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To what extent can information in the term structure of interest rates predict macroeconomic variables in Norway?

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## **Abstract**

This research paper examines to what extent information in the term structure can predict macroeconomic variables. We use Norwegian interest rates in the period April 2001 to December 2018 to investigate the forecast performance on inflation and unemployment. By using the Nelson-Siegel model as the core of the analysis, we derive a factor model. Further, by comparing this to an autoregressive benchmark model, we find that the level and slope factors are the most valuable factors when forecasting inflation and that the curvature factor is the most valuable factor when forecasting unemployment.

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# 1 Introduction

To what extent can information in the term structure of interest rates predict macroeconomic variables? Previous studies on the relation between the yield curve and the macroeconomy show that the behavior of the yield can be relevant for forecasting business cycles (Soares, Martins, and Aguiar-Conraria, 2010). Business cycles are hard to predict, but certain measures, such as economic indicators, can provide signals about the progress of business cycles. In recessions, yields on short-term bonds tend to be low while yields on long-term bonds tend to be high, which results in upward sloping yields curves (Ang, Piazzesi, and Wei, 2004). Since recessions are usually followed by periods of rapid growth, upward sloping yields can also indicate better times tomorrow. Based on this intuition, many researchers have used the shape of the yield curve as a leading economic indicator in models to predict the main macroeconomic variables.

In this paper, we will investigate the relation between the yield curve and macroeconomic variables such as the rate of inflation and the unemployment rate. This relation can be interesting for policymakers in Norway, as it can be valuable for the forecasting of potential forthcoming movements in the business cycle. We will use the yields on Norwegian government bonds and Norwegian Interbank Offered Rates (NIBOR) to represent the Norwegian yield curve.

Building on the classic work of Nelson and Siegel (1987), extended to a dynamic framework by Diebold and Li (2006), we will start to measure the shape of the Norwegian yield curve by adopting a decomposition of the curve into three latent factors, called level, slope, and curvature. This method has a long tradition in the finance literature as the factors together explain almost all of the cross-sectional variation of interest rates (Moench, 2012). Further, we will perform ordinary least squares (OLS) regressions with the latent factors as the independent variables, and then use the estimates to predict the macroeconomic variables. Our goal is to provide a characterization of the interactions between the yield curve and the macroeconomy by focusing on out-of-sample forecasts rather than in-sample forecasts. This is because an in-sample fit does not necessarily say much about a model's forecast performance, which is what we want to evaluate.

We compare our model's forecast performance to the performance of an autoregressive benchmark model, by looking at the Root Mean Squared Forecast Error (RMSFE) and the Cumulative Sum of Squared Forecast Error Difference (CSSFD). These are two different forecast evaluation statistics that can give us information on whether or not we should include information of the yield curve when forecasting inflation and unemployment. We perform iterative forecasts one to twelve months ahead in a rolling window scheme.

First, we find that our model has an overall poor out-of-sample performance relative to the benchmark model when including all the three Nelson-Siegel factors. However, when looking at the relative RMSFE, we find the level factor and the slope factor to be linked to inflation when forecasting at certain horizons. The relative RMSFE also show that the curvature factor is related to unemployment, independent of the forecast horizon. The latter contrasts with what is found in most previous research regarding the curvature factor, which tend to conclude that this factor happens to be the least predictive. Further, when looking at the CSSFD over time, our results show that there are only specific periods in which the factor model and the autoregressive model show clearly different results.

There is a large literature that investigates the forecasting of future business cycles using the term structure of interest rates. When examining the correlations between the three Nelson-Siegel factors and macroeconomic variables, Diebold, Rudebusch, and Aruoba (2005) finds that the level factor is highly correlated with inflation, and the slope factor is highly correlated with real activity. Aguiar-Conraria, Martins, and Soares (2012) also finds the slope factor to have some relevance to unemployment. It is harder to establish a relation between the curvature factor and the macroeconomic variables, but this factor has received increased attention in recent research. Modena (2008) suggests that the curvature factor could be a coincident indicator of economic activity. A change in the curvature could affect both the slope and the level factors and lead to a fall in real output. Other researchers, such as Aguiar-Conraria, Martins, and Soares (2012), claim that there is a reason to believe that the curvature factor relates significantly to unemployment. This statement is also supported by our results, in which the model including the curvature factor shows better performance when forecasting unemployment at all horizons.

Our research paper will proceed as follows. We review earlier literature on the relation between the factors and macroeconomic variables in section 2 and present the theoretical framework in section 3. In section 4, we present the data we use, and in section 5 we extract the three Nelson-Siegel factors level, slope, and curvature. Section 6 describes the empirical framework we use when forecasting the macroeconomic variables, focusing on the two forecast evaluation statistics. We present our forecasting results in section 7 and conclude in section 8. Finally, the references are listed in section 9.

## 2 Literature Review

Some of the literature regarding the financial perspective is presented by Dai and Singleton (1998), who introduces the affine class of asset pricing. However, this way to describe the yield curve does not give proper insight to the underlying economic forces that cause movements. Researchers, therefore, started to include macroeconomic variables into the yield curve models. In fact, it is found that the yield curve conveys information about the development of economic activity, inflation, and monetary policy (Moench, 2012). The yield curve variation is captured by the three unobservable factors level, slope, and curvature, which will be featured in the following.

The level factor is strongly associated with the target of inflation (Afonso and Martins, 2012). Diebold et al. (2005) shows that a surprise increase in actual inflation will result in a long-run boost to the level factor. This comes from the fact that a surprise increase in inflation indicates an expectation of higher future inflation, and that an increase in the level factor may be indicated as higher inflation expectations. According to Barr and Campbell (1997), long-term expected inflation explains almost 80% of the movements in long yields.

The slope factor has a close relationship to monetary policy instruments. Also, Estrella and Mishkin (1998) and Wheelock and Wohar (2009) find that the slope factor outperforms other factors when forecasting output growth and recessions. The relation between the slope and the output growth is shown through the fact that unexpected increases of the slope factor are followed by an immediate decline in output (Moench, 2012). The term spread, i.e., the difference between long rates and short rates, is also useful to predict future GDP growth according to Hamilton and Kim (2000). The higher the slope or term spread, the larger GDP growth is expected to be in the future (Ang, Piazzesi, and Wei, 2004).

The literature does not provide a clear interpretation of the curvature factor. Some researchers, such as Moench (2012), finds the curvature factor to have predictability about the future evolution of the yield curve and the macroeconomy. Other researchers find that it has a poor significance in case of forecasting macroeconomic variables.



These three factors are important to understand future economic activity. The level factor captures expected long-run inflation, and the slope factor may be used to forecast GDP growth, monetary policy instruments, recessions, and expansions. Although the curvature factor has no clear interpretation in the literature, the sum of the three factors gives the best overview of the expectation information contained in the yield curve. Chauvet and Senyuz (2016) finds that the components of the yield curve contain information that is useful for the forecasting of recessions and expansions. Also, Marcelle and Zeynep (2016) shows that components of the yield curve, especially the slope factor, provides information that is useful for forecasting business cycle turning points. However, even though the yield curve is a statistically significant predictor of future activity, the predictive power of the term spread is not stable over time, as found in Chauvet and Potter (2001).

## 3 Theoretical Framework

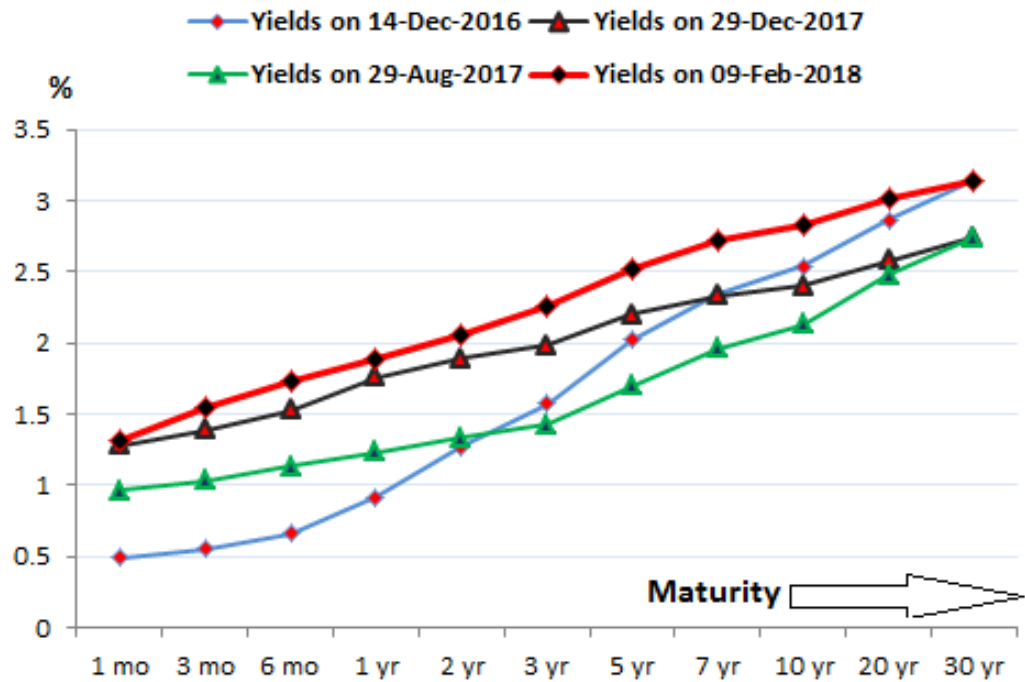
### 3.1 The Term Structure of Interest Rates

The term structure of interest rates measures the relationship among interest rates or bond yields that have different terms to maturity. It has become one of the most popular leading indicators of economic activity, and it plays a central role in an economy. The term structure of interest rates can be described using the yield curve, which is the line that plots maturity against yields for different bonds on a given date. Thus, yield to maturity is used to compare bonds of different coupons and maturity. Fixed-income securities that make a single payment at a specified future date are known as zero-coupon securities (Campbell, 1995). The price of a zero-coupon bond can be found using the formula

$$\text{Zero Coupon Bond Value} = \frac{F}{(1+r)^t} \quad (3.1)$$

where  $F$  is the face value of the bond,  $r$  is the yield, and  $t$  is the time to maturity. A zero-coupon bond is a bond that pays one lump sum at maturity, called the face value, instead of paying coupon payments. Zero-coupon bonds always show yields to maturity equal to their normal rates of return, in which the yield to maturity is often referred to as the “spot-rate”.

U.S. Treasury securities is a frequently reported yield curve. It is often used as a benchmark for other debt in the market, comparing the three-month, two-year, five-year, and thirty-year U.S Treasury debt (Campbell, 1995). **Figure 3.1** shows U.S Treasury yield curves for four different dates, taken from Wolf Street (2018).

**Figure 3.1 – US Treasury Yield Curves**

The horizontal axis shows the maturity of the different bonds, with the short maturities to the left and the long maturities to the right. The vertical axis is the annualized interest rates, which makes it easy to see how much you will get by investing bonds with different maturities. For instance, by looking at the figure, one can see that the spread between the one-month yield and the five-year yield on the 9<sup>th</sup> of February 2018 was 1.23%.

The shape of the yield curve may reflect expectations of future interest rate changes and business cycles. The yield curve is normally upward-sloping since long-term rates tend to be higher than short-term rates, representing positive yield spreads. This is typically the case when an economy seems to be in a good state. An upward-sloping yield curve is also a hallmark at the end of recessions and in the early stages of economic expansions when short term interest rates are at relatively low levels. The additional interest in the higher long-term rates is to compensate for the risk that strong economic growth could set off a rise in prices, i.e., inflation. Hence, the slope of the yield curve will stay positive in the case when the market expects inflationary pressures in the future (Phillips, 2018). However, at the end of expansions, the slope of the curve tends to flatten out or become inverted, which indicates a market expectation of weak economic activity in the future, and thereby a fall in interest rates (Chauvet and Senyuz, 2016).

### 3.2 Macro-finance

Macro-finance addresses the link between asset prices and macroeconomic variables (Cochrane, 2016). One popular way to analyze the movements in the yield curve is to do it within a financial perspective, in which the changes are captured in a no-arbitrage framework where yields are linear functions of a few unobservable or latent factors (Rudebusch and Wu, 2008). However, these no-arbitrage models do not provide sufficient information about the underlying forces that drive movements in interest rates. To include this information into the fundamental drivers of the yield curve, macroeconomic variables can be combined with the financial model (Rudebusch, 2010). This combination of the two perspectives results in the basis of the macro-finance literature.

In a no-arbitrage model of the term structure, the model is estimated using data on yields but not on macroeconomic variables. This financial perspective states that the short-term interest rate is a linear function of some latent and unobserved factors, but with no economic interpretation. The long rates are risk-adjusted averages of expected future short rates and are related to the same unobserved factors (Rudebusch, 2010). This is different from the macroeconomic perspective. To understand the yield curve and its movements from a macroeconomic point of view, one uses the short-term interest rates set by the central bank and the expectation hypothesis of the term structure. Here, long-term yields are driven by expectations of future short-term interest rates, which depends on the expectations of the macroeconomic variables (Rudebusch, 2010).

Together, these two perspectives provide an explanation of the movements in the short rates, by the understanding of how central banks control the short rate in response to fundamental macroeconomic shocks. Additionally, because of the consistency between the short rates and the long rates, the expected future macroeconomic variations should be a good predictor for movements later in the yield curve (Rudebusch and Wu, 2008).

To explain the link between asset prices and the macroeconomy, the predictive effect for future asset returns is central. It is known that financial market variables have predictive power for future macroeconomic variables. Higher current expected returns are consistent with improving conditions, such as higher future output and

consumption growth. Correspondingly, a higher risk is consistent with declining conditions, like lower future economic activity (McMillan, 2018).

### 3.3 Nelson-Siegel Model

When it comes to investigating yield-curve dynamics, researchers have produced a vast amount of literature with many different models. Nelson and Siegel (1987) introduced a parametrically parsimonious model for yield curves that is able to represent the shapes – monotonic, humped, and S-shaped – that are generally associated with yield curves. During a certain period, they found that the model explains 96% of the variation in bill yields across different maturities. These results suggest that the model captures some essential features of the relationship between yield and maturity. This model, known as the Nelson-Siegel model, has become widely used among financial market practitioners and central banks as it provides an extremely good fit to the cross-section of yields in many countries (Christensen, Diebold, and Rudebusch, 2008). The Nelson-Siegel curve can be represented with the functional form

$$y(\tau) = \beta_0 + \beta_1 \left( \frac{1-e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_2 \left( \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \quad (3.2)$$

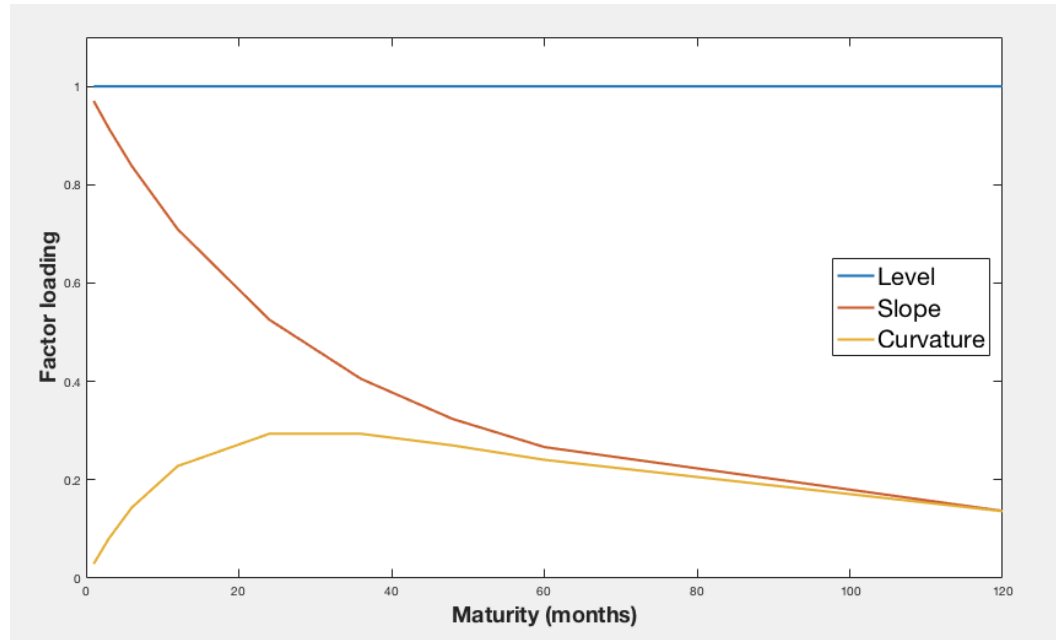
where  $y(\tau)$  is the zero-coupon yield, represented as a function of maturity  $\tau$ , while  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\lambda$  are model parameters (Christensen, Diebold, and Rudebusch, 2008).  $\beta_0$  is independent of time to maturity, and it is often interpreted as the long-term yield.  $\beta_1$  is weighted by a function of time to maturity. This function exponentially decays to zero as  $\tau$  grows, which makes  $\beta_1$  impact mostly at the short end of the curve. This function is also unity for  $\tau = 0$ .  $\beta_2$  is also weighted by a function of  $\tau$ , but this function is zero for  $\tau = 0$ , before it increases and then decreases back to zero as  $\tau$  grows. Thus, this function adds a hump to the curve. The functions multiplied with the coefficients are called loadings and show how much a one-percentage point increase in either of the coefficients, for a given maturity, will affect the zero-coupon yield. The parameter  $\lambda$  determines the position of the hump, i.e., it affects the loadings for  $\beta_1$  and  $\beta_2$  (Gilli, Grosse, and Schumann, 2010).

Based on the work of Nelson and Siegel (1987), Diebold and Li (2006) introduced a dynamic model to describe the yield curve over time and to show that it corresponds exactly to a statistical three-factor model. They show that the three coefficients in the Nelson-Siegel curve may be interpreted as latent level, slope, and curvature factors of the yield curve. Such a dynamic Nelson-Siegel model is easy to estimate and has resulted in good empirical performance (Christensen, Diebold, and Rudebusch, 2008). The dynamic representation of Diebold and Li (2006) contained a replacement of the beta parameters with factors that varies over time:

$$y(\tau) = L_t + S_t \left( \frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + C_t \left( \frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right) \quad (3.3)$$

where they interpret  $L_t$ ,  $S_t$ , and  $C_t$  as the Nelson-Siegel factors level, slope, and curvature, which have different impact responses to the yield curve. The names describe how the yield curve shifts or changes shape in response to a shock. Including such time-varying factors makes it possible to understand the evolution of the bond market over time (Christensen, Diebold, and Rudebusch, 2008).

In this representation of the model,  $\lambda_t$  determines the exponential decay rate of the loading of the level factor. Small values of  $\lambda_t$  results in a slow decay, which will be a better fit to the curve at long maturities. Large values of  $\lambda_t$ , however, will generate faster decay and will better fit the curve at short maturities. Additionally, the parameter also determines the maturity where the curvature factor has its maximum loading. Following standard practice in the literature, we have chosen  $\lambda_t$  to equal 0.0609 in our model (Diebold and Li, 2006). This is the value that maximizes the loading of the curvature factor at exactly 30 months, which corresponds to the line showing the curvature factor in **figure 3.2**. Fixing parameter  $\lambda_t$  at a prespecified value makes it easier to estimate the remaining parameters  $L_t$ ,  $S_t$ , and  $C_t$ . We plot the three factor loadings in **figure 3.2**, using  $\lambda = 0.0609$ .

**Figure 3.2 – Factor loadings from the Nelson-Siegel model**

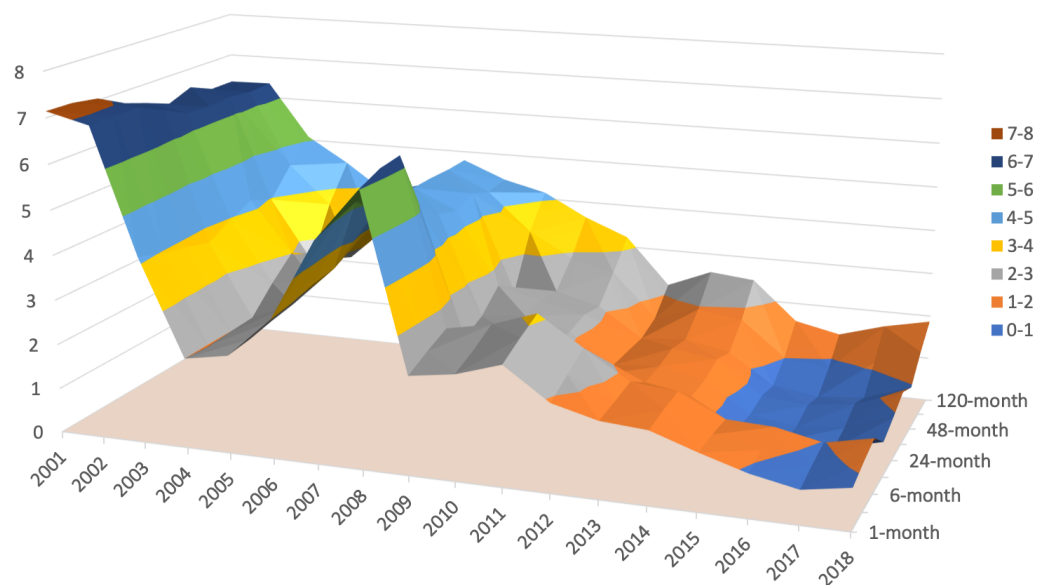
The level factor,  $L_t$ , is the blue line in the figure, which influences interest rates at all maturities in the same way with its loading equal to one. Hence, a one percentage point increase in the level factor will lead to a one percentage point increase in all interest rates.  $L_t$  may, therefore, be interpreted as the overall level of the yield curve (Martins and Afonso, 2010). The red line is the slope factor,  $S_t$ , which has a maximum loading equal to one at the shortest maturity,  $\tau = 0$ . The loading will then decline towards zero as the maturity increases. This means that a change in  $S_t$  will have a larger impact on the interest rates on short-term bonds relative to the interest rates on long-term bonds, which leads to a change in the slope of the yield curve. The last factor is the curvature,  $C_t$ , shown as the yellow line in the figure. This factor has a loading equal to zero at the shortest maturity and increases until an intermediate maturity. After that, the loading gets smaller and smaller as the maturity increases. A change in  $C_t$  will, therefore, lead to a change in the shape of the yield curve.

## 4 Data

### 4.1 Yield Data

We use two different interest rates in our study. For the interest rates with short maturities, we use Norwegian Interbank Offered Rate (NIBOR). NIBOR is a collective term for Norwegian money market rates with different maturities (NoRe, 2017). For the long maturities, we use data on Norwegian government bond yields. All yields are continuously compounded and collected from Bloomberg, which is a platform that provides real-time and historical data on different topics. The data is collected end-of-month spanning from April 2001 until December 2018, including a total of 213 monthly observations. We have collected yields for maturities 1, 3, 6, 12, 24, 36, 48, 60, and 120 months. Hence, the term structure information is extracted from a wide range of maturities.

*Figure 4.1 – A surface plot of the yield data*



In **figure 4.1**, we provide a three-dimensional plot of our yield curve data. The plot represents simultaneously the cross-section point of view, i.e., the yield on a given date as a function of the time to maturity, as well as the time series point of view, i.e., the yield of a given maturity as a function of the date. The first thing to notice is that yields vary significantly over time, with a maximum value of 7-8% in 2001. Especially in the period 2007-2009, the interest rates appeared to be volatile, and remarkable high during the financial crisis in 2008. This is clearly visible in the



figure, showing a gradual rise in all yields before and during the crisis. We also see that there is a stronger correlation between rates with similar maturities. This indicates that some common factors affect the movements of the yield curve. The descriptive statistics for the monthly yields at different maturities are shown in **table 4.1**.

*Table 4.1 – Descriptive statistics*

Maturity (months)	Mean	Std. dev	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
1	2.850	1.987	0.620	7.720	0.975	0.558	0.006
3	2.962	1.964	0.770	7.690	0.977	0.551	-0.010
6	3.061	1.929	0.860	7.810	0.977	0.553	-0.007
12	2.612	1.916	0.381	7.337	0.977	0.619	0.124
24	2.666	1.830	0.328	7.125	0.978	0.676	0.231
36	2.795	1.753	0.440	7.041	0.979	0.713	0.304
48	2.666	1.830	0.328	7.125	0.978	0.676	0.231
60	3.074	1.667	0.954	6.954	0.979	0.749	0.398
120	3.520	1.516	1.021	6.822	0.980	0.762	0.448

We see from **table 4.1** that the long rates are less volatile and more persistent than the short rates. It is often the case that the mean is increasing in maturity, but this is not fully present among the medium-term rates in our data. The reason for this may be that the interval from 2001-2018 includes several volatile periods regarding the Norwegian economy. Especially the financial crisis in 2008 had a major economic impact, resulting in a recession.

Long yields are a risk-adjusted average of expected short yields. Hence, there exists a reason to assume that the standard deviation should decrease as maturity increases (Rudebusch and Wu, 2008). We see that this corresponds to our results as well, with the exception of 48 months, showing a standard deviation of 1.830. Our results also show a decreasing interval between minimum and maximum yields as the maturity increases.

The last three columns display the autocorrelation coefficients for each maturity. Yields for all maturities seem to be persistent, with the short-term yields being the least persistent, showing first-order autocorrelations of 0.975 and 0.977. However, these are still highly persistent, which is essential for econometric analyses (Koopman, Mallee, and van der Wel, 2007).

**Table 4.2 – Correlation Matrix**

<b>Maturity (months)</b>	<b>1</b>	<b>3</b>	<b>6</b>	<b>12</b>	<b>24</b>	<b>36</b>	<b>48</b>	<b>60</b>	<b>120</b>
1	1								
3	0.997	1							
6	0.991	0.997	1						
12	0.971	0.970	0.972	1					
24	0.948	0.944	0.945	0.992	1				
36	0.925	0.919	0.920	0.978	0.995	1			
48	0.948	0.944	0.945	0.992	1,000	0.995	1		
60	0.887	0.880	0.880	0.946	0.975	0.990	0.975	1	
120	0.842	0.832	0.831	0.902	0.939	0.962	0.939	0.990	1

In **table 4.2**, showing the correlation between yields with different maturities, we see that all the yields are relatively highly correlated, with correlation coefficients between 0.831 and 1. As substantiated by the plot of the yield data in **figure 4.1**, the correlation matrix also indicates that rates with similar maturities are stronger related.

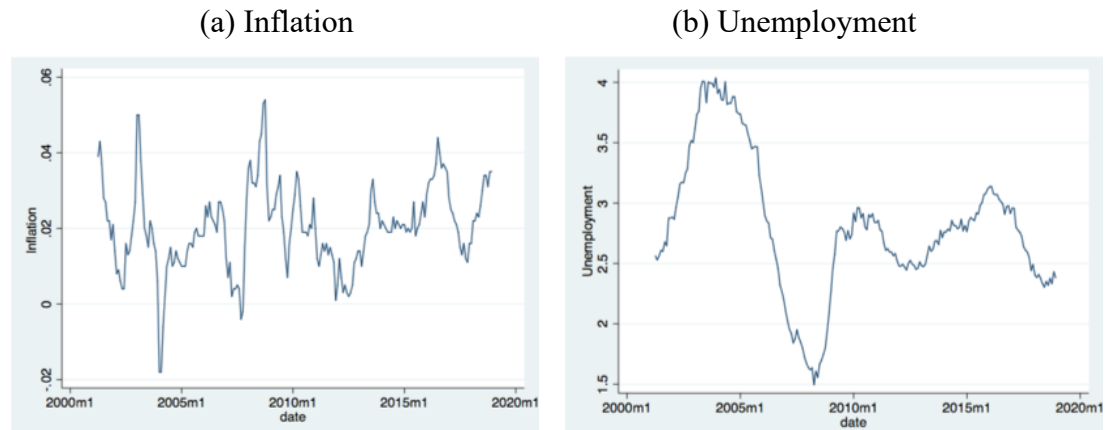
## 4.2 Macro Data

Our macroeconomic data includes a measure of unemployment and a measure of price inflation. The specific sample period, starting in 2001, is chosen based on the monetary policy in Norway. Norway introduced an inflation target as part of its monetary policy framework at the end of March 2001. The previous years were, therefore, a period of transition and instability when it came to the monetary policy in Norway, as the Norwegian authorities replaced a fixed exchange rate regime with an inflation-targeting regime. Using data after this period will increase the probability that our data is collected from a stable period and give us more representative data.

Inflation is defined as the yearly percentage change in the consumer price index (CPI), collected from the Organization for Economic Co-operation and Development (OECD). The data shows the monthly CPI, from April 2001 to December 2018. Regarding unemployment in Norway, there are two different measures to take into account. One applies to those registered as job seekers at NAV, while the other is a number based on a questionnaire published by Statistics Norway, called the Labour Force Survey (LFS). LFS provides the most comprehensive picture of total unemployment as it also includes the part that is job seekers, but not registered at NAV. Hence, LFS will show higher unemployment.

However, we will use data for registered unemployment to avoid any sample uncertainty. In fact, NAV unemployment has shown smaller short-term fluctuations than LFS unemployment (Nordbø, 2016). The development of the variables during the sample period are shown in **figure 4.2**.

**Figure 4.2 – Inflation and unemployment during the sample period**



As we see from **figure 4.2**, both variables have a visible variation over the period 2001-2018. These variations can be explained by the many unexpected events during the sample period, influencing both inflation and unemployment.

## 5 Estimation of the Nelson-Siegel Model

### 5.1 Extracting the factors

We start by adopting a decomposition of the yield curve into three latent factors – level, slope, and curvature – using the Nelson-Siegel model shown in equation 3.3. Fixing parameter  $\lambda_t$  at the prespecified value substantiated in section 3.3,  $\lambda = 0.0609$ , makes us able to compute the values of the two factor loadings. Therefore, instead of estimating all the parameters using nonlinear least squares, we can now use ordinary least squares to estimate only the factors –  $L_t, S_t, C_t$  – for each month  $t$ . This gives us a time-series of estimates of the three factors. Doing it this way has several advantages. In addition to making it easier to estimate the three factors, it also facilitates highly precise estimation by replacing a large number of potentially challenging optimizations with trivial least-squares regressions. This will, however, depend on lambda being an appropriate value.

**Figure 5.1** reports the estimated factors as obtained from the Nelson-Siegel latent factor model. We see that the level factor is most persistent, while the slope and the curvature factors are more volatile.

*Figure 5.1 – Level, slope, and curvature*



The descriptive statistics of the three estimated factors are presented in **table 5.1**, with the last three columns showing sample autocorrelations at displacements of 1, 12, and 30 months. We see that the level factor is more persistent relative to the slope and the curvature factors.

*Table 5.1 – Descriptive statistics for level, slope, and curvature*

Factor	Mean	Std. Dev	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
Level	3.996	1.457	1.336	6.906	0.977	0.756	0.555
Slope	-0.901	1.370	-4.132	3.645	0.955	0.138	-0.448
Curvature	-2.993	1.923	-8.986	1.832	0.901	0.244	-0.168

These observations are in line with some of the historical stylized facts concerning the yield curve, which state that spread dynamics are less persistent than yield dynamics and that long rates are more persistent than short rates (Diebold and Li, 2006). In our framework, a strong persistence of the level factor corresponds to persistent yield dynamics. Correspondingly, weaker persistence of the slope factor can be interpreted as less persistent spread dynamics. We also know that the level factor is closely related to long-term yields. Since the level factor appears to be the most persistent factor, our results suggest that long-term yields will be more persistent than short-term yields.

In the literature, the three factors have been interpreted as the long-term, short-term, and medium-term components of the yield curve (Diebold and Li, 2006). The long term factor is closely related to the yield curve level, which many researchers define as the 10-year yield,  $y_t(120)$ . The slope factor, however, can be interpreted as the short-term factor. Some authors such as Frankel and Lown (1994) represent the yield curve slope as the spread between long-term yields and short-term yields,  $y_t(120) - y_t(1)$ , which turns out to give a number close to the slope factor, but with opposite sign. Finally, the medium-term factor is related to the curvature factor, which is defined in the literature as twice the 2-year yield minus the sum of the 1-month and 10-year yield. In **figure 5.2**, the estimated factors obtained from the Nelson-Siegel model are compared with the common empirical level, slope, and curvature from our data.

**Figure 5.2 – Model-based level, slope, and curvature (i.e., estimated factors) vs. data-based level, slope, and curvature**

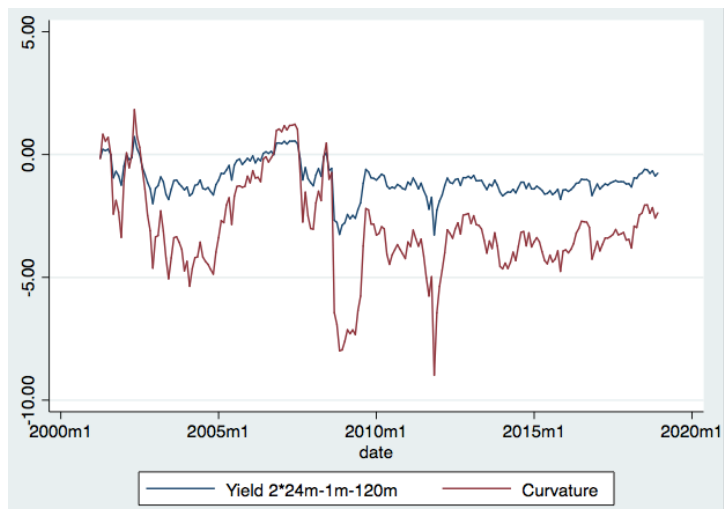
(a) Level, with proxy from data



(b) Slope, with proxy from data



(c) Curvature, with proxy from data



The figure confirms our assertion about the high correlation between our three estimated factors and our data-based proxies. Panel (a) shows that the level factor is very close to the 120-months yield, with a correlation of 0.9717. Panel (b) shows that the slope factor is highly related to the spread of 120- and 1-month yields, with a correlation of -0.9807. Lastly, panel (c) displays a close relationship between the curvature factor and the 24-months yield minus the 1- and 120-months yield, with a correlation of 0.9847. More significant deviations are, however, observed in certain periods, probably due to more volatile periods in the Norwegian economy, such as the financial crisis.

As discussed earlier in this paper, it has been shown in past research (e.g., Diebold and Li, 2006) that the three Nelson-Siegel factors together explain almost all of the cross-sectional variation of interest rates of different maturities over time. This conclusion is confirmed by our results reported in **table 5.2**, which lists the nine different maturities of yields used in our analysis. The three columns to the right contain the cumulative shares of variance for each maturity, explained by the estimated factors level (L), slope (S), and curvature (C). It is clear that all yields, independent of maturity, are almost entirely explained by the three factors, with the level factor explaining the largest share and the curvature factor explaining the smallest. This indicates that almost no information about yield curve dynamics is left out when using the three factors further in our analysis.

*Table 5.2 – Yields and share of variance explained by the estimated factors*

<b>Maturity</b>	<b>L</b>	<b>L,S</b>	<b>L, S, C</b>
1 month	0.5597	0.9939	0.9968
3 months	0.5380	0.9963	0.9996
6 months	0.5260	0.9867	0.9944
12 months	0.6248	0.9337	0.9975
24 months	0.7058	0.9175	0.9994
36 months	0.7676	0.9150	0.9987
48 months	0.7058	0.9175	0.9994
60 months	0.8697	0.9417	0.9988
120 months	0.9442	0.9705	0.9992

Further, **table 5.2** suggests that the longer the maturity of the yield, the more of the variation is explained by the level factor. This result should be expected since the level factor is closely related to long-term yields. In contrast, we see that the slope factor explains more of the variation in yields with shorter maturities. Therefore,

the two factors together contain valuable information about movements in yields, especially for yields with a very short or a very long maturity, which is shown in the third column of **table 5.2**. By adding the curvature factor, the results indicate that between 99.44-99.66% of the variation in yields, dependent on maturity, is explained by the three factors. These observations are in line with what has already been documented in previous studies, in which the three factors have been interpreted as long-term, short-term, and medium-term.

### 5.2 Relation Between the Factors and the Macroeconomic Variables

In the previous section, we extracted the three Nelson-Siegel factors and showed that these factors capture almost all of the variation in the yield curve. In this section, we will examine whether or not the yield curve conveys information about inflation and unemployment by looking at the correlation between the three factors and the two macroeconomic variables.

The statistics reported in **table 5.3** give us information about the degree of co-movement between each of the macroeconomic variables and the yield curve’s three components – level, slope, and curvature – over different lag/lead lengths.

**Table 5.3 – Cross-correlation tables of inflation and unemployment**

(a) Cross Correlation of inflation with															
Variable x	x(t-7)	x(t-6)	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)	x(t+6)	x(t+7)
Level	-0.219	-0.223	-0.224	-0.212	-0.195	-0.178	-0.154	-0.130	-0.113	-0.101	-0.086	-0.061	-0.034	-0.010	0.018
Slope	0.258	0.291	0.311	0.315	0.308	0.309	0.314	0.305	0.271	0.225	0.163	0.091	0.017	-0.043	-0.097
Curvature	0.095	0.071	0.052	0.031	-0.003	-0.005	-0.027	-0.055	-0.081	-0.114	-0.140	-0.161	-0.167	-0.157	-0.163

(b) Cross Correlation of unemployment with															
Variable x	x(t-7)	x(t-6)	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)	x(t+6)	x(t+7)
Level	0.364	0.353	0.341	0.327	0.313	0.298	0.281	0.266	0.247	0.227	0.205	0.184	0.163	0.144	0.124
Slope	-0.324	-0.374	-0.422	-0.469	-0.511	-0.550	-0.582	-0.607	-0.621	-0.628	-0.628	-0.621	-0.609	-0.591	-0.569
Curvature	-0.274	-0.282	-0.285	-0.284	-0.275	-0.260	-0.237	-0.216	-0.184	-0.145	-0.099	-0.056	-0.012	0.022	0.053

The contemporaneous correlation coefficients are shown in the x(t) column and tell us how much the time series are related to each other at the same point in time. A number close to one in absolute value indicates that there is a strong co-movement between the two time series. Whether the co-movement is positive or negative depends on the sign of the correlation coefficient. A value close to zero implies that the yield factor does not vary contemporaneously with the particular



macroeconomic variable in any systematic way. In that case, the two time series happen to be uncorrelated. We can see from the table that the slope factor is the yield component that has the strongest contemporaneous co-movement with both inflation and unemployment, relative to the level and curvature factor.

The remaining columns of the table also report correlation coefficients, but with a lead/lag relationship. That is, the time series of the yield factors have been shifted one to seven months either backward or forward, relative to the macroeconomic variables. These numbers carry some information about the co-movements with the macroeconomic variables, but they also indicate which of the two time series that leads the other.

By looking at the level factor in panel (b), we can see that the largest coefficient is in column  $x(t-7)$  and that the correlation decreases as we move to the right. This indicates that the level factor is stronger related to future unemployment, and we say that the level factor leads the macroeconomic variable. In panel (a), the slope appears to be positively correlated to inflation, with the largest coefficient in column  $x(t-4)$ . This indicates that the slope factor leads inflation and tend to peak about four months before inflation. The level factor also leads inflation, but by about five months. In contrast, a series that lags the macroeconomic variable would have the largest correlation coefficient in the column corresponding to  $x(t+j)$ , where  $j > 0$ . This will be the case for the slope factor in panel (b), which tend to peak 2-3 months after unemployment. The curvature factor is the yield component that is least correlated with both inflation and unemployment. However, one cannot expect to find any systematic relation between this factor and the two macroeconomic variables, as the curvature factor tends to capture the smallest part of the variation in yields.

## 6 Empirical Framework

In this section, we will turn to the specifics related to the methodology of our forecasting and explain the two criteria we use to evaluate the forecasting performance of the models.

### 6.1 Out-of-sample Forecasting

While the in-sample statistics cover  $t = 1, \dots, T$  for which we have observations, the out-of-sample statistics cover  $T+h$  where  $h = 1, \dots, H$ , for which we do not yet know the true values (Bjørnland and Thorsrud, 2015). Researchers are usually more interested in looking at a model's out-sample forecast performance rather than its in-sample performance when measuring a model's ability to predict. In fact, a good in-sample fit of forecasting models has in many settings proven to be a poor indicator of forecast performance. Thus, we choose to focus on out-of-sample performance when predicting inflation and unemployment, using an iterative forecasting model.

### 6.2 Rolling Window

One of the most accurate ways to compare models and forecast out-of-sample is using rolling windows. By using rolling window estimation, one uses a fixed number of the most recent observations rather than all available observations (Inoue, Jin, and Rossi, 2016). This method is often used when parameter instability is suspected, which is considered as a crucial issue in forecasting.

A common alternative to rolling statistics is to choose an expanding window, using all available data at any point of time. In general, it is not easy to say that one of these methods is better than the other, as it depends on the specific empirical application and on the properties of the time series data (Bjørnland and Thorsrud, 2015). Our choice to adopt a rolling window approach is motivated by the fact that expanding windows do not take into account the possibility of structural breaks. If structural breaks characterize a particular time series, such as the financial crisis, using the full historical data series to estimate a forecasting model may lead to forecast errors that are no longer unbiased (Pesaran and Timmermann, 1999).

For rolling estimation, different window lengths lead to various forecast performances. More extended rolling window sizes are in general in the position to yield more precise estimates. However, a large window may result in too few estimates to enable us to test the forecast accuracy of our model. It will, therefore, be necessary to find a balance to fulfill this trade-off, since there is no strict criterion for selecting the window size in rolling window estimation (Balcilar, Ozdemir, and Arslanturk, 2010). We therefore use 60 observations, which we believe will give us an appropriate window size. Our goal is to forecast inflation and unemployment between time  $t$  and  $t+h$ , using information up to time  $t$ .

### 6.3 Out-of-sample Forecast Criteria

To evaluate the out-of-sample forecast, we applicate measures to be able to assess how good the model is. In the following, we present two measures used to examine the accuracy of the forecasting model when it comes to the overall forecast performance, as well as its forecast performance over time.

#### 6.3.1 RMSFE

To evaluate whether or not we have a good forecasting model, we need to compare the value of our loss function with the loss function of another model. The root mean squared forecast error (RMSFE) is a symmetric loss function that is by far the most commonly used evaluation method for forecast accuracy. It simply measures the size of the forecast error by taking the square root of the mean squared error (Bjørnland and Thorsrud, 2015):

$$RMSFE = \sqrt{E[(e_{t+h})^2]} = \sqrt{E[(y_{t+h} - \hat{y}_{t+h})^2]} \quad (6.1)$$

where the forecast error,  $e_{t+h}$ , is a measure of the difference between the actual and the predicted value of a time series. A low RMSFE value indicates a better forecast performance, as we want the forecast error to be as small as possible.

#### 6.3.2 CSSED

In addition to measuring the overall forecast performance using RMSFE, we will also evaluate the model's out-of-sample performance over time relative to the AR( $p$ ) model. This will be done by looking at the Cumulative Sum of Squared

Forecast Error Difference (CSSED), which is another forecast evaluation statistic. Our CSSED statistics are computed as

$$CSSED_{m,t+h} = \sum_{t=60}^{T-h} (\hat{e}_{bm,t+h}^2 - \hat{e}_{m,t+h}^2) \quad (6.2)$$

where  $\hat{e}_{bm,t+h}$  is the forecast error of the benchmark model and  $\hat{e}_{m,t+h}$  is the forecast error of the factor model, in period  $t+h$ . The time span of the forecasting is denoted by  $t$  and  $T-h$ , with  $t$  being the start of the forecasting period and  $T-h$  being the end. For this measure, an increasing curve of CSSED means that the factor model outperforms the benchmark model in the particular period, while a falling curve of CSSED means that the benchmark model has the best forecasting performance.

## 7 Empirical Results

### 7.1 Benchmark Model

The yield curve is only one set of predictive instruments available to forecast macroeconomic variables. To compare our out-of-sample forecasts, we choose to use the autoregressive (AR) process as the benchmark model in our analysis, since this is a model that has been widely used for forecasting. In fact, it is shown that if one uses a stable autoregressive process like the  $AR(p)$ , and use the conditional expectations to derive the predictor, the forecasts will on average be equal to the true value. The  $AR(p)$  is defined as a time series process that links the value of a variable  $y$ , at time  $t$ , to its value in previous periods,  $t-i$ , and a random disturbance  $e$ , also at time  $t$  (Bjørnland and Thorsrud, 2015):

$$y_t = \sum_{i=1}^p \phi_i y_{t-i} + e_t \quad (7.1)$$

Under the assumption of stationarity, we will estimate  $AR(p)$  models using OLS with 60 observations at a time in a rolling window scheme, explained earlier. We will evaluate whether or not our model is better at forecasting the macroeconomic variables relative to the  $AR(p)$  model by comparing the values of the RMSFE, as well as looking at the CSSED.

#### 7.1.1 Model Selection

The model selection involves the task of selecting a model from a set of candidate models. The most commonly used methods to select the best model is the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). These methods give a measure of how good our models are in regard to what we aim to find (Bjørnland and Thorsrud, 2015). AIC and BIC can be written as:

$$AIC(p) = \ln \left( \frac{SSR(p)}{T} \right) + (p + 1) \frac{2}{T} \quad (7.2)$$

$$BIC(p) = \ln \left( \frac{SSR(p)}{T} \right) + (p + 1) \frac{\ln(T)}{T} \quad (7.3)$$

where  $SSR(p) = \sum_{t=1}^T \hat{\varepsilon}_t \hat{\varepsilon}_t$ .

In the process of selecting the number of lags to include in the AR( $p$ ) model, we have to choose a lag-length that contributes to remove autocorrelation in the residuals. Too few lags might result in autocorrelated residuals and omission of valuable information. On the other hand, too many lags might lead to additional estimation error in the model, which means that the parameter estimates become more uncertain (Bjørnland and Thorsrud, 2015). Using the two criteria together is suggested to give the best result in the model selection process.

**Table 7.1 - AIC and BIC for the different models with different lag-lengths**

		AIC	BIC
<b>Inflation</b>	1-lag	-10.4670	-10.4986
	2-lags	<b>-10.5479</b>	<b>-10.5956</b>
	3-lags	-10.5203	-10.5841
	4-lags	-10.4941	-10.5740
	5-lags	-10.4734	-10.5696
	6-lags	-10.4448	-10.5575
	7-lags	-10.4159	-10.5451
	8-lags	-10.4219	-10.5678
<b>Unemployment</b>	1-lag	-14.3699	-14.4015
	2-lags	-14.3599	-14.4075
	3-lags	-14.4526	-14.5163
	4-lags	<b>-14.4930</b>	-14.5730
	5-lags	-14.4847	-14.5810
	6-lags	-14.4688	-14.5815
	7-lags	-14.4529	-14.5821
	8-lags	-14.4693	<b>-14.6151</b>

The results shown in **table 7.1** suggest that the most accurate way to forecast inflation is by using a model with two lags when forecasting inflation, emphasized by the lowest values for AIC, -10.5479, and BIC, -10.5956. This indicates that two lags should be enough to remove autocorrelation in the residuals.

When forecasting unemployment, the recommended lag length from AIC and BIC is mismatched. The AIC suggests a model including four lags, with an AIC value of -14.4930, while the lowest BIC value of -14.6151 indicates that a model including eight lags should be used. Since several researchers argue that one should not choose only one of the criteria in favor of the other, we choose to continue with both the model with four lags and the model with eight lags in our analysis. As provided in the literature, more lags may imply less parameter uncertainty. The case where either AIC or BIC suggests a model of more lags could therefore indicate a more robust model (Aguar-Conraria, Martins, and Soares, 2012).

## 7.2 Augmented Dickey-Fuller Test

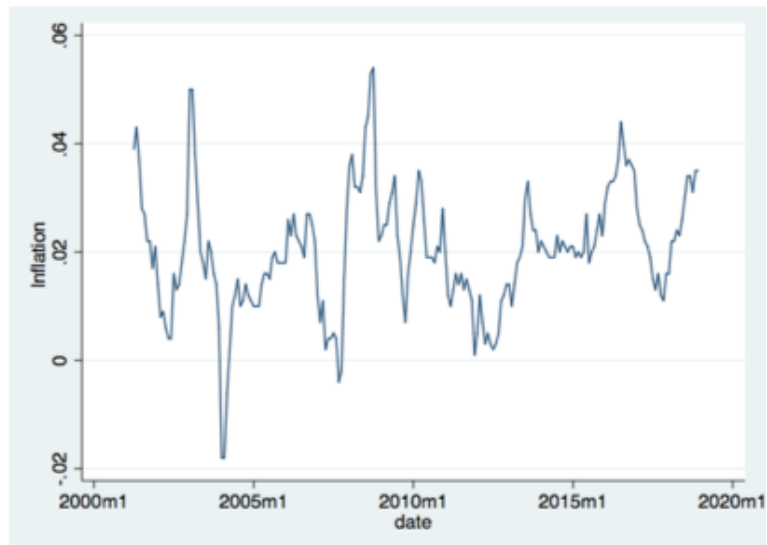
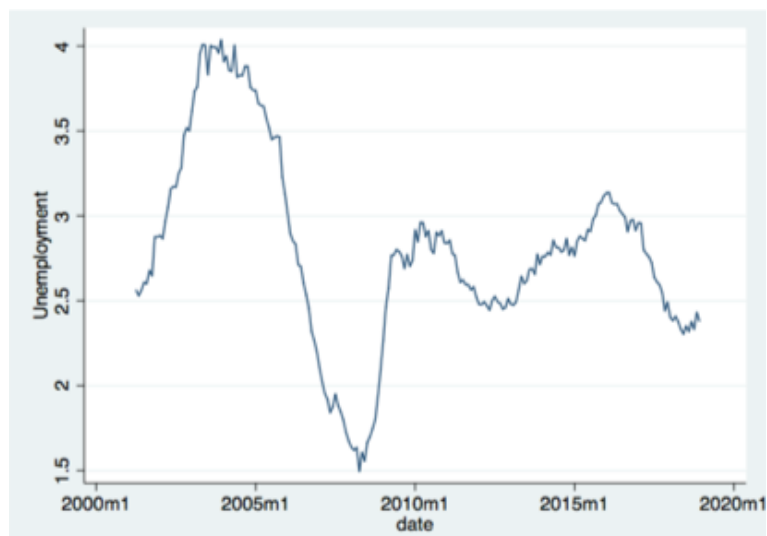
A large amount of macroeconomic time series turns out to be non-stationary in the sense of having one or more unit roots. We will test for such unit roots by applying the augmented Dickey-Fuller (ADF) test, which is a widely used method. The ADF test examines the null hypothesis that a unit root is present in a time series sample against stationary alternatives. Once a value for the test statistic is computed, it can be compared to the relevant critical value for the ADF test. The ADF statistic used in the test is a negative number, and the more negative the number is, the stronger is the rejection of the null hypothesis at some level of confidence (Cheung and Lai, 1995). We start by applying the ADF test to each individual time series from 2001-2018, including the lag-lengths proposed by the AIC.

**Table 7.2 – Augmented Dickey-Fuller test for unit root, 2001-2018**

	<i>Test Statistic</i>	<i>1% Critical Value</i>	<i>5% Critical Value</i>	<i>10% Critical Value</i>	<i>P-Value</i>
<b>Inflation</b>	-4.480	-3.473	-2.883	-2.573	0.0002
<b>Unemployment</b>	-2.395	-3.473	-2.883	-2.573	0.1431

The coefficient estimates are reported in **table 7.2**. The 10%, 5%, and 1% critical values are -2.573, -2.883, and -3.473, respectively. The test statistics, -4.480 and -2.395, show that a unit root hypothesis can be rejected at the 1% level for inflation but cannot be rejected at any significance level for unemployment. The same conclusion can be drawn by looking at the p-values. These results suggest that inflation is stationary, while unemployment is non-stationary. Non-stationary behaviors can include trends, cycles, and random walks, which could make the time series unpredictable. Thus, traditional regression and forecasting results regarding unemployment can lead to incorrect inferences.

There are different ways to make the times series of unemployment stationary in order to get more credible results. One alternative is to detrend the series by using a Hodrick-Prescott (HP) filter. The disadvantage of this method is that we do not have the information regarding the trend, which makes it necessary to compute the trend of the time series. This may result in a poor presentation of the series, which will make the estimates less trustworthy. One may also choose to estimate the time series using an expanding window, but this could result in large variations in the trend, which is not desirable in our case.

**Figure 7.2 – Inflation and unemployment over time****(a) Inflation****(b) Unemployment**

Instead, we look at other options to get the series stationary. Even though the null hypothesis cannot be rejected, we see from **figure 7.2 (b)** that the time series of unemployment looks quite close to stationary. Therefore, we choose to see if including more observations will change the results. Applying the ADF test to the expanded time series of unemployment, spanning from 1995-2018, we get the coefficient estimates presented in **table 7.3**.



**Table 7.3 – Augmented Dickey-Fuller test for unit root, 1995-2018**

	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value	P-Value
<b>Unemployment</b>	-3.791	-3.457	-2.879	-2.570	0.0030

Using a longer time series of unemployment results in a p-value close to zero and a test statistic that is higher than the critical values in absolute value. We can, therefore, reject the null hypothesis stating the presence of a unit root at a 1% significance level. Although the time series seems to be non-stationary in specific periods, we choose to use our original sample size in the forecasting evaluation as the time series appears to be stationary in the long term.

### 7.3 Overall Forecast Performance

In this section we will analyze out-of-sample forecast performances by comparing our benchmark model,  $AR(p)$ , with our factor model.

$$\text{Factor model: } y_{t+h} = \beta_{0,t} + \beta_{1,t}L_t + \beta_{2,t}S_t + \beta_{3,t}C_t + \phi_{1,t}y_t + \phi_{2,t}y_{t-1} + \dots + \phi_{p,t}y_{t-p} + \varepsilon_t$$

$$\text{AR}(p) \text{ model: } y_{t+h} = \phi_{0,t} + \phi_{1,t}y_t + \phi_{2,t}y_{t-1} + \dots + \phi_{p,t}y_{t-p} + \varepsilon_{t+h}$$

When doing forecast evaluation, we use RMSFE to measure the overall forecast performance. We will present the RMSFE ratios of our factor model relative to the benchmark model when forecasting one, three, six, and twelve months ahead. A ratio of less than one indicates that our model has a smaller RMSFE compared to the benchmark model and thus is better at forecasting the macroeconomic variable. We start by presenting the overall performance of the factor model. Then we show the results of the single factors analysis, in which we test the forecast performance of each factor.

### 7.3.1 Factor Model

The overall performance of the factor model is shown in **table 7.4**.

**Table 7.4 – Relative RMSFE**

	<i>1 month</i>	<i>3 months</i>	<i>6 months</i>	<i>12 months</i>
<b>Inflation (2-lags)</b>	1.043	1.054	1.034	1.168
<b>Unemployment (4-lags)</b>	1.067	1.071	1.071	1.189
<b>Unemployment (8-lags)</b>	1.087	1.154	1.107	1.088

These forecast results confirm that including the yield factors in the model do not lead to clear forecast improvement compared to the benchmark model. This is the case for both inflation and unemployment when forecasting one, three, six, and twelve months ahead.

We find the lowest relative RMSFE, 1.034, in the case of forecasting inflation six months ahead, including two lags. Still, the benchmark model performs better by a small margin. We also see that forecasting one- and twelve months ahead differs in that the factor model has a slightly better forecast performance at the shorter horizon. One reason why the factor model does not outperform the autoregressive model could be that the factor model makes bad forecasts in certain periods. This could have a negative impact on the average forecasts, which is reflected in the overall performance.

### 7.3.2 Analyzing the Single Factors

We have also tested the forecast performance of each factor for both inflation and unemployment. The results are shown in **table 7.5**. In most of the cases, the factor model is beaten by the benchmark model. We also see that in several cases, the two models have an equal forecast performance, showing a relative RMSFE equal to one.

**Table 7.5 – Relative RMSFE of the single factors**

		1 month	3 months	6 months	12 months
Inflation (2-lags)	<b>Level</b>	1.000	<b>0.989</b>	1.000	1.107
	<b>Slope</b>	1.000	1.000	<b>0.983</b>	1.053
	<b>Curvature</b>	1.000	1.022	1.059	1.076
Unemployment (4-lags)	<b>Level</b>	1.016	1.000	1.000	<b>0.981</b>
	<b>Slope</b>	1.028	1.000	1.036	1.151
	<b>Curvature</b>	<b>0.989</b>	<b>0.929</b>	<b>0.929</b>	<b>0.981</b>
Unemployment (8-lags)	<b>Level</b>	1.044	1.077	1.071	1.018
	<b>Slope</b>	1.042	1.077	1.107	1.123
	<b>Curvature</b>	<b>0.985</b>	<b>0.923</b>	<b>0.893</b>	<b>0.895</b>

When forecasting inflation, the benchmark model is beaten by the factor model in two cases. In the case of forecasting three months ahead, the model including the level factor performs better than the benchmark model, while in the case of forecasting six months ahead, the model including the slope factor performs better.

These results indicate that the forecast accuracy can be improved by including the level factor in the model when forecasting inflation three months ahead, which is consistent with previous findings. For instance, it is found that the level factor is highly correlated to long-run inflation expectations. Hence, an increase in the level factor indicates higher inflation in the future. The slope factor, on the other hand, is according to earlier research able to predict inflation, since a rise in the short-term interest rate flattens the slope of the yield curve (Chen and Tsang, 2013). One reason why the slope factor is a good predictor when forecasting inflation comes from the fact that a downward sloping yield curve reflects expectations of a falling rate of inflation, while a steeply upward sloping yield curve indicates expectations of a rising rate of inflation (Mishkin, 1990). This is explained by the variation of term premiums over time.

Forecasts of unemployment indicate that the factor model performs better than the benchmark model more often than what we found for the inflation forecasts. When forecasting unemployment using four lags, the model including the curvature factor performs better than the benchmark model at all horizons. This is also the case when forecasting unemployment with eight lags. Aguiar-Conraria, Martins, and Soares (2012) verify a link between the curvature factor and unemployment, in which increases in the curvature factor is associated with lagged decreases in

unemployment. The benchmark model is also beaten by the model including the level factor when forecasting unemployment 12 months ahead, using four lags.

Even though **table 7.5** shows that the  $AR(p)$  model has an overall better performance relative to our factor model, there is a reason to believe that the factor model performs better than the benchmark model in several situations when forecasting unemployment. According to Aguiar-Conraria, Martins, and Soares (2012), level, slope, and curvature is related to unemployment. The yield curve level leads unemployment. This can be one of the reasons why the model including the level factor beats the benchmark model in one of the cases when forecasting unemployment. Further, an increase in unemployment anticipates a decrease in the slope, i.e., a steepening of the yield curve. This evidence can be interpreted as capturing the lead of higher unemployment to ease the monetary policy, which would result in steeper yield curves. Regarding the third factor, increases in the curvature are, as mentioned, associated with lagged decreases in unemployment.

#### **7.4 Forecast Performance Over Time**

The previous section showed that if we include only the level factor or the slope factor, the factor model turns out to predict inflation better than the benchmark model at a three- and six-months horizon, respectively. For unemployment, the model including only the curvature factor outperforms the benchmark model independent of the forecast horizon. Although RMSFE is a good measure to determine the accuracy of a forecast, a disadvantage is the high influence of outliers in the data. This means that more substantial errors in specific periods can have an excessively large effect on the value of RMSFE and thereby skew the results. It may, therefore, be interesting to analyze the models' forecast performance over time, which we will do by computing the cumulative sum of squared forecast error difference, CSSSED. As explained earlier in the paper, increasing values of CSSSED indicate that the factor model outperforms the benchmark model. Correspondingly, decreasing values means that the benchmark model predicts better.

Since we estimated several different models, we choose to comment on the forecast performance over time for particular models that we believe have some interesting intuitions. First, we start by analyzing the forecast performance for the models that,

according to the RMSFE measures, outperform the AR(2) model when predicting inflation.

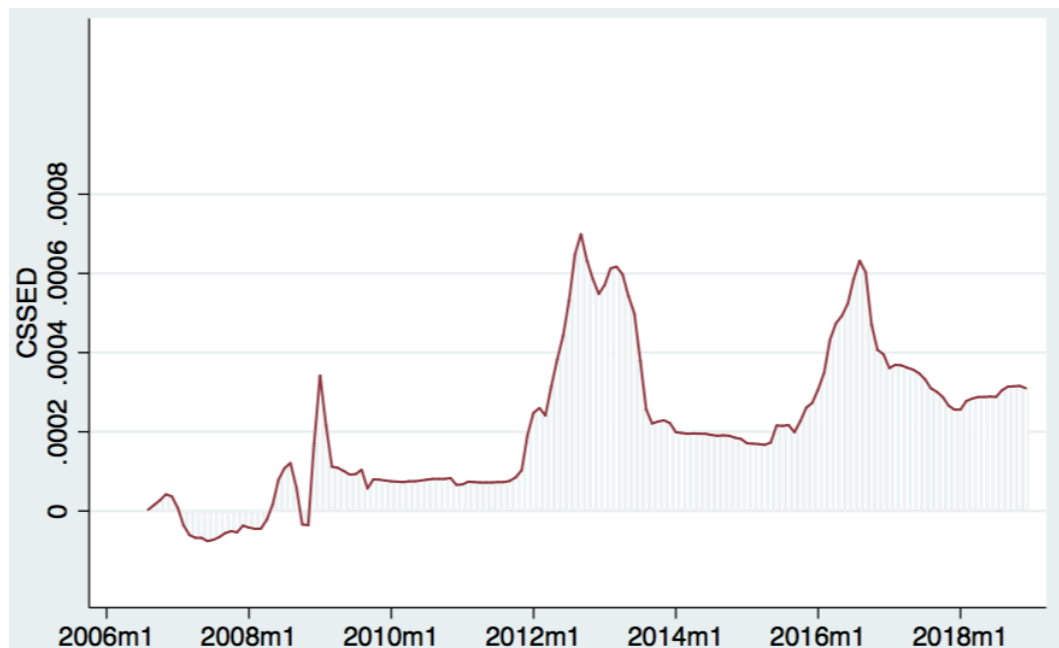
**Figure 7.3 – The CSSED of the model including the slope factor at six months horizon, forecasting inflation using two lags**



**Figure 7.3** displays the forecast performance over time for the model including the slope factor, forecasting inflation six months ahead. The CSSED starts to increase at the end of 2007 and continues to rise for about a year up to the end of 2008. This period includes the start of the financial crisis in Norway, in which inflation increased until the fourth quarter of 2008. In fact, in October 2008, the year-on-year rise in the consumer price index was as high as 5.4% before it decreased in the following periods. The model appears to predict the increase in inflation better than the autoregressive model. To be able to interpret these results, we need to explore the slope factor in the model in more detail. As mentioned earlier, the slope factor can be interpreted as the short-term factor and has been documented to be an excellent indicator to forecast GDP growth and monetary policy instruments, as well as recessions and expansions. By looking at the movements in the slope factor, our estimates show that the slope factor continued to increase until September 2008 when the crisis took a new and dramatic turn, due to the bankruptcy of the Lehman Brothers. An increase in the slope factor may indicate a forthcoming recession, which was the case for Norway during this period.

Although the slope factor helps predict inflation better than the benchmark model in certain periods, Kozicki (1997) finds that the level of yields is a more useful predictor of inflation. As mentioned earlier, the level factor has been proven to contain information about future inflation. **Figure 7.4** shows the overall forecast performance over time for the model including the level factor, at a forecasting horizon of three months.

**Figure 7.4 – The CSSED of the model including the level factor at three months horizon, forecasting inflation using two lags**



There are three periods, in which we see a significant increase in the CSSED followed by decreasing values of similar magnitude. The first period starts at the end of 2008. As substantiated when interpreting **figure 7.3**, this period refers to the financial crisis, which had a major impact on inflation. Another highly important element within this period is the exceptionally high price of oil. During the summer of 2008, the price of oil per barrel reached a peak compared to prior years, which was followed by a fear of a rise in future inflation. Together, these two events resulted in periods of highly volatile inflation in Norway. However, examining the movements of the level factor, our results do not show a clear pattern that indicates the level factor having a significant relevance in the forecasting of inflation during this period.

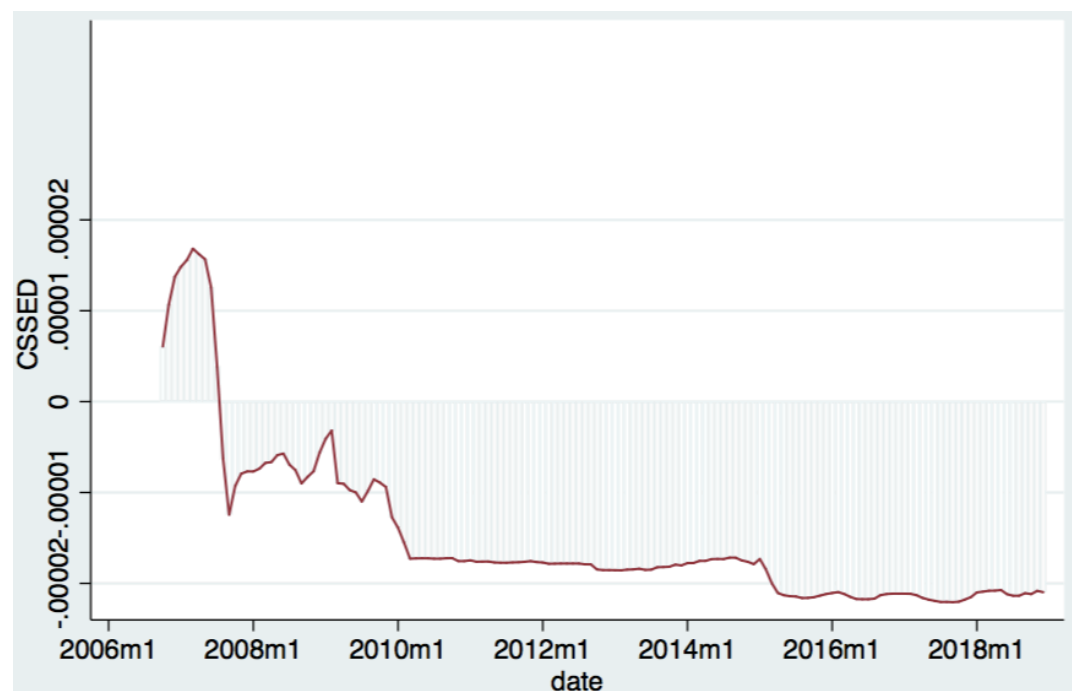
The next period containing an increase in the CSSSED starts at the end of 2011, continuing until the third quarter of 2012. The inflation in Norway was unusually low in 2012. One of the features of the period preceding this was a decrease in the interest rate, which occurred over the period 2011-2012. This made people consume more, resulting in a rise in inflation the upcoming year. Also, the price of electricity in Norway faced a downturn throughout the summer of 2012. This decrease is attributed to large volumes of rainfall prior to, and during, this period, in addition to lower consumption linked to the summer season. From our data, we can see a fairly consistent relationship when looking at the level estimates over the 2011-2012 period. The estimates show a decreasing trend, with the highest measure of the level factor at the start of 2011, reduced to a measure half the size at the end of 2012. This could reflect expectations of lower future inflation and may signify why the factor model outperforms the autoregressive model in this particular period.

The third peak in the CSSSED starts in 2016. An essential event preceding this period was the sudden decrease in the key policy rate in Norway, in which the rate got reduced to the historical low value of 0.75% during September 2015. In general, the reason why the key policy rate is adjusted down by Norges Bank is to make people consume more and thereby avoid too low economic activity and inflation relative to the inflation target. In 2015, the Norwegian oil industry experienced a significant slow-down due to the oil price decrease and the decline in oil investments in Norway, and the consequences started to appear. A rapid increase in unemployment and an economic activity lower than desired dominated the Norwegian economy. As a response to this, Norges Bank lowered the key policy rate, which resulted in a rise in inflation. Another event characterizing this period was the expensive electricity price. In 2016, Norway experienced an extraordinary increase in the electricity price, which occurred due to the developments in the Norwegian water reservoirs during the period, justified by abnormally low rainfalls (Dagens Næringsliv, 2017). Norwegian consumers of electricity paid on average 12.1% more in 2016 than what they did the previous year (Pedersen, 2019). This caused an increase in inflation. Therefore, the rise in inflation can, among other things, be seen in the context of both the reduction in the key policy rate and the increased electricity prices.

To interpret how the level factor attributes to this, suggesting that an increasing level factor indicates higher future inflation, we look closer at how the level factor developed before and during this period. Our estimates at the beginning of this phase could resemble a pattern, containing decreasing numbers of the level factor. However, we cannot see a fully consistent pattern to be able to claim that it exists a relation between the level factor and inflation in this particular case. Still, the positive CSSED during this period indicates that the factor model happens to predict the increase in inflation better than the benchmark model.

Now, we turn to the forecasts of unemployment and will continue the analysis by interpreting the CSSED of the model for the slope factor including four lags, forecasting unemployment three months ahead. This is one of the models that, according to the RMSFE measures, predicts unemployment as good as the AR(4) model. The CSSED is shown in **figure 7.5**.

**Figure 7.5 - The CSSED of the model including the slope factor at three months horizon, forecasting unemployment using four lags**



**Figure 7.5** illustrates the forecast performance over time for the slope factor at a three months horizon when forecasting unemployment. There are two periods, in which an increase in CSSED is very noticeable. The first interval lasts from the



beginning of the forecasting period until the beginning of 2007, while the second period persists from the last quarter in 2007 until the first quarter of 2009.

Unemployment is a result of the difference between labor supply and demand and is therefore strongly related to the economic situation within a country. An increase in demand usually leads to a rise in production and employment, and thereby a decrease in unemployment. Correspondingly, insufficient demand tends to cause a reduction in production, followed by an increase in unemployment. This was the case during the financial crisis in Norway, in which the economy faced a recession. Business sales and revenues decreased, which drove unemployment up. As the slope factor is known to be a good indicator of predicting both output growth and recessions, it also contains valuable information of future unemployment.

The yield curve is typically upward sloping. Many empirical studies have found, however, that most recessions are preceded by a sharp decline in the slope of the yield curve and often by an inversion of the yield curve (i.e., by short-term yields exceeding long-term yields) (Wheelock and Wohar, 2009). This could be related to the fact that individuals prefer stable consumption over time, which is the central assumption of Harvey (1988). The desire to hedge will, therefore, lead consumers to buy long-term securities and sell short-term securities when expecting a recession in the future. The increased demand for long-term securities will increase the respective price and lead to a decrease in the yield to maturity. Simultaneously, the selling pressure of short-term securities cause a reduction in the respective price and, as a result of this, a rise in yields. Hence, if a recession is expected, long rates will decrease while short rates increase, resulting in flat or inverted yield curves. This is reflected in our data in the period 2007-2008, in which yields with one-month maturity exceeds yields with a ten-year maturity, and the slope factor changes sign. Therefore, the change in the slope factor can indicate a forthcoming recession and, thereby, a rise in unemployment. This could be the reason for why our model predicts unemployment better than the benchmark model in the period preceding the financial crisis. However, Norway had a very mild recession relative to other countries, and unemployment was kept below 4% also in the years following the crisis.

By examining the movements of the slope factor and unemployment in Norway during the first interval, there is no clear relationship that corresponds to the increase in CSSED. Although the values of CSSED look very volatile up to 2010, it is important to point out that the differences are very small, reflected in the y-axis. An interesting aspect of **figure 7.5** is, however, that it seems to be irrelevant whether or not to include the slope factor in the model in the period of 2010-2018, with the exception of 2015.

A model that outperforms the AR(4) according to the RMSFE measures is the model including the curvature factor using four lags, forecasting unemployment three months ahead. The CSSED regarding this model is shown in **figure 7.6**.

**Figure 7.6 - The CSSED of the model including the curvature factor at three months horizon, forecasting unemployment using four lags**



The figure shows an increase in the CSSED starting in 2008 and lasts until the latter part of 2009. This indicates a period in which the model including the curvature factor has a better performance than the autoregressive model when forecasting unemployment three months ahead. The increase in the CSSED may have a relevance to the financial crisis in 2008, which was followed by a recession and thereby a rise in unemployment. Including information of interest rates may improve the forecast performance in this specific period since the interest rates

happened to be volatile during the crisis in 2008. To substantiate this, we look at how the curvature estimates develop relative to unemployment. As mentioned earlier, Aguiar-Conraria, Martins, and Soares (2012) finds that increasing measures of the curvature factor relates to lagged decreases in unemployment. From our estimates, we see that reductions in the curvature factor move in line with increased unemployment, which could justify why our factor model performs better than the benchmark in this particular period.

As we can see from the figure, the CSSED also increases in the period from 2010 to the first part of 2011. This period is characterized by the European debt crisis, which started in 2009 before it got more severe at the beginning of 2010. The crisis occurred as an effect of the financial crisis, resulting in several European countries being unable to repay their government debt. The European debt crisis, therefore, led many countries within the European Union (EU) into a recession, followed by a sudden increase in unemployment. The European Central Bank (ECB) contributed to solve the crisis by providing cheap loans and lower the interest rates, aiming to minimize the impacts of the recession. Although Norway is not a member of the EU, Norway was impacted by the crisis in several ways. Some of the affected EU-countries were important trading partners to Norway, which was the reason why the crisis indirectly caused a slowdown in the Norwegian economy, followed by an increase in unemployment. One of the main impacts on the economy was the global downturn in the oil price due to reduced investments and exports. As mentioned earlier, the curvature factor can be interpreted as the medium-term component of the yield curve. By looking at our data, the medium-term interest rates were reduced over the interval from 2010-2012. However, our estimates of the curvature factor do not show an evident pattern, which implies that we cannot see any clear connection between unemployment and the curvature factor during this period.

## 8 Conclusion

This paper addresses the task of forecasting inflation and unemployment using the term structure of interest rates. Recent studies have shown that movements in the yield curve reflect expectations regarding future economic conditions and that it in some way leads the macroeconomic variables. To investigate this, we developed and estimated a factor model by using the three Nelson-Siegel factors level, slope, and curvature. Then we compared the forecasting performance of the factor model relative to an autoregressive benchmark model, AR(p), by computing the RMSFE and CSSED.

From the comparison of the two models, we find that the factor model does not outperform the benchmark model when including all three factors. This applies for both inflation and unemployment at all forecast horizons used in our analysis. However, when analyzing the forecast performance of each individual factor, especially the curvature factor stands out in the case of forecasting unemployment. For instance, the model including the curvature factor beats the benchmark model when forecasting unemployment at all horizons. Regarding the forecasting of inflation, the model including either the level- or the slope factor beats the benchmark model at certain horizons.

Lastly, by looking at the forecast performance over time, we find that our model performs better than the benchmark model in certain periods of time. Especially around the time of the financial crisis in Norway, including information of the yield curve will improve the out-of-sample forecasts of inflation and unemployment. However, all of the increases in the CSSED lasts for a considerably short time, which may be the reason why our model does not show an overall better performance when including all the three factors.

Regarding our research, the results we have found may be coincidences and lack some important aspects. Looking at a larger dataset, containing several financial crises or volatile periods, may lead to different results. Also, future research of this topic should include GDP in their analysis to have a wider ranch of macroeconomic variables. This may increase the factors' predictive power.

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## 10 Appendix

### *Estimation of the Nelson-Siegel Model*

```
clear all

EstimationData = xlsread('YieldData.xlsx')
EOMDates = xlsread('Dates.xlsx')
```

#### Plot

```
lambda_t = .0609
TimeToMat = [1 3 6 12 24 36 48 60 120]';
X = [ones(size(TimeToMat)) (1 - exp(-
lambda_t*TimeToMat))./(lambda_t*TimeToMat) ...
((1 - exp(-lambda_t*TimeToMat))./(lambda_t*TimeToMat) - exp(-
lambda_t*TimeToMat))]
plot(TimeToMat,X,'LineWidth',2)
xlabel('Maturity (months)', 'fontsize',18,'fontweight','bold')
ylabel('Factor loading', 'fontsize',18,'fontweight','bold')
ylim([0 1.1])
legend({'Level', 'Slope', 'Curvature'}, 'location', 'east', 'fontsize',20)
```

#### Extracting level, slope and curvature

```
Beta = zeros(size(EstimationData,1),3);

for jdx = 1:size(EstimationData,1)
    tmpCurveModel =
    DieboldLi.fitBetasFromYields(EOMDates(jdx),lambda_t*12,daysadd(EOMDates(j
dx),30*TimeToMat),EstimationData(jdx,:));
    Beta(jdx,:) = [tmpCurveModel.Beta1 tmpCurveModel.Beta2
tmpCurveModel.Beta3];
end
```

### *Out-of-sample Forecasts*

```
% Inflation (similar codes for unemployment)
clear;
close all;

% Load data
Inflation = xlsread('Inflation.xlsx');
```

#### AIC and BIC

```
p = 8;
Bic = NaN(p,1);
Aic = NaN(p,1);

for i = 1:p
    [yt, xt] = constructArModel(Inflation,i, 1, 213);
    [~, betas, errors] = estimateArModel(yt(1+i:end),xt(1+i:end,:));
    Bic(i) = bic(errors,i);
    Aic(i) = aic(errors,i);
    if i == p
        AICBIC = [Bic Aic];
    end
end

% The number of 2 lags is chosen based on both aic and bic
```

**Factor model – 2 lags**

```

clear;
close all

macro = readtable('betasmacro.xlsx');

Inflation =
fints(datenum(macro.Date, 'yyyymm'),macro.Inflation, 'Inflation', 'M');
Level = fints(datenum(macro.Date, 'yyyymm'),macro.Beta1, 'Beta1', 'M');
Slope = fints(datenum(macro.Date, 'yyyymm'),macro.Beta2, 'Beta2', 'M');
Curvature = fints(datenum(macro.Date, 'yyyymm'),macro.Beta3, 'Beta3', 'M');

X = [fts2mat(lagts(Level)) fts2mat(lagts(Slope))
fts2mat(lagts(Curvature)) fts2mat(lagts(Inflation))
fts2mat(lagts(Inflation,2))];
y = fts2mat(Inflation);

% OLS for 60 observations at a time
windowlength = 60;
n = length(y);
p = n - windowlength;
betaHatAlle = [];
Forecasts1 = [];
Forecasts2 = [];
Forecasts3 = [];
Forecasts4 = [];
Forecasts5 = [];
Forecasts6 = [];
Forecasts7 = [];
Forecasts8 = [];
Forecasts9 = [];
Forecasts10 = [];
Forecasts11 = [];
Forecasts12 = [];

for i = (3:p);

    % Forecast 1 step ahead
    lmK = fitlm(X(i:(windowlength+i-1), :),y(i:(windowlength+i-1)));
    betaHat = lmK.Coefficients.Estimate;
    X1 = X(i+windowlength, 1);
    X2 = X(i+windowlength, 2);
    X3 = X(i+windowlength, 3);
    X4 = X(i+windowlength, 4);
    X5 = X(i+windowlength, 5);
    Yhat1 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
betaHat(5)*X4 + betaHat(6)*X5;
    Forecasts1 = [Forecasts1, Yhat1];
    betaHatAlle = [betaHatAlle, betaHat];

    % 2 steps ahead
    Yhat2 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
betaHat(5)*Yhat1 + betaHat(6)*X4;
    Forecasts2 = [Forecasts2, Yhat2];

    % 3 steps ahead
    Yhat3 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
betaHat(5)*Yhat2 + betaHat(6)*Yhat1;
    Forecasts3 = [Forecasts3, Yhat3];

    % 4 steps ahead
    Yhat4 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
betaHat(5)*Yhat3 + betaHat(6)*Yhat2;
    Forecasts4 = [Forecasts4, Yhat4];

    % 5 steps ahead
    Yhat5 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
betaHat(5)*Yhat4 + betaHat(6)*Yhat3;
    Forecasts5 = [Forecasts5, Yhat5];

    % 6 steps ahead

```

```

    Yhat6 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat5 + betaHat(6)*Yhat4;
    Forecasts6 = [Forecasts6, Yhat6];

    % 7 steps ahead
    Yhat7 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat6 + betaHat(6)*Yhat5;
    Forecasts7 = [Forecasts7, Yhat7];

    % 8 steps ahead
    Yhat8 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat7 + betaHat(6)*Yhat6;
    Forecasts8 = [Forecasts8, Yhat8];

    % 9 steps ahead
    Yhat9 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat8 + betaHat(6)*Yhat7;
    Forecasts9 = [Forecasts9, Yhat9];

    % 10 steps ahead
    Yhat10 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat9 + betaHat(6)*Yhat8;
    Forecasts10 = [Forecasts10, Yhat10];

    % 11 steps ahead
    Yhat11 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat10 + betaHat(6)*Yhat9;
    Forecasts11 = [Forecasts11, Yhat11];

    % 12 steps ahead
    Yhat12 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3 +
    betaHat(5)*Yhat11 + betaHat(6)*Yhat10;
    Forecasts12 = [Forecasts12, Yhat12];

    End

```

## Root Mean Squared Forecast Error

```

% 1 step ahead
TrueValue1 = y(63:213);
ForecastError1 = TrueValue1 - Forecasts1';
RMSFE1 = sqrt(sum(ForecastError1.^2)/(length(ForecastError1)+1))

% 3 steps ahead
TrueValue3 = y(65:213);
ForecastError3 = TrueValue3 - Forecasts3(1:end-3+1)';
RMSFE3 = sqrt(sum(ForecastError3.^2)/(length(ForecastError3)+1))

% 6 steps ahead
TrueValue6 = y(68:213);
ForecastError6 = TrueValue6 - Forecasts6(1:end-6+1)';
RMSFE6 = sqrt(sum(ForecastError6.^2)/(length(ForecastError6)+1))

% 12 steps ahead
TrueValue12 = y(74:213);
ForecastError12 = TrueValue12 - Forecasts12(1:end-12+1)';
RMSFE12 = sqrt(sum(ForecastError12.^2)/(length(ForecastError12)+1))

```

## AR(2)

```

clear;
close all;

macro = readtable('betasmacro.xlsx');

Inflation =
fints(datenum(macro.Date, 'yyyymm'), macro.Inflation, 'Inflation', 'M');
Level = fints(datenum(macro.Date, 'yyyymm'), macro.Beta1, 'Beta1', 'M');
Slope = fints(datenum(macro.Date, 'yyyymm'), macro.Beta2, 'Beta2', 'M');
Curvature = fints(datenum(macro.Date, 'yyyymm'), macro.Beta3, 'Beta3', 'M');

```

```

y = fts2mat(Inflation);
LX = fts2mat(lagts(Inflation));
L2X = fts2mat(lagts(Inflation,2));
X = [LX L2X];

windowlength = 60;
n = length(y);
p = n - windowlength;
betaHatAlle = [];
Forecasts1 = [];
Forecasts2 = [];
Forecasts3 = [];
Forecasts4 = [];
Forecasts5 = [];
Forecasts6 = [];
Forecasts7 = [];
Forecasts8 = [];
Forecasts9 = [];
Forecasts10 = [];
Forecasts11 = [];
Forecasts12 = [];

for i = (3:p);

    % 1 step ahead
    lmK = fitlm(X(i:(windowlength+i-1), :),y(i:(windowlength+i-1)));
    betaHat = lmK.Coefficients.Estimate;
    betaHatAlle = [betaHatAlle, betaHat];
    X1 = X(i+windowlength, 1);
    X2 = X(i+windowlength, 2);
    Yhat1 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2;
    Forecasts1 = [Forecasts1, Yhat1];

    % 2 steps ahead
    Yhat2 = betaHat(1) + betaHat(2)*Yhat1 + betaHat(3)*X1;
    Forecasts2 = [Forecasts2, Yhat2];

    % 3 step ahead
    Yhat3 = betaHat(1) + betaHat(2)*Yhat2 + betaHat(3)*Yhat1;
    Forecasts3 = [Forecasts3, Yhat3];

    % 4 steps ahead
    Yhat4 = betaHat(1) + betaHat(2)*Yhat3 + betaHat(3)*Yhat2;
    Forecasts4 = [Forecasts4, Yhat4];

    % 5 steps ahead
    Yhat5 = betaHat(1) + betaHat(2)*Yhat4 + betaHat(3)*Yhat3;
    Forecasts5 = [Forecasts5, Yhat5];

    % 6 steps ahead
    Yhat6 = betaHat(1) + betaHat(2)*Yhat5 + betaHat(3)*Yhat4;
    Forecasts6 = [Forecasts6, Yhat6];

    % 7 steps ahead
    Yhat7 = betaHat(1) + betaHat(2)*Yhat6 + betaHat(3)*Yhat5;
    Forecasts7 = [Forecasts7, Yhat7];

    % 8 steps ahead
    Yhat8 = betaHat(1) + betaHat(2)*Yhat7 + betaHat(3)*Yhat6;
    Forecasts8 = [Forecasts8, Yhat8];

    % 9 steps ahead
    Yhat9 = betaHat(1) + betaHat(2)*Yhat8 + betaHat(3)*Yhat7;
    Forecasts9 = [Forecasts9, Yhat9];

    % 10 steps ahead
    Yhat10 = betaHat(1) + betaHat(2)*Yhat9 + betaHat(3)*Yhat8;
    Forecasts10 = [Forecasts10, Yhat10];

    % 11 steps ahead
    Yhat11 = betaHat(1) + betaHat(2)*Yhat10 + betaHat(3)*Yhat9;
    Forecasts11 = [Forecasts11, Yhat11];

```

```

% 12 steps ahead
Yhat12 = betaHat(1) + betaHat(2)*Yhat11 + betaHat(3)*Yhat10;
Forecasts12 = [Forecasts12, Yhat12];

End

```

### Root Mean Squared Forecast Error

```

% 1 step ahead
TrueValue1 = y(63:213);
ForecastError1 = TrueValue1 - Forecasts1';
RMSFE1 = sqrt(sum(ForecastError1.^2)/(length(ForecastError1)+1))

% 3 steps ahead
TrueValue3 = y(65:213);
ForecastError3 = TrueValue3 - Forecasts3(1:end-3+1)';
RMSFE3 = sqrt(sum(ForecastError3.^2)/(length(ForecastError3)+1))

% 6 steps ahead
TrueValue6 = y(68:213);
ForecastError6 = TrueValue6 - Forecasts6(1:end-6+1)';
RMSFE6 = sqrt(sum(ForecastError6.^2)/(length(ForecastError6)+1))

% 12 steps ahead
TrueValue12 = y(74:213);
ForecastError12 = TrueValue12 - Forecasts12(1:end-12+1)';
RMSFE12 = sqrt(sum(ForecastError12.^2)/(length(ForecastError12)+1))

```

### Forecasting inflation using a single factor

```

% Level (similar codes for slope and curvature)

clear;
close all;

macro = readtable('betasmacro.xlsx');

Inflation =
fints(datumum(macro.Date, 'yyyymm'), macro.Inflation, 'Inflation', 'M');
Level = fints(datumum(macro.Date, 'yyyymm'), macro.Beta1, 'Beta1', 'M');
Slope = fints(datumum(macro.Date, 'yyyymm'), macro.Beta2, 'Beta2', 'M');
Curvature = fints(datumum(macro.Date, 'yyyymm'), macro.Beta3, 'Beta3', 'M');

X = [fts2mat(lagts(Level)) fts2mat(lagts(Inflation))
fts2mat(lagts(Inflation, 2))];
y = fts2mat(Inflation);

% OLS for 60 observations at a time
windowlength = 60;
n = length(y);
p = n - windowlength;
betaHatAlle = [];
Forecasts1 = [];
Forecasts2 = [];
Forecasts3 = [];
Forecasts4 = [];
Forecasts5 = [];
Forecasts6 = [];
Forecasts7 = [];
Forecasts8 = [];
Forecasts9 = [];
Forecasts10 = [];
Forecasts11 = [];
Forecasts12 = [];

for i = (3:p);

% Forecast 1 step ahead
lmK = fitlm(X(i:(windowlength+i-1), :), y(i:(windowlength+i-1)));
betaHat = lmK.Coefficients.Estimate;

```

```

X1 = X(i>windowlength, 1);
X2 = X(i>windowlength, 2);
X3 = X(i>windowlength, 3);
Yhat1 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3;
Forecasts1 = [Forecasts1, Yhat1];
betaHatAlle = [betaHatAlle, betaHat];

% 2 steps ahead
Yhat2 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat1 +
betaHat(4)*X2;
Forecasts2 = [Forecasts2, Yhat2];

% 3 steps ahead
Yhat3 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat2 +
betaHat(4)*Yhat1;
Forecasts3 = [Forecasts3, Yhat3];

% 4 steps ahead
Yhat4 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat3 +
betaHat(4)*Yhat2;
Forecasts4 = [Forecasts4, Yhat4];

% 5 steps ahead
Yhat5 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat4 +
betaHat(4)*Yhat3;
Forecasts5 = [Forecasts5, Yhat5];

% 6 steps ahead
Yhat6 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat5 +
betaHat(4)*Yhat4;
Forecasts6 = [Forecasts6, Yhat6];

% 7 steps ahead
Yhat7 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat6 +
betaHat(4)*Yhat5;
Forecasts7 = [Forecasts7, Yhat7];

% 8 steps ahead
Yhat8 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat7 +
betaHat(4)*Yhat6;
Forecasts8 = [Forecasts8, Yhat8];

% 9 steps ahead
Yhat9 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat8 +
betaHat(4)*Yhat7;
Forecasts9 = [Forecasts9, Yhat9];

% 10 steps ahead
Yhat10 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat9 +
betaHat(4)*Yhat8;
Forecasts10 = [Forecasts10, Yhat10];

% 11 steps ahead
Yhat11 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat10 +
betaHat(4)*Yhat9;
Forecasts11 = [Forecasts11, Yhat11];

% 12 steps ahead
Yhat12 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat11 +
betaHat(4)*Yhat10;
Forecasts12 = [Forecasts12, Yhat12];

end

```

### Root Mean Squared Forecast Error

```

% 1 step ahead
TrueValue1 = y(63:213);
ForecastError1 = TrueValue1 - Forecasts1';
RMSFE1 = sqrt(sum(ForecastError1.^2)/(length(ForecastError1)+1))

% 3 steps ahead

```

```

TrueValue3 = y(65:213);
ForecastError3 = TrueValue3 - Forecasts3(1:end-3+1)';
RMSFE3 = sqrt(sum(ForecastError3.^2)/(length(ForecastError3)+1))

% 6 steps ahead
TrueValue6 = y(68:213);
ForecastError6 = TrueValue6 - Forecasts6(1:end-6+1)';
RMSFE6 = sqrt(sum(ForecastError6.^2)/(length(ForecastError6)+1))

% 12 steps ahead
TrueValue12 = y(74:213);
ForecastError12 = TrueValue12 - Forecasts12(1:end-12+1)';
RMSFE12 = sqrt(sum(ForecastError12.^2)/(length(ForecastError12)+1))

```

## *Forecast Performance Over Time*

```

% 2 lags - Factor model with a single factor
% Level (similar codes for slope and curvature)

clear;
close all;

macro = readtable('betasmacro.xlsx');

Inflation =
fints(datenum(macro.Date, 'yyyymm'), macro.Inflation, 'Inflation', 'M');
Level = fints(datenum(macro.Date, 'yyyymm'), macro.Betal, 'Betal', 'M');

X = [fts2mat(lagts(Level)) fts2mat(lagts(Inflation))
fts2mat(lagts(Inflation, 2))];
y = fts2mat(Inflation);

% OLS for 60 observations at a time
windowlength = 60;
n = length(y);
p = n - windowlength;
betaHatAlle = [];
Forecasts1 = [];
Forecasts2 = [];
Forecasts3 = [];
Forecasts4 = [];
Forecasts5 = [];
Forecasts6 = [];

for i = (3:p);

    % Forecast 1 step ahead
    lmK = fitlm(X(i:(windowlength+i-1), :), y(i:(windowlength+i-1)));
    betaHat = lmK.Coefficients.Estimate;
    X1 = X(i+windowlength, 1);
    X2 = X(i+windowlength, 2);
    X3 = X(i+windowlength, 3);
    Yhat1 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*X2 + betaHat(4)*X3;
    Forecasts1 = [Forecasts1, Yhat1];
    betaHatAlle = [betaHatAlle, betaHat];

    % 2 steps ahead
    Yhat2 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat1 +
betaHat(4)*X2;
    Forecasts2 = [Forecasts2, Yhat2];

    % 3 steps ahead
    Yhat3 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat2 +
betaHat(4)*Yhat1;
    Forecasts3 = [Forecasts3, Yhat3];

    % 4 steps ahead
    Yhat4 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat3 +
betaHat(4)*Yhat2;

```



```

Forecasts4 = [Forecasts4, Yhat4];

% 5 steps ahead
Yhat5 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat4 +
betaHat(4)*Yhat3;
Forecasts5 = [Forecasts5, Yhat5];

% 6 steps ahead
Yhat6 = betaHat(1) + betaHat(2)*X1 + betaHat(3)*Yhat5 +
betaHat(4)*Yhat4;
Forecasts6 = [Forecasts6, Yhat6];

end

```

## AR(2)

```

XAR = [fts2mat(lagts(Inflation)) fts2mat(lagts(Inflation,2))];

betaHatAlleAR = [];
Forecasts1AR = [];
Forecasts2AR = [];
Forecasts3AR = [];
Forecasts4AR = [];
Forecasts5AR = [];
Forecasts6AR = [];

for i = (3:p);

% 1 step ahead
lmKAR = fitlm(XAR(i:(windowlength+i-1), :), y(i:(windowlength+i-1)));
betaHatAR = lmKAR.Coefficients.Estimate;
betaHatAlleAR = [betaHatAlleAR, betaHatAR];
X1AR = XAR(i+windowlength, 1);
X2AR = XAR(i+windowlength, 2);
Yhat1AR = betaHatAR(1) + betaHatAR(2)*X1AR + betaHatAR(3)*X2AR;
Forecasts1AR = [Forecasts1AR, Yhat1AR];

% 2 steps ahead
Yhat2AR = betaHatAR(1) + betaHatAR(2)*Yhat1AR + betaHatAR(3)*X1AR;
Forecasts2AR = [Forecasts2AR, Yhat2AR];

% 3 steps ahead
Yhat3AR = betaHatAR(1) + betaHatAR(2)*Yhat2AR + betaHatAR(3)*Yhat1AR;
Forecasts3AR = [Forecasts3AR, Yhat3AR];

% 4 steps ahead
Yhat4AR = betaHatAR(1) + betaHatAR(2)*Yhat3AR + betaHatAR(3)*Yhat2AR;
Forecasts4AR = [Forecasts4AR, Yhat4AR];

% 5 steps ahead
Yhat5AR = betaHatAR(1) + betaHatAR(2)*Yhat4AR + betaHatAR(3)*Yhat3AR;
Forecasts5AR = [Forecasts5AR, Yhat5AR];

% 6 steps ahead
Yhat6AR = betaHatAR(1) + betaHatAR(2)*Yhat5AR + betaHatAR(3)*Yhat4AR;
Forecasts6AR = [Forecasts6AR, Yhat6AR];

end

```

## Forecast error

```

% 1 step ahead
TrueValue1 = y(63:213);
ForecastError1 = TrueValue1 - Forecasts1';
ForecastError1AR = TrueValue1 - Forecasts1AR';

% 3 steps ahead
TrueValue3 = y(65:213);
ForecastError3 = TrueValue3 - Forecasts3(1:end-3+1)';
ForecastError3AR = TrueValue3 - Forecasts3AR(1:end-3+1)';

```

```

% 6 steps ahead
TrueValue6 = y(68:213);
ForecastError6 = TrueValue6 - Forecasts6(1:end-6+1)';
ForecastError6AR = TrueValue6 - Forecasts6AR(1:end-6+1)';

```

### CSSED - 3 steps ahead

```

SED3 = (ForecastError3AR.^2-ForecastError3.^2);
CSSED3cumsum = cumsum(SED3);

% Cumulative sum plot
k = (1:length(SED3))';
Dates = (2006+8/12:1/12:2018+12/12)';
subplot(1,2,1); plot(Dates,SED3); axis('tight');
subplot(1,2,2); plot(Dates,CSSED3cumsum); axis('tight');

% import to excel
xlswrite('CSSED3cumsum.xls',CSSED3cumsum)

```

### CSSED - 6 steps ahead

```

SED6 = (ForecastError6AR.^2-ForecastError6.^2);
CSSED6cumsum = cumsum(SED6);

% Cumulative sum plot
k = (1:length(SED6))';
Dates = (2006+11/12:1/12:2018+12/12)';
subplot(1,2,1); plot(Dates,SED6); axis('tight');
subplot(1,2,2); plot(Dates,CSSED6cumsum); axis('tight');

```