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Volatility-Managed Portfolios: Evidence from the Norwegian Equity Market

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by

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ABSTRACT

We study the results of volatility-managed portfolios on the Norwegian market to examine Moreira and Muir's (2017) findings. We replicate their methodology and implemented it on data from the Oslo stock exchange. We found that most of our results mirrored Moreira and Muir's (2017) report. We concluded that there is a clear indication of positive alphas in most cases.

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The risk-return relationship is highly regarded as one of the most robust theories in the finance world. Ever since Markowitz's (1952) paper on portfolio. In a paper published in the *Journal of Finance*, Moreira and Muir (2017) suggested a new portfolio investment strategy based on volatility timing, that let investors take the same risk, but gaining higher returns and avoiding big drawdowns in financial crises which typically have high volatility. In other words the utility for an investor following this strategy will increase. Moreira and Muir found a 65% increase in utility for an investor investing only in the market portfolio (Moreira & Muir, 2017).

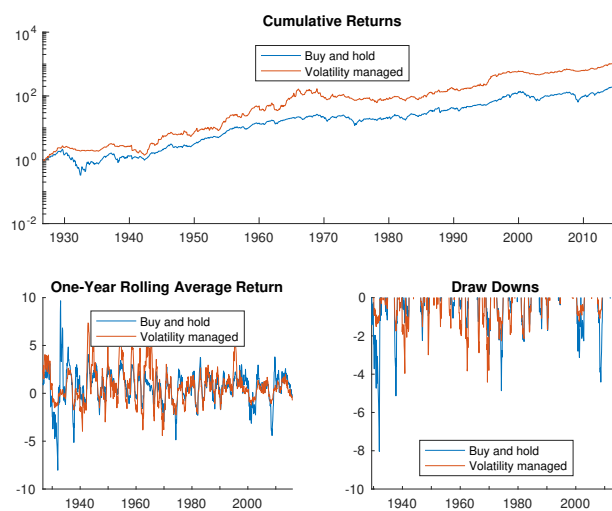


Figure 1: The top panel plots the cumulative returns from the buy-and-hold strategy for the market portfolio versus the volatility-managed market portfolio. The Y-axis is log scaled, and both strategies have the same standard deviation. The lower panel plots rolling one-year average returns. For this panel the y-axis in percentage

As seen in Figure 1, the investor following this strategy can have the same monthly standard deviation as the market, but still, have substantially better returns. We study the volatility-managed portfolio framework from Moreira and Muir(2017), which is the one plotted in Figure 1.

We will take a closer look at the approach to see if the strategy holds for the Norwegian market. We find this an important, and highly intriguing, topic as it is a way for investors to increase their utility measured in Sharpe-ratio, implied by significant positive alphas. This is in contrast to well-established economic theory, where returns typically are considered compensation for risk, but Moreira and Muir (2017) introduces a strategy that has the same volatility but higher returns. To make sure that the numbers we present from the

Moreira and Muir's (2017) paper.¹ One of the main contributions this paper presents is to increase the strategy's validity as the research question: Can investors implement the strategy of Moreira and Muir on the Norwegian market to increase utility?

We include the data from the Norwegian market and ran the same tests as in the original paper (Moreira & Muir, 2017). In the same way that Moreira and Muir test their results (2017), we choose to test the strategy on well-known risk factors proven to be a part of the Norwegian market by Ødegaard (2016a) and also Naes (2011). The risk factor we use are the Fama-French three-factor model (1993), namely market risk, small minus big (SMB) and high minus low (HML) in addition we also include the momentum factor from Fama and French (2015), the momentum factor from Carhart (1997) and the liquidity factor from Naes, Skjeltorp and Odegaard (2011). We find that most alphas are both positive and significant, and that HML is the only factor that provides a negative alpha. A very interesting example is the managed market portfolio as it generates an alpha of 4.49, implying utility gains for investors.

In addition, we create the optimal mean-variance efficient (MVE) portfolio and test these results against the non-managed factors and the Fama-French three-factor model to make sure that the results we found are not skewed because of well known factors. The MVE portfolio creates positive and significant yearly alphas varying from 3.604 to 10.286. To examine how the strategy works under shorter and more realistic time horizons for investors, we split the sample to ten-year periods. The subsamples show us that the strategy is not sufficient for the ten-year investment horizon.

The paper is organized as follows: In Section 2, we review the relevant literature. Section 3 includes the models we have used and related research. In Section 4, we explain the empirical methods used. Section 5 contains the data. In Section 6, we report the results, and Section 7 concludes.

¹The results from the replication are placed in Appendix B.

The idea of a volatility-based trading strategy gained momentum when Fleming, Kirby, and Ostdiek enlightened the economic significance of time-varying, predictable volatility in their papers (2001; 2003). They found that volatility timing strategies outperform the unconditionally efficient static portfolios that have the same target expected return and volatility.² Their base has been a building block for several volatility timing strategies, most notable are the paper from Barroso and Santa-Clara (2015) on how a momentum strategy nearly doubles the Sharpe ratio.³ The momentum factor has a considerable downside risk regarding market crashes, but as the risk of momentum is highly predictable, it is a problem that should be held under control.

The attempt to characterize the nature of the linear relation between the conditional mean and the conditional variance of the excess return on stocks proved to be challenging. The reports on the subject were conflicting, with Campbell and Hentschel (1992) and French, Schwert, and Stambaugh (1987) reporting that the data is consistent with a positive relationship between conditional expected excess return and conditional variance, whereas several other papers concluded with the opposite (C. Campbell, 1987; Fama & Schwert, 1977; Pagan & Hong, 1991; Turner et al., 1989). The difference in results may be a consequence of the varied models used to explain the relationship. The GARCH-in-mean (GARCH-M) framework gives, at best, weak results on the tradeoff between the conditional volatility and the market's risk premium.

⁴ The GARCH methods require large datasets spanning 100+ years, and if the correlation were stronger, a vast amount of data would not be needed to prove explanatory power (Lundblad, 2007). In the paper from French, Schwert, and Stambaugh (1987), they found a statistically significant positive relation between expected returns and anticipated volatility, only when using the GARCH-M. Other models yield negative and insignificant relationships (Whitelaw, 1994).

²Also robust to transaction cost and estimation risk

³Momentum strategy means investing in assets showing an upward-trending price and short the assets with downward-trending prices.

⁴For example, French, Schwert and Stambaugh (1987), Chou (1988), Campbell and Hentschel (1992), and Bansal and Lundblad (2002) find a positive relationship between the expected excess return and conditional variance, whereas Baillie and DeGennaro (1990), Nelson (1991), and Glosten, Jagannathan and Runkle (1993) find the opposite.

time-varying; predictable volatility rather than evaluating the analytical performance of volatility models, which the existing literature centered around. The evidence from this and several other articles⁵ overwhelmingly suggests that volatility is to some extent, predictable. However, the explanatory power of the standard volatility models typically only explains a fraction of the variation in squared returns, which led some researchers to question the variation in squared returns (Fleming et al., 2001).

Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) proposed a new approach⁶ called "realized" volatility, which exploits the information in high-frequency returns. They sum the squares of intra daily returns sampled at very short intervals to estimate volatility. This, in turn, makes the volatility observable. Realized volatility seems to be log-normally distributed, and daily returns standardized by realized volatility are approximately average (2003). In 2003 Fleming, Kirby and Ostdiek updated their 2001 paper to include realized variance, and their results indicated that the economic value of the realized volatility approach is substantial.

Busse (1999) found that a significant percentage of mutual fund managers usually reduced their market exposure during periods of high volatility when he examined the behavior of active portfolio managers. Even though this is a sign that managers try to behave as volatility timers, their trading decisions may be driven by other factors than volatility modeling.

The estimated gains for an investor implementing the volatility-timing strategy was so incremental that investors should be willing to pay on the order of 50 to 200 basis points per year to implement the strategy (Fleming et al., 2003). Furthermore, they found that the volatility timing strategy at a daily level leads to performance gains over longer horizons.

In 2002 Engle proposed a dynamic conditional correlation (DCC) model⁷. The model was often found to be the most accurate model when compared with simple multivariate GARCH and several other estimators (Engle, 2002). Build-

⁵See Bollerslev, Chou, and Kroner (1992), Bollerslev, Engle, and Nelson (1994), Diebold and Lopez (1995) and Palm (1996).

⁶Built on earlier work by Schwert (1989) and Hsieh (1991).

⁷The estimation of DCC can be divided into two stages. The first step is to estimate univariate GARCH, and the second is to utilize the transformed standardized residuals to estimate time-varying correlations (Engle, 2002).

and out-of-sample results show that a risk-averse investor should be willing to switch to a DCC strategy from a buy-and-hold, with substantially high switching fees. These results are backed up by Boguth, Carlson, Fisher, and Simultin (2011), where they proved that returns from high volatility periods are more influential in an OLS time-series regression.

Consequently, favorable volatility timing produces unconditional betas that overstate average portfolio risk and understate unconditional alphas. Thus, unfavorable volatility timing will give the opposite effect. Given the significant potential magnitude of this bias, volatility bias should be considered whenever evaluating investment performance (Boguth et al., 2011).

Another interesting take on the risk-reward payoff is the paper from Pastor and Stambaugh (2012) where they found that stock is more volatile over the long horizon from an investor's perspective. Their claim is based on that investor's believe the expected return is "persistent". They do admit that their finding is not robust, as the conclusion may be reversed with different models or with perfect predictors. Even with biased results, they contribute to encouraging further studies on the previous research on long- vs. short horizon investments.

Two studies that do show the upside of volatility timing is Zhou and Zhu (2012) and Bollerslev, Hood, Huss, and Pedersen (2018). Zhou and Zhu use a two-factor volatility model, whereas Bollerslev et al. focus explicitly on volatility forecasting. Zhou and Zhu show that papers based on a one-factor model will produce significantly different results than with a two-factor model⁸. They expand this to prove that the effect is in place with a two-factor model with estimation errors in the parameters. The effect of errors in the parameters is not as substantial but still economically significant. Hence, investors using a one-factor model instead of a two-factor model will incur significant economic losses; the same applies when there is an incorrect estimated parameter (Zhou & Zhu, 2012). In Bollerslev et al. (2018) paper, they provide results that it is possible to produce a robust dynamic risk model that is worth 48 ba-

⁸A one-factor model captures the short-run volatility movement primarily, while a two-factor volatility model can capture both the short- and the long-run components (they are essential in affecting the dynamics of the asset returns; see, e.g., Adrian and Rosenberg (2008)), but both models are still widely used in practice. Consequently, the elasticity of intertemporal substitution (EIS) impact on volatility trading will be more prominent in a two-factor model.

commonality in risk everywhere has a statistically and economically significant impact. Their new risk models, new panel-based estimation techniques, and their global volatility factor result in statistically significant out-of-sample forecast improvements and essential utility gains compared to more conservative estimated asset-specific risk models.

The theories surrounding volatility timing accelerates when looking into which investors sold during the crash in 2008-2009. It has been shown that the top investors - those in the top one percent - are, along with older investors, much more likely to sell the stock when the market is experiencing tumult than other investors (Nagel et al., 2016). That older investors that are close to retirement are more sensitive to bad times in the market is consistent with prior research (Chai et al., 2011). The tendency of high-income investors to sell during high-volatility periods might be a consequence of their tendency to pay more attention to their portfolios, and it might indicate that they perceive themselves better able to time the market (Nagel et al., 2016).

Moreira and Muir (2019), keep building on the idea that long-term investors should time volatility. They studied the portfolio of a long-term investor in a framework that is flexible enough to fit essential facts about the aggregate stock market. Intuitively, prices become more volatile only in the short run but not in the long term because of the higher volatility of expected returns leads to an increased degree of mean reversion. Thus, when volatility is substantially low, returns are singularly driven by permanent shocks. As the findings show significant gains from volatility timing, a long-time investor should react relatively aggressively to changes in volatility. The gains are measured using an annualized per period fee the investor is willing to pay to switch from a static buy-and-hold portfolio to a volatility timing portfolio. Then, for the baseline estimates, the naive buy-and-hold investor would willingly pay a 2.36% per period annualized fee to time volatility. In terms of wealth, this is a 60% increase relative to the buy-and-hold portfolio. The gains are about 80% of the total gain of switching from the buy-and-hold strategy to the optimal strategy.

⁹ Hence, ignoring variation in volatility comes with a substantial cost, and the

⁹The optimal strategy also conditions on expected return, as well as the volatility.

(Moreira & Muir, 2019).

Moreira and Muir's (2017) findings are quite provocative, so it is not surprising that multiple papers try to disprove their hypothesis. The two working papers highlighted in this review were the papers that were found to be most reliable, and most applicable.

Cederburg, O'Doherty, Wang, and Yan (2019) contribute by studying 103 equity trading strategies. They found no economic or statistical evidence to prove volatility-managed portfolios systematically will earn higher Sharp ratios than the unmanaged portfolios. However, they acknowledge potential economic gains from the investment strategy. Furthermore, they found that the trading strategies implied by spanning regressions are not possible to implement in real-time, as they require investors to combine non-scaled and volatility-scaled versions of a given portfolio using ex-post optimal weights. When they reproduce the in-sample spanning regression from Moreira and Muir (2017) with their broader sample of equity strategies they found that 77 out of the 103 volatility-scaled portfolios earn positive alphas in spanning tests, with 23 significantly positive estimates, compared to only three significantly negative. When controlling spanning regressions for exposure to the market, size, and value factors, they produce 70 positive intercepts. The fact that there is a significant volume of positive alphas offers confirmation of the potential economic gains from volatility-managed portfolios (Cederburg et al., 2019).

Over the entire sample of 103 equity portfolios, volatility management degrades the performance at about the same frequency as it improves it. Practically that suggests that direct investments in volatility-managed portfolios are not the solution for improved performance. Economically, the approximately equal split between positive and negative performance differences is suggestive of a generally positive risk-return tradeoff for the individual factors and anomaly portfolios. However, they also demonstrate that the structural instability in the regression parameters limits the appeal of the positive alphas to the investors, as they are reliant on real-time information. The Sharp ratios and certainty equivalent ratios for the out-of-sample combination are considerably less impressive than the in-sample versions. Furthermore, there are more straightforward strategies made to invest in the original, unscaled portfolios

al., 2019).

The second working paper reviewed is by Liu, Tang, and Zhou (2018) and is a direct answer to the Moreira and Muir (2017) paper. In Liu, Tang and Zhou's (2018) paper, they identify a look-ahead bias in their procedure, and after correcting said bias they discover that the strategy becomes challenging to implement in practice as its maximum drawdown changes to 68-93% in almost all cases, compared to the maximum drawdown of 56% without correcting the bias¹⁰. Also, the strategy outperforms the market only during times of financial crisis (Liu et al., 2018).

In the period from 1936 to 2017, they uncovered that the Sharpe ratio of the volatility-managed portfolio does not outperform the market. For all estimation cases, the Sharpe ratio is only minimally higher, lower in one case, and the difference is never statistically significant (Liu et al., 2018).

When they broke down the sample period to approximately 20-year subplots, they found that the volatility-managed portfolio underperforms the market in almost half of the cases. However, it does outperform the market in the last period, due to the financial crisis that appeared in that period and the significant spike in volatility that it created. This alone will not justify the strategy as superior, due to the fact that financial crises are rare and difficult to predict. In addition to that, investment strategies that will not deliver superior performances over more extended periods are not likely to be implemented in practice. The large drawdowns are also of concern, as the strategy may suffer a forced liquidation before a financial crisis hits the market (Liu et al., 2018).

¹⁰The market drawdown is 50%

We replicated the methodology from Moreira and Muir (2017) when we constructed the volatility-managed portfolio for well-established risk factors such as the Fama-French 3 factor model (Fama & French, 1993), Carhart Momentum PR1YR (Carhart, 1997) and Fama-French momentum (Fama & French, 2015).

This entire thesis tests the hypothesis that the strategy does not increase utility and expands the efficient frontier for the investor, meaning that the hypothesis is as follows:

$$H_0 : \alpha = 0 \quad H_A : \alpha > 0$$

3.1 Volatility Adjusted Portfolio

The volatility adjusted portfolio is created for each factor, f_{t+1}^σ , as follows:

$$f_{t+1}^\sigma = \frac{c}{\hat{\sigma}_t^2(f)} * f_{t+1} \quad (1)$$

where f_{t+1} is the excess return for the buy-and-hold strategy of each factor. The constant c controls the exposure of the strategy so that the standard deviation of the buy-and-hold strategy is equal to the standard deviation of the volatility-managed portfolio, for the entire sample. Moreira and Muir(2017) point out that the value of c does not affect the strategies Sharpe ratio; hence, the fact that the whole sample is used to calculate c will not affect the results. $\hat{\sigma}^2$ is a proxy for the realized variance in the previous month. Calculated as:

$$\hat{\sigma}_{t-1}^2 = \frac{22}{D_{t-1}} \sum_{d=1/D_{t-1}}^1 \left(r_{t-1,d} - \frac{1}{D_{t-1}} \sum_{d=1/D_{t-1}}^1 r_{t-1,d} \right)^2 \quad (2)$$

where D_{t-1} is the number of trading days in month $t - 1$, $r_{t-1,d}$ is the excess return of the high-risk portfolio in month $t - 1$ on date d , and the multiplier of 22 is added to convert the daily variances into monthly values.

As shown in (3), the formula used by Moreira and Muir is not exactly the same, we have chosen to use (2) because it is more precise and it also takes into

in the formula are that (2) standardize the variance based on trading days in the previous month so that the number of trading days does not affect the variance. For standard months containing 22 trading days, the formula is equal. Formula (2) is the same formula that Liu uses in his working paper: "Volatility-Managed Portfolio: Does It Really Work?" (Liu et al., 2018). They also are used the same strategy as Moreira and Muir.¹¹

$$\hat{\sigma}^2(f) = RV_t^2(f) = \sum_{d=1/22}^1 \left(f_{t+d} - \frac{\sum_{d=1/22}^1 f_{t+d}}{22} \right)^2 \quad (3)$$

Shortened, c is a constant that controls the exposure and is created so that the standard deviation of the buy-and-hold strategy is equal to the standard deviation of the managed portfolio. To do this, we first create the portfolio assuming $c = 1$ and after that creates c as follows,

$$c = \frac{\sigma(f)}{\sigma(f^c)} \quad (4)$$

where $\sigma(f)$ is the standard deviation for the buy-and-hold strategy, and $\sigma(f^c)$ is the standard deviation of the volatility-managed portfolio. Moreira and Muir (2017) claim the creation of c does not have an impact on the Sharpe ratio, and there appears to be no problems regarding the formula. However, when reviewing the formula (1) for a second time while focusing on the fact that the entire sample period is used to calculate c , we understand that c is subject to a look-ahead bias. To make sure that the strategy is transferable to use in practice, one must use an out-of-sample method, i.e., only including information available at the given point of time. With these criteria, the whole strategy will have a different outcome, as the estimation points now are altered. However as Moreira and Muir(2017) point out, c has no effect on the results.

3.2 Evaluating

We evaluated the performance of the volatility-managed portfolios based on mean-variance with focus on the risk-return trade-off,

¹¹Tyler Muir has confirmed that this is the method they use, for non-standard months.

$$\frac{E[R_{t+1}]}{\text{Var}[R_{t+1}]}, \quad (5)$$

where R_{t+1} is the excess return. To test our results, we made the same time-series analysis as Moreira and Muir (2017) and regress the volatility-managed factor on the buy-and-hold portfolio. That is:

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon \quad (6)$$

A positive intercept (α) implies that the volatility-managed portfolio expands the efficient-frontier, which means that it also increases the Sharpe Ratio. As the Sharpe Ratio is calculated as the excess return of the portfolio divided by the standard deviation of the portfolio. Besides, we control the results against other well-established risk factors such as the Fama-French three-factor model (Fama & French, 1993) plus the original factor. So for Carhart momentum($PR1YR$), the regression will be:

$$f_{t+1}^{\sigma} = \alpha + \beta_1(mkt - rf) + \beta_2SMB + \beta_3HML + \beta_4PR1YR + \epsilon \quad (7)$$

For consistency we also report the annualized appraisal ratio (AR 8) for each managed portfolio, as a perspective to how much the managed portfolio increases the Sharpe ratio. Where α is the unconditional alpha and RMSE is Root Mean Squared Error of the regression as in (6).

$$AR = \sqrt{12} * \frac{\alpha}{RMSE} \quad (8)$$

3.3 MVE portfolio

In this section, we create different portfolios that optimize the Sharpe ratio. Thus, we found the portfolio with the optimal static weights of each factor, based on historical data. To construct these portfolios, we adjust the weights so

as follows:

$$SR = \frac{Re}{\sqrt{12} * \sigma(R)} \quad (9)$$

Where Re is the average annualized excess return of the portfolio constructed as $F_{t+1}^{MVE} = b' * F_{t+1}$ where b' is a vector of the static weights that maximizes equation (9), and F_{t+1} is a vector of the factors returns. We then created the volatility-managed portfolio as:

$$f_{t+1}^{MVE\sigma} = \frac{c}{\hat{\sigma}_t^2(f)} * f_{t+1}^{MVE} \quad (10)$$

Where c again controls the average exposure so that both portfolios have the same standard deviation, and $\hat{\sigma}^2$ is the realized variance as in equation (3). We have created four different portfolios consisting of different factors. The MVE portfolio noted as FF3 is the original Fama-French(1993) factors: MKT, SMB, and HML. In FF3LIQ and FF3MOM, we include the liquidity factor and the Fama-French momentum factor, respectively. The last MVE portfolio we created consists of both liquidity and momentum factor(MOM) in addition to the three Fama-French factors.

It is important to note that we only included the Fama-French momentum factor (2015) and not Carhart-momentum (Carhart, 1997) PR1YR. The reason why we excluded the PR1YR factor is because it is highly correlated¹² with the MOM factor. If we were to have included both it might lead to multicollinearity. Thus, to avoid these issues, we chose only to include the MOM factor as this was the same factor used by Moreira and Muir (2017) as well.

To control the results, we ran similar regression as for the single-factor portfolios. First, we ran an univariate regression for the volatility-managed portfolio on the original MVE portfolio. Also, we controlled these results against the Fama-French three-factor model plus the non-managed MVE portfolio.

¹²correlation between MOM and PR1YR is 0.781 as shown in 3

In response to the Norwegian market we utilized monthly and daily data from Bernt Arne Ødegaards website on the factors: market return (MKT), size factor (SMB), value factor (HML), Fama-French momentum (MOM), Carhart momentum factor (PR1YR), liquidity factor (LIQ) and the risk-free rate (RF).¹³ All the portfolios are value-weighted and calculated utilizing data from the Oslo stock exchange.

The risk-free rate does need some extra attention as this is an estimation done by Bernt Arne Ødegaard (Ødegaard, 2016b). For the data period after 1986, the interbank rate, NIBOR is used as an estimation of the risk-free rate. For the period 1982 to 1986, B.A Ødegaard has used the overnight NIBOR as an approximation for monthly interest rates. While for the remaining years from 1981 to 1982, it was used as the shortest possible bond yield for Treasuries in Eitrheims et al. (2004). The data is available from January 1981 to December 2018 for all included factors. The period from January 1981 to December 2018 produced a data sample of 9411 daily observations. This translates to 450 months. All the regressions run will state 449 observations, as the first month in the sample will not be reported when the whole series is lagged. The 449 observations should be enough to show clear tendencies and give robust results. Closer descriptive statistics are demonstrated in the following tables:

¹³<http://finance.bi.no/bernt>

Summary statistics monthly observations

Norwegian market								US. market					
	MKT	SMB	HML	MOM	PR1YR	LIQ	RF		MKT	SMB	HML	MOM	RF
Mean	1.34	0.235	-0.155	0.19	0.395	-0.407	0.559	Mean	0.645	0.213	0.385	0.688	0.281
Max	18.6	21.3	17.4	25.3	14.9	15.6	2.07	Max	38.8	36.7	35.5	18.4	1.35
Min	-25.0	-17.4	-17.2	-25.4	-17.7	-18.0	0.052	Min	-29.1	-16.9	-13.3	-52.3	-0.06
N	450	450	450	450	450	450	450	N	1073.0	1068.0	1073.0	1073.0	1073

Table 1: Mean, max, min, and the number of observations for all factors for monthly observations on both the Norwegian market and the US. market used for replication. Mean, max, and min is a number for monthly return in % while N is the number of observations.

Summary statistics daily observations

Norwegian market								US. market					
	MKT	SMB	HML	MOM	PR1YR	LIQ	RF		MKT	SMB	HML	MOM	RF
mean	0.102	0.0532	0.0333	0.034	0.0223	0.0876	0.0267	Mean	0.0284	0.00518	0.0165	0.0276	0.0127
Max	11.4	9.89	11.4	8.09	10.3	13.6	0.15	Max	15.8	8.21	8.43	7.01	0.061
Min	-17.8	-11.3	-10.3	-17.2	-17.2	-8.83	0.00188	Min	-17.4	-11.6	-5.98	-18.3	-0.003
N	9411	9411	9411	9411	9411	9411	9411	N	23612	23612	23612	23487	23612

Table 2: Mean, max, min, and the number of observations for all factors for daily observations on both the Norwegian market and the US. market. The US. factors are used in replication. Mean, max, and min is a number for daily return in % while N is the number of observations.

Correlation matrix

Norwegian market								US market					
	MKT	SMB	HML	MOM	PR1YR	LIQ	RF		MKT	SMB	HML	RF	MOM
MKT	1.0	-0.411	0.046	-0.114	-0.070	-0.579	-0.009	MKT	1.0	0.319	0.2409	-0.064	0.0001
SMB	-0.411	1.0	-0.137	0.130	0.136	0.560	-0.030	SMB	0.3191	1.0	0.124	-0.052	-0.1294
HML	0.046	-0.137	1.0	-0.033	-0.041	0.026	0.104	HML	0.241	0.125	1.0	0.024	-0.133
MOM	-0.114	0.130	-0.033	1.0	0.781	-0.046	-0.125	RF	-0.064	-0.052	0.024	1.0	0.008
PR1YR	-0.070	0.136	-0.040	0.781	1.0	-0.057	-0.137	MOM	0.001	-0.129	-0.132	0.008098	1.0
LIQ	-0.579	0.56	0.026	-0.046	-0.057	1.0	0.035						
RF	-0.009	-0.030	0.104	-0.125	-0.137	0.035	1.0						

Table 3: Correlation matrix calculated on monthly observations for both the Norwegian market, and the US. factors used for replication.

factors PR1YR and MOM, but this is a natural effect. The two factors are two different measures for the same thing. Both factors discern that previous winners tends to outperform previous losers. It is also worth noting that the liquidity factor (LIQ) is negatively correlated with the market.

4.1 Subsample analysis

Since more than 30-years can be an unrealistic investment horizon for many investors, except funds and pension funds, we wanted to see how the strategy performs over shorter periods that are more reasonable for smaller investors. Besides, when checking the results for subsamples, it is also possible to see how the strategy works in different periods with different characteristics.

To check this, we run the same tests as before, but we restrict the periods to 10-year samples. As the Norwegian data started in 1981, we split the data into the following periods: 1981-1990,1991-2000,2001-2010, and,2011-2018. Meaning that the sample size varies from 96 observations, for the last subsample, to 120 observations for the second and third subsample. We did not analyze even shorter subsamples as we feel the statistical power already is weak enough. For the daily observations used to calculate the RV as in (2) the sample size ranges from 2007 to 2510.

Moreira and Muir (2017) analyzed their results in three separate 30-year subsamples; 1926 to 1955, 1956 to 1985, and, 1986 to 2015. The mid-period was the one with the weakest results, not surprisingly, as that was the period with the least volatility. We wanted to check the strategy in an even shorter horizon, as Moreira and Muir (2019) claim that investors with risk aversion of a 5- and a 20-year horizon should time volatility. We could chose to split the period into 5-year intervals, but decided not to do that as it would probably mean that we would need to shorten the interval of the data sample, i.e., daily data from stocks would not be sufficient, and we would have to find another data sample that reports results in a more continuous matter. We can already see some problems when shorting the period down to ten years, as we have fewer observations the statistical power will be weaker. We present the results for this in Section 5.2.

5.1 Single-Factor Portfolio

According to well-established finance theory such as (1952), it should not be possible to generate positive alpha values and at the same time take lower/the same risk. To check if the volatility-managed single-factor portfolios generate abnormal returns, we run the regression from (6). With the hypothesis that they do not generate abnormal returns ($\alpha = 0$).

To control our approach and code, we have made sure that we replicate the results of Moreira and Muir (2017) for the three original Fama-French factors (MKT, SMB, HML) as well as MOM for the US market. Appendix B shows very similar results. The difference in results may be due to changes from the CRSP in the data from Kenneth French's website.¹⁴ For more cooperation and details see Appendix B and table

In table 4, we report the result from the univariate regression for the Norwegian market, where we regress each volatility-managed portfolio on the same unmanaged factor, as in equation (6). As we can see, the volatility-managed portfolio generates positive alpha values for the factors MKT, SMB, MOM, and PR1YR. Also, the SMB factor is only marginally significant (significant at the 10% level), and the liquidity factor is not significant at all. For the factors MKT, SMB MOM, and PR1YR the alpha is significantly different from zero

¹⁴We thank Kenneth French for clarification on this matter.

¹⁵0.00041852

¹⁶1.9891e-05

	MKT^σ	SMB^σ	HML^σ	MOM^σ	$PR1YR^\sigma$	LIQ^σ
MKT	0.774 (0.064)					
SMB		0.796 (0.071)				
HML			0.748 (0.065)			
MOM				0.765 (0.059)		
PR1YR					0.716 (0.071)	
LIQ						0.776 (0.059)
alpha(α)	4.490	2.476	-2.786	7.422	8.311	2.137
p-value	0.043	0.095	0.122	0.000 ¹⁵	0.000 ¹⁶	0.193
AR	0.340	0.206	-0.253	0.581	0.707	0.214
N	449	449	449	449	449	449
R^2	0.592	0.633	0.558	0.585	0.511	0.602
RMSE	3.81	2.61	3.18	3.69	3.39	2.88

Table 4: *Results from regression (6) in the Norwegian market. The alpha is a monthly alpha multiplied by 12 to get an annual alpha. AR is calculated as in (8). The top panel reports the β (not annualized). The standard error reported in parenthesis are adjusted for heteroskedasticity*

Another difference between the U.S market (Moreira and Muir (2017)) and the Norwegian market worth mentioning is that for the U.S. market the SMB generates negative alpha, while for the Norwegian market the HML factor generates negative alpha. However, in both instances, these negative alphas are not significant.

The most extensive alpha is 8.311 and is generated from the momentum factor PR1YR. This is consistent with Moreira and Muir (2017) as they also get the highest alpha for momentum. However, they do not test the momentum factor PR1YR, but only the momentum factor MOM, which is also the next highest alpha in our results. Besides, Barroso and Santa-Clara (2015) also finds similar results and finds that managing volatility of momentum almost increases the Sharpe ratio of 100%. Moreira and Muir point out in their paper (2017) that a positive alpha implies that the strategy expands the mean-variance frontier compared to the non-managed portfolios. That is, the risk-adjusted performance of the managed portfolio is better than for the non-managed portfolio.

of the mean-variance efficient frontier compared to the non-managed factors, by looking at the appraisal ratio (AR) from equation (8), which also means an increase in the Sharpe ratio. As we can see from table 4 it is clear that all the significant factors increase the utility and the appraisal ratios range from 0.34 to 0.707, with the biggest at 0.707 being for the momentum factor PR1YR, followed by MOM on 0.581 and MKT on 0.340. The other non-significant appraisal ratios are: SMB¹⁷ at 0.206 HML at -0.253 and LIQ at 0.214.

Among the significant factors for the volatility-managed portfolios, it should be paid extra attention to the MKT-portfolios as this is relatively easy to implement for investors, as this is a long-only portfolio. While all the other portfolios either requires access to sophisticated portfolios that are allowed to short, or they require the investor to create these factors by themselves, which requires shorting quite frequent rebalancing, which again leads to higher transaction cost, which is not considered in this paper, however Moreira and Muir(2017) shows that the strategy also works when implementing transaction costs.

As a robustness test, we control our results for exposure to the well known Fama-French three-factor model we also regress the volatility-managed portfolio on the well known Fama-French three-factor model. For those factors not included in the three-factor model, we add this to the model.

Controlling for Fama-French three-factors

	MKT	SMB	HML	MOM	PR1YR	LIQ
Alpha (α)	3.870	2.476	-1.854	5.350	6.530	2.909
P-value	0.084	0.108	0.320	0.012	0.001	0.086

Table 5: *Results from a robustness check against the Fama-French three-factor model*

As we see in table 5, the significance of the results is reduced, and also the magnitude of the alphas are reduced. However, this is natural, and the strategy still produces positive alphas for all of the factors except HML. Among the other factors, SMB is the only positive alpha that is not significant on a 10% level. Even though these results are not as significant as earlier, the results are

¹⁷significant on 10% level

managed market portfolio generates a positive alpha. As Moreira and Muir point out, this implies an increase in utility by following the strategy also in the Norwegian market. When it comes to the two momentum factors, MOM and PR1YR, they are both significant at a 5% level. Daniel and Moskowitz (2016) has also shown earlier that volatility timing for momentum factor can increase utility. Now we display that this also holds for the Norwegian Market, also with a different strategy, namely the one developed by Moreira and Muir.

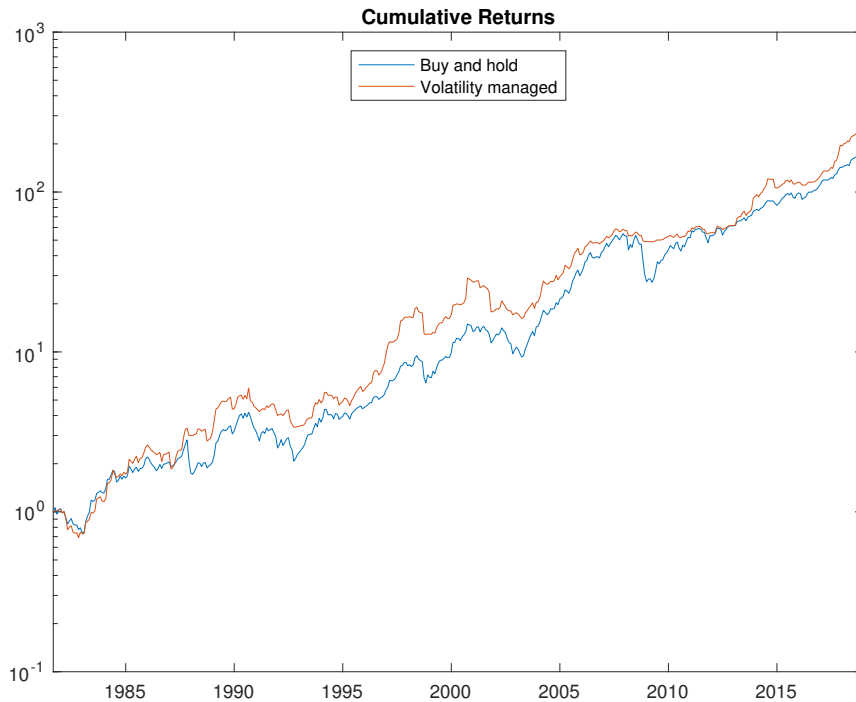


Figure 2: Cumulative returns from the buy-and-hold strategy for the market portfolio versus the volatility-managed market portfolio for the Norwegian market

From Figure 2 we can see that the strategy handles financial crisis very well, as it has no dip during the global financial crisis around year 2009. We can also see that the strategy outperforms the market portfolio during the dot-com bubble (1994-2000) and the Norwegian bank crisis (1987-1990). A typical characteristic of financial crisis is high volatility, which we can see both in Figure 3 and Figure 4

5.2 Subsample Analysis

In this part of the thesis, we do the same analysis as described in Section 5.1, but we split the data set into subsamples consisting of approximately ten years. That is, the first subsample is from 31 of July 1981 and until the end

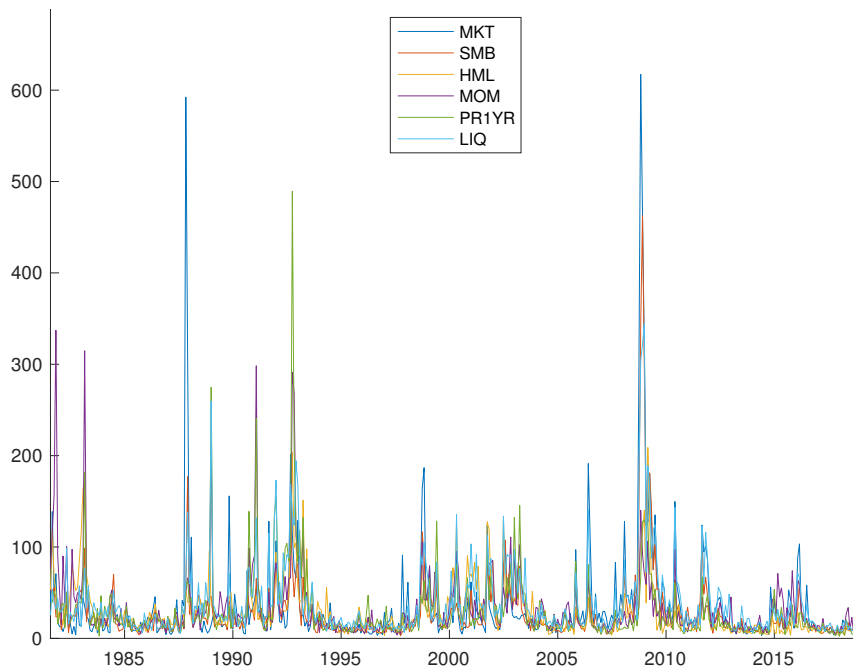


Figure 3: Time series of volatility on the Norwegian market, by factor. As we can see there was very high volatility around 1987, 1994 and 2009, which is the beginning of financial crisis.

of 1990, and the last subsample is from the end of 2011 until the end of 2018. While the middle two split the years 1990 to 2010 into two sample periods of 10 years each.

As we can see from table 6, the strategy does not perform as well for shorter periods. It is especially interesting that the managed market portfolio does not produce significant alpha for any of the ten-year periods. In addition to this, none of the other portfolios produces significant positive alphas for all of the sub-periods. This can be a sign of weakness for this strategy since a 10-year investment horizon is more realistic for investors. On the other side, Moreira and Muir (2017) find that most of their portfolios generate positive alphas in subsample periods of 30 years. However, we argue that a 30-year investment horizon also is unrealistic, unless for specific investment targets such as college savings, or pension funds, but for private investors a 30 years long investment horizon is uncommon. However the statistical power of these tests are weaker as the sample size is reduced as described in Section 4.1.

Another point worth mentioning about the subsample analysis is that for the latest period none of the alphas are significant on the 5 percent level. Also, some of the most significant alphas are negative, which gives an opposite result

the investor.

Further, Moreira and Muir (2017) find reliable results for excellent performance during recessions. However, our subsample analysis does not give any clear indication of this. For instance, there are not extraordinary good results for the subsample period 2001 - 2010, which include the global financial crisis. However, it should not be put to much emphasis on this as the subsample analysis is not meant to analyze periods of recessions. Furthermore, we can see from Figure 2 that the volatility-managed market portfolio outperforms the market during this period.

Subsample analysis

	MKT	SMB	HML	MOM	PR1YR	LIQ
1981 - 1990 α	0.276	-10.5***	2.84	-4.05	-0.517	5.7
1991 - 2000 α	5.17	5.97*	-10.7**	10.3**	9.29**	-14.5**
2001 - 2010 α	-0.0971	3.78	3.89	10.4***	11.5**	5.52
2011 - 2018 α	4.89	2.7	0.991	5.91*	4.71*	-2.48

Table 6: * Significant at 10% level, ** Significant at 5%level, *** Significant at 1% level

In this section, we report the results for the mean-variance efficient portfolios created as in section 3.3.

As we can see from tables 7 and 8, these portfolios generate large positive and significant alphas. In table 7, we run a univariate regression for the managed portfolio on the non-managed portfolio. Moreover, as we see the alphas are all positive and also more significant compared to the single-factor portfolio.

In table 8, we also control the results against the well known Fama-French 3 risk factors (1993) plus the non-managed MVE portfolio. However, the way these portfolios are constructed and the complexity of the portfolios makes it impossible for investors to invest in them. On the other hand, the idea behind this part of the research is to see if volatility timing generates positive alphas and expands the efficient frontier for investors already invested in multifactor portfolios.

The alphas for the MVE portfolios range from 3.604 to 3.982, and the portfolio consisting of the Fama-French three-factor and the Fama-French momentum is the one generating the highest alpha value. Also, the portfolio consisting of both momentum, liquidity, and Fama-French three-factor is the one generating the lowest alpha. Another exciting aspect of the MVE portfolios is that all the alphas generated are significant and positive. Here the single-factor portfolio and the multifactor portfolio differs as the HML single-factor portfolio generated negative and not significant alpha.

Also, we see from table 8 that even when controlling for Fama-French three-factors, the multifactor portfolios generate positive and significant alphas. Thus, the alpha values now range from 3.635 to 10.286. The significance level of the alphas is also increased, as all alphas now are significant at 1% level. This is contrary to the single-factor portfolios where both the significance level, and the alphas were decreased.

	$FF3^\sigma$	$FF3LIQ^\sigma$	$FF3MOM^\sigma$	$FF3MOMLIQ^\sigma$
FF3	0.780 (0.057)			
FF3LIQ		0.776 (0.062)		
FF3MOM			0.784 (0.055)	
FF3MOMLIQ				0.798 (0.055)
alpha(α)	3.890	3.654	3.982	3.604
p-value	0.0032	0.0064	0.0013	0.0106
AR	0.502	0.465	0.547	0.435
N	449	449	449	449
R^2	0.603	0.605	0.608	0.632
RMSE	2.24	2.27	2.1	2.39

Table 7: Results from regression (10) on the Norwegian market. The alpha is a monthly alpha multiplied by 12 to get an annual alpha. AR is calculated as in (8). The top panel reports the β (not annualized). The standard error reported in parenthesis are adjusted for heteroskedasticity.

MVE portfolio regression, with the Fama-French factors

	$FF3^\sigma$	$FF3LIQ^\sigma$	$FF3MOM^\sigma$	$FF3MOMLIQ^\sigma$
alpha(α)	10.286	3.744	3.987	3.635
p-value	0.0000 ¹⁸	0.0048	0.0014	0.0098
AR	0.502	0.465	0.547	0.435
N	449	449	449	449
R^2	0.615	0.614	0.619	0.637
RMSE	2.21	2.25	2.11	2.39

Table 8: Results from regressing MVE portfolios on the Fama-French three-factor model plus the non-managed portfolio. Alpha is a monthly alpha multiplied by 12 to get annual alpha. AR is calculated as in (8)

¹⁸5.4635e-06

In this thesis, we examined the methodology of Moreira and Muir's "Volatility-Managed Portfolios" (2017) on the Norwegian market. We researched whether it is possible to gain positive alpha values using volatility timing based on previous months variance, i.e., we found a way to increase returns while having the same risk, and thereby increase utility and the efficient frontier for investors in the Norwegian market.

We see clear indications of positive alpha values and therefore, also increased Sharpe ratios for the market factor (MKT), Fama-French momentum (MOM) and Carhart momentum factor (PR1YR). For the value factor HML, the strategy did not work in the Norwegian market and generated negative alphas. For the size factor, the alpha values were only marginally significant¹⁹. Especially the positive alpha values from the volatility-managed market portfolio is exciting as this is easy to implement for investors and also at the same time is a very diversified portfolio showing implying that the management of volatility expands the efficient frontier for investors on the Norwegian market.

However, we do believe that the strategy might be subject to look-ahead bias, as the creation of the constant controlling the exposure of the strategy is calculated using the whole sample period. How the strategy works when avoiding this bias is a subject for further research, where we suggest to start with finding other easy ways for investors to control the exposure.

¹⁹significant on the 10% level.

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A Fama French Factors

The variables published at Ken French's webpage are created as follows. Mkt is the excess market return meaning return on the market portfolio minus the risk-free rate. We will use the Oslo stock index as a proxy for the market return, and we will use the return on the Norwegian government bonds as a proxy for the risk-free rate. SMB is small minus big, and "is the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks" (Fama & French, 2015) HML is high minus low and "is the difference between the returns on diversified portfolios of high and low B/M stocks. RMW is short for robust minus weak and "is the difference between the returns on diversified portfolios of stocks with robust and weak profitability" (Fama & French, 2015). CMA is short for conservative minus aggressive and "is the difference between the returns on diversified portfolios of stocks of low and high investment firms, which we call conservative and aggressive." (Fama & French, 2015)

In this section, we report the results from our code using data for the same period as Moreira and Muir (2017), for the factors MKT, SMB, HML, and MOM. We have chosen only to replicate these factors, meaning that we do not replicate the strategy for the factors: RMW, CMA, FX, ROE, IA, and BAB. We do not include these factors as the result of the initial replication is very strong, and we wanted to test other factors in the Norwegian market. Furthermore, these factors had to be calculated both for the replication and for the Norwegian market, which is a master thesis itself. See the work of Mads Aurvåg and Rasmus Stenebråten(2015).

For the replication of Moreira and Muir (2017), we use both daily and monthly data from Kenneth French's website on the Fama-French three-factor model (Fama & French, 1993) as well as momentum (Fama & French, 2015).²⁰ The data collected is for the same period; however, due to CRSP continually updating and correcting their data, the observations will not be precisely equal.²¹ Nevertheless, this strengthens the anomaly.

²⁰<https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html>

²¹Thanks to Kenneth French for clarification on this matter.

Univariate regression

Replication					Original				
	MKT^σ	SMB^σ	HML^σ	MOM^σ		MKT^σ	SMB^σ	HML^σ	MOM^σ
MKT	0.609 (0.093)				MKT	0.61 (0.05)			
SMB		0.612 (0,094)			SMB		0.62 (0,08)		
HML			0.574 (0.138)		HML			0.57 (0.07)	
MOM				0.468 (0.101)	MOM				0.47 (0.07)
alpha(α)	4.783	-0.430	1.782	12.558	alpha(α)	4.86 (1.56)	-0.58 (0,91)	1.97 (1.02)	12.51 (1.71)
R^2	0.371	0.374	0.325	0.218	R^2	0.37	0.38	0.32	0.22
RMSE	51.36	30.48	34.56	50.28	RMSE	51.39	30.44	34.92	50.37
N	1072	1072	1072	1067	N	1065	1065	1065	1060
FF3-Alpha	5.323	-0.191	2.464	10.540	FF3-Alpha	5.45 (1.56)	-0.33 (0.89)	1.97 (1.02)	12.51 (1.60)
P-value	0.001	0.019	0.839	0.000 ²²					

Table 9: Results from our replication of Moreira and Muir(2017) and the results reported in their paper(table I). It is important to note that in stead of p-values Moreira and Muir(2017) reports standard errors also in the twi lower panels. We choose to report p-values as this gives an easier interpretation. The upper panel reports β and standard error adjusted for heteroskedasticity in both tabels

²²3.0996e-11

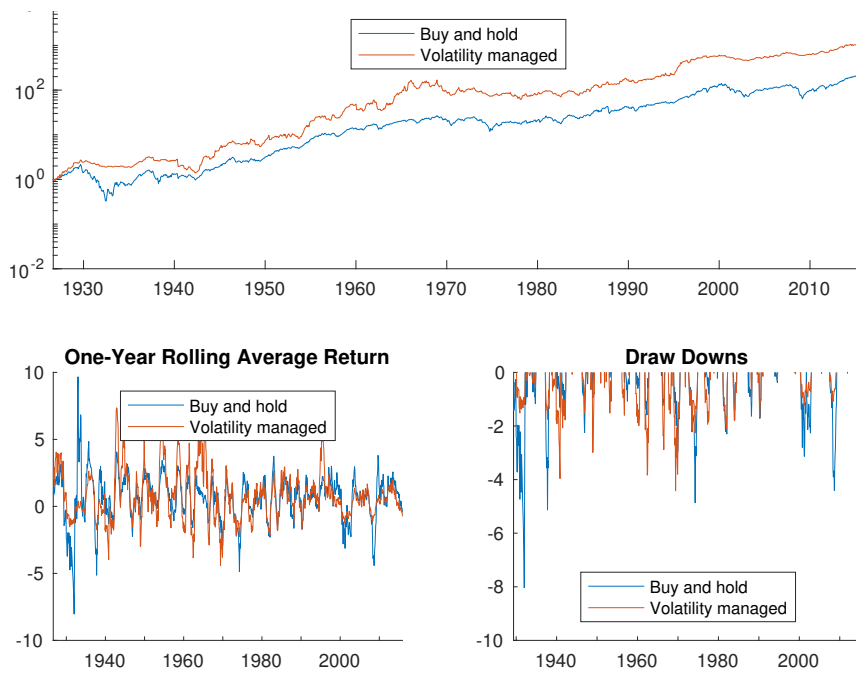


Figure 1: The top panel plots the cumulative returns from the buy-and-hold strategy for the market portfolio versus the volatility-managed market portfolio. The Y-axis is on log scale, and both strategies have the same standard deviation. The lower panel plots rolling one-year average returns. For this panel the y-axis in percentage

Our replication generates almost the same beta coefficient as Moreira and Muir (2017). While for the alpha values, there are small differences. For MKT, Moreira and Muir get 4.86, while we get 4.783 from the univariate regression. When controlling for the Fama-French three-factor model, they get an alpha of 5.45, and we get 5.32. The MKT alpha is significant at the 5% level.

For the SMB factors, we also get similar results, however, this factor is not significantly different from zero and, it is also the only negative alpha both for our replication and in the original paper (Moreira & Muir, 2017). This is also the factor from our replication that differs the most (in %). Our replication generates an alpha of -0.43 while Moreira and Muir get -0.58 as can be seen .

For the HML factor, our replication generates an alpha of 1.78 while the original alpha is 1.97, it is also worth mentioning that this factor only is marginally significant (significant on 10% level).

If we compare Figure 1 to Figure 3 from the original paper by Moreira and Muir(2017), we can see that our replication generates almost the exact plots. The same goes for Figure 4. It is clear to see that our Figure reports the same

have not replicated their strategy for all the factors, as mentioned.

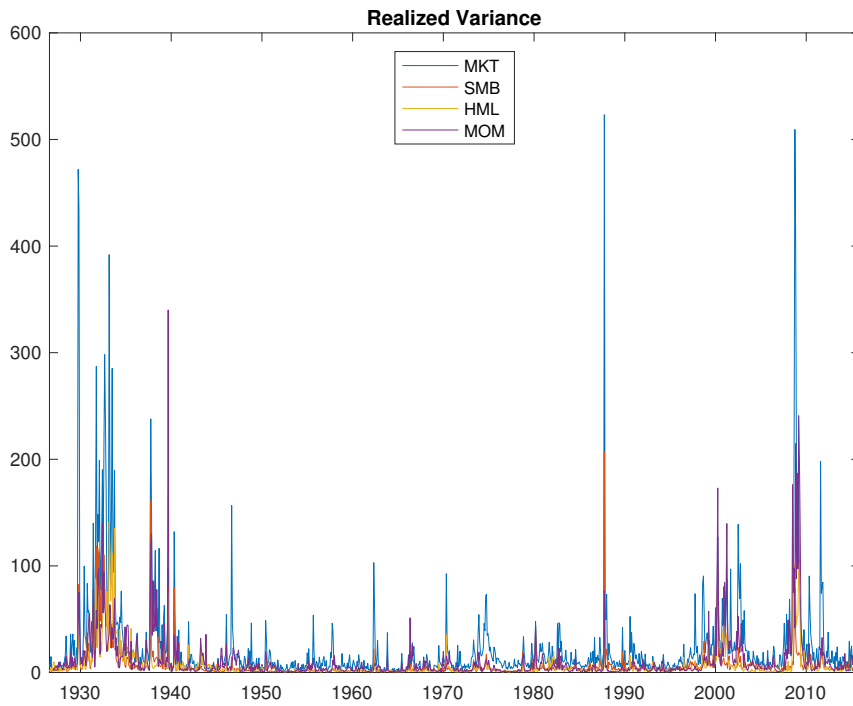


Figure 4: The trend in our Figure matches Figure 2 from Moreira and Muir (2017). We have replicated four out of seven factors, and we can see that we have replicated most of the outliers.

²³Figure 2