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A New Economic Framework: A DSGE Model with Cryptocurrency*

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Abstract

This paper develops a Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the economic repercussions of cryptocurrency. We assume that cryptocurrency offers an alternative currency option to government currency for households and we have an endogenous supply and demand for cryptocurrency. We estimate our model with Bayesian techniques using monthly data for the period 2013:M6-2019:M3. Our results indicate a substitution effect between the real balances of government currency and cryptocurrency in response to technology, preferences and monetary policy shocks. In addition, real balances of cryptocurrency exhibit a countercyclical reaction to these shocks. Moreover, we find that government currency demand shocks have larger effects on the economy than shocks to cryptocurrency demand. Our results also show that cryptocurrency productivity shocks have negative effects on output and on the exchange rate between government currency and cryptocurrency, with a more pronounced negative reaction to output if the central bank increases its weight to government currency growth. Overall, our results provide novel insights on the underlying mechanisms of cryptocurrency and spillover effects to the economy.

Keywords: DSGE Model, Government Currency, Cryptocurrency, Bayesian Estimation.

JEL classification: E40, E41, E51, E52.

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1 Introduction

Cryptocurrency has recently gained considerable interest from investors, central banks and governments worldwide. There are numerous reasons for this intensified attention. For example, Japan and South Korea have recognised Bitcoin as a legal method of payment (Bloomberg, 2017a; Cointelegraph, 2017). Some central banks are exploring the possibility of using cryptocurrency (Bloomberg, 2017c). Moreover, a large number of companies and banks created the Enterprise Ethereum Alliance¹ in order to customize Ethereum for industry players (Forbes, 2017). Finally, the Chicago Mercantile Exchange (CME) started the Bitcoin futures on 18th December 2017 (Chicago Mercantile Exchange, 2017).²

In this paper, we develop and estimate a Dynamic Stochastic General Equilibrium (DSGE) model in order to evaluate the economic repercussions of cryptocurrency. Our model includes demand and supply of cryptocurrency by extending and reformulating standard DSGE models with money (see, among the others, Nelson, 2002, Christiano et al., 2005, Ireland, 2004) with the new sector of the economy related to cryptocurrency. Our analysis allows us to compare the responses of real money balances for government currency and cryptocurrency to several demand and supply shocks driving the economy. Moreover, we are able to evaluate the response of main macroeconomic fundamentals to productivity shocks for production of cryptocurrency.

In 2017 the value of cryptocurrencies experienced an exponential growth and their market capitalization substantially increased. However, the volatility of cryptocurrencies has been very significant with regular daily swings up to 30%. Figure 1 provides evidence of these characteristics by

¹Source: <https://entethalliance.org/members/>.

²Nasdaq and Tokyo Financial Exchange followed in 2018 (Bloomberg, 2017b; Tokyo Financial Exchange, 2017).

showing the Coinbase Index (CBI).³

Bitcoin, the first decentralized cryptocurrency created in 2009 and documented in Nakamoto (2009), had grown in 2017 to a maximum of about 2,700% price return, and in the same year, some cryptocurrencies had achieved far higher growth than Bitcoin. Some economists, famous investors, and finance professionals warned that rapidly increasing prices of cryptocurrencies could cause the “bubble” to burst. In fact, in early 2018, a large sell-off of cryptocurrencies occurred. From January to February 2018, the price of Bitcoin fell 65%, and by the end of the first quarter of 2018, the entire cryptocurrency market fell by 54%, with losses in the market topping 500 USD billion. The decline of the cryptocurrency market was larger than the bursting of the Dot-com bubble in 2002. In November 2018, the total market capitalization for Bitcoin fell below 100 USD billion for the first time since October 2017, and the Bitcoin price fell below 5,000 USD. More recently, the Bitcoin price has partially recovered and, in summer 2019, it traded at levels higher than 10,000 USD. As we can observe from Figure 1, such dynamics have been shared by all types of cryptocurrencies.

Cryptocurrency is the private sector counterpart of government-issued currency (Nakamoto, 2009; Ethereum, 2014; Ripple, 2012) and is issued in divisible units that can be easily transferred in a transaction between two parties. Digital currencies are intrinsically useless electronic tokens that travel through a network of computers. Advances in computer science have allowed for the creation of a decentralized system for transferring these electronic tokens from one person or firm to another. The key innovation of the cryptocurrency system is the creation of a payments system across a network of computers that does not require a trusted third party to update balances and keep track of the ownership of the virtual units. The technology

³The CBI tracks the combined financial performance of all of the digital assets listed for trading in the US region by Coinbase. The components of CBI are weighted by market capitalization, defined as price multiplied by supply.

behind the system is called Blockchain.⁴

The characteristics of cryptocurrency are the following ones. The first characteristic relates to the fact that cryptocurrency is not based on a central authority that has private information. On the contrary, it relies on public information, such as computation, from a large number of individual distributed computers and servers connected to each other by the network and not by a recognized authority. Secondly, the issue of new currency and operations are validated by the network via complex pre-defined mathematical operations, algorithm defined as proof of work. This kind of network approves pre-defined, encrypted and immutable operations, so history cannot be changed and manipulated. The last characteristic refers to the easiness of payment and management. Cryptocurrency is by definition computer-based, and when linked to a portfolio the only requirement to transfer value or pay bills is an internet connection.

Most of previous studies have analysed cryptocurrency empirically. For example, Hencic and Gourieroux (2014) applied a non-causal autoregressive model to detect the presence of bubbles in the Bitcoin/USD exchange rate. Sapuric and Kokkinaki (2014) measured the volatility of the Bitcoin exchange rate against six major currencies. More recently, Catania et al. (2018) have analysed and predicted cryptocurrency volatility, whereas Catania et al. (2019) predicted the full distribution of cryptocurrency. Both Bianchi (2018) and Giudici and Pagnottoni (2019) have investigated the structural relationships between cryptocurrency and other macroeconomic as well as financial time-series.

However, there have been only a few theoretical studies that have modelled cryptocurrency. In this regard, Boehme et al. (2015) introduced economics, technology and governance of Bitcoin, whereas Fernández-Villaverde and Sanches (2016) developed a model of competition among

⁴Cryptocurrency is just one of the many applications of Blockchain.

privately-issued fiduciary currencies. More recently, Garratt and Wallace (2018) and Schilling and Uhlig (2019) focused on the exchange rate of Bitcoin and its theoretical determinants. As we will explain in the next section, these studies have assumed partial equilibrium models and did not examine the economic repercussions from the introduction of cryptocurrency to the overall economy and its different sectors.

We try to fill this gap and we develop a Dynamic Stochastic General Equilibrium (DSGE) where cryptocurrency is considered as an alternative to government currency. This assumption is in line with Gans and Halaburda (2019) that have defined cryptocurrency as a private digital currency.⁵ Therefore, in our model, we include two separate demand shocks to government currency and cryptocurrency, respectively. We estimate our model with Bayesian techniques using US and cryptomarkets monthly data for the period 2013:M6-2019:M3. Specifically we construct two new series to proxy the quantity of cryptocurrency and the technological development, respectively. To the best of our knowledge, our work is the first attempt to provide a general equilibrium model with cryptocurrency and to estimate its parameters with Bayesian techniques.

Our empirical analysis indicates that the reaction of the economy to shocks in preferences, technology and monetary policy are in line with the findings of previous literature (see, for example, Ireland, 2004 and Andrés et al., 2009). In addition, the reaction of real balances for cryptocurrency is countercyclical to output in response to these shocks. Moreover, in response to technology and monetary policy shocks, we find a strong substitution effect between the real balances of government currency and the real balances of cryptocurrency. Our results also show that the economy responds differently

⁵Central banks often define cryptocurrency as cryptoasset because they are not yet a full “money-like” due to their current limitations and have more uses than a form of money payment including investment purposes (see <https://www.bankofengland.co.uk/knowledgebank/what-are-cryptocurrencies>). We agree on both points but we prefer to stick to the common terminology of cryptocurrency that has been more frequently used.

to shocks in the demand for government currency and cryptocurrency. In particular, government currency demand shocks have larger effects on output, inflation and nominal interest rate. We also find that cryptocurrency productivity shocks imply a fall in the nominal exchange rate between government currency and cryptocurrency. The increase in the supply of cryptocurrency leads to lower real balances of government currency due to the substitution effect. In turn, both output and inflation fall, whereas the inflation rate increases. However, the magnitude of these effects is much lower than in the case of preference, technology and monetary policy shocks.

We are also able to quantify the contributions of each shock in our model through a variance decomposition analysis. Our findings indicate that technology, preferences and monetary policy have the highest contribution in terms of variations in the key endogenous variables of our model. We also find that specific supply shocks play an important role in the variation of real balances of cryptocurrency and nominal exchange rate between government currency and cryptocurrency. Finally, we assess the role of monetary policy in the presence of shocks to cryptocurrency productivity. Our robustness analysis indicates that the larger is the response of the monetary policy rule to a change in government currency growth, the stronger is the decline in output.

Our study also provides two policy recommendations. Firstly, we show that an increase in cryptocurrency supply has a negative effect on output. Therefore, the monetary authority could adjust its policy rate in response to changes in the real balances for cryptocurrency and include a weight for cryptocurrency growth in its policy reaction function. Secondly, we provide evidence that the response of the nominal interest rate to changes in government currency growth needs to be gradual in order for the central bank to mitigate the fall in output.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 outlines the new DSGE model on which our

study is based. In Section 4, we present the data used for the analysis and our Bayesian estimates. Section 5 presents the impulse response functions based on our estimated model. In Section 6, we focus on the variance decomposition analysis, whereas Section 7 provides a robustness on different assumptions about the monetary policy rule. The concluding remarks are found in Section 8.

2 Literature review

Our paper refers to two different streams of literature. On one hand, we contribute to studies that have developed theoretical models to analyse and describe cryptocurrency dynamics. However, these studies have focused mainly on partial equilibrium models. In our work, we develop a general equilibrium framework introducing cryptocurrency as an alternative to government currency. On the other hand, our study also contributes to the DSGE literature that has analysed the role of government currency in the economy.

Regarding the first strand of literature and the theoretical models, Boehme et al. (2015) presented a platform's design principles and properties of Bitcoin for a non-technical audience. They reviewed past, present, and future uses of Bitcoin pointing out risks and regulatory issues as Bitcoin interacts with the conventional financial system and the real economy.

Furthermore, Fernández-Villaverde and Sanches (2016) built a model of competition among privately-issued fiduciary currencies.⁶ They found that the lack of control over the total supply of money in circulation has critical implications for the stability of prices across the economy. In other words, the economy ends up in a state of hyperinflation. These authors also showed that in the short and medium terms, the value of digital currencies goes up

⁶More specifically, Fernández-Villaverde and Sanches (2016) extended the Lagos and Wright (2005) model by including entrepreneurs who can issue their own currencies to maximize profits following a predetermined algorithm (as in Bitcoin).

and down unpredictably as a result of self-fulfilling prophecies.

Another theoretical model analysing the exchange rate between fiat currency and Bitcoin was developed by Athey et al. (2016). In particular, they argued that Bitcoin exchange rates can be fully determined by two market fundamentals: the steady state transaction volume of Bitcoin when used for payments, and the evolution of beliefs about the likelihood that the technology survives. Garratt and Wallace (2018) also studied the behaviour of the Bitcoin-to-Dollar exchange rate. They used the model introduced by Samuelson (1958) with identical two-period lived overlapping generations with one good per date. After exploring the problems of pinning down money prices in the one-money model, these authors expanded their analysis to include a competing outside fiat money (Bitcoin), and they also discussed other aspects of competing cryptocurrencies.

More recently, Schilling and Uhlig (2019) also used a model in the spirit of Samuelson (1958) assuming that there are two types of money: Bitcoins and Dollars. Both monies can be used for transactions. These authors found a “fundamental condition”, which is a version of the exchange-rate indeterminacy result in Kareken and Wallace (1981) showing that the Bitcoin price in Dollar terms follows a martingale, adjusted for the pricing kernel. Schilling and Uhlig (2019) also found that there is a “speculative condition”, in which the Dollar price for the Bitcoin is expected to rise, and some agents start hoarding Bitcoin in anticipation of the price increase.

Finally, Sockin and Xiong (2018) developed a model in which the cryptocurrency has two main roles: *(i)* to facilitate transactions of certain goods among agents; *(ii)* as the fee to compensate coin miners for providing clearing services for the decentralized goods transactions on the platform. As a consequence of the first role of cryptocurrency, households face difficulty in making such transactions as a result of severe search frictions. In turn, such rigidity induced by the cryptocurrency price leads to either no or two equilibria.

However, the aforementioned theoretical studies have utilised partial equilibrium models. In our work, we develop a general equilibrium set-up. Many DSGE models have analysed the role of government currency in the economy. For example, Nelson (2002) presented empirical evidence for the US and the UK that real money base growth matters for real economic activity. In particular, they have shown that the presence of the long-term nominal rate in the money demand function increases the effect of nominal money stock changes on real aggregate demand when prices are sticky.

In addition, Christiano et al. (2005) developed a model embodying nominal and real rigidities that accounts for the observed inertia in inflation and persistence in output. They included money among the variables of interest and found that the interest rate and the money growth rate move persistently in opposite directions after a monetary policy shock.

A small monetary business cycle model which contains three equations summarizing the optimizing behaviour of the households and firms that populate the economy was developed by Ireland (2004). This author found that, if changes in the real stock of money have a direct impact on the dynamics of output and inflation, then that impact must come simultaneously through both the IS and the Phillips curve. In the same spirit, Andrés et al. (2009) have analysed the role of money in a general equilibrium framework focusing on the US and the EU. Their findings uncovered the forward-looking character of money demand.

Therefore, our work can be seen as an extension to these studies redefining the standard DSGE model with money with the inclusion of a new sector of the economy related to cryptocurrency generating endogenous supply and demand in a general equilibrium framework. In the next section, we present in detail our structural model of monetary business cycle with cryptocurrency.

3 Model

3.1 Households

The representative household of the economy maximizes the following expected stream of utility:

$$\max_{\{C_t, H_t, B_t, M_t^g, M_t^c\}} E \sum_{t=0}^{\infty} \beta^t A_t \left[u \left(C_t, \frac{M_t^g}{P_t}, \chi_t \frac{M_t^c}{P_t} \right) - \eta H_t \right] \quad (1)$$

where $0 < \beta < 1$ and $\eta > 0$. The budget constraint each period is given by:

$$M_{t-1}^g + \chi_{t-1} M_{t-1}^c + T_t + B_{t-1} + W_t H_t + D_t = P_t C_t + \frac{B_t}{R_t} + M_t^g + \chi_t M_t^c \quad (2)$$

The variable $\frac{M_t^g}{P_t}$ represents the real balance for government currency, whereas $\frac{M_t^c}{P_t}$ denotes the real balance for the cryptocurrency. Moreover, χ_t indicates the nominal exchange rate between the government currency and the cryptocurrency. In both equations (1) and (2) cryptocurrency enters as an alternative currency with respect to government currency. Our assumption is in line with the definition of cryptocurrency as private digital currency (Gans and Halaburda, 2019). In particular, holding cryptocurrency gives utility to the representative household. Moreover, since cryptocurrency is not an asset and it does not pay any interest, the representative household purchases cryptocurrency at $t - 1$ in terms of government currency, $M_{t-1}^c = \frac{M_{t-1}^g}{\chi_{t-1}}$, and holds cryptocurrency at time t as $M_t^c = \frac{M_t^g}{\chi_t}$.⁷

In equations (1) and (2) C_t and H_t denote household consumption and labour supply during the period t . The shocks A_t , E_t^g and E_t^c follow the

⁷In this regard, our modelling differs from standard open economy DSGE models with multiple currencies (see, among the others, Bodenstein et al., 2011). In these models the exchange rate is used to convert the interest rate received by the representative household in holding foreign bonds. On the contrary, in our model the exchange rate allows to convert two currencies (i.e., government currency and cryptocurrency) that are used in the same economy.

autoregressive processes:

$$\ln(A_t) = \rho^a \ln(A_{t-1}) + \varepsilon_t^a \quad (3)$$

$$\ln(E_t^g) = \rho^{eg} \ln(E_{t-1}^g) + \varepsilon_t^{eg} \quad (4)$$

$$\ln(E_t^c) = \rho^{ec} \ln(E_{t-1}^c) + \varepsilon_t^{ec} \quad (5)$$

where $0 < \rho^a, \rho^{eg}, \rho^{ec} < 1$ and the zero-mean, serially uncorrelated innovations ε_t^a , ε_t^{eg} and ε_t^{ec} are normally distributed with standard deviations σ^a , σ^{eg} and σ^{ec} . As we are going to show below, the shock A_t translates, in equilibrium, into disturbances to the model's IS curve, whereas E_t^g and E_t^c indicate disturbances to government money and cryptocurrency demand curves.

In the budget constraint, the household's sources of funds include T_t , a lump-sum nominal transfer received from the monetary authority at the beginning of period t , and B_{t-1} , the value of nominal bonds maturing during period t . The household's sources of funds also include labor income, $W_t H_t$, where W_t denotes the nominal wage, and nominal dividend payments, D_t , received from the intermediate goods-producing firms. The household's uses of funds consist of consumption, C_t , of finished goods, purchased at the nominal price, P_t , newly-issued bonds of value $\frac{B_t}{R_t}$, where R_t denotes the gross nominal interest rate.

It is convenient in what follows to denote by $m_t^g = \frac{M_t^g}{P_t}$ and $m_t^c = \frac{M_t^c}{P_t}$ the household's real balances for government currency and cryptocurrency, respectively. Moreover, we denote by $\pi_t = \frac{P_t}{P_{t-1}}$ the gross inflation rate during period t .

3.2 Entrepreneurs

We assume that there is a continuum of entrepreneurs indexed by n , where $n \in [0, 1]$, producing cryptocurrency. Each representative entrepreneur operates under a perfect competition. Following Sockin and Xiong (2018),

we introduce a cost of producing cryptocurrency given by: $\kappa^{-\phi_t} Q_t^c$, where Q_t^c is the amount of tokens that the entrepreneur is producing. In addition:

$$\phi_t = \xi_t + \nu_t \quad (6)$$

is the entrepreneur's productivity, which depends on the productivity of the other entrepreneurs via the common component, ξ_t , as well as on the specific programming skills of the entrepreneur, ν_t . We assume that ξ_t and ν_t represent the common and specific supply shocks to producing costs following the autoregressive processes:

$$\ln(\xi_t) = \rho^\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi \quad (7)$$

$$\ln(\nu_t) = \rho^\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu \quad (8)$$

where $0 < \rho^\xi, \rho^\nu < 1$, and the zero-mean, serially uncorrelated innovations, ε_t^ξ and ε_t^ν , are normally distributed with standard deviations σ^ξ and σ^ν . Entrepreneurs also gain a fraction $(1 - \rho) \in (0, 1)$ from selling the cryptocurrency to households at price $\frac{P_t}{\chi_t}$. Thus, entrepreneurs maximise their profits with respect to Q_t^c :

$$\Pi_t = \max_{\{Q_t^c\}} \left((1 - \rho) \frac{P_t}{\chi_t} - \kappa^{-\phi_t} \right) Q_t^c \quad (9)$$

3.3 Production Goods Firms

We assume a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$ producing differentiated varieties of intermediate production goods, and a single final production good firm combining the variety of intermediate production goods under perfect competition. During each period $t = 0, 1, 2, \dots$, the representative final goods-producing firm uses $Y_t(i)$ units of each intermediate good purchased at the nominal price, $P_t(i)$, to manufacture $Y_t(i)$ units of the final goods according to the constant-returns

to-scale technology described by:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{(\theta-1)}{\theta}} di \right]^{\frac{\theta}{(\theta-1)}} \quad (10)$$

where $\theta > 1$. The final goods-producing firm maximizes its profits by choosing:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \quad (11)$$

which reveals that θ measures the constant price elasticity of demand for each intermediate good. Competition drives the final goods-producing firm's profits to zero in equilibrium, determining P_t as:

$$P_t = \left[\int_0^1 (P_t(i))^{1-\theta} di \right]^{\frac{1}{1-\theta}} \quad (12)$$

During each period $t = 0, 1, 2, \dots$, the representative intermediate goods-producing firm hires $H_t(i)$ units of labor from the representative household to manufacture $Y_t(i)$ units of intermediate good i according to the linear technology:

$$Y_t(i) = Z_t H_t(i) \quad (13)$$

where the aggregate productivity shock, Z_t , follows the autoregressive process:

$$\ln(Z_t) = \rho^z \ln(Z_{t-1}) + \varepsilon_t^z \quad (14)$$

where $0 < \rho^z < 1$, and the zero-mean, serially uncorrelated innovation, ε_t^z , is normally distributed with standard deviation σ^z . In equilibrium, this supply-side disturbance acts as a shock to the Phillips curve. Since the intermediate goods substitute imperfectly for one another in producing the final goods, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market: the firm acts as a price-setter, but must satisfy the representative final goods-producing firm's demand at its chosen

price. Similar to Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in terms of the final goods and given by:

$$\frac{\phi}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t \quad (15)$$

with $\phi > 0$ and π measures the gross steady-state inflation rate. This cost of price adjustment makes the intermediate goods-producing firm's problem dynamic: it chooses $P_t(i)$ for all $t = 0, 1, 2, \dots$ to maximize its total market value. At the end of each period, the firm distributes its profits in the form of a nominal dividend payment, $D_t(i)$, to the representative household.

3.4 Monetary Policy

We assume that the central bank sets the nominal interest rate following a modified version of the Taylor (1993) rule given by:

$$\begin{aligned} \ln \left(\frac{R_t}{R} \right) = & \rho^r \ln \left(\frac{R_{t-1}}{R} \right) + (1 - \rho^r) \rho^y \ln \left(\frac{Y_t}{Y} \right) + \\ & (1 - \rho^r) \rho^\pi \ln \left(\frac{\pi_t}{\pi} \right) + (1 - \rho^r) \rho^{\mu^g} \ln \left(\frac{\mu_t^g}{\mu^g} \right) + \varepsilon_t^r \end{aligned} \quad (16)$$

where:

$$\mu_t^g = \frac{\frac{M_t^g}{P_t}}{\frac{M_{t-1}^g}{P_{t-1}}} \quad (17)$$

In equation (16), ρ^r , ρ^y , ρ^π and ρ^{μ^g} are non-negative parameters, and the zero-mean, serially uncorrelated policy shock, ε_t^r , is normally distributed with standard deviation σ^r . The monetary authority adjusts the short-term nominal interest rate in response to deviations of output and inflation from their steady-state levels as well as government currency growth as shown in equation (17). Andrés et al. (2009) have argued that an interest-rate rule that depends on the change in real balances for government currency may be motivated as part of an optimal reaction function when money growth variability appears in the central bank's loss function. As an alternative

explanation, the response to money growth can be justified by money's usefulness in forecasting inflation.

3.5 Equilibrium

The symmetric equilibrium of the model can be log-linearized to obtain the following set of equations⁸:

$$\hat{y}_t = \hat{y}_{t+1} - \omega_1 (\hat{r}_t - \hat{\pi}_{t+1}) + \omega_2 [(\hat{m}_t^g - \hat{e}_t^g) - (\hat{m}_{t+1}^g - \hat{e}_{t+1}^g)] + \quad (18)$$

$$\omega_3 [(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c) - (\hat{\chi}_{t+1} + \hat{m}_{t+1}^c - \hat{e}_{t+1}^c)] + \omega_1 (\hat{a}_t - \hat{a}_{t+1})$$

$$\hat{m}_t^g = \gamma_1 \hat{y}_t - \gamma_2 \hat{r}_t + \gamma_3 \hat{e}_t^g - \gamma_4 \hat{\chi}_t - \gamma_4 \hat{m}_t^c + \gamma_4 \hat{e}_t^c \quad (19)$$

$$\hat{m}_t^c = \gamma_5 \hat{y}_t - \gamma_6 \hat{r}_t + \gamma_7 \hat{e}_t^c - \gamma_8 \hat{\chi}_t - \gamma_8 \hat{m}_t^g + \gamma_8 \hat{e}_t^g \quad (20)$$

$$\hat{\pi}_t = \left(\frac{\pi}{R}\right) \hat{\pi}_{t+1} + \psi \begin{bmatrix} \left(\frac{1}{\omega_1}\right) \hat{y}_t - \left(\frac{\omega_2}{\omega_1}\right) (\hat{m}_t^g - \hat{e}_t^g) \\ - \left(\frac{\omega_3}{\omega_1}\right) (\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c) - \hat{z}_t \end{bmatrix} \quad (21)$$

$$\hat{\chi}_t = -\varrho \hat{\phi}_t \quad (22)$$

$$\hat{\phi}_t = \left(\frac{\xi}{\phi}\right) \hat{\xi}_t + \left(1 - \frac{\xi}{\phi}\right) \hat{\nu}_t \quad (23)$$

$$\hat{r}_t = \rho^r \hat{r}_{t-1} + (1 - \rho^r) \rho^y \hat{y}_t + (1 - \rho^r) \rho^\pi \hat{\pi}_t + (1 - \rho^r) \rho^{\mu^g} \hat{\mu}_t^g + \varepsilon_t^r \quad (24)$$

Equation (18) represents a log-linearized version of the Euler equation that links the household's marginal rate of intertemporal substitution to the real interest rate. When utility is non-separable, real balances for government currency and cryptocurrency affect the marginal rate of intertemporal substitution; hence, they also appear in the IS curve.

Equation (19) takes the form of a money demand relationship for government currency, with income elasticity (γ_1), interest semi-elasticity (γ_2), elasticity of \hat{m}_t^g with respect to government currency demand shocks (γ_3), and cross-elasticity with cryptocurrency (γ_4). Moreover, equation (20) reveals the form of a money demand relationship for cryptocurrency, with income elasticity (γ_5), interest semi-elasticity (γ_6), elasticity of \hat{m}_t^c with respect to

⁸We denote by small letters with hat, \hat{x}_t , the deviation of a given variable, X_t , from its steady state value, X . The full derivation of the model is shown in the Appendix A.

cryptocurrency demand shocks (γ_7), and cross-elasticity with government currency (γ_8).

Equation (21) is a forward-looking Phillips curve that also allows real balances for government currency (\hat{m}_t^g) and cryptocurrency (\hat{m}_t^c), to enter the specification when ω_2 and ω_3 are non-zero. Equations (18) and (21) also reveal that, wherever the real balances for government currency (\hat{m}_t^g) and cryptocurrency (\hat{m}_t^c) appear in the IS and Phillips curve relationships, they are followed immediately by the money demand disturbances, \hat{e}_t^g and \hat{e}_t^c .

Equation (22) is the log-linearized first order condition derived from the profit maximization problem of entrepreneurs that shows a negative relationship between the entrepreneurs' productivity and the exchange rate between government currency and cryptocurrency. Equation (23) is the log-linearized expression for the entrepreneurs' productivity that depends on the common productivity in the cryptocurrency sector as well as on the specific productivity of the entrepreneur. Equation (24) shows the log-linearized relation for the monetary policy rule indicating that the interest rate adjusts to output, inflation and government currency growth.

The cryptocurrency market is in equilibrium if the quantity of cryptocurrency supplied by entrepreneurs is equal to the demand of cryptocurrency by households. The goods market clearing condition implies that the output produced by production goods firms is equal to households' consumption. The model is closed by adding the log-linearized versions of the AR(1) processes for the preferences shock to consumption, the demand shocks for government currency and cryptocurrency, the common and specific supply shocks of cryptocurrency as well as the aggregate technology shock.

4 Estimating the Model

In this section, we estimate the model described in Section 3 using Bayesian techniques. In what follows, we initially describe the data used in order to

estimate the model (Section 4.1). Successively, we present the parameters of the model (Section 4.2) and their identification (Section 4.3). Finally, we describe the estimation results (Section 4.4).

4.1 Data

The main challenge in estimating our model is the relatively short sample for the macroeconomic series related to the market of cryptocurrency because of its recent development. Accordingly, in order to have a sufficient number of observations for our estimated model, we decided to use US data at monthly frequency. Foroni and Marcellino (2014) have dealt with DSGE models estimated with mixed frequency data including monthly data. Our sample period corresponds to 2013:M6-2019:M3. We use seven data series in the estimation because there are seven shocks in the theoretical model (see Table 1).⁹

The seven data series include the industrial production index, the natural log of real private consumption, the natural log of real money stock, the real bitcoin price, the real cumulative initial coin offering (ICO), the real Nvidia volume weighted average price and the effective federal funds rate. All the real variables are deflated by the consumer price index (CPI). Real private consumption and real M2 money stock are expressed in per capita terms dividing them by working-age population.

Focusing on monetary variables, we follow Ireland (2004) by considering money stock M2 as an indicator that includes a broader set of financial assets held principally by households. Real bitcoin price is obtained from monthly average of daily data assuming that the daily price is the average between opening and closing prices. We consider the bitcoin price as representative of the cryptocurrency price. Our choice is related to the longer sample period that is available for the Bitcoin price compared to the CBI. Our assumption

⁹The data sources and the construction of all observed variables are reported in the Appendix B.

is plausible since, for the same sample period, the correlation between CBI and the bitcoin price corresponds to the 99%.

The ICO or initial currency offering is a type of funding which uses cryptocurrency. In an ICO, a quantity of cryptocurrency is sold in the form of tokens to speculator investors, in exchange for legal tender or other cryptocurrency. The tokens sold are promoted as future functional units of currency if the ICO's funding goal is met and the project is launched. The Nvidia volume weighted average price is obtained as monthly average from daily data. Nvidia Corporation is the most important American technology company that designs graphics processing units (GPUs) for the gaming and professional markets, as well as systems on chip units (SoCs) for the mobile computing and automotive market.

4.2 Model Parameters

We decided to split the parameters of the model into two groups. The first group of parameters is fixed and consistent with data at a monthly frequency. In line with Ireland (2004), we assume ω_1 equal to one implying the same level of risk aversion as a utility function that is logarithmic in consumption. The parameter ψ is fixed equal to 0.1 following King and Watson (1996), Ireland (2000) and Ireland (2004). Such value implies that the fraction of the discounted present value and future discrepancies between the target price and the actual price of production goods is equal to 10%. The steady state values for the nominal interest rate and inflation are computed from monthly data of the effective federal funds rate and natural log changes in CPI. For our sample period they are equal to 0.70 % and 0.13%, respectively.

The second group of parameters is estimated with the Bayesian technique (Tables 2 and 3). To the best of our knowledge, our study is the first attempt to estimate a DSGE model including cryptocurrency. Hence, this is one of our main contributions and we rely on our judgement and the findings of previous

DSGE models considering government currency (e.g., Ireland, 2000, Ireland, 2004 and Andrés et al., 2009).

Table 2 shows the prior distributions for the endogenous parameters of our model. For the parameter indicating the output elasticity with respect to real balances of government currency (ω_2) we assume that its prior mean is in line with the range of estimates by Ireland (2004). On the other hand, we assume that the prior mean of the elasticity of output with respect to real balances of cryptocurrency (ω_3) is one fourth lower than that of government currency.

In order to set up the priors for the income elasticity of government currency demand (γ_1), the interest semi-elasticity of government currency demand (γ_2) and the elasticity of real balances of government currency with respect to government currency demand shocks (γ_3) we follow the estimated results of Ireland (2004) for the US economy. Moreover, we assume a prior mean value for γ_4 such that changes in the demand of cryptocurrency can affect the real balances for government currency.

Focusing on the parameters that characterize the demand relationship for cryptocurrency, we assume that γ_6 has a higher prior mean value than γ_5 . Moreover, we assume that the real balances for cryptocurrency are strongly affected by exogenous changes in cryptocurrency demand, which corresponds to a large prior mean for γ_7 . Moreover, following Gans and Halaburda (2019), we believe that cryptocurrency is a valuable alternative to government currency and assume a high prior mean value for γ_8 .

In line with Athey et al. (2016) and Garratt and Wallace (2018), we acknowledge that the exchange rate between government currency and cryptocurrency is an important determinant of the cryptocurrency supply and, in turn, we assume a high prior mean value for ϱ .

Turning to the parameter measuring the relative importance of common productivity with respect to specific productivity in the production of cryptocurrency ($\frac{\xi}{\phi}$), we are agnostic about its prior and, in turn, we assume

that it covers a reasonable range of values.

Regarding the parameters of the monetary policy rule, the prior for the degree of interest rate smoothing (ρ^r), the reaction coefficient of output (ρ^y), the interest-rate response to inflation (ρ^π) and government currency growth (ρ^{μ^g}) are all in line with the estimates by Andrés et al. (2009).

Table 3 reports the priors of the parameters related to the exogenous processes driving the economy. We set the persistence parameters of all autoregressive exogenous processes to be Beta distributed. We assume that the technology shock is more persistent than consumption preference and government currency demand shocks. We also assume that the prior for the persistence of the cryptocurrency demand shock has a relatively low value. For both productivity shocks to cryptocurrency, we assume that their prior means and standard deviations correspond to 0.60 and 0.05, respectively. Finally, we use Inverse Gamma distributions for standard errors of all exogenous shocks with means equal to 0.01 and infinite degrees of freedom which correspond to rather loose priors.

4.3 Parameter Identification

We estimated our model using a sample of 5,000,000 draws and we dropped the first 1,250,000.¹⁰ Our acceptance rate corresponds to 37%. In order to test the stability of the sample, we used the Brooks and Gelman (1998) diagnostics test, which compares within and between moments of multiple chains. Moreover, we performed other diagnostic tests for our estimates, such as the Monte Carlo Markov Chain (MCMC) univariate diagnostics and the multivariate convergence diagnostics.¹¹

As it is well known, the lack of identification in the parameter values is a potentially serious problem for the quantitative implications of DSGE

¹⁰In order to perform our estimation analysis, we used Dynare (<http://www.dynare.org/>).

¹¹The plots for MCMC univariate and multivariate convergence diagnostics are shown in the Appendix C.

models (see, for example Canova and Sala, 2009). Accordingly, we compared the prior and posterior distributions of the model parameters. For most of the parameters we found that prior probability density functions are wide, and posterior distributions are different from the priors.¹² Moreover, we performed the test proposed by Iskrev (2010).¹³ This test checks the identification strength and sensitivity component of the parameters using a rank condition based on the Fischer information matrix and the moment information matrix normalized by either the parameter at the prior mean or by the standard deviation at the prior mean. Our results show that the derivative of the vector of predicted autocovariogram of observables with respect to the vector of estimated parameters has full rank when we evaluate it at the posterior mean estimate. This implies that all the parameters are identifiable in the neighbourhood of our estimates.

4.4 Estimated Results

Tables 2 and 3 show the posterior means for the endogenous and exogenous parameters with their 90% confidence intervals.

We start by focusing on the estimated parameters of the IS curve. From Table 2, we note that estimated posterior of ω_2 does not vary substantially from its prior mean whereas ω_3 is well identified. The estimated values for these parameters imply that the output response to changes in real balances of government currency is more than six times higher compared to variations in real balances of cryptocurrency. As we will see in the next section, this result has important consequences for the effects of cryptocurrency productivity shocks on the economy.

Turning to the parameters of the money demand equation for government currency, our estimated values of γ_1 , γ_2 and γ_3 are in line with the ranges

¹²We report the plots for prior and posterior density functions of all parameters in the Appendix C.

¹³The plots showing the results for this test are reported in the Appendix D.

of estimates provided by Ireland (2004) implying that the demand shock (\hat{e}_t^g) has the highest influence on the movements in the real balances for government currency. Moreover, the estimated posterior of γ_4 is well identified and indicates an important degree of substitution between the demand of government currency and cryptocurrency.

Now we focus on the estimated parameters included in the money demand equation for cryptocurrency. From Table 2, it is possible to note that the posterior mean of γ_6 is much higher than γ_5 , implying that real balances for cryptocurrency respond more to changes in nominal interest rate than to variations in output. As will be shown in Section (5), this result has an important effect in terms of the response of cryptocurrency demand to the preference shock. Moreover, we find that the posterior means of γ_7 and γ_8 are above unity. These results have two main implications. Firstly, they suggest that the demand shock (\hat{e}_t^c) plays a substantial role in terms of variation in the real balances for cryptocurrency. Secondly, our estimates indicate a strong elasticity of substitution between cryptocurrency and government currency. This result will be discussed further in the next section. In particular, we are going to show that the change in government currency demand greatly affects the demand for cryptocurrency.

Focusing on the parameters related to the production of cryptocurrency, the estimated posterior of ϱ is well identified and has a value slightly below unity. Our result confirms the studies by Garratt and Wallace (2018) and Athey et al. (2016) who found that the exchange rate between government currency and cryptocurrency is an important determinant of cryptocurrency production. Moreover, the estimated value of $\frac{\xi}{\phi}$ suggests that common productivity has a stronger impact than the specific productivity in terms of cryptocurrency production. This implies that common productivity shocks have larger effects on the economy than specific productivity shocks.

Turning to the estimates of the monetary policy reaction function, we observe that in our sample period there is significant interest-rate smoothing.

In addition, the nominal interest rate appears to react more strongly to variations in the inflation rate than to output changes. Interestingly, our estimated parameter for the interest-response to government currency growth (ρ^{μ^g}) has a higher value than in Andrés et al. (2009). This result suggests that the central bank relies on the government currency growth to set up its policy rate.

Table 3 shows the posterior estimates for the exogenous processes. In general, the posteriors of these parameters are well identified. We note that technology and preference shocks are more persistent than government currency and cryptocurrency demand shocks. Moreover, we find that the specific productivity shock to cryptocurrency production is slightly more persistent than the common productivity shock to cryptocurrency production. Finally, our posterior estimates show that shocks to specific cryptocurrency productivity, cryptocurrency and government currency demand are much more volatile than the remaining shocks.

5 Impulse Response Functions

In this section, we show the results of impulse response functions (IRFs) for the estimated model considering some of the exogenous shocks driving the economy. Firstly, we focus on the “traditional” shocks to preferences, technology and monetary policy. Secondly, we analyse the shocks to the demand of households for real balances of government currency and cryptocurrency. Finally, we consider the “new” shocks to cryptocurrency common and specific productivity. We consider a positive 1% shock for each of these exogenous processes and we set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution.¹⁴

¹⁴Accordingly, our strategy allows us to compare the impulse responses among the different shocks. In the Appendix E, we present the estimated impulse responses together with their confidence intervals.

5.1 “Traditional” Shocks

Figures 2–4 present the responses of output, real balances for government currency and cryptocurrency, nominal exchange rate between government currency and cryptocurrency, inflation rate, and nominal interest rate.

From Figure 2, we note that, on impact, the preferences shock increases output and inflation by about 0.6% and 0.1%, respectively. The monetary authority responds by increasing the nominal interest rate that achieves its peak after two months. On impact, the real balances for government currency increase but, only after two months they fall exhibiting a strong inverse relationship with the nominal interest rate. These results are in line with the findings by Ireland (2004) and Andrés et al. (2009). Focusing on the real balances of cryptocurrency, we observe that they decrease in response to this shock.¹⁵ This result is a consequence of the larger estimated value for the interest semi-elasticity (γ_6) than the income elasticity of cryptocurrency demand (γ_5). Finally, we observe that the response of the nominal exchange rate between government currency and cryptocurrency remains almost unchanged in response to the preferences shock.

Figure 3 shows the IRFs for the technology shock. We find that a 1% positive shock to technology increases output and the peak is achieved after seven months and corresponds to about 0.97%. Inflation decreases on impact by about 0.16% and it remains negative for all the periods considered in the graph. Accordingly, the monetary authority decreases its policy rate. Real balances for government currency exhibit an inverse relationship with the nominal interest rate and have their peak response seven months after the occurrence of the shock. These findings are in line with the results reported by Ireland (2004) and Andrés et al. (2009). Furthermore, we observe a strong substitution effect between the real balances of cryptocurrency and government currency. This result is a consequence of the large estimated

¹⁵On impact, the demand of cryptocurrency drops by 0.01%.

value for cross elasticity of cryptocurrency demand and government currency demand (γ_8). Finally, our results indicate that the nominal exchange rate between government currency and cryptocurrency is not affected by the technological shock.

Figure 4 shows that a positive shock of 1% to monetary policy induces an increase in the nominal interest rate by 0.7%. In response to the shock both output and inflation fall.¹⁶ The negative response of output and the positive response of nominal interest rate induce the fall in the demand for government currency. These results confirm the findings of Ireland (2004) and Andrés et al. (2009). Moreover, our results suggest a strong substitution effect between real balances of cryptocurrency and government currency, with the former increasing by 0.02% on impact. However, the impulse response of the nominal exchange rate between government currency and cryptocurrency shows a negligible change.

Our interesting and novel results indicate that when cryptocurrency is considered in the economy as an alternative currency option, we observe a strong substitution effect between real balances of cryptocurrency and government currency. In particular, our estimated model suggests that real balances of cryptocurrency are countercyclical to output, whereas government currency is procyclical in response to preferences, technology and monetary shocks.”

5.2 Government Currency Demand Shocks vs. Cryptocurrency Demand Shocks

Figure 5 presents the impulse responses to real balances of government currency (blue lines) and cryptocurrency demand shocks (red lines).

The positive shock on government currency demand induces both real balances of government currency and cryptocurrency to rise. This result is a

¹⁶On impact, output decreases by 2.3% and inflation by 0.5%.

consequence of the large estimated values of γ_3 and γ_8 . As described above, in the IS and Phillips curves, the real balances for government currency are immediately followed by the government currency demand shock.¹⁷ Since the response of the shock to government currency demand is systematically higher than that of the real balances for government currency, output decreases and inflation rate increases.

Furthermore, we find that the nominal interest rate drops in response to this shock. This may be explained by the fall in the government currency growth that induces the central bank to decrease its policy rate. Finally, we observe that the nominal exchange rate between government currency and cryptocurrency does not move in response to the shock.

Now we focus on the effects of a positive shock to cryptocurrency demand. We begin by noticing that the real balances of cryptocurrency increase in response to this shock. Moreover, because of the large estimated value of γ_7 , the positive response of real balances for cryptocurrency is systematically higher than the shock to cryptocurrency demand. This implies that the real balances for government currency fall.

From Figure 5, we also observe that the effects of this shock on output, inflation and nominal interest rate are weak.¹⁸ This finding can be explained by the low estimated value of ω_3 . In particular, on the impact of the shock, output increases, whereas inflation rate falls from the second month onwards. Moreover, the increase in the government currency growth leads the central bank to raise its policy rate. Also in this case, the response of the exchange rate between government currency and cryptocurrency is almost unchanged.

Overall, the above results indicate that shocks to government currency demand have larger spillover effects to the economy than shocks to cryptocurrency demand. To the best of our knowledge, this is the first time

¹⁷In particular, equations (18) and (21) show a difference between the real balances for government currency and the government currency demand disturbance.

¹⁸On impact, output increases by only the 0.01%.

that such a result is documented in a general equilibrium framework and it is mainly driven by our key estimated parameters.

5.3 Shocks to Cryptocurrency Productivity

The shocks to common and specific productivity of cryptocurrency are presented in Figure 6. The impulse responses to the former shock are shown in blue lines, whereas the impulse responses to the latter shock are in red lines.¹⁹

In general, a positive shock to the productivity of entrepreneurs producing cryptocurrency implies a fall in the nominal exchange rate between government currency and cryptocurrency.²⁰ The decrease in the exchange rate induces an increase in the real balances of cryptocurrency.²¹ The demand of government currency drops as a consequence of the substitution effect with cryptocurrency demand. However, we note that, in terms of magnitude, the fall in the real balances of government currency is much lower than the increase in the real balances of cryptocurrency.²²

From Figure 6 we note that output falls in response to cryptocurrency productivity shocks. This result is a consequence of the larger estimated value of the output elasticity to real balances of government currency (ω_2) than the output elasticity with respect to cryptocurrency (ω_3). Our findings also indicate that the change in output in response to these shocks is much less pronounced than in the case of the “traditional” shocks.²³ The reduction

¹⁹Although the magnitude of effects of common and specific productivity shocks differ, the responses of the several macroeconomic variables to these shocks are qualitatively the same.

²⁰On impact, the nominal exchange rate falls by 0.45% and 0.33% in response to the common and the specific shocks, respectively.

²¹In the case of a common shock the increase corresponds to 0.46%, whereas it is equal to 0.34% for the specific shock.

²²The government currency demand decreases only by 0.006% and 0.005% in response to common and specific shocks, respectively.

²³Common and specific productivity shocks induce a fall in output of only 0.002% and 0.001%, respectively.

in aggregate output induces a decrease in the inflation rate. Moreover, the increase in the government currency growth leads the central bank to raise the nominal interest rate. We note that, in terms of magnitude, the changes in both inflation and nominal interest rate are negligible compared to their responses in the case of “traditional” shocks.²⁴

To summarise, the common and specific productivity shocks generate qualitatively similar reactions to the economy. In particular, the nominal exchange rate decreases due to the higher cryptocurrency supply. This leads to lower real balances of government currency, due to the substitution effect, which in turn reduces the inflation rate. However, the impact to the economy from these shocks is not as strong in comparison to the “traditional” shocks presented earlier.

6 Variance Decomposition Analysis

Table 4 shows the importance of each shock in terms of fluctuations in the key endogenous variables of the model. In particular, the variance decomposition analysis is based on the simulation of the estimated model (10,000 iterations).²⁵ More specifically, our strategy consists of two steps. As a first step we run the model estimation and we obtain that the parameters and the variance matrix of the shocks are set to the mode for the maximum likelihood estimation or posterior mode computation. As a second step, we simulate the model so that our simulation of the estimated model is based on the posterior modes of the model.²⁶

In Table 4, we observe that “traditional” shocks explain most of the

²⁴On impact, both common and specific productivity shocks induce a fall in the inflation rate of only 0.0002%. The peak responses of nominal interest rate to common and specific productivity shocks are 0.00002% and 0.00001%, respectively.

²⁵Our simulation results are detrended using the HP filter with a smoothing parameter equal to 1,600.

²⁶In general, it is preferable to follow this approach because the exact distributions of the posteriors are not known. Consequently, in the presence of irregular posteriors the mode is preferred to the mean as a measure of the central tendency of the distribution.

variations in output and inflation. In particular, the contributions of technology shocks on both output and inflation changes are almost 90%. The “traditional” shocks have also an important influence on the nominal interest rate. More specifically, 89% of the variation in the nominal interest rate is explained by a combination of technology and monetary policy shocks. The remaining 11% is explained by preferences, government currency and cryptocurrency demand shocks.

As expected, our results also show that government currency and cryptocurrency demand shocks contribute to most of the variations in the real balances for government currency and cryptocurrency (98% and 84%, respectively). Moreover, we find that the shock to cryptocurrency specific productivity accounts for 13% in terms of variation in the real balances for cryptocurrency. Interestingly, the variation in the nominal exchange rate between government currency and cryptocurrency is almost entirely explained by shocks to cryptocurrency specific productivity.

These results are confirmed by the forecast error variance decomposition, which we show for 1, 5, 12 and 30 periods ahead (Table 5). The “traditional” shocks (technology, preferences and monetary policy) have the highest contribution in terms of variations in the key endogenous variables of our model. We also find that specific supply shocks play an important role in the variation of cryptocurrency demand and exchange rate between government currency and cryptocurrency.

7 Robustness Analysis: Different Assumptions about the Taylor Rule

In this section, we investigate the role of monetary policy in the presence of the shocks to cryptocurrency productivity. In particular, we provide a counterfactual analysis with three different scenarios of the Taylor rule (24). More specifically, the parameter measuring the response of the policy rate to

government currency growth (ρ^{μ^g}) is assumed to be: equal to its estimated value (benchmark scenario), equal to zero (counterfactual scenario 1),²⁷ and equal to the double of its estimated value in our model (counterfactual scenario 2).²⁸

Figures 7 and 8 show the responses of the key variables of our model in the cases of cryptocurrency common and specific productivity shocks, respectively. The solid lines represent the impulse responses of the variables in the benchmark scenario, whereas the dashed and dotted lines show the impulse responses for the same variables in counterfactual scenarios 1 and 2, respectively.

In general, the increase in the entrepreneurs' productivity induces a drop in the nominal exchange rate between government currency and cryptocurrency. Accordingly, the real balances of cryptocurrency increase. We also observe a substitution effect between cryptocurrency demand and government currency demand with a reduction of the latter. As explained above, these effects induce output to fall. However, from Figures 7 and 8, we note that the magnitude of this decrease is different between the three scenarios. This result clearly depends on the response of the central bank to cryptocurrency (common and specific) productivity shocks. When the monetary authority does not consider government currency growth in the Taylor rule (counterfactual scenario 1), the nominal interest rate falls. In turn, the fall in the output is less pronounced than in the benchmark case. On the contrary, when the weight of government currency growth in the Taylor rule is higher (counterfactual scenario 2), the increase in the nominal interest rate is larger than in the benchmark case. In turn, this effect induces a larger fall in output.

The different magnitude of the fall in output between the three alternative

²⁷This assumption implies no weight of government currency growth in the Taylor rule.

²⁸This assumption implies a higher weight of government currency growth in the Taylor rule.

scenarios has also consequences on the inflation rate. As it can be observed from Figures 7 and 8, in counterfactual scenario 2, inflation falls more than in the benchmark case whereas, in counterfactual scenario 1 slightly increases.²⁹

8 Conclusion

In this paper, we have developed and estimated a Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the economic repercussions of cryptocurrency. Our model assumed that the representative household maximizes its utility accounting also for cryptocurrency holdings. Moreover, in our theoretical framework, we included entrepreneurs that determine the supply of cryptocurrency in the economy. We estimated our model using US monthly data and we compared our empirical findings with the “state-of-art” models without cryptocurrency.

We provided an impulse response analysis to show the effects of preferences, technology and monetary policy shocks on the real balances of government currency as well as to the real balances of cryptocurrency. Moreover, we evaluated the responses of main macroeconomic fundamentals to productivity shocks for production of cryptocurrency.

We found a strong substitution effect between the real balances of government currency and the real balances of cryptocurrency in response to technology, preferences and monetary policy shocks. Moreover, government currency demand shocks had larger effects on the economy than shocks to cryptocurrency demand. We also found that cryptocurrency productivity shocks imply a fall in the nominal exchange rate. Output and inflation fall whereas the nominal interest rate increases. However, the magnitude of the effects of these shocks was much lower than the “traditional” shocks.

Overall, our work provides novel insights and new evidence on the

²⁹The small increase in inflation in counterfactual scenario 1 is also due to the fall in the nominal interest rate.

underlying mechanisms of cryptocurrency and the spillover effects it has on the economy. This can provide guidance to investors, policy makers, central bankers and researchers, on how to act towards cryptocurrency and its ecosystem in the future. In particular, two policy recommendations emerge from our analysis. Firstly, we have shown that an increase in cryptocurrency supply has a negative effect on output. Therefore, the monetary authority could decide to adjust its policy rate in response to changes in the real balances for cryptocurrency, including a weight for cryptocurrency growth, in its policy reaction function. Secondly, we provided evidence that the response of the nominal interest rate to changes in government currency growth needs to be gradual if the central bank wants to avoid a fall in output.

Our analysis opens several extensions. For example, our estimated DSGE framework could be extended to a two-country exercise, extending studies on global cryptocurrency such as Benigno et al. (2019), or even to a heterogeneous household setup.

References

- Andrés, J., López-Salido, J. D., and Nelson, E. (2009). Money and the natural rate of interest: Structural estimates for the united states and the euro area. *Journal of Economic Dynamics and Control*, 33(3):758–776.
- Athey, S., Parashkevov, I., Sarukkai, V., and Xia, J. (2016). Bitcoin pricing, adoption, and usage: Theory and evidence. Technical report, SSRN.
- Benigno, P., Schilling, L., and Uhlig, H. (2019). Cryptocurrencies, currency competition, and the impossible trinity. Technical report, NBER Working Paper No. w26214.
- Bianchi, D. (2018). Cryptocurrencies As an Asset Class? An Empirical Assessment. Technical report, SSRN Working Paper.
- Bloomberg (2017a). Japan’s BITPoint to Add Bitcoin Payments to Retail Outlets. <https://www.bloomberg.com/news/articles/2017-05-29/japan-s-bitpoint-to-add-bitcoin-payments-to-100-000s-of-outlets>.
- Bloomberg (2017b). Nasdaq Plans to Introduce Bitcoin Futures. <https://www.bloomberg.com/news/articles/2017-11-29/nasdaq-is-said-to-plan-bitcoin-futures-joining-biggest-rivals>.
- Bloomberg (2017c). Some Central Banks Are Exploring the Use of Cryptocurrencies. <https://www.bloomberg.com/news/articles/2017-06-28/rise-of-digital-coins-has-central-banks-considering-e-versions>.
- Bodenstein, M., Erceg, C. J., and Guerrieri, L. (2011). Oil shocks and external adjustment. *Journal of International Economics*, 83(2):168–184.
- Boehme, R., Christin, N., Edelman, B., and Moore, T. (2015). Bitcoin: Economics, technology, and governance. *Journal of Economic Perspectives*, 29(2):213–38.

- Brooks, S. P. and Gelman, A. (1998). General methods for monitoring convergence of iterative simulations. *Journal of computational and graphical statistics*, 7(4):434–455.
- Canova, F. and Sala, L. (2009). Back to square one: Identification issues in dsge models. *Journal of Monetary Economics*, 56(4):431–449.
- Catania, L., Grassi, S., and Ravazzolo, F. (2018). Predicting the Volatility of Cryptocurrency Time—Series. Technical report, Mimeo.
- Catania, L., Grassi, S., and Ravazzolo, F. (2019). Forecasting cryptocurrencies under model and parameter instability. *International Journal of Forecasting*, 39(2):485–501.
- Chicago Mercantile Exchange (2017). CME Group Announces Launch of Bitcoin Futures. http://www.cmegroup.com/media-room/press-releases/2017/10/31/cme_group_announceslaunchofbitcoinfutures.html.
- Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45.
- Cointelegraph (2017). South Korea Officially Legalizes Bitcoin, Huge Market For Traders. <https://cointelegraph.com/news/south-korea-officially-legalizes-bitcoin-huge-market-for-traders>.
- Ethereum (2014). Ethereum Wiki. <https://github.com/ethereum/wiki/wiki/White-Paper/>.
- Fernández-Villaverde, J. and Sanches, D. (2016). Can currency competition work? Technical report, National Bureau of Economic Research.
- Forbes (2017). Emerging Applications For Blockchain. <https://www.forbes.com/sites/forbestechcouncil/2017/07/18/emerging-applications-for-blockchain>.

- Foroni, C. and Marcellino, M. (2014). Mixed-frequency structural models: Identification, estimation, and policy analysis. *Journal of Applied Econometrics*, 29(7):1118–1144.
- Gans, J. S. and Halaburda, H. (2019). Some economics of private digital currency. Technical report, Rotman School of Management Working Paper No. 2297296.
- Garratt, R. and Wallace, N. (2018). Bitcoin 1, bitcoin 2,...: An experiment in privately issued outside moneis. *Economic Inquiry*, 56(3):1887–1897.
- Giudici, P. and Pagnottoni, P. (2019). Vector error correction models to measure connectedness of bitcoin exchange markets. *Applied Stochastic Models in Business and Industry*, forthcoming.
- Hencic, A. and Gourieroux, C. (2014). Noncausal Autoregressive Model in Application to Bitcoin/USD Exchange Rate. *Proceedings of the 7th Financial Risks International Forum*, pages 1–25.
- Ireland, P. N. (2000). Interest rates, inflation, and federal reserve policy since 1980. *Journal of Money, Credit and Banking*, pages 417–434.
- Ireland, P. N. (2004). Money’s role in the monetary business cycle. *Journal of political Economy*, 36(6):969–983.
- Iskrev, N. (2010). Local identification in dsge models. *Journal of Monetary Economics*, 57(2):189–202.
- Kareken, J. and Wallace, N. (1981). On the indeterminacy of equilibrium exchange rates. *The Quarterly Journal of Economics*, 96(2):207–222.
- King, R. and Watson, M. (1996). Money, prices, interest rates and the business cycle. *The Review of Economics and Statistics*, 78(1):35–53.

- Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of political Economy*, 113(3):463–484.
- Nakamoto, S. (2009). Bitcoin: A Peer-to-Peer Electronic Cash System. <https://Bitcoin.org/Bitcoin.pdf>.
- Nelson, E. (2002). Direct effects of base money on aggregate demand: theory and evidence. *Journal of Monetary Economics*, 49(4):687–708.
- Ripple (2012). Welcome to Ripple. <https://ripple.com/>.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211.
- Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy*, 66(6):467–482.
- Sapuric, S. and Kokkinaki, A. (2014). Bitcoin is Volatile! Isn't That Right? *Business Information Systems Workshops*, pages 255–265.
- Schilling, L. and Uhlig, H. (2019). Some simple bitcoin economics. *Journal of Monetary Economics*, 106:16–26.
- Sockin, M. and Xiong, W. (2018). A model of cryptocurrencies. Technical report, Working paper.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214.
- Tokyo Financial Exchange (2017). Tokyo Financial Exchange Plans for Bitcoin Futures Launch. <https://www.cryptocoinsnews.com/breaking-tokyo-financial-exchange-plans-bitcoin-futures-launch/>.

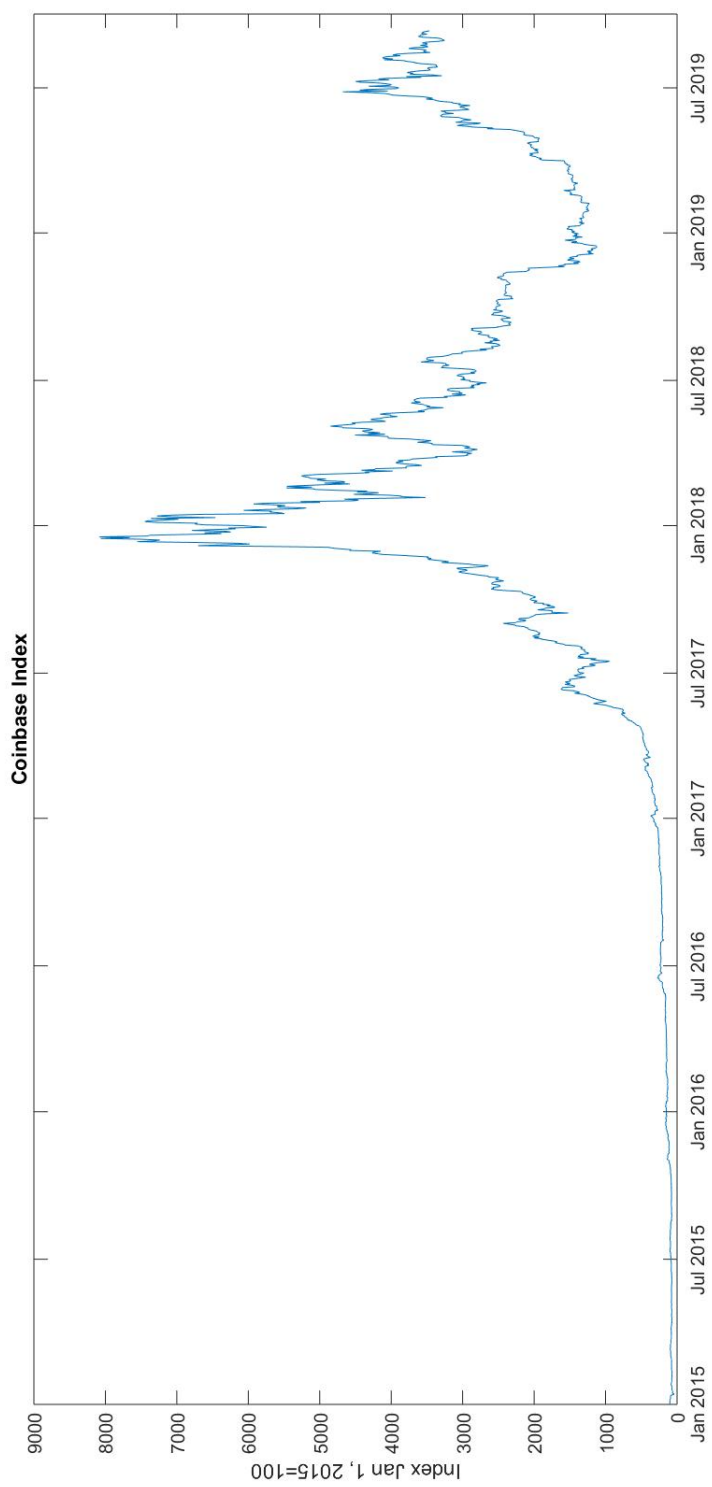


Figure 1: Coinbase Cryptocurrency Index

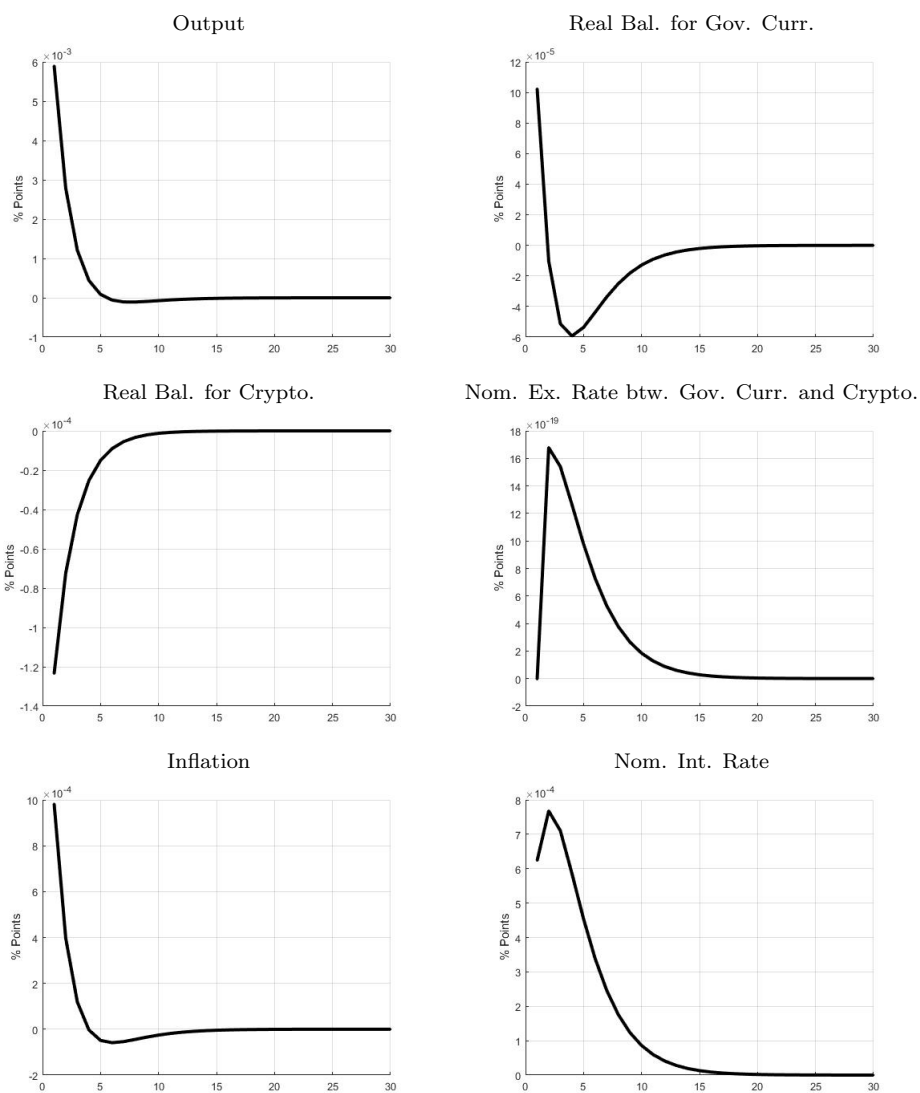


Figure 2: Responses to Preferences Shock

Notes: Simulated 1% shock to household preference.

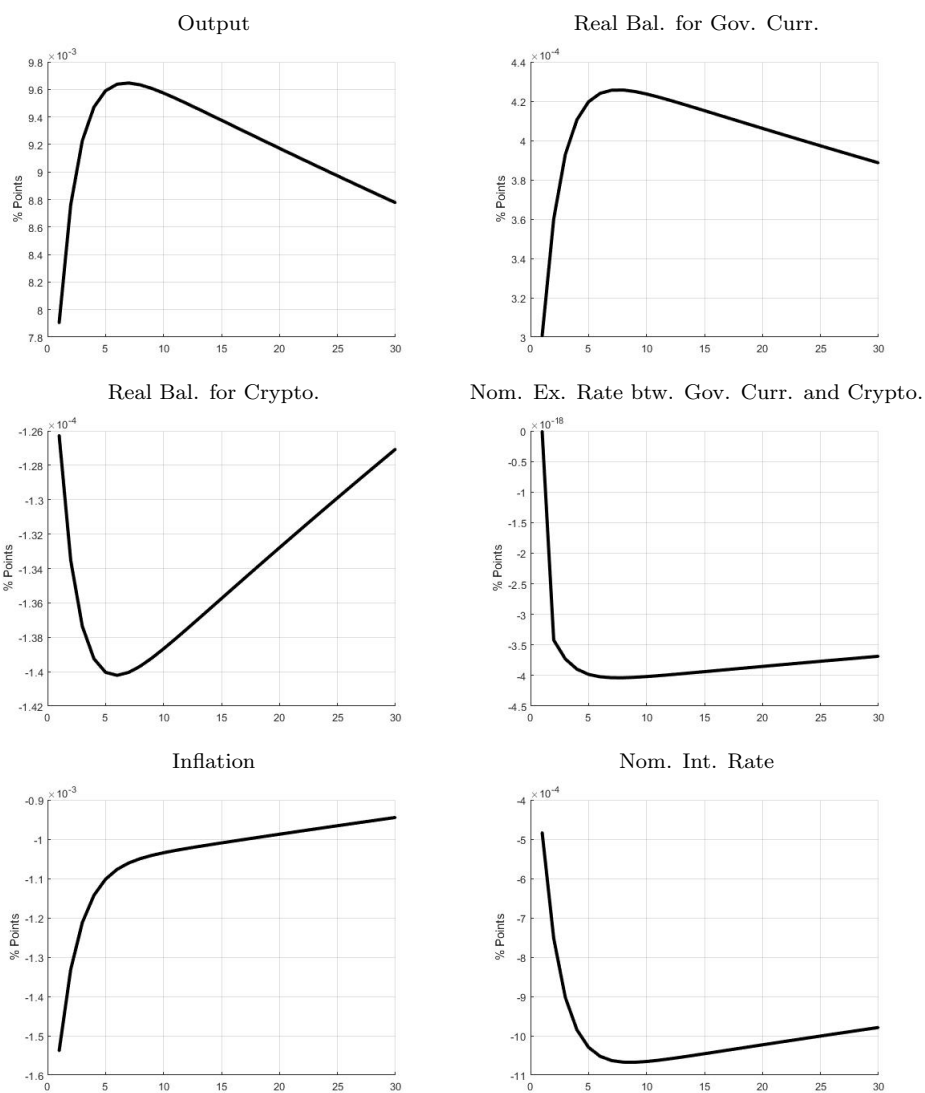


Figure 3: Responses to Technology Shock

Notes: Simulated 1% shock to technology.

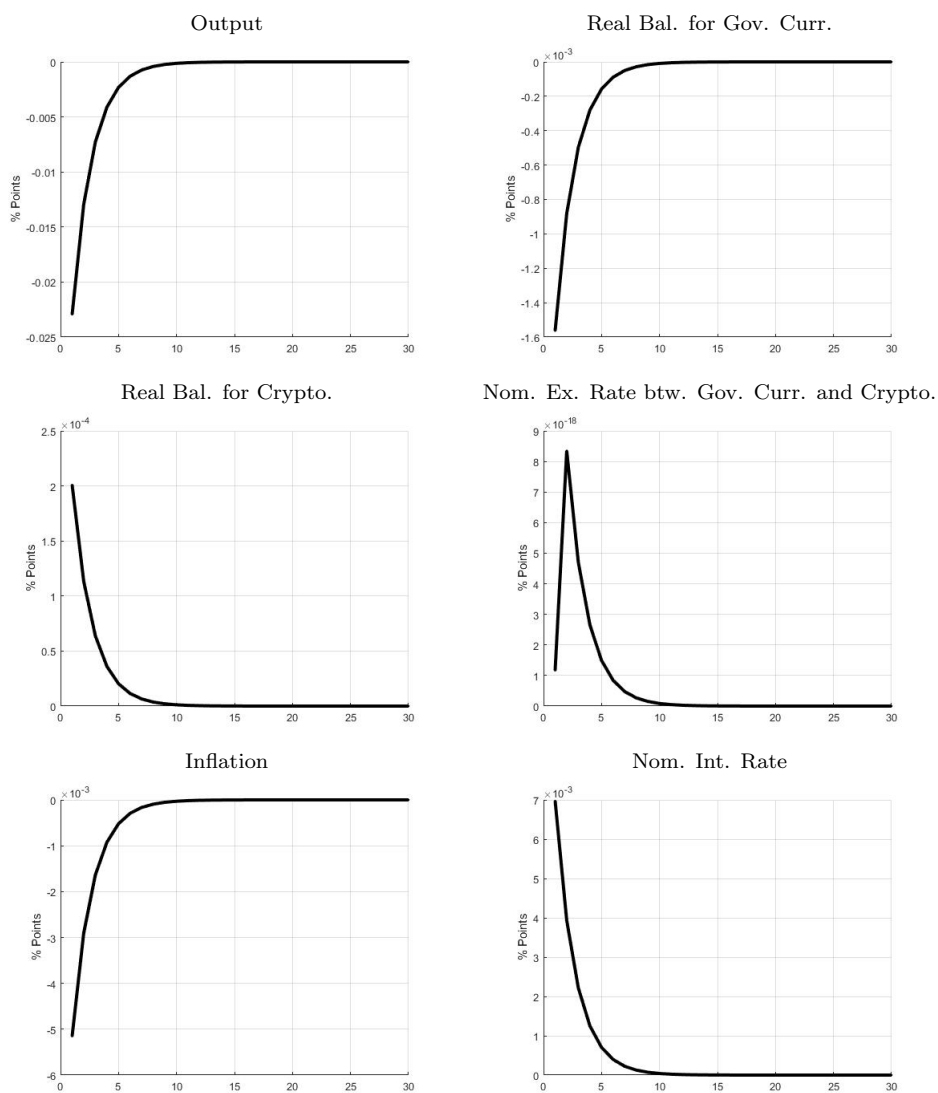


Figure 4: Responses to Monetary Policy Shock

Notes: Simulated 1% shock to monetary policy.

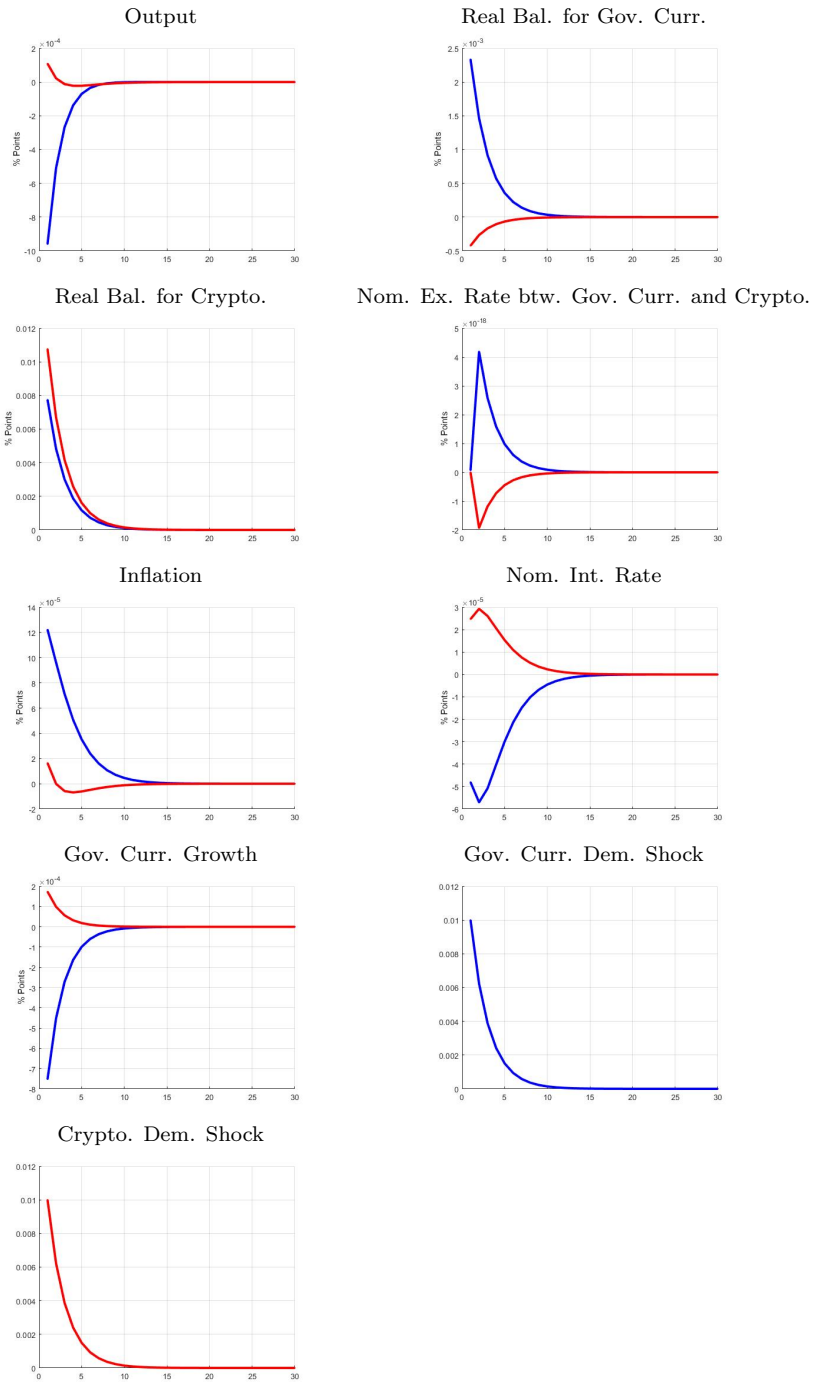


Figure 5: Responses to Government Currency and Cryptocurrency Demand Shocks

Notes: Simulated 1% shocks to government currency and cryptocurrency demands. Blue lines denote the responses to a government currency demand shock, whereas red lines represent the responses to a cryptocurrency demand shock.

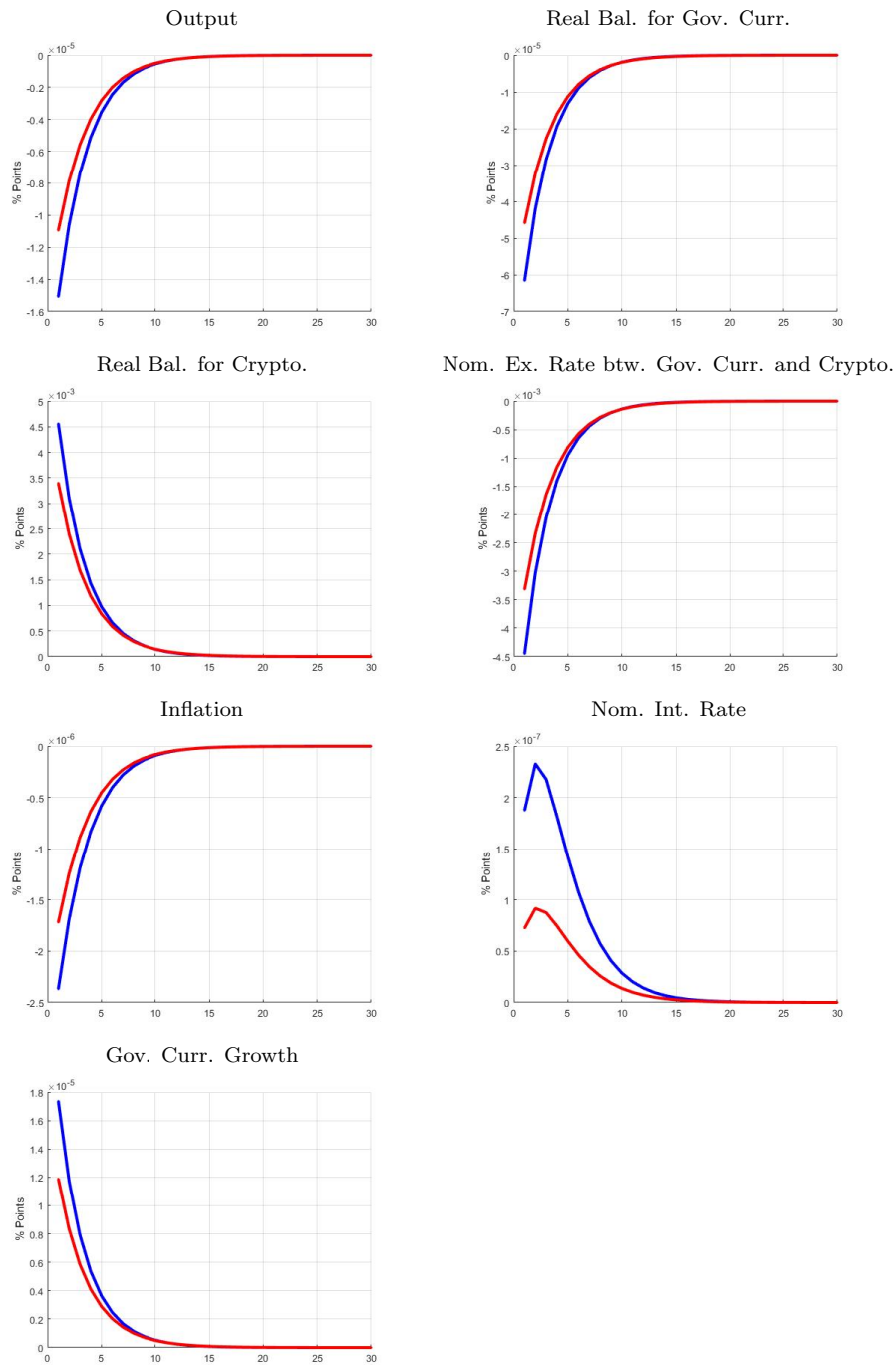


Figure 6: Responses to Cryptocurrency Productivity Shocks

Notes: Simulated 1% shocks to common and specific productivity of cryptocurrency. Blue lines denote the responses to a common productivity shock of cryptocurrency, whereas red lines represent the responses to a specific productivity shock of cryptocurrency.

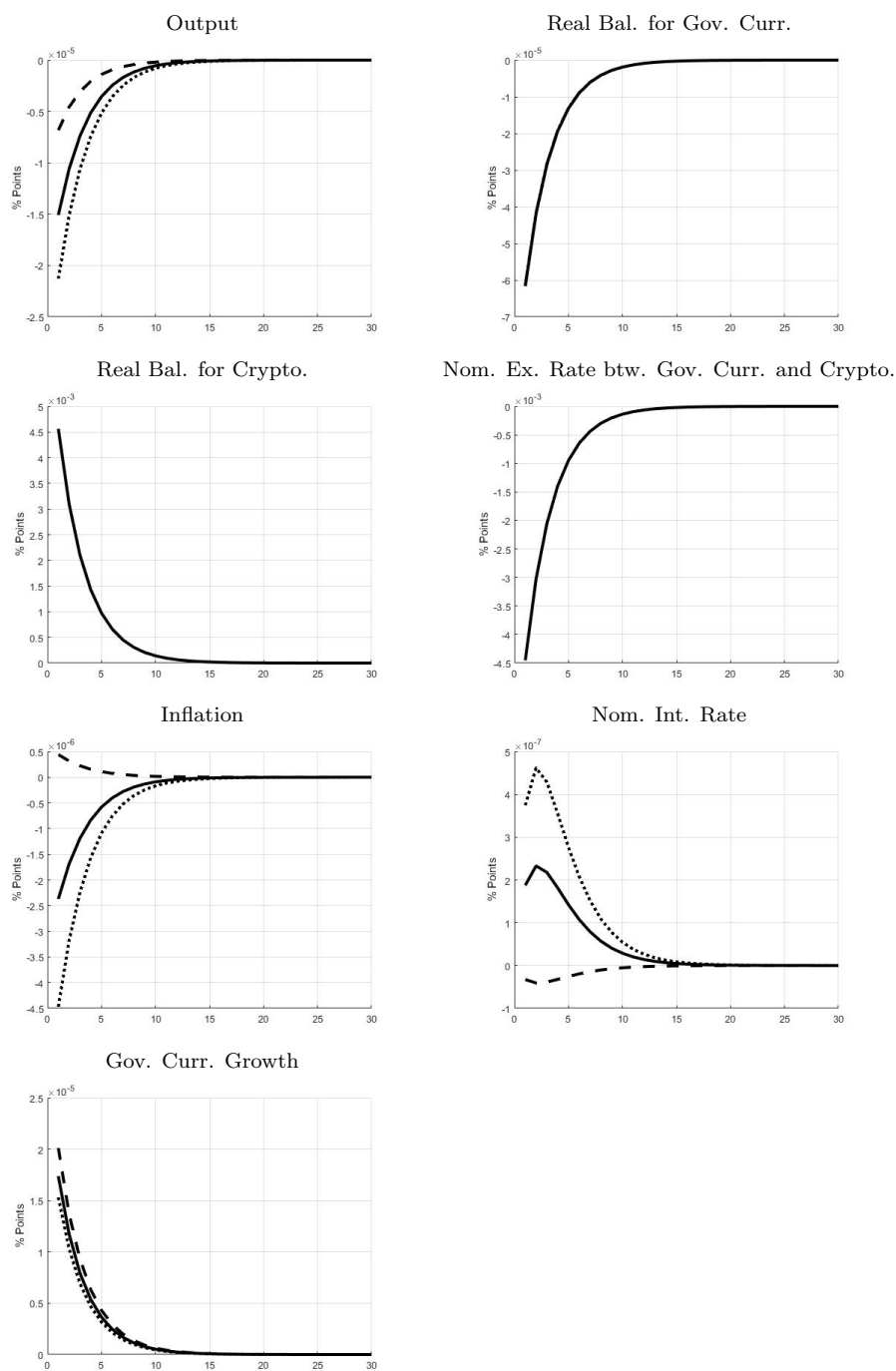


Figure 7: Robustness: Responses to Cryptocurrency Common Productivity Shock

Notes: Solid lines denote the IRFs of the benchmark model, whereas the dashed and dotted lines represent the responses of the model in counterfactual scenarios 1 and 2, respectively.

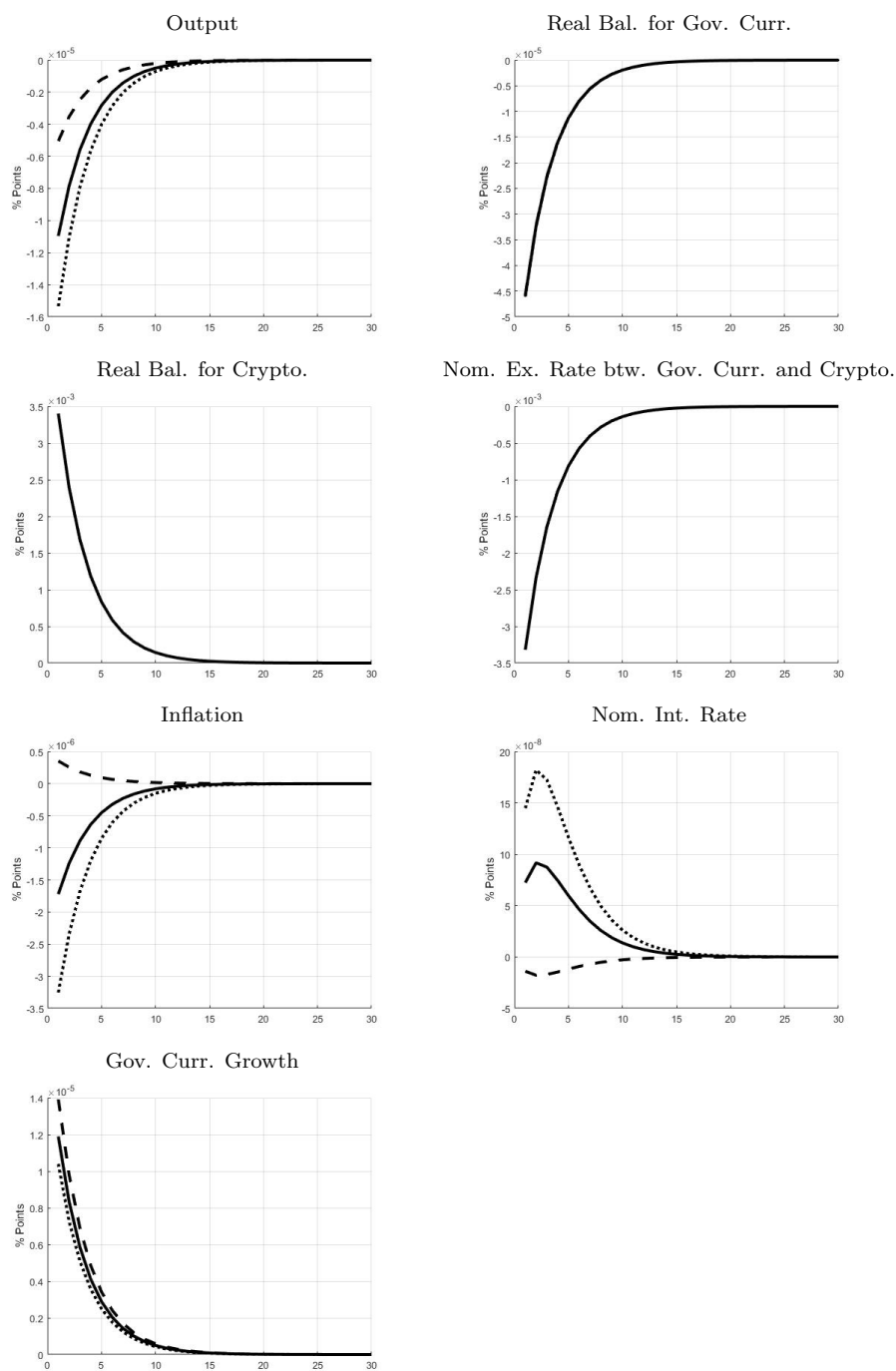


Figure 8: Robustness: Responses to Cryptocurrency Specific Productivity Shock

Notes: Solid lines denote the IRFs of the benchmark model, whereas the dashed and dotted lines represent the responses of the model in counterfactual scenarios 1 and 2, respectively.

Table 1: Exogenous Shocks and Observed Variables

Shocks	Observed Variables
Technology Shock	US Industrial Production Index
Shock to Household's Preferences	US Real Private Consumption
Shock to Household's Demand for Government Currency	US Real Balances for Government Currency
Shock to Household's Demand for Cryptocurrency	Real Bitcoin Price
Common Supply Shock of Cryptocurrency	Real Cumulative Initial Coin Offering (ICO)
Specific Supply Shock of Cryptocurrency	Real Nvidia Volume Weighted Average Price
Monetary Policy Shock	US Nominal Interest Rate

Notes: The data sources and the construction of all observed variables are reported in the Appendix B.

Table 2: Priors and Posteriors for the Endogenous Parameters

<i>Parameter</i>	<i>Symbol</i>	<i>Priors</i>			<i>Posteriors</i>		
		Dist.	Mean	St. Dev.	Mean	Conf.	Inter.
Output El. to Real Bal. of Gov. Currency	ω_2	G	0.200	0.050	0.195	0.102	0.284
Output El. to Real Bal. of Cryptocurrency	ω_3	G	0.050	0.010	0.035	0.024	0.046
Income El. of Gov. Currency Demand	γ_1	G	0.015	0.005	0.021	0.009	0.032
Interest Semi-El. of Gov. Currency Demand	γ_2	G	0.150	0.050	0.140	0.066	0.214
El. of Real Bal. of Gov. Curr. wrt Gov. Curr. Dem. Shock	γ_3	G	0.900	0.100	0.664	0.593	0.733
Cross El. of Gov. Cur. Dem. and Crypto. Dem.	γ_4	G	0.500	0.050	0.554	0.467	0.638
Income El. Cryptocurrency Demand	γ_5	G	0.015	0.005	0.013	0.006	0.020
Interest Semi-El. of Cryptocurrency Demand	γ_6	G	0.150	0.050	0.155	0.073	0.236
El. of Real Bal. of Crypto. wrt Crypto. Dem. Shock	γ_7	G	0.800	0.100	1.034	1.014	1.053
Cross El. of Crypto. Dem. and Gov. Cur. Dem.	γ_8	G	0.600	0.100	1.011	0.985	1.037
Ex. Rate Crypto. / Gov. Cur. El. wrt Prod.	ϱ	G	0.900	0.100	0.777	0.638	0.916
Share of Crypto. Common Prod. on Crypto. Tot. Prod.	ξ_ϕ	G	0.500	0.050	0.572	0.482	0.662
Interest. Rate Smoothing	ρ^r	B	0.800	0.050	0.808	0.765	0.852
Taylor Rule Coef. on Output	ρ^y	B	0.200	0.010	0.153	0.142	0.163
Taylor Rule Coef. on Inflation	ρ^π	G	1.800	0.050	1.980	1.900	2.063
Taylor Rule Coef. on Gov. Currency Growth	ρ^{μ^g}	B	0.200	0.050	0.459	0.368	0.555

Table 3: Priors and Posteriors for the Shock Processes Parameters

<i>Parameter</i>	<i>Symbol</i>	<i>Priors</i>			<i>Posteriors</i>		
		Distr.	Mean	St. Dev.	Mean	Conf.	Inter.
Household's Preference Shock Pers.	ρ^a	B	0.700	0.050	0.668	0.586	0.751
Gov. Cur. Demand Shock Pers.	ρ^{eg}	B	0.650	0.050	0.623	0.548	0.700
Crypto. Demand Shock Pers.	ρ^{ec}	B	0.550	0.050	0.622	0.554	0.690
Technology Shock Pers.	ρ^z	B	0.900	0.050	0.996	0.992	0.999
Crypto. Common Prod. Shock Pers.	ρ^ξ	B	0.600	0.050	0.679	0.616	0.742
Crypto. Specific Prod. Shock Pers.	ρ^ν	B	0.600	0.050	0.703	0.642	0.765
Household's Preference Shock St. Err.	σ^a	I-G	0.010	Inf	0.278	0.238	0.315
Gov. Cur. Demand Shock St. Err.	σ^{eg}	I-G	0.010	Inf	1.578	0.824	2.320
Crypto. Demand Shock St. Err.	σ^{ec}	I-G	0.010	Inf	3.799	3.065	4.538
Technology Shock St. Err.	σ^z	I-G	0.010	Inf	0.734	0.611	0.853
Crypto. Common Prod. Shock St. Err..	σ^ξ	I-G	0.010	Inf	0.047	0.041	0.054
Crypto. Specific Prod. Shock St. Err..	σ^ν	I-G	0.010	Inf	4.763	4.071	5.436
Monetary Policy Shock St. Err.	σ^r	I-G	0.010	Inf	0.076	0.059	0.091

Table 4: Variance Decomposition (%)

	\hat{y}_t	$\hat{\pi}_t$	\hat{r}_t	\hat{m}_t^g	\hat{m}_t^c	$\hat{\chi}_t$
σ^a	3.52	4.04	8.07	0.01	0.00	0.00
σ^{eg}	3.08	2.78	1.41	83.08	7.09	0.00
σ^{ec}	0.24	0.30	2.15	15.85	79.29	0.00
σ^z	89.02	84.20	60.74	0.66	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.32	13.61	99.98
σ^r	4.14	8.68	27.63	0.08	0.00	0.00

Table 5: Forecast Error Variance Decomposition (%)

<i>Period 1</i>						
	\hat{y}_t	$\hat{\pi}_t$	\hat{r}_t	\hat{m}_t^g	\hat{m}_t^c	$\hat{\chi}_t$
σ^a	6.41	4.82	6.68	0.00	0.00	0.00
σ^{eg}	5.49	2.41	1.29	83.45	7.17	0.00
σ^{ec}	0.41	0.25	1.97	15.86	80.23	0.00
σ^z	80.49	82.69	28.03	0.30	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.01	0.00	0.00	0.29	12.60	99.98
σ^r	7.19	9.82	62.04	0.09	0.00	0.00
<i>Period 5</i>						
	\hat{y}_t	$\hat{\pi}_t$	\hat{r}_t	\hat{m}_t^g	\hat{m}_t^c	$\hat{\chi}_t$
σ^a	1.48	1.84	6.04	0.01	0.00	0.00
σ^{eg}	1.38	1.72	1.02	82.39	7.02	0.00
σ^{ec}	0.08	0.12	1.56	15.76	78.35	0.00
σ^z	95.14	91.69	75.72	1.43	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.34	14.62	99.98
σ^r	1.91	4.63	15.65	0.08	0.00	0.00
<i>Period 12</i>						
	\hat{y}_t	$\hat{\pi}_t$	\hat{r}_t	\hat{m}_t^g	\hat{m}_t^c	$\hat{\chi}_t$
σ^a	0.59	0.99	2.54	0.01	0.00	0.00
σ^{eg}	0.55	0.95	0.42	80.39	7.00	0.00
σ^{ec}	0.04	0.07	0.64	15.38	78.11	0.00
σ^z	98.06	95.49	90.49	3.81	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.34	14.89	99.98
σ^r	0.76	2.49	5.92	0.07	0.00	0.00
<i>Period 30</i>						
	\hat{y}_t	$\hat{\pi}_t$	\hat{r}_t	\hat{m}_t^g	\hat{m}_t^c	$\hat{\chi}_t$
σ^a	0.25	0.49	1.03	0.01	0.00	0.00
σ^{eg}	0.23	0.46	0.17	76.08	7.00	0.00
σ^{ec}	0.01	0.03	0.26	14.56	78.10	0.00
σ^z	99.19	97.80	96.13	8.97	0.01	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.32	14.89	99.98
σ^r	0.32	1.22	2.41	0.07	0.00	0.00

Appendices to “A New Economic Framework:
A DSGE Model with Cryptocurrency”

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1 Appendix A: Model Solution

1.1 First Order Conditions

The representative household chooses C_t , H_t , B_t , M_t^g and M_t^c for all $t = 0, 1, 2, \dots$ to maximize its expected utility, subject to its budget constraints. The first-order conditions for this problem can be written as follows.

The first order condition for C_t is given by:

$$\lambda_t = -A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) \frac{1}{P_t} \quad (\text{A1})$$

$m_t^g = \frac{M_t^g}{P_t}$ and $m_t^c = \frac{M_t^c}{P_t}$, whereas λ_t is the Lagrange multiplier associated with the representative household budget constraint. Moreover, u_1 denotes the derivative of the utility function, u , with respect to its first argument.

The first order condition for H_t is given by:

$$-A_t \eta - \lambda_t w_t = 0 \quad (\text{A2})$$

where $w_t = \frac{W_t}{P_t}$.

Combining equations (A1) and (A2) we obtain:

$$\eta = u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) w_t \quad (\text{A3})$$

The first order condition for B_t is given by:

$$A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) = \beta R_t \left[A_{t+1} u_1 \left(C_{t+1}, \frac{m_{t+1}^g}{E_{t+1}^g}, \frac{\chi_{t+1} m_{t+1}^c}{E_{t+1}^c} \right) \right] \frac{1}{\pi_{t+1}} \quad (\text{A4})$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

The first order condition for M_t^g is given by:

$$R_t u_2 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) = (R_t - 1) E_t^g u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) \quad (\text{A5})$$

where u_2 denotes the derivative of the utility function, u , with respect to its second argument.

The first order condition for M_t^c is given by:

$$R_t u_3 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) = (R_t - 1) E_t^c u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) \quad (\text{A6})$$

where u_3 denotes the derivative of the utility function, u , with respect to its third argument.

The market clearing conditions imply that:

$$\begin{aligned} M_t^g &= M_{t-1}^g + T_t \\ M_t^c &= M_{t-1}^c \\ B_t &= B_{t-1} = 0 \end{aligned}$$

Therefore, from the household's budget constraint we obtain that:

$$w_t H_t + d_t = C_t \quad (\text{A7})$$

where $d_t = \frac{D_t}{P_t}$.

The representative entrepreneur chooses Q_t^c for all $t = 0, 1, 2, \dots$ to maximize its profit given by:

$$\Pi_t = \max_{\{Q_t^c\}} \left((1 - \rho) \frac{P_t}{\chi_t} - \kappa^{-\phi_t} \right) Q_t^c$$

The first-order condition for this problem is:

$$(1 - \rho) \frac{1}{\chi_t} = \frac{1}{P_t} \kappa^{-\phi_t} \quad (\text{A8})$$

Moreover, from equation (6) in the maintext, we know that the entrepreneur's productivity is given by:

$$\omega_t = \xi_t + \nu_t \quad (\text{A9})$$

The representative intermediate goods-producing firm chooses $P_t(i)$ for all $t = 0, 1, 2, \dots$ to maximize its total market value, given by:

$$E \sum_{t=0}^{\infty} \beta A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right) \left[\frac{D_t(i)}{P_t} \right]$$

where $\beta A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right)$ measures the marginal utility value to the representative household of an additional dollar in profits received during period t . Moreover:

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \left(\frac{w_t Y_t}{Z_t} \right) - \frac{\phi}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t \quad (\text{A10})$$

for all $t = 0, 1, 2, \dots$

The expression (A10) for the firm's real dividend payment incorporates the linear production function along with the requirement that the firm supply output on demand; it also shows how the cost of price adjustment subtracts from profits. The first-order conditions for this problem are:

$$\begin{aligned} 0 = & (1 - \theta) \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \left(\frac{Y_t}{P_t} \right) + \\ & \theta \left[\frac{P_t(i)}{P_t} \right]^{-\theta-1} \left(\frac{Y_t w_t}{Z_t P_t} \right) - \phi \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right] \left[\frac{Y_t}{\pi P_{t-1}(i)} \right] + \\ & \beta \phi E \left\{ \left[\frac{A_{t+1} u_1 \left(C_{t+1}, \frac{m_{t+1}^g}{E_{t+1}^g}, \frac{\chi_{t+1} m_{t+1}^c}{E_{t+1}^c} \right)}{A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right)} \right] \left[\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right] \left[\frac{Y_{t+1} P_{t+1}(i)}{\pi P_t(i)^2} \right] \right\} \end{aligned} \quad (\text{A11})$$

for all $t = 0, 1, 2, \dots$

In a symmetric equilibrium:

$$\begin{aligned} Y_t(i) &= Y_t \\ H_t(i) &= H_t \\ P_t(i) &= P_t \\ D_t(i) &= D_t \end{aligned}$$

and:

$$Y_t = Z_t H_t$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$ equations (A7) and (A10) can be combined to derive the economy's aggregate resource constraint:

$$Y_t = C_t + \frac{\phi}{2} \left[\frac{\pi_t}{\pi} - 1 \right]^2 Y_t \quad (\text{A12})$$

Combining equations (A3) and (A11) we obtain:

$$\theta - 1 = \theta \left[\frac{\eta}{Z_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right)} \right] - \phi \left(\frac{\pi_t}{\pi} - 1 \right) \left(\frac{\pi_t}{\pi} \right) + \quad (\text{A13})$$

$$\beta \phi E \left\{ \left[\frac{A_{t+1} u_1 \left(C_{t+1}, \frac{m_{t+1}^g}{E_{t+1}^g}, \frac{\chi_{t+1} m_{t+1}^c}{E_{t+1}^c} \right)}{A_t u_1 \left(C_t, \frac{m_t^g}{E_t^g}, \frac{\chi_t m_t^c}{E_t^c} \right)} \right] \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{\pi_{t+1}}{\pi} \right) \right\}$$

1.2 Steady State Relations

In the absence of shocks, the economy converges to a steady state, in which:

$$\begin{aligned}
 Y_t &= Y \\
 C_t &= C \\
 m_t^g &= m^g \\
 \chi_t &= \chi \\
 m_t^c &= m^c \\
 \pi_t &= \pi \\
 R_t &= R
 \end{aligned}$$

From equation (A4) we have that:

$$R = \frac{\pi}{\beta} \quad (\text{A14})$$

From equation (A12) we have that:

$$Y = C \quad (\text{A15})$$

From equation (A5) we have that:

$$R_t u_2 \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) = (R - 1) E^g u_1 \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) \quad (\text{A16})$$

From equation (A6) we have that:

$$R_t u_3 \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) = (R - 1) E^c u_1 \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) \quad (\text{A17})$$

From equation (A8) we have that:

$$(1 - \rho) \frac{1}{\chi} = \kappa^{-\phi} \quad (\text{A18})$$

From equation (A9) we have that:

$$\frac{\nu}{\phi} = 1 - \frac{\xi}{\phi} \quad (\text{A19})$$

From equation (A13) we have that:

$$(\theta - 1) Z u_1 \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) = \theta \eta \quad (\text{A20})$$

1.3 Log-linearized Equations

We denote:

$$\begin{aligned}\hat{y}_t &= \ln\left(\frac{Y_t}{Y}\right) \\ \hat{c}_t &= \ln\left(\frac{C_t}{C}\right) \\ \hat{m}_t^g &= \ln\left(\frac{m_t^g}{m^g}\right) \\ \hat{\chi}_t &= \ln\left(\frac{\chi_t}{\chi}\right) \\ \hat{m}_t^c &= \ln\left(\frac{m_t^c}{m^c}\right) \\ \hat{\pi}_t &= \ln\left(\frac{\pi_t}{\pi}\right) \\ \hat{r}_t &= \ln\left(\frac{R_t}{R}\right) \\ \hat{a}_t &= \ln\left(\frac{A_t}{A}\right) \\ \hat{e}_t^g &= \ln\left(\frac{E_t^g}{E^g}\right) \\ \hat{e}_t^c &= \ln\left(\frac{E_t^c}{E^c}\right) \\ \hat{\xi}_t &= \ln\left(\frac{\xi_t}{\xi}\right) \\ \hat{\nu}_t &= \ln\left(\frac{\nu_t}{\nu}\right) \\ \hat{z}_t &= \ln\left(\frac{Z_t}{Z}\right) \\ \hat{\mu}_t^g &= \ln\left(\frac{\mu_t^g}{\mu^g}\right)\end{aligned}$$

The first-order Taylor approximation to equation (A12) gives:

$$\hat{y}_t = \hat{c}_t \tag{A21}$$

The first-order Taylor approximation to equation (A4) gives:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \omega_1 (\hat{r}_t - \hat{\pi}_{t+1}) + \omega_2 [(\hat{m}_t^g - \hat{e}_t^g) - (\hat{m}_{t+1}^g - \hat{e}_{t+1}^g)] + \\ &\omega_3 [(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c) - (\hat{\chi}_{t+1} + \hat{m}_{t+1}^c - \hat{e}_{t+1}^c)] + \omega_1 (\hat{a}_t - \hat{a}_{t+1})\end{aligned}\quad (\text{A22})$$

where:

$$\omega_1 = -\frac{u_1\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{Y u_{11}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \quad (\text{A23})$$

$$\omega_2 = -\frac{\frac{m^g}{E^g} u_{12}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{Y u_{11}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \quad (\text{A24})$$

$$\omega_3 = -\frac{\frac{\chi m^c}{E^c} u_{13}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)}{Y u_{11}\left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c}\right)} \quad (\text{A25})$$

The first-order Taylor approximation to equation (A5) gives:

$$\hat{m}_t^g = \gamma_1 \hat{y}_t - \gamma_2 \hat{r}_t + \gamma_3 \hat{e}_t^g - \gamma_4 \hat{\chi}_t - \gamma_4 \hat{m}_t^c + \gamma_4 \hat{e}_t^c \quad (\text{A26})$$

where:

$$\gamma_1 = \left(R - 1 + \frac{Y R \omega_2}{m^g}\right) \left(\frac{\gamma_2}{\omega_1}\right) \quad (\text{A27})$$

$$\gamma_2 = \frac{R}{(R-1) \frac{m^g}{E^g}} \left[\frac{u_2\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}{(R-1) E^g u_{12}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right) - R u_{22}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)} \right] \quad (\text{A28})$$

$$\gamma_3 = 1 - (R-1) \gamma_2 \quad (\text{A29})$$

$$\gamma_4 = \frac{\frac{\chi m^c}{E^c}}{\frac{m^g}{E^g}} \left[\frac{\frac{u_{23}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}{u_{22}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right) - \left(\frac{R-1}{R} E^g\right) u_{12}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)}{u_{13}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)} - \frac{1}{\left(\frac{R}{R-1} \frac{1}{E^g}\right) u_{22}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right) - u_{12}\left(Y, \frac{m^g}{E^g}, \frac{m^c}{E^c}\right)} \right] \quad (\text{A30})$$

The first-order Taylor approximation to equation (A6) gives:

$$\hat{m}_t^c = \gamma_5 \hat{y}_t - \gamma_6 \hat{r}_t + \gamma_7 \hat{e}_t^c - \gamma_8 \hat{\chi}_t - \gamma_8 \hat{m}_t^g + \gamma_8 \hat{e}_t^g \quad (\text{A31})$$

where:

$$\gamma_5 = \left(R - 1 + \frac{Y R \omega_3}{\chi m^c} \right) \left(\frac{\gamma_6}{\omega_1} \right) \quad (\text{A32})$$

$$\gamma_6 = \frac{R}{(R-1) \frac{\chi m^c}{E^c}} \left[\frac{u_3 \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right)}{(R-1) E^c u_{13} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) - R u_{33} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right)} \right] \quad (\text{A33})$$

$$\gamma_7 = 1 - (R-1) \gamma_6 \quad (\text{A34})$$

$$\gamma_8 = \frac{\frac{m^g}{E^g}}{\frac{\chi m^c}{E^c}} \left[\frac{\frac{u_{32} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right)}{u_{33} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) - \left(\frac{R-1}{R} E^c \right) u_{13} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right)}{u_{12} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right)} - \frac{1}{\left(\frac{R}{R-1} \frac{1}{E^c} \right) u_{33} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right) - u_{13} \left(Y, \frac{m^g}{E^g}, \frac{\chi m^c}{E^c} \right)} \right] \quad (\text{A35})$$

Since in steady-state $P = 1$, the log-linearized expression for (A8) is given by:

$$\hat{\chi}_t = -\varrho \hat{\phi}_t \quad (\text{A36})$$

where $\varrho = \phi \ln(\kappa)$.

The log-linearized expression for (A9) is given by:

$$\hat{\omega}_t = \left(\frac{\xi}{\omega} \right) \hat{\xi}_t + \left(1 - \frac{\xi}{\omega} \right) \hat{\nu}_t \quad (\text{A37})$$

The first-order Taylor approximation to equation (A13) gives:

$$\hat{\pi}_t = \left(\frac{\pi}{R} \right) \hat{\pi}_{t+1} + \psi \left[\begin{array}{l} \left(\frac{1}{\omega_1} \right) \hat{y}_t - \left(\frac{\omega_2}{\omega_1} \right) (\hat{m}_t^g - \hat{e}_t^g) \\ - \left(\frac{\omega_3}{\omega_1} \right) (\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c) - \hat{z}_t \end{array} \right] \quad (\text{A38})$$

where:

$$\psi = \frac{(\theta - 1)}{\phi} \quad (\text{A39})$$

Equation (A23) shows that ω_1 depends inversely on the household's relative risk aversion. Equation (A24) and (A25) indicate that $\omega_2 > 0$ and $\omega_3 > 0$, so that changes in real balances for government currencies and cryptocurrencies enter into the IS and Phillips curves, if and only if $u_{12} > 0$ and $u_{13} > 0$, so that utility is non-separable across consumption and real balances for government currency and cryptocurrencies. Finally, equation

(A40) reveals that the parameter ψ in the Phillips curve, equation (A39), is inversely related to the cost-of-price-adjustment parameter, ϕ .

Finally, the log-linearization of the Taylor rule (16) in the main text gives:

$$\hat{r}_t = \rho^r \hat{r}_{t-1} + (1 - \rho^r) \rho^y \hat{y}_t + (1 - \rho^r) \rho^\pi \hat{\pi}_t + (1 - \rho^r) \rho^{\mu^g} \hat{\mu}_t^g + \varepsilon_t^r \quad (40)$$

2 Appendix B: Data Construction and Sources

As we described in the main body of the paper, the data is monthly and the model is estimated for the sample period 2013:M6-2019:M3. Here, we provide the sources and construction methods of the observed series. Unless otherwise noted, all original series are seasonally adjusted.

US Industrial Production Index. The US industrial production index, index 2012=100, is taken from the Federal Reserve Bank of St. Louis (code INDPRO in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>).

US Real Private Consumption. It is obtained from the series of personal consumption expenditures, billions of Dollars, and it is taken from the Federal Reserve Bank of St. Louis (code PCE in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>). The original series is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>), divided by the civilian employment level, thousands of persons (code CE16OV in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>) and expressed in log terms.

US Real Balances of Government Currency. It is obtained from the series of M2 money stock, billions of Dollars, and it is taken from the Federal Reserve Bank of St. Louis (code M2 in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>). The original series is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>), divided by the civilian employment level, thousands of persons (code CE16OV in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>) and expressed in log terms.

Real Bitcoin Price. It is obtained as the average of the series of

opening and closing prices and it is taken from CoinMarketCap (link: <https://coinmarketcap.com>). The monthly series is obtained as average from daily data and is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>).

Real cumulative ICO funding. It is obtained from the series of cumulative initial coin offering (ICO) funding, millions of Dollars, and it is taken from the CoinDesk ICO Tracker (link: <https://www.coindesk.com/ico-tracker>). For 2013:M6-2014:M1, we applied the growth rate of the series of Bitcoin average market cap (link: <https://coinmarketcap.com>, the monthly series was obtained as average from daily data). The final series of cumulative ICO funding was deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>).

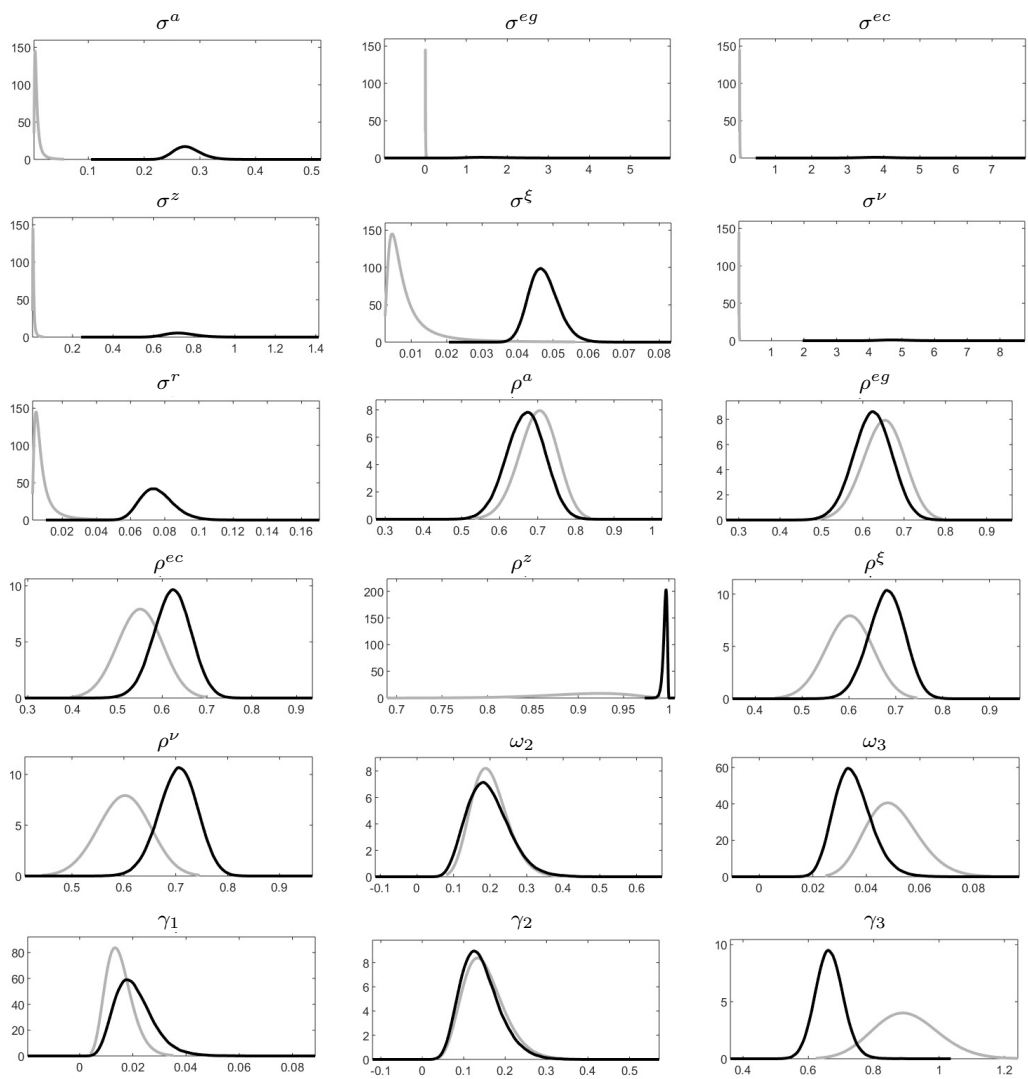
Real Nvidia Volume Weighted Average Price. It is obtained from the series of Nvidia volume weighted average price and it is downloaded from Thomson Reuters Eikon. The monthly series was obtained as average from daily data and is deflated by the consumer price index for all urban consumers, all items, index 1982-1984=100 (code CPIAUCSL in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>).

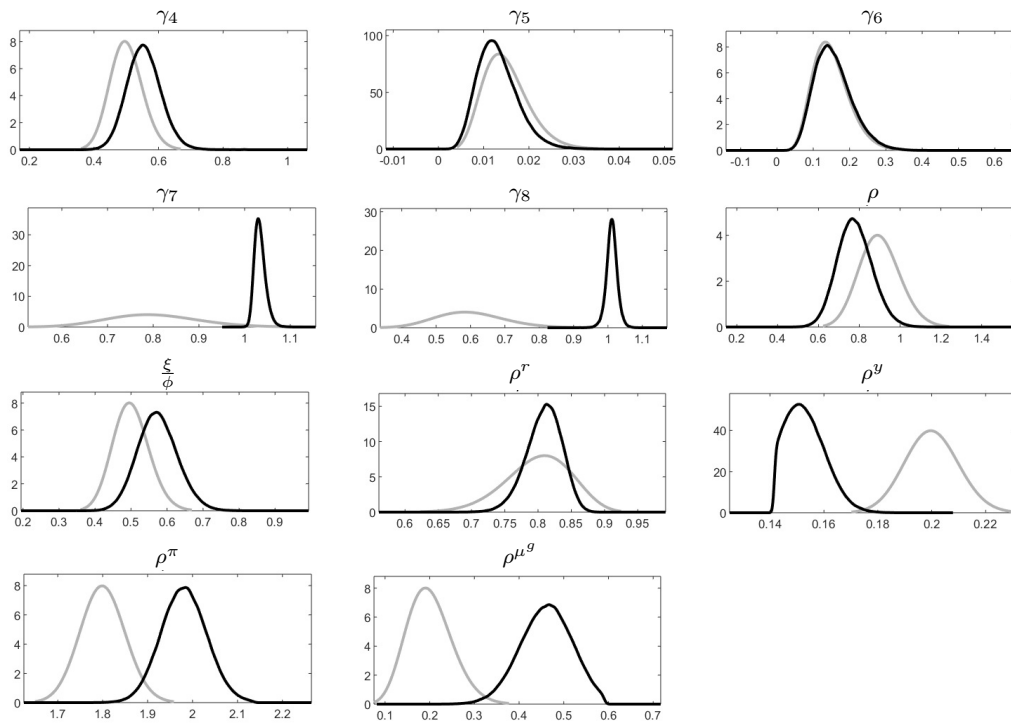
US Nominal Interest Rate. The US nominal interest rate is the series of effective Federal funds rate, %, and it is taken from the Federal Reserve Bank of St. Louis (code FEDFUNDS in Federal Reserve Economic Data, link: <https://fred.stlouisfed.org>).

3 Appendix C: Diagnostic Tests

3.1 Prior and Posterior Distributions

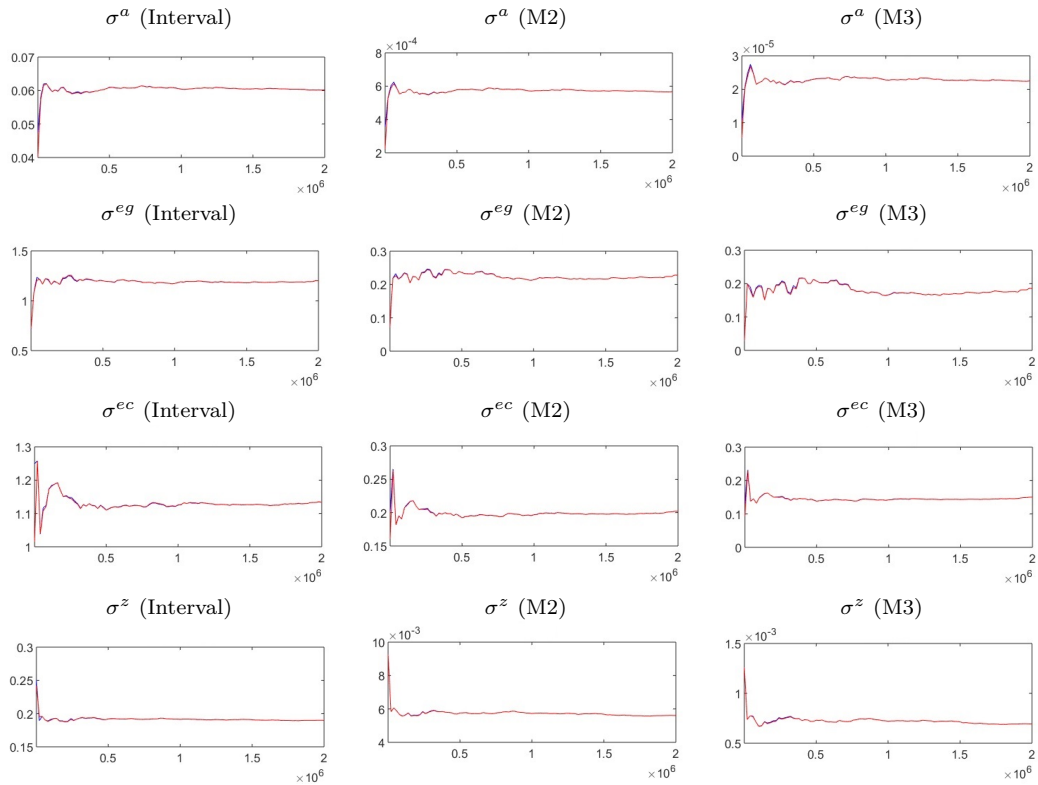
In the graphs below, the gray lines represent the prior distributions while the black lines correspond to the posterior distributions.

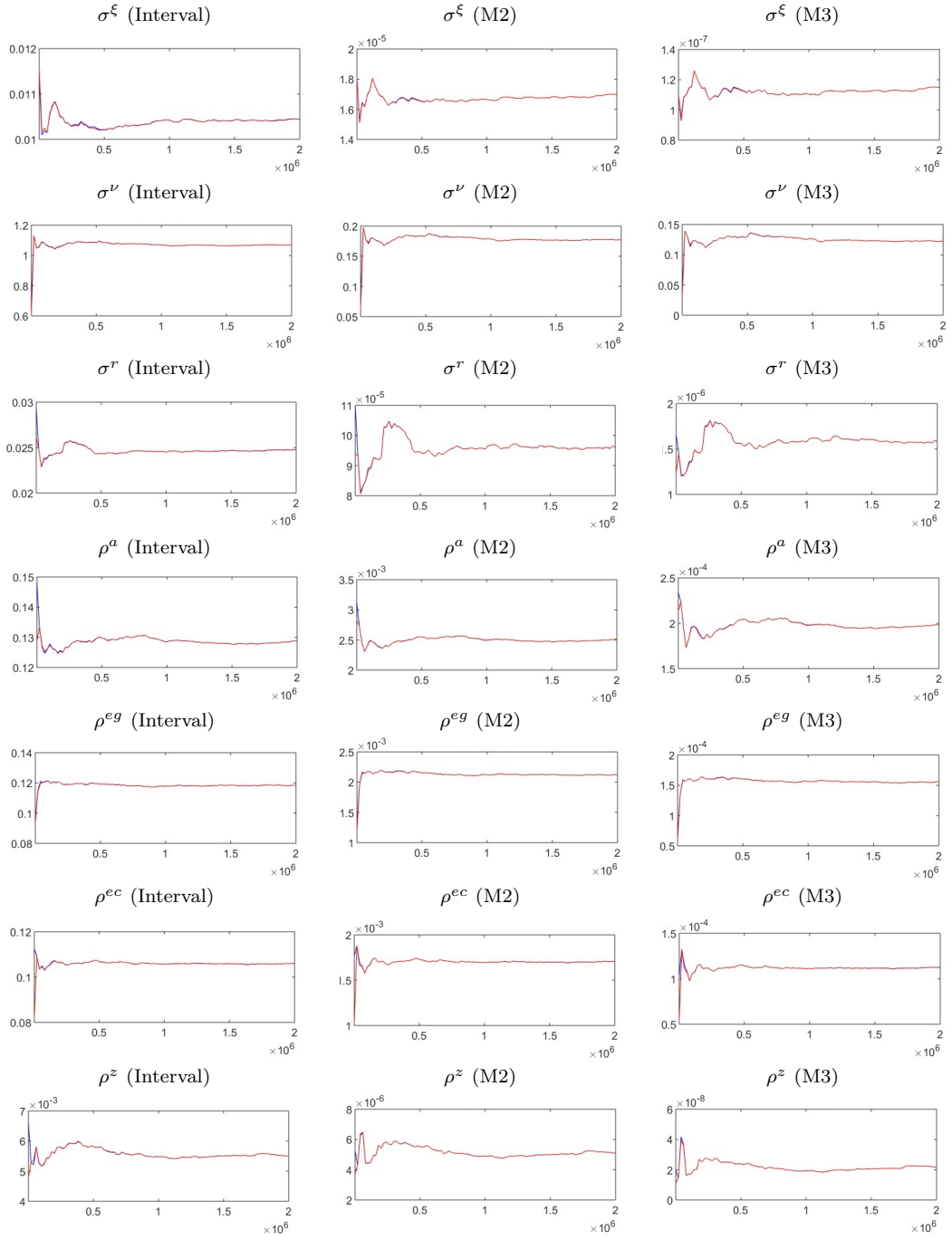


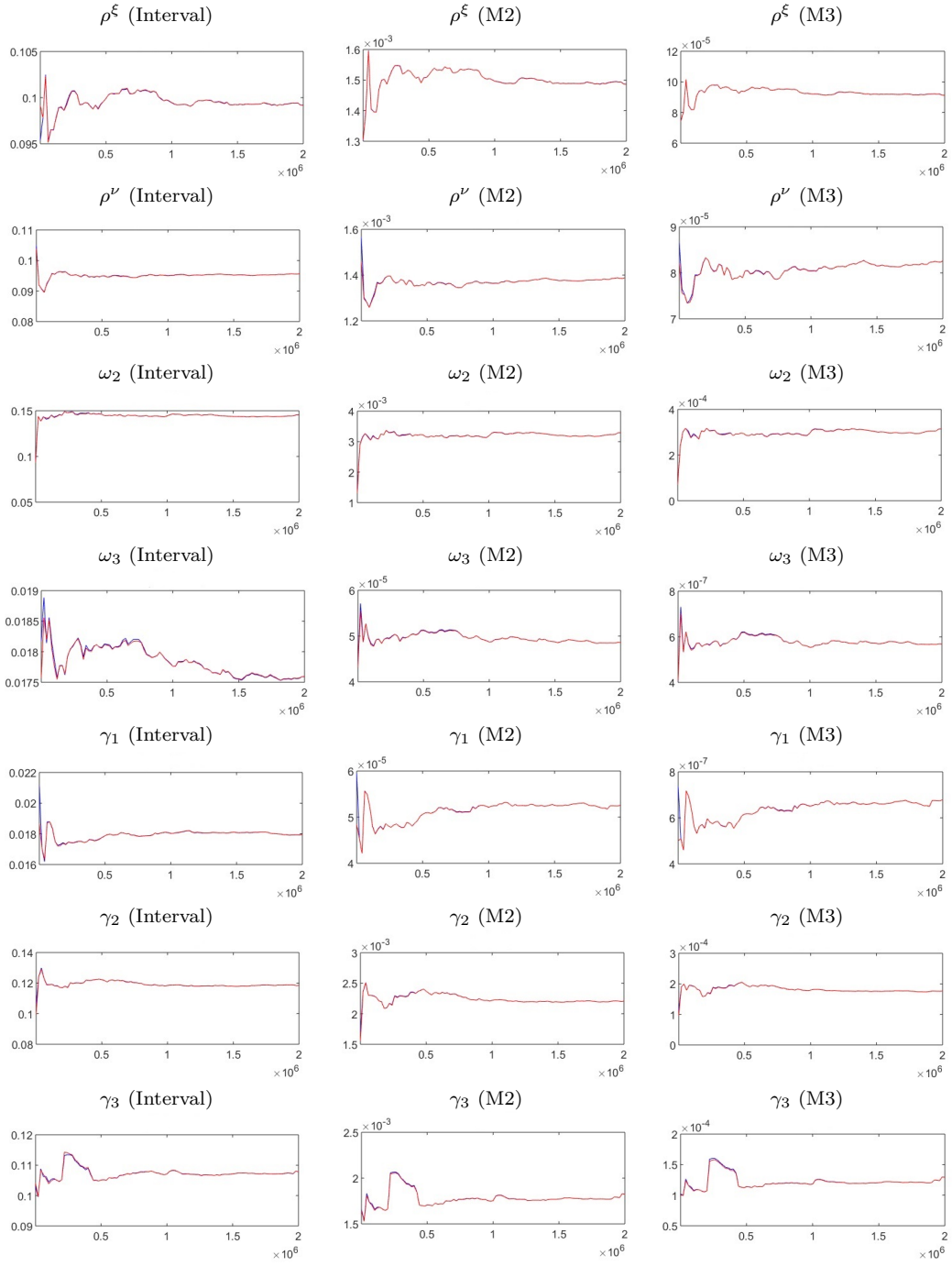


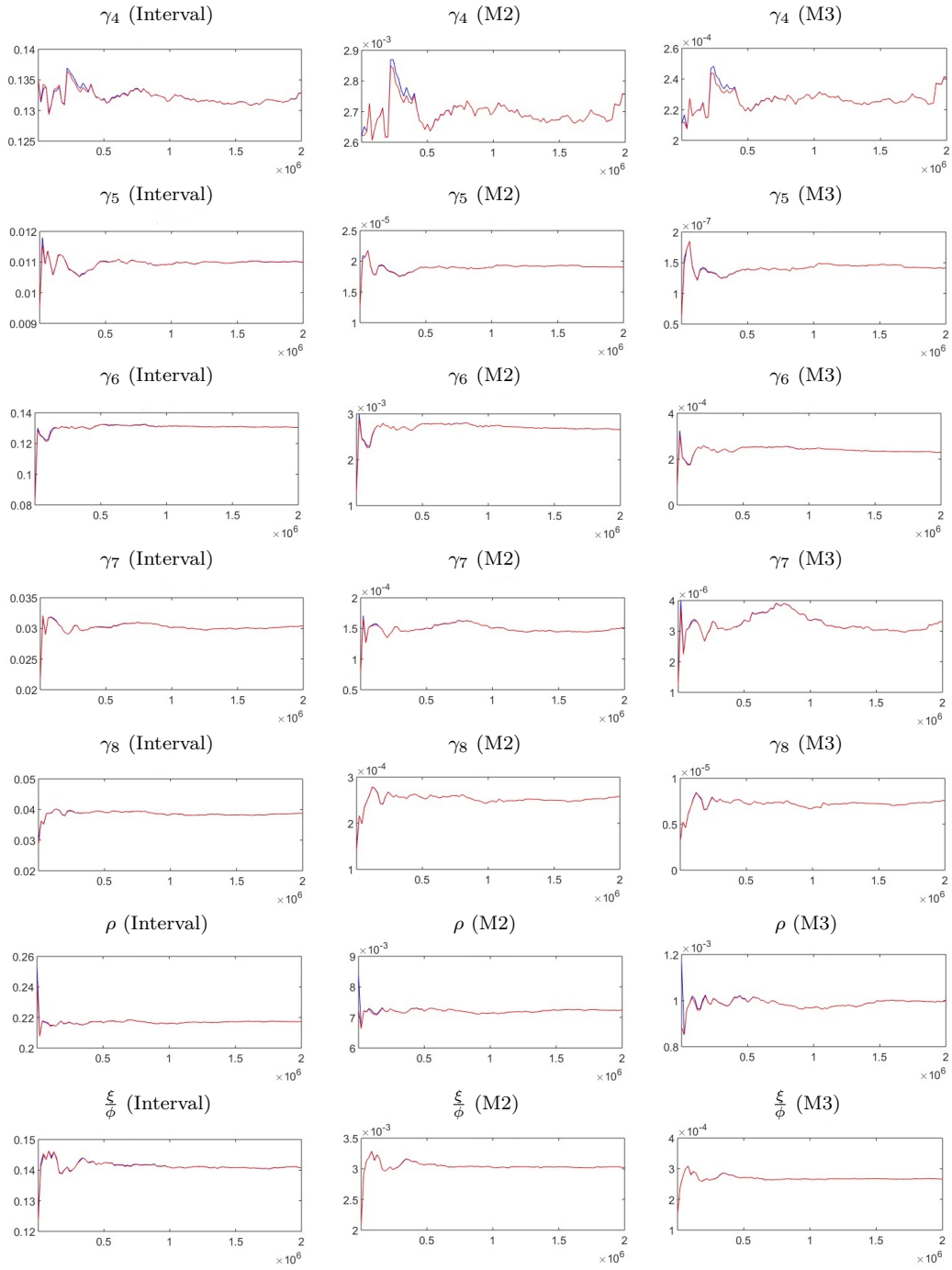
3.2 Monte Carlo Markov Chain Univariate Diagnostics

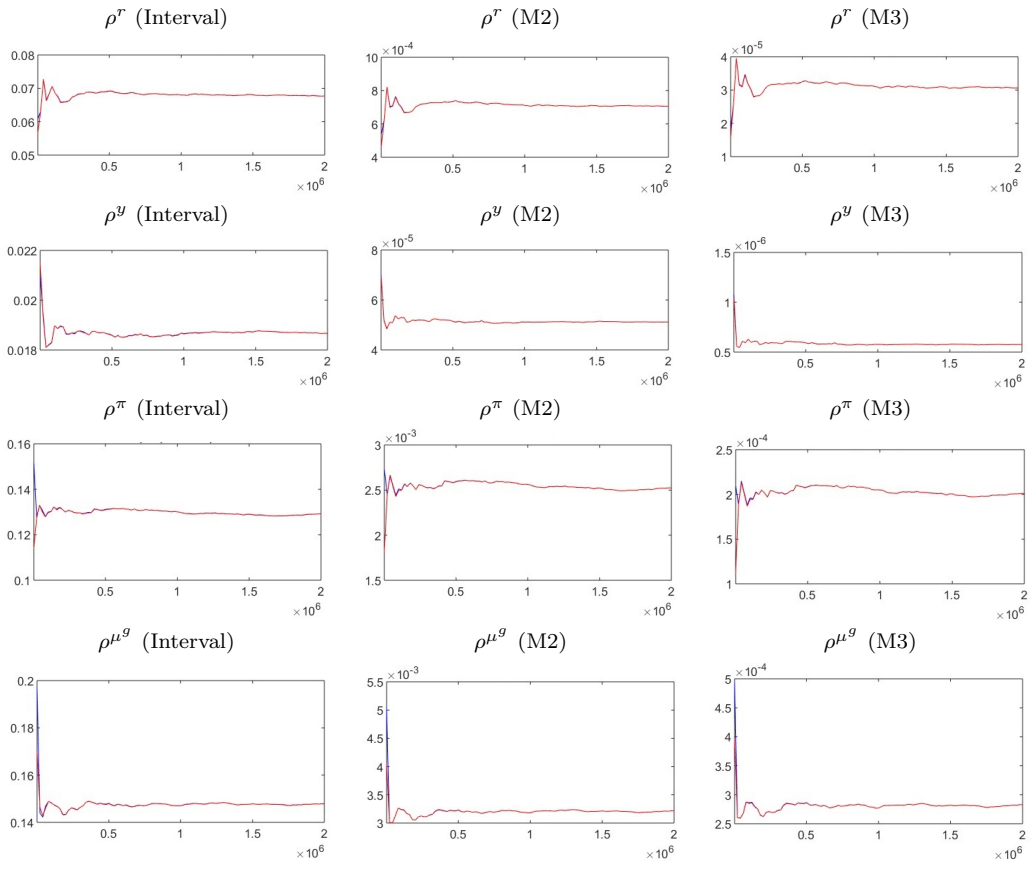
In the graphs below, the first column with the label “Interval” shows the Brooks and Gelman (1998) convergence diagnostics for the 80% interval. The blue line represents the 80% interval range based on the pooled draws from all sequences, whereas the red line indicates the mean interval based on the draws of the individual sequences. The second and the third column with labels “M2” and “M3” denote an estimate of the same statistics for the second and third central moments.





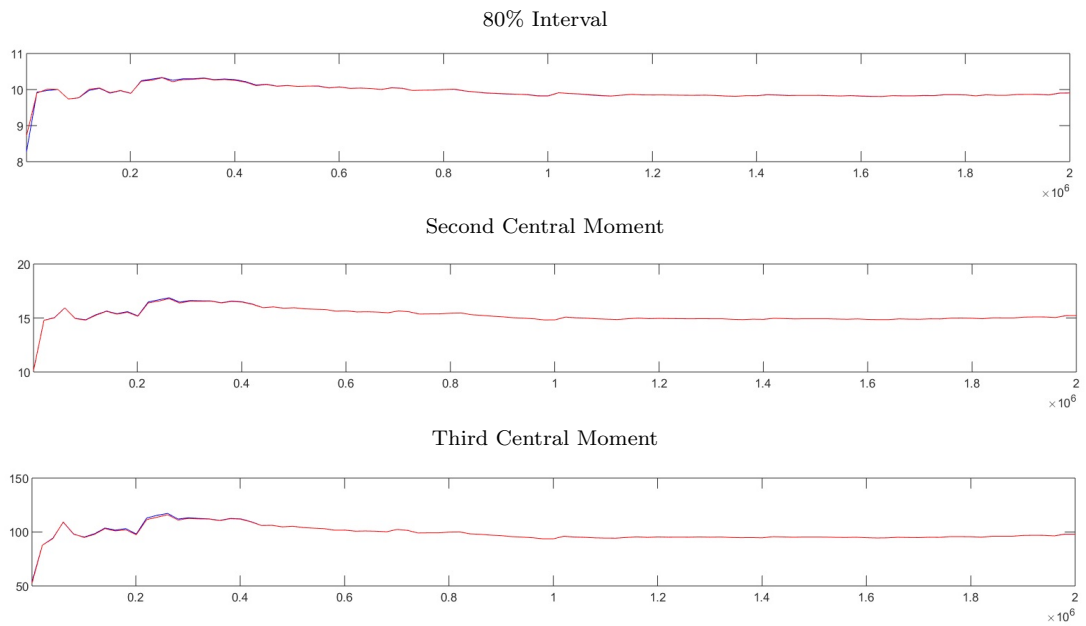






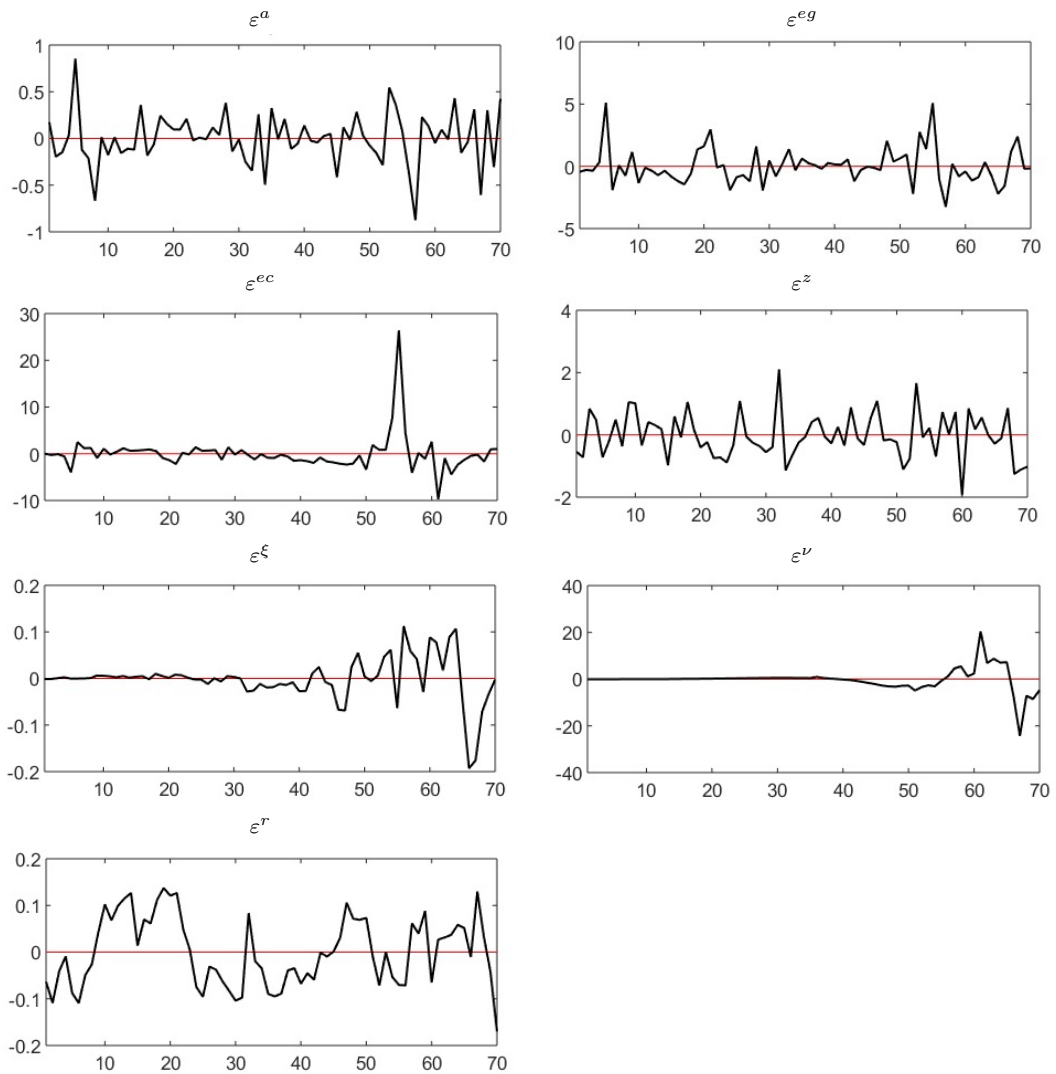
3.3 Multivariate Convergence Diagnostics

In the graphs below, the diagnostics is based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters.



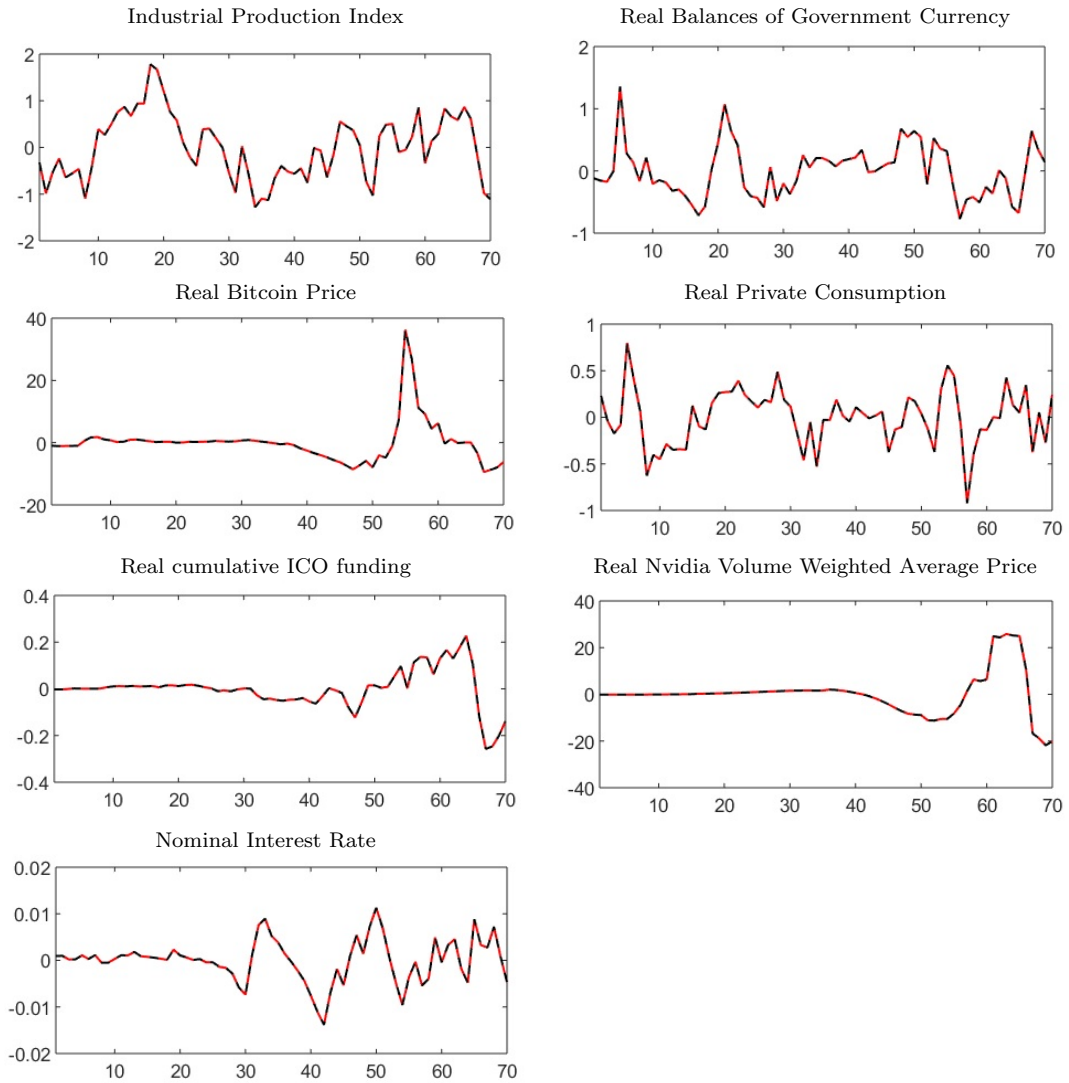
3.4 Smoothed Shocks

In the graphs below, the black lines represent the estimates of the smoothed structural shocks derived from the Kalman smoother.



3.5 Historical and Smoothed Variables

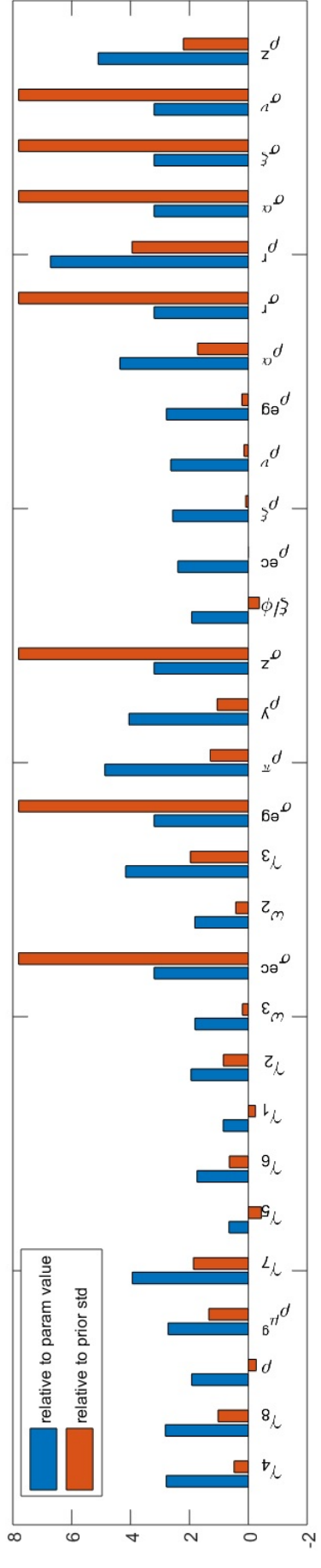
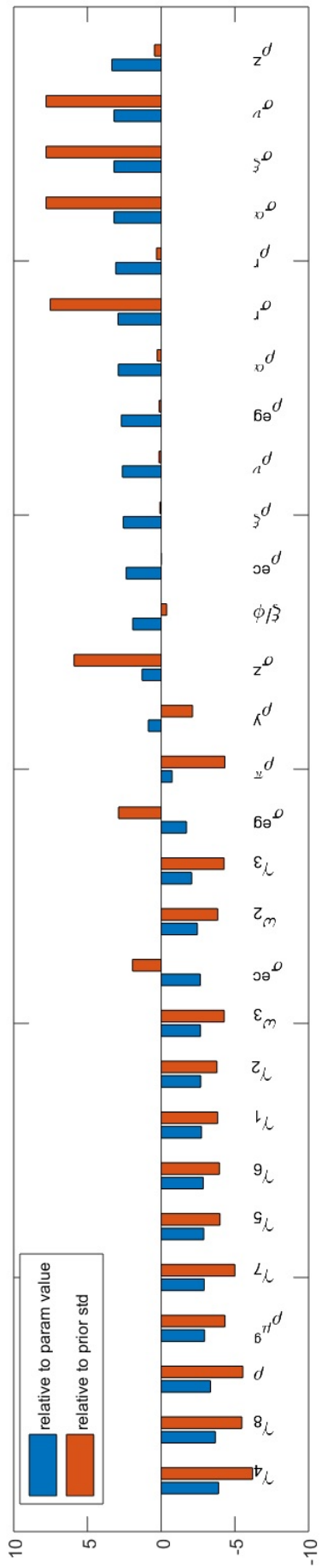
In the graphs below, the dotted black lines indicate the observed data whereas the red lines indicate the estimates of the smoothed variables derived from the Kalman smoother.



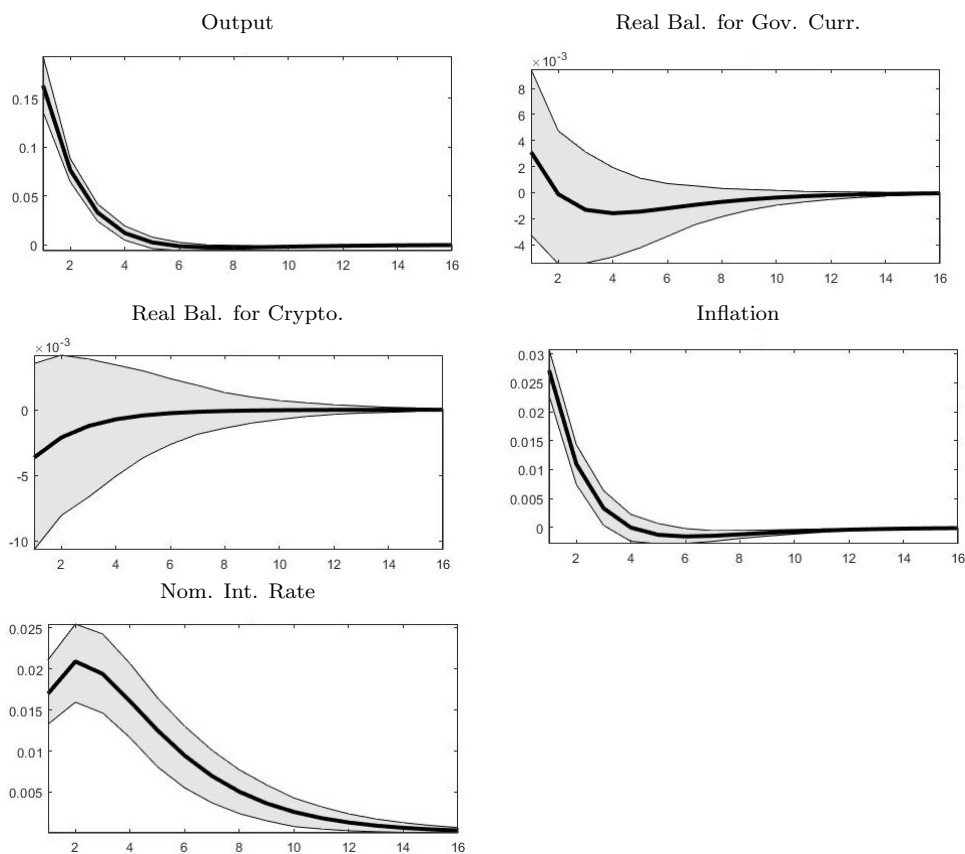
4 Appendix D: Identification Tests

In the top panel, the bar charts represent the identification strength of the parameters based on the Fischer information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

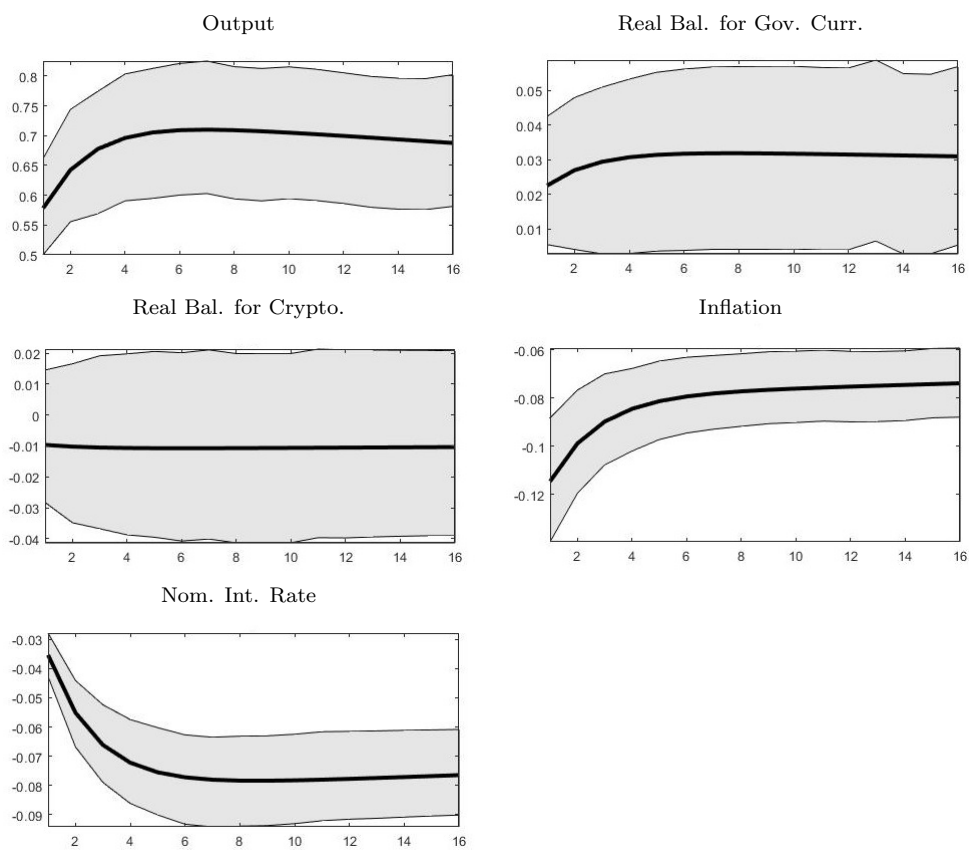
In the bottom panel, we show the sensitivity component of the parameters based on the moments information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).



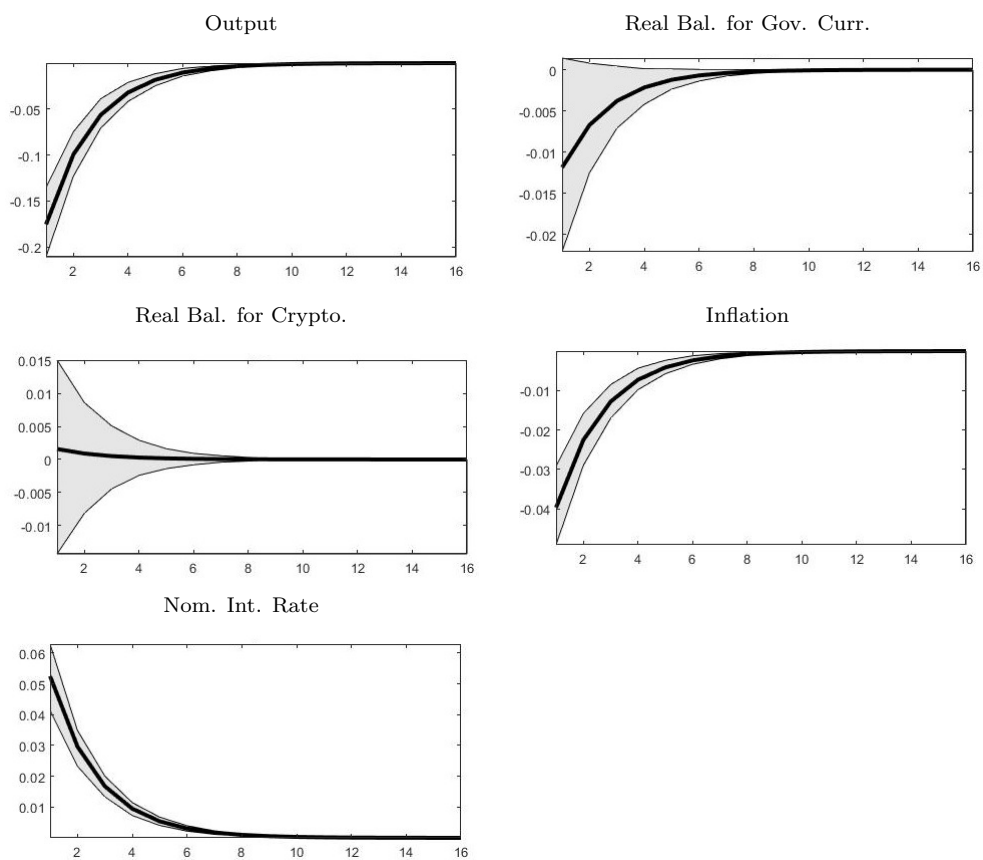
5 Appendix E: Estimated Impulse Response Functions



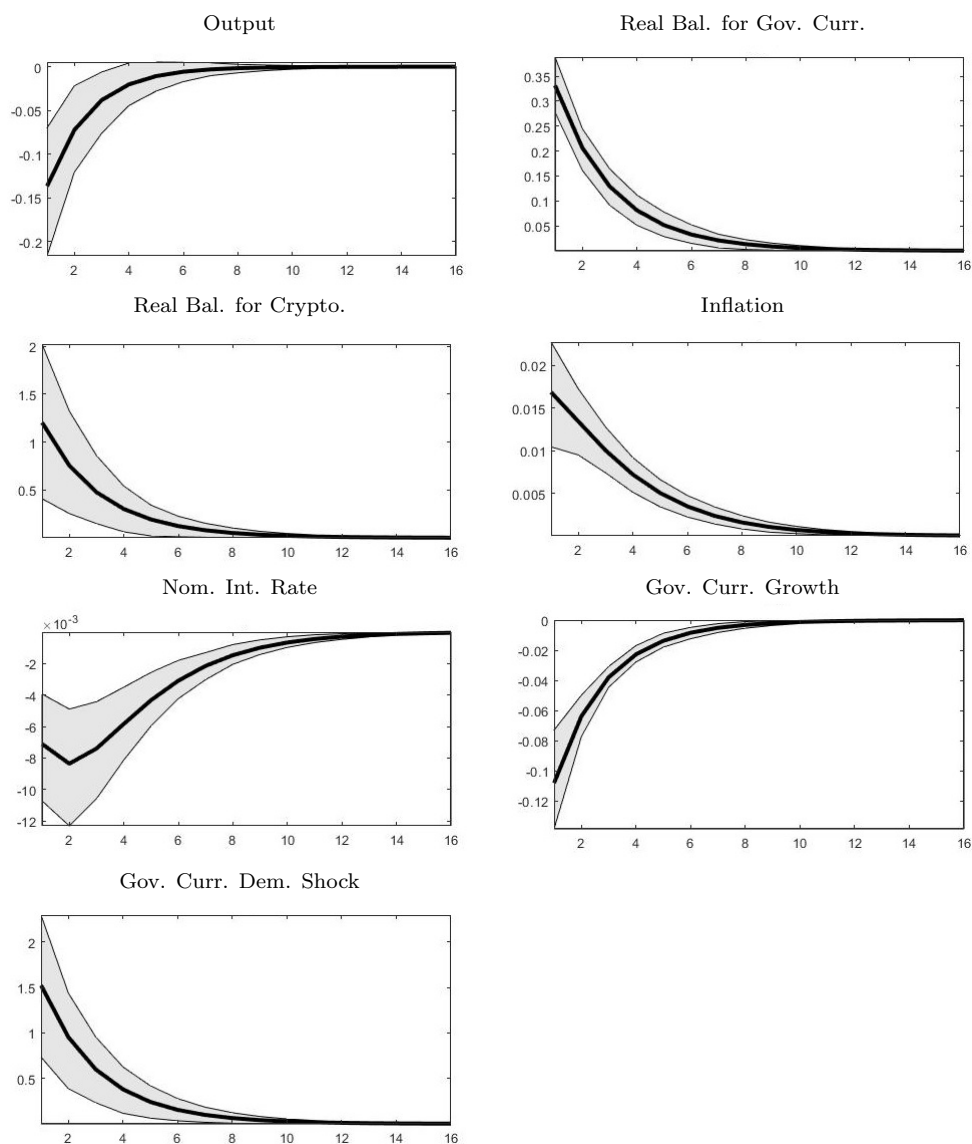
Responses to preferences shock. The graph shows the responses of the key variables together with the 95% confidence intervals.



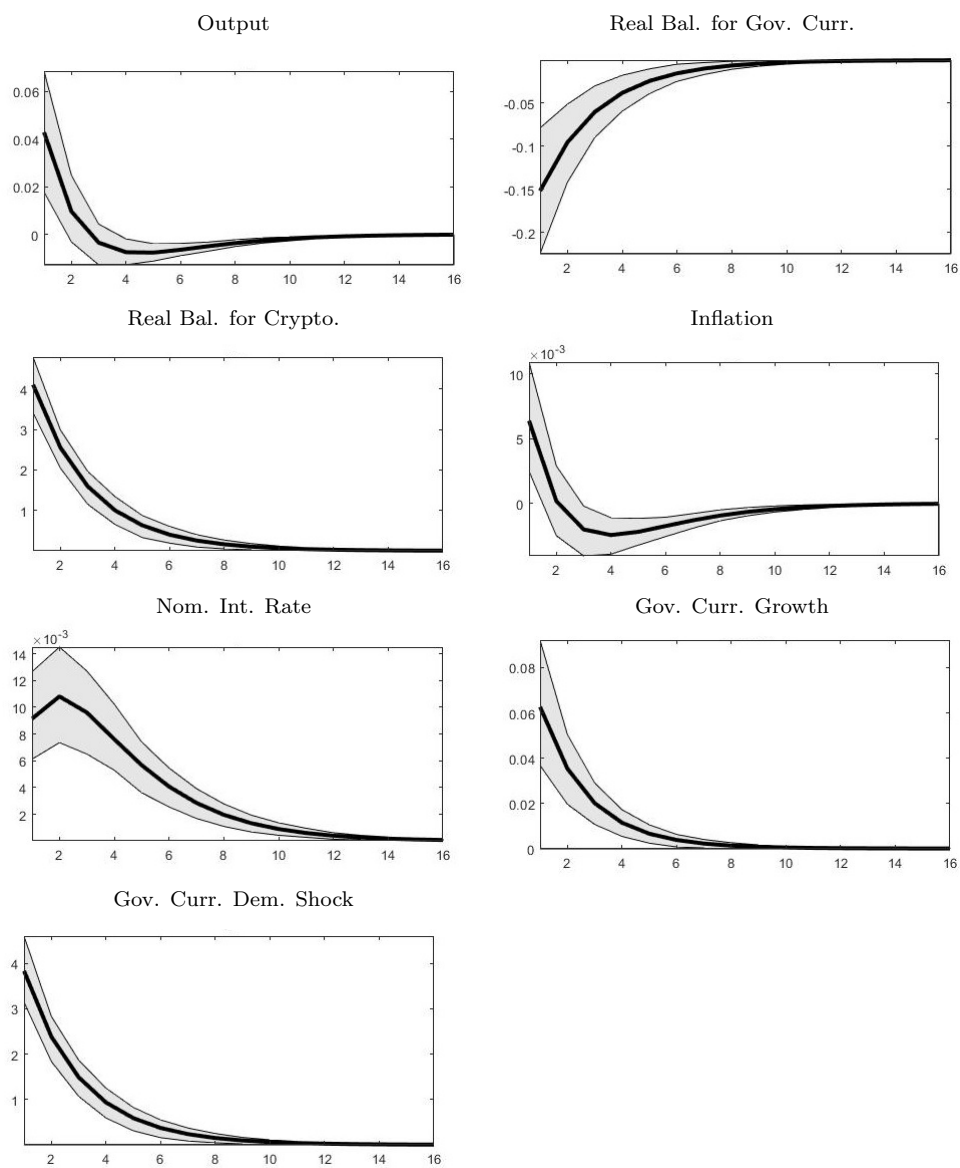
Responses to technology shock. The graph shows the responses of the key variables together with the 95% confidence intervals.



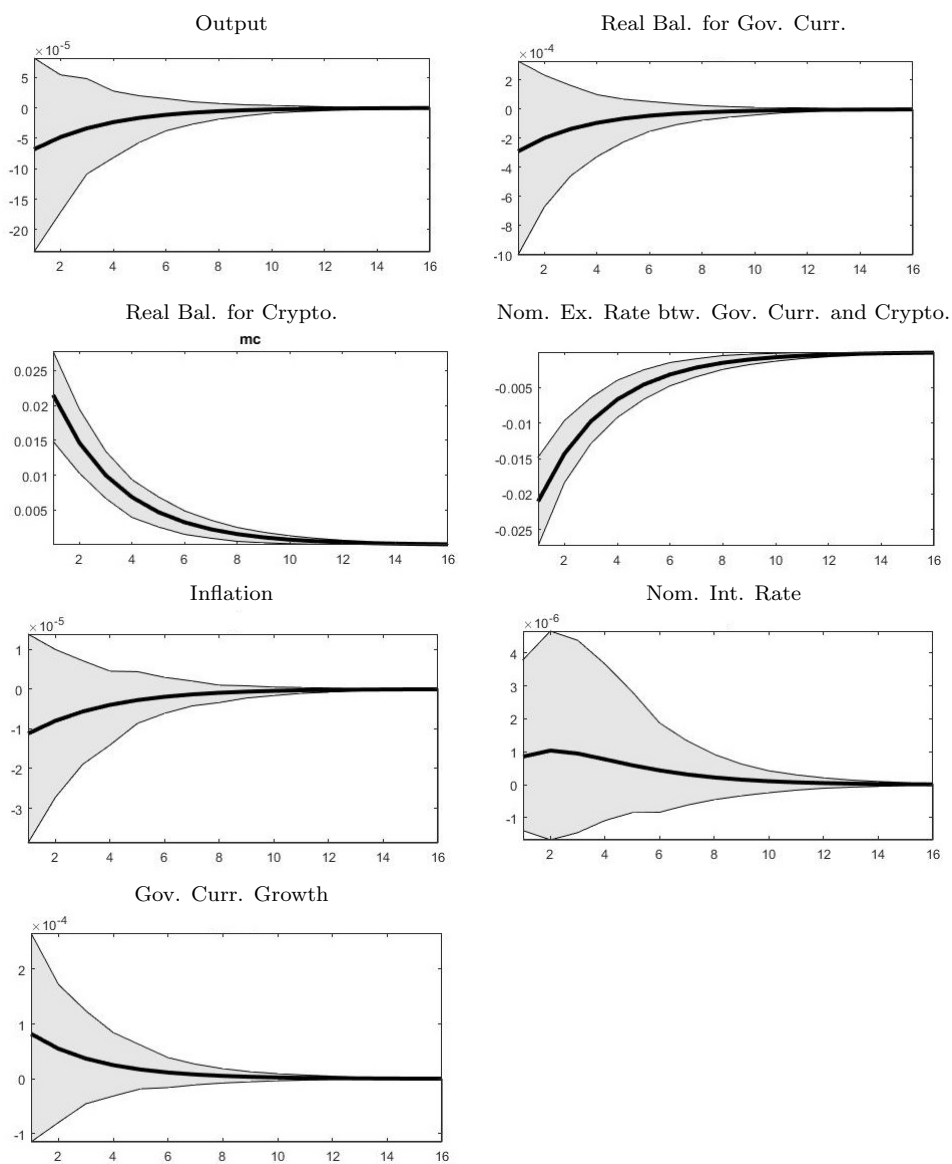
Responses to monetary policy shock. The graph shows the responses of the key variables together with the 95% confidence intervals.



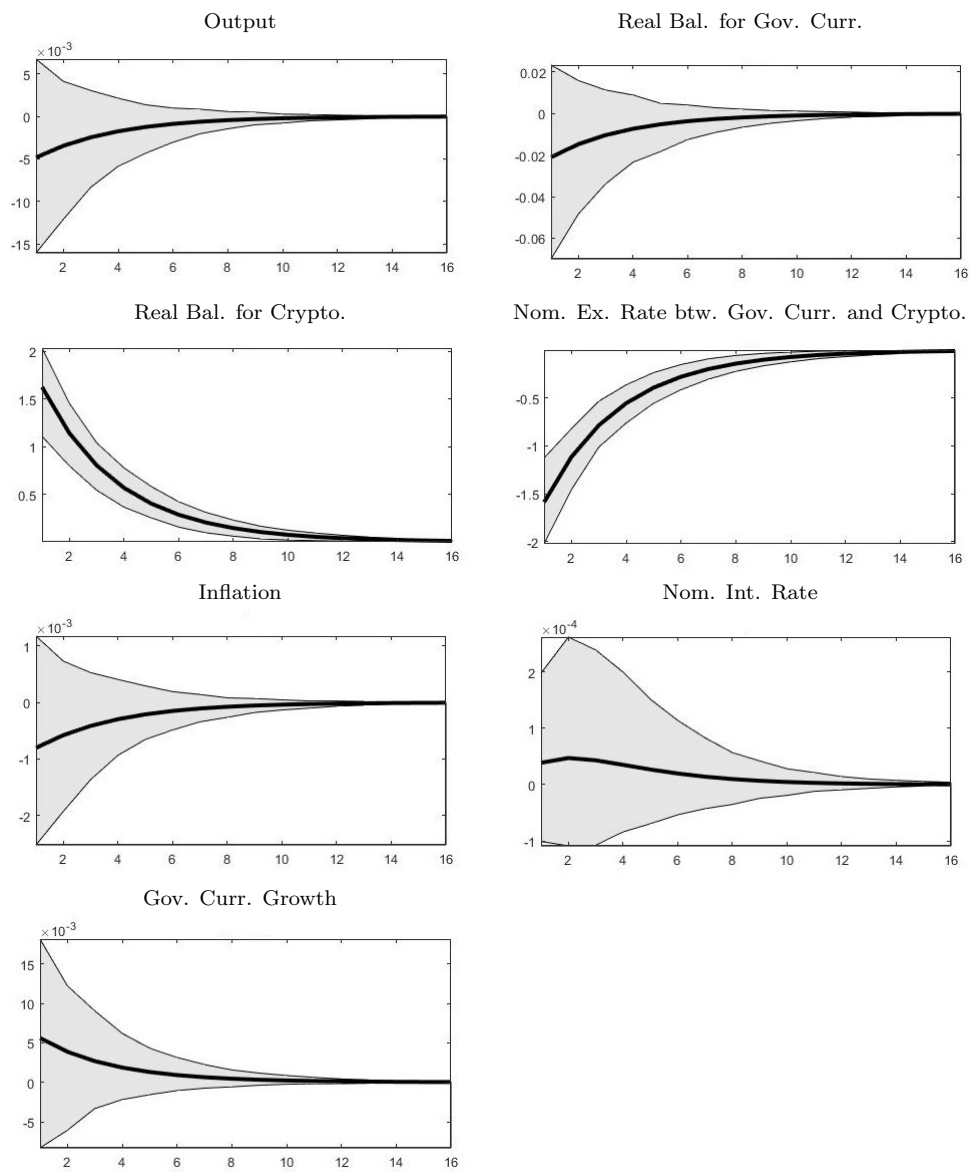
Responses to government currency demand shock. The graph shows the responses of the key variables together with the 95% confidence intervals.



Responses to cryptocurrency demand shock. The graph shows the responses of the key variables together with the 95% confidence intervals.



Responses to cryptocurrency common productivity shock. The graph shows the responses of the key variables together with the 95% confidence intervals.



Responses to cryptocurrency specific productivity shock. The graph shows the responses of the key variables together with the 95% confidence intervals.

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