences, an optimist and a pessimist with a low and a high subjective disaster probability, respectively. We solve each model for the equilibrium on the complete and the incomplete market. Completeness means that the consumption claim, the risk-free asset, and all necessary insurance contracts needed to trade jump risk independent from diffusion risk are available to the investors. When the market is incomplete, we assume that the insurance products are no longer available. By comparing the results, we see whether market incompleteness matters for the speculation between investors (i.e., the local consumption share dynamics), investor survival (i.e., the long-term evolution of the consumption share over time), and aggregate asset pricing moments as the risk-free rate, the risk premium, and the return volatility.[1]

In our calibrated examples, we find that the relative importance of jump and diffusive risks is a key determinant for differences between complete and incomplete markets. In model (1), jump risk is the dominant source of risk and market incompleteness barely matters, irrespective of whether we look at speculation, investor survival, or asset pricing moments. On the *complete* and the *incomplete* market, the investors speculate against each other using jump risk, and depending on the true jump intensity, either of the two agents or both survive. In model (2), jump and diffusive risks are more balanced regarding their importance and here market incompleteness matters with respect to speculation and investor survival, but not regarding asset pricing moments. On the *complete* market, the agents almost exclusively share diffusion risk and use jump risk to speculate. Depending on the true measure, the optimist, the pessimist, or both survive. On the *incomplete* market, however, the investors trade mainly diffusive risks, and irrespective of the true jump intensity, the pessimist is driven out of the market in the long run.

The first step to understanding our findings regarding speculation is that the relative importance of the different sources of risk will be reflected in the exposures of the consumption claim. In model (1), a jump leads to a 40% drop in aggregate consumption growth, while the diffusive consumption volatility is 2%. Thus, jump risk is substantially more important than diffusive risk, so that the consumption claim loads almost exclusively on jump risk. Hence, the consumption claim is rather similar to the jump insurance product. So even when the market is incomplete, this dominant source of risk remains separately tradable via the consumption claim, and, loosely speaking, incompleteness represents a *non-binding* restriction. As a consequence, market incompleteness barely matters for speculation and we find virtually no differences regarding investor survival between the two market structures.

In model (2), the volatility of aggregate consumption is 2.52%, the volatility of the

[1]With homogeneous preferences and beliefs, the agents share risks perfectly. Introducing disagreement causes the investors to take speculative bets against each other.

long-run growth rate is 1.14%, and a jump leads to a 3% drop in the expected growth rate. Hence, jump and diffusive risks are more balanced with respect to their importance. This is reflected in the exposures of the consumption claim, so that the two types of risk can only be traded jointly via this asset. On the incomplete market, an investor cannot adjust her jump risk exposure without simultaneously affecting her exposure to the diffusive risks, so that incompleteness represents a *binding* restriction. Selling the consumption claim is the pessimist's only way to reduce her jump exposure, which also implies less diffusive exposure. This has two effects. First, by holding a smaller position in this asset than the optimist, the pessimist misses out on its excess returns. Second, with respect to the consumption-savings decision, saving becomes more attractive for the optimist and less attractive for the pessimist, and in total the optimist ends up saving more than the pessimist. Taking the two effects together, the pessimist is driven out of the market over time.

From a technical perspective, we use a solution method that is innovative in a number of ways. First, we solve the individual investors' optimization problems, which are linked through market clearing conditions, instead of applying the social planner approach. Formally, this amounts to solving a system of coupled partial differential equations for the individual investors' wealth-consumption ratios. We then extend this approach to the incomplete market case, where the model solution has to satisfy additional partial differential equations and constraints related to the investors' subjective price-cash flow ratios as well as their individual exposures to the risk factors. Compared to, e.g., the technique suggested by Collin-Dufresne, Lochstoer, and Johannes (2013, 2017), our approach is more flexible, since it can be applied to problems featuring incomplete markets. Furthermore, an advantage relative to the solution method developed by Dumas and Lyasoff (2012) is that our method does not require the discretization of continuous-time stochastic processes and also works for infinite-horizon problems.

Our paper contributes to three strands of the literature. The first one deals with natural selection in financial markets, dating back to Alchian (1950) and Friedman (1953). The question of investor survival has been addressed on a *complete* market in a general equilibrium framework with CRRA investors, for instance, by Sandroni (2000), Blume and Easley (2006), and Yan (2008). For i.i.d. consumption growth and time-additive utility, it can be shown analytically that the investor whose belief is further away from the true measure will lose all her consumption in the long run and disappear from the economy. However, Borovicka (2018) shows that this is not necessarily true anymore in models with recursive utility. Two investors with identical EZ preferences, who differ with respect to their beliefs concerning the expected growth rate of consumption, can both have non-zero expected consumption shares in the long run, even when the beliefs of one of them represent the true model. It may even happen that only the investor with the "worse" belief survives in the long-run. In our two examples, the complete market results show that, depending on the true jump intensity, either of two agents or both can survive, in line with Borovicka (2018).

The second strand of the literature addresses the question of investor survival on an *incomplete* market. The two models we look at in this paper are quite different from the ones studied in the related literature by Sandroni (2005) and Blume and Easley (2006). In particular, our model (2) is the first to combine heterogeneous investors in a long-run risk economy with incomplete markets. Sandroni (2005) argues that the agent with the more accurate beliefs will dominate the economy, as on the complete market. In contrast to this, Blume and Easley (2006) find under the assumption of time-separable preferences that the market selection hypothesis *may* fail, if the market is incomplete.[2] In this paper, we study market incompleteness by means of two examples. In model (1), we obtain the same result as on the complete market, i.e., depending on how we set the true jump intensity, either of two agents or both can survive. In model (2), however, the pessimist is driven out of the

[2]Schneider (2018) takes a different approach. Starting from observable bid ask spreads for S&P500 options, he constructs a model with an optimist, a pessimist, and a pragmatist who have different views on the distribution of the S&P500 and trade option portfolios and forwards. In his setup, the pessimist is the most successful agent, while the optimist loses consistently.

market, irrespective of the true jump intensity and even if her beliefs are correct. Although our framework is different from theirs, our findings support the view offered by Blume and Easley (2006) substantially more than the conclusions drawn by Sandroni (2005).

The third strand of the literature studies asset pricing moments on an *incomplete* market and can be roughly sorted into two categories.[3] On the one hand, there are papers like Levine and Zame (2002) or Dieckmann (2011) arguing that incompleteness matters. Levine and Zame (2002) point out that "in the presence of aggregate risk, market incompleteness alone may have substantial effects" (p. 1807). However, they do not provide any quantitative results. Dieckmann (2011) is the paper closest to ours in the sense that he studies a model very similar to our model (1). He considers the case of jumps in aggregate consumption, and two investors with log preferences who disagree about their likelihood. In his model, the equity premium (conditional on no disasters) on the complete market is up to 30% higher than on the incomplete market. However, due to log utility, the level of both premia as well as the difference between them are fairly small in absolute terms and the latter amounts to 24 bps. On the other hand, papers as Telmer (1993), Heaton and Lucas (1996), or Krüger and Lustig (2010) indicate that market incompleteness does not have a significant impact on asset pricing moments. In the two models we look at, it seems difficult to conclude that incompleteness matters significantly for standard asset pricing moments, irrespective of whether we look at the risk-free rate, the risk premium, or the return volatility, be it locally or on the basis of a long time series of simulated data.

The remainder of the paper consists of three parts. In Sections II and III, we apply the method described in Appendix A and check when and how market incompleteness matters

[3]There is a large literature on models with heterogeneous agents matching aggregate asset pricing moments on the *complete* market. Examples focussing on CRRA utility include Basak (2005), and Chen, Joslin, and Tran (2012), while Garleanu and Panageas (2015), Chabakauri (2015), and Collin-Dufresne, Lochstoer, and Johannes (2017) use recursive preferences. Since our focus is on market incompleteness, we do not contribute to this literature.

in the two types of models mentioned above. Section IV concludes.

II. Model (1): Jumps in aggregate consumption

A. Model setup

We consider two investors with identical recursive preferences.[4] β is the subjective time preference rate, γ is the coefficient of relative risk aversion, ψ denotes the elasticity of intertemporal substitution (EIS), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. In the following, we assume $\gamma > 1$ and $\psi > 1$, which implies $\gamma > \frac{1}{\psi}$, $\theta < 0$, and that both investors exhibit a preference for early resolution of uncertainty.

Under the true probability measure $\mathbb{P}$, aggregate consumption follows

$$\frac{dC_t}{C_t} = \mu_C \, dt + \sigma_C \, dW_t + L_C \, dN_t(\lambda),$$

where W_t is a standard Brownian motion, and N_t represents a Poisson process with constant intensity λ and constant jump size L_C. The investors agree on all parameters of the model except the intensity of the Poisson process. This implies that under investor i's subjective probability measure $\mathbb{P}^i$ $(i = 1, 2)$ aggregate consumption evolves as

$$\frac{dC_t}{C_t} = \mu_C \, dt + \sigma_C \, dW_t + L_C \, dN_t(\lambda_i).$$

Let investor 1 be the pessimistic investor, i.e., $\lambda_1 > \lambda_2$.

Since it is central to our analysis whether the market is complete or not, we fix the set of traded assets. On the complete market, the investors can trade the claim on aggregate consumption, the money market account, and an insurance product linked to the jump

[4] See Epstein and Zin (1989) for the discrete-time setup and Duffie and Epstein (1992) for the extension to continuous-time stochastic differential utility.

component N_t, which investors use to speculate against each other.[5] The consumption claim is in unit net supply, while the other two assets are in zero net supply. On the incomplete market, the insurance product is not available.

All equilibrium quantities are functions of the pessimist's share of aggregate consumption, $w_t = \frac{C_{1,t}}{C_t}$, as the endogenous state variable. Its dynamics are:

$$dw_t = \mu_w(w_t)\, dt + \sigma_w(w_t)\, dW_t + L_w(w_t)\, dN_t(\lambda), \tag{1}$$

where $\mu_w \equiv \mu_w(w_t)$, $\sigma_w \equiv \sigma_w(w_t)$, and $L_w \equiv L_w(w_t)$ are determined in equilibrium. Solving the model for the equilibrium on the complete and the incomplete market follows the procedure outlined in Appendix A, details are provided in Online Appendix I.

B. Quantitative analysis

1. Calibration

The parameters for the quantitative analysis of the first model are given in Table 1. They are taken from Chen et al. (2012), except for the elasticity of intertemporal substitution ψ, since Chen et al. (2012) use CRRA utility, while we want to employ recursive preferences. The investor has a subjective discount rate of $\beta = 0.03$, a relative risk aversion of $\gamma = 4$, and an elasticity of intertemporal substitution of $\psi = 1.5$. The expected growth rate of aggregate consumption, not including the compensation for the jump process, is $\mu_C = 0.025$. The diffusive volatility is $\sigma_C = 0.02$, and in case of a jump, aggregate consumption drops by $L_C = -0.40$.[6] The investors disagree about the frequency of these consumption disasters. The pessimist assumes an intensity of $\lambda_1 = 0.017$, i.e., on average one jump every 59 years,

[5] The cash flow of the insurance product, $\frac{dI_t}{I_t} = \mu_I\, dt + L_I\, dN_t(\lambda)$, is given exogenously, but its price-to-cash flow ratio is determined in equilibrium.

[6] In Online Appendix II.C, we report additional results for $L_C = -0.20$.

while the optimist thinks there will be on average one jump only every 1,000 years, i.e., $\lambda_2 = 0.001$.[7] Following Chen et al. (2012), we assume that the pessimist's belief represents the true model.[8]

[Insert Table 1 around here]

2. Consumption share dynamics

Since the two investors disagree about the likelihood of jumps in aggregate consumption, they engage in risk sharing and speculation with each other. The dynamics of the pessimist's consumption share depicted in Figure 1 describe the equilibrium. Specifically, the volatility σ_w and the jump size L_w, respectively, summarize how investors use diffusive shocks and jumps to share risks and speculate.[9] The drift μ_w in turn shows how investors compensate each other. The portfolio weights in Figure 2 show the corresponding positions in the consumption claim, the jump insurance product, and the money market account.

[Insert Figure 1 around here]

[Insert Figure 2 around here]

We start with the intuition on the *complete* market (grey line) in Figure 1. Since the investors disagree about the likelihood of jumps, they speculate against each other using jump risk. The optimist considers jumps to be less risky than the pessimist, hence she sells the insurance product to the pessimist (see the top row of Figure 2). While the investors use jump risk to speculate against each other, they share exclusively diffusive risk, i.e., $L_w > 0$ and $\sigma_w = 0$. As a result of this transaction, the pessimist receives the insurance payoff in

[7] In Online Appendix II.D, we present additional results when we decrease λ_1 to 0.012, while keeping λ_2 unchanged. After that we increase λ_2 to 0.06 and keep λ_1 fixed.

[8] In Section II.B.5, we analyze how a variation in the true jump intensity affects the results.

[9] Risk sharing is characterized by $\sigma_w = 0$ for diffusive risk and $L_w = 0$ for jump risk.

case of a jump, but she also has to pay the insurance premium to the optimist in calm times, i.e., $\mu_w < 0$.

When the market is *incomplete* (black line), the investors still want to speculate using jump risk, but since the insurance product is not available, the consumption claim is the only instrument they can trade jump risk with. The pessimist sells the consumption claim (see the bottom row of Figure 2), since she considers jumps to be more risky than the optimist. The investors speculate so aggressively that the pessimist ends up selling the consumption claim short when she is small (for $w_t < 0.25$). Compared to the situation on the complete market, the investors speculate on the incomplete market only slightly less on jump risk, and this comes along with some speculation on diffusion risk, so that $L_w > 0$ and $\sigma_w < 0$. The reason is the risk structure of the consumption claim. Although the investors can no longer trade jump risk separately through the insurance product, they can still trade the consumption claim which mainly reflects jump risk and only a very small fraction of diffusion risk. This also explains the small decrease in the plot for σ_w, relative to the complete market. Finally, by selling the consumption claim to the optimist, the pessimist misses out on the returns from holding the consumption claim in calm times. Hence, Figure 1 shows that μ_w is negative.

3. Investor survival

Given the local dynamics of the pessimist's consumption share, its evolution over time determines the investors' long-run survival. Table 2 shows the results from a Monte Carlo simulation of the pessimist's consumption share over a period of 50, 100, 200, 500, 1,000, and 10,000 years. The pessimist's belief coincides with the true jump intensity.

[Insert Table 2 around here]

On the *complete* market, the pessimist looses a fraction of her consumption share at the beginning, but even after 10,000 years she still survives. The pattern we observe in the simulations corresponds to the discussion in Section II.B.2 regarding Figure 1. On average, the pessimist's consumption share falls, but a jump in aggregate consumption takes place roughly every 59 years, and whenever it occurs, the payoff from the insurance product leads to a significant increase in her consumption share. On the *incomplete* market, we observe in Figure 1 that the increase in the pessimist's consumption share following a jump, L_w, is a bit smaller than on the complete market.

To understand what drives these results, we follow Borovicka (2018) who uses the difference in expected log wealth growth rates under the true measure to analyze investor survival:

$$\begin{aligned} & \lim_{w \to 0} \left\{ \frac{1}{dt} \mathbb{E}_t \left[d \log V_{1,t} \right] - \frac{1}{dt} \mathbb{E}_t \left[d \log V_{2,t} \right] \right\} \\ (2) \qquad = & \lim_{w \to 0} \left\{ \left(\mu_{V_1} - \frac{1}{2} \sigma_{V_1}^2 + \log\left(1 + L_{V_1}\right) \lambda \right) - \left(\mu_{V_2} - \frac{1}{2} \sigma_{V_2}^2 + \log\left(1 + L_{V_2}\right) \lambda \right) \right\}, \end{aligned}$$

where μ_{V_i}, σ_{V_i}, and L_{V_i} are the drift, volatility, and jump size of investor i's individual wealth process. If this expression is negative, the pessimist cannot survive. Moreover, the above equation can be rewritten using the fact that the expected return on individual wealth under investor i's probability measure is equal to the risk-free rate plus the compensation for diffusive shocks and jump risk:

$$\begin{aligned} \frac{1}{dt} \mathbb{E}_{i,t} \left[dR_{i,t}^V \right] &= r_{i,t}^f + \sigma_{V_i} \eta_{i,t}^W - L_{V_i} \eta_{i,t}^N \lambda_i \\ \mu_{V_i} &= r_{i,t}^f + \sigma_{V_i} \eta_{i,t}^W - L_{V_i} \lambda_{i,t}^{\mathbb{Q}} - e^{-v_{i,t}}, \end{aligned}$$

where $r_{i,t}^f$ denotes investor i's risk-free rate, $\eta_{i,t}^W$ her market prices of diffusion risk, and $\lambda_{i,t}^{\mathbb{Q}}$ her risk-neutral jump intensity, which is determined through $\lambda_{i,t}^{\mathbb{Q}} = \lambda_i \left(1 + \eta_{i,t}^N\right)$ from her market price of jump risk $\eta_{i,t}^N$. Plugging this expression, which holds on the complete and on the incomplete market, into Equation (2) yields

$$
\begin{aligned}
& \lim_{w \to 0} \left\{ \frac{1}{dt} \mathbb{E}\left[d \log V_{1,t}\right] - \frac{1}{dt} \mathbb{E}\left[d \log V_{2,t}\right] \right\} \\
= & \lim_{w \to 0} \Bigg\{ \left(e^{-v_{2,t}} - e^{-v_{1,t}}\right) + \left(\frac{1}{2}\, \sigma_{V_2}^2 - \frac{1}{2}\, \sigma_{V_1}^2\right) + \left[\log\left(1 + L_{V_1}\right) - \log\left(1 + L_{V_2}\right)\right] \lambda \\
& \qquad + \left[\left(\sigma_{V_1}\, \eta_{1,t}^W - L_{V_1}\, \lambda_{1,t}^{\mathbb{Q}}\right) - \left(\sigma_{V_2}\, \eta_{2,t}^W - L_{V_2}\, \lambda_{2,t}^{\mathbb{Q}}\right)\right] \Bigg\}. \qquad (3)
\end{aligned}
$$

The expression in curly brackets on the right-hand side of Equation (3) has four terms: (i) the savings channel, $e^{-v_{2,t}} - e^{-v_{1,t}}$; (ii) the speculative volatility channel, $\frac{1}{2}\, \sigma_{V_2}^2 - \frac{1}{2}\, \sigma_{V_1}^2$; (iii) the speculative jump channel, $\left[\log\left(1 + L_{V_1}\right) - \log\left(1 + L_{V_2}\right)\right] \lambda$, and; (iv) the "risk premium" channel, $\left(\sigma_{V_1}\, \eta_{1,t}^W - L_{V_1}\, \lambda_{1,t}^{\mathbb{Q}}\right) - \left(\sigma_{V_2}\, \eta_{2,t}^W - L_{V_2}\, \lambda_{2,t}^{\mathbb{Q}}\right)$.[10]

The aggregate effect. Figure 3 presents the results.[11] The *savings channel* is negative and essentially the same for the two markets, implying that the pessimist consumes more out of her wealth than the optimist. Since the optimist beliefs jumps will happen only once every 1,000 years, saving is more attractive for her than for the pessimist. The *speculative volatility channel* is exactly zero on the complete and close to zero on the incomplete market. On the complete market, this follows from the fact that both investors share diffusive risk. On the incomplete market, they speculate by trading the consumption claim, but since this asset reflects diffusive risk only to a small degree, the channel is not very important quantitatively. The *speculative jump channel* is positive for the two market structures, since the pessimist is less exposed to jump risk than the optimist. Through trading the insurance product on the complete market or the consumption claim on the incomplete market, the investors take speculative bets against each other. Hence, they end up with rather different exposures, so that the resulting speculative jump channel is substantial.

[10]We put the term "risk premium" in quotes, since the expression given here does not coincide with the difference between the pessimist's and the optimist's *actual* risk premium given by $\left(\sigma_{V_1}\, \eta_{1,t}^W - L_{V_1}\, \left[\lambda_{1,t}^{\mathbb{Q}} - \lambda\right]\right) - \left(\sigma_{V_2}\, \eta_{2,t}^W - L_{V_2}\, \left[\lambda_{2,t}^{\mathbb{Q}} - \lambda\right]\right)$.

[11]In Online Appendix II, we present additional results breaking each channel down into its components.

Finally, the *"risk premium" channel* is negative on both markets. This indicates that the "risk premium" earned by the optimist is higher than the pessimist's. The large importance of jump risk implies that the jump part of the "risk premium" is sizeable. On the complete market, the two investors agree on the risk-neutral jump intensity, so this part of the premium reflects the difference in jump exposures discussed previously. On the incomplete market, the investors' will have different subjective risk-neutral jump intensities, but the key point is that the huge importance of jump risk is still reflected in the consumption claim. Therefore, this asset is fairly similar to the insurance product with respect to its sensitivity to jump risk, and the investors are not materially restricted in terms of their options to trade. Consequently, the differences between the investors' risk-neutral jump intensities are rather small, and the jump part of the "risk premium" channel is caused mainly by the jump exposures, as on the complete market. While the pessimist wants to be less exposed to jumps, the optimist gathers more jump exposure, and she gets compensated for that through a higher "risk premium".

[Insert Figure 3 around here]

Overall, Figure 3 shows that the optimist saves more, takes on a higher jump exposure through trading, and, hence, earns a higher "risk premium" than the pessimist on both the complete and the incomplete market. Therefore, the pessimist is driven out of the market, in line with the results shown in Table 2.

Finally, we relate our findings on the *complete* market to Chen et al. (2012). They find that "the agent with correct beliefs will dominate in the long run" (p. 2205), which is at odds with our results. The reason is that Chen et al. rely on CRRA utility with $\gamma = 4$, implying $\psi = \frac{1}{4}$, while we use recursive preferences with the same γ, but with $\psi = 1.5$. The choice of ψ affects the savings behavior of the investors, and thus the savings channel. For $\psi < 1$, it is positive, i.e., the pessimist saves more than the optimist, and the savings channel is so pronounced that it dominates the other three channels. For $\psi > 1$, it is negative and smaller in magnitude, as pointed out above.

4. Aggregate asset pricing moments

Before we can study aggregate asset pricing moments, we have to take a closer look at aggregate wealth. It is given as $V_t = C_t\, e^{v_t}$, where $v_t \equiv \log\left(w_t\, e^{v_{1,t}} + (1-w_t)\; e^{v_{2,t}}\right)$ is the aggregate log wealth-consumption ratio. The total return (including consumption) R_t^V on aggregate wealth evolves as

$$dR_t^V \;=\; \left(\mu_V + e^{-v_t}\right)\, dt + \sigma_V\, dW_t + L_V\, dN_t, \tag{4}$$

where μ_V, σ_V, and L_V are the drift, volatility, and jump size of the aggregate wealth process.

Next, we take a look at the implications of market incompleteness on the following three aggregate asset pricing moments: the risk-free rate which the investors have to agree on, $r_{1,t}^f = r_{2,t}^f = r_t^f$[12], the risk premium on the levered claim to aggregate consumption calculated as

$$EER_t \;=\; \phi\left(\frac{1}{dt}\,\mathbb{E}_t\left[dR_t^V\right] - r_t^f\right), \tag{5}$$

with leverage factor ϕ, and the return volatility of the levered claim to aggregate consumption given as

$$RV_t \;=\; \phi\sqrt{\sigma_V'\,\sigma_V + \lambda\,\left(L_V\right)^2}. \tag{6}$$

The three quantities are shown in Figure 4. While there is virtually no difference between the risk premia on the two markets, the risk-free rate is slightly higher on the incomplete market, and, depending on w_t, the return volatility on the incomplete can be

[12]The expression for the risk-free rate is given in Equation (I.10) in the Online Appendix.

lower than, equal to, or higher than that on the complete market. Overall, however, the differences appear too small to conclude that market incompleteness matters for aggregate asset pricing moments in this model.

[Insert Figure 4 around here]

Finally, we compare our results for the *complete* market to Chen et al. (2012). They only provide results for the risk premium, which is quantitatively similar to ours. In this model, the impact of the intertemporal elasticity of substitution, ψ, on the risk premium is negligible, but it is highly relevant for survival. Concerning the *incomplete* market case, the paper by Dieckmann (2011) is closest to ours.[13] In terms of aggregate asset pricing moments, he focuses on the risk premium. In an economy with two log utility investors, he finds that the risk premium (conditional on no jump) on the complete market can be up to 30% higher than on the incomplete market. However, due to log preferences, the level of the risk premium in the two markets as well as the difference between them are fairly small in absolute terms, with the difference amounting to only roughly 24 bps. The fact that we find a similar result with a more flexible preference specification indicates that the preference parameters might not be of first-order importance when it comes to explaining differences between the risk premiums on the complete and the incomplete market.

5. *Varying the true jump intensity*

The results presented so far are based on the assumption that the true measure coincides with the pessimist's belief, i.e., $\lambda = \lambda_1 = 0.017$. Since the probability of rare disasters is by definition hard to estimate, we study in this section, how varying the true jump intensity affects investor survival.[14]

[13] In his model, a jump leads to 20% drop in aggregate consumption and the consumption volatility is 4%. The pessimist beliefs jumps take place every 10 years and every 100 years according to the optimist.

[14] We look at the corresponding aggregate asset pricing moments in Online Appendix II.

Table 3 shows the pessimist's expected consumption share $\mathbb{E}[w_T]$ simulated T years into the future, for λ varying between 0.01 and 0.10, i.e., for the range of individual beliefs spanned by Dieckmann (2011) and Chen et al. (2012). Varying the true belief on the *complete* market makes jumps more or less frequent, which affects the likelihood of a positive payoff from the insurance product. Since the pessimist holds a long position in this asset, less frequent jumps lead to a decrease in her consumption share, and the opposite is true, when jumps become more frequent.

[Insert Table 3 around here]

Looking at the *incomplete* market, we find almost the same pattern. The fact that there are only minor differences between the complete and the incomplete market is in line with our results from Section II.B.3. The consumption claim is rather similar to the insurance product, and by trading the consumption claim the investors can take almost the same positions as on the complete market.

III. Model (2): Long-run risk model with jumps in the long-run growth rate

A. Model setup

As in Section II.A, we consider two investors with identical recursive preferences. Under the true probability measure $\mathbb{P}$, aggregate consumption C_t and the stochastic component of its expected growth rate X_t follow a system of stochastic differential equations

$$\begin{aligned}\frac{dC_t}{C_t} &= (\bar{\mu}_C + X_t)\, dt + \sigma_C'\, dW_t, \\ dX_t &= -\kappa_X\, X_t\, dt + \sigma_X'\, dW_t + L_X\, dN_t(\lambda),\end{aligned}$$

where $W_t = (W_t^C, W_t^X)'$ is a two-dimensional standard Brownian motion, and N_t represents a non-compensated Poisson process with constant jump size $L_X < 0$ and true constant intensity λ. With the exception of the jump component, this is the classical long-run risk setup from Bansal and Yaron (2004) in continuous time. The volatility vectors are specified as $\sigma_C' = (\sigma_c, 0)$ and $\sigma_X' = (0, \sigma_x)$, so that consumption and the long-run growth rate are locally uncorrelated. The key feature of this model is that jumps represent disasters, which do not occur in the consumption process itself, but in the state variable X_t.

The investors agree on all parameters of the model except the intensity of the Poisson process, i.e., roughly speaking, they disagree about the likelihood of a disaster in the growth rate over the next time interval. This implies that under investor i's subjective probability measure $\mathbb{P}^i$ $(i = 1, 2)$ the stochastic growth rate evolves as

$$dX_t = -\kappa_X \, X_t \, dt + \sigma_X' \, dW_t + L_X \, dN_t \left(\lambda_i \right).$$

Hence, the investors also disagree on the expectation $\frac{1}{dt}\mathbb{E}_t^i \left[dX_t \right] = -\kappa_X \, X_t + L_X \, \lambda_i$. Let investor 1 be the pessimistic investor, i.e., $\lambda_1 > \lambda_2$.

For parsimony, the expected growth rate X_t is assumed to be observable as in Bansal and Yaron (2004), while the jump intensity λ is not. Specifically, it seems reasonable to assume that the investors do not have perfect knowledge about λ. In fact, it is basically impossible for the agents to infer the unknown intensity exactly from observations of X_t. Especially for low values of the true intensity, as they are commonly used in the literature, the uncertainty around estimates even from long samples can be rather large.[15]

When the market is complete, the investors can trade the claim on aggregate consumption, the money market account, and two insurance products linked to the Brownian motion

[15] With an unobservable parameter, one could consider adding learning to the model. However, since our focus is on the implications of market incompleteness, we require an otherwise parsimonious setting.

W_t^X and the jump component N_t, respectively.[16] The consumption claim is in unit net supply, while the other three assets are in zero net supply. When the market is incomplete, the insurance products are not available.[17]

All equilibrium quantities are functions of the pessimist's relative share in aggregate consumption $w_t = \frac{C_{1,t}}{C_t}$ and the expected growth rate X_t. Similar to Equation (1), the dynamics of the consumption share can be written as a jump-diffusion process

$$dw_t = \mu_w (w_t, X_t)\, dt + \sigma_w (w_t, X_t)'\, dW_t + L_w (w_t, X_t)\, dN_t (\lambda)\,, \tag{7}$$

where the coefficient functions $\mu_w \equiv \mu_w (w_t, X_t)$, $\sigma_w \equiv \sigma_w (w_t, X_t)$, and $L_w \equiv L_w (w_t, X_t)$ are determined in equilibrium. Solving the model for the equilibrium on the complete and the incomplete market follows the procedure outlined in Appendix A, details are provided in Online Appendix III.

B. Quantitative analysis

1. Calibration

The calibration used in the quantitative analysis is presented in Table 4. The parameters represent standard values from the long-run risk literature, except for the mean reversion speed $\kappa_X = 0.1$, i.e., shocks have a half-life of about 6.9 years, which is chosen to match the equity premium.[18] When a disaster occurs, expected consumption growth drops

[16]Online Appendix IV contains details on the insurance products. Their cash flow exposures to the risk factors merely represent scaling factors, which do not have an impact on the equilibrium.

[17]We have also analyzed the case of "intermediate incompleteness", where only one of the insurance products is available to the investors. These results typically lie between the cases of completeness and market incompleteness presented in Section III.B below.

[18]When we follow Benzoni, Collin-Dufresne, and Goldstein (2011) and set $\kappa_X = 0.2785$, the results remain qualitatively unchanged.

by $L_X = -0.03$, which is about the same as in Benzoni et al. (2011).[19] The pessimistic investor assumes an intensity of $\lambda_1 = 0.02$, i.e., on average one jump in X_t every 50 years, while the optimist thinks there will be on average one jump only every 1,000 years, i.e., $\lambda_2 = 0.001$.[20] So the probabilities for rare events are about the same as in Chen et al. (2012). Moreover, we assume that the pessimist's belief represents the true model.[21]

Equity is considered a claim on levered aggregate consumption with a leverage factor of $\phi = 1.3$. All the model results are shown for the case when the stochastic part of the expected growth rate of consumption is at its long-run mean of -0.006.[22]

[Insert Table 4 around here]

2. Consumption share dynamics

As in model (1), the disagreement between the investors causes them to trade with each other. Figure 5 presents the dynamics of the pessimist's consumption share, which summarize how the investors share the different types of risks and speculate against each other. The corresponding portfolio weights in the consumption claim, the diffusion insurance product Z, the jump insurance product I, and the money market account are shown in Figure 6.

[Insert Figure 5 around here]

[Insert Figure 6 around here]

[19] When a jump occurs, these parameter values lead to respective wealth losses of up to 37% and 7% for the optimist and the pessimist on the complete market. Although aggregate consumption itself remains unchanged, the price of the claim on levered aggregate consumption falls by up to 9%.

[20] In Online Appendix VI.D, we present additional results when we decrease λ_1 to 0.015, while keeping λ_2 unchanged, and for the case when we increase λ_2 to 0.06 and keep λ_1 fixed.

[21] In Section III.B.5, we analyze how varying the true jump intensity affects the results.

[22] The long-run mean is computed as the value of X_t, for which $\frac{1}{dt}\mathbb{E}[dX_t] = 0$. This value is given by $\frac{\lambda L_X}{\kappa_X}$ and equal to -0.006 for our choice of parameters.

On the *complete* market (gray line), the mechanism is basically the same as for the model with jumps in aggregate consumption discussed in Section II.B. The investors want to speculate against each other because they disagree about the likelihood of jumps. The optimist thinks these are less likely, and so she speculates on her belief by selling the insurance product linked to those jumps to the pessimist (see the top row of Figure 6). The pessimist, in turn, sells the diffusion insurance product to the optimist. Consequently, we observe almost perfect risk sharing in diffusive risks, i.e., $\sigma_{w,C}, \sigma_{w,X} \approx 0$, and the investors speculate against each other using jump risk, i.e., $L_w > 0$. As a result of this transaction, the pessimist receives in case of a jump the insurance payoff from the optimist. Upon a jump, the pessimist's consumption share increases significantly implying that L_w is positive and quantitatively sizeable. However, in calm times, the pessimist has to pay the insurance premium to the optimist. This leads to a drop in w_t, so that μ_w is negative.

On the *incomplete* market (black line), the investors can only trade the money market account and the consumption claim. As before, the agents want to take speculative bets on jump risk against each other based on their beliefs. Since the optimist considers jumps to be less frequent than the pessimist, she buys the consumption claim from the pessimist (see the bottom row of Figure 6). Holding a smaller fraction of the consumption claim affects the pessimist in several ways. In case of a positive diffusion shock, the pessimist's smaller position in the consumption claim hurts her in relative terms and, thus, leads to a decrease in her consumption share, i.e., $\sigma_{w,C}, \sigma_{w,X} < 0$. Similarly, the pessimist is better protected against a negative shock and, hence, is better off than the optimist after such a shock. In case of a jump, which is negative by definition, the pessimist is less negatively affected than the optimist who holds a larger fraction of the consumption claim. Therefore, w_t goes up slightly following a jump, i.e., $L_w > 0$. Finally, reducing the position in the consumption claim also implies that the pessimist earns less than the optimist in calm times, i.e., $\mu_w < 0$.

The results on the incomplete market here are strikingly different from those for the case of a jump in aggregate consumption analyzed in Section II.B. In model (1), jump risk is substantially more important than diffusive risk, so that the consumption claim loads almost exclusively on jump risk. Hence, the consumption claim resembles an insurance product against jump risk and market incompleteness barely matters. In model (2), jump and diffusive risks are more balanced regarding their importance. Hence, the consumption claim reflects both types of risk so that they can only be traded jointly. Therefore, an investor cannot speculate on jump risk without simultaneously speculating on diffusive shocks. The fact that the consumption claim is mainly exposed to diffusive shocks and to a lesser degree to jump risk, explains the magnitude of the effects we observe in Figure 5.

3. Investor survival

As in model (1), looking at the local consumption share dynamics motivates us to take a look at the evolution of the pessimist's consumption share over time. Table 5 presents the pessimist's expected consumption share simulated T years into the future under the assumption that the true jump intensity λ coincides with the pessimist's belief $\lambda_1 = 0.02$. Over time, the pessimist's expected consumption share becomes smaller and smaller. Whether the market is complete or incomplete does not impact the overall results on survival, it only has an effect on the speed of extinction. Starting from a share of 50%, the pessimist's expected consumption decreases to about 37% of total output after 100 years on the complete and to 12% on the incomplete market, i.e., it decreases significantly faster on the latter. To understand what drives these results, we rely on the same analysis as for model (1) and investigate the four channels from Equation (3).

[Insert Table 5 around here]

The aggregate effect. Figure 7 presents the difference in expected log wealth growth rates and its break down into the different channels for the two market structures. It shows that, irrespective of whether the market is complete or incomplete, the optimist's expected log wealth growth rate is higher than the pessimist's, which is the reason why the pessimist is driven out of the market as shown in Table 5. The optimist always ends up saving more, and earning a higher "risk premium" than the pessimist. However, as stated above, when the market is incomplete, the pessimist's consumption share decreases faster than on the complete market for two reasons. First, the two investors end up with jump exposures which are rather similar implying that the speculative jump channel is close to zero, i.e., the pessimist can no longer speculate on jump risk as much as on the complete market and this reduces her chances of survival. Second, selling the consumption claim implies less compensation for the pessimist. Therefore, the optimist's excess return on wealth is higher than the pessimist's. Next, we discuss each channel in more detail.

[Insert Figure 7 around here]

The savings channel as defined in Equation (3) is the difference between the optimist's and the pessimist's respective consumption-wealth ratios, $e^{-v_{2,t}} - e^{-v_{1,t}}$.[23] Figure 8 shows the individual consumption-wealth ratios and the resulting savings channel.

[Insert Figure 8 around here]

The *complete* market is presented in the left column. When the optimist (dashed line) becomes small (i.e., when w_t tends to 1), she consumes less and saves more. The analogous logic (for w_t going to 0) applies for the pessimist (dotted line). So if the threat of extinction becomes more pronounced, each investor increases her savings rate to increase her future wealth and, thus, to avoid extinction. However, this behavior is more pronounced for the optimist than for the pessimist. Under the optimistic investor's belief, jumps occur once every 1,000 years, and this makes saving more attractive for her, in relative terms. For the

[23]Since an investor can either save or consume, a *decreasing* consumption-wealth ratio translates into an *increasing* savings-wealth ratio. In Online Appendix VI, we present additional results related to, e.g., the aggregate and individual wealth-consumption ratios and their dependence on w_t and X_t.

same reason, the dashed line is always located below the dotted one, implying that the optimist consumes less and saves more out of her wealth than the pessimist, irrespective of the consumption share. Consequently, the savings channel (solid line) which is the difference between the dashed and the dotted line is negative across the whole range of w_t.

For the *incomplete* market, we find that the optimist reacts in a similar fashion as on the complete market. As she becomes smaller (w_t close to 1), she saves more. In contrast, the pessimist behaves strikingly different than on the complete market. Instead of trying to save her way out of extinction, when she is faced with the threat of extinction, she actually *consumes more*. So the fact that she is materially restricted in terms of her options to trade makes her *save less*. When w_t tends to zero, the expected growth rate of the pessimist's individual consumption shown by the dotted line on the right of Figure 9 is *negative* on the incomplete market.[24] As on the complete market, the optimist consumes less and saves more than the pessimist, so that the savings channel is negative. However, the resulting solid line is almost constant and does not have the same clear decreasing shape as on the complete market.

[Insert Figure 9 around here]

The speculative volatility channel is given in Equation (3) as one half times the difference between the optimist's and the pessimist's variance of individual wealth, $\frac{1}{2}\,\sigma_{V_2}'\,\sigma_{V_2} - \frac{1}{2}\,\sigma_{V_1}'\,\sigma_{V_1}$. Figure 10 shows the exposures of the investors' respective wealth, $\sigma_{V_i,C}$ and $\sigma_{V_i,X}$, as well as their difference.

[Insert Figure 10 around here]

The graphs in the top row representing the *complete* market confirm our finding from above that the investors share diffusive risks almost perfectly. Since the optimist takes on

[24] The expected growth rate of investor i's individual consumption under the true measure is given by $\frac{1}{dt}\,\mathbb{E}\left[\frac{dC_{i,t}}{C_{i,t}}\right] = \mu_{C_i} + \lambda\, L_{C_i}$.

more jump risk, her exposure to diffusive long-run growth rate shocks is lower than the pessimist's. Therefore, the speculative volatility channel is negative, but rather small in magnitude.

Again, on the *incomplete* market, the pessimist sells the consumption claim to the optimist to speculate on jump risk. Since the consumption claim is driven mainly by diffusive shocks, we observe in the bottom row that the optimist's exposure is higher than the pessimist's. Consequently, the speculative volatility channel is positive, but not very large.

The speculative jump channel captures the difference between the pessimist's and the optimist's log jump sizes in individual wealth scaled by the true jump intensity, $[\log(1 + L_{V_1}) - \log(1 + L_{V_2})]\,\lambda$, as given in Equation (3). Figure 11 shows the investors' exposures to jump risk, L_{V_i}, and the difference between them.

[Insert Figure 11 around here]

On the *complete* market, the optimist sells the insurance product against jumps to the pessimist and, thus, takes on more negative jump exposure. Interestingly, the pessimist does not stop trading once she has reached an exposure of zero. When she is small enough, she keeps on buying the insurance product to speculate on jump risk so that such a jump would lead to a significant increase in her wealth.[25] The optimist's wealth drops in this case, but she thinks the event can happen only once every 1,000 years. Consequently, she ends up with a more negative exposure to jump risk than the pessimist, implying that the speculative jump channel is positive and large.

On the *incomplete* market, the picture is different. By trading the consumption claim which represents jump risk only to a small degree the investors still speculate. However, due to the restricted set of trading instruments, it is impossible for the pessimist to obtain an exposure larger or equal to zero. The optimist will have a more negative exposure again so

[25] When w_t is close to zero, this behavior could be described as "gambling for resurrection."

that the speculative jump channel is still positive. The difference between the two investors is fairly small, however, and much less pronounced than on the complete market.

The "risk premium" channel is defined in Equation (3) as $\left(\sigma'_{V_1}\,\eta^W_{1,t} - L_{V_1}\,\lambda^{\mathbb{Q}}_{1,t}\right) - \left(\sigma'_{V_2}\,\eta^W_{2,t} - L_{V_2}\,\lambda^{\mathbb{Q}}_{2,t}\right)$. The first term in parentheses is the *actual* diffusive risk premium, while the second term has some similarities to the *actual* jump risk premium.[26] Figure 12 presents the break down of the "risk premium" channel into the parts due to diffusive shocks and jump risk.

[Insert Figure 12 around here]

On the *complete* market, we solve for the equilibrium using the fact that the investors have to agree on the different components of the pricing kernel. Since then $\eta^W_{1,t} = \eta^W_{2,t} = \eta^W_t$ and $\lambda^{\mathbb{Q}}_{1,t} = \lambda^{\mathbb{Q}}_{2,t} = \lambda^{\mathbb{Q}}_t$, the "risk premium" channel expression simplifies to $(\sigma_{V_1} - \sigma_{V_2})'\,\eta^W_t - (L_{V_1} - L_{V_2})\,\lambda^{\mathbb{Q}}_t$.[27] Hence, the differences between the two investors in the first two graphs in the top row of Figure 12 are caused entirely by their different wealth exposures, i.e., by the expressions in parentheses. Consequently, the pessimist earns the higher diffusive risk premium, so that this part of the "risk premium" channel is positive, but rather small in magnitude. For the jump part, on the other hand, the situation is reversed. Through selling the insurance product, the optimist is taking over a lot of jump exposure from the pessimist and earns a large "risk premium", while for the pessimist this "risk premium" is smaller and can become even negative when she starts speculating on jump risk. Therefore, the jump part of the channel is negative and quite sizeable, so that it essentially represents the "risk premium" channel.

When the market is *incomplete*, the investors still have to agree on the prices of all traded assets, but no longer on the different components of the pricing kernel. The market

[26] The *actual* (unlevered) risk premium is given by $\sigma'_{V_i}\,\eta^W_{i,t} - L_{V_i}\left(\lambda^{\mathbb{Q}}_{i,t} - \lambda\right)$.

[27] The market prices of risk $\eta^W_{i,t}$ and the risk-neutral jump intensities $\lambda^{\mathbb{Q}}_{i,t}$ for $i = 1, 2$ are shown in the top row of Figure 13.

prices of risk and the risk-neutral jump intensities from the equilibrium solution are presented in the bottom row of Figure 13. We see that the optimist's market price for diffusive shocks is higher than the pessimist's. Since the consumption claim reflects mostly diffusion risk, this explains why the optimist is willing to buy it from the pessimist, thus ending up with a higher exposure to this type of shocks. Consequently, the optimist earns the higher diffusive risk premium, implying that, in contrast to the complete market result from Figure 12, this part of the "risk premium" channel is negative.

[Insert Figure 13 around here]

Again, the situation is reversed for jump risk, which the pessimist considers to be riskier than the optimist. Hence, the pessimist's risk-neutral jump intensity is higher than the optimist's which is close to zero. This is the reason why the pessimist ends up with the higher "jump risk premium", although her individual wealth is exposed less to jump risk (see the bottom row of Figure 11). Therefore, the jump part of the channel is positive, but not very large. Since the investors trade mainly diffusive risk, it is the sizeable diffusive part, which drives the negative "risk premium".

4. Aggregate asset pricing moments

The expressions for the risk-free rate, the risk premium, and the return volatility are given in Equation (I.10) in the Online Appendix, and in Equations (5) and (6). Figure 14 shows these three quantities. When the market is incomplete, we find that the risk-free rate is lower, while the expected excess return and the return volatility are higher than in the case of market completeness. However, the differences are fairly small. Hence, in this model, it seems difficult to conclude that market incompleteness matters when it comes to aggregate asset pricing moments.

[Insert Figure 14 around here]

In terms of absolute magnitudes, the risk-free rate varies between 0.6% and 1.2%, and the risk premium between 3.9% and 5%. Both quantities are consistent with the data, while the return volatility of slightly more than 5% is too low. This is a commonly known problem for long-run risk models which can be mitigated by introducing stochastic volatility à la Bansal and Yaron (2004) or stochastic intensity as in Wachter (2013). However, since our focus is on the implications of market incompleteness, we require an otherwise parsimonious setting.[28]

5. Varying the true jump intensity

In this section, we relax the assumption that the true jump intensity coincides with the pessimist's belief. As highlighted in Section III.A, all equilibrium quantities are functions of two state variables, the pessimist's consumption share w_t and the expected growth rate X_t. Since X_t follows an exogenous jump diffusion process with mean-reversion, and variations in λ have a straightforward impact on X_t, we focus on the impact of λ on the endogenous state variable w_t. Varying the true λ does not affect the *local* dynamics of w_t presented in Figure 5, but it does affect its *evolution* over time. Again, we focus in the following on the impact of λ on investor survival.[29]

Table 6 shows the pessimist's expected consumption share $\mathbb{E}\left[w_T\right]$ for T years into the future for λ varying between 0.01 and 0.10, i.e., for the range for the individual beliefs spanned by Dieckmann (2011) and Chen et al. (2012).[30] For the *complete* market, we observe

[28]In the context of a long-run risk model without jumps, Pohl, Schmedders, and Wilms (2018) use the insights from Borovicka (2018) and modify the calibration of Bansal and Yaron (2004) in a way which allows them not only to generate a large equity premium, but also to explain further asset pricing puzzles.

[29]The corresponding asset pricing moments are shown in Online Appendix VI.

[30]For each λ, we follow the procedure exemplified for $\lambda = 0.02$ in Online Appendix V. First, we simulate X_t to obtain the quantiles for the long-run growth rate. Then, we solve the model numerically over the resulting interval for X_t.

the expected result according to Borovicka (2018). By adjusting the true belief, jumps happen more or less frequently, and this determines how often the pessimist receives a positive payoff from the corresponding insurance product. When jumps under the true measure happen less often than considered so far, i.e., when $\lambda < 0.02$, the pessimist's consumption share decreases even faster over time. On the other hand, if jumps take place more often, i.e., $\lambda > 0.02$, the pessimist benefits from this and her consumption share tends to decrease more slowly. For $\lambda \geq 0.05$, this positive impact becomes even stronger, so that her consumption share increases over time and, after 1,000 years, becomes larger than 0.5. Therefore, adjusting the true jump intensity determines which of the two investors or whether both survive. The picture on the *incomplete* market is strikingly different. Adjusting the true belief has an almost negligible impact on the speed of extinction and the pessimist dies out after 500 years, irrespective of whether the true intensity is 0.01 or 0.10.[31]

[Insert Table 6 around here]

To understand the results for the two market structures, it is helpful to go back to the interpretation of Equation (3). Changing the true λ does not have an impact on the savings channel, the speculative volatility channel, or the "risk premium" channel, but it of course affects the speculative jump channel. Hence, as can be seen from Equation (3), increasing or decreasing λ corresponds to an upward or downward shift of the line representing the speculative jump channel in Figure 7.

When the market is *complete*, the presence of the insurance products allows the investors to speculate on jump risk. Consequently, their jump exposures will be quite different. Scaling this *large* difference in logs with varying λ's results in a *sizeable* shift for the speculative jump channel which then translates into a significant shift of the line representing the difference in expected log wealth growth rates on the right in Figure 7. So if we choose a "large enough" true jump intensity, the speculative jump channel will dominate the other

[31] If the true jump intensity is close to $\lambda = 1.00$, the optimist will die out.

three channels. In this case, the optimist still saves more and earns the higher "risk premium", but there are "so many" jumps leading to insurance payoffs from the optimist to the pessimist, that the pessimist ends up taking over the economy.

On the *incomplete* market, the investors can only rely on the consumption claim which is much less exposed to jump risk than the corresponding insurance product. Consequently, the differences between the investors are much smaller than on the complete market or even close to zero. Multiplying this *very small* difference in logs by varying values for λ will hardly affect the speculative jump channel. Due to the restricted set of traded assets, the investors' jump exposures are just "too similar" for the pessimist to benefit significantly from jumps. Hence, in these scenarios in Table 6, the pessimist is still driven out of the market by the optimist, because the optimist saves more and earns the higher "risk premium."

IV. Conclusion

We study under what circumstances market incompleteness matters for two investors with recursive preferences who disagree about the likelihood of jumps. We tackle this research question using two models. With jumps in aggregate consumption, market incompleteness barely matters, irrespective of whether we look at speculation, investor survival, or aggregate asset pricing quantities like the risk-free rate, the risk premium, or the return volatility. In a long-run risk model with jumps in the long-run growth rate, market incompleteness matters with respect to speculation, and investor survival, but not regarding asset pricing moments.

To get the intuition behind our results, suppose, on the complete market, the investors use a jump insurance product to speculate on the source of risk they disagree on. When the market is incomplete, the insurance product is no longer available and the investors can only trade the consumption claim and the money market account. Whether this spanning incompleteness poses a material problem for the investors or not, depends on the similarity between the exposures of the unavailable insurance product and the consumption claim. (i)

When the two asset are *very similar* in terms of their exposures, the investors will not care too much whether they can trade one or the other since both assets allow them to take almost the same positions. Loosely speaking, in this case, spanning incompleteness becomes a *non-binding* restriction and would thus have barely an impact. (ii) When the two assets are *very different*, in the sense that the insurance product loads exclusively on jump risk and the consumption claim loads almost equally on jump risk and diffusion risk, the investors cannot trade the two sources of risk separately, but only jointly in the relative composition represented by the consumption claim. This makes it impossible for them to take basically the same positions as on the complete market, i.e., incompleteness represents a *binding* restriction and, hence, it matters.

The model with jumps in aggregate consumption in Section II turns out to be rather similar to case (i), and thus market incompleteness barely matters. The long-run risk model with jumps in the long-run growth rate in Section III is an example for case (ii) in which incompleteness matters. Overall, our results highlight that the relative importance of jump and diffusive risks is a key determinant regarding the differences between complete and incomplete markets. In line with Blume and Easley (2006), we show that the market selection hypothesis *may indeed fail*, if the market is incomplete, although our framework is different from theirs.

The question whether market incompleteness is relevant or not, is discussed, e.g., by Lustig and Verdelhan (2018) in an international context. Based on a preference based general equilibrium analysis, we show under what circumstances incompleteness may or may not matter for speculation, investor survival, and asset pricing moments.

Appendix

A. The solution method

Solving asset pricing models with recursive utility investors is challenging, since the wealth-consumption ratio affects the pricing kernel and thus prices, and vice versa. Combining recursive utility with heterogeneous investors makes the problem even more difficult, since the investors' consumption share, i.e., how much each investor consumes relative to aggregate consumption, represents an *endogenous* state variable.

In the literature, different approaches have been suggested to solve this problem. For instance, Eraker and Shaliastovich (2008) combine a log-linear approximation à la Campbell and Shiller (1988) with the guess that the log wealth-consumption ratio of the representative investor is an affine function of the state variables. However, we want to treat the log wealth-consumption ratios of the heterogeneous investors as a general, *not necessarily affine* function of the consumption share. We determine the investors' individual (log) wealth-consumption ratios, v_1 and v_2, numerically as solutions to two partial different equations (PDEs).[32]

We specify all equilibrium quantities as functions of investor 1's share of aggregate consumption, $w_t \equiv \frac{C_{1,t}}{C_t}$, and assume its dynamics are given as

$$dw_t \quad = \quad \mu_w (w_t)\; dt + \sigma_w (w_t)'\; dW_t + L_w (w_t)\; dN_t (\lambda)\,,$$

where W_t represents an n-dimensional Brownian motion, and N_t is a non-compensated Poisson process with constant intensity λ. The coefficient functions $\mu_w \equiv \mu_w (w_t)$, $\sigma_w \equiv \sigma_w (w_t)$, and $L_w \equiv L_w (w_t)$ will be determined in equilibrium.

When the market is *complete*, the investors agree on the pricing kernel and its different

[32] Similar approaches have been used, e.g., by Benzoni et al. (2011) to solve a model with a representative investor, long-run risk, stochastic volatility, and learning. The results in Online Appendix VI confirm that the log wealth-consumption ratios of the investors are *non-affine* functions of the consumption share.

components, i.e., on the risk-free rate $r^f_{1,t} = r^f_{2,t}$, the market prices of diffusion risk $\eta^W_{1,t} = \eta^W_{2,t}$, and on the risk-neutral jump intensity $\lambda^{\mathbb{Q}}_{1,t} = \lambda^{\mathbb{Q}}_{2,t}$, which is determined through $\lambda^{\mathbb{Q}}_{i,t} = \lambda_i \left(1 + \eta^N_{i,t}\right)$ from the market price of jump risk $\eta^N_{i,t}$. We use these restrictions to solve for: (i) μ_w which follows from $r^f_{1,t} = r^f_{2,t}$, (ii) σ_w which is obtained from $\eta^W_{1,t} = \eta^W_{2,t}$, and (iii) L_w which is found using $\lambda^{\mathbb{Q}}_{1,t} = \lambda^{\mathbb{Q}}_{2,t}$. Technically, this amounts to a system of two PDEs, one for each investor's log wealth-consumption ratio, $v_{1,t}$ and $v_{2,t}$, where the solutions are connected through μ_w, σ_w, and L_w, i.e., each investor decides on how much to save and to consume so that markets clear.

When the market is *incomplete*, the investors decide on their individual log wealth-consumption ratios, $v_{1,t}$ and $v_{2,t}$, as before. However, determining μ_w, σ_w, and L_w using the different components of the pricing kernel no longer works, since the investors do not have to agree on one pricing kernel. Nevertheless, the investors still have to agree on the prices of all traded assets, i.e., on the risk-free rate, $r^f_{1,t} = r^f_{2,t} = r^f_t$, and on the log price-cash flow ratio of the consumption claim, $\nu_{1,t} = \nu_{2,t} = \nu_t$, which is calculated as the solution of a PDE for each investor. Furthermore, on the incomplete market, the investors can only take positions which are obtainable via holdings in a smaller set of traded assets.[33] This implies that the investors' individual wealth and the return on their portfolios have to react identically to diffusive risks and jumps, respectively. The resulting restrictions together with the ones from the traded assets mentioned above give us μ_w, σ_w, L_w, and also investor 1's portfolio weight for the claim on aggregate consumption, $\pi_{1,C,t}$. This leaves us with seven equations for seven unknowns ($v_{1,t}$, $v_{2,t}$, ν_t, μ_w, σ_w, L_w, and $\pi_{1,C,t}$), and these equations have to be solved numerically.

[33]In contrast, market completeness implies that both investors can take individual wealth exposures with respect to the different risk factors without any limitations.

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Table 1. Parameters for model (1)

The table reports the calibration for the model with jumps in aggregate consumption presented in Section II. We assume that the pessimist's belief λ_1 represents the true measure λ, except in Section II.B.5 in which we vary the true jump intensity.

Relative risk aversion	γ	4
Elasticity of intertemporal substitution	ψ	1.5
Subjective discount rate	β	0.03
Expected growth rate of aggregate consumption (excluding jump compensation)	μ_C	0.025
Volatility of aggregate consumption	σ_C	0.02
Jump size in aggregate consumption	L_C	-0.40
Leverage factor	ϕ	1.0
Jump intensity of the pessimistic investor 1	λ_1	0.017
Jump intensity of the optimistic investor 2	λ_2	0.001
Leverage factor	ϕ	1.00
Drift of insurance product I	μ_I	-0.10
Jump size of insurance product I	L_I	0.40

Table 2. Investor survival in model (1)

The table shows for the model with jumps in aggregate consumption presented in Section II, the pessimist's expected consumption share $\mathbb{E}[w_T]$ for T years into the future under the true measure $\lambda = 0.017$. The second (third) column gives the results on the complete (incomplete) market. The pessimist's expected consumption share is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (1) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 1.

T (years)	Complete market	Incomplete market
50	0.3866	0.3767
100	0.3265	0.3096
200	0.2694	0.2402
500	0.2283	0.1759
1,000	0.2108	0.1513
10,000	0.2128	0.1418

Table 3. Impact of varying the true measure on investor survival in model (1)

The table shows for the model with jumps in aggregate consumption presented in Section II, the pessimist's expected consumption share $\mathbb{E}[w_T]$ for T years into the future under the true measure λ which we vary in each column. The top (bottom) panel gives the results on the complete (incomplete) market. The pessimist's expected consumption share is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (1) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 1.

T	λ									
(years)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Complete market										
50	0.3239	0.4095	0.4964	0.5732	0.6475	0.7099	0.7640	0.8106	0.8457	0.8796
100	0.2399	0.3634	0.4932	0.6114	0.7135	0.7914	0.8527	0.9006	0.9327	0.9555
200	0.1671	0.3187	0.4866	0.6459	0.7623	0.8586	0.9147	0.9536	0.9758	0.9885
500	0.1008	0.2853	0.4801	0.6567	0.7958	0.8916	0.9514	0.9816	0.9945	0.9988
1,000	0.0771	0.2784	0.4851	0.6549	0.7923	0.8976	0.9576	0.9881	0.9976	0.9996
Incomplete market										
50	0.3189	0.3992	0.4823	0.5572	0.6268	0.6949	0.7451	0.7991	0.8334	0.8702
100	0.2303	0.3441	0.4692	0.5864	0.6850	0.7759	0.8376	0.8908	0.9234	0.9501
200	0.1474	0.2867	0.4504	0.6140	0.7378	0.8409	0.9050	0.9472	0.9740	0.9565
500	0.0714	0.2339	0.4333	0.6231	0.7703	0.8737	0.9457	0.9783	0.9934	0.9984
1,000	0.0376	0.2167	0.4367	0.6209	0.7749	0.8798	0.9516	0.9853	0.9968	0.9995

Table 4. Parameters for model (2)

The table reports the calibration for the long-run risk model with jumps in the long-run growth rate presented in Section III. We assume that the pessimist's belief λ_1 represents the true measure λ, except in Section III.B.5 in which we vary the true jump intensity.

Relative risk aversion	γ	10
Elasticity intertemporal of substitution	ψ	1.5
Subjective discount rate	β	0.02
Expected growth rate of aggregate consumption	$\bar{\mu}_C$	0.02
Volatility of aggregate consumption	σ_C	0.0252
Mean reversion speed of the long-run growth rate	κ_X	0.1
Volatility of the long-run growth rate	σ_x	0.0114
Jump size of the long-run growth rate	L_X	-0.03
Jump intensity of the pessimistic investor 1	λ_1	0.020
Jump intensity of the optimistic investor 2	λ_2	0.001
Leverage factor	ϕ	1.3
Drift of insurance product Z	μ_Z	-0.10
Volatility of insurance product Z	σ_Z	0.001
Drift of insurance product I	μ_I	-0.10
Jump size of insurance product I	L_I	0.01

T (years)	Complete market	Incomplete market
50	0.4185	0.2598
100	0.3669	0.1188
200	0.3075	0.0216
500	0.2431	0.0001
1,000	0.2176	0.0000
10,000	0.2089	0.0000

Table 5. Investor survival in model (2)

The table shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the pessimist's expected consumption share $\mathbb{E}[w_T]$ for T years into the future under the true measure $\lambda = 0.02$. The second (third) column gives the results on the complete (incomplete) market. The pessimist's expected consumption share is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (7) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 4.

Table 6. Impact of varying the true measure on investor survival in model (2)

The table shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the pessimist's expected consumption share $\mathbb{E}[w_T]$ for T years into the future under the true measure λ which we vary in each column. The top (bottom) panel gives the results on the complete (incomplete) market. The pessimist's expected consumption share is computed via a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (7) with a starting value of $w_0 = 0.5$. The coefficients μ_w, σ_w, and L_w are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 4.

T	λ									
(years)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Complete market										
50	0.3818	0.4185	0.4573	0.4911	0.5294	0.5655	0.5984	0.6330	0.6662	0.6922
100	0.3091	0.3669	0.4271	0.4862	0.5429	0.5979	0.6510	0.7013	0.7433	0.7806
200	0.2261	0.3075	0.3943	0.4770	0.5575	0.6270	0.6982	0.7517	0.8030	0.8420
500	0.1287	0.2431	0.3614	0.4697	0.5647	0.6460	0.7168	0.7772	0.8282	0.8692
1,000	0.0790	0.2176	0.3534	0.4709	0.5649	0.6474	0.7148	0.7774	0.8276	0.8709
Incomplete market										
50	0.2555	0.2598	0.2637	0.2672	0.2711	0.2747	0.2784	0.2826	0.2869	0.2909
100	0.1145	0.1188	0.1228	0.1269	0.1306	0.1352	0.1397	0.1444	0.1491	0.1541
200	0.0200	0.0216	0.0232	0.0250	0.0268	0.0287	0.0310	0.0332	0.0355	0.0383
500	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004
1,000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Figure 1. Consumption share dynamics in model (1)

The figure depicts for the model with jumps in aggregate consumption presented in Section II, the coefficients in the dynamics of the pessimist's consumption share as defined in Equation (1). From left to right, the graphs show the drift (μ_w), and the coefficients for diffusive consumption shocks (σ_w) and jumps in aggregate consumption (L_w), respectively. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.017$ and shown as functions of the pessimist's consumption share w_t. The parameters are given in Table 1.

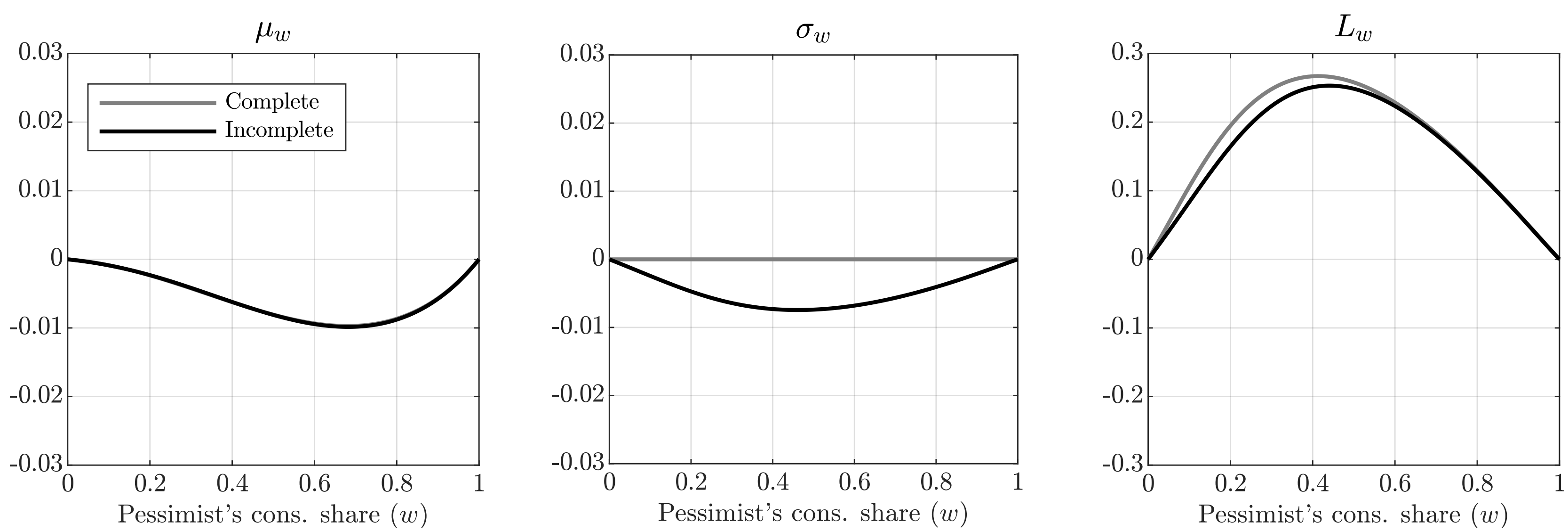

Figure 2. Portfolio weights in model (1)

The figure shows for the model with jumps in aggregate consumption presented in Section II, the investors' portfolio weights on the complete (top row) and on the incomplete market (bottom row), respectively. From left to right, the graphs show the fraction of wealth invested in the consumption claim ($\pi_{i,C}$), the jump insurance product I ($\pi_{i,I}$), and the money market account ($\pi_{i,M}$). The pessimist's (optimist's) portfolio weights are indicated by the dotted (dashed) line. All quantities are determined under the true measure $\lambda = 0.017$ and shown as functions of the pessimist's consumption share w_t. The parameters are given in Table 1.

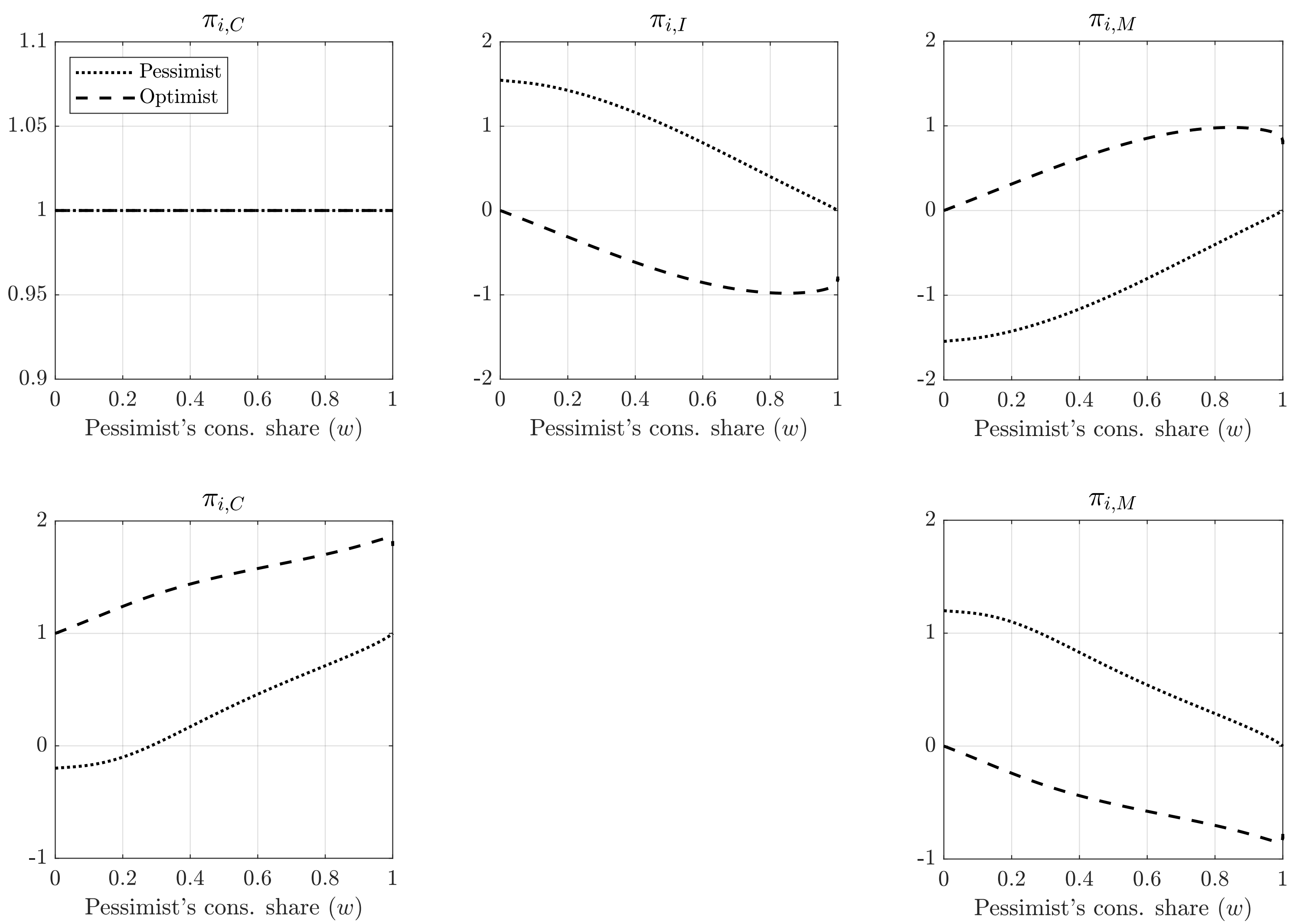

Figure 3. Borovicka (2018) decomposition in model (1)

The figure depicts for the model with jumps in aggregate consumption presented in Section II, (from left to right) the following channels we defined in Equation (3): (1) the savings channel, $e^{-v_{2,t}} - e^{-v_{1,t}}$; (2) the speculative volatility channel, $\frac{1}{2}\,\sigma_{V_2}'\,\sigma_{V_2} - \frac{1}{2}\,\sigma_{V_1}'\,\sigma_{V_1}$; (3) the speculative jump channel, $\log\left(1 + L_{V_1}\right)\,\lambda - \log\left(1 + L_{V_2}\right)\,\lambda$; (4) the "risk premium" channel, $\left(\sigma_{V_1}\,\eta_1^W - L_{V_1}\,\lambda_1^{\mathbb{Q}}\right) - \left(\sigma_{V_2}\,\eta_2^W - L_{V_2}\,\lambda_2^{\mathbb{Q}}\right)$. The plot on the furthest to the right labeled with "Total" shows the sum over these four channels, i.e., the difference in the investors' expected log wealth growth rates. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.017$ and shown as functions of the pessimist's consumption share w_t. The parameters are given in Table 1.

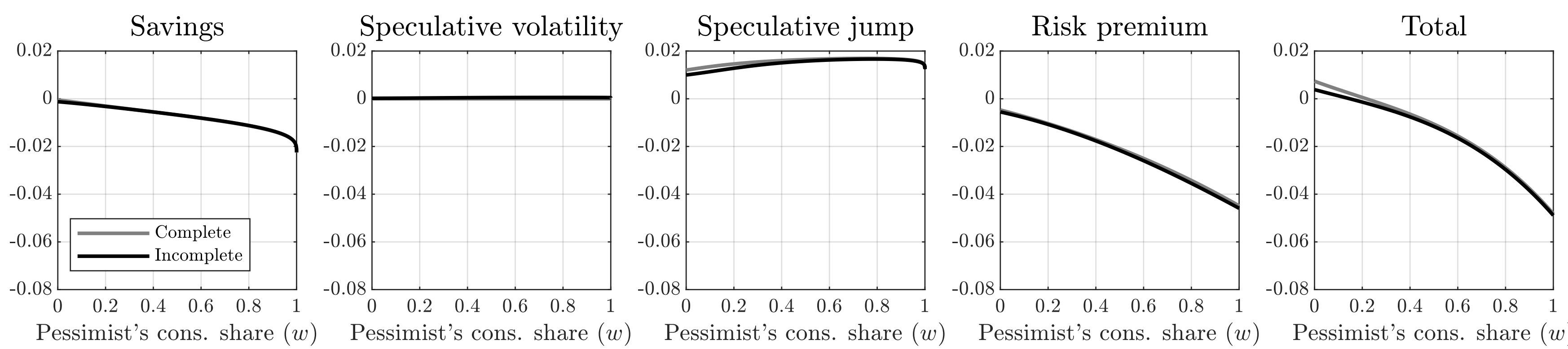

Figure 4. Risk-free rate, risk premium, and return volatility in model (1)

The figure shows for the model with jumps in aggregate consumption presented in Section II, (from left to right) the risk-free rate, the risk premium, and the return volatility. The gray (black) line represents the results on the complete (incomplete) market. The risk-free rate is given in Equation (I.10) in the Online Appendix, the risk premium in (5), and the return volatility in (6). All quantities are determined under the true measure $\lambda = 0.017$ and shown as functions of the pessimist's consumption share w_t. The parameters are given in Table 1.

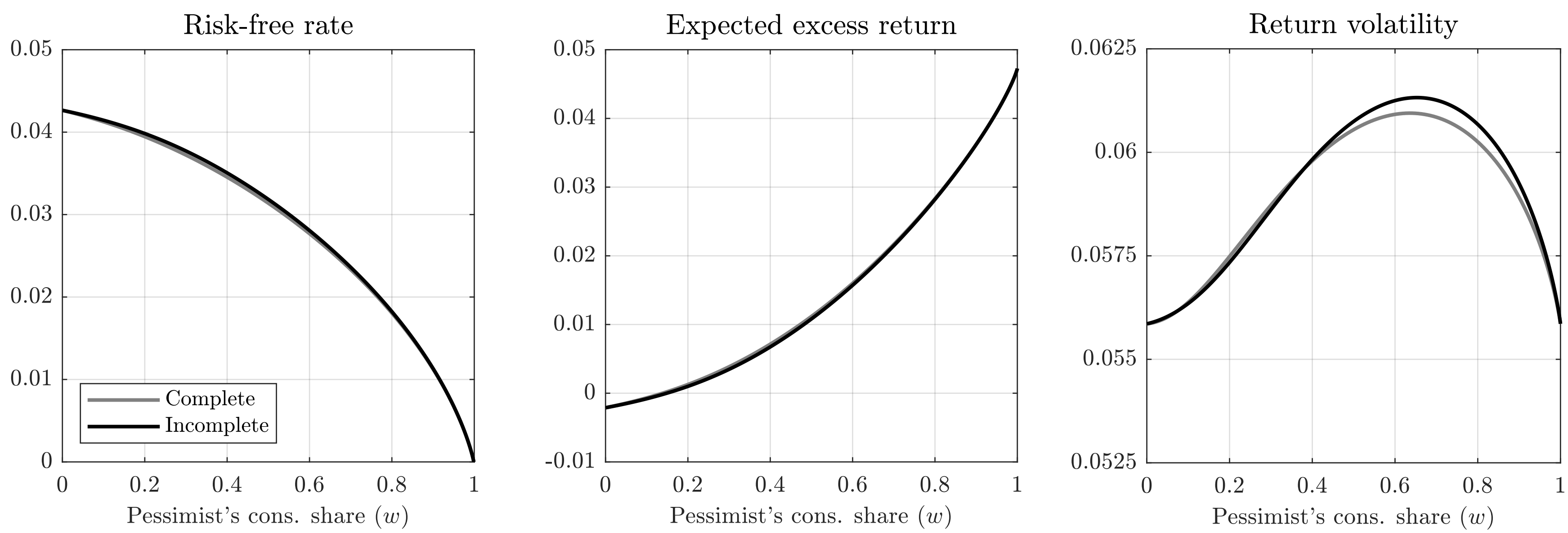

Figure 5. Consumption share dynamics in model (2)

The figure depicts for the long-run risk model with jumps in the long-run growth rate presented in Section III, the coefficients in the dynamics of the pessimist's consumption share as defined in Equation (7). The gray (black) line represents the results on the complete (incomplete) market. From left to right the graphs show the drift (μ_w), and the coefficients for diffusive consumption shocks ($\sigma_{w,C}$), diffusive expected growth rate shocks ($\sigma_{w,X}$), and jumps in the expected growth rate (L_w), respectively. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

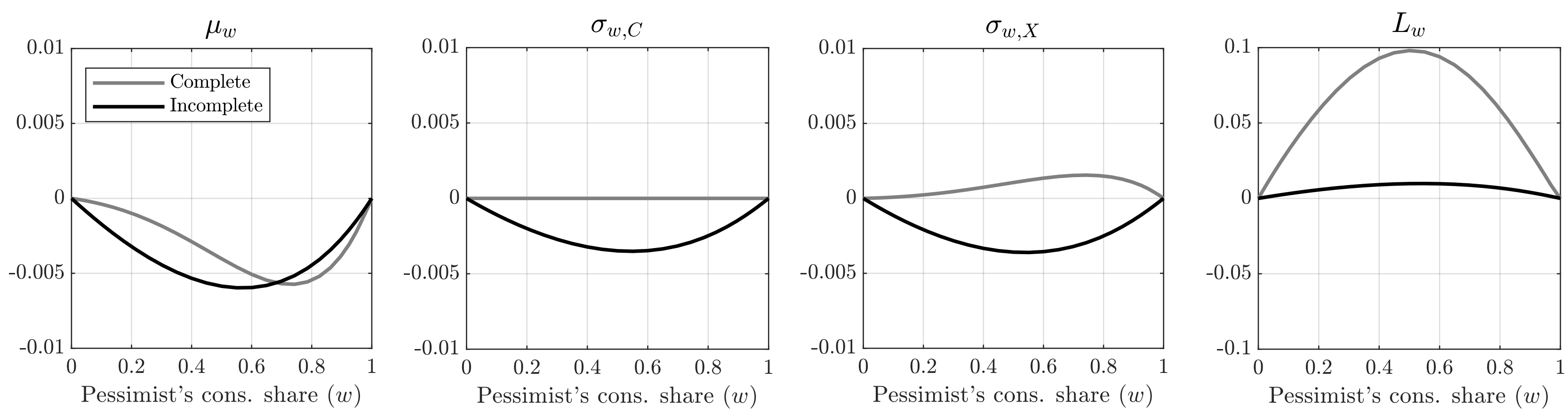

Figure 6. Portfolio weights in model (2)

The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the investors' portfolio weights on the complete (top row) and on the incomplete market (bottom row), respectively. From left to right, the graphs show the fraction of wealth invested in the consumption claim ($\pi_{i,C}$), the diffusion insurance product Z ($\pi_{i,Z}$), the jump insurance product I ($\pi_{i,I}$), and the money market account ($\pi_{i,M}$). The pessimist's (optimist's) portfolio weights are indicated by the dotted (dashed) line. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the stochastic part of the expected growth rate of consumption fixed at $X_t = -0.0060$. The parameters are given in Table 4.

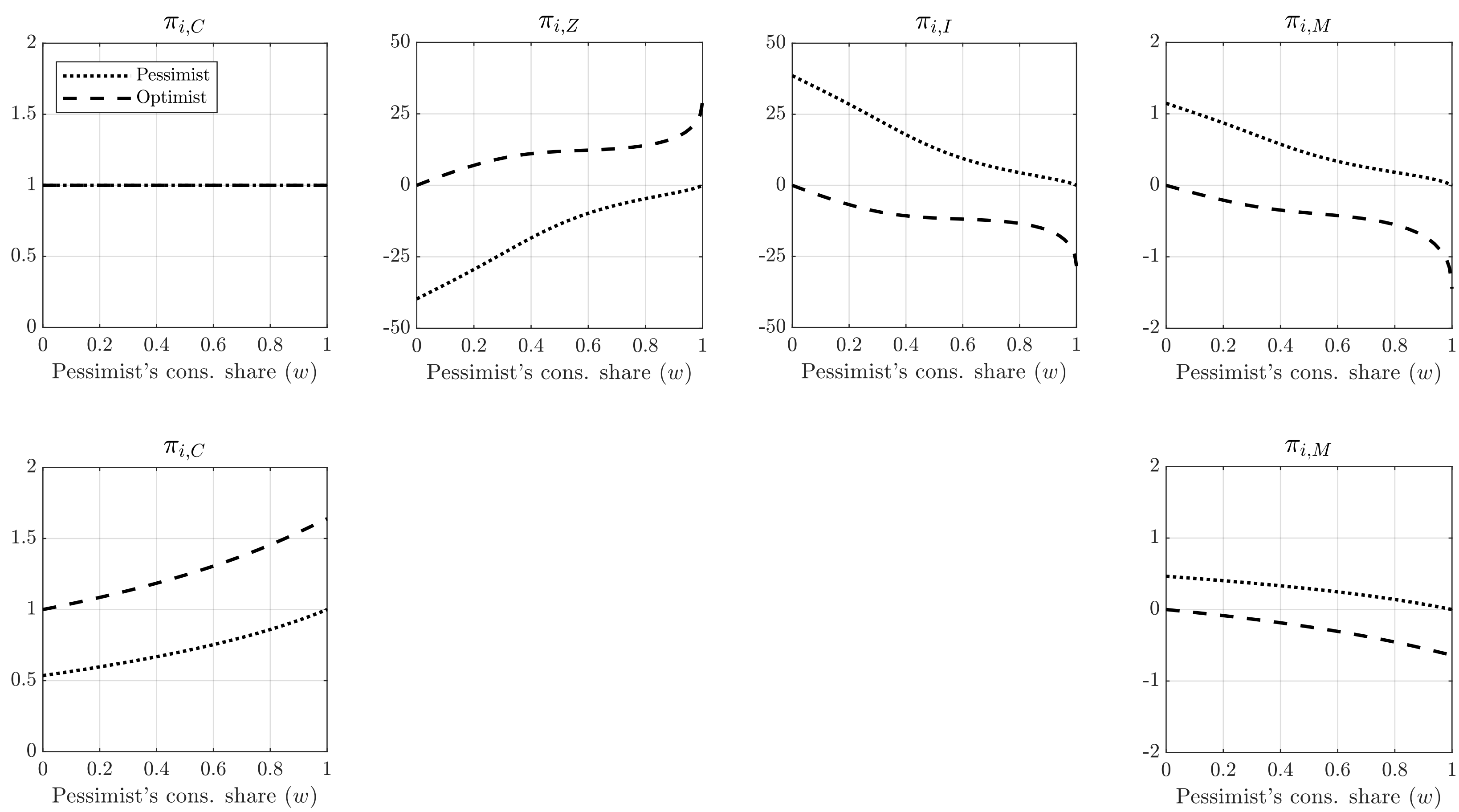

Figure 7. Borovicka (2018) decomposition in model (2)

The figure depicts for the long-run risk model with jumps in the long-run growth rate presented in Section III, the following channels (from left to right) we defined in Equation (3): (1) the savings channel, $e^{-v_{2,t}} - e^{-v_{1,t}}$; (2) the speculative volatility channel, $\frac{1}{2}\,\sigma_{V_2}'\,\sigma_{V_2} - \frac{1}{2}\,\sigma_{V_1}'\,\sigma_{V_1}$; (3) the speculative jump channel, $\log(1+L_{V_1})\,\lambda - \log(1+L_{V_2})\,\lambda$; (4) the "risk premium" channel, $\left(\sigma_{V_1}\,\eta_1^W - L_{V_1}\,\lambda_1^{\mathbb{Q}}\right) - \left(\sigma_{V_2}\,\eta_2^W - L_{V_2}\,\lambda_2^{\mathbb{Q}}\right)$. The plot on the furthest to the right labeled with "Total" shows the sum over these four channels, i.e., the difference in the investors' expected log wealth growth rates. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

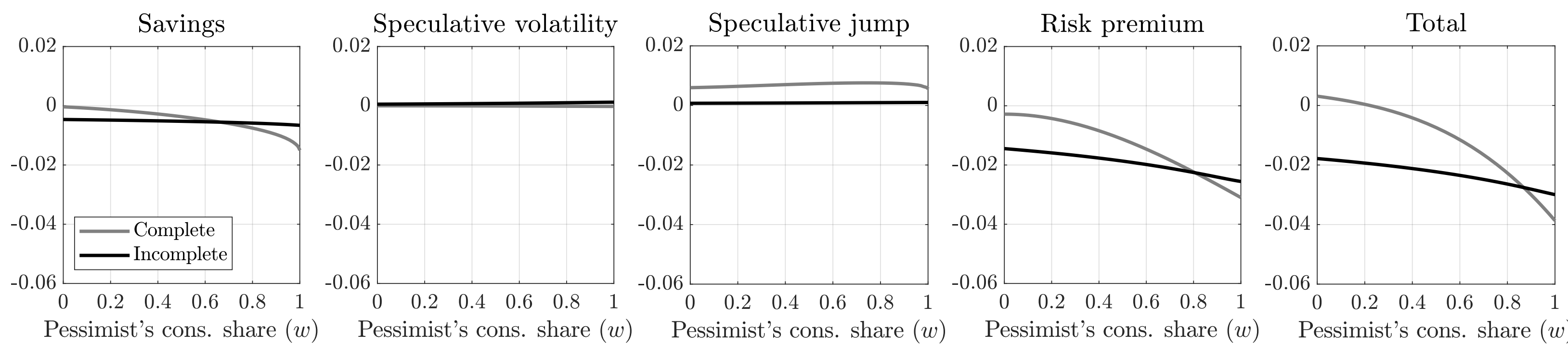

Figure 8. Savings channel and consumption-wealth ratios in model (2)

The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the individual wealth-consumption ratios, $e^{-v_{i,t}}$ for $i = 1, 2$, and the resulting savings channel, $e^{-v_{2,t}} - e^{-v_{1,t}}$, as defined in Equation (3). The dotted (dashed) line shows the pessimist's (optimist's) individual consumption-wealth ratio, whereas the solid line indicates the savings channel. The graphs on the left (right) show the results for the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

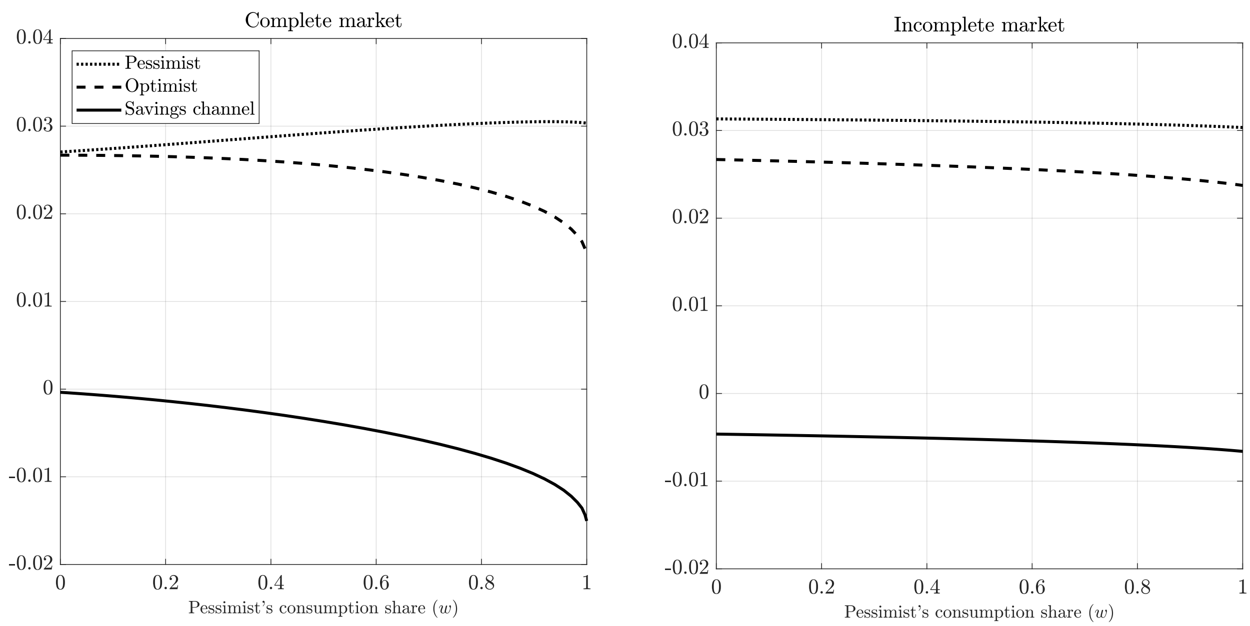

Figure 9. Expected growth rate of individual consumption in model (2)

The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, both investors' expected growth rate of individual consumption, i.e., $\frac{1}{dt}\,\mathbb{E}_{1,t}\left[\frac{dC_{i,t}}{C_{i,t}}\right] = \mu_{C_i} + \lambda\, L_{C_i}$. The dotted (dashed) line represents the pessimist's (optimist's) expected consumption growth. The graphs on the left (right) show the results for the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

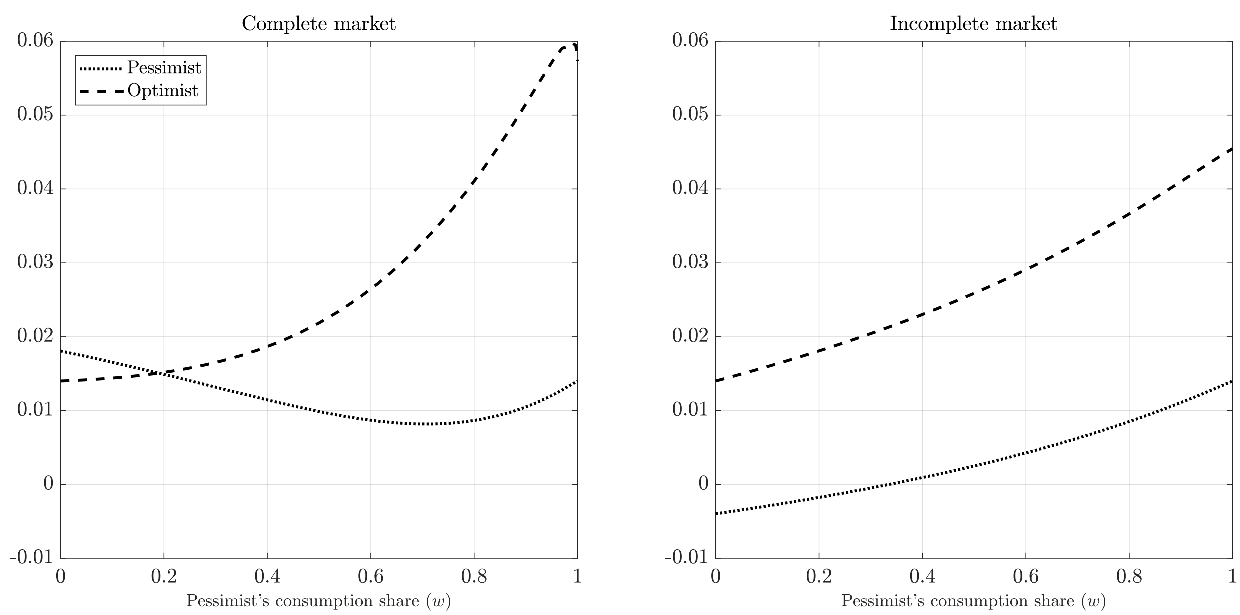

Figure 10. Speculative volatility channel and exposures of individual wealth to diffusive risks in model (2)

The figure depicts for the long-run risk model with jumps in the long-run growth rate presented in Section III, (from left to right) each investor's exposure of individual wealth to diffusive consumption shocks ($\sigma_{V_i,C}$) and diffusive expected growth rate shocks ($\sigma_{V_i,X}$ for $i = 1, 2$) and the resulting speculative volatility channel, $\frac{1}{2}\,\sigma_{V_2}'\,\sigma_{V_2} - \frac{1}{2}\,\sigma_{V_1}'\,\sigma_{V_1}$ as defined in Equation (3). The dotted (dashed) line represents the results for the pessimist (optimist), whereas the solid line indicates the speculative volatility channel. The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

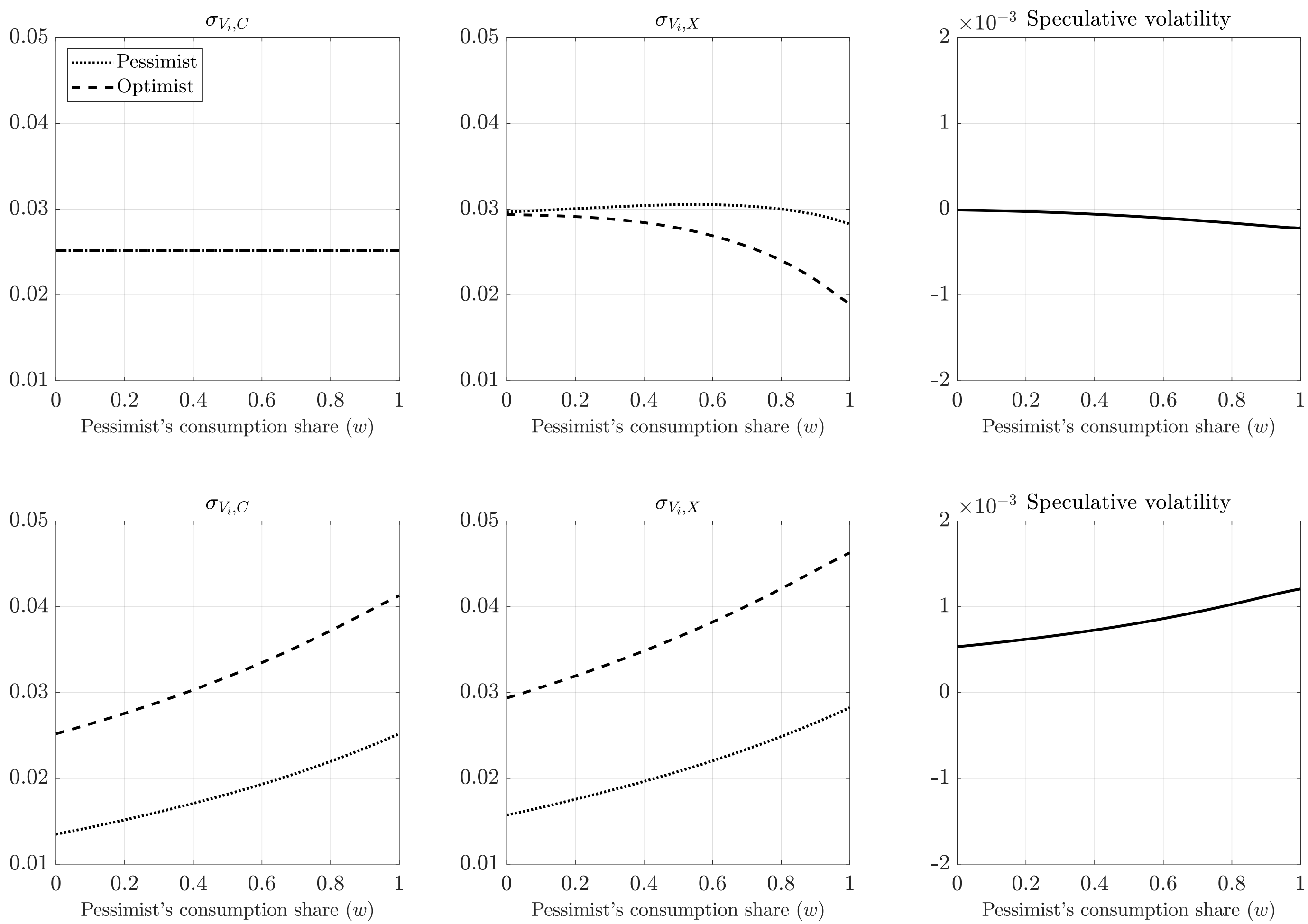

Figure 11. Speculative jump channel and exposures of individual wealth to jump risk in model (2)

The figure depicts for the long-run risk model with jumps in the long-run growth rate presented in Section III, (from left to right) each investor's exposure of individual wealth to jumps in the long-run growth rate (L_{V_i} for $i = 1, 2$), and the resulting speculative jump channel, $\log(1 + L_{V_1})\,\lambda - \log(1 + L_{V_2})\,\lambda$, as defined in Equation (3). The dotted (dashed) line represents results for the pessimist (optimist), whereas the solid line gives those for the speculative jump channel. The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

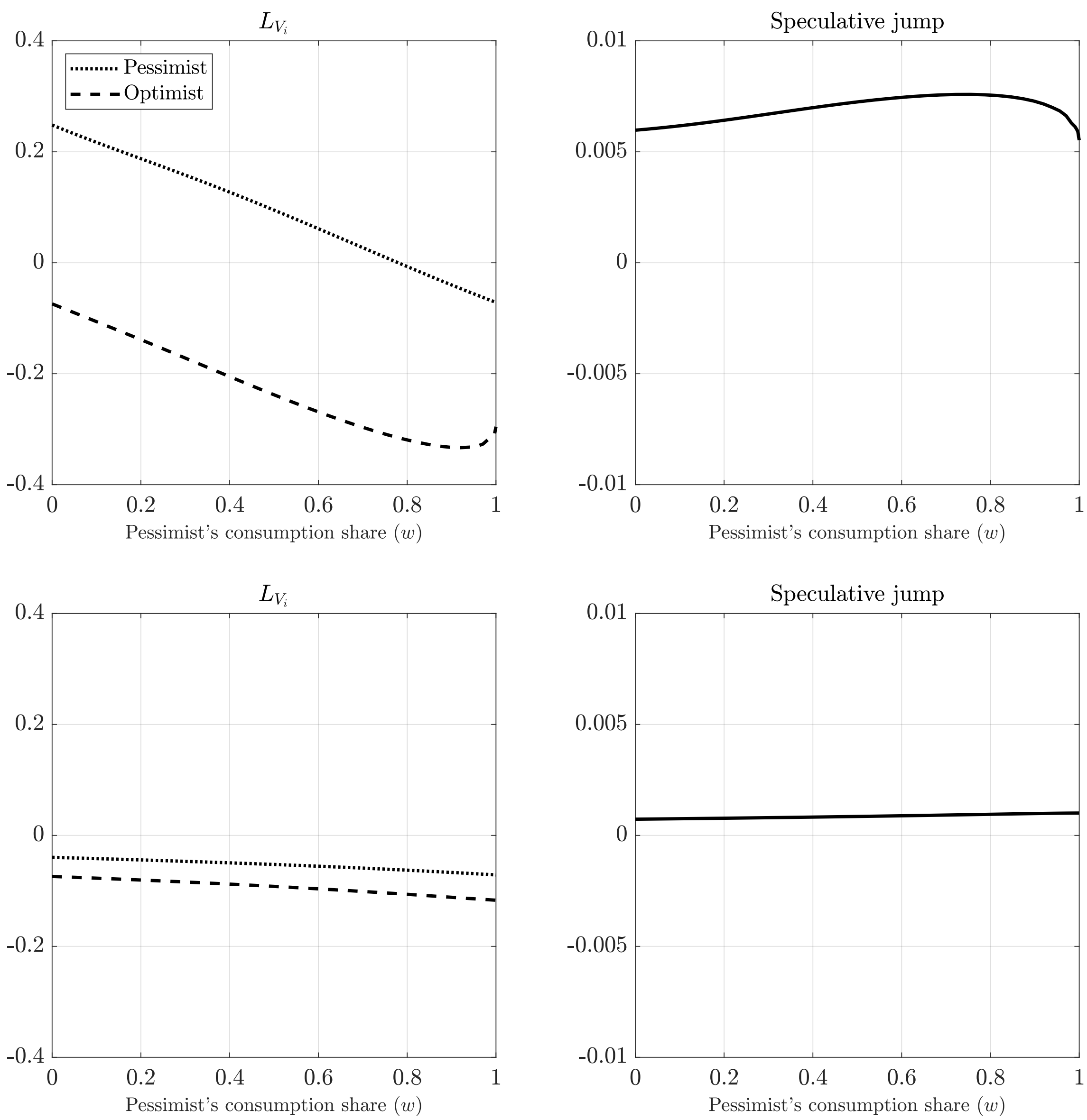

Figure 12. "Risk premium" channel in model (2)

The figure depicts for the long-run risk model with jumps in the long-run growth rate presented in Section III, (from left to right) the part of the "risk premium" channel that is due to diffusive shocks, $\sigma_{V_1}\,\eta_1^W - \sigma_{V_2}\,\eta_2^W$, due to jump risk, $\left(-L_{V_1}\,\lambda_1^{\mathbb{Q}}\right) - \left(-L_{V_2}\,\lambda_2^{\mathbb{Q}}\right)$, and the sum over both parts, $\left(\sigma_{V_1}\,\eta_1^W - L_{V_1}\,\lambda_1^{\mathbb{Q}}\right) - \left(\sigma_{V_2}\,\eta_2^W - L_{V_2}\,\lambda_2^{\mathbb{Q}}\right)$, as defined in Equation (3). The dotted (dashed) line represents the results for the pessimist (optimist), whereas the solid line gives those for the difference between the pessimist and the optimist. The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

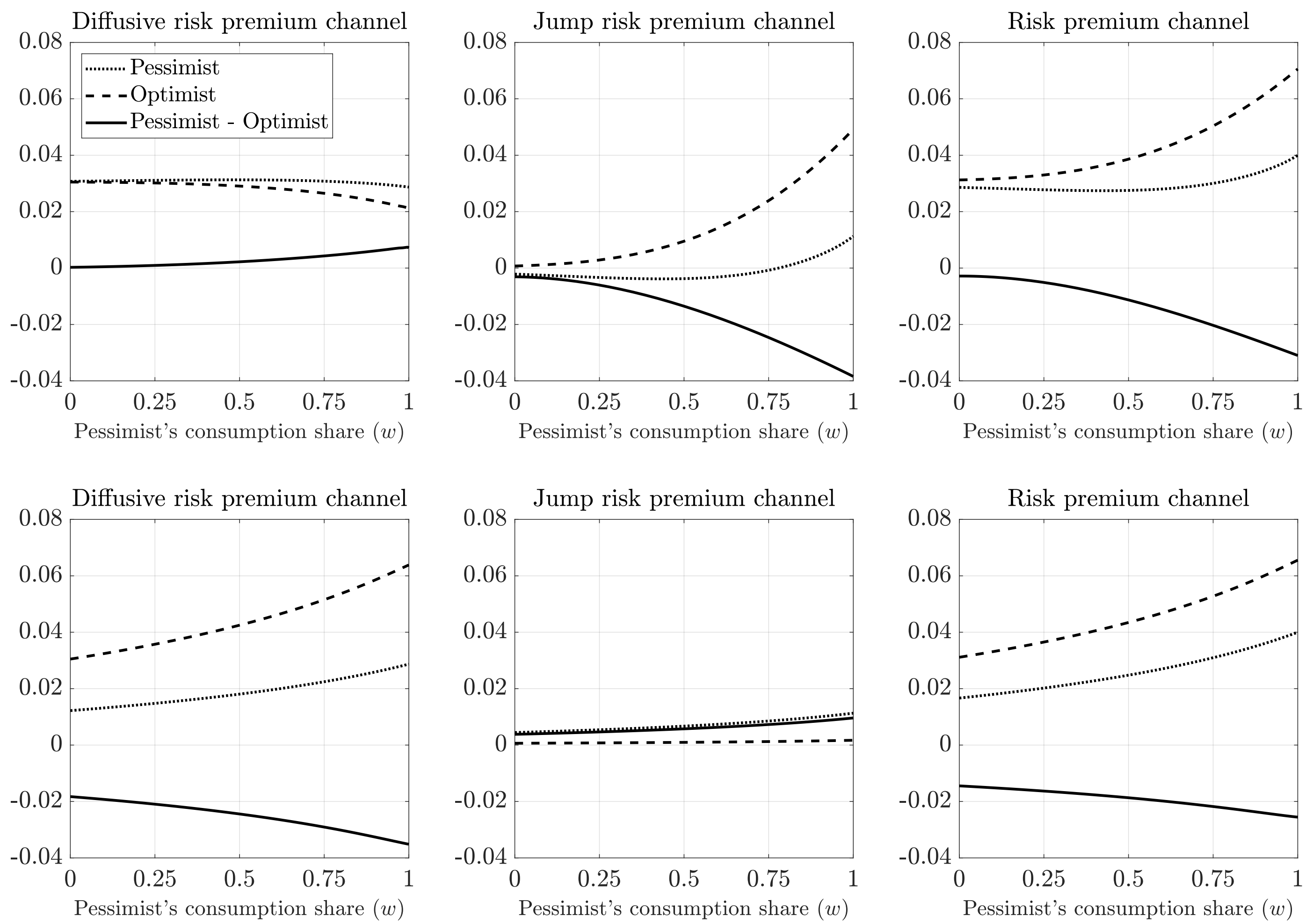

Figure 13. Market prices of risk and risk-neutral jump intensities in model (2)

The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, (from left to right) the market prices of risk for diffusive consumption and diffusive growth rate risk, and risk-neutral jump intensities. The dotted (dashed) line represents the results for the pessimist (optimist). The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.

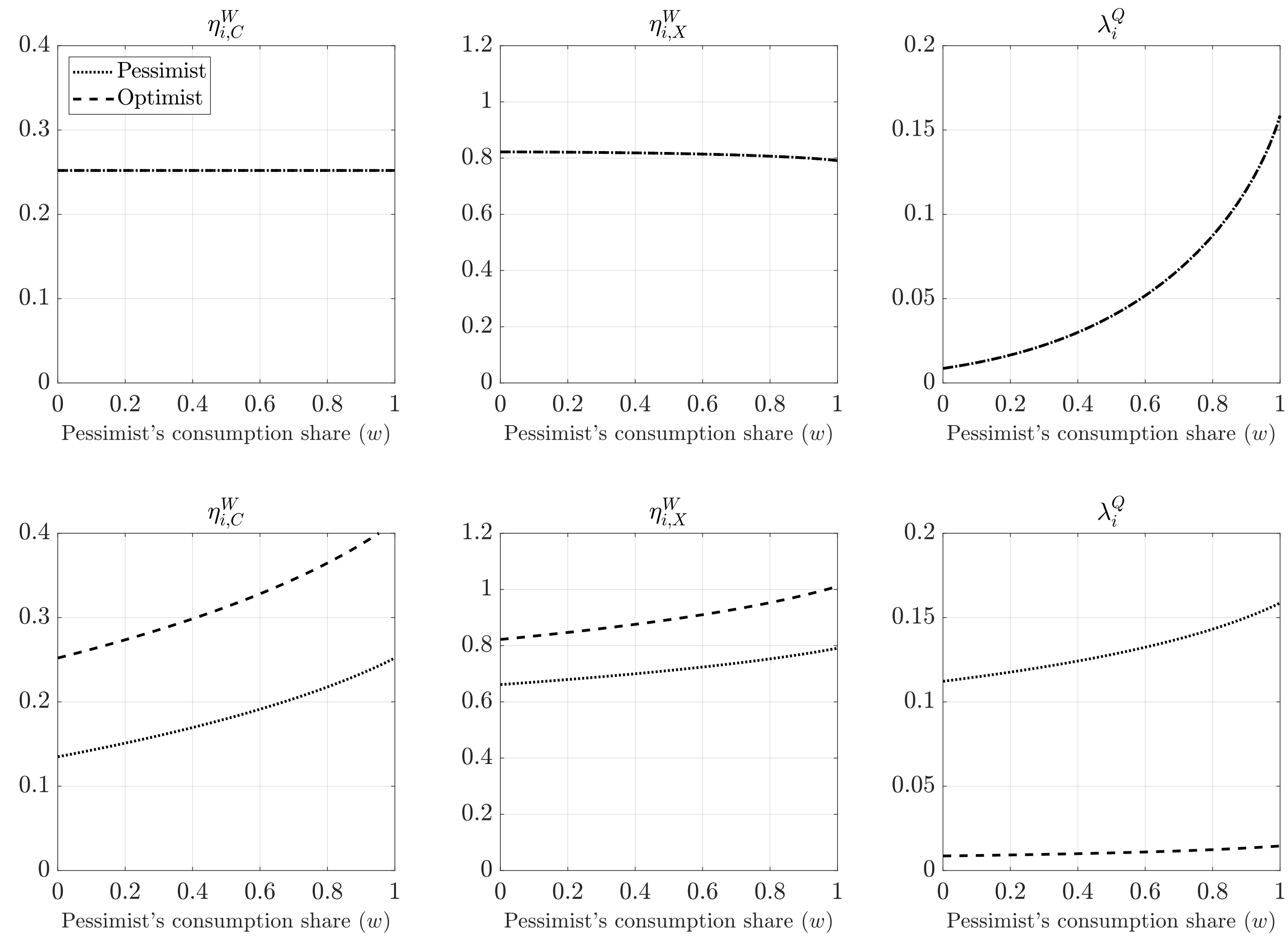

Figure 14. Risk-free rate, risk premium, and return volatility in model (2)

The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, (from left to right) the risk-free rate, the risk premium, and the return volatility. The gray (black) line represents the results on the complete (incomplete) market. The risk-free rate is given in Equation (I.10) in the Online Appendix, the risk premium in (5), and the return volatility in (6). All quantities are determined under the true measure $\lambda = 0.02$ and shown as functions of the pessimist's consumption share w_t with the long-run growth rate being fixed at $X_t = -0.0060$. The parameters are given in Table 4.