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An estimate of the financial value added of the Norwegian Petroleum Fund mechanism

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An estimate of the financial value added of the Norwegian Petroleum Fund mechanism

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Abstract

In this thesis, we quantify the financial value added from re-allocating oil and gas resources into a globally diversified financial portfolio. We do this by introducing an alternative approach to management of the oil and gas at the Norwegian continental shelf. We perform the analysis ex-ante, assuming that the Norwegian government at the time before the establishment of the Oil Fund had two choices; To slow down the rate of production and accumulate oil and gas reserves, or to extract these resources at high pace and invest the proceeds in financial assets across the globe. We simulate the expected future values of these approaches over 21 years. Our simulations are highly sensitive to the Brent crude price volatility, and investors risk aversion levels, and this affects the value added significantly. We find that the value added by utilizing the fund mechanism is substantial.

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1 INTRODUCTION

The question we ask in this thesis is: What is the financial value added from establishing the Government Pension Fund Global (the Oil Fund) and, by that, diversifying parts of national wealth from oil and gas reserves to a claim on the value added of listed firms around the world. To get a quantitative answer to this question, we compare two possible approaches to the management of the oil and gas wealth on the Norwegian continental shelf (NCS). Since 1996, the Norwegian government has extracted oil and gas at a high rate and reallocated the proceeds into the Oil Fund, which is a globally diversified financial portfolio. The alternative approach could have been to extract oil and gas at a much slower rate and only in the amount that would have been necessary for the Norwegian government to finance the spending path they have chosen. Instead of reallocating oil and gas reserves to a global financial portfolio, we would have had more abundant remaining oil and gas reserves. In this thesis, we define value added as the monetary premium the Norwegian government receive by reducing the risk of a proportion of the Norwegian national wealth. Figure 1 depicts an illustration of the Norwegian national balance sheet. We aim to find a quantitative estimate of the value of diversification and risk reduction by the petroleum fund mechanism, and to our knowledge, a quantitative estimate of the financial value added from the fund strategy has not been found before.

The Norwegian National Balance Sheet with the Alternative Approach	
<i>Assets</i>	<i>Liabilities</i>
PV of Future Labor Physical Capital in Norway	PV of Private and Government Consumption
Oil and Gas Reserves	
Alternative Approach	
The Norwegian National Balance Sheet with the Oil Fund	
<i>Assets</i>	<i>Liabilities</i>
PV of Future Labor Physical Capital in Norway	PV of Private and Government Consumption
Oil and Gas Reserves	
The Oil Fund	

Figure 1: The Norwegian national balance sheet

This subject is interesting due to the Oil Fund's importance for the Norwegian economy. The Oil Fund's performance and value have direct effects on the government's ability to fund its activities. Withdrawals from the fund towards the national budget reduce the amount of capital the government requires from other sources, such as taxes/fees and government debt. The Oil Fund has since its inception yielded sizeable cash flows, enabling the government to use these cash flows to cover deficits in the national budget, known as the oil-adjusted budget deficit. As we will outline in chapter 2 of this thesis, the Oil Fund was initially established with an official objective to preserve wealth generated from petroleum-related activities for future generations. However, a not so prominent argument made in favor of establishing the Oil Fund was how re-allocation of the national wealth would lead to substantial value creation through diversification effects. We believe that the re-allocation of resources from oil and gas reserves into a globally diversified financial portfolio had had considerable value adding effects, and this effect is what we seek to quantify in this thesis. To perform this analysis, we mainly utilize an ex-ante approach, meaning that we simulate a range of possible future values of the two approaches. We simulate the value development of the two strategies from 1997 up to 2018 by introducing a stochastic process. Further, we utilize a utility framework to investigate whether the establishment of the Oil Fund and diversification of national wealth is value adding in an ex-ante context.

We find that the value added of re-allocating oil and gas resources into a globally diversified financial portfolio as of year-end 2018 is substantial. The value added is, however, highly sensitive towards the volatility of the returns of Brent crude and risk preference levels.

The paper is organized as follows: In chapter 2, we introduce background information that will serve as a good foundation in understanding why the Oil Fund was established. In chapter 3, we outline the theoretical framework used to quantify the financial value added and introduce a stochastic process we will use in our ex-ante analysis to simulate asset prices from 1997 and up to 2018. Additionally, we present two simplified examples of how we utilize the theoretical framework in chapter 5. Chapter 4 provides an overview of our computational methodology and

the proxies we have used for asset prices. In chapter 5, we present our findings and use our theoretical framework to discuss the implications of these findings.

2 BACKGROUND

In this chapter, we highlight some events that took place in the period of 1959 up to 1998. The period before the establishment of the Oil Fund in 1990 was prone to considerable discussion on how the income from petroleum-related activities should be treated. Up to 1990, all national income from petroleum-related operations were used on an ongoing basis. In 1983, Norwegian economists recognized the importance of thinking long-term and suggested the establishment of a "buffer fund." However, there were significant concerns related to the establishment of a buffer fund, as it was believed that Norwegian politicians would spend the money either way. A suggestion proposed was to put constraints on licenses on the NCS to slow down the rate of production, effectively preserve this "buffer fund" in the ground as oil and gas reserves. This suggestion is similar to the alternative approach we will investigate in this thesis.

2.1 Discovery of oil and gas at the NCS

In 1959 oil was discovered off the Dutch coast, which led to increasing interest from American petroleum companies. Rumors circulated regarding the possibility of discovering oil under the seabed further north. In 1962, Phillips Petroleum was one of the companies reaching out to Norwegian politicians for exclusive rights to large parts of the NCS. This approach induced the Norwegian government to lay claim to the NCS and its resources due to governing laws being absent. The Norwegian government communicated to the participants on the NCS that the Norwegian government would make the rules and incorporate a licensing system – with Norwegian governmental control. The NCS would not be given exclusively to just one company (Norsk Olje & Gass, 2017).

The Norwegian oil adventure commenced in 1969 with the discovery of the Ekofisk oil field on the NCS. However, news headlines at the time questioned the Ekofisk fields profitability and oil exploration in general. When Mobil discovered Statfjord, a massive oil field, in 1974, a paradigm shift appeared, and Norway rapidly became an "oil-nation." Oil resources became an essential contributor to the Norwegian economy, and the findings of sizeable subsea oil fields on the NCS sparked

optimism and resulted in increased investments into exploration, complemented by multiple periods of significant increases in oil prices.

Discovering oil led to substantial government spending during the 1970s- and 80s and expensive reforms were pushed through at record pace. Many financial decisions made were reasoned on the expected value of cash flows from petroleum-related activities. The dependency on income from petroleum-related operations became prominent during the oil crisis in the mid-1980s, which resulted in years of constrained budgets that ended in deficit, regardless. The Norwegian government recognized the need for a long-term approach to the usage of income from petroleum-related activities.

2.2 Official Norwegian Reports

During the period of increasingly substantial monetary contributions from oil production, a question that arose was, "what to should we do with the money?". Official Norwegian Reports ("NOU" - Norsk Offentlig Utredning) are utilized to answer questions beyond the government's expertise or to obtain supplementary documentation. Two of the most prominent NOU's to answer this posed question were the Tempo selection (NOU 1983; 27 – *Petroleumsvirksomhetens Fremtid*) and the Steigum selection (NOU 1988; 21 – *Norsk økonomi i forandring*).

2.2.1 The Tempo Selection – the future of the petroleum industry

The Tempo selection was appointed by the Norwegian government in 1982 to study "all circumstances that are of significant impact on the future development of the petroleum industry." The group was managed by Hermod Skånland, a Norwegian economist who served as governor of the Norwegian central bank from 1985-1993. The group weighted that production of oil and gas was different than production in a traditional sense due to it mainly being a re-allocation of assets and that there should be a distinction between income and how you spend it (Skånland, et al., 1983, s. 96). It was suggested that the development on the NCS should be paced according to the oil industry itself and that the government should instead implement long-term guidelines for the usage of the income that was dependent on the activity on the shelf. To achieve this, the group proposed that the government

should establish a “buffer fund” in the central bank. It was reasoned by making the petroleum wealth less dependent on the development of the oil price and currency fluctuations, and to separate income and how the revenue was spent. However, the selection argued that such a fund should not become so large that it endangered the objective of petroleum-related income mainly being used domestically (Skånland, et al., 1983, s. 100). The group also suggested different ways to manage a potential fund. One way could be to let the central bank place the inflows as claims abroad, similar to the currency reserves, and the other suggestion was an actively managed fund with its own administration. The group also discussed a budgetary rule such that the expenditure of petroleum-related income towards the national budget would not exceed the fund's real return.

However, due to rapid turnover in the Norwegian political system since the discovery of petroleum resources at the NCS, the group deemed it unrealistic that the government would allocate funds towards investments abroad. “From the attitudes we know both in the political environment and the general public, it is hard to imagine the allocation of hundreds of billions (NOK) to be placed as claims abroad, at the same time we face uncovered needs domestically” (Skånland, et al., 1983, s. 90). Based on this argument, the Tempo selection proposed to drain the petroleum wealth at a slower rate, due to the assumption that if political authority received money, the money would be spent immediately. The idea of the governmental body to save money was distant. The most realistic way of saving oil money was not to produce it in abundance. According to Skånland, the solution was simple, if the government spent money based on the future expected value of the petroleum resources, the answer was to manage the oil and gas exploration licenses instead more actively. Or more directly, control the activity on the NCS.

2.2.2 The Steigum Selection – the Norwegian economy in change

The Steigum Selection weighted the troubles on the supply-side of the Norwegian economy and wanted to engage in measures to strengthen the production-basis, increase productivity and the level of wealth by utilizing the country's production factors and resources more efficiently (Steigum, et al., 1988, s. 18). The Steigum selection noted that the rapid extraction of the reserves combined with budget deficits meant that the future generations were neglected (Steigum, et al., 1988, s.

65). The group advocated for a financial petroleum fund and emphasized that the total wealth would be a combined pool of petroleum resources and financial assets. Due to the aforementioned, there should be budgetary rules for proper management of the combined wealth. The Oil Fund could be viewed as a re-allocation of wealth. By placing the wealth generated from petroleum resources into globally diversified financial assets, the exposure to fluctuations in the oil price could be reduced. The group proposed expenditures based on the estimated real return on remaining petroleum reserves in addition to the return on the Oil Fund. This differs from the budgetary rule that was implemented in 2001, where the use of oil-related income over time should follow the expected real return of the Oil Fund only.

Former Norwegian prime minister Kåre Willoch stated in a recent documentary: “I think it was beneficial that there was so much money in such a short time; all those who wanted to spend it thought “We can’t spend all of it now”, so we were able to build up this very valuable Oil Fund” (Norsk Olje & Gass, 2017).

Both the Tempo selection and the Steigum selection reached the same conclusions among many of the central questions. The arguments brought forward by the groups played a pivotal role in the establishment of the Fund in 1990. Restrictions on income and how the revenue should be distributed could not be ignored if a financial fund was to be established. Both groups recognized the positive prospects of a globally diversified financial fund to lower the exposure towards the development in oil prices, given that there were constraints on the Norwegian government spending.

2.3 Establishment of the oil fund

In 1990, the Oil Fund was established with the official objective to save income stemming from petroleum-related activities for future generations. The first surplus eligible to be deposited into the Oil Fund was not until 1996 when the Norwegian Central Bank handled the fund. In 1998, Norges Bank Investment Management (NBIM), an asset management unit of the central bank, was established to manage the fund on behalf of the Ministry of Finance. The funds before the establishment of NBIM (1996-1998) was handled in the same way as Norway's currency reserves, i.e., invested in fixed-income securities. Based on a recommendation from Norges

Bank in a letter to the Ministry of Finance in 1997, a decision was made to convert 40% of the Oil Fund into equity investments, and, by that, lay a claim on the future value added by listed companies across the globe (Norges Bank, 1997). At this point, there was a belief that the Oil Fund would grow much faster than previously anticipated. Within the first half of 1998, approximately 40% of the fund was converted into equity investments, primarily in large- and mid-cap stocks in developed markets across the globe. The remaining 60% was invested in investment grade government debt (Norges Bank Investment Management, 2019b).

3 THEORETICAL FRAMEWORK

In this chapter, we outline the conceptual framework we use to estimate the value added from the petroleum fund mechanism quantitatively. We have presented two approaches; investing in the Oil Fund or investing in the alternative approach, which is a portfolio of oil and gas reserves. We apply a parsimonious power utility function, and to capture the value-added effect of choosing an approach that exhibits a lower level of risk we introduce a value-added parameter, μ . When a decision maker, in this case, the Norwegian government, is faced with the option to invest in one of two portfolios, the attitude towards risk will be decisive as to which portfolio exhibits the more desirable characteristics.

As we outline in the following section, a risk-averse decision maker will prefer a portfolio where the range of possible future outcomes is minimized, or put differently, where the standard deviation of returns is small. As we elaborate in the subsequent section, a decision maker will, in the case of uncertainty and aversion to risk, be willing to accept a certain payout that is lower than the expected payout of an uncertain bet. To illustrate how the theoretical framework is utilized, we introduce two examples in section 3.2 and 3.3, respectively. These examples are based on simple assumptions but provide an accurate indication as to how we will approach the main analysis in chapter 5.

3.1 Utility Function

To investigate the value added, that is the value of diversification, we introduce the following relationship,

$$E[U(W_{1,t}, \gamma)] = E[U(W_{2,t}, \gamma)] \quad (1)$$

Where

W_1 = Value of the alternative approach

W_2 = Value of the Oil Fund

In this relationship, we assume that the Oil Fund, denoted by W_2 , yields a level of utility that is equal to the utility of the alternative approach. U denotes a power

utility function first introduced by Arrow (1965, 1971) and Pratt (1964). It has the following form,

$$U(W_{n,t}, \gamma) = \frac{W_{n,t}^{1-\gamma}}{1-\gamma} \quad (2)$$

Where W denotes the market value of portfolio n at time t , and γ is a measure of constant relative risk aversion. The functional properties of the power utility function are discussed in Appendix A. By substituting the utility function into our null hypothesis, we get,

$$E \left[\frac{W_{1,t}^{1-\gamma}}{1-\gamma} \right] = E \left[\frac{W_{2,t}^{1-\gamma}}{1-\gamma} \right] \quad (3)$$

Further, we expand $W_{2,t}$, and define this as $W_{2,t} \times \mu$, where μ is a measure of value-added from re-allocating oil and gas reserves into a globally diversified financial portfolio. μ is interpreted as a factor that measures the relative relationship between the market values of the two portfolios. By introducing μ , the expression is expanded to,

$$E \left[\frac{W_{1,t}^{1-\gamma}}{1-\gamma} \right] = E \left[\frac{(W_{2,t} \times \mu)^{1-\gamma}}{1-\gamma} \right] \quad (5)$$

Where we find μ by using a goal-seek function. To find the NOK denoted value-added of the implemented strategy, that is the Oil Fund, we solve the following equation,

$$E(W_t) \times (1 - \mu) = \text{Value Added in NOK} \quad (6)$$

Hence, if μ equals 1, the value-added effect will be zero. If μ differs from 1, the value-added effect will be non-zero. This model features desirable characteristics, as a utility function provides a way of valuing and comparing different risky alternatives by considering risk preferences. The risk preferences of the decision maker are scaled by the constant relative risk aversion parameter γ .

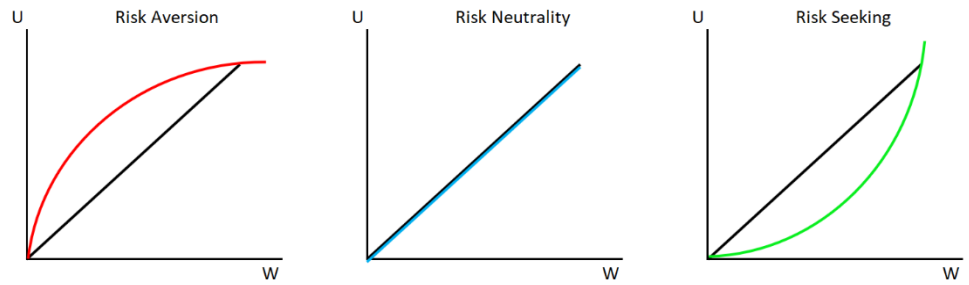


Figure 2: Risk preferences

Figure 2 shows the shape of the power utility function for different attitudes towards risk. For risk-averse decision makers, the utility function is concave ($\gamma > 0, \gamma \neq 1$). For risk seeking decision makers, the utility function is convex ($\gamma < 0$), and for a risk-neutral decision maker, the utility curve is linear ($\gamma = 0$). The convexity or concavity of the utility curve is determined by the magnitude of the risk aversion parameter. In our analysis, we assume that the decision maker that is the Norwegian government is exhibiting risk-averting behavior, which means that the aversion to uncertainty about future outcomes of wealth is high. This uncertainty is characterized by the volatility of the portfolios we investigate, as higher volatility will broaden the range of possible future wealth levels.

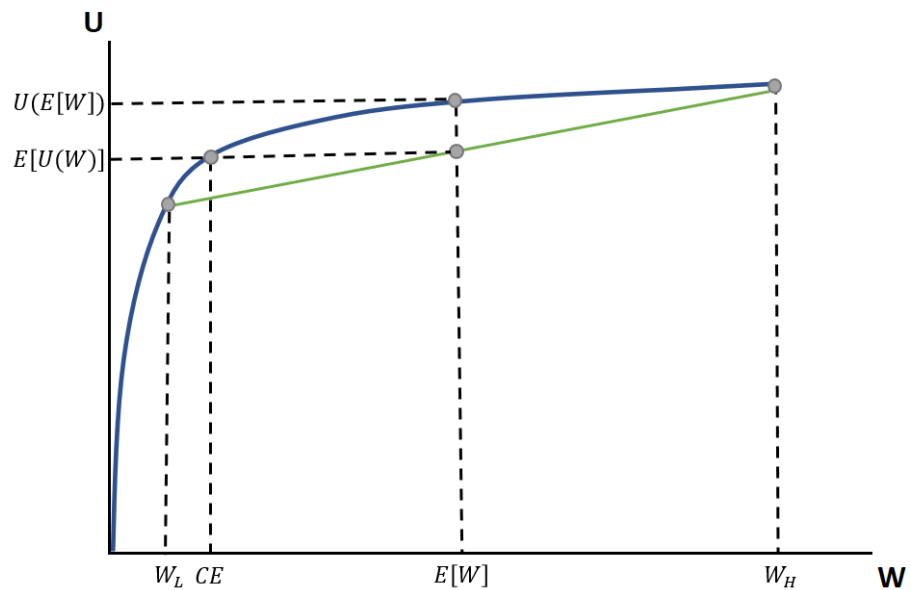


Figure 3: Value creation through risk reduction

In Figure 3, we illustrate a scenario where the decision maker exhibits risk aversion, shown by the concavity of the utility curve. U denotes utility and W denotes wealth. In this case, the decision maker faces two options, a sure payoff CE or participation in a lottery where the outcomes, denoted by W_L and W_H , are equally probable. The point CE denotes the certainty equivalent, that is the amount the decision maker is willing to accept with certainty to avoid participation in the lottery. Hence, due to risk aversion, the decision maker is willing to accept a certain payoff that is smaller than the expected outcome of the lottery, denoted $E[W]$. We see that the utility of the expected outcome and the expected utility of the outcome denoted $U(E[W])$ and $E[U(W)]$ respectively, differs due to the concavity of the utility curve.

However, in this thesis, the decision maker, that is the Norwegian government, has an option to participate in one of two lotteries, or more specifically to invest in one of two portfolios. This means that the possible future outcomes of both lotteries are uncertain. As stated at the beginning of this chapter, we aim to quantify the value added to choosing the less risky lottery, which is to invest in a portfolio of globally diversified financial assets. By equalizing the expected return on the portfolios, we ensure that,

$$U(E[W_{\text{ALTERNATIVE APPROACH}}]) = U(E[W_{\text{OIL FUND}}]) \quad (7)$$

Meaning that the utility of the expected future values of the two portfolios are equal. By utilizing the theoretical framework, we effectively transform the less risky portfolio, that is the Oil Fund, into a certain payoff by applying equation (5). In this manner, the certain payoff will be situated at point $(CE, E[U(W)])$ in Figure 3. By transforming the Oil Fund portfolio into a guaranteed payoff, we identify the value added of diversification as,

$$\text{Value added in NOK} = E[W] - CE \quad (8)$$

The next section introduces a fictional example to depict how we utilize the theoretical framework in chapter 5, by adding a scenario where the decision maker has the option to invest in one of two portfolios.

3.2 Simplified example

We assume that the decision maker is exhibiting risk-averting behavior, which means that the utility curve shows concavity and that the aversion to uncertainty about future outcomes of wealth is high. In chapter 5, this uncertainty is characterized by the volatility of the portfolios we investigate, as this will broaden the range of possible future wealth levels. To illustrate how we utilize the theoretical framework, we provide a simple example based on fictional variables. In this simplified example, the decision maker has to choose between investing in the Oil Fund or investing in the alternative approach. Hence no certain payout will be available. Both portfolios have two equally probable outcomes,

Table 1: Possible future outcomes

Range (B NOK)	Outcome 1	Outcome 2
<i>Alternative Approach</i>	10	90
<i>Oil Fund</i>	40	60
<i>Risk Aversion Parameter</i>	2	

The expected outcome of both portfolios equals NOK 50 billion. However, the range of possible outcomes in the alternative approach is wider than for the Oil Fund. Intuitively, a risk-averse decision maker will choose to invest in the Oil Fund. Going forward, we will apply our theoretical framework to quantify the value added of choosing the less risky portfolio. The utility of the expected outcome equals,

$$U(E[W]) = \frac{50^{1-2}}{1-2} = -0.02$$

Hence, the utility of the expected outcome is equal in both portfolios. However, we seek to investigate the expected utility of the portfolios,

$$E[U(W_1)] = \frac{1}{2} \left[\frac{10^{1-2}}{1-2} \right] + \frac{1}{2} \left[\frac{90^{1-2}}{1-2} \right] = -0.06$$

$$E[U(W_2)] = \frac{1}{2} \left[\frac{40^{1-2}}{1-2} \right] + \frac{1}{2} \left[\frac{60^{1-2}}{1-2} \right] = -0.02$$

We see that the expected utility of the outcomes in the portfolios differs from the utility of the expected outcome. This difference arises due to the concavity of the utility curve and indicates an aversion towards risk. To quantify the value-added parameter μ , we apply equation (5) from our theoretical framework,

$$\frac{1}{2} \left[\frac{10^{1-2}}{1-2} \right] + \frac{1}{2} \left[\frac{90^{1-2}}{1-2} \right] = \frac{1}{2} \left[\frac{(40 \times \mu)^{1-2}}{1-2} \right] + \frac{1}{2} \left[\frac{(60 \times \mu)^{1-2}}{1-2} \right]$$

By utilizing a goal seek function we find that,

$$\mu = 0.37$$

And finally, we find the value added denominated in NOK by applying equation (6),

$$50 \times (1 - 0.37) = \text{NOK } 31.50 \text{ billion}$$

The NOK denominated value added can be interpreted as the amount a risk-averse decision maker will be willing to pay to avoid investing in the riskier portfolio or put differently, the value added from choosing a portfolio exhibiting lower risk and equal expected payoff. By paying an amount equal to NOK 31.50 billion, the decision maker arrives at a level of expected utility where he or she will be indifferent between the two portfolios. The utility of the certainty equivalent in the two-portfolio case will be,

$$E[U(\text{Certainty Equivalent})] = \frac{(50 - 31.50)^{1-2}}{1-2} = -0.06$$

Which equals the expected utility of the alternative approach. Hence, by receiving a certain amount equal to NOK 50 billion – NOK 31.50 billion = NOK 18.50 billion by investing in the Oil Fund, the decision maker will be indifferent between the two portfolios. This example is illustrated in Figure 4.

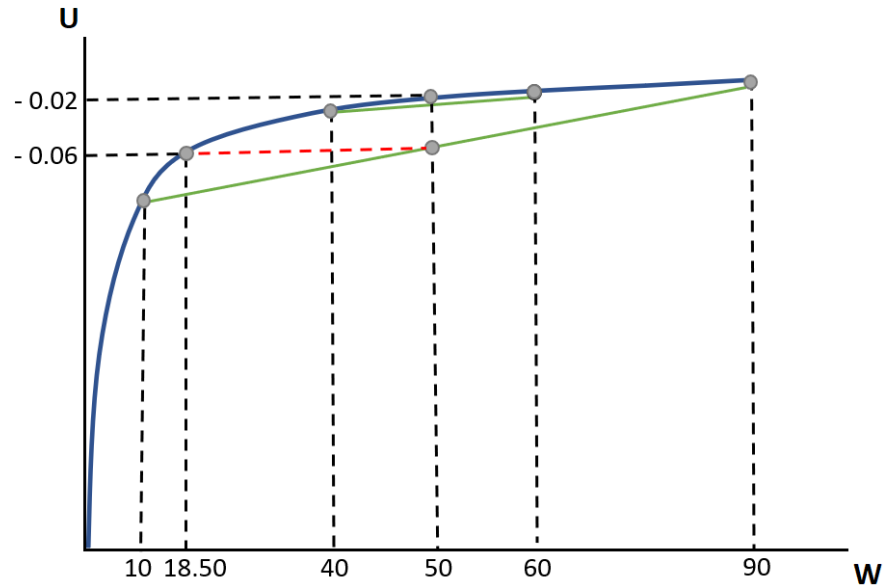


Figure 4: Illustration of the two-portfolio example

In Figure 4, we see that the utility curve is strongly concave, indicating a high degree of risk aversion. The red line illustrates the NOK denominated value added of choosing the safer portfolio, which is the Oil Fund.

3.3 Simulation of possible future outcomes

The example delineated in the previous section serves as a good guideline of how the theoretical framework will be utilized in chapter 5. However, in the case in section 3.2, we simplify by introducing only two possible outcomes for each portfolio. Most financial time-series do not exhibit this property, as the range of possible future outcomes can be infinite. To estimate a more realistic range of possible future outcomes for the Oil Fund and the alternative approach, we use a stochastic process called the Geometric Brownian motion (Hull, 2018). The following stochastic differential equation denotes the process,

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \text{ where } W_t \sim N(0,1) \tag{9}$$

Where μ denotes a drift parameter and σ denotes volatility. These parameters remain constant across time. W_t is a Wiener process or Brownian motion process and

depicts the stochastic component of the time-series. According to Hull (2018), equation (9) transforms into the closed form solution,

$$S_t = S_0 \exp \left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right) \quad (10)$$

Where S_t denotes price at time t , S_0 denotes initial price, μ denotes expected arithmetic return, and σ denotes expected volatility.

In section 3.2, we introduced a simple example of how the theoretical framework will be utilized. In this section, we will complicate our example somewhat by adding the Geometric Brownian motion. Rather than assuming that each portfolio has only two possible outcomes, we set our model to yield a total of 10.000 possible future outcomes. The parameter assumptions are summarized in Table 2.

Table 2: Parameter assumptions

	Alternative Approach	Oil Fund
<i>Expected Return</i>	10 %	10 %
<i>Expected Volatility</i>	20 %	10 %
<i>Initial Value</i>	NOK 50 billion	
<i>Periods</i>	21	
<i>Risk Aversion Parameter</i>	2	

We set the level of volatility in the alternative approach at 20%, and the volatility of the Oil Fund at 10%, defined as the annual standard deviation of returns. We set the expected yearly return equal for both portfolios to capture the value-added effect of diversification and risk reduction alone. Additionally, we assume that there are no regular cash inflows in either portfolio. By applying equation (10), we create a modeling tool in MatLab. We simulate for a total of 21 years, that is from 31/12/1997 up to 31/12/2018, which is equivalent to the measurement period we use in chapter 5. The range of the possible future prices as of 31/12/2018 are summarized in Table 3.

Table 3: Range of possible future values

Range (B NOK)	Lowest Outcome	Highest Outcome
<i>Alternative Approach</i>	6.58	8,601.80
<i>Oil Fund</i>	45.11	2,065.61

By investigating the data in Table 3, we see that the range of possible future outcomes of the alternative approach exceeds the range of the possible future outcomes of the alternative approach considerably. The different levels of volatility explain this difference.

Further, we investigate the levels of utilities of expected future values and the expected utilities of future values of the two portfolios. In the simple example outlined in section 3.2, we computed these sizes by hand to illustrate how we utilize the theoretical framework. However, as we simulate a total of 10.000 possible future outcomes of each portfolio, we summarize these sizes in Table 4 rather than doing the calculations manually.

Table 4: Summary of utilities and value added

	Alternative Approach	Oil Fund
$E(W)$	NOK 367.00 billion	NOK 367.00 billion
$U(E(W))$	-0.0027	-0.0027
$E(U(W))$	-0.0060	-0.0033
μ	0.55	
<i>Value Added</i>	NOK 165.00 billion	
<i>Certainty Equivalent</i>	NOK 202.00 billion	
$U(\text{Certainty Equivalent})$	-0.0060	

We see that the expected values as of 31/12/2018 are equal for both portfolios. We also find that the value of diversification is NOK 165.00 billion, interpreted as the premium the decision maker receives from choosing a portfolio with lower risk.

The certainty equivalent is NOK 202.00 billion, equal to the amount where the expected utility of the certainty equivalent is similar to the expected utility of the alternative approach.

4 DATA AND METHODOLOGY

In the previous chapter, we outlined the theoretical framework and illustrated how this framework would be utilized in chapter 5 by introducing two hypothetical examples. A simplification we proposed was that there were no regular cash inflows in either portfolio in these two examples. In reality, the Oil Fund receives steady cash inflows that are gradually invested according to the investment mandate. The Norwegian state's income from petroleum-related activities is partially used to cover deficits on the national budget yearly, known as the oil-adjusted budget deficit. Any residual amount is placed in the Oil Fund (Lund & Stiansen, 2017). In our analysis, we assume that the Norwegian government has the choice of investing in A) the Oil Fund, which is a globally diversified financial portfolio, or B) the alternative approach, which builds on the assumption that these residual cash flows are placed back at the NCS in the form of oil and gas reserves, comparable to the suggestion proposed by the Tempo selection in 1983. In section 4.1, we describe the different components of the state's net cash flow from petroleum-related activities in addition to the oil-adjusted budget deficit.

In the latter example in section 3.3, we introduced a stochastic process where we set the parameters of the model at arbitrary sizes to emphasize the value of diversification. However, to accurately depict the value of diversification in our primary analysis in chapter 5, we gather and analyze a range of historical data to create more reasonable estimates of these parameters. In section 4.2, we outline the proxies we use for the equity and bond proportions of the Oil Fund. We also explain some simplifications we do concerning the calculation of the value of the alternative approach. In section 4.3, we outline how we estimate the values of the Oil Fund and the alternative approach on an ongoing basis.

4.1 Government related components

To properly understand the relationship between the SNCF from petroleum-related activities, the oil-adjusted national budget deficit and the cash inflows into the Oil Fund, we explain this in detail in this section. We gather the data from several sources, primarily through regjeringen.no, norskipetroleum.no and through contact with the Norwegian Ministry of Petroleum and Energy. Data on the SNCF from

petroleum-related activities and the oil-adjusted national budget deficit are denoted yearly.

4.1.1 State's net cash flow

The state's net cash flows from petroleum-related activities stem from the petroleum industry. The Norwegian government claims all oil and gas resources found on or under the sea of the geographical area known as the NCS. The assets at the NCS are placed in a portfolio called the State's Direct Financial Interest (SDFI). The Norwegian government generates income from petroleum-related activities through its direct ownership (SDFI), through capital- and environmental taxes and fees in addition to dividends from Equinor. See Figure 5 (Norsk Petroleum, 2018) for a detailed overview of the different components on an annual basis. The oil and gas sector is Norway's largest industry measured in value added, government revenues, investments, and export value (Norsk Petroleum, 2019a). The SNCF totaled NOK 256.1 billion in 2018 (Norsk Petroleum, 2019b). We gather the SNCF from petroleum-related activities data through the database of the Norwegian Ministry of Finance. We transform these amounts into USD at an exchange rate of NOK/USD 7.25, further elaboration on this is given in section 4.2.

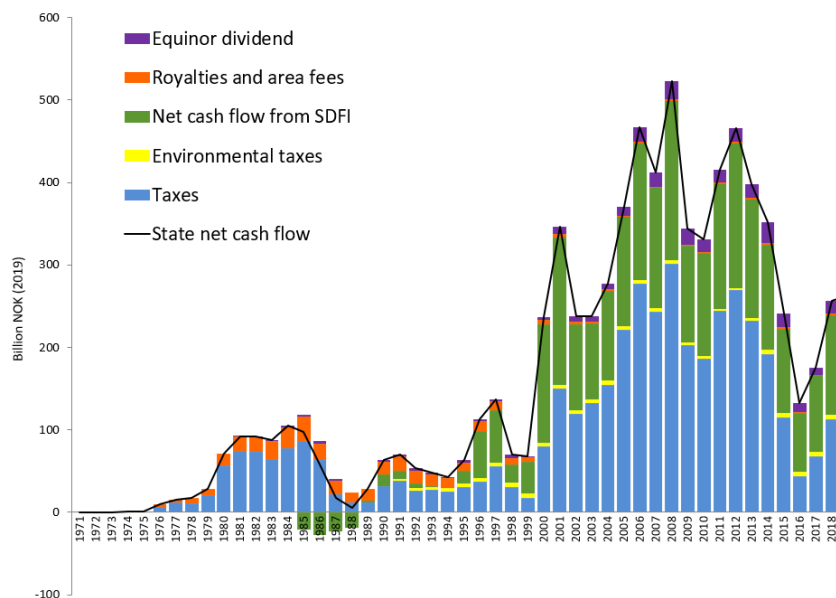


Figure 5: SNCF components 1971-2018 in billion NOK (Norsk Petroleum, 2018)

In the following subsections, we provide a summary of the different sources of contribution towards the SNCF.

4.1.1.1 The state's direct financial interest

The State's Direct Financial Interest (SDFI) is a portfolio consisting of the directly owned exploration and production licenses on the NCS. The cash flows from the SDFI are large and have averaged approximately 40% of the total SNCF in the period 1998-2018. Since 2001 the resources have been managed by Petoro AS, a wholly government-owned company controlled by the Ministry of Petroleum and Energy, that oversees all commercial aspects of the SDFI. Before this, the SDFI was handled by Statoil (now Equinor); however, due to the privatization of Statoil, it was deemed necessary that the SDFI was given new government management. Petoro's main objective is to maximize government income from the SDFI portfolio. Through this chain of command, the Norwegian government awards production licenses to eligible companies to search for and extract petroleum resources in certain areas on the NCS (Norsk Petroleum, 2019a). The SDFI contributed with NOK 121.1 billion in 2018 (Norsk Petroleum, 2019b).

4.1.1.2 Taxes and fees

A priority for the Norwegian government has been to ensure that a large as possible part of the value from petroleum production accrues to the state, this to benefit the Norwegian society. The ordinary company tax-rate is 22%; however, due to extraordinary profitability in oil and gas production, the companies are subject to an individual tax of 56%. This means that these companies are subject to a marginal tax rate of 78% following the petroleum taxation act of 1975. Additionally, participants in the oil industry are also subject to fees such as environmental taxes and area fees while operating on the NCS to ensure that awarded acreage is explored in an efficient way (Norsk Petroleum, 2019a). The different taxes and fees totaled NOK 119.7 billion in 2018 (Norsk Petroleum, 2019b).

4.1.1.3 Dividends from Equinor

In 2001 Statoil (Equinor) was partially privatized, previously being wholly owned by the Norwegian state. When Statoil was registered on the Oslo Stock Exchange and NYSE, the Norwegian state had a goal to retain a 2/3 ownership in the company, being a majority shareholder. The Ministry of Petroleum and Energy manages this position of equity. Equinor has historically been a highly profitable

company that pays regular dividends. The dividends are part of the SNCF and totaled NOK 15.3 billion in 2018 (Norsk Petroleum, 2019b).

4.1.2 Oil-adjusted budget deficit

The oil-adjusted budget deficit shows the difference between the income and expenses on the national budget without incorporating the effects of petroleum-related activities,

$$\text{Income without oil} - \text{Expenses without oil} = \text{Oil adjusted budget deficit}$$

The structural oil-adjusted budget deficit will not be utilized in our calculations, as it accounts for the budget being affected by booms or recessions. The number displays the underlying use of oil income and could be used to see the development in usage and oil-dependency over time. By this logic, the oil adjusted budget deficit is a better measurement for our analysis, because it quantifies the exact amount needed every year to cover the budget deficit. Thereby, we can more firmly quantify the value of the remaining cash flow that is placed in the Oil Fund. The data are retrieved through the Ministry of Finance and transformed into USD at an exchange rate of NOK/USD 7.25.

4.2 Proxies for asset prices

To assess reasonable estimates of the future expected returns and volatilities of the Oil Fund and the alternative approach, we analyze historical data. We do not have access to detailed data on the different components of the Oil Fund as of 31/12/1997, so we use representative benchmarks as proxies. As outlined in section 2.3, the fund's mandate as of 31/12/1997 was to invest up to 40% in equities, primarily in large- and mid-cap stocks in developed markets across the globe. The remaining 60% was invested in investment grade government debt. On this basis, we gather price data on two indices that replicate these compositions, the MSCI World Total Return Index (the equity Index) and the Merrill Lynch Global Government Bond Total Return Index (the bond index).

For oil, natural gas, NGL, and condensate prices, we use historical spot prices of Brent crude (Oil) as a proxy. We simplify by assuming that the price of one oil equivalent of natural gas, NGL, and condensate equals the price of one oil equivalent of oil. One oil equivalent is defined as 1 Sm³ of oil, NGL, and condensate, or 1000 Sm³ of natural gas. We will elaborate further in subsection 4.2.3.

All price-data sets are collected through Global Financial Data (GFD), denominated in USD on a yearly basis. By utilizing annual prices, we experience that the number of observations for each time-series of prices is somewhat low for any descriptive statistics to be sufficiently robust. However, the focal point of this thesis is not to create ideal estimates of future performance of both real and financial assets. Descriptive statistics of the historical data are given in Appendix B.

4.2.1 The MSCI World Total Return Index

The MSCI World Total Return Index tracks the performance of approximately 1,600 large- and mid-cap stocks across 23 developed markets (MSCI, 2019). As such, it is representative of the equity investment universe initially available at the point of establishment of the Oil Fund in 1998 (Norges Bank Investment Management, 2019c). The index measures the total return, which means that all dividends are reinvested in the index. This is equivalent to the practice implemented by the Oil Fund. We collect year-end prices for the MSCI World Total Return Index from 1969 up to 2018.

4.2.2 The Merrill Lynch Global Government Bond Total Return Index

The Merrill Lynch Global Government Bond Total Return Index tracks the performance of government debt issued by investment grade issuers (ETFdb, 2019). In 1998, the Oil Fund's investment mandate stated that the fixed-income investments were restricted to investment grade government debt, so the benchmark is representative of the Oil Fund's accessible debt investments as of 1998 (Norges Bank Investment Management, 2019c). The index measures the total return, which means that all coupon payments are re-invested in the index. We collect year-end

closing prices for the Merrill Lynch Global Government Bond Total Return Index from 1985 up to 2004, as the index existed only during this period.

4.2.3 Brent crude, natural gas, NGL, and condensate

We collect year-end spot closing prices for Brent Crude from 1969 to 2018, denominated in USD per barrel. We convert these prices USD per oil equivalent by multiplying with the rate of exchange between barrels and oil equivalents, 6.2898 (Norsk Petroleum, 2019d). For simplicity, we assume that the price of one oil equivalent of natural gas, NGL and condensate are equal to one oil equivalent of Brent crude. This is a substantial simplification, but as natural gas is priced in regional markets, assessing the correct market price for natural gas is difficult.

4.2.4 USD/NOK

In chapter 5, we present our findings as of 31/12/2018 denominated in NOK. As all simulations are performed with USD denominated price-series, we use an exchange rate of 7.25 NOK/USD to translate these values into NOK, which is equal to the exchange rate as of 31/12/1997. We gather exchange rates from the Norwegian Central bank (Norges Bank, 2019b).

4.3 Estimation of parameters

In section 3.3, we introduced the Geometric Brownian motion and computed the value added of diversification in a simplified example. We simulated the performance of an initial investment of NOK 50 billion, with no cash inflows during the simulation period. In reality, periodical cash inflows occur, and this will affect the range of possible future outcomes. To more realistically assess the expected future performance of the Oil Fund and the alternative approach, we account for these regular cash inflows. To make the portfolios directly comparable, we assume that the expected return, the initial investment, and the regular cash inflows are identical. If we do not do this, our simulations will yield estimates where the expected future values of the two portfolios are contrasting, and we will not be able to quantify the value added of diversification and risk reduction alone. Additionally, if the portfolios exhibit different levels of expected returns, a decision maker would prefer the portfolio with the higher expected return. For our theoretical framework

to be applicable, we assume that the expected returns on both portfolios are equal, while the portfolios exhibit different levels of volatility. In this manner, the range of possible future outcomes will differ, as illustrated in Figure 6.

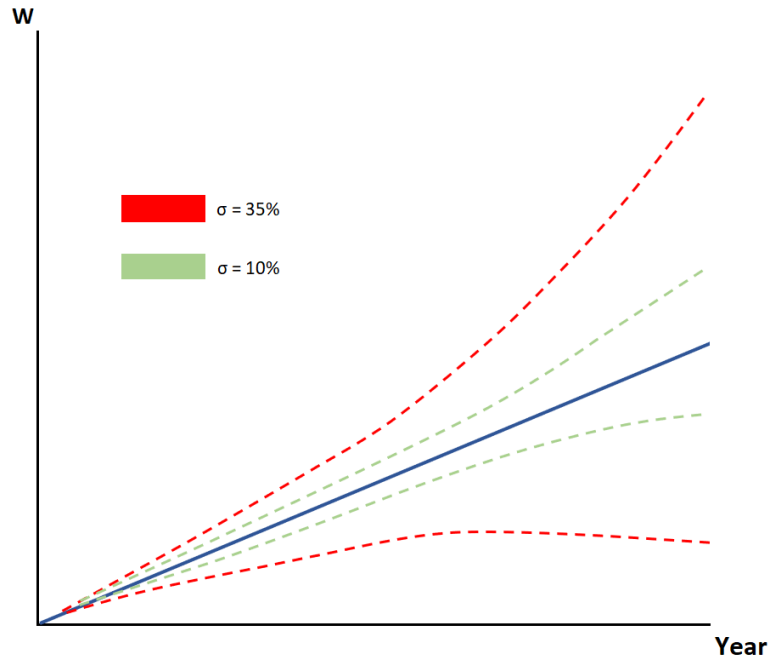


Figure 6: Outcome intervals

σ denotes the standard deviation, and the blue line illustrates the drift parameter, which is the expected return. We utilize the historical data to create an estimate of the expected return. We estimate the historical arithmetic mean return on the equity index, the bond index, and the oil price and use these as proxies for future returns. We compute the yearly arithmetic mean return by the following formula,

$$\bar{r}_A = \frac{1}{n} \sum_{i=1}^n r_i \tag{11}$$

Where n denotes the number of observations and r denotes return at time i . To establish a common drift parameter for the Oil Fund and the alternative approach, we simplify by computing this as the expected arithmetic return on a *combined* portfolio, that is the expected arithmetic return on a portfolio consisting of both the Oil Fund and the alternative approach. We apply the following formula,

$$E[\bar{r}_A] = \frac{(w_E \times \bar{r}_{A, E} + w_B \times \bar{r}_{A, B}) + \bar{r}_{A, OIL}}{2}, \quad (12)$$

where $\sum_{i=1}^n w_i = 1$

Where w denotes the weight of equities and bonds in the Oil Fund portfolio. In this manner, we determine an average expected arithmetic return, and we will be able to capture the effect of diversification in the theoretical framework. The historical standard deviation of returns of the equity index, the bond index, and oil are computed as the yearly standard deviation of historical returns. We apply the following formula,

$$s = \left(\sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r})^2} \right) \quad (13)$$

Where s denotes the sample standard deviation. For the alternative approach, we set the expected standard deviation using the historical standard deviation as a proxy. For the Oil Fund, we have to compute the portfolio standard deviation, as this portfolio consists of two financial assets. The portfolio standard deviation is computed as,

$$s_{OIL\ FUND} = \sqrt{w_E^2 \times s_E^2 + w_B^2 \times s_B^2 + 2 \times \rho_{E,B} \times s_E \times s_B} \quad (14)$$

where $\sum_{i=1}^n w_i = 1$

Where s denotes historical sample standard deviation, w denotes weight and ρ denotes the historical correlation between equities and bonds. Equation (12) and (13) and (14) are applied to estimate the parameters we insert in the Geometric Brownian motion. In the following section, we elaborate on how we compute the values of the Oil Fund and the alternative approach by accounting for periodical cash inflows.

4.4 Calculation of expected future wealth

As described in section 4.3, we make the characteristics of the alternative approach directly comparable to the Oil Fund by assuming equal expected arithmetic returns, same initial investments, and equivalent cash inflows. As explained in section 2.2.1, the Tempo selection argued in favor of slowing down the rate of production and instead produce what was needed to cover the Norwegian national budget deficit (Skånland, et al., 1983). However, if we calculate the value of the alternative approach as the value of “over-production” of oil and gas reserves by assuming that these reserves were never extracted, it would not be directly comparable to the Oil Fund strategy. The value of the alternative approach would be falsely inflated. From the Norwegian government’s perspective, extracting and liquidating oil and gas reserves comes at a transaction cost towards intermediaries who operate on the NCS. The Norwegian state effectively only has a claim on a fraction of reserves at the NCS, as the companies responsible for extracting oil and gas also have claims on the reserves. Hence, the SNCF represents the total cash flow resulting from the Norwegian state’s ownership and management on the NCS. In some sense, we can compare this logic by selling a stock and repurchasing it immediately. If this were the case, an investor would lose money due to transaction fees to brokers. In our case, the brokers are the intermediaries operating at the NCS, as these companies liquidate the assets on behalf of the Norwegian state.

To make the Oil Fund and the alternative approach directly comparable, we assume that the SNCF less the amount needed to cover the national budget deficit represents the cash inflow in both portfolios, which is wealth stored at the NCS in oil and gas reserves. This is approximately equivalent to how the Oil Fund mechanism works (Lund & Stiansen, 2017). We calculate the range of future possible market values of both the Oil Fund and the alternative approach by applying the following formula,

$$MV_t = MV_0 \exp \left(\left(\bar{r}_A - \frac{\sigma^2}{2} \right) t + \sigma W_t \right) + \text{Cash Inflow}_t \quad (16)$$

where $\text{Cash Inflow}_t = \text{SNCF}_t - \text{Oil Adjusted Budget Deficit}_t$

Where MV denotes market value, the first link in the equation represents the geometric Brownian motion, where \bar{r}_A denotes the expected arithmetic return, σ denotes expected standard deviation, t denotes time step, and W_t denotes the stochastic component.

The only parameter that differs when calculating the range of possible future market values of the Oil Fund and the alternative approach is the volatility or the standard deviation of returns. In this manner, we can capture the effect of diversification alone. For both portfolios, we perform a total of 50.000 simulations over 21 years, that is from 31/12/1997 up to 31/12/2018. We assume that the cash inflows occur at a year-end basis and that the amount invested at initiation is the cash inflow as of 31/12/1997. Additionally, we simulate the USD denominated price series and transform these into NOK by utilizing an exchange rate of USD/NOK 7.25, equal to the exchange rate as of 31/12/1997.

4.5 Assumptions

As the previous sections incorporate a substantial amount of information about the assumptions we make to create an estimate of the value added of the petroleum fund mechanism, we summarize the assumptions in this section.

Assumption 1: Following the Norwegian national balance sheet illustrated in Figure 1; regardless of the approach chosen by the Norwegian state, the asset side of the Norwegian national balance sheet contains a fraction of oil and gas reserves, that is reserves that are not yet extracted. We only account for the fraction of oil and gas reserves that have been liquidated.

Assumption 2: We utilize ex-post observations of the SNCF and the oil-adjusted budget deficit for simplicity. These variables are equal in the computation of the future range of expected market values for both the Oil Fund and the alternative approach, so if these variables were to be simulated ex-ante the relative relationship between the two portfolios would not have been affected.

Assumption 3: As explained in section 4.4, the SNCF represents government ownership at the NCS. This means the total value production of oil and gas is not representative of the value of the alternative approach. Intermediaries at the NCS liquidate the assets on behalf of the Norwegian state, and for that, the Norwegian state pays a transaction cost. Hence, the SNCF represents the total cash flow from the Norwegian state's claim of the reserves. This is transformed back into oil and gas reserves in the alternative approach.

Assumption 4: For simplicity, we assume that the price of one oil equivalent of oil equals the price of one oil equivalent of natural gas, NGL, and condensate. Prices for natural gas, NGL and condensate are hard to obtain since these assets are priced in regional markets.

Assumption 5: We use the historical annual mean arithmetic return and the historical standard deviation of the MSCI World Total Return Index in the period 1969 up to 2018 as proxies for the equity proportion of the Oil Fund.

Assumption 6: We use the historical annual mean arithmetic return and the historical standard deviation of the Merrill Lynch Global Government Bond Total Return Index in the period 1985 up to 2018 as proxies for the fixed-income proportion of the Oil Fund.

Assumption 7: We convert USD denominated prices from simulations into NOK at an exchange rate of 7.25 NOK/USD, equal to the spot exchange rate as of 31/12/1997.

Assumption 8: The cash inflows in both the Oil Fund and the alternative approach occur as lump sums at the end of each year.

5 DATA ANALYSIS AND RESULTS

In section 3.2 and 3.3, we introduced two examples of how we will utilize the theoretical framework to estimate the value added of the petroleum fund mechanism quantitatively. However, in these examples, we incorporated simple assumptions about the parameters. In chapter 4, we elaborated on how we will proceed to introduce more reasonable estimates of the parameters in the stochastic processes, and how we will account for periodical cash inflows in our simulations.

In this chapter, we provide a detailed analysis of our findings. In section 5.1, we present the assumptions we make about the expected returns and the expected levels of volatilities in the stochastic simulations. In section 5.2, we present the findings from our primary analysis. However, we find that the expected market values of the portfolios differ substantially from ex-post observations. On this basis, we introduce an additional examination in section 5.4, where we incorporate ex-post observed parameters. In section 5.5, we discuss the shortcomings of the analysis.

5.1 Simulation of possible future outcomes

We apply the geometric Brownian motion introduced in section 3.3 to simulate the range of possible future outcomes. For estimates of the expected return and the expected standard deviation, we use historical data parameters as proxies, computed by utilizing the methodology outlined in chapter 4. The descriptive statistics of the historical data are given in Appendix B. The parameter assumptions are summarized in Table 5.

Table 5: Parameter assumptions

Simulations	Alternative Approach	Oil Fund
<i>E[Mean Return]</i>	12.21%	12.21%
<i>E[Standard Deviation]</i>	46.56%	9.36%
<i>Simulations</i>	50 000	50 000

At a level of volatility in returns of 46.56%, the alternative approach exhibits a level of volatility more than 4x higher than the level of volatility of the Oil Fund. We argue that the expected returns and the expected levels of volatility might not be representative of the future, but as our focal point is to quantify the value added of investing in a portfolio with a lower level of volatility, we do not engage in additional measures to optimize these parameters in this section.

We run simulations for a total of 21 years, that is from 31/12/1997 up to 31/12/2018. The simulated value paths are illustrated in Figures 7 and 8. The figures depict the development of the USD denominated prices series.

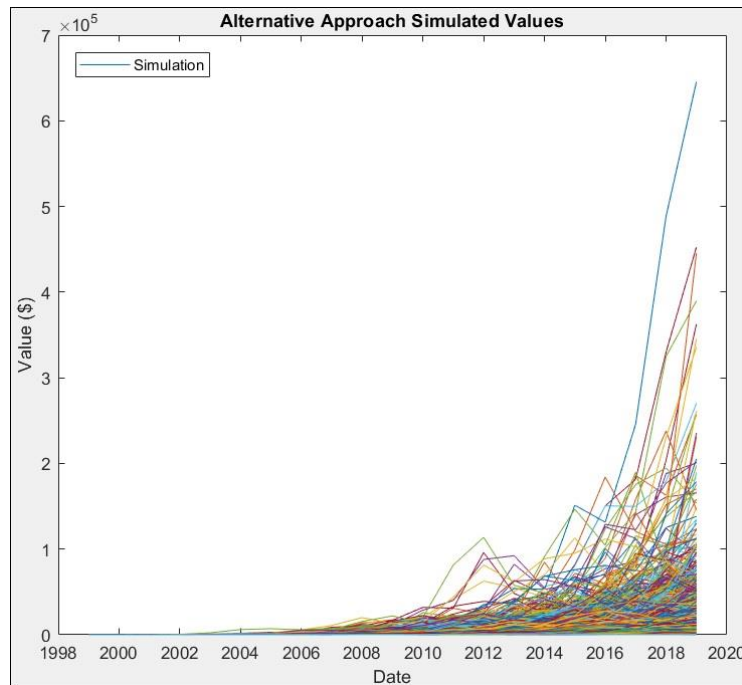


Figure 7: Value paths of the alternative approach

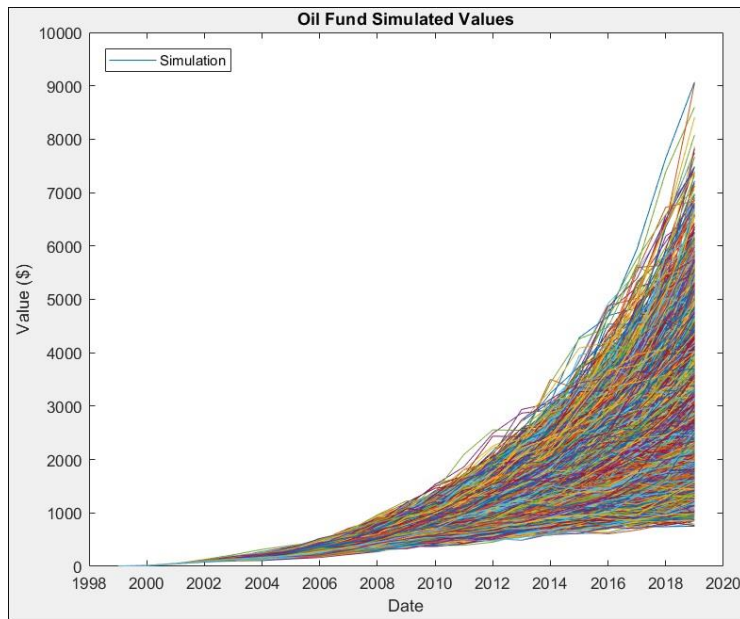


Figure 8: Value paths of the Oil Fund

The alternative approach exhibits a level of volatility that is higher than the Oil Fund. Table 6 reports the range of the possible future outcomes of both portfolios in NOK denominated values.

Table 6: Range of possible future values

Range (B NOK)	Lowest Outcome	Highest Outcome
<i>Alternative Approach</i>	-61.71	6,970,263.04
<i>Oil Fund</i>	5,055.79	66,760.96

The range of possible future outcomes of the alternative approach is considerably more extensive than the range of possible future outcomes of the Oil Fund. This difference is explained by the different levels of volatility in returns. We emphasize that this range does not represent a normally distributed range of possible future values, but an approximately log-normal distribution of future values. We see that the lowest observed value of the alternative approach is negative; this occurs due to the cash inflows in 2016 and 2017 being negative.

5.2 Investigating the value-added

To find the value added, that is the value of diversification; we apply the theoretical framework outlined in section 3.1. We estimate the utility of expected wealth, the expected utilities of wealth and the value added as of 31/12/2018, the last period in our simulations. We assume a risk aversion parameter of 2 in the following calculations. All reported results are given in Table 7.

Table 7: Summary of utilities and value added

	Alternative Approach	Oil Fund
$E(W)$	NOK 18,572.20 billion	NOK 18,572.20 billion
$U(E(W))$	-0.0001	-0.0001
$E(U(W))$	-0.0004	-0.0001
μ	0.14	
Value Added	NOK 16,024.94 billion	
Certainty Equivalent	NOK 2,547.26 billion	
$U(\text{Certainty Equivalent})$	-0,0004	

We find that the expected values of both portfolios are NOK 18,572.20 billion, which is the expected market value as of 31/12/2018. If the levels of volatility were equal for both portfolios, the Norwegian state would be indifferent between investing in the Oil Fund or the alternative approach as both portfolios would exhibit the same risk and return characteristics. We proceed to find the utilities of expected wealth, denoted $U(E[W])$. Since the expected wealth are equal for both portfolios, the utilities of expected wealth levels will, by definition, be similar. The utilities of the expected wealth levels are -0.0001 for both portfolios, as reported in Table 7.

The volatility of returns in the alternative approach is 46.56%, more than 4x higher than the level of volatility of returns in the Oil Fund (See Table 5). As the two portfolios display different levels in the volatility of returns, the expected utilities

of wealth, denoted $E[U(W)]$, differ. For the alternative approach and the Oil Fund, the reported utilities are -0.0004 and -0.0001 , respectively. This suggests that the Norwegian state will prefer the Oil Fund, as the expected utility of wealth is higher. To find the value of μ and the value added of diversification, we utilize equations (5) and (6). The value-added parameter μ is 0.14, and the value added is NOK 16,024.94 billion. The certainty equivalent that is the amount the Norwegian state will accept to avoid investing in the alternative approach is NOK 2,547.26 billion. This indicates that the Norwegian state receives a premium of NOK 16,024.94 billion by re-allocating oil and gas reserves into a globally diversified financial portfolio.

The value of diversification measured as value added is substantial. As the model is highly sensitive to the relative relationship of volatility levels between the two portfolios, the value added of diversification is significantly affected when the value of this relative relationship increases. Additionally, in this section, we calculate the value added by assuming a risk aversion parameter of 2, which indicates an aversion to risk, hence the value added is positive. In the following section, we perform a sensitivity analysis to illustrate how value added is affected by changes in risk preferences and the volatility of returns of the two portfolios. We set the levels of the volatility of returns in the alternative approach at arbitrary sizes across multiple risk preference levels while keeping the volatility of the Oil Fund constant. In this aspect, we see how the value added of diversification changes when we change the underlying parameters.

5.3 Sensitivity analysis

Table 8 depicts how the value added is affected by different levels of risk aversion and how the value added is affected by the volatility level of the alternative approach.

Table 8: Sensitivity analysis

<i>Risk Aversion</i>	3	2	1.1	0	-1	-2	-3
<i>Oil Fund/Alternative Approach Volatility</i>							
9,36%/10%	0.97 578.25	0.98 466.33	0.99 186.53	1.00 0.00	1.00 (186.53)	1.02 (373.06)	1.03 (559.60)
9,36%/15%	0.80 3,805.25	0.86 2,760.67	0.91 1,678.79	1.00 0.00	1.06 (1,149.48)	1.14 (2,693.00)	1.23 (4,328.68)
9,36%/20%	0.61 7,237.43	0.71 5,409.42	0.82 3,273.63	1.00 0.00	1.18 (3,291.13)	1.41 (7,658.52)	1.69 (12,800.10)
9,36%/30%	0.30 13,019.92	0.44 10,539.04	0.62 7,124.77	1.00 0.00	1.58 (10,735.12)	2.53 (28,461.19)	3.78 (51,785.86)
9,36%/40%	0.11 16,657.29	0.23 14,428.23	0.42 10,729.53	1.00 0.00	2.61 (29,956.50)	6.55 (103,432.83)	12.38 (212,330.69)
9,36%/50%	0.01 18,447.99	0.11 16,694.59	0.24 14,226.32	1.00 0.00	3.94 (54,789.49)	11.57 (197,154.34)	22.93 (408,992.66)

*The reported values are μ (upper) and value added in billion NOK (lower)

The leftmost vertical axis depicts the volatility levels of the Oil Fund and the alternative approach. We keep the volatility of the Oil Fund constant at 9.36% while we set the volatility level of the alternative approach at a range of different levels. The reported values are μ (upper) and the value added of diversification denominated in billion NOK (lower). We see that when the relative relationship between the volatility levels of the Oil Fund and the alternative approach is close to 1, a change in the risk aversion parameter has a minor impact on the value added. However, as we increase the volatility of the alternative approach, the value added changes significantly across the different levels of risk aversion. We see that if the Norwegian state exhibits risk neutrality ($\gamma = 0$), the value added of diversification is zero. If the Norwegian state shows risk-seeking preferences ($\gamma < 0$), the effect of diversification is negative. This is intuitive, as the alternative approach offers a broader range of possible future values.

In section 5.2, we assume that the Norwegian state exhibits risk aversion of 2. For instance, if we assume the volatility of returns in the alternative approach is 20%, the value added decreases to billion NOK 5,409.42 billion at the same level of risk aversion, which is significantly lower than the reported value in Table 7. For high levels of volatility in the alternative approach and risk seeking preferences the

estimates of value added become somewhat meaningless. As we simulate over 21 years, we argue that the substantial values added for high values of risk preferences and high levels of relative volatility occur due to the exponential growth factor of the range of possible future outcomes. Thus, we argue that the uncertainty of possible future outcomes grows somewhat exponentially to time. This is illustrated in Figures (8) and (9) and Table (7), where the range of possible future outcomes in the alternative approach is considerably more extensive than the range of possible future outcomes in the Oil Fund. Additionally, both portfolios exhibit an exponential development in the range of possible future values.

5.4 Utilizing ex-post realized returns

In our primary analysis, we set the expected return on the alternative approach and the Oil Fund to 12.21%. By using this rate of return into our stochastic processes, we find that the expected market values of the portfolios as of 31/12/2018 are NOK 18,572.20 billion. However, by analyzing ex-post observations of the market value of the Oil Fund, we see that the market value as of 31/12/2018 is NOK 8,256.00 billion, and the ex-post annualized rate of return from the fund's establishment in 1998 up to 2019 is 5.80% (Norges Bank Investment Management, 2019a). Hence, the rate of return we utilize in our primary analysis is not equal to the ex-post realized return. To account for this difference, we simulate the range of possible future outcomes by using the ex-post realized rate of return of 5.80% on the alternative approach and the Oil Fund. In section 5.2, we experience that the range of possible future outcomes of the alternative portfolio is too extensive based on economic reasoning. To reduce this range, we deliberately set the expected standard deviation of the alternative approach at a lower level. Additionally, we increase the level of volatility of the Oil Fund. Hence, we do not estimate expected volatility by using historical levels of volatilities as proxies. The parameter assumptions are summarized in Table 9.

Table 9: Parameter assumptions

Simulations	Alternative Approach	Oil Fund
<i>E[Mean Return]</i>	5.80%	5.80%
<i>E[Standard Deviation]</i>	25.00%	15.00%
<i>Simulations</i>	50 000	50 000

Ex-post observations of the price of Brent crude show that the price per barrel as of 31/12/1997 was USD 15.86. By applying an expected return of 5.80% on the alternative approach, we get a more appropriate measure of the expected price per barrel as of 31/12/2018, as this can be computed as $USD\ 15.86(1.058)^{21} = USD\ 51.82$. This is approximately synonymous with the ex-post observed price per barrel as of 31/12/2018 at USD 50.52.

As in section 5.2, we estimate the utility of expected wealth, the expected utilities of wealth and the value added as of 31/12/2018, the last period in our simulations. We assume a risk aversion parameter of 2 in the following calculations. All reported results are given in Table 10.

Table 10: Summary of utilities and value added

	Alternative Approach	Oil Fund
<i>E(W)</i>	NOK 8,284.94 billion	NOK 8,284.94 billion
<i>U(E(W))</i>	-0.0001	-0.0001
<i>E(U(W))</i>	-0.0002	-0.0001
μ	0.70	
<i>Value Added</i>	NOK 2,531.95 billion	
<i>Certainty Equivalent</i>	NOK 5,752.99 billion	
<i>U(Certainty Equivalent)</i>	-0,0002	

We find that the expected market values of both portfolios are NOK 8,284.94 billion, which are the expected market values as of 31/12/2018. This is almost synonymous with the ex-post observed market value of the Oil Fund as of 31/12/2018 at NOK 8,256.00 billion. As outlined in chapter 3, we simulate the development of both portfolios using USD denominated prices and convert these into NOK denominated prices by multiplying with the NOK/USD exchange rate as of 31/12/1997 at 7.25.

We proceed to find the utilities of expected wealth, denoted $U(E[W])$. The utilities of the expected wealth levels are -0.0001 for both portfolios, as reported in Table 10. As the two portfolios display different levels in the volatility of returns, the expected utilities of wealth, denoted $E[U(W)]$, differ. For the alternative approach and the Oil Fund, the reported utilities are -0.0002 and -0.0001 , respectively. This suggests that the Norwegian state will prefer the Oil Fund, as the expected utility of wealth is higher. To find the value of μ and the value added of diversification, we utilize equations (5) and (6). The value-added parameter μ is 0.70, and the value added is NOK 2,531.95 billion. The certainty equivalent that is the amount the Norwegian state will accept to avoid investing in the alternative approach is NOK 5,752.99 billion. This indicates that the Norwegian state receives a premium of NOK 2,531.95 billion by re-allocating oil and gas reserves into a globally diversified financial portfolio, considering the aversion to risk.

In this example, we utilize the ex-post realized rate of return on the Oil Fund as a proxy for expected future returns on both portfolios. This is contrary to the methodology we employ in section 5.2, where we use historical parameters of representative benchmarks as proxies for expected future returns and volatility. This latter example is meant to more appropriately illustrate the value added of diversification of the ex-post observed market value of the Oil Fund, as the rate of return utilized in section 5.2 results in an expected market value as of 31/12/2018 of NOK 18,572.20 billion. As outlined at the beginning of this section, we set the expected standard deviations of returns at arbitrary levels. To get a more realistic picture of the value added effect, a more appropriate methodology could be to use historical volatilities in the period 1998 up to 2018.

However, we argue that utilizing a greater extent of ex-post data to find an ex-ante value added effect will lead to overfitted results. In section 5.2, we use the historical rates of return and volatility in the period 1970 up to 2018 on the three respective benchmarks as proxies. We do this to increase the number of observations to more accurately create a representative picture of the historical rates of returns and volatilities.

5.5 Shortcomings of the analysis

Throughout this thesis, we assume that the decision maker, that is the Norwegian state, has the option of investing the proceeds from oil production into a globally diversified portfolio or investing into a hypothetical oil and gas portfolio. We assume that both portfolios exhibit the same fundamental characteristics, except for the levels of volatility. The underlying assumption that the expected returns on the portfolios are equal is spurious. However, as our goal is to quantify the value added of diversification alone, this is an appropriate assumption. As explained in section 4.3, an investor would, in the case of non-equal expected returns, choose the portfolio exhibiting the highest rate of expected return. In this manner, we would not be able to quantify the value added effect of diversification alone.

Furthermore, we assume that the weights of equity and fixed-income in the Oil Fund are constant. Ex-post, the asset weights were revised at two occasions, and the asset weights as of 31/12/2018 were approximately 66% equity, 41% bonds and 3% real estate investments (Norges Bank Investment Management, 2019c). If we implement these weights, we experience that the volatility of the Oil Fund increase, and the value added effect is less. Additionally, we perform simulations on USD denominated price series and converted these into NOK at an exchange rate of NOK/USD 7.25, equal to the exchange rate as of 31/12/1997. If we convert the yearly simulated prices at annual ex-post spot exchange rates, we would have seen an additional increase in volatility of the Oil Fund and the alternative approach in NOK denominated values, as the spot exchange rate has fluctuated significantly from 1997 up to 2018.

As explained, we assume that both portfolios exhibit equal expected rates of returns. A standard measure of portfolio risk and return characteristics is the Sharpe-ratio,

introduced by financial economist William Sharpe in 1966 (Sharpe, 1966). In our analysis, a more appropriate assumption could be to assume different expected rates of return on the portfolios. In this case, the Sharpe-ratio could be a more appropriate measure of investor portfolio preferences. However, as the Sharpe-ratio does not explicitly account for investor risk preferences, we believe utilizing a utility-based framework is the proper methodology.

6 CONCLUSION

In our analysis, we consider an alternative approach to the management of the oil and gas resources at the Norwegian continental shelf to quantify the value added by reallocating assets into a globally diversified financial portfolio. By creating a hypothetical oil and gas reserve portfolio that represents the alternative approach, we have been able to find the financial value added through diversification.

We find that the μ , our factor of value adding effects, is 0,14. Based on simulations using historical data as proxies, the expected future value as of 31/12/2018 is NOK 18,572.20 billion. The expected future value of the Oil Fund is composed of financial value added and the certainty equivalent. The certainty equivalent is NOK 2,547.26 billion and the financial value added of diversification we find is NOK 16,094.24 billion. These estimations are based on a risk aversion level of 2.

However, we recognize that the range of possible future outcomes in the alternative approach is too broad and economically unrealistic. Given this extensive range of outcomes, the value added effect is abnormally large. We argue that this issue arises due to the volatility of the alternative approach being exceedingly large and that the uncertainty about the future development of the portfolio values increases somewhat exponentially with respect to time. To account for these issues, we adjust the volatility of the portfolios so that the range of future expected outcomes are reduced. We then find the value added parameter μ is 0.70, and the expected market value of the two approaches is NOK 8,284.94 billion. The value added is NOK 2,531.95 billion, and the certainty equivalent is NOK 5,752.99.

Our results show that μ and value added of diversification are highly sensitive to the relative relationship between the volatility levels of the portfolios. By performing a sensitivity analysis, we find that the value added of diversification varies by large amounts across different levels of risk preferences and volatilities of the two portfolios. This is a strong indication that the value added is profoundly affected by fluctuations in the Brent spot price. However, we do find that the value added through the fund mechanism is significant.

In our primary analysis, we have presented some shortcomings in our investigation and computational methodology. For future research, we recommend looking further into the factors outlined because these may have a significant impact on the findings presented in our analysis. The price of Brent crude has been the single parameter for the volatility of the oil reserves. By introducing prices for natural gas, NGL, and condensate, we argue that the volatility of the alternative approach would decrease due to diversification effects within the portfolio.

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Appendices

Appendix A: The Power Utility Function

To establish a framework for finding the value added from reallocating oil and gas reserves into global financial assets, we will apply a parsimonious power utility function, first introduced by Arrow (1965, 1971) and Pratt (1964). The power utility function has the following form,

$$U(W_{n,t}, \gamma) = \frac{W_{n,t}^{1-\gamma}}{1-\gamma}$$

Where W_n denotes the value of portfolio n at time t , and γ is a measure of constant relative risk aversion (CRRA). To investigate the functional form of this utility function, we compute the first derivative with respect to wealth,

$$\frac{\partial U(W_{n,t}, \gamma)}{\partial W_{n,t}} = \frac{(1-\gamma)W_{n,t}^{(1-\gamma)-1}}{1-\gamma} = W_{n,t}^{-\gamma} \quad \text{or} \quad \frac{1}{W_{n,t}^\gamma} > 0$$

Which indicates that the utility function is increasing. Hence, utility is increasing as the wealth level is increasing. This indicates a desire for more wealth. We compute the second derivative with respect to wealth,

$$\frac{\partial^2 U(W_{n,t}, \gamma)}{\partial W_{n,t}} = \frac{\partial W_{n,t}^{-\gamma}}{\partial W_{n,t}} = -\gamma W_{n,t}^{-\gamma-1} = -\gamma W_{n,t}^{-(1+\gamma)} < 0$$

Showing that as wealth increases, the utility is marginally decreasing. The intuition behind this is that an increase in wealth when the initial wealth is low leads to a higher increase in utility than if the wealth is high beforehand. "The last bite is never as satisfying as the first."

In the power utility setting, the marginal utility of wealth is characterized as the additional utility an individual gets from a change in wealth. Economists base their models on the assumption that more wealth is always desirable, hence the marginal utility function will always be positive. As shown above, the marginal utility

function can be obtained by deriving the initial utility function with respect to wealth. This utility function is capturing the fundamental desire for more wealth rather than intermediate objectives such as the mean-variance properties of wealth. γ exhibits two essential features:

1. It determines the concavity of the utility curve, so an increase in γ indicates an increase in risk aversion.
2. It is also capturing aversion to volatility in wealth over time. Hence, an investor would care to smooth the development of wealth over time, i.e., avoid large fluctuations.

Further, we can derive the constant absolute risk aversion measure (ARA) by the following equation,

$$A(W_{n,t}, \gamma) = -\frac{\partial^2 U(W_{n,t}, \gamma) / \partial W_{n,t}}{\partial U(W_{n,t}, \gamma) / \partial W_{n,t}} = \frac{\gamma}{W_{n,t}} \quad , \quad \begin{array}{l} \text{Positive if } \gamma > 0 \\ \text{Negative if } \gamma < 0 \end{array}$$

Which is a measure of how an individual's preference for risk changes with a change in wealth. By taking the first derivative with respect to wealth we obtain the slope of the absolute risk aversion measure,

$$\frac{\partial A(W_{n,t}, \gamma)}{\partial W_{n,t}} = \frac{-\gamma}{W_{n,t}^2} \quad , \quad \begin{array}{l} \text{Positive if } \gamma < 0 \\ \text{Negative if } \gamma > 0 \end{array}$$

Following from the function above, if the marginal ARA measure is negative, this indicates that the investor desires to increase his or her investments in risky assets as wealth increases, and if the marginal ARA measure is positive, the investor would want to decrease his or her investments in risky assets as wealth increases. Additionally, if the proportion of risky assets remains unchanged if wealth changes, the investor displays constant ARA, or CARA. Under the assumption of rational investors, decreasing ARA is the desired characteristic.

In addition to the absolute risk aversion measure (ARA), we can also compute the relative risk aversion measure (RRA),

$$R(W_{n,t}, \gamma) = W_{n,t} * A(W_{n,t}, \gamma) = \frac{W_{n,t}\gamma}{W_{n,t}} = \gamma$$

Showing how the proportion of wealth invested in risky assets changes with a change in the level of wealth. By taking the partial derivative with respect to wealth of this function, we can illustrate that the power utility function exhibits constant relative risk aversion (CRRA), i.e. that the proportion of wealth invested in risky assets is independent in the level of wealth,

$$\frac{\partial R(W_{n,t}, \gamma)}{\partial W_{n,t}} = 0$$

According to Blume and Friend (1975), a reasonable explanation of investor behavior is constant relative risk aversion. To conclude, the power utility function does fulfill all assumptions necessary to be consistent with rational investment behavior,

1. Investors prefer higher rather than lower wealth, $\partial(U)/\partial(W) > 0$
2. Investors are risk averse, $\partial^2(U)/\partial(W)^2 < 0$
3. Investors display decreasing absolute risk aversion, $\partial(A)/\partial(W) < 0$
4. Investors display constant relative risk aversion, $\partial(R)/\partial(W) = 0$

Indicating that the power utility function is well fitted to explain rational investment behavior.

Appendix B: Descriptive Statistics of Historical Data

Historical (Yearly 1970-2018 for equity and oil, 1985-2004 for bonds)	<i>Oil</i>	<i>Equity Index</i>	<i>Bond Index</i>
<i>Arithmetic Mean</i>	14,54 %	10,81 %	9,25 %
<i>Median</i>	2,51 %	14,71 %	9,98 %
<i>Standard Deviation</i>	46,56 %	17,57 %	7,49 %
<i>Skewness</i>	1,41	-0,67	0,14
<i>Excess Kurtosis</i>	2,19	0,25	-1,27
<i>Observations</i>	49	49	18
<i>Correlation (Bonds , Equity)</i>	0,29		