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The effects of skewness and kurtosis on excess return based on CAPM

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Abstract

The main aim of our research is to investigate how higher order moments of distribution such as systematic skewness and systematic kurtosis influence the investors' behaviour and their expected returns. In our study, we followed Fama Macbeth two-step procedure for US stock market over the period 1963 to 2019. Firstly, we proved that CAPM should be expanded by the measures of conditional co-skewness and co-kurtosis and found that investors require a higher return for bearing higher systematic variance, negative systematic skewness, and higher systematic kurtosis. Secondly, considering the effect between systematic skewness and systematic kurtosis simultaneously in addition to the main risk factors provides more accurate results. Thirdly, we identified empirical evidence that investors' behaviour changed significantly after the financial crisis of 2008, which signalizes about the necessity of improving current asset pricing theory.

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Last, but not the least, we would like to thank our families, boyfriends and closest friends for your spiritual support, which means the world for us.

1. Introduction

Nowadays the topic of investor's expected return and defining factors that affect this return is popular and relevant for economists, researchers, and investors. The significant part of research in finance is related to the question of how investors value risky cash flows. It is generally accepted that investors demand a higher premium for investment with higher risk. Moreover, the risk-return relationship is one of the most important factors which influence investors' decisions. Since investors try to create a portfolio with a maximum expected return, they should care about different risk factors that are likely to affect their profit. However, there are still many issues on how investors evaluate risk and premium for this risk, which is still a crucial question in capital markets in terms of both theory and practice.

The main model which explains the return is the single factor capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), which is used by most financial managers for assessing the risk of the cash flow from a project and for arriving at the appropriate discount rate to use in valuing the project. According to the CAPM, the risk of a project is measured by the beta of the cash flow with respect to the return on the market portfolio of all assets in the economy, and the relation between required expected return and beta is linear (Jagannathan, 1996). Although the CAPM is very popular, it is only a static model which is unable to explain the cross-sectional variation in average returns among portfolios sometimes (Jagannathan, 1996). Moreover, this theory has serious difficulties to explain the past superior performance of stocks and it is obvious that expected returns cannot be explained by the market beta alone. The whole mean-variance theory has become more and more questioned. Furthermore, CAPM considers the normal distribution of expected returns. Nevertheless, the normality of asset returns has been widely rejected in most empirical tests. As a result, in this research, we will analyse additional risk factors that are likely to influence the expected return of investors. So, we will expand CAPM adding the factors of skewness and kurtosis as higher moments in order to define whether these risks carry statistically significant risk premiums. By adding new variables to standard CAPM, we want to investigate how skewness and kurtosis influence the investors' behaviour and expected returns.

Since it is likely that investors have become more concerned about crash risk after the financial crisis of 2008, we will investigate whether investors' behaviour changed after the financial crisis.

In our study, we followed Harvey&Siddique's (2000) and Fang&Lai's (1997) methodological approach by using Fama Macbeth two-step procedure (1973). However, we experimented with different types of data and time-horizons to define the best specification of the model. Moreover, we incorporated the interaction term between skewness and kurtosis to find whether this factor influences the expected return by investors.

The rest of the paper is structured as follows. The second part summarizes academic literature dedicated to our topic. Section three presents the relevant theoretical background and the methodology to proceed with our research. In addition, section four contains a description of the data used and preliminary analysis. In part five the main results and findings of our research are presented. Finally, section six concludes this investigation and contains some suggestions for future studies.

2. Literature review

2.1 Non-normality of asset returns

Initially, investors have been constrained in their ability to incorporate non-normality of asset returns. Nevertheless, it's generally accepted now that stock market returns have negative skewness and severe excess kurtosis (this occurs when extreme negative returns are observed, with a magnitude and frequency greater than implied by the normal distribution.). This stylized fact has been supported by a huge collection of empirical studies. Campbell and Hentschel (1992) developed a formal model of the volatility feedback effect (an increase in stock market volatility raises required stock returns, and thus lowers stock prices). Their model is asymmetric and helps to explain the negative skewness and excess kurtosis of US monthly and daily stock returns over the period 1926-1988. So, their model of volatility feedback creates negative skewness and excess kurtosis in returns. Also, Salomons and Grootveld (2003) found that the distribution of equity risk premium in an emerging market is neither normally nor symmetrically distributed, which suggests that investors should focus more on downside risk (or negative skewness) instead of standard deviations. Moreover, Lettau, Maggiori, and Weber (2014) found that the unconditional CAPM (without considering downside risk) cannot explain the cross-section of currency returns because the spread in currency betas is not sufficiently large to match the cross-sectional variation in expected returns. In addition, they said that the downside risk CAPM (DR-CAPM) explains currency returns because the difference in beta between high- and low-yield currencies is higher conditional on bad market returns when the market price of risk is also high than it is unconditionally. The DR-CAPM can also jointly explain the cross-section of currencies, equity, equity index options, commodities, and sovereign bond returns.

Sheikh and Qiao (2010) stated that researchers worldwide empirically observe non-normality with much greater frequency than current mean-variance frameworks allow them. Moreover, ignoring non-normality in equity return distributions significantly understates downside portfolio risk - in the worst of the worst-cases, potentially posing a solvency risk for the investor. Researchers believed that investors need to allow for downside risk in a more robust fashion than standard deviation measures have traditionally assumed.

Kim and White (2004) did the research on S&P data and demonstrated that in robust normalizes data skewness and excess kurtosis are closed to normal. Moreover, they have found that looking beyond the standard skewness and kurtosis measures can provide deeper and more accurate insight into market returns behaviour. Chung et al. (2006) rejected normality of returns for daily, weekly, monthly, quarterly, and semi-annual intervals. So, in the absence of normality, investors should be very concerned with the shape of the tails of the distribution of portfolio returns, which can be measured with a set of higher-order moments.

Karoglou (2010) investigated the reasons for the existence of non-normality observed in daily stock-market returns. The findings suggest that a substantial element of the observed deviations from normality might indeed be due to the co-existence of breaks and GARCH effects. Also, there is still some remaining excess kurtosis that is unlikely to be linked to the specification of the conditional volatility or the presence of breaks.

Neuberger and Payne (2018) showed that short-horizon (i.e. daily) returns can be used to make more much precise estimates of long-horizon (e.g. annual) moments without making strong assumptions about the data generating process. Skewness comprises two components: the skewness of short-horizon returns, and a leverage effect, which is the covariance between contemporaneous variance and lagged returns. Applying the technology to US stock index returns, authors showed that skew is large and negative and does not significantly attenuate with the horizon as one goes from monthly to multi-year horizons.

It could be summarized that since the distribution of returns is not normal, higher-order moments matter to risk-averse investors, concerned about extreme outcomes, and require further investigation.

2.2 Skewness as the third moment of distribution and its influence on excess return

There are many different researchers that investigated the effects of skewness and kurtosis on excess return. A lot of researchers in the empirical, as well as theoretical articles, have argued that the higher moments of the rates of return distributions, such as skewness, cannot be neglected unless there is a reason to believe that the rates of return have a normal (symmetrical) probability distribution and/or quadratic utility function. This is equivalent to the assertion that the higher (more than two) moments are irrelevant to the investors' decision under uncertainty. Arditti (1967) for the first time expanded expected utility function in a Taylor series with the higher moments of a probability distribution, namely skewness. Attention had been concentrated on the second (variance) and third moments (skewness) of r's distribution. This had been done because previously the higher moments of r add little if any information about the distribution's physical features. The researcher concluded that the second and third moments of the probability distribution are reasonable risk measures while the market correlation coefficient of returns is not. Arditti argued that the first three moments contain all the income information, while the omitted variables must then relate to some non-income information. Jean (1971) attempted to extend the analysis to three and many dimensions by deriving risk premiums as functions of higher order moment and summarized that one of the reasons for ignoring skewness at least has to do with the form of distribution of cash flows. If the cash returns have a symmetric distribution, then the third moment will be zero and the corresponding term in the expansion, therefore, will be zero. Nevertheless, Jean showed that leveraged capital structures will most likely result in nonsymmetric, skewed payments to security holders. Ingersoll (1975) derived a normative, individual pricing model for risky securities analogous to the capital market line within the framework of a perfect market. Ingersoll proofed that positive skewness is desired and we must expect to forego expected return in order to increase skewness. These findings are the reverse of the situation in the mean-variance model where high covariance is compensated for by higher expected return.

Kraus and Litzenberger (1976) started to work with CAPM and extended the Sharpe-Lintner capital asset pricing model to incorporate the effect of skewness on valuation. Their evidence suggested that prior empirical findings that are interpreted as inconsistent with the traditional theory can be attributed to misspecification of the capital asset pricing model by the omission of systematic skewness. Also, researchers showed that when the capital asset pricing model is extended to include systematic skewness, the prediction of a significant price of systematic skewness is confirmed (and the price has the predicted sign) and the prediction of a zero intercept for the security market line in excess return space is not rejected. Sears (1985) examined the importance of skewness in the pricing of risky assets, finding the results of such tests to be influenced by the market risk premium. The researcher explored a not so obvious theoretical relationship within such models that are intrinsically nonlinear in the market risk premium. Failure to account for this interaction may lead to erroneous conclusions regarding the empirical results of the models. Singleton and Wingender (1986) stated that the frequency of positive skewness in their study is found to be relatively stable over varying time periods from 1961 to 1980. However, the skewness of individual stocks and portfolios of stocks do not persist across different time periods. Positively-skewed equity portfolios in one period are not likely to be positively-skewed in the next time period. Past positively-skewed returns do not predict future positively-skewed returns.

The researches mentioned above provided mixed results of the effect of skewness on the equilibrium asset pricing. Nevertheless, in all these studies kurtosis and its effect on asset returns received relatively little attention.

2.3 Skewness and kurtosis as the third and fourth moments of the distribution

Further, researchers started to extend their models with the fourth moment of distribution and investigate its effects on excess returns. Scott and Horvath (1980) formulated and proofed two theorems, the second of which was related to the fourth moment of distribution (kurtosis). The first theorem says that investors exhibiting positive marginal utility of wealth for all wealth levels, consistent risk aversion at all wealth levels, and strict consistency of moment preference will have a positive preference for positive skewness (negative preference for negative skewness). While the second theorem states that consistent risk aversion, strict consistency of moment preference and positive preference for positive skewness imply a negative preference for the kurtosis. Gibbons, Ross, and Shanken (1989) documented that skewness and kurtosis cannot be diversified by increasing the size of the portfolio. Thus, the non-diversified skewness and kurtosis became important considerations in the security valuation. Hwang and Satchell (1999) suggested that emerging markets are better explained with higher-moments CAPMs based on some test statistics such as the adjusted R^2 and the LM statistics reported in their study.

Fang and Lai (1997) derived a four moment CAPM where, as well as variance, skewness, and kurtosis contribute to the risk premium of an asset. They showed that in the presence of skewness and excess kurtosis in asset distribution, the expected excess rate of return is related to systematic variance as well as skewness and systematic kurtosis. They concluded that investors have a preference for positive skewness in their portfolios and thus require a higher expected return for assets when the market portfolio is negatively-skewed and vice versa. Also, investors are compensated with a higher expected return for bearing the systematic variance and the systematic kurtosis risk. So, in general, investors require a higher expected return for assets with systematic variance, assets which are negatively skewed and have systematic kurtosis.

Harvey and Siddique (2000) proved that conditional skewness helps explain the cross-sectional variation of expected returns across assets and is significant even when factors based on size and book-to-market are included. Therefore, systematic skewness is economically important and commands a risk premium, on average, of 3.60 percent per year. So, the low expected return momentum portfolios have higher skewness than high expected return portfolios. While Hung, Shackleton, and Xu (2003) investigated UK data and showed limited evidence for the existence of higher order pricing factors (gamma and delta) associated with market co-skewness and co-kurtosis respectively.

Prakash, Chang, and Pactwa (2003) suggested that the incorporation of skewness into an investor's portfolio decision causes a major change in the final optimal portfolio. The empirical evidence, based on the optimal portfolio from Latin American, US and European capital markets, indicates that investors trade expected return of the portfolio for skewness, especially those in Latin America.

Adrian and Rosenberg (2008) explore the cross-sectional pricing of volatility risk by decomposing equity market volatility into short- and long-run components. For short-run volatility, they used skewness of market returns as an indicator of the tightness of financial constraints since returns skewness arises endogenously in pricing theories with financial constraints. Intuitively, shocks to market skewness are particularly costly when financial constraints of investors are binding. The long-run component relates to business cycle risk. They found that the risk premium of the short-run component is highly correlated with the risk premium of market skewness, while the risk premium of the long-run component has a high level of correlation with the risk premium of industrial production growth. On one hand, market skewness is a significant pricing factor in the cross-section of size- and book-to-market–sorted portfolio. On the other hand, including the short-run volatility component as an additional factor makes skewness insignificant.

Chang, Christoffersen, and Jacobs (2009) argued that the market skewness risk premium is statistically and economically significant and cannot be explained by other common risk factors such as the market excess return or the size, book-to-market, momentum, and market volatility factors, or by firm characteristics. They also found that stocks with high exposure to innovations in implied market skewness exhibit low returns on average, whereas stocks with high exposure to innovations in implied market kurtosis exhibit somewhat higher returns on average.

Conrad, Dittmar, and Ghysels (2013) used option prices to estimate ex-ante higher moments of the underlying individual securities' risk-neutral returns distribution. Researchers found that individual securities' risk-neutral volatility, skewness, and kurtosis are strongly related to future returns. They also found a strong negative relation between skewness and subsequent returns and positive relation between kurtosis and returns. These relations are robust to controls for differences in firm characteristics, such as firm size, book-to-market ratios, and betas, as well as liquidity and momentum. However, when authors control for interactions between volatility, skewness, and kurtosis, they found that the evidence for an independent relation between kurtosis and returns is relatively weak.

Amaya et al. (2015) used intraday data to compute weekly realized moments for equity returns and studied their time-series and cross-sectional properties. Their findings suggested that buying stocks in the lowest realized skewness decile and selling stocks in the highest realized skewness decile generates an average return of 19 basis points the following week with a t-statistic of 3.70.

Treating returns as a function of their mean, variance, skewness, and kurtosis enabled Chen (2016) to ascribe behavioural significance to the odd and even moments of the distribution of returns. The researcher showed that investors like mean return and positive skewness and dislike variance and kurtosis. The odd moments, mean and skewness, advance returns, while the even moments "produce a drag on expected compound return". The presence of a positive third derivative in the most commonly employed models, therefore, predicted that investors are more willing to indulge their taste for positively skewed outcomes as their wealth grows. The alternating treatment of odd- and even-numbered mathematical moments represented a logical extension of an essential non-linear feature of observed investor behaviour already captured by the treatment of semi variance: most investors perceive infrequent large losses or shortfalls to be far more risky than more frequent smaller losses or shortfalls. Chen concluded that positive skewness indicates the presence of outsized gains; it suggests the tantalizing possibility that certain holdings in the portfolio will offer disproportionately large pay-outs, as though they were winning lottery tickets. By contrast, even moments measure dispersion, and therefore volatility, something undesirable that increases the uncertainty of returns.

To sum up, the investors are significantly concerned about negative skewness, kurtosis and downside risk that increase the uncertainty of their expected returns. In addition, the popularity of lotteries, sweepstakes, out-of-the-money options implies that investors also care about the right tail of the distribution. Based on the literature review, we can conclude that the topic of the third and fourth moments is important for the investors and it is necessary to investigate it more deeply with new data, factors, and interpretations.

3. Theory and methodology

3.1 The Capital Asset Pricing Model

The main model which is basic in our research is CAPM. This model represents the relationship between the risk of the assets and expected returns. It is based on simplifying assumptions about investor behaviour and the presence of a single common risk factor:

- Firstly, investors care only about expected returns and risk (volatility). Furthermore, investors are rational, so they will always choose the maximum point of expected return for any given level of expected volatility.
- Secondly, all investors in the market have homogeneous beliefs about the risk (volatility) and expected return trade-offs.
- Thirdly, there is only one risk factor which is common to a market portfolio. It is the systematic market risk which drives non-diversifiable volatility. Since the market does not reward investors for the bearing of diversifiable risk, it is assumed that investors hold only diversified portfolios.
- Fourthly, it is assumed that asset returns are distributed by the normal distribution (Womack, 2003).

Consequently, according to CAPM if investors know the beta of security (the risk of security), they can calculate the corresponding expected return. The equation of the CAPM model is presented in such a way:

$$E(r_A) = r_f + \beta_A\big(E(r_m) - r_f\big) \text{ (1)},$$

where $E(r_A)$ is expected return of an asset, r_f is the risk-free rate, $(E(r_m) - r_f)$ is the expected excess return of the market portfolio beyond the risk-free rate, often called the equity risk premium, β_A is a systematic risk which is measured as the degree to which its returns vary relative to those of the overall market and is calculated as:

$$\beta_A = \frac{cov(r_A, r_m)}{\sigma_m^2} \text{ (2)},$$

where r_A is the return of the asset, r_m is the return of the market, σ_m^2 is the variance of the return of the market, $cov(r_A, r_m)$ is covariance between the return of the market and the return of the asset (Womack, 2003).

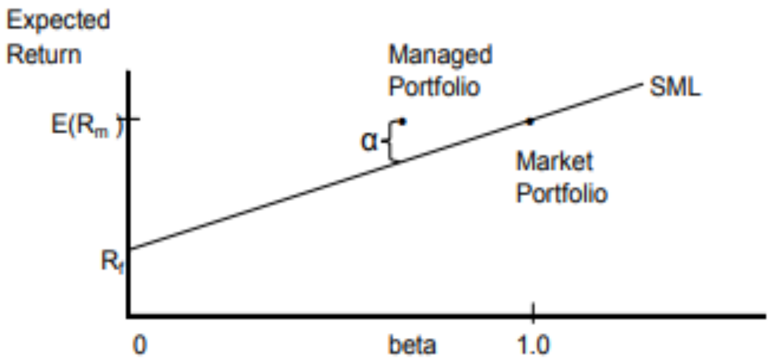
Important to note that now the world market portfolio, which consists of all assets in the world, is not observable, so it is necessary to use a proxy (Bartholdy, 2015). Usually, the Standard & Poor's 500 Index is used as a proxy for the market return in models, as it is a stock market index that includes the stocks of 500 US companies based on weights of their market capitalization. It represents the stock market's performance by reporting the risks and returns of the biggest companies (www.thebalance.com).

Also, CAPM is used for evaluation of fund managers. Since this model helps to predict the expected return based on a given risk, we can evaluate whether fund managers outperform or not. If a realized return is higher than expected return, it means that fund manager outperforms and there is “adding value”. In other case, there is “just collecting fees” and no added value.

On the one hand, the fund managers often take a higher risk (beta or volatility) in order to get a higher return. On the other hand, it is possible to create real value when a fund manager can achieve higher returns at the same or reduced level of risk. The difference between realized return and expected return based on CAPM is the excess return or α (see Figure 1). Positive α means that portfolio is located beyond the security market line (SML) and the manager has good performance.

Figure 1: Security Market Line

Figure 1 presents the relationship between risk and expected return, SML, or graph representation of the CAPM formula. The intercept is the risk-free rate and the slope represents the market premium. Individual securities’ expected return and risk are plotted on the SML graph. For one security, if it is plotted above the SML, it is undervalued as the investors are expecting a greater return for the same amount of risk (beta). If it is plotted below the SML, it is overvalued as the investors would accept a lower return for the same amount of risk (beta).



Source: Womack, 2003

Nevertheless, the CAPM has a lot of limitations which cause the problem in the model. First of all, the model is based on non-realistic assumptions. For example, it is assumed that the investment returns are normally distributed, although it is not true in real life. Also, this model includes risk-free rate, although even government bonds contain risk as well. In addition, the assumption, that all investors have homogeneous expectations and are rational, is also quite unrealistic (Shefrin, 2008).

Secondly, the CAPM is based on forward-looking data such as the expected rate of return and the expected beta that are calculated on historical data. It means that these numbers can't be exactly précised, as time is changing. Moreover, there is no reason to believe that realized rates of return over the past holding periods are necessarily equal to the expected rates of return (Brigham & Gapenski, 1994).

Thirdly, the model includes the return of market portfolio which should consist of all types of assets, although in real economy it is not possible to calculate the return of such a market portfolio. That's why we always take the proxy for market return such as S&P 500 Index, which may cause deformation of the CAPM. Fourthly, CAPM is a single period model, so investors care only about the year return. Nevertheless, investors care about their return during the whole period (www.businesswritingservices.org).

Finally, according to the model the expected return depends on only one factor - the stock market beta. Nonetheless, it is obvious that other macroeconomic factors may influence a security's return also (www.businesswritingservices.org).

To sum up, the CAPM has some limitations, some of which we are going to solve in our research.

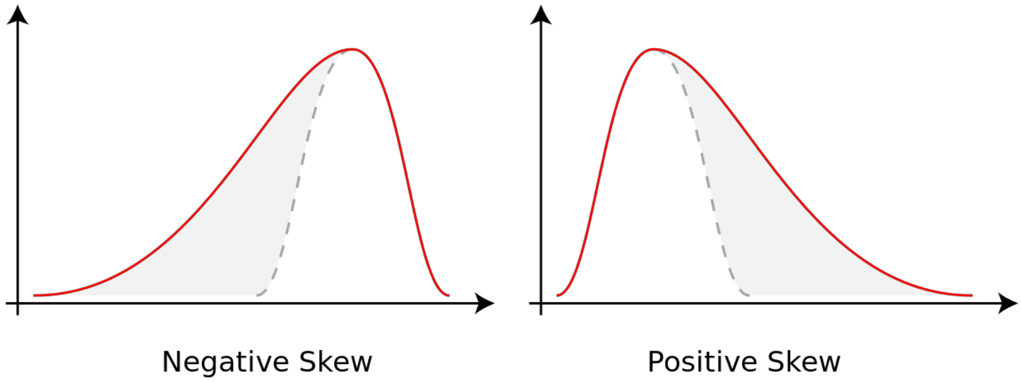
3.2 Skewness and kurtosis

Skewness is a measure of the asymmetry of the distribution of a variable. A positive skew value indicates that the tail on the right side of the distribution is longer than the left side and the bulk of the values lie to the left of the mean. In contrast, a negative skew value indicates that the tail on the left side of the distribution is longer than the right side and the bulk of the values lie to the right of the mean (Figure 2). Kurtosis is a measure of the peakedness of distribution, while excess kurtosis obtained by subtracting 3 from the kurtosis measure. Distributions with positive excess kurtosis are called leptokurtic distribution meaning high peak, and distributions with negative excess kurtosis are called platykurtic distribution meaning flat-topped curve (Figure 3) (Kim, 2013).

Systematic skewness or kurtosis are defined as components of an asset's skewness (kurtosis) that is related to the market portfolio's skewness (kurtosis). In such context, the systematic skewness (kurtosis) is considered as a non-diversifiable measure of skewness (kurtosis) and therefore it is consistent with the assumption of portfolio theory that only systematic risk is relevant to the investor's decision (Doan, 2011).

Figure 2: Negative and positive skewness of the distribution

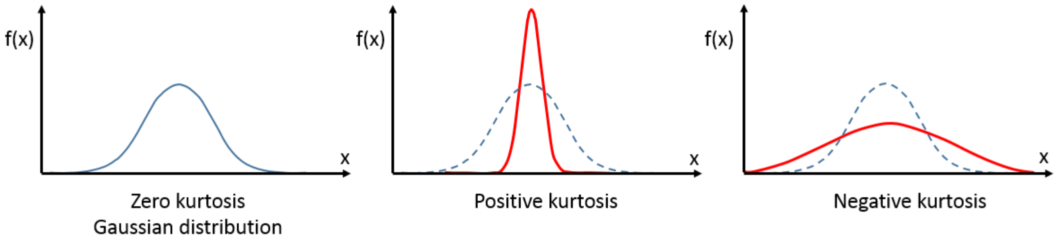
Figure 2 presents two types of asymmetry of the distribution of a variable.



Source: (www.statisticshowto.datasciencecentral.com)

Figure 3: Kurtosis of the distribution

Figure 3 presents two types of peakedness of a distribution.



Source: (www.statisticshowto.datasciencecentral.com)

Also, a lot of researchers consider co-skewness and co-kurtosis. First, co-skewness was used by Krauss and Litzenberger in 1976, and then by Harvey and Siddique in 2000. It was defined as a measure of securities' risk in relation to market risk and is similar to covariance which measures systematic risk in CAPM. Investors like positive co-skewness more since it provides a higher probability of positive returns by two assets in excess of market returns. In other words, the assets with higher co-skewness increase the systematic skewness of an investor's portfolio (Adesi et al., 2004). Co-kurtosis is also used to measure a security's risk in relation to the market risk based on historical price data and market data, but it considers extreme events. The risk-averse investors like low co-kurtosis, as it means no significant difference between securities' returns and market returns, while risk-lovers prefer high co-kurtosis in order to win in the case of extreme positive returns (Fang & Lai, 1997).

To calculate skewness and kurtosis, such formulas are used:

$$skew_A = E\left[\frac{(r_A - E(r_A))^3}{Var(A)^{\frac{3}{2}}}\right] (3)$$

$$kurt_A = E\left[\frac{(r_A - E(r_A))^4}{Var(A)^2}\right] (4),$$

where $skew_A$ is skewness of asset A, $kurt_A$ is kurtosis of asset A, $Var(A)$ is the variance of asset A (Jondeau, 2003).

Also, it is important to define the excess kurtosis, which is equal to

$$excesskurt_A = E\left[\frac{(r_A - E(r_A))^4}{Var(A)^2}\right] - 3 (5).$$

In the CAPM, investors care only about two moments - mean and variance - for portfolio returns and one co-moment - covariance - for security returns. In general, investors may care about higher moments such as skewness, kurtosis, and higher co-moments such as co-skewness, co-kurtosis (Chung, 2006). On the one hand, the way to extend traditional CAPM is to add Fama - French factors such as the excess return of the companies with small capitalization over the companies with large capitalization (SMB), the excess return of the companies with high Book-to-Market ratio over the companies with low ratio (HML), or the difference between the returns on diversified portfolios of stocks with robust and weak profitability (RMW), the difference between the returns on diversified portfolios of the stocks of low and high investment firms (CMA) (Fama & French, 2015). On the other hand, as normality of returns is the crucial assumption for CAPM, but in reality, it does not work, it is very important to add higher moments of distribution such as skewness and kurtosis in the CAPM. In our study, we will extend classical CAPM with these higher moments of the distribution.

In the next part, we will explain our hypothesis and the main methodology which we use for the model.

3.3 The hypothesis to be formally tested

In order to investigate the effects of skewness and kurtosis on excess return based on CAPM, we will test such hypothesis:

- Expected excess rate of return is explained by systematic skewness and systematic kurtosis as well as systematic variance:
    - ➔ investors are compensated with a higher expected return for bearing the systematic kurtosis risk;
    - ➔ investors forego the expected excess return for taking the benefit of increasing the systematic skewness.

- Whether and how after the financial crisis of 2008, the relations between the expected excess rate of return and systematic skewness and kurtosis have changed.

3.4 The importance of conducting the test for normality

Since we decided to test the hypothesis described above, we need to start by checking the data for normality. The higher moments such as skewness and kurtosis signalize whether data is normal or not. If the skewness is equal to 0 and excess kurtosis is also 0, the data will be normally distributed. As we will expand the CAPM with these higher moments, we need to be sure that the data is really abnormal, which also will be the confirmation one of the CAPM's limitations. In the case of normal data, our model does not have any sense. In order to check the data for normality, we will use Jarque-Bera test. This method is used for testing the residuals for normality by testing whether the coefficient of skewness and the coefficient of excess kurtosis are jointly zero (Jarque & Bera, 1987). We have already written the formulas for excess kurtosis and skewness, but it can be shown with residuals estimated in OLS regression also:

$$b1 = skew_A = \frac{E({u_A}^3)}{Var(A)^{\frac{3}{2}}} \, (6)$$

$$b2 = kurt_A = \frac{E({u_A}^4)}{Var(A)^2} \, (7).$$

The null-hypothesis H_0 in this test: the coefficients of skewness and excess kurtosis are jointly zero. In order to check this hypothesis, we need to calculate the Jarque-Bera test statistics:

$$W = N\left[\frac{b_1^2}{6} + \frac{(b_2 \ - 3)^2}{24}\right] \, (8),$$

where N is the number of observations.

If the test-statistics exceeds a critical value, the null hypothesis will be rejected.

So, it is important to conduct the test on normality in our model, and after the confirmation of the abnormal data, we can move to the main method of our research.

3.5 Fama Macbeth two-step procedure as a method for a model

Skewness and kurtosis effects cannot be neglected unless there is a reason to believe that the rates of return have a normal probability distribution. Only if the excess returns of companies have a symmetric distribution, then the third and the fourth moments, i.e. skewness and excess kurtosis, will be zero and the corresponding term in the expansion, therefore, will be zero. In order to test the effects of skewness and kurtosis on excess return, we will use Fama Macbeth two-step procedure. Fama Macbeth regressions perform in two steps, as it seems from the name of the procedure. Firstly, the estimated betas for different risk factors of each stock could be found by using time-series regressions. Before step two can be carried out, it is assumed that the estimated betas from the first step agree with the actual unknown betas. Secondly, it will be run the cross-sectional regression in order to find the estimates of the risk premium for different risk factors using estimated betas from the previous step.

Moreover, Fama and Macbeth test the CAPM with a two-pass procedure that first sorts stocks into portfolios on the basis of historical beta estimates and then estimates the mean cross-sectional relationship between the portfolio returns and portfolio betas for each period. By sorting on beta they are able to maximize the cross-sectional variation in the variable of interest. However, Sylvain (2013) noted that portfolios can contain one or more securities (Fama & Macbeth, 1973; Bartholdy, 2005; Chung, 2006; Sylvain, 2013). In order to run Fama Macbeth regressions, portfolio data or stock data could be used. The discussion about using stock versus portfolio data is provided in Section Data.

On the first stage, we will run traditional CAPM with Fama Macbeth two-step procedure. In such a way we are going to find a risk premium for holding a systematic risk and look whether a systematic risk is the only factor that explains the average excess return of companies. Firstly, a time-series OLS regression, equation (9), is run on each asset to generate an intercept, an estimate of the asset's beta and residuals (Grauer et al., 2010; Bartholdy et al., 2005).

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it} (9),$$

where $R_{it} - R_{ft}$ is excess return of asset i, $R_{mt} - R_{ft}$ is a market excess return, β_i and α_i are parameters estimates.

By using $\beta_i s$ (systematic risk) we can move toward to the second step where risk premium for holding systematic risk will be generated by running cross-sectional regression, equation (10):

$$r_i = \gamma_0 + \gamma_1 \widehat{\beta_i} + \varepsilon_{it} \text{ (10)},$$

where r_i is the average excess return of company i, $\widehat{\beta_i}$ is systematic risk generated from equation (9), γ_0 and γ_1 are parameters estimates (Fama & Macbeth, 1973; Bartholdy, 2005).

If the CAPM holds, only the second-order systematic co-moment (beta) should be priced (Chung, 2006). Moreover, γ_1 should be positive and significant. This is a direct measure of the ability of the beta estimate to explain differences in returns on individual stocks in the period following estimation (Bartholdy, 2005). If γ_0 is significantly different from zero, it will mean that systematic risk is not an only factor that explains the average excess return of companies.

On the second stage, we will extend traditional CAPM with additional factors. The extension will make sense only in case if traditional CAPM faced omitted variables problem. We will run the cubic market model consistent with four-moment CAPM derived by Fang and Lai (1997) (equation 11):

$$R_{it} - R_{ft} = \alpha_i + \beta_{1i}(R_{mt} - R_{ft}) - \beta_{2i}(R_{mt} - R_{ft})^2 + \beta_{3i}(R_{mt} - R_{ft})^3 + \varepsilon_{it} \text{ (11)},$$

where $(R_{mt} - R_{ft})^2$ are $(R_{mt} - R_{ft})^3$ are stochastic discount factors in the market return, $\beta_{1i}, \beta_{2i}, \beta_{3i}$ are market prices of the systematic variance, systematic skewness, and systematic kurtosis, respectively.

The motivation for the model is that investors may care about skewness and kurtosis in addition to mean and variance. If so, then investors who hold the market portfolio would evaluate a marginal change in the holding of an asset in terms of its effect on variance, skewness, and kurtosis, and these marginal effects are captured by covariance, co-skewness, and co-kurtosis (Back, 2004). The quadratic form for the marginal rate of substitution implies an asset pricing model where the expected excess return on an asset is determined by its conditional covariance with both the market return and the square of the market return (conditional co-skewness) (Harvey et al., 2000). A cubic pricing kernel is consistent with investors' preferences for higher order moments, specially kurtosis (Christoffersen et al., 2017). The signs of the coefficients in equation (11) are based on the assumption that investors dislike variance, prefer positive skewness to negative skewness, and dislike kurtosis. Thus, high covariance/ negative co-skewness/ high co-kurtosis assets are undesirable and consequently sell at low prices, producing high expected returns (Back, 2004).

So, β_1, β_2, β_3 are obtained by running time-series regression for each stock and depict systematic variance, systematic skewness, and systematic kurtosis, respectively. We will use these estimated coefficients for running cross-sectional regression to find risk premiums that investors should demand for holding stocks with different numbers for variance, skewness and kurtosis, equation (12):

$$r_i = \gamma_0 + \gamma_1\widehat{\beta_{1\iota}} - \gamma_2\widehat{\beta_{2\iota}} + \gamma_3\widehat{\beta_{3\iota}} + \varepsilon_{it} \ (12),$$

where γ_1, γ_2, γ_3 are systematic market risk premia for an increase in systematic variance, a decrease in systematic skewness, and an increase in systematic kurtosis, respectively (Fama & Macbeth, 1973; Bartholdy, 2005). γ_0 should be zero, which means that we added all factors that define the excess return of the company.

One important concern with our empirical approach is that the Fama MacBeth two-step method may be biased since the right-hand-side variables in the equation (12) are the estimates from the first-step time-series equation (11). We understand that the error-in-variables problem results in an underestimation of the price of beta risk and an overestimation of the other cross-sectional regression coefficients associated with variables observed without error (Kim, 1995). Chung (2006) suggested recalculating all standard errors using the Shanken adjustment in order to test for the errors-in-variables bias. Shanken (1992) modifies the traditional two-pass procedure and derives an asymptotic distribution of the cross-sectional regression estimator within a multifactor framework in which asset returns are generated by portfolio returns and prespecified factors. Chung (2006) stated that because of the way higher-order right-hand-side variables are created, the Shanken adjustment appears to be inappropriate for specifications that include such variables. Therefore, Chung concluded that although his estimates from Fama Macbeth two-step procedure are likely to have an errors-in-variables bias, the researcher did not believe that the bias is large enough to negate his overall conclusions.

Instead of employing the two-pass procedure, Gibbons (1982) used the maximum likelihood estimation approach to eliminate the error-in-variables problem by simultaneously estimating betas and beta risk prices. Kim (1995) stated that Gibbons' estimator is thus still subject to an error-in-variables bias. Moreover, researcher performed the correction for the error-in-variables problem by incorporating the extracted information about the relationship between the measurement error variance and the idiosyncratic error variance into the maximum likelihood estimation under either homoscedasticity or heteroscedasticity of the disturbance term of the market model. Kim (1995) concluded about the importance of implementation of correction when firm size is included as an additional explanatory variable. Moreover, the multivariate normality assumption imposed in Gibbons model is improper in cases when the stock returns are characterized by significant departures from normality as revealed by the high skewness and kurtosis of returns (Fang & Lai, 1997). A generalized method of moments used by Lim (1989) avoids the measure error problem and distributional assumption and it is subject to the sample size and computer time limitations (Fang & Lai, 1997).

Barthodly (2005) mentioned that one possibility is the use of more sophisticated estimation techniques to deal with problems such as errors in variables which arise when a simple technique, namely Fama Macbeth two-step procedure, is used. However, the researcher stated that such techniques are probably cost prohibitive for individual firms, in particular, in relation to the amount of data required. This suggests that individual firms should use professional beta providers for obtaining their beta estimates instead of estimating them themselves and that professional beta providers should use more complex techniques than Fama Macbeth two-step procedure.

Important to mention that OLS estimators could be not efficient. This is due to the fact that with the presence of high kurtosis in stock return distribution, the OLS estimators are fairly sensitive to outliers as pointed out by Lee and Wu (1985). An instrumental variable estimation can help to avoid the error-in-variables problem. An instrumental variable estimator is known to yield consistent estimators if a matrix of instrumental variables can be found which is uncorrelated with the disturbance term in the model (Fang & Lai, 1997). An instrumental variable estimator is used in cases when the second assumption about disturbances does not hold (Greene, 1997). So, it is used as an instrumental variable z, which is uncorrelated with the error term. If the second assumption about the unobservable error term does not hold, we will use slightly different adjustment which will be discussed in the section Model diagnostic tests.

Conrad, Dittma, and Ghysels (2013) examined the importance of higher moments using a dramatically different approach as an alternative to adjusted or original Fama Macbeth two-step procedure. They exploited the fact that, if option and stock prices reflect the same information, then it is possible to use options market data to extract estimates of the higher moments of the securities' probability density function. Firstly, the advantages of that method could be that option prices are a market-based estimate of investors' expectations. So, option market prices can capture the information of market participants. Secondly, the use of option prices eliminates the need for a long time series of returns to estimate the moments of the return distribution. Thirdly, options reflect a true ex-ante measure of expectations, they do not give us, as Battalio and Schultz (2006) note, the "unfair advantage of hindsight."

Based on all previous findings, we will run traditional Fama Macbeth two-step procedure and will not proceed with the correction for the error-in-variables problem due to lack of argumentation that this correction will implement significant changes to our models. However, we will check for holding all assumptions underlying the classical linear regression model. If one of it does not hold, we will perform corresponding adjustments. In such a way we are going to eliminate the error-in-variables problem. A comparison between the different specification of models or using option prices is a topic for future research.

3.6 Specification of the model

As it was previously mentioned, we will expand the CAPM with higher moments such as skewness and kurtosis. Higher-order moments have been criticized for being unreliable and lacking intuition. Nevertheless, Chung et al. (2006) believe that both criticisms can be answered by looking at several co-moments. Each co-moment may individually be unreliable, but the set of co-moments should be relevant. That is why we will not test the effects of skewness and kurtosis separately, only both factors together.

Also, it is important to consider the interaction term in our model. Interaction term shows how the effect of one variable changes due to another variable change (Buis, 2010). On the one hand, if two independent variables affect the outcome of the dependent variable in a non-additive way, an interaction term needs to be included in the model to capture this effect (Field, 2009). Moreover, in the case of statistically significant interaction terms, we need to interpret the main effects with considering the interactions. Also, sometimes the interaction term is used to liquidate the problem of multicollinearity between independent variables. In such a way, the researches take the interaction term of these variables instead of them. On the other hand, Morris, Sherman, and Mansfield (1986) had noted a persistent failure of psychologists to detect interaction effects between continuous variables in multiple regression analysis. They mentioned that multicollinearity between interaction term and its components may exist and be the source of the problem in the model. In order to solve this problem, Cronbach (1987) suggested performing an additive transformation for a given predictor. However, based on Cronbach's article, multicollinearity will only be a problem when it leads to computational errors within current computer algorithms. To sum up, it is unlikely that most empirical researches have such a high degree of multicollinearity, so there may be other factors that make it difficult to correctly detect moderated relationships (Jaccard et al, 1990).

Based on mentioned above, we decided to proceed with the interaction term between skewness and kurtosis and try it in our model. It will be added in such a way:

$$R_{it} - R_{ft} = \alpha_i + \beta_{1i}(R_{mt} - R_{ft}) - \beta_{2i}(R_{mt} - R_{ft})^2 + \beta_{3i}(R_{mt} - R_{ft})^3 + \beta_{4i}(R_{mt} - R_{ft})^2(R_{mt} - R_{ft})^3 + \varepsilon_{it} \quad (13),$$

where $(R_{mt} - R_{ft})^2(R_{mt} - R_{ft})^3$ is the interaction term between stochastic discount factors in the market return, $\beta_{1i}, \beta_{2i}, \beta_{3i}, \beta_{4i}$ are market prices of the systematic variance, systematic skewness, systematic kurtosis and combination of systematic skewness and systematic kurtosis respectively.

3.7 Model diagnostic tests

The validity of a model can only be trusted when the few required assumptions are true. As it is a classical linear regression model, which is based on the OLS method, we need to check the next five assumptions about the unobservable error term. We need to be sure that we have BLUE (Best Linear Unbiased Estimators) what is desirable properties for estimated slopes and intercept.

For all diagnostic tests, we cannot observe the disturbances and so perform the tests of the residuals (Brooks, 2019). The next assumptions will be checked:

1. $E(u_t) = 0$

The mean of the residuals is zero. Generally, the mean of the residuals will always be zero provided that there is a constant term in the regression (Brooks, 2019).

2. $Var(u_t) = \sigma^2 < \infty$

It is assumed that the variance of the residuals is constant and equal to σ^2, which is called as homoscedasticity. If the errors do not have a constant variance, we say that they are heteroscedastic, which is the violation of the second assumption. In order to check it, we apply White's test for heteroscedasticity, which is a very good method as it makes few assumptions about the form of the heteroscedasticity.

The null-hypothesis H_0 in White's test for heteroscedasticity: the disturbances (we consider residuals) are homoscedastic (Brooks, 2019).

Firstly, we run our regression, estimate the model and obtain the residuals. Secondly, we need to run the auxiliary regression using the residuals:

$$\hat{u}_t^2 = \alpha_1 + \alpha_2 x_{2t} + \alpha_3 x_{3t} + \alpha_4 x_{2t}^2 + \alpha_5 x_{3t}^2 + \alpha_6 x_{2t} x_{3t} + v_t \ (14),$$

where $\hat{u}_t$ is the obtained residuals from the previous regression, x_{nt} – the independent variables from the previous regression.

Thirdly, the null-hypothesis will be rejected or not rejected after calculation of chi-square test statistic:

$$TR^2 \sim \chi^2(m) \ (15),$$

where T is the number of observations, R^2 is taken from the auxiliary regression; m is the number of regressors in the auxiliary regression excluding the constant term.

If chi-square test statistic is greater than the corresponding value from the statistical table, H_0 about homoscedasticity will be rejected (Brooks, 2019).

The heteroscedasticity is the problem in the model, as our estimators are still unbiased, but they are no longer BLUE. Moreover, the standard errors can be wrong, which causes the irrelevant interpretation of the estimators.

In order to solve the problem, White's heteroscedasticity consistent standard error estimates can be used. Due to this method, we receive new standard errors for the slope coefficients, and we would need more evidence against the null hypothesis before we would reject it (White, 1980).

3. $Cov(u_i, u_j) = 0 \ for \ i \neq j$

This assumption means that there is no pattern between disturbances (residuals), in other case the residuals are autocorrelated. In order to check it, the Breusch-Godfrey test can be used. It is the test for r^{th} order autocorrelation, where such a regression will be run:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \cdots + \rho_r u_{t-r} + v_t \ (16),$$

where ρ_r – autocorrelation between residuals, $v_t \sim N(0, \sigma_v^2)$.

The null-hypothesis H_0 in this test: there is zero autocorrelation between error terms or $\rho_1 = 0\ and\ \rho_2 = 0\ and\ \rho_3 = 0\ and\ ...\ and\ \rho_r = 0$ (Brooks, 2019).

After running the linear regression using OLS and obtaining the residuals, we should put these residuals in the equation above. Then we obtain R^2 and calculate the test statistics:

$$(T-r)R^2 \sim \chi_r^2 \ (17),$$

where T is the number of observations, r is the number of lags, R^2 is taken from the regression of residuals.

If the test statistic is larger than the critical value from the statistical tables, we need to reject the null hypothesis of no autocorrelation, which means the violation of the third assumption of the classical linear regression model (Brooks, 2019). In this case, we can receive inappropriate standard errors and wrong conclusion, as the estimators are unbiased, but not BLUE. To deal with it, it is possible to move to the model in first differences.

4. The x_i are not stochastic.

This assumption means that there is no correlation between the residuals and the independent variables, as in other way the OLS estimators will not be even consistent. In order to check this assumption, we need to calculate the correlation between the residuals and each independent variable.

5. The disturbances are normally distributed.

As we have already described in the section about testing on normality, the Jarque-Bera test is used to check the residuals on normality. In addition, it is very good to see on the histogram and time series plot of the estimated residuals. The fifth assumption is very important to make relevant conclusions about the model.

Also, we need to check the model on multicollinearity between explanatory variables, as high multicollinearity may become the problem in the model. In order to check it, we need just to calculate the correlation between independent variables. If the problem is ignored, there can be such results:

1. High R^2 but the coefficients can have large standard errors;
2. If there are small changes in the specification, the regression can be very sensitive to this;
3. The confidence intervals become very wide, so testing can give wrong conclusions (Brooks, 2019).

To sum up, it is important to make all diagnostics tests in order to check the estimators. We will conduct these tests for our model in the next sections.

4. Data and Preliminary Analysis

4.1 Data

In this section, we will describe the data we used, sources, the way of collection and explain the variables.

Generally, there are different opinions about data which we need to take for investigation of return-risk relationship: portfolios' returns or stocks returns. The first approach is to aggregate stocks into portfolios for testing. The motivation for creating portfolios is originally stated by Blume (1970) that betas are estimated with error and this estimation error is diversified away by aggregating stocks into portfolios. Numerous authors, Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and Fama and French (1993) have used this motivation to use portfolios as base assets in factor model tests. The second approach is to use the whole stock universe and run cross-sectional tests directly on all individual stocks. In creating portfolios, estimates of beta become more precise, but the dispersion of beta shrinks. It causes potentially larger efficiency losses in using portfolios versus individual stocks. In addition, using individual stocks permit more powerful tests of whether factors are priced (Ang et al., 2008). As a result, we decided to proceed with individual stocks.

In order to find the data, we used The Center for Research in Security Prices (CRSP) from Wharton Research Data Services (WRDS). This database provides us with returns of individual securities traded on the NYSE, the AMEX, or The Nasdaq markets. This database includes more than 10000 stocks that were listed in different time periods. We analysed 3000 stocks and chose only 100 US securities that have information from January of 1986 to December of 2018, which gave us 396 periods. Other securities don't have information exactly for all periods from January of 1986 to December of 2018, so companies with missed data for one of the periods were excluded. On the one hand, it is enough to take 100 random stocks from the perspective of econometric science, as we have maximum four independent variables. On the other hand, it is better to analyse all stocks and choose stocks that have data for 1986-2018 years or take the portfolios' returns which can also perfectly present the whole market, as 100 securities can be not enough to represent the market. We will proceed with both chosen 100 stocks and portfolios data in order to capture the whole market and to define which is a better data approach in our model.

This empirical research is based on monthly Holding Period Returns including dividends. The HPR for each company's stock is calculated by using the next formula:

$$HPR_{t,t+1} = \frac{D_{t+1} + P_{t+1}}{P_t} - 1 \ (18),$$

where D_{t+1} is the dividends paid out, P_t and P_{t+1} are the prices of stock for 2 periods.

The CAPM includes risk-free rate and market portfolio which should consist of all companies (assets) in the world. Since it is impossible to consider all companies, the market proxy is always used. As it has already been described in the theory section, the common market proxy is value-weighted S&P Composite Index (Gómez, 2003). Nevertheless, we used Kenneth French's data library to extract market proxy and the risk-free rate. In this case, market proxy is value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares and price data at the beginning of t, and good return data for t. Generally, Fama and MacBeth (1973) used the equal-weighted NYSE portfolio as a proxy for the market portfolio, although in theory, the market portfolio should be the value-weighted portfolio of total investors wealth which includes human capital and other assets not tradeable or readily measurable (Sylvain, 2013). That's why we decided to proceed with the return of the value-weighted market portfolio. Moreover, Bartholdy (2005) proved that for the estimation of beta, it is irrelevant whether or not dividend adjusted indexes are used. So, we will use a value-weighted market portfolio including dividends. Risk-free is the one-month Treasury bill rate. All returns are presented in percentage.

As it was discussed early in this section, we will use securities that have information from January of 1986 to December of 2018, which gave us 396 periods. On the one hand, on the first stage of our model, we will run time-series regressions in order to generate estimated betas for selected securities. In general, for estimation, the more observations, the better. This suggests using as long time period as possible. Bai and Zhou (2015) demonstrated analytically and using Fama Macbeth two-step risk premia estimates that the standard OLS estimators can contain large bias when the time series sample size is small. On the other hand, with a long estimation period for the beta, however, it is likely that the value of the true beta changes over the period. The resulting estimate for the beta will, therefore, be biased (Bartholdy, 2005). That fact motivated us to short the period. One way of obtaining more observations, over a shorter time period, is to increase the sampling frequency. However, moving from monthly to daily returns, for example, results in an increase in the amount of noise in the data, which reduces the efficiency of the estimates. Bartholdy (2005) suggested that using 5 years of monthly data appears to be appropriate. His findings were based on a comparison of R-square for models with different time horizons and frequency of data.

That's why we will run Fama Macbeth two-step procedure for the whole sample (396 periods) and for 5 year-horizons separately. We will use adjusted R-square to rank different model specification and this criterion will be crucial for choosing the best model specification. Adjusted R-square is better than original R-square because it takes into account not only goodness of fit statistics, but also the loss of degrees of freedom associated with adding extra variables (Brooks, 2019). The significance of β_{1i}, β_{2i}, β_{3i}, β_{4i} and correct signs will also be noted as this provides a necessary condition for the model to be of any use.

In order to obtain portfolio data, we used Kenneth French's data library. This database provides us with value weighted monthly returns of 32 Portfolios Formed on Size, Operating Profitability, and Investment. We used monthly returns from July 1963 to January 2019 (667 periods), since it was proven previously in our research that the more observations, the better. That's why it suggests using as long time period as possible.

The portfolios, which are constructed at the end of each June, are allocated to two Size groups (Small and Big) using NYSE median market cap breakpoint. Stocks in each Size group are allocated independently to four operating profit groups (Low OP to High OP for fiscal year t-1) and four Investment groups (Low Inv to High Inv for fiscal year t-1) using NYSE quartile breakpoints specific to the Size group. Operating profit for June of year t is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in t-1. Investment for June of year t is the change in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1, divided by t-2 total assets. The portfolios for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for June of t, (positive) book equity data for t-1, total assets data for t-2 and t-1, non-missing revenues data for t-1, and non-missing data for at least one of the following: cost of goods sold, selling, general and administrative expenses, or interest expense for t-1 (Kenneth R. French, 2019).

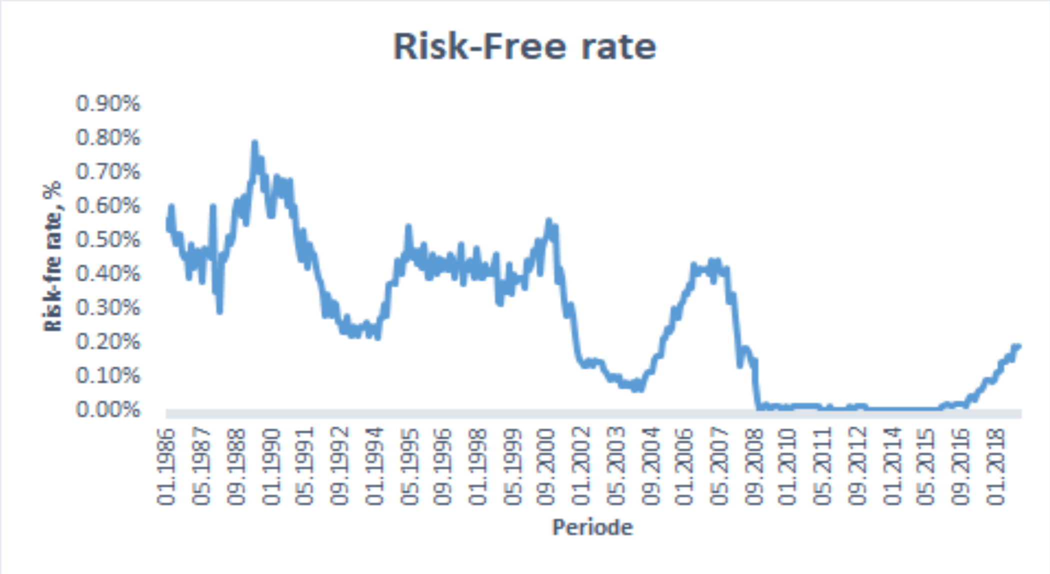
The column titled "SMALL LoOP LoINV" contains the return in year t on a value-weighted portfolio that consists of the US stocks with small size (bottom 50%), low profitability (bottom 25%), and low investment (bottom 25%) in year t- 1 (the low investment means low growth rate of total assets from year t-2 to year t-1). The next column is the return on a value-weighted portfolio with small size (bottom 50%), low profitability (bottom 25%) and the second lowest investment group (firms with investment in the 25-50 percentile) in year t-1, the following column is of firms with small size, low profitability and thirds lowest investment (firms with investment in the 50-75 percentile), the next one is small size, low profitability and firms in the top 25% percentile of investment), the next one is small firms, profitability in the 25-50 percentile, bottom 25% investment, etc (Computer assignment from course GRA 6534 Investments).

4.2 Descriptive statistics

The data for our research includes 100 US companies from different industries such as banking, investment, oil and gas, furniture and others. All descriptive statistics for stocks are presented in the Appendix. The average risk-free rate for 1986-2018 years is 0.26% with variation from 0.00% to 0.79% (see Figure 4). Market return varied from -22.64% to 12.89% with mean value of 0.91% (see Figure 5). The average return of all companies for this period is equal to 1.28%, although there is a large variance between the maximum value of 60.17% and minimum value -37.01%. The average standard deviation of all companies is 10.49%. The highest volatility of returns (31.82%) is presented in Immunomedics International company from Biotech and Pharma industry when Adam Express Co from Investment industry has the lowest standard deviation (4.47%). Also, the returns of these companies for 1986-2018 have average skewness of 0.57 and average kurtosis 9.20, which may signalize about non-normality of data, but we will check it in the next sections (see Figure 6 and 7).

Figure 4: Dynamic of monthly risk-free rate during 1986-2018 years

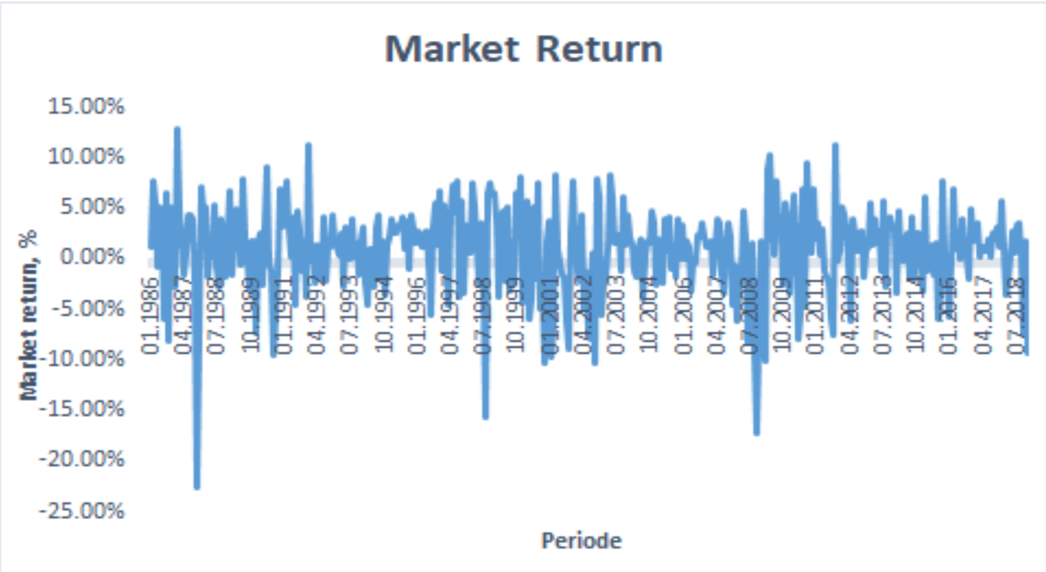
Figure 4 presents the monthly measured average risk-free rate for 1986-2018 years in the US. Risk-free is the one-month Treasury bill rate. All returns are presented in percentage.



Source: Kenneth French's data library

Figure 5: Dynamic of monthly market return during 1986-2018 years

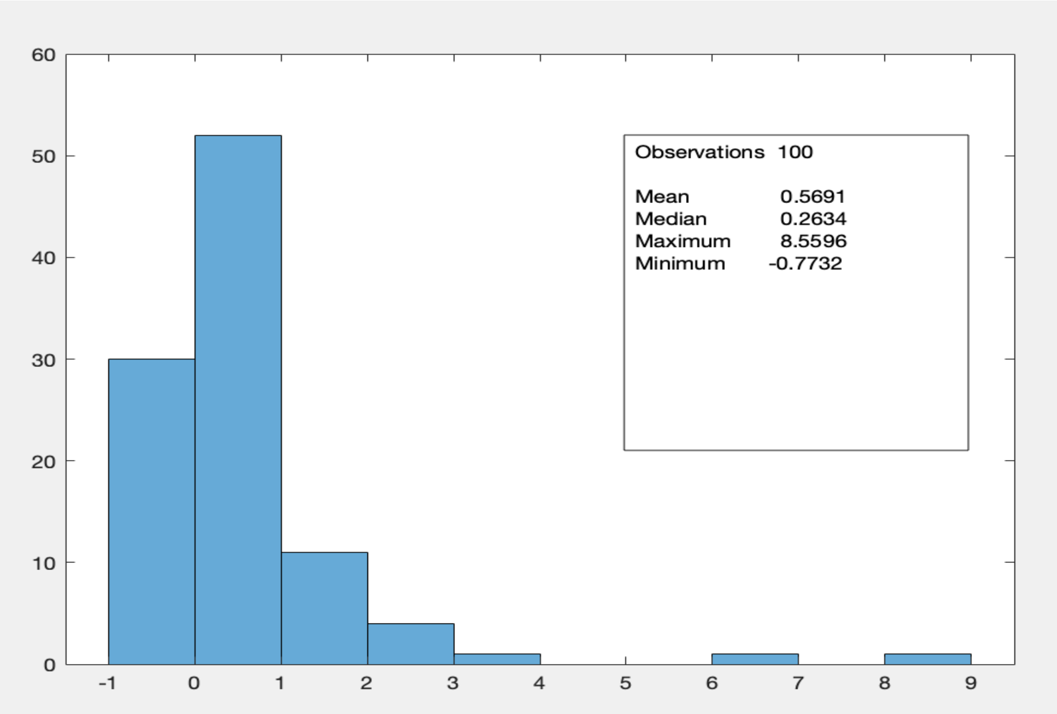
Figure 5 presents the monthly market return during 1986-2018 years. The market return is the value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares, and price data at the beginning of t, and good return data for t.



Source: Kenneth French's data library

Figure 6: Histogram of returns' skewness during 1986-2018 years

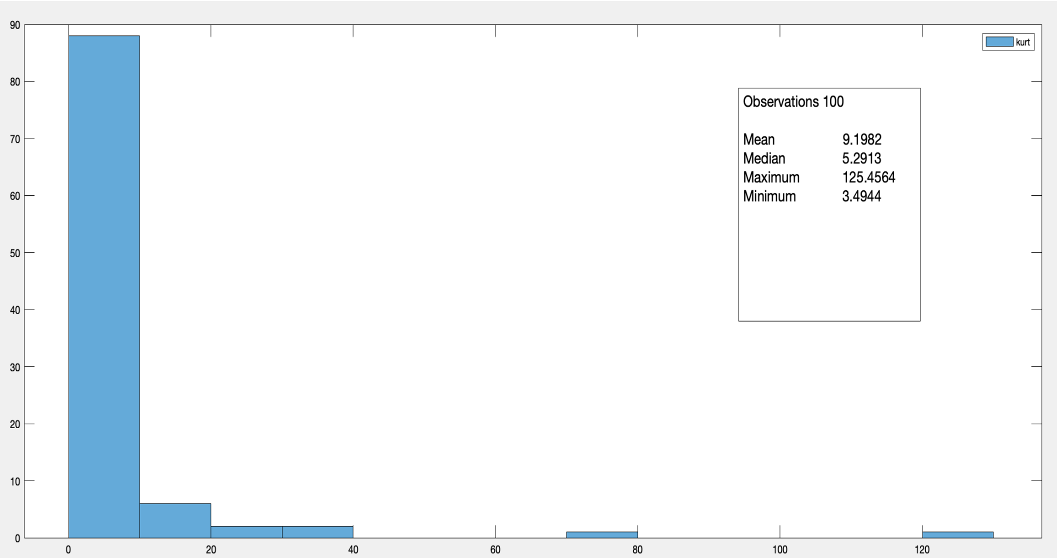
Figure 6 presents the calculated skewness for 100 individual stocks based on historical data 1986-2018 years.



Source: based on data from the Center for Research in Security Prices (CRSP)

Figure 7: Histogram of returns' kurtosis during 1986-2018 years

Figure 7 presents the calculated kurtosis for 100 individual stocks based on historical data 1986-2018 years.



Source: based on data from the Center for Research in Security Prices (CRSP)

Table with descriptive statistics for 32 value-weighted portfolios is presented in the Appendix.

4.3 Preliminary analysis

As described in the section about methodology, we will run the model based on the Fama MacBeth procedure. In order to do it, first, excess returns are calculated in Excel by the following formulas: $R_{it} - R_{ft}$- the difference between the stock return of each company and risk-free rate; $R_{mt} - R_{ft}$- the difference between the return of the market portfolio and the risk-free rate.

In the model, we will use the second equation above to calculate stochastic discount factors for skewness and kurtosis. To sum up, the main goal of this research is to check whether the higher moments influence the expected return by the investor. We defined four steps and potential results of them:

1. We will run time-series models for each company in order to generate betas.
2. We will put these generated betas in cross-section CAPM to receive risk-premium for systematic volatility. We expect to get a significant and positive risk-premium estimate and significant intercept to show that the volatility is not only one factor that explains expected return.
3. If the second step holds, we will repeat the first step adding stochastic discount factors for systematic skewness and systematic kurtosis.
4. In the last step, we will put the betas from previous models to generate risk-premiums for different risk factors. We expect to receive insignificant intercept, which means that we included all factors that explain expected return. Also, the risk-premium for systematic skewness should have a negative sign, but the risk-premium for systematic kurtosis should have a positive sign.
5. All these steps will be done for different time-periods and we will also consider the interaction term between systematic skewness and systematic kurtosis in order to find the best model. Moreover, we will use stock data and portfolios data separately. All steps will be done in order to choose the best model specification.

5. Results and analysis

As it was described in the previous sections, the main goal of our research is to define the relationships between the expected return and higher-order moments. In this section, we will describe the main results of our models. Firstly, we will check our data on normality. Secondly, we will run the CAPM with only skewness and kurtosis, and after we will expand this model with the interaction term. Also, we will consider different time-periods in the models.

5.1 Test for normality

As we have already explained in the section about the value of the normality test, we need to do it to be sure that the data is abnormal. It is the main point of our model. In order to check that point, we used Jargue-Bera test and conducted the test in both Matlab and Excel. Firstly, we calculated skewness and kurtosis for each stock in Excel. Secondly, we used them to define Jargue-Bera t-statistics based on the formula from the previous section. Thirdly, we found critical value for 90%, 95%, and 99% confidence intervals and rejected the null hypothesis about normality when Jargue-Bera t-statistics were higher than critical values. These results are presented in the Appendix. We can see that it was rejected the null hypothesis about normality for 99 stocks out of 100 based on 99%-confidence interval and for one stock we rejected the null hypothesis based on 90%-confidence interval. In addition, we repeated the same in Matlab and proved that data is abnormal for 99% of stocks. As a result, due to the non-normality of data, we can move to the model.

5.2 CAPM with skewness and kurtosis

This section presents and discusses the results of our research based on the CAPM and the expansion of this model with skewness and kurtosis in order to check whether these stochastic factors influence expected stock returns. It is done based on the methodology described above. It is used the returns of 100 stocks for 396 time periods for regressions.

Firstly, we run the time-series regression for each stock where excess returns of stock depend on excess market returns. The results of 100 regressions are presented in the Appendix and summary is shown in Table 1 below.

Table 1: Summary of generated market prices for a systematic variance for 100 stocks

This table reports a summary of the estimated market prices (β_i) for a systematic variance under the two-moment CAPM based on time-series regressions for each of 100 stocks during 1986-2018. It includes mean, maximum value, minimum value and standard deviation for all betas.

	Mean	Min	Max	Std.Dev
Two-Moment CAPM: $R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it}$				
β_i	1.022	0.296	2.355	0.404

Notes: $R_{it} - R_{ft}$ denotes excess return of asset i; $R_{mt} - R_{ft}$ is a market excess return; β_i and α_i are parameters estimates.

We can see that all betas are significantly different from zero on 99 % - confidence interval (see the Appendix). We will use these betas for the next step in order to generate a risk premium for holding systematic variance (see Table 2).

Table 2: Output from running traditional CAPM for generating risk premium for holding systematic variance

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step of time-series regressions for 1986-2018. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1 \widehat{\beta_i} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00341	0.00109	3.12080	0.00237
γ_1	0.00665	0.00665	6.68790	0.00000

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.004

R-squared: 0.313, Adjusted R-Squared: 0.306

F-statistic vs. constant model: 44.7, p-value = $1.4 * e^{-09}$

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

From Table 2, we see that systematic risk explains the excess return of chosen companies (p-values is approximately equal to zero). Moreover, systematic risk has a positive impact on the excess return of companies, which is compliant with economic theory. Furthermore, the systematic variance is not the only factor that explains the average excess return of companies, since we have significantly different from zero intercept on 99 % - confidence level (p-value is 0.002). So, we have found empirical evidence that the traditional CAPM should be extended with additional factors due to omitted variables problem. The R-squared is equal to 31.3%, which could seem too high, but according to the analysis of traditional CAPM models based on CRSP securities for 1929-2004 years, the R-squared varied from approximately 0.1% to 40% (Sanchez, 2015). As a result, we can conclude that this value of R-squared is relevant.

Secondly, we run a cubic market model consistent with four-moment CAPM derived by Fang and Lai (1997) for each stock separately to extend the model with systematic skewness and systematic kurtosis. Market prices of the systematic variance, systematic skewness and systematic kurtosis obtained by running time-series regressions for each stock are presented in the Appendix and summary is shown in Table 3 below.

Table 3: Summary of generated market prices for systematic variance, systematic skewness and systematic kurtosis for 100 stocks

This table reports a summary of the estimated market prices for systematic variance (β_{1i}), systematic skewness (β_{2i}) and systematic kurtosis (β_{3i}) under the four-moment CAPM based on time-series regressions for each of 100 stocks during 1986-2018. It includes mean, maximum value, minimum value and standard deviation for all betas.

	Mean	Min	Max	Std.Dev
Four-Moment CAPM: $R_{it} - R_{ft} = \alpha_i + \beta_{1i}(R_{mt} - R_{ft}) - \beta_{2i}(R_{mt} - R_{ft})^2 + \beta_{3i}(R_{mt} - R_{ft})^3 + \varepsilon_{it}$				
β_{1i}	0.967	0.275	2.473	0.450
β_{2i}	0.739	-9.074	9.892	2.571
β_{3i}	7.254	-33.109	64.333	14.019

Notes: $R_{it} - R_{ft}$ denotes excess return of asset i; $R_{mt} - R_{ft}$ is a market excess return; $(R_{mt} - R_{ft})^2$ are $(R_{mt} - R_{ft})^3$ are stochastic discount factors in the market return.

The betas from Appendix will be used for running cross-sectional regression in order to find risk premiums that investors will require for holding stocks considering variance, skewness, and kurtosis. The results of this regression are shown in the Table 4.

Table 4: Output from running expanded CAPM for generating risk premiums for holding systematic variance, systematic skewness and systematic kurtosis

This table reports the estimated risk premium for holding systematic variance, systematic skewness and systematic kurtosis under the four-moment CAPM based on cross-section regression and betas generated in the previous step of time-series regressions for 1986-2018. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Four-Moment CAPM: $r_i = \gamma_0 + \gamma_1 \widehat{\beta_{1\iota}} - \gamma_2 \widehat{\beta_{2\iota}} + \gamma_3 \widehat{\beta_{3\iota}} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00237	0.00133	1.77850	0.07849
γ_1	0.00745	0.00119	6.25090	0.00000
γ_2	-0.00050	0.00027	-1.83110	0.07019
γ_3	0.00014	0.00005	2.80130	0.00616

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.00399

R-squared: 0.328, Adjusted R-Squared: 0.307

F-statistic vs. constant model: 15.6, p-value = $2.43 * e^{-08}$

Notes: r_i denotes average excess return of company i;$\widehat{\beta_\iota}$, $\widehat{\beta_{2\iota}}$, $\widehat{\beta_{3\iota}}$ depict systematic variance, systematic skewness, and systematic kurtosis, respectively; γ_1, γ_2, γ_3 are systematic market risk premia for an increase in systematic variance, a decrease in systematic skewness, and an increase in systematic kurtosis, respectively. All rates in the table are measured in percent per month.

In order to interpret the results of this model, we need to check the next five assumptions about the unobservable error term which were described in the section about model diagnostics tests.

Firstly, the mean of the residuals is equal to zero, as there is an interaction term in the model. In addition, we conducted the test and found out that we can't reject the null hypothesis about a zero mean of the residuals. So, the first assumption is not violated. Secondly, we need to reject the null hypothesis about homoscedasticity of the residuals' variance, as we received p-value of 0.00002 in White's test. So, the model has a heteroscedasticity problem. In order to solve the problem, we will apply White's heteroscedasticity consistent standard error estimates. The new results are presented in Table 5. Due to fixing the heteroscedasticity problem, we received changes in the interpretation of the estimators. Systematic skewness became insignificant although it was significantly different from zero on 90%-confidence interval.

Table 5: Output from running expanded CAPM for generating risk premium for holding systematic variance, systematic skewness, and systematic kurtosis respectively with White's heteroscedasticity consistent standard error estimates

This table reports new values of standard errors, test-statistics and p-values due to conducting White's heteroscedasticity consistent standard error estimates in order to eliminate the heteroscedasticity problem in the model. The estimators are the same as in the previous table.

White's heteroscedasticity consistent standard error estimates

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00237	0.00141	1.67700	0.09674
γ_1	0.00745	0.00133	5.59620	0.00000
γ_2	-0.00050	0.00035	-1.43300	0.15504
γ_3	0.00014	0.00008	1.76590	0.08052

Thirdly, we can conclude that there is no pattern between disturbances (residuals) as the p-value is equal to 0.7250, so we cannot reject the null-hypothesis about zero autocorrelation in Breusch-Godfrey test. In addition, there is no correlation between residuals and independent variables, so the independent variables are not stochastic (see Table 6).

Table 6: Correlation matrix between the residuals and the independent variables

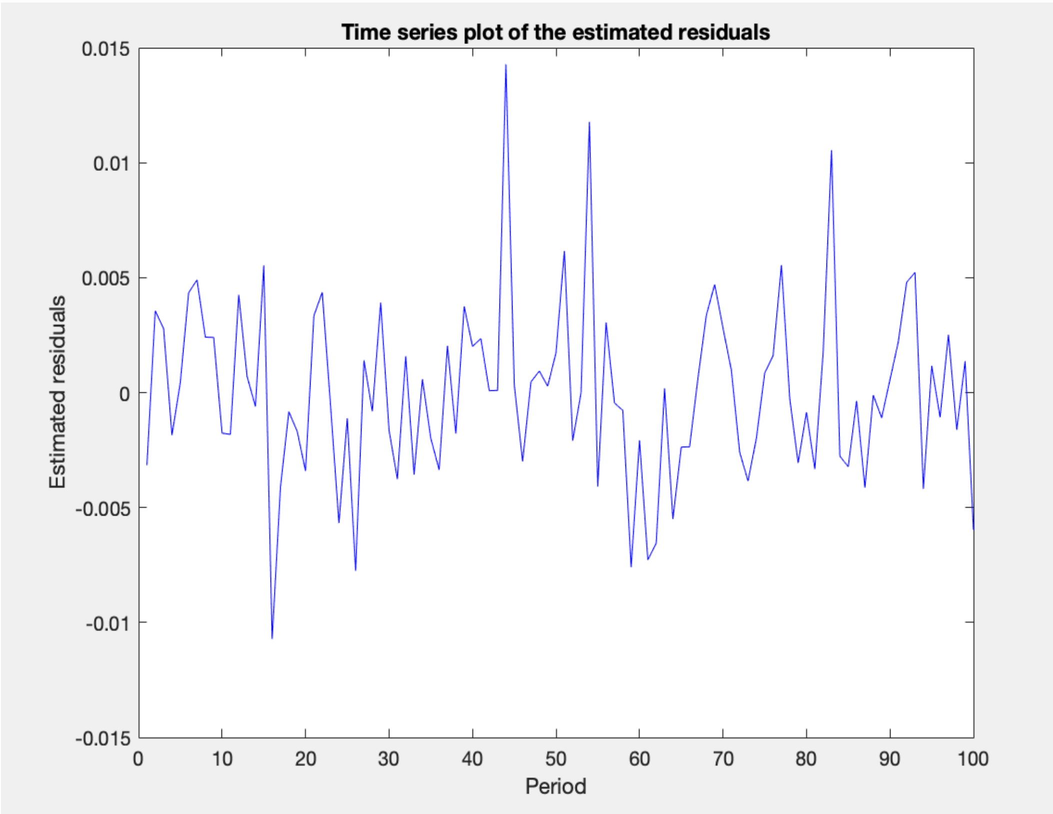
This table reports the values of correlation between the residuals and the independent variables such as systematic variance, systematic skewness, and systematic kurtosis in order to check the third assumption about the residuals in the model.

Correlation matrix			
	Systematic variance	Systematic skewness	Systematic kurtosis
Residuals	-0.1187*e^{-15}	0.2356*e^{-15}	0.4067*e^{-15}

Nevertheless, the results of Jarque-Bera test showed that the disturbances are not normally distributed as p-value is equal to 0.00013 and we should reject the null hypothesis about the normal distribution of the residuals. Also, the histogram and time series plot of the estimated residuals are presented in Figures 8 and 9 that proves non-normality of the residuals (positive-skewed disturbances).

Figure 8: Time series plot of the estimated residuals in the model

The figure presents the estimated residuals of the model for different US-listed stocks.



Finally, we need to check the independent variables on multicollinearity with the correlation matrix. Based on the results presented in Table 7, we can conclude that there is no problem of multicollinearity between the explanatory variables. The correlation between systematic skewness and systematic kurtosis achieves 67%, but it is not too high number for model

Figure 9: Histogram of the estimated residuals in the model

The figure presents the frequency of different estimated residuals for US-listed stocks in the model.

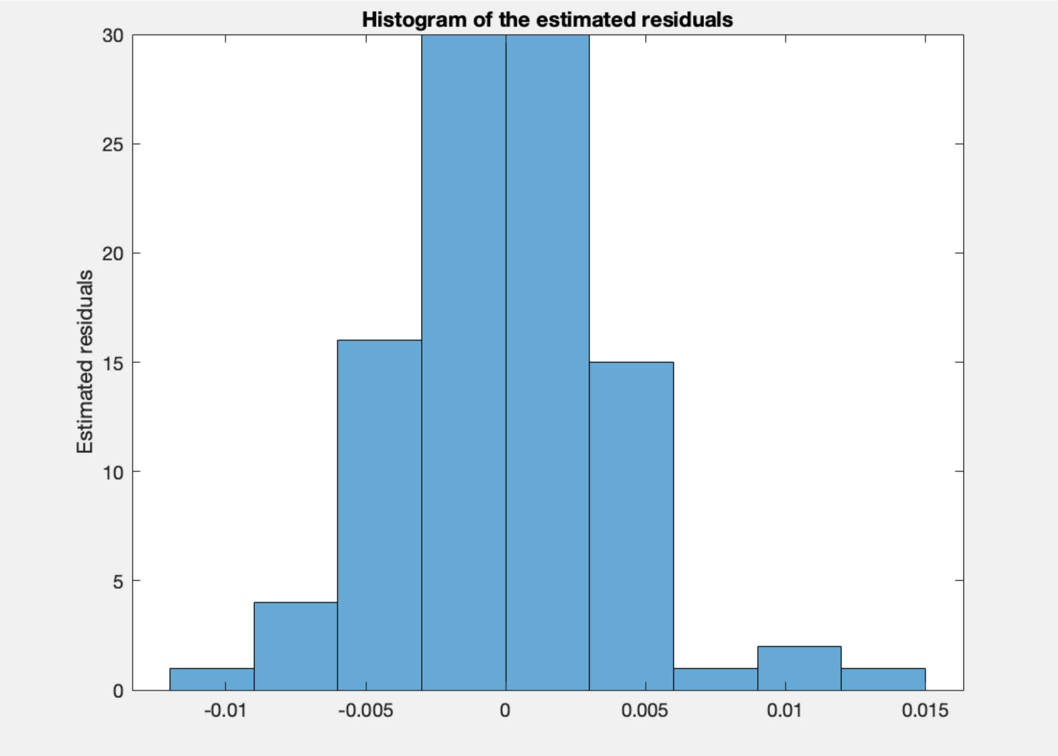


Table 7: Correlation matrix of independent variables

This table reports the values of correlation between the independent variables such as systematic variance, systematic skewness, and systematic kurtosis in order to check the problem of multicollinearity in the model.

Correlation matrix			
	Systematic variance	Systematic skewness	Systematic kurtosis
Systematic variance	**1.0000**	0.2759	-0.2626
Systematic skewness	0.2759	**1.0000**	0.6706
Systematic kurtosis	-0.2626	0.6706	**1.0000**

After conducting all diagnostics tests, we can sum up that this model has some problems such as heteroscedasticity of the residuals which was fixed and abnormal distribution of the residuals. Nonetheless, all other assumptions such as zero residuals' mean, zero autocorrelation between residuals, zero correlation between the independent variables and residuals are not violated. Moreover, independent variables do not have a high level of correlation too.

After conducting White's heteroscedasticity consistent standard error estimates in order to solve the problem of heteroscedasticity, we can interpret the results of this model in a such way: the systematic market risk premia for an increase in systematic variance is significantly different from zero on 99%-confidence interval and positively correlated with expected return by investors; the systematic market risk premia for a decrease in systematic skewness is not significantly different from zero; and the systematic market risk premia for an increase in systematic kurtosis is significantly different from zero on 90%-confidence interval and positively correlated with expected return, which means that investors will require a higher premium for stock with excess kurtosis. Moreover, the intercept in the model is not significantly different from zero on 95%-confidence interval as p-value is equal to 0.097, which means that there are no other factors that also define expected excess return of the company. Nevertheless, if we take 90%-confidence interval, we will conclude that the intercept is not zero. In addition, these factors explain the average excess return of companies on 31 % (look at adjusted R-square).

To sum up, as this model has the problem in diagnostics test and not all factors are significant on a 95%-confidence interval, we need to try other specification of the model, which is likely to give a better result. One of such specifications is CAPM with skewness, kurtosis, and an interaction term between them, which is presented in the next section.

5.3 CAPM with skewness, kurtosis, and an interaction term between them (1986-2018)

This section presents and contains a discussion of the results of our research based on the expanded CAPM with skewness, kurtosis and interaction term between them. It is used the excess monthly return of 100 US companies for 1986-2018 (396 periods) in regressions.

Firstly, on the previous stage of our research, we have found empirical evidence that systematic risk is not the only factor that explains the excess return of companies.

Secondly, we will extend traditional CAPM with systematic skewness, systematic kurtosis and an interaction term between these risk factors. In order to do that, we will run 100 cubic market models consistent with four-moment CAPM derived by Fang and Lai (1997) for each stock separately. Market prices of the systematic variance, systematic skewness, systematic kurtosis, and interaction term between systematic skewness and kurtosis, respectively for each stock are presented in the Appendix. The summary for each type of betas is shown in Table 8. These betas are obtained by running time-series regressions and only depict different risk factors since stochastic discount factors in the market return were used.

Table 8: Summary of generated market prices for systematic variance, systematic skewness, systematic kurtosis and an interaction term for 100 stocks

This table reports a summary of the estimated market prices for systematic variance (β_{1i}), systematic skewness (β_{2i}), systematic kurtosis (β_{3i}) and the interaction term between systematic skewness and systematic kurtosis (β_{4i}) based on time-series regressions for each of 100 stocks during 1986-2018. It includes mean, maximum value, minimum value and standard deviation for all betas.

	Mean	Min	Max	Std.Dev
Four-Moment CAPM with interaction term:				
$R_{it} - R_{ft} = \alpha_i + \beta_{1i}(R_{mt} - R_{ft}) - \beta_{2i}(R_{mt} - R_{ft})^2 + \beta_{3i}(R_{mt} - R_{ft})^3 + \beta_{4i}(R_{mt} - R_{ft})^2(R_{mt} - R_{ft})^3 + \varepsilon_{it}$				
β_{1i}	0.967	0.298	2.710	0.456
β_{2i}	0.739	-9.023	9.526	2.553
β_{3i}	7.339	-112.256	114.057	35.191
β_{4i}	-1.661	-1484.060	1824.518	585.182

Notes: $R_{it} - R_{ft}$ denotes excess return of asset i; $R_{mt} - R_{ft}$ is a market excess return; $(R_{mt} - R_{ft})^2$ are $(R_{mt} - R_{ft})^3$ are stochastic discount factors in the market return.

We will use these estimated coefficients for running cross-sectional regression to find risk premiums that investors should demand for holding stocks with variance, skewness, kurtosis, and interaction term between skewness and kurtosis (see Table 9).

Before we can interpret the results of the model, we should check for holding of all assumptions underlying the classical linear regression model. So, we will check the validity and adequacy of the model.

Table 9: Output from running expanded CAPM for generating risk premium for holding systematic variance, systematic skewness, systematic kurtosis, and an interaction term between systematic skewness and kurtosis respectively

This table reports the estimated risk premium for holding systematic variance, systematic skewness, systematic kurtosis, and systematic skewness and systematic kurtosis at once under the four-moment CAPM with interaction term based on cross-section regression and betas generated in the previous step of time-series regressions for 1986-2018. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Four-Moment CAPM with interaction term:

$$r_i = \gamma_0 + \gamma_1\widehat{\beta_{1\iota}} - \gamma_2\widehat{\beta_{2\iota}} + \gamma_3\widehat{\beta_{3\iota}} + \gamma_4\widehat{\beta_{4\iota}} + \varepsilon_{it}$$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00135	0.00127	1.06830	0.28808
γ_1	0.00829	0.00113	7.33330	0.00000
γ_2	-0.00059	0.00025	-2.32520	0.02219
γ_3	0.00018	0.00005	3.74680	0.00031
γ_4	0.00001	0.00000	3.91450	0.00017

Number of observations: 100, Error degrees of freedom: 95

Root Mean Squared Error: 0.00372

R-squared: 0.423, Adjusted R-Squared: 0.398

F-statistic vs. constant model: 17.4, p-value = $9.88 * e^{-11}$

Notes: r_i denotes average excess return of company i; $\widehat{\beta}_i$, $\widehat{\beta_{2\iota}}$, $\widehat{\beta_{3\iota}}$, $\widehat{\beta_{4\iota}}$ depict systematic variance, systematic skewness, systematic kurtosis, and an interaction term between systematic skewness and kurtosis respectively; γ_1,γ_2, γ_3, γ_4 are systematic market risk premia for an increase in systematic variance, a decrease in systematic skewness, and an increase in systematic kurtosis, and an increase in interaction between systematic skewness and systematic kurtosis, respectively. All rates in the table are measured in percent per month.

The mean of the residuals is equal to zero because there is a constant term in the regression. Also, we rejected the assumption that the variance of the residuals is constant on 99 % - confidence interval (p-values is equal to 0.0025). That means that our model faces with heteroscedasticity problem. We will fix this problem by applying White's heteroscedasticity consistent standard error estimates (see Table

10). Moreover, there is no pattern between disturbances (residuals) (p-value is equal to 0.7027 and we cannot reject the null-hypothesis in Breusch-Godfrey test that there is zero autocorrelation between error terms). Furthermore, there is no correlation between the residuals and the independent variables (see Table 11). So, the fourth assumption is not violated. Finally, the disturbances are normally distributed (p-value is equal to 0.1957 and we cannot reject the null in Jarque-Bera test) and the histogram and time series plot of the estimated residuals are presented in Figures 10 and 11. These two figures graphically prove that the disturbances are normally distributed.

Table 10: Output from running expanded CAPM for generating risk premium for holding systematic variance, systematic skewness, systematic kurtosis, and an interaction term between systematic skewness and kurtosis respectively with White's heteroscedasticity consistent standard error estimates

This table reports new values of standard errors, test-statistics and p-values due to conducting White's heteroscedasticity consistent standard error estimates in order to eliminate the heteroscedasticity problem in the model. The estimators are the same as in the previous table.

White's heteroscedasticity consistent standard error estimates

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00135	0.00146	0.92323	0.35815
γ_1	0.00829	0.00139	5.97970	0.00000
γ_2	-0.00059	0.00030	-1.97310	0.05130
γ_3	0.00018	0.00007	2.54570	0.01247
γ_4	0.00001	0.00000	2.72430	0.00763

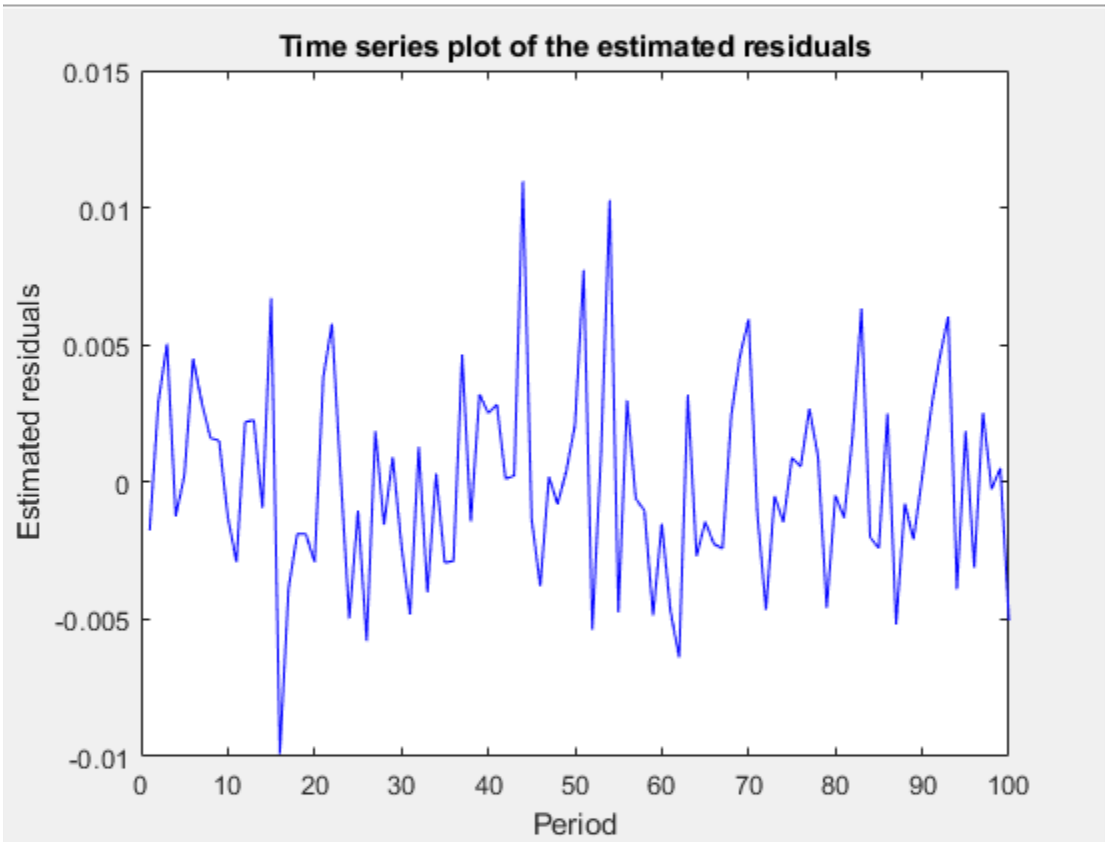
Table 11. Correlation matrix between the residuals and the independent variables

This table reports the values of correlation between the residuals and the independent variables such as systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and systematic kurtosis in order to check the third assumption about the residuals in the model.

Correlation matrix				
	Systematic variance	Systematic skewness	Systematic kurtosis	Interaction term between systematic skewness and kurtosis
Residuals	$-0.0116{*}e^{-14}$	$0.0116{*}e^{-14}$	$0.1058{*}e^{-14}$	$-0.1156{*}e^{-14}$

Figure 10: Time series plot of the estimated residuals in the model

The figure presents the estimated residuals of the model for different US-listed stocks.



Also, we checked the model on multicollinearity between explanatory variables (see Table 12). We can observe high correlation only between systematic kurtosis and interaction term between systematic skewness and kurtosis. Nevertheless, in our case high correlation between these two risk factors can be ignored because interaction term directly consists of systematic kurtosis.

Figure 11: Histogram of the estimated residuals in the model

The figure presents the frequency of different estimated residuals for US-listed stocks in the model.

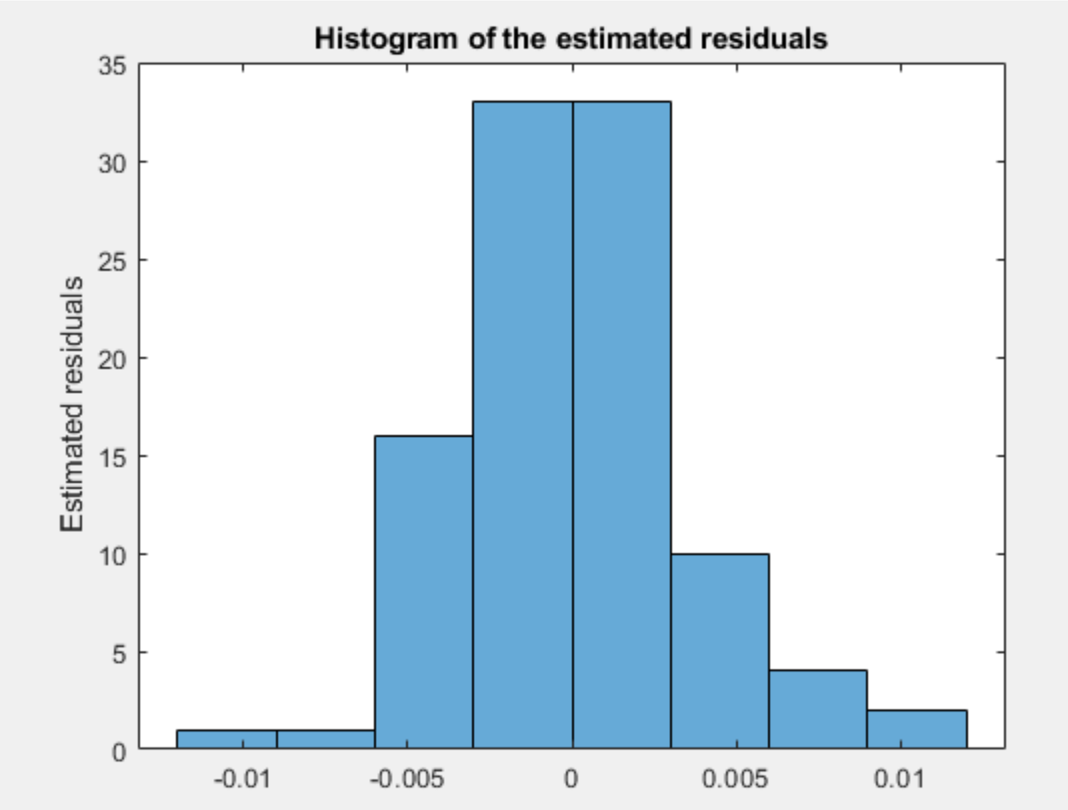


Table 12: Correlation matrix of independent variables

This table reports the values of correlation between the independent variables such as systematic variance, systematic skewness, systematic kurtosis and an interaction term between the systematic skewness and systematic kurtosis in order to check the problem of multicollinearity in the model.

	Systematic variance	Systematic skewness	Systematic kurtosis	The interaction term between systematic skewness and kurtosis
Systematic variance	**1.0000**	0.2429	-0.2870	0.1912
Systematic skewness	0.2429	**1.0000**	0.3331	-0.0806
Systematic kurtosis	-0.2870	0.3331	**1.0000**	-0.9199
Interaction term between systematic skewness and kurtosis	0.1912	-0.0806	-0.9199	**1.0000**

To sum up, we made all the diagnostics tests in order to check the estimators. Only the second assumption about the constant variance of the residuals is violated, but we calculated White's heteroscedasticity consistent standard error estimates in order to solve this problem. We will interpret the model after the implementation of White's heteroscedasticity consistent standard error estimates, because we received new increased standard errors for the coefficients, and we would need more evidence against the null hypothesis before we would reject it. White's heteroscedasticity consistent standard error estimates only change standard errors for coefficients. Firstly, the intercept is equal to zero (p-value is higher than 0.1), which means that we added all factors that define excess return charged by investors. Moreover, our factors explain the average excess return of companies on 40 % (look at adjusted R-square). Secondly, all factors are significantly different from zero on 95 % - confidence interval (p-value for systematic skewness is approximately equal to 0.05). The signs of different risk factors are consistent with theory (systematic variance and systematic kurtosis have positive signs, while systematic skewness has a negative sign).

Based on that we can conclude that CAPM with skewness, kurtosis, and an interaction term between them is a better specification of the model in comparing to the previous one. Firstly, the model with interaction term has higher adjusted R-square, which means a better explanation of the average excess return of companies. Secondly, we have to take into consideration that after calculation of White's heteroscedasticity consistent standard error estimates in the model without interaction term, systematic skewness and systematic kurtosis are equal to zero on 95 % - confidence interval. So, these two factors do not explain the average excess return of companies. Nevertheless, in this model, the systematic skewness becomes significant after adding the interaction term. It is caused by considering both systematic skewness and systematic kurtosis together. In other words, this interaction term helps to find how the required return by the investors will be changed when the stock has the problems of skewness and excess kurtosis together. Based on Appendix, we can say that most of the stocks have skewness and excess kurtosis simultaneously. That's why considering the interaction term between systematic skewness and systematic kurtosis improved our model and made the factor of systematic skewness significant.

Moreover, the results of the investigation show that investors expect a lower return when the distribution of stock returns demonstrates positive co-skewness.

The economic interpretation of this phenomena is very important. Our model specifications can also be viewed as competing approximations for the discount factor or the intertemporal marginal rate of substitution. A beta coefficient near co-skewness can be considered as relative risk aversion. A negative beta implies that with an increase in the next period's market return, the marginal rate of substitution declines. This decline in the marginal rate of substitution is consistent with decreasing marginal utility. According to Arrow (1964), nonincreasing absolute risk aversion is one of the essential properties for a risk-averse individual. Nonincreasing absolute risk aversion for a risk-averse utility-maximizing agent can also be linked to prudence as defined by Kimball (1990). Prudence relates to the desire to avoid disappointment and is usually linked to the precautionary savings motive. Nonincreasing absolute risk aversion implies that in a portfolio an increase in total skewness is preferred. Since adding an asset with negative co-skewness to a portfolio makes the resultant portfolio more negatively skewed (i.e., reduces the total skewness of the portfolio), assets with negative co-skewness must have higher expected returns than assets with identical risk-characteristics but zero-co-skewness. Thus, in a cross section of assets, the slope of the excess expected return on conditional co-skewness with the market portfolio should be negative. Thus, the premium for skewness risk over the risk-free asset's return (assuming that the risk-free asset possesses zero betas with respect to all the factors being examined to explain the cross-section of returns) should also be negative (Harvey &Siddique, 2000).

To sum up, the second model with the interaction term is better and has a practical implication in investors' relationship. That's why in the next section we will use this model to explore different time-horizons.

5.4 CAPM with skewness, kurtosis, and an interaction term between them for 5 year-horizons

In this section, we will repeat exactly the same procedure as in the previous section. The only difference is that we will use 5 year-horizons in order to find the best specification of the model. Based on the theory, there is an assumption that the model can give better results if it is based on 5 year-horizons. So, we will check it and repeat all steps from the previous section 5 times (1993-1997, 1998-2002, 2003-2007, 2008-2012, 2013-2017 year). Each model will have 60 observations.

The common feature for different time horizons: percentage of abnormal data (vary from 34 percent to 53 percent) and percentage of significant betas on the first stage is relatively low (if to compare with full data sample where we have 100 % of significant betas). The first phenomenon looks strange, as theoretically the more sample size, the more probably sample distributions will follow a normal distribution. We observed a reversed trend. 99 % of monthly stock returns are non-normally distributed when we increase the sample size to 396 periods. Now we will consider 5 different periods separately.

1993-1997. We can see that systematic risk explains the excess return of chosen companies (p-values is approximately equal to zero) (see Table 13). Furthermore, the systematic variance is not the only factor that explains the average excess return of companies, since we have significantly different from zero intercept on 95 % - confidence level (p-value is 0.02). After expanding traditional CAPM with systematic skewness, systematic kurtosis and an interaction term between these risk factors, we can observe that systematic skewness is equal to zero (p-value is equal to 0.5) and has a wrong positive sign (see Table 14).

Table 13: Output from running traditional CAPM for generating a risk premium for holding systematic variance (1993-1997)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step of time-series regressions for 1993-1997. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1 \widehat{\beta_i} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00504	0.00213	2.36540	0.01998
γ_1	0.01051	0.00198	5.31110	0.00000

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.0115

R-squared: 0.224, Adjusted R-Squared: 0.216

F-statistic vs. constant model: 28.2, p-value = $6.81 * e^{-07}$

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

Table 14: Output from running expanded CAPM for generating risk premiums for holding systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and kurtosis (1993-1997)

This table reports the estimated risk premium for holding systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and systematic kurtosis at once under the four-moment CAPM with interaction term based on cross-section regression and betas generated in the previous step of time-series regressions for 1993-1997. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Four-Moment CAPM with interaction term:

$$r_i = \gamma_0 + \gamma_1 \widehat{\beta_{1\iota}} - \gamma_2 \widehat{\beta_{2\iota}} + \gamma_3 \widehat{\beta_{3\iota}} + \gamma_4 \widehat{\beta_{4\iota}} + \varepsilon_{it}$$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00485	0.00219	2.20840	0.02962
γ_1	0.01071	0.00207	5.17880	0.00000
γ_2	0.00007	0.00010	0.67536	0.50109
γ_3	0.00003	0.00001	4.39010	0.00003
γ_4	0.00000	0.00000	3.14470	0.00222

Number of observations: 100, Error degrees of freedom: 95

Root Mean Squared Error: 0.0116

R-squared: 0.232, Adjusted R-Squared: 0.199

F-statistic vs. constant model: 7.16, p-value = $4.41 * e^{-05}$

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$, $\widehat{\beta_{2\iota}}$, $\widehat{\beta_{3\iota}}$, $\widehat{\beta_{4\iota}}$ depict systematic variance, systematic skewness, systematic kurtosis, and an interaction term between systematic skewness and kurtosis respectively; γ_1,γ_2, γ_3, γ_4 are systematic market risk premia for an increase in systematic variance, a decrease in systematic skewness, and an increase in systematic kurtosis, and an increase in interaction between systematic skewness and systematic kurtosis, respectively. All rates in the table are measured in percent per month.

1998-2002. We can see exactly the same results for traditional CAPM as in 1993-1997 (see Table 15). In expanded CAPM we observe that all additional risk factors are not significantly different from zero (see Table 16). That means that systematic risk is not the only factor that explains the average excess return of companies, but expanded CAPM with systematic skewness, systematic kurtosis and an interaction term between them looks like a wrong model specification for that time period (all additional factors do not contain any significantly different from zero information).

Table 15: Output from running traditional CAPM for generating a risk premium for holding systematic variance (1998-2002)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step for 1998-2002. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1\widehat{\beta}_i + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	-0.00285	0.00149	-1.91850	0.05795
γ_1	0.01196	0.00143	8.37690	0.00000

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.0107

R-squared: 0.417, Adjusted R-Squared: 0.411

F-statistic vs. constant model: 70.2, p-value = $6.81 * e^{-07}$

Notes: r_i denotes average excess return of company i; $\widehat{\beta}_i$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

Table 16: Output from running expanded CAPM for generating risk premiums for holding systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and kurtosis (1998-2002)

This table reports the estimated risk premium for holding systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and systematic kurtosis at once under the four-moment CAPM with interaction term based on cross-section regression and betas generated in the previous step for 1998-2002. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Four-Moment CAPM with interaction term:

$$r_i = \gamma_0 + \gamma_1\widehat{\beta_{1\iota}} - \gamma_2\widehat{\beta_{2\iota}} + \gamma_3\widehat{\beta_{3\iota}} + \gamma_4\widehat{\beta_{4\iota}} + \varepsilon_{it}$$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	-0.00117	0.00171	-0.68806	0.49309
γ_1	0.00934	0.00168	5.57580	0.00000
γ_2	0.00027	0.00030	0.91347	0.36331
γ_3	0.00005	0.00004	1.29580	0.19817
γ_4	0.00000	0.00000	0.46761	0.64114

Number of observations: 100, Error degrees of freedom: 95

Root Mean Squared Error: 0.01

R-squared: 0.502, Adjusted R-Squared: 0.481

F-statistic vs. constant model: 24, p-value = 1.01e-13 $1.01 * e^{-13}$

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$, $\widehat{\beta_{2\iota}}$, $\widehat{\beta_{3\iota}}$, $\widehat{\beta_{4\iota}}$ depict systematic variance, systematic skewness, systematic kurtosis, and an interaction term between systematic skewness and kurtosis respectively; γ_1,γ_2, γ_3, γ_4 are systematic market risk premia for an increase in systematic variance, a decrease in systematic skewness, and an increase in systematic kurtosis, and an increase in interaction between systematic skewness and systematic kurtosis, respectively. All rates in the table are measured in percent per month.

2003-2007. While we have evidence for extending traditional CAPM with additional risk-factors (see Table 17), adding systematic skewness, systematic kurtosis and an interaction term between them does not bring any significantly different from zero information (see Table 18).

Table 17: Output from running traditional CAPM for generating a risk premium for holding systematic variance (2003-2007)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step for 2003-2007. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1\widehat{\beta_i} + \varepsilon_{it}$				
Estimated Coefficients:				
	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	-0.00285	0.00149	-1.91850	0.05795
γ_1	0.01196	0.00143	8.37690	0.00000

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.0147

R-squared: 0.075, Adjusted R-Squared: 0.0655

F-statistic vs. constant model: 7.94, p-value = 0.00584

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

Table 18: Output from running expanded CAPM for generating risk premiums for holding systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and kurtosis (2003-2007)

This table reports the estimated risk premium for holding systematic variance, systematic skewness, systematic kurtosis and an interaction term between systematic skewness and systematic kurtosis at once under the four-moment CAPM with interaction term based on cross-section regression and betas generated in the previous step of time-series regressions for 2003-2007. It includes estimated coefficients, their standard errors, test-statistics,
and p-values.

Four-Moment CAPM with interaction term:

$$r_i = \gamma_0 + \gamma_1\widehat{\beta_{1i}} - \gamma_2\widehat{\beta_{2i}} + \gamma_3\widehat{\beta_{3i}} + \gamma_4\widehat{\beta_{4i}} + \varepsilon_{it}$$

Estimated Coefficients:				
	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00693	0.00298	2.32410	0.02225
γ_1	0.00444	0.00210	2.10970	0.03752
γ_2	-0.00002	0.00012	-0.19344	0.84703
γ_3	0.00000	0.00001	-0.03059	0.97566
γ_4	0.00000	0.00000	-1.22620	0.22314

Table 18 (continued)

Number of observations: 100, Error degrees of freedom: 95

Root Mean Squared Error: 0.0143

R-squared: 0.154, Adjusted R-Squared: 0.118

F-statistic vs. constant model: 4.31, p-value = 0.003

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$, $\widehat{\beta_{2i}}$, $\widehat{\beta_{3i}}$, $\widehat{\beta_{4i}}$ depict systematic variance, systematic skewness, systematic kurtosis, and an interaction term between systematic skewness and kurtosis respectively; γ_1,γ_2, γ_3, γ_4 are systematic market risk premia for an increase in systematic variance, a decrease in systematic skewness, and an increase in systematic kurtosis, and an increase in interaction between systematic skewness and systematic kurtosis, respectively. All rates in the table are measured in percent per month.

2008-2012, 2013-2017. We can observe that systematic variance does not explain the average excess return of companies for both periods as betas are not significantly different from zero (see Tables 19 and 20). As Bartholdy (2015) proved that if this value is not significant and positive, then beta is not able to explain the excess return on the left-hand side. Thus, the significance of this value is a necessary condition for the model to be of any use. Based on that we are not allowed to extend the non-working model as even beta alone is not able to explain the excess return of chosen companies. That's why we cannot move to the next step of the Fama-MacBeth procedure. One of the reasons that CAPM does not hold may be the fact that returns are not normally distributed.

Table 19: Output from running traditional CAPM for generating a risk premium for holding systematic variance (2008-2012)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step for 2008-2012. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1 \widehat{\beta_i} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00627	0.00220	2.84540	0.00540
γ_1	0.00252	0.00159	1.58650	0.11584

Table 19 (continued)

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.00947

R-squared: 0.025, Adjusted R-Squared: 0.0151

F-statistic vs. constant model: 2.52, p-value = 0.116

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

Table 20: Output from running traditional CAPM for generating a risk premium for holding systematic variance (2013-2017)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step of time-series regressions for 2013-2017. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1\widehat{\beta_i} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00962	0.00240	3.99860	0.00012
γ_1	0.00317	0.00186	1.70460	0.09143

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.01

R-squared: 0.0288, Adjusted R-Squared: 0.0189

F-statistic vs. constant model: 2.91, p-value = 0.0914

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

Moreover, one of the hypotheses we wanted to test was whether the relations between the expected excess rate of return and systematic skewness and kurtosis have changed after the financial crisis of 2008. So, we tried to run the Fama-MacBeth procedure based on data for 2008-2018. The first step gave us evidence that traditional CAPM does not work as the market price for systematic variance is not significantly different from zero (see Table 21). As a result, we cannot move to the next steps.

Table 21: Output from running traditional CAPM for generating risk premium for holding systematic variance after the financial crisis (2008-2018)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in the previous step for 2008-2018. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1 \widehat{\beta_i} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.00905	0.00186	4.87110	0.00000
γ_1	0.00026	0.00141	0.18154	0.85632

Number of observations: 100, Error degrees of freedom: 98

Root Mean Squared Error: 0.00706

R-squared: 0.000336, Adjusted R-Squared: -0.00986

F-statistic vs. constant model: 0.033, p-value = 0.856

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and $\gamma_1$are parameters estimates. All rates in the table are measured in percent per month.

To sum up, we have found empirical evidence that it can be concluded that CAPM is not a suitable descriptor of asset prices, especially after the financial crisis. Beta stopped to predict excess return at all after the financial crisis. It signalizes that investors' behaviour has changed significantly after the crisis. It could be explained by the development of behavioural economics after the financial crisis of 2008. It does not mean that investors became irrational, it could be evidence that investors started to price other market risk factors.

5.5 CAPM with skewness, kurtosis, and an interaction term between them (1967-2019) for portfolio data

In order to consider the whole market, we performed the same procedure as with 100 individual stocks, but in this case, we used data of value-weighted monthly returns of 32 Portfolios Formed on Size, Operating Profitability, and Investment. We took monthly returns from July 1963 to January 2019 (667 periods). Firstly, we run a normality test and found that 94 % of data are not normally distributed with a 95 % confidence interval (see Appendix). Secondly, we run the time-series regression for each portfolio where excess returns of stock depend on excess market returns. After we used these betas for the next step in order to generate a risk premium for holding systematic variance (see Table 22). As we can see beta is not significant and positive, which means that beta is not able to explain the excess return on the left-hand side. As Bartholdy (2015) proved that the significance of this value is a necessary condition for the model to be of any use, we are not allowed to move to the next steps of Fama McBeth two-step procedure.

Table 22: Output from running traditional CAPM for generating risk premium for holding systematic variance using portfolio data (July 1963-January 2019)

This table reports the estimated risk premium for holding systematic variance under the two-moment CAPM based on cross-section regression and betas generated in time-series regressions for value weighted monthly returns of 32 Portfolios Formed on Size, Operating Profitability, and Investment during July 1963-January 2019. It includes estimated coefficients, their standard errors, test-statistics, and p-values.

Two-Moment CAPM: $r_i = \gamma_0 + \gamma_1 \widehat{\beta_i} + \varepsilon_{it}$

Estimated Coefficients:

	Estimate	Std.error	t-stat	p-value
Intercept (γ_0)	0.01039	0.00307	3.38030	0.00203
γ_1	-0.00327	0.00287	-1.14050	0.26311

Number of observations: 32, Error degrees of freedom: 30

Root Mean Squared Error: 0.00255

R-squared: 0.0416, Adjusted R-Squared: 0.00961

F-statistic vs. constant model: 1.3, p-value = 0.263

Notes: r_i denotes average excess return of company i; $\widehat{\beta_i}$ is systematic risk generated from equation (9); γ_0 and γ_1 are parameters estimates. All rates in the table are measured in percent per month.

We can see that portfolio data is not the best variant for running four-moment CAPM since CAPM does not hold at all. We understand that 100 stocks are not enough to describe the whole market, but from econometrics point of view these stocks were chosen randomly and we have maximum four independent variables. So, portfolio data provide a picture of the whole market, but it causes potentially larger efficiency losses versus individual stocks.

6. Conclusions

The main aim of our research was to investigate how higher order moments such as skewness and kurtosis influence investors' behaviour and their expected returns. The motivation of this research question is based on previous investigations mentioned in the literature review, namely Harvey&Siddique and Fang&Lai, and the importance of defining additional risk factors since traditional CAPM, which considers only systematic variance, was criticized a lot. In addition, capital markets are developing and the role of risk-return relationship for investors is becoming more and more significant.

Firstly, we found empirical evidence that CAPM is not enough for explaining investors' behaviour in the US context. The intercept that is expected to be zero in CAPM is found to be significant, which means that only the variance cannot explain the expected return by investors and there are other factors that should be significant. This fact signalizes that traditional CAPM should be expanded with additional risk factors.

Secondly, we proved that returns do not follow a normal distribution, so, measuring risk requires more than just measuring covariance, and higher order systematic co-moments should be important to risk-averse investors who are concerned about the extreme outcomes of their investments. If investors know that the asset returns have conditional co-skewness and co-kurtosis, expected returns should include a component attributable to conditional co-skewness and co-kurtosis. That's why we expanded CAPM with systematic skewness and systematic kurtosis as additional higher-moments risk factors.

Our model formalizes this intuition by incorporating measures of conditional co-skewness and co-kurtosis. In the absence of normality, investors should be very concerned with the shape of the tails of the distribution of portfolio returns, which can be measured with a set of higher-order co-moments. Also, we found out that investors charge market prices for not only systematic variance, but also systematic skewness and kurtosis. Based on our investigation, investors require a higher return for bearing higher systematic variance, negative systematic skewness, and higher systematic kurtosis. If investors observe positive systematic skewness, investors will require a lower return, since positive systematic skewness increases the probability of gains. So, investors are significantly concerned about negative systematic skewness, systematic kurtosis and downside risk that increase the uncertainty of their expected returns. High covariance, or negative co-skewness, or high co-kurtosis assets are undesirable and consequently sell at lower prices, producing high expected returns. Moreover, since returns are not normally distributed, investors should focus more on downside risk (or negative skewness) instead of standard deviations.

Thirdly, we proved that CAPM with systematic skewness, systematic kurtosis, and an interaction term between them is a better specification of the model in comparing to the model without the interaction term. The model with the interaction term has higher adjusted R-square, which means a better explanation of the average excess return of companies. Moreover, after adding the interaction term all risk factors became significant. What is more, the interaction term helps to find how the required return by the investors will be changed when the stock has the problems of skewness and excess kurtosis together. Based on the analysis, most of the stocks have skewness and excess kurtosis simultaneously. That’s why considering the interaction term between systematic skewness and systematic kurtosis was important to improve our model.

Fourthly, we ran regressions on 5-year time-horizons and a full sample to find the best model specification. Since 5-year time-horizons models gave us mostly insignificant result or inappropriate estimators, we conclude that the whole sample model is a better description of investors’ behaviour. Moreover, we performed the same procedure with portfolio data and concluded that even though portfolio data provide a picture of the whole market, it causes potentially larger efficiency losses versus individual stocks.

Fifthly, we identified empirical evidence that CAPM is not a relevant explanation of asset prices especially after the financial crisis of 2008, systematic variance stopped to predict excess return at all after the financial crisis. As Bartholdy (2015) proved that if the systematic variance is not significant and positive, then beta is not able to explain the excess return. Thus, the significance of this value is a necessary condition for the model to be of any use. It signalizes that investors' behaviour has changed significantly after the crisis. This phenomenon could be explained by the development of behavioural economics after the financial crisis of 2008.

Implications: If investors' preferences contain portfolio skewness and kurtosis measures, each stock's contribution to systematic skewness (co-skewness) and kurtosis (co-kurtosis) may determine a stock's relative attractiveness and therefore required return. Our model will be useful for investors as it helps them to define the required returns based on historical skewness and kurtosis as now they know market premiums of these co-moments. In addition, investors can define whether stocks are underpriced or overpriced, which helps them to buy underpriced stocks and receive gains in the future.

As we described in the section about Fama MacBeth procedure, this method could have the errors-in-variables problem. However, we did not find enough argumentation that the correction for the errors-in-variables problem will implement the significant changes to our model. That's why we recommend trying the instrumental variable method for future research as an alternative method to OLS estimation. Moreover, we recommend investigating investors' behaviour after the financial crisis of 2008 using another model, namely Fama-French three- or five-factor models. Furthermore, since CAPM based on portfolio data did not work, we recommend considering all possible stocks with the appropriate time horizon in future research.

Appendix

Table 1: Descriptive statistics of companies and stocks' returns

This table reports the descriptive statistics of 100 US companies and their stocks' returns for 1986-2018 years which are used for our regression analysis. It includes the company name, industry where the company operates, mean, maximum and minimum value and standard deviation of returns. In addition, the table presents the calculated returns' skewness, kurtosis, and test-statistics for Jarque-Bera test.

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
ADAMS EXPRESS CO	Investment	0.8	21	-17	4	-0.3	5.4	98.9	***
ABBOTT LABORATORIES	Health care	1.3	22	-21	6	-0.2	3.9	16.1	***
FIRST FINANCIAL BANCORP OHIO	Banking	1.1	44	-34	7	0.3	8.1	434.6	***
FIRST MIDWEST BANCORP DE	Commercial Banking, Financial Services	0.9	26	-50	8	-0.8	8.9	620.6	***
ROGERS CORP	Advanced Specialty Materials	1.3	54	-33	11	0.4	4.9	69.7	***
ROLLINS INC	Commercial Services	1.3	25	-23	6	0.1	4.5	35.8	***
SEABOARD CORP	Agriculture and Shipping Conglomerate	1.6	74	-33	11	1.1	8.6	596.8	***
1ST SOURCE CORP	Banking	1.2	45	-26	9	0.6	7.0	288.7	***
SHERWIN WILLIAMS CO	General building materials	1.5	23	-33	7	-0.3	5.1	76.7	***
FLEXSTEEL INDUSTRIES INC	Home & Office Products	0.9	39	-25	9	0.4	4.7	55.6	***
FOSTER L B CO	Manufactured Goods	1.2	65	-41	14	0.7	5.4	124.6	***
FRANKLIN ELECTRIC INC	Electrical/Electronic Manufacturing	1.5	39	-30	8	0.4	5.7	130.0	***
FRANKLIN RESOURCES INC	Financial Services	1.6	50	-30	10	0.5	5.7	142.8	***
FULLER H B CO	Adhesives	1.2	39	-35	9	0.1	4.9	57.3	***

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
GENTEX CORP	Auto Parts	2.1	48	-43	12	0.6	5.0	87.1	***
DONNELLEY R R & SONS CO	Commercial printing, Logistics and Supply Chain, Digital Marketing	0.3	62	-37	10	0.4	8.4	495.1	***
AMREP CORP	Land Development, Media Services	0.9	98	-40	15	1.5	9.3	807.0	***
STANDARD MOTOR PRODUCTS INC	Automobile	1.3	57	-45	12	0.3	5.2	86.1	***
APACHE CORP	Petroleum	1.1	36	-33	10	0.1	3.5	5.3	*
MATTEL INC	Entertainment	0.9	47	-39	10	0.1	5.2	83.5	***
BECTON DICKINSON & CO	Medical equipment, Consulting	1.4	27	-22	7	0.1	4.6	44.0	***
GRACO INC	Diversified Industrials	1.8	45	-30	9	0.5	7.1	291.0	***
BARNES GROUP INC	Manufacturing, Aerospace	1.2	32	-28	8	-0.1	4.2	24.1	***
WEYERHAEUSER CO	Real estate investment trust	0.9	29	-37	8	-0.2	4.5	38.3	***
INTERNATIONAL FLAVORS & FRAG INC	Specialty chemicals, Research and development	1.0	26	-28	7	-0.3	4.5	43.1	***
AVON PRODUCTS INC	Personal	0.7	42	-43	11	-0.1	5.5	107.4	***
HAWAIIAN ELECTRIC INDUSTRIES INC	Utilities	0.9	19	-35	5	-0.7	9.7	777.3	***
HAVERTY FURNITURE COS INC	Furniture	1.1	41	-34	11	0.3	4.3	34.1	***
HEALTHCARE SERVICES GROUP INC	Hospitality	1.5	52	-45	10	-0.1	6.4	185.7	***

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
PARKER HANNIFIN CORP	Motion and control technologies	1.3	37	-37	9	0.2	4.9	63.3	***
WEIS MARKETS INC	Retail	0.6	25	-25	6	0.5	5.0	80.1	***
WOLVERINE WORLD WIDE INC	Textile	1.5	34	-35	10	-0.1	3.9	14.5	***
BARNWELL INDUSTRIES INC	Oil, Gas & Coal	0.9	107	-43	13	1.9	16.8	3409.4	***
SMUCKER J M CO	Packaged Foods	1.0	30	-23	7	0.4	4.7	54.8	***
MOOG INC	Aerospace, Defense, Industrial Automation, and Motion Control	1.0	33	-32	8	0.1	5.0	70.1	***
MEREDITH CORP	Mass media	1.0	51	-31	8	0.7	7.6	381.9	***
HUNT J B TRANSPORT SERVICES INC	Transportation & Logistics	1.5	30	-44	10	-0.1	5.1	70.5	***
HUNTINGTON BANCSHARES INC	Banking	1.0	68	-62	9	-0.2	18.0	3705.1	***
HURCO COMPANY	Scientific & Technical Instruments	1.9	85	-48	17	1.4	7.7	499.1	***
SYNALLOY CORP	Steel	1.5	73	-45	13	0.5	7.2	307.5	***
MCDONALDS CORP	Restaurants	1.3	18	-26	6	-0.3	4.0	21.2	***
STANDEX INTERNATIONAL CORP	Food service equipment, electronics and hydraulics	1.2	51	-28	9	0.1	6.4	186.6	***
V F CORP	Apparel, Accessories	1.3	35	-30	7	-0.1	4.9	62.1	***
IMMUNOMEDICS INC	Biotech & Pharma	3.5	423	-55	32	6.3	78.4	96548.5	***
BEMIS CO INC	Packaging	1.2	26	-26	7	0.2	4.3	33.3	***
KENNAMETAL INC	Metals	1.1	50	-34	10	0.0	4.8	54.8	***

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
P & F INDUSTRIES INC	Logistics and Supply Chain	1.2	80	-41	14	1.3	8.4	596.7	***
TELEFLEX INC	Medical Device Manufacturing	1.4	28	-36	8	-0.3	4.6	47.4	***
INTEGRATED DEVICE TECHNOLOGY	Semiconductor industry	1.9	66	-47	17	0.4	3.8	21.6	***
AUTOMATIC DATA PROCESSING INC	Business services	1.3	27	-18	6	0.4	4.4	40.8	***
DIODES INC	Semiconductors	2.5	109	-53	18	1.4	7.9	522.9	***
INVACARE CORP	Medical Devices	1.0	39	-57	11	0.0	5.1	72.1	***
MARSH & MCLENNAN COS INC	Insurance brokers, Professional services	1.1	43	-39	7	0.0	8.9	576.2	***
ISRAMCO INC	Oil and gas	2.6	167	-62	21	2.3	14.9	2701.1	***
SAFEGUARD SCIENTIFICS INC	Venture Capital & Private Equity	1.6	90	-45	17	1.3	8.9	688.6	***
CLOROX CO	Consumer household products, Healthcare, Food	1.3	33	-24	6	0.1	5.3	89.7	***
GENUINE PARTS CO	Automotive	1.0	38	-18	6	0.6	7.9	426.3	***
CABOT CORP	Chemicals	1.3	45	-28	10	0.3	4.7	55.1	***
RITE AID CORP	Retail	0.7	150	-53	16	2.2	23.3	7116.3	***
KAMAN CORP	Aerospace industry	1.0	46	-39	9	0.0	5.9	138.8	***
KELLY SERVICES INC	Staffing and Recruiting	0.6	43	-30	9	0.1	5.5	100.6	***
NEW YORK TIMES CO	Newspapers	0.7	46	-32	9	0.5	6.1	178.1	***
KEY TRONICS CORP	Computer peripherals	1.7	373	-58	25	8.6	125.5	252299	***

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
C N A FINANCIAL CORP	Insurance	0.6	44	-41	8	-0.2	7.8	383.6	***
FEDERAL SIGNAL CORP	Public safety	1.1	47	-38	9	0.2	5.0	64.8	***
KIMBALL INTERNATIONAL INC	Furniture	1.0	32	-31	10	0.3	4.2	29.1	***
KULICKE & SOFFA INDS INC	Semiconducto rs	2.2	94	-52	19	0.7	5.0	94.3	***
LAM RESH CORP	Semiconducto rs	2.4	115	-40	17	1.3	9.6	824.7	***
LANCASTER COLONY CORP	Consumer Products	1.5	31	-27	7	-0.1	4.5	39.8	***
HUMANA INC	Managed health care	1.6	50	-52	11	-0.3	6.2	180.1	***
UNION PACIFIC CORP	Transportation	1.3	22	-34	7	-0.3	4.9	70.0	***
LAWSON PRODUCTS INC	Logistics and Supply Chain	0.8	30	-33	10	0.1	4.0	16.2	***
LINCOLN NATIONAL CORP	Insurance, Asset management	1.2	69	-59	10	0.7	14.6	2238.0	***
SIFCO INDUSTRIES INC	Metal	1.2	113	-33	15	1.5	11.3	1270.8	***
ALEXANDERS INC	Real estate investment trust	1.1	71	-39	10	1.2	12.9	1719.6	***
BLOCK H & R INC	Commercial Services	1.2	27	-26	8	-0.2	3.7	9.8	***
DANAHER CORP	Conglomerate	2.0	37	-52	8	0.0	9.0	597.6	***
SERVOTRONICS INC	Machinery	1.1	75	-41	14	1.4	7.4	452.1	***
LEE ENTERPRISES INC	Media	1.1	166	-61	18	3.9	37.8	20955.0	***
M T S SYSTEMS CORP	Scientific & Technical Instrument	1.2	35	-31	9	0.1	3.8	10.6	***
PENNSYLVANIA REAL ESTATE INVT TR	Real estate investment trust	0.9	118	-60	11	2.8	38.4	21248.0	***

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
LILLY ELI & CO	Pharmaceuticals	1.2	31	-29	7	0.2	4.7	51.4	***
SEMTECH CORP	Semiconductors	2.4	95	-42	16	1.0	6.2	239.4	***
WINNEBAGO INDUSTRIES INC	Manufacturing	1.4	66	-54	14	0.8	6.0	187.6	***
SONY CORP	Conglomerate	1.0	55	-30	10	0.5	4.8	68.1	***
MANITOWOC CO INC	Manufacturing	1.6	82	-40	14	1.0	8.1	485.2	***
TERADYNE INC	Test & Automation	1.6	78	-46	15	0.5	5.2	100.9	***
MARCUS CORP	Hospitality, Entertainment	1.3	50	-36	9	0.4	5.8	140.3	***
MARINE PETROLEUM TRUST	Oil, Gas & Coal	0.8	35	-28	9	0.2	4.3	28.4	***
SERVICE CORP INTERNATIONAL	Death care	1.3	120	-58	13	2.3	26.0	9058.8	***
SYSCO CORP	Wholesale	1.3	26	-24	6	-0.1	3.9	14.5	***
MCCORMICK & CO INC	Processed & Packaged goods	1.4	28	-21	6	0.0	4.8	51.2	***
JACOBS ENGINEERING GROUP INC	Engineering, Architect, Construction	1.7	58	-33	10	0.6	6.0	176.6	***
SUPERIOR INDUSTRIES INTL INC	Automotive	0.9	41	-42	10	-0.1	4.5	39.2	***
EQUIFAX INC	Credit risk assessment	1.3	34	-26	8	0.0	4.5	36.0	***
COCA COLA BOTTLING CO CONS	Food processing	0.8	33	-28	8	0.1	4.3	27.2	***
DYCOM INDUSTRIES INC	Telecommunications	2.0	74	-55	16	0.5	5.0	80.5	***
G A T X CORP	Railway Equipment Leasing	1.2	49	-28	9	0.4	6.0	161.8	***
GENERAL DYNAMICS CORP	Aerospace, Defense, Shipbuilding	1.3	34	-28	7	-0.1	5.3	84.6	***

Company	Industry	Mean (%)	Max Value (%)	Min Value (%)	Std Dev(%)	Skewness	Kurtosis	Jarque-Bera	Significance
GENERAL ELECTRIC CO	Conglomerate	0.7	25	-27	7	-0.3	4.5	44.2	***
Average	-	1	60	-37	10	0.6	9.2	4383.7	-

Notes: * indicates a significance level of 0.1; ** indicates a significance level of 0.05; *** indicates a significance level of 0.01.

Table 2: Descriptive statistics of 32 portfolios' returns

This table reports the descriptive statistics of 32 portfolios' returns for 1963-2019 years which are used for our regression analysis. It includes the portfolio name, mean, maximum and minimum value and standard deviation of returns. In addition, the table presents the calculated returns' skewness, kurtosis, and test-statistics for Jarque-Bera test.

Portfolio type	Mean (%)	Max Value (%)	Min Value (%)	Std Dev (%)	Skewness	Kurtosis	Jarque-Bera	Significance
SMALL LoOP LoINV	1,1	43	-32	7	0,1	3,7	15,0	***
ME1 OP1 INV2	1,3	32	-32	6	-0,2	3,0	4,0	
ME1 OP1 INV3	1,0	28	-33	7	-0,2	0,8	138,8	***
SMALL LoOP HiINV	0,4	35	-35	8	-0,2	2,6	8,6	**
ME1 OP2 INV1	1,3	27	-33	6	-0,5	3,1	30,1	***
ME1 OP2 INV2	1,3	25	-26	5	-0,5	3,4	34,9	***
ME1 OP2 INV3	1,3	24	-28	5	-0,4	1,2	107,4	***
ME1 OP2 INV4	1,0	27	-33	6	-0,5	2,0	54,0	***
ME1 OP3 INV1	1,5	24	-28	6	-0,5	2,3	37,3	***
ME1 OP3 INV2	1,3	25	-22	5	-0,4	1,6	77,0	***
ME1 OP3 INV3	1,3	26	-25	5	-0,4	1,0	137,0	***
ME1 OP3 INV4	1,1	27	-32	6	-0,5	2,0	52,8	***
SMALL HiOP LoINV	1,6	27	-26	6	-0,3	3,0	13,5	***
ME1 OP4 INV2	1,4	25	-29	5	-0,5	3,3	28,9	***
ME1 OP4 INV3	1,4	28	-26	5	-0,5	2,8	24,1	***
SMALL HiOP HiINV	1,1	29	-32	6	-0,5	3,5	30,2	***
BIG LoOP LoINV	1,0	19	-27	5	-0,3	1,3	92,6	***
ME2 OP1 INV2	0,7	20	-36	5	-0,8	3,6	83,5	***
ME2 OP1 INV3	0,9	22	-21	5	-0,3	4,0	36,4	***
BIG LoOP HiINV	0,8	18	-26	6	-0,5	1,0	137,1	***
ME2 OP2 INV1	1,0	20	-18	4	-0,1	3,3	3,4	
ME2 OP2 INV2	0,9	14	-18	4	-0,4	2,7	18,3	***
ME2 OP2 INV3	1,0	18	-27	5	-0,5	2,1	48,6	***
ME2 OP2 INV4	0,7	22	-22	5	-0,2	1,9	37,7	***
ME2 OP3 INV1	1,1	22	-20	4	-0,2	5,9	240,4	***
ME2 OP3 INV2	1,0	17	-18	4	-0,3	2,8	9,4	***
ME2 OP3 INV3	0,8	20	-21	4	-0,3	1,0	116,7	***
ME2 OP3 INV4	0,9	25	-27	5	-0,3	2,1	35,8	***
BIG HiOP LoINV	1,1	17	-23	4	-0,3	0,5	183,9	***
ME2 OP4 INV2	1,0	14	-23	4	-0,4	0,4	205,4	***
ME2 OP4 INV3	0,9	19	-23	4	-0,4	0,6	180,2	***
BIG HiOP HiINV	1,0	20	-23	6	-0,3	3,1	9,6	***

Notes: * indicates a significance level of 0.1; ** indicates a significance level of 0.05; *** indicates a significance level of 0.01.

Table 3: Generated market prices for a systematic variance for 100 stocks

This table reports the estimated market prices for systematic variance (β_{1i}) based on time-series regressions of traditional CAPM for each of 100 stocks during 1986-2018.

Two-Moment CAPM: $R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it}$

Estimated Coefficients (β_i):

0,882***	0,58***	0,559***	0,806***	1,027***	0,472***	0,804***	0,669***
1,112***	0,717***	1,428***	0,973***	1,279***	1,198***	0,913***	1,247***
0,7***	1,034***	1,028***	1,247***	0,813***	1,136***	0,296***	0,881***
0,515***	1,006***	0,823***	0,482***	0,794***	1,051***	1,053***	0,986***
0,715***	0,795***	0,941***	2,355***	0,821***	1,264***	0,66***	0,928***
1,868***	0,883***	0,822***	0,953***	2,072***	0,607***	0,677***	1,158***
1,03***	1,138***	1,57***	1,012***	1,01***	0,894***	2,308***	2,105***
0,854***	0,756***	1,382***	1,164***	0,718***	0,635***	1,063***	0,687***
1,051***	0,697***	1,364***	1,599***	1,115***	1,624***	2,058***	1,022***
0,709***	0,527***	0,979***	1,089***	0,893***	0,553***	1,673***	1,119***
0,918***	0,896***	0,614***	1,216***	2,052***	1,28***	0,605***	1,295***
0,683***	0,832***	1,242***	1,086***	0,845***	0,886***	0,925***	0,901***
0,493***	0,832***	1,086***	1,131***				

Notes: $R_{it} - R_{ft}$ denotes excess return of asset i; $R_{mt} - R_{ft}$ is market excess return; β_i and α_i are parameters estimates.

Table 4: Generated market prices for systematic variance, systematic skewness and systematic kurtosis for 100 stocks

This table reports the estimated market prices for systematic variance (β_{1i}), systematic skewness (β_{2i}) and systematic kurtosis (β_{3i}) based on time-series regressions of expanded CAPM for each of 100 stocks during 1986-2018.

Four-Moment CAPM:

$$R_{it} - R_{ft} = \alpha_i + \beta_{1i}(R_{mt} - R_{ft}) - \beta_{2i}(R_{mt} - R_{ft})^2 + \beta_{3i}(R_{mt} - R_{ft})^3 + \varepsilon_{it},$$

Estimated Coefficients (β_{1i}):

0.911***	0.552***	0.717***	0.809***	1.084***	0.397***	0.816***	0.701***
0.779***	0.589***	0.946***	0.652***	1.419***	0.817***	1.054***	1.187***
0.784***	1.238***	0.75***	0.555***	0.692***	0.87***	1.002***	1.172***
0.716***	1***	0.275***	0.693***	0.3680	1.096***	0.413***	0.966***
0.681***	0.427***	0.714***	0.98***	1.055***	1.077***	1.006***	1.273***
0.729***	0.689***	0.828***	2.473***	0.724***	1.297***	0.544***	0.806***
2.262***	0.798***	1.959***	0.805***	0.813***	0.4040	2.192***	0.552***
0.631***	1.085***	1.08***	0.801***	0.922***	1.036***	1.381***	1.128***
0.954***	0.831***	2.373***	2.089***	0.42***	0.733***	0.765***	0.744***
1.367***	1.186***	0.739***	0.582***	0.815***	0.554***	1.376***	0.816***
0.981***	0.627***	1.616***	1.421***	1.148***	1.699***	2.051***	0.993***
0.421***	1.159***	0.559***	0.441***	1.077***	1.029***	0.835***	0.516***
1.719***	1.036***	0.766***	1.081***				

Estimated Coefficients (β_{2i}):

-0.061	1.399	0.378	-1.255	-3.092*	0.421	-0.671	-1.477
1.897*	0.953	3.166	-0.153	2.858*	1.603	0.492	2.11
-4.071	2.714	0.587	1.8	-2.408**	2.848**	-0.365	2.345**
2.73***	-0.036	-0.045	2.887	-0.459	2.507*	0.313	-2.782*
-3.558	-0.762	-1.723	1.235	-0.988	0.504	2.631	-5.79***

Table 4 (continued)

-0.557	0.65	-0.256	6.581	3.426***	-0.11	-9.074***	0.029
0.814	3.081***	-0.677	0.201	1.418	4.52	-0.937	1.72
1.648*	3.31**	9.892***	-0.089	2.225	3.818**	7.656*	-0.431
-1.073	0.38	3.927	4.309	1.034	-1.189	0.369	0.491
-0.719	-2.916	-1.557	-0.225	1.061	-0.044	3.067	-2.36
3.237*	0.789	1.918	6.077**	-1.262	1.537	3.541	-0.742
2.453	-1.488	2.179**	-0.036	0.72	0.425	-0.661	-1.673
0.358	1.626	0.108	2.7***				
Estimated Coefficients (β_{3i}):							
-2.974	6.415	-14.118*	-3.591	-13.578	8.244	-2.845	-6.956
18.26***	11.49	24.151	5.87	8.343	19.086*	22.814*	6.621
1.607	7.985	15.485	31.236***	-5.602	23.142***	1.475	13.336*
16.411**	12.959	1.892	25.551**	22.28*	20.498**	10.562	-3.52
4.212	3.265	3.094	10.022	-2.738	-7.389	26.998	-33.109**
-2.81	11.78	10.179	6.115	18.253**	-3.395	-12.858	11.831
-17.955	12.588**	-10.442	8.013	4.583	64.333**	-13.939	9.755
8.745	15.636	45.147**	7.859	16.232*	19.858**	38.24	-12.232
2.537	6.994	4.136	12.878	20.409**	15.249	9.499	2.408
-0.446	-9.769	-6.047	4.443	26.511***	12.558	0.304	1.944
15.21	8.764	-19.047	33.069**	-6.414	-3.173	10.07	0.816
13.303	-10.918	20.136***	8.147	-7.469	6.905	3.737	-0.9
-3.41	12.176	6.552	11.812*				

Notes: $R_{it} - R_{ft}$ denotes excess return of asset i; $R_{mt} - R_{ft}$ is market excess return; $(R_{mt} - R_{ft})^2$ are $(R_{mt} - R_{ft})^3$ are stochastic discount factors in the market return.

Table 5: Generated market prices for systematic variance, systematic skewness, systematic kurtosis and combination of systematic skewness and systematic kurtosis for 100 stocks

This table reports the estimated market prices for systematic variance (β_{1i}), systematic skewness (β_{2i}), systematic kurtosis (β_{3i}) and a combination of systematic skewness and systematic kurtosis (β_{4i}) based on time-series regressions of expanded CAPM with an interaction term for each of 100 stocks during 1986-2018.

Four-Moment CAPM with interaction term:

$$R_{it} - R_{ft} = \alpha_i + \beta_{1i}(R_{mt} - R_{ft}) - \beta_{2i}(R_{mt} - R_{ft})^2 + \beta_{3i}(R_{mt} - R_{ft})^3 + \beta_{4i}(R_{mt} - R_{ft})^2(R_{mt} - R_{ft})^3 + \varepsilon_{it}$$

Estimated Coefficients (β_{1i}):

0,817***	0,65***	0,602***	0,787***	1,13***	0,421***	1,03***	0,824***
0,845***	0,559***	1,009***	0,859***	1,257***	0,825***	0,885***	1,11***
0,755**	1,327***	0,756***	0,468***	0,687***	0,713***	0,934***	1,085***
0,713***	0,794***	0,298***	0,736***	0,623***	1,114***	0,535***	1***
0,718***	0,493***	0,813***	0,931***	0,832***	1,085***	1***	1,287***
0,726***	0,691***	0,807***	2,71***	0,878***	1,373***	0,587**	0,953***
2,238***	0,779***	1,79***	1,106***	0,744***	0,418	2,217***	0,589***
0,673***	1,093***	0,772***	0,754***	0,674***	0,997***	1,031**	0,92***
0,872***	0,83***	2,578***	2,098***	0,429***	0,424**	0,957***	0,96***
1,05***	1,149***	0,779***	0,706***	1,021***	0,45	1,525***	0,787***
0,791***	0,652***	2,046***	1,274***	1,09***	1,427***	2,077***	1,062***
0,545***	1,221***	0,523***	0,497***	1,046***	0,997***	0,783***	0,742***
1,694***	0,898***	0,855***	0,99***				

Estimated Coefficients (β_{2i}):

-0,173	1,515	0,241	-1,281	-3,037	0,449	-0,417	-1,331
1,976*	0,917	3,241	0,093	2,665*	1,612	0,292	2,018

Table 5 (continued)

-4,105	2,819	0,594	1,697	-2,415**	2,662*	-0,446	2,242*
2,726***	-0,28	-0,017	2,938	-0,156	2,528*	0,459	-2,743*
-3,515	-0,683	-1,606	1,176	-1,253	0,513	2,624	-5,773***
-0,56	0,652	-0,281	6,864	3,609***	-0,02	-9,023***	0,205
0,785	3,059***	-0,877	0,558	1,336	4,537	-0,907	1,764
1,697*	3,319**	9,526***	-0,145	1,931	3,772**	7,24	-0,679
-1,171	0,378	4,171	4,321	1,045	-1,556	0,597	0,748
-1,096	-2,96	-1,51	-0,077	1,307	-0,167	3,244	-2,395
3,011	0,818	2,429	5,903**	-1,33	1,214	3,572	-0,66
2,601	-1,415	2,136**	0,03	0,682	0,388	-0,722	-1,405
0,328	1,462	0,214	2,591**				
Estimated Coefficients (β_{3i}):							
17,399**	-14,842	10,861	1,212	-23,597	3,239	-49,209	-33,442
3,936	18,039	10,431	-38,985*	43,444*	17,352	59,299*	23,427
7,801	-11,131	14,27	49,989*	-4,455	57,171**	16,221	32,063*
17,132	57,593*	-3,09	16,324	-32,873	16,556	-15,99	-10,768
-3,761	-11,045	-18,211	20,79	45,521*	-9,175	28,245	-36,094
-2,238	11,482	14,643	-45,408	-15,122	-19,845	-22,088	-20,12
-12,785	16,647	26,111	-57,208*	19,493	61,355	-19,459	1,722
-0,3	14,022	111,962**	18,056	69,819***	28,207	114,057	32,91
20,424	7,322	-40,318	10,737	18,403	82,106***	-32,185*	-44,382
68,279**	-1,723	-14,705	-22,442	-18,296	35,102	-32,118	8,32
56,345*	3,35	-112,256**	64,832	6,049	55,712	4,365	-14,026

Table 18 (continued)

-13,57	-24,264	27,86*	-3,89	-0,66	13,798	14,998	-49,853**
1,963	42,111*	-12,785	31,694*				
Estimated Coefficients (β_{4i}):							
-398,8***	416,109	-488,95	-94,017	196,11	97,968	907,553	518,427
280,37	-128,197	268,559	878,02**	-687,09	33,94	-714,18	-328,956
-121,24	374,186	23,77	-367,077	-22,444	-666,08	-288,66	-366,564
-14,12	-873,688	97,53	180,605	1079,58*	77,166	519,73	141,885
156,063	280,111	417,04	-210,789	-944,66*	34,948	-24,412	58,419
-11,208	5,835	-87,39	1008,529	653,3*	322,01	180,673	625,43*
-101,204	-79,451	-715,5	1276,67**	-291,84	58,293	108,05	157,236
177,05	31,595	-1307,9	-199,605	-1048,9**	-163,42	-1484,06	-883,63**
-350,133	-6,422	870,16	41,908	39,256	-1308,7**	815,9**	915,891*
-1345,3***	-157,497	169,47	526,27	877,07**	-441,293	634,646	-124,817
-805,207	105,975	1824,5**	-621,73	-243,953	-1152,63*	111,664	290,523
526,026	261,247	-151,19	235,61	-133,294	-134,94	-220,439	958,21**
-105,189	-585,967	378,52	-389,17				

Notes: $R_{it} - R_{ft}$ denotes excess return of asset i; $R_{mt} - R_{ft}$ is market excess return; $(R_{mt} - R_{ft})^2$ are $(R_{mt} - R_{ft})^3$ are stochastic discount factors in the market return.

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