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The Efficiency of Structured Equity Products as Investment Vehicles

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The Efficiency of Structured Equity Products as Investment Vehicles

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Abstract

In this paper we find that in the Swedish market principal protected notes (PPN) or an alternative replicating strategy of such kind, can be an adequate investment vehicle for retail investors with different sources of liquidity in periods of stable volatility and low interest rates. We assessed the fair value of such notes and compared the offered participation rates by the issuing financial institutions and found tendencies of overpricing for the index PPN at issuance. Our results illustrate how in addition of the capital protection, this type of structured products can be as profitable or more than other alternative investing choices.

Table of contents

List of tables and figures

1 Introduction

Structured products have existed in Europe since the 1970s and were developed in response to investors demand for achieving risk-return objectives and for issuers risk distribution needs (Beder & Marshall, 2011). More recently these types of securities have become increasingly popular in the United States and Asia. In the 4th 2018 Nordic conference for structured products and derivatives, the data presented evidence of a global tendency to an increasing market for structured products.

Structured products are investment vehicles where you pre-package two or more financial asset classes together to be comprised as a single pay-out structure. The market of structured products has undergone much criticism especially in the aftermath of the financial crisis in 2008, where major investors and financial institutions were affected by a special kind of structured products backed by credit and mortgage obligations such as CDO and MBS, which derived in stricter regulation for all types of structured products. In countries like Norway new regulations were introduced after the crisis which in practice constrained issuers to commercialise structured products to retail investors. In Sweden structured product issuers need to comply with regulations such as MiFIDǁ and PRIIP. While MiFIDǁ covers the general trading amongst financial products, PRIIP is mostly directed against the marketing of structured products to retail investors.

There are several different types of structured products in the market, among them are the equity index linked notes. Equity index linked notes are divided into two different type of notes: Principal protected and yield enhancement notes. Principal protected note (PPN) is an instrument with fixed income security where the interest coupon or principal is linked to movements in equity market indexes. This type of instrument is popular among retail investors since it can enable them to create a differentiated exposure to an entire index with relatively low capital investment at a small transaction cost.

The investment motives for this type of products arise from an environment of low-interest rates and the demand from retail investors to generate higher returns. Another important motive is a “rule buster”, which takes views on markets where the asset class is not available to the interested party, either for regulatory or market motives. The third reason for buying this type of product is simply because the investor wants to limit the risk exposure of their equity investment.

Our main objective is to determine if equity linked products in the Swedish market are convenient investment instruments, and if they are priced fairly at primary issuance. We look at the Swedish market of structured products since among the Nordic countries, Sweden has the largest market. Principal protected notes are the most common structured product investment in Sweden, we therefore investigate if those products, offer an adequate risk adjusted return to retail investors. We determined the return of expired PPN and investigate if they were fairly priced at issuance. This was done by setting up a Monte Carlo pricing model, following a geometric Brownian motion. The volatility parameter in the model was generated from an EGARCH (1,1). Furthermore, we calculated the compounded annualized returns and compare it to different benchmarks, and thus, determine if PPNs in the Swedish market are an efficient investment vehicle for retail investors.

From our research we find that investing in PPN has been as profitable or more than investing in similar alternative investment strategies, in periods of stable volatility and low interest rates. Retail investors with certain characteristics such as low liquidity needs and limited access to derivative products, could benefit from investing in PPNs. Should their market views and investment characteristics be appropriate, investors with the possibility to invest in these products should consider the opportunity.

The analysis of our research question is further divided into five more sections. Section 2 contains the literature review covering the topic of our research question. Section 3 shows the relevant data used for this purpose. Section 4 contains a detailed description of the methodology and theory used in our thesis. Section 5 shows the empirical evidence and the analysis of our results. Ultimately, section 6 contains the conclusion of our research and recommendations about further research.

2 Literature Review

Previous research from different countries on structured products pricing suggest, that these kinds of instruments are generally overpriced in the market. Most of the research on the matter was performed before the financial crisis in 2008 and has been mostly stalled ever since. In parallel, the structured products market size stagnated due to exacerbated fears and prejudices about the use of these instruments. Recently, the market has started to show some revitalized interest in these sophisticated asset classes, which makes it a compelling moment to resume its study.

Jørgensen, Nørholm, & Skovmand (2011) look at the price efficiency and cost structure for the Danish retail market of principal protected notes (PPNs). They find that on average the PPNs are 6% overpriced and that only half of that overpricing is disclosed by the sellers at the time of issuance (hidden costs). The writers of the paper also find that the degree of overpricing has declined over time but not the hidden costs. To come to this conclusion, they sum the present value of the bond element and use an extension of Black and Scholes to determine the price of the option element in the structure of the principal protected note.

We can see further that structured products overpricing is not exclusive to the Nordic market. Benet, Giannetti & Pissaris (2006) conclude in their paper that reverse exchangeable securities are generally overpriced, and that there is a marked bias in the pricing of these products, in favour of the issuing financial institution in the United States. Additionally Chen & Wu (2007) were testing the pricing of bullish underlying linked securities (equity linked notes, with a similar structure to principal protected notes) in the US market, and concluded that BULS issued in 2001 were overpriced during seasonal periods (the day after issuance and four months forward) but fairly priced afterwards.

We find similar patterns in other major European countries such as Germany and Switzerland, where extensive research on this matter has been done due to the large market for structured products in that region. Stoimenov & Wilkens (2005) look at the German market for equity linked structured products and find that in the primary market on average instruments of this sort are overpriced at issuance, a clear detriment for investors who choose to hold their position until maturity. Their explanation for this is that the degree of overpricing is related to the hedging costs from the issuers. The same pattern of overpricing can be detected in the Swiss and Dutch market for structured products in research by Wohlwend, Burth & Kraus (2001), Wohlwend & Grünbichler (2003), and Szymanowska, Horst, & Veld (2009).

To assess the efficiency and profitability of an investment instrument, we cannot limit in pricing considerations at issuance, but we need to analyse the return achieved by the instrument. In an article by Henderson & Pearson (2011) they provide analysis on structured equity product SPARQS and its initial pricing and return behaviour. They provided evidence that the expected return of these asset is lower than the risk-free return. Their explanation is that this is due to a large overpricing of the SPARQS (8%) and that the call option is of short-term. Edwards & Swidler (2005), provide evidence that equity linked certificates of deposit in a sample period ranging from December 1981-2004 almost generates the same average return as the American treasury bill, even when these are much riskier. The standard deviation is almost 65% higher than the treasury bill.

After the examination of earlier research, we expect that the structured products in the Swedish market will be overpriced at issuance. Furthermore, we expect that the structured notes will generate a similar or lower return as other less risky asset classes. By investigating and testing the research question that structured products in the Swedish market are correctly priced at issuance and if they generate a fair return. We will also be able to determine if the Swedish market for structured products shares the same characteristics as other larger markets. Individual research has been performed on either pricing or performance of structured products. Only Henderson & Pearson (2011), are incorporating both pricing and performance in their research. To our knowledge there has not been an exhaustive examination of pricing and performance of structured products in the Swedish market, which will differentiate our thesis report from earlier research in the area.

3 Data

In this section of the thesis we describe the data collected and used to perform the research. A principal protected note is a so-called capital protected investment with the underlying of a fixed income bond where the coupon or return is forgone and utilized to buy a call option usually written on an index. The calculation of the option is the most complex part in our pricing model and parameters such as volatility, risk-free rate and dividend can be hard to determine.

3.1 Principal protected notes

The Swedish market has two main types of principal protected notes. The first is a safe product, where the whole investment is capital protected and the investor can expect to get the nominal amount back at time to maturity. And the other is a riskier product, since it is sold at a premium and only the nominal is capital protected. The riskier feature will accelerate the participation rate since the structure of the product allows to incorporate, or buy, more options on the underlying index. Hence, it increases the end value of the PPN if the index experiences a positive development from the start value.

The return on the investment will depend on the underlying index the call option is written on. It will also depend on the participation rate of the investment. If the index's development is negative, the investor will only retain the full nominal amount. Hence there will be no return on the investment. If the development of the index is positive the investor will retain the full nominal amount and a percentage of the positive development of the index, which is determined by the participation rate.

Most of the Swedish principal protected notes market experience a feature where the call option is European with an Asian tail. That means that the end value of the index is calculated as the arithmetic average over specified dates. Usually the measure time is one year before maturity with prespecified dates each month. The general form for this type of feature on the principal protected notes is:

$$PPN(T) = N + N * \varphi * \max\left(\frac{\frac{1}{M}\sum_{i=1}^{M} S_{ti} - S_0}{S_0}, 0\right) \qquad (3.1)$$

Where N is the nominal amount, φ is the participation rate, M is the number of prespecified dates, t is time and S_0 is the start value of the index.

The characteristics of each of the PPNs is obtained from the website for structured products from each of the four main banks in Sweden. We collected information from 40 different expired PPN issued in the Swedish market between 2011 and 2015, 19 are safe and 21 are risky. For each of the safe notes, there is usually an equivalent risky note with the same embedded option. A large part of the sample 65% involves PPNs with an underlying that is denominated in a different currency than the domestic currency SEK, and thereby have returns that are also dependent on the currency fluctuations. More than half of the sample 55 % where issued in the year 2014.

3.2 Volatility Modelling

The determination of the volatility estimate is the most complex parameter to add to the model. The greatest available approximation for the volatility measure is the implied volatility. But this type of measure is not available to us, since there are no call options written on the indices with maturities matching the PPNs. Instead it is possible to use the historical volatility or more sophisticated time-series model. According to (Brooks, 2014) the usage of a more sophisticated time-series model to determine the volatility usually gives a more accurate option value. Hence, we chose GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1) to forecast future volatility.

The GARCH-model with the best goodness of fit is chosen to forecast volatility. It is determined by using Akaike and Bayesian information criterion. In the determination of which model fits the data best we choose the GARCH model with the lowest BIC and AIC measure. For all the return series in our sample the EGARCH (1,1) gave the best fit, hence are model used for the purpose (table 8.1).

EGARCH was developed to overcome weaknesses of GARCH to handle financial time series. This model allows for asymmetric effects between positive and negative asset returns. The formula for an EGARCH (1,1) model can be written as:

$$\log(\sigma_t^2) = \omega + \beta_1 \log(\sigma_{t-1}^2) + \alpha_1 \left[\frac{|r_{t-1}|}{\sigma_{t-1}} - E\left\{\frac{|r_{t-1}|}{\sigma_{t-1}}\right\}\right] + \xi_1 \left(\frac{r_{t-1}}{\sigma_{t-1}}\right) \qquad (3.2)$$

Where σ_{t-1} is the last estimate of variance rate, r_{t-1} is the last estimate of squared return, ω is the weighted long-run average variance rate, α and β are the respective weights for each factor and ξ_1 will capture the size effect of asymmetry.

3.2.1 Procedure to fit the data to GARCH models

To fit the return data to the GARCH model we perform statistical tests for stationarity, autocorrelation and conditional heteroscedasticity.

3.2.1.1 Stationarity

An important basis for a time-series analysis is stationarity. Time-series says to be strictly stationary if the joint distribution of the time-series variables is invariant over time-shifts (Tsay, 2001). This condition is strong and hard to verify. Therefore, it is possible to assume a weaker version of stationarity. A test used for checking the time-series for stationarity is Augmented Dickey Fuller (ADF) test. The ADF tests the null hypothesis that a unit root is present in the time-series sample. If this is the case the time-series sample is non-stationary. Index prices are collected from Bloomberg and are typically non-stationary. Therefore, we use the log-returns of the prices for modelling volatility:

$$u_i = log\left(\frac{S_i}{S_{i-1}}\right) \qquad (3.3)$$

We use the Econometric Modeler App in MATLAB to perform the volatility modelling. To describe the process, we will display the modelling for one of the PPNs written on the Swedish index OMXS30. The rest of the results from the volatility modelling can be find in table 8.1. We start by adding the log-return time-series into the app.

Figure 3.1: Historical return OMXS30. This graph shows the historical returns from 2005-01-04 and up to the issuance of the PPN 2013-05-06.

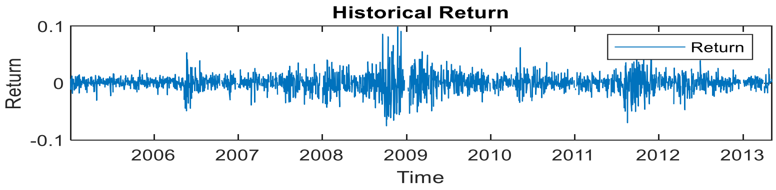
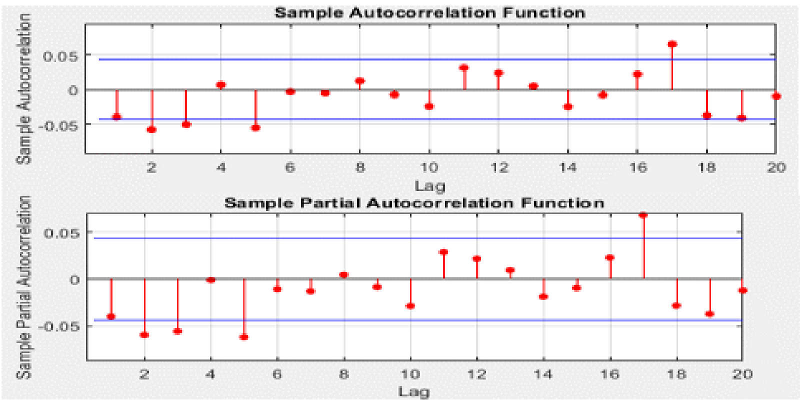


Figure 3.1 shows that the historical returns seem to be mean reverting to zero. Furthermore, the figure of the historical returns displays some clustering effect especially in the end of 2008. To be able to fit the data to the GARCH models we need to check for stationarity in the return series. We perform an ADF-test. The null hypothesis is rejected; hence we have a time-series without a unit root which is stationary (table 8.2).

3.2.1.2 Autocorrelation

The residuals in a conditional volatility model need to be described as a white noise process, they should be random and experience no pattern. The autocorrelation function (ACF) is commonly used to investigate these patterns and to give a visual overview of the structure in the residual return. To further examine if there exist any autocorrelation in the residuals, we can use Ljung- Box Q test. The Ljung-Box Q tests for autocorrelation in multiple lags jointly. The null hypothesis is stated as: The autocorrelation up to lag *m* are jointly zero.

Figure 3.2: Autocorrelation and partial autocorrelation function historical returns. The plot displays the sample autocorrelation and partial autocorrelation function of historical returns. The blue line states the confidence level at 5%.



The next step is to plot the autocorrelation function of the sample this to see if the residuals in the returns follow a white noise process. In figure 3.2 the residuals in the returns seem to experience autocorrelation for lags larger than one. This is confirmed by the Ljung-Box Q test (table 8.2) where the null hypothesis for no autocorrelation is rejected. Since the residuals are autocorrelated it can indicate that the return is not only determined by an intercept and an error term:

$$r_t = \mu + \varepsilon_t \tag{3.4}$$

Where r_t is the return at time t, μ is the intercept and ε_t is the residual at time *t*. The structure in the residuals need to be modelled separately with a conditional mean model before we can estimate the conditional variance with the GARCH-models. The plot of the autocorrelation and partial autocorrelation seem to be geometrically declining. The autocorrelation in the residuals can be successfully removed by assuming that the returns follow an ARMA (2,2) process (equation 8.1). The new property indicates that the residuals follow a white-noise process and that the null hypothesis is no longer rejected (table 8.2).

3.2.1.3 Autocorrelation in squared residuals

The residuals in the return can be uncorrelated but can still experience conditional heteroscedasticity. This would say that the squared residuals are autocorrelated. (Engle, 1982) A time-series that experience this type of autocorrelation in the squared returns is said to have ARCH-effect. The Engle ARCH test can be used to investigate if the residuals experience this type of characteristic. The null hypothesis in is stated as: there are no ARCH effects in the residuals.

Figure 3.3: Autocorrelation function squared historical returns. The plot displays the sample autocorrelation function of the squared historical returns. The blue line states the confidence level at 5 %.

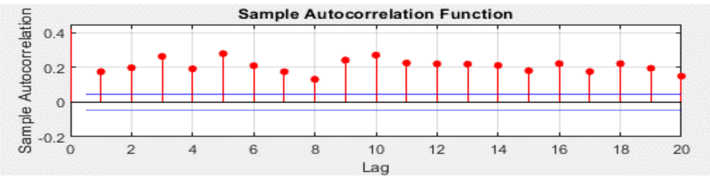


Figure 3.3 depicts the autocorrelation function for the squared returns. From the figure one can see that the residuals of the squared returns seem to be autocorrelated at every lag. This result also coincide with the results from the Engle ARCH test (table 8.2). The null hypothesis is rejected hence there are ARCH effects in the squared residuals.

3.2.2 Historical volatility

The simplest model used for forecasting volatility is the usage of historical volatility where it is assumed that the recent realized volatility will continue into the future. We calculate the lognormal returns from historical stock or index prices and obtain the volatility from the historical lognormal returns as follows:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \tag{3.5}$$

Where S_i is the stock price, u_i is the lognormal return, n is the number of observations, $\bar{u}$ is the mean of u_i and σ is the historical volatility.

For the Basket options we chose to use historical volatility due to the complexity of performing a multi-variate GARCH- model. The results from the procedure are depicted in table 8.3.

3.3 Risk-free rate

To find the most appropriate measure for the risk-free rate we obtained the government bond yields from the issuing countries of the indices. The government bonds are used for borrowing money in the countries own currency and by assumption can be considered risk-free securities. This given to the probability of a government defaulting on a loan denominated in its own currency is highly unlikely since they have the possibility to increase its money supply. (Hull, 2017). We obtain the government bond yield data from each of the countries' central banks. The data obtained from the central banks did not contain all maturities matching the data sample, we therefore used linear interpolation to adjust the yield for the absent maturities (table 8.4 and 8.5).

3.4 Dividend

The principal protected notes in our data sample are written on one or several indices. The indices consist of a portfolio of stocks that pay out dividends to shareholders. The indices themselves do not pay out any dividend, but the price of the indeces will be adjusted after an ex-dividend date of an underlying stock. Hence, we need to add the dividend parameter to the model. We obtain the historical annualized dividend yields from Bloomberg. The average of the historical dividend yield for each of the indices are calculated and added to the pricing model (table 8.4 and 8.5).

4 Methodology

In this section we explain in detail the different pricing models that we used for the call option pricing embedded in the principal protected notes (PPN). To resolve the convenience of investing in this type of structured products, we first wanted to determine if we could replicate some of the marketed PPN and yield a higher participation rate than the one offered by the sampled Swedish banks. Subsequently, we determined their efficiency by comparing their performance against other alternative investment vehicles as benchmarks. To replicate the PPN and eventually obtain its potential participation rate, we calculated both legs of the structured product, the underlying call option and the zero-coupon bond.[1]

4.1 Call Option Pricing

The option within the principal protected notes that we priced were European Call Options with Asian tails, and the following characteristics:

- Up to 5 years to maturity from the issuance date.
- The strike price K is equal to the spot price at issuance S_0.
- Usually one year before the expiry of the PPN, the closing price of the index is registered. This process is repeated every month until the end date, totalling 13 different observation dates.
- The observed registered values are averaged to determine the final value of the underlying, S_{avg}.
- The pay-off of the option is equal to the maximum between the appreciation of the underlying $(S_{avg} – K)$ and zero.

4.1.1 Index Option Pricing

We estimated the index call option price at issuance using a Monte Carlo simulation approach, based on a risk-neutral valuation framework where the underlying index follows a geometric Brownian Motion. For each option pricing we sampled 1,000,000 different paths, to obtain the expected pay-off of the option under risk neutral conditions, and then discounted it with its corresponding risk-free rate. Additionally, we compared the computed option price with standard closed-form solutions such as Black-Scholes-Merton model for European call options (Black & Scholes, 1973), Kemna-Vorst approach for options based on average asset values

[1] The MATLAB codes used to price the PPNs can be found in the Appendix 8.1.1 and 8.1.2

(Kemna & Vorst, 1990), and with the Levy pricing model for continuous arithmetic averaging options (Levy, 1992).

4.1.2 Monte Carlo Simulation

Presumably one of the most widely used approaches for valuing derivative securities, Monte Carlo simulation, is especially useful for pricing complex path-dependent exotic options, such as the ones that we find in the principal protected notes structure. The Monte Carlo simulation of a geometric Brownian Motion is a robust method to sample a possible outcome for the process, with the possibility to create as many different random paths as desired. To perform a Monte Carlo simulation, and thus price the option pay-off of our path dependent option. We followed the 5 steps suggested by Hull (2017):

- Sample a random path for S in a risk-neutral world, which in our case is generated following a geometric Brownian motion.
- Compute the pay-off from the option.
- Repeat the previous steps and get as many sample values as desired. In our case 1,000,000 different paths.
- Calculate the sample mean of the pay-off to obtain an estimate of the option expected pay-off in a risk-neutral world.
- Lastly, discount the obtained expected pay-off at its corresponding risk-free rate, to obtain the estimated value of the option.

4.1.3 Geometric Brownian Motion Index options

In financial modelling, a common assumption is that stock prices follow a stochastic process in the form of a geometric Brownian motion. In our case, we furthermore extended this assumption to stock market indexes. The return to the asset holder, under this process in a time interval is considered normally distributed, with independent returns at each different period. An asset price following a geometric Brownian motion, has a lognormal geometric average price and an approximately lognormal arithmetic average price. The equation to determine the index price change with geometric Brownian motion is as follows:

$$\Delta S = e^{([r-\delta]-\frac{\sigma^2}{2})}\Delta T + \varepsilon\sigma\sqrt{\Delta T} \tag{4.1}$$

Where ΔS is the stochastic price change of the index, r is the risk-free rate corresponding to the underlying asset, δ is the expected dividend yield of the index, the subtraction r-δ represents the percentage drift of the process; σ^2 is the volatility of the index, ΔT is the time step of the process, and ε is a random draw of a normally distributed number. The result of multiplying the geometric Brownian motion stochastic factor with the previous time instant spot price is the simulated predicted price one step ahead.

4.1.4 Geometric Brownian Motion Basket options

In the case of the basket option, the derivative price dependents in more than one underlying asset. For this, we need to determine a correlated stochastic process, among the paths of all these assets. As expressed by Hull (2017), if we consider a situation where the option pay-off depends on n different variables θ_i, with volatility σ_i,, and expected growth μ_i in a risk-neutral world (in this case the difference between the risk-free rates and dividend yields) ,with a correlation ρ_{ik} between the Wiener processes θ_i and θ_k, where the life of the option is divided into n subintervals of length Δ_t, and ε_i is a random sample from a standard normal distribution. These adjustments result in another version of the GBM. The discrete version of a process for θ_i equal to:

$$\Delta\theta i = e^{(\mu i - \frac{\sigma i^2}{2})} \Delta T + \varepsilon i \sigma i \sqrt{\Delta T} \qquad (4.2)$$

Each simulation implicates obtaining n samples of different ε_i from a multivariate standardized normal distribution, to eventually generate the desired simulated path for each θ_i. This process is repeated as many times as needed to obtain a sample value to compute the option value.

To produce a n number of correlated samples $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ from a standard normal distribution, for the basket option computation, we implemented the Cholesky decomposition procedure. Hull (2017) explained this procedure as follows:

In a situation like this, where we need n correlated samples ε_n from normal distribution with the correlation between sample i and sample j being ρ_{ij}. We start by sampling n different variables xi, from univariate normal distributions. The required samples ε_1, are thus defined as:

$$\varepsilon 1 = \alpha 11 x 1$$

$$\varepsilon 2 = \alpha 21 x 1 + \alpha 22 x 2$$

$$\varepsilon \mathrm{i} = \alpha i 1 x 1 + \alpha i 2 x 2 + \cdots + \alpha i j x j$$

We adjust the coefficients αij in a way that the variances and correlations are correct. So, if we set $\alpha_{11} = 1$, we choose α_{21} so that $\alpha_{11}\ \alpha_{21} = \rho_{21}$, and so on.

The obtained correlated random sample εi, was plugged to the aforementioned geometric Brownian motion formula, and produced the stochastic factor needed to predict the correlated future prices of the basket option underlying assets. Subsequently this process was repeated, until the needed price path was completed. The finalized option calculation was then compared with the Longstaff - Schwartz Monte Carlo model for basket options (Longstaff & Schwartz, 2001).

4.1.5 Computation of the option pay-off

Once we obtained the entire predicted path, we filtered the estimated prices in the pre-specified observation dates. With the filtered estimated values, we then proceeded to compute the arithmetic average for the option, which is computed by dividing the sum of the estimated observed prices S_{ti}, by the number of observation dates.

$$Savg = \frac{1}{N}\sum_{t=1}^{N} Sti \tag{4.3}$$

For the basket option, we repeated this procedure in each different underlying asset, and computed a weighted average of these values. We then calculated the call option pay-off as the maximum value between zero, and the difference of S_{avg} minus the strike value K (which is set to be equal to the initial price S_0).

4.1.6 Repetitions, confidence interval and option pricing

We decided to use 1,000,000 repetitions in our model, to obtain an acceptably small standard error, at a cost of significantly more computation time. The reason behind this is that the standard error of the estimates depends on the sample size. As explained by Hull (2017), the accuracy of the estimates generated by a Monte Carlo simulation depend on the number of different trials performed in its estimation. We computed the standard deviation ω, and the mean μ of the payoffs derived from the simulation trials. The mean variable μ represents the estimated value of the derivative, and the standard deviation ω the squared root of the variance of the different path outcomes sampled; the last together with the square root of the variable M, representing the number of different trails, will be used to calculate the standard error SE of the estimate:

$$SE = \frac{\omega}{\sqrt{M}} \tag{4.4}$$

As we can see in the previous equation, the larger the sample size of the trials the smaller the size of the standard error of our result. The obtained standard error helped us to achieve a narrower confidence interval for our estimated pay-off values. The 95% confidence interval for the option pay-off value P_{avg} is given then by the following formula:

$$\mu - \frac{1.96\,\omega}{\sqrt{M}} < \mu < \mu + \frac{1.96\,\omega}{\sqrt{M}} \qquad (4.5)$$

The 1,000,000 repetitions that we performed, provided more certainty to our estimates, than for instance a facile to compute 10,000 iteration model which would be ten times more inaccurate than the one we obtained.

Once we obtained the estimated pay-off μ and it's 95% confidence interval, we continued by discounting these values, with their respective risk-free rate.

$$Pcall = \mu \times e^{(-rf \times T)} \qquad (4.6)$$

Finally, the obtained discounted pay-off and its confidence interval resulted in our final estimated Asian tailed Call option price and confidence interval. Overall, this component of the principal protected note is the factor determining the return of the security. The profitability level of the PPN will depend on how many of these call options can be bought with the pre-invest proceeds of the fixed leg of the PPN, the zero-coupon bond.

4.2 Zero Coupon Bond Pricing

To compute the zero-bond, we used the issuing bank borrowing cost at the time of issuance of the PPN, for a maturity equal to the one from the priced security. This connotes an enhanced yield, at the cost of taking some credit risk from the bank, compared to the usage of a risk-free rate security. The following formula, where *100* represents the par value of the security, *zy* the zero-coupon yield and *TMY* the time to maturity, computes the present value of the zero-coupon bond:

$$Zero\ bond = 100e^{-zy*TMY} \qquad (4.7)$$

This component of the principal protected note is the one that vouches for a minimum return on the security at maturity. We find two different classes of principal protected notes depending on its riskiness. Of these, the safe notes guarantee an investment return not less to the initial capital invested, while the risky notes guarantee at least a significant part of it. In both cases the guaranteed amount is equal to the face value of the PPN, which is standardized to SEK 100.

4.3 Principal Protected Note Participation Rate computation

The participation rate is the percentage over the appreciation of the underlying asset that the investor is entitled to claim at the maturity of the note. Once we had the call option and the zero-coupon prices, we proceed to compute the participation rate *PR* of the principal protected note. We obtained this by computing the ratio of the disposable investment capital *DIC* (the remaining capital after subtracting the cost of the zero-coupon bond) and the price of the call option *Pcall*.

$$PR = \frac{DIC}{Pcall} \tag{4.8}$$

After computing the participation rates derived from our pricing model, we compared them with the participation rates offered by the banks.

4.4 CVA Computation

An important factor to determine the fair-value of a principal protected note, is the credit value adjustment *CVA*. It is true, that the PPN is not entirely a risk-free security, since there is always the possibility, that the underwriting bank could default and fail to pay back the expected value of the note to the holder. An investment in PPN will also contain liquidity risk, since the secondary market for PPNs in Sweden is not very liquid.

The CVA reflects the expected loss from a default by the counterparty. Consequentially the value of the security is adjusted by this factor, and part of its value is subtracted (Hull, 2018). We opted to use a method based on the implicit default probability derived from the credit default swaps spreads (Hull, 2018), since this reflects better the market sentiment, at the time of issuance, towards the default risk of the counterparty. The CVA computation is obtained by first obtaining the risk-neutral default probability q_i, which is estimated from the counterparty credit spread. The first step to compute q_i was to estimate the average hazard rate ƛi, which we obtained with the following formula:

$$ƛi = \frac{Si}{1-R} \tag{4.9}$$

Where S_i is the credit default swap spread at the time, and *R* is the estimated recovery rate, which we estimated to be at 40%. Once we computed the hazard rate ƛi, we obtained the risk-neutral default probability derived from:

$$qi = e^{-(ƛi-1)(ti-1)} - e^{(-ƛi)(ti)} \tag{4.10}$$

With the risk-neutral default probability q_i, in addition to the present value of the expected exposure *vi* (in this case SEK 100) and the estimated recovery rate *R* in the event of the counterparty default (in this case 40%) defined, we continued with the final CVA computation:

$$CVA = \sum_{i=1}^{n}(1-R)\ qi\ vi \qquad (4.11)$$

By accounting for the CVA and the fixed brokerage fee (applied on the SEK 100 and SEK 110 PPN's prices), we were able to approximate better the actual fair value of the sampled principal protected notes.

4.5 Comparing the PPNs with other benchmarks

To continue our analysis, we computed the returns obtained by the investors of the principal protected notes at maturity based on the contract conditions of the issuing banks. The annual holding period return was calculated as the compounded return according to the equation:

$$Annualized\ return = \left(\frac{End\ value - Start\ value}{Start\ value}^{\frac{1}{TMY}} - 1\right) \qquad (4.12)$$

Markowitz (1952) classic Modern Portfolio Theory, and the Capital Asset Pricing Model (Sharpe, 1964) are based on the assumption that financial assets returns are normally distributed, and that investors are always mean-variance oriented. Structured products such as the PPN are a different case, since their return distributions have important levels of kurtosis and skewness (Nørholm, 2012). Because of this, standard risk-adjusted performance measures such as, the Sharpe ratio or Jensen's alpha are not optimal methods to measure the performance of the principal protected notes that we priced. Thus, we decided to compare the realized returns of our samples PPNs with a number of alternative investment strategies that a PPN investor would consider. The benchmarks considered contain securities such as risk-free debt (government zero-coupon bond from Sweden), risky debt (zero-coupon bond from the issuing bank) and the equity index investment (investment in the underlying index).

5 Empirical Results / Analysis

This section of our paper illustrates how efficient principal protected notes are as investment vehicles. The efficiency of the product is assessed by comparing the embedded European Asian option of the PPN with other types of options, by comparing the participation rate offered by the bank with a replication strategy, and ultimately, by comparing the return obtained by the investor of the PPN with other investment alternatives.

The methods to compute the index options and the basket options are significantly different one to the other. To better explain the difference in prices between these two, we separated this part of the analysis for each of these types[2].

5.1 PPN Pricing Analysis

5.1.1 Comparing the option prices with different pricing methods

In this section, we asses which option type results in the most economically efficient, among Arithmetic Asian options and European options of similar kind. This to see if a different option type than the one chosen by the bank could be better for a retail investor to incorporate in a PPN structure.

To better reflect the option prices, we followed the general practice of a standardized level of 100 units as the spot price at time zero S_0 for all the different indexes. This facilitated the process of matching the obtained call option price with the standard value of 100 of the principal protected notes. We would like to highlight that in reality most of the index have contrasting different levels and multipliers, thus contract prices may vary. Nevertheless, with the appropriate adequation it is possible to obtain a value proportional to the one we present.

Table 5.1: Index Option price comparison. Shows the approximated call option prices that we obtained from distinct methods for the index options. In column 1, we observe the underlying asset, in column 2 and 3 the issue and maturity dates, in column 4 the option price of the replicated embedded option, in column 5 the complete arithmetic Asian option from our Monte Carlo simulation model , in column 6 the arithmetic Asian with the Levy method, and in column 7 the price of a plain vanilla European option computed with the Black-Scholes-Merton method *BSM*.

Underlying Asset	Issue Date	Maturity Date	Replicated Option	MC Arithmetic Asian	Kemna-Vorst Method	Levy Method	Black Scholes Merton
OMXS30	17/02/2011	17/02/2016	17.7700	10.7075	9.5404	10.8402	18.3363
S&P 500 Index	26/05/2011	19/05/2014	11.1790	7.0095	6.5461	7.0529	11.8243
OMXS30	20/12/2012	22/11/2017	12.1425	7.9590	7.2211	8.0532	12.8712
S&P 500 Low Volatility Index	07/02/2013	07/02/2018	7.5944	4.8271	4.5418	4.8298	7.7684
OMXS30	07/05/2013	23/04/2018	12.7547	8.3062	7.4931	8.3838	13.4806
S&P 500 Index	03/06/2013	21/05/2018	13.3336	8.5025	7.7636	8.5698	14.2064
S&P Nordic Low Volatility	10/01/2014	27/12/2018	8.9525	5.9019	5.4858	5.9379	9.5005
S&P 350 Europe Low Volatility Index	05/02/2014	05/02/2019	5.8355	3.9833	3.7496	3.9812	6.1204
Hang Seng Index	03/03/2014	18/02/2019	14.9233	9.7291	8.6620	9.8271	16.0012
S&P 350 Europe Low Volatility Index	05/03/2014	05/03/2019	5.6951	3.8822	3.6435	3.8620	5.9331
OMXS30	09/04/2014	10/04/2018	11.4028	7.5281	6.8897	7.5871	12.2900
OMXS30	07/05/2014	07/05/2018	11.2050	7.4079	6.7918	7.4737	12.0715
S&P 350 Europe Low Volatility Index	27/08/2014	15/08/2018	4.7141	3.2752	3.1123	3.2652	4.9907
OMXS30	28/08/2014	15/08/2018	10.1254	6.7786	6.2423	6.8357	10.8793
S&P 350 Europe Low Volatility Index	03/12/2014	21/11/2018	4.5564	3.1712	3.0209	3.1660	4.8210
OMXS30	03/12/2014	21/11/2018	9.8910	6.6527	6.1046	6.6827	10.5585

From table 5.1 and 5.2 we can see that the results from the different pricing methods strength the prime motivation of using the chosen settings for the Asian option that

[2] We did not price any American option alternatives since the characteristics of these, are not compatible with the features of the principal protected notes we priced.

we replicated instead of a plain vanilla European option, or a standard averaging option over the whole period between the issue and the maturity of the contract. We can see that the replicated option price in general is lower than the European option with the BSM method and Longstaff-Schwartz, but larger than the other arithmetic average options.

Table 5.2: Basket Option pricing comparison. Shows the approximated call option prices that we obtained from distinct methods for the basket options. In column 1, we observe the underlying assets, in column 2 and 3 the issue and maturity dates, in column 4 the option price of the replicated embedded option, in column 5 the complete arithmetic Asian option from our Monte Carlo simulation model , and in column 6 the price of a plain vanilla European option computed with the Longstaff-Schwartz method.

Underlying Asset	Issue Date	Maturity Date	Replicated Option	MC Arithmetic Asian	Longstaff-Schwartz European
Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/2012	23/11/2016	17.1458	10.4378	17.9415
DAX , Hang Seng, OMXS30 & S&P 500	08/07/2013	25/06/2018	17.7561	11.0943	19.1843
MSCI Singapore & Taiwan Stock Exchange Index	09/04/2014	28/03/2017	11.4737	7.5541	13.0885
MSCI Singapore & MSCI Taiwan	08/10/2015	10/10/2018	11.2603	7.4014	13.2106

The benefits of choosing the replicated option over a common European option are that the investor would pay less for the option and will be subject to lower volatility in the expected payoff of the option thanks to the averaging of the 13 different observed spot prices. The obvious drawback is that in case of a continuous appreciation and favourable volatile movements in the underlying asset, the investor would obtain a lower yield.

Furthermore, the motivation to invest in the replicated option over the other arithmetic Asian options, relies in the fact that these alternatives are arguably better for hedging purposes, with lower expected pay-offs and hence option prices; as we can see their objective is not in line with the motivation of the common retail investor. The standard price of the arithmetic Asian option computed from the same paths produced by the original pricing model based on Monte Carlo simulation, and the Levy model are very similar. The result was expected since in both cases the averaging of the Asian options is over the whole predicted path.

The outcome of the results points favourable for the banks chosen alternative, as this seem to be adequate for an individual looking to benefit from a possible appreciation of the underlying, compared to the other computed alternatives.

5.1.2 Call Options Prices and Confidence Intervals

From the replication of the embedded option in the PPN we experienced differences between the PPN with index and basket option (table 5.3). PPNs with basket options are usually more expensive than the ones from a single index option. But a basket option is often less expensive than buying multiple single options. Table 5.3 also display the confidence interval for the replicated option at the 95% confidence interval. The confidence interval for the PPN with index option is often narrower and can be explained by the iterations in the simulation of the options.

In general, we can see that the market conditions at the time of the analysis where very permissive and delivered very low option prices. We know that during the years 2012-2015, the world was experiencing record low interest rate yields after the implementation of Quantitative Easing policies by the different central banks around the world, and capital markets were enjoying a bullish period with sustained low-volatility conditions. Both of these factors were determinant for the low pricing of the equity options.

Table 5.3: Call Options Prices and Confidence Intervals. Shows the approximated call option prices that we obtained from the Monte Carlo simulation and it is respective confidence level at 95 %. In column 1, we observe the type of option embedded in the PPN. In column 2, we observe the underlying asset, in column 3 the issue date, in column 4 the option price of the replicated embedded option, in column 5 the option price at the 95 % confidence level.

Type	Underlying Asset	Issue Date	Replicated Option	Confidence Interval
Index	OMXS30	17/02/2011	17.7700	[17.7094, 17.8306]
Index	S&P 500	26/05/2011	11.1790	[11.1435, 11.2146]
Index	OMXS30	20/12/2012	12.1425	[12.0966, 12.1885]
Index	S&P 500 Low Volatility	07/02/2013	7.5944	[7.5676, 7.6213]
Index	OMXS30	07/05/2013	12.7547	[12.707, 12.8024]
Index	S&P 500	03/06/2013	13.3336	[13.2885, 13.3788]
Index	S&P Nordic Low Volatility	10/01/2014	8.9525	[8.9200, 8.9850]
Index	S&P 350 Europe Low Volatility	05/02/2014	5.8355	[5.8125, 5.8586]
Index	Hang Seng	03/03/2014	14.9233	[14.8687, 14.978]
Index	S&P 350 Europe Low Volatility	05/03/2014	5.6951	[5.6725, 5.7177]
Index	OMXS30	09/04/2014	11.4028	[11.3621, 11.4434]
Index	OMXS30	07/05/2014	11.2050	[11.1646, 11.2453]
Index	S&P 350 Europe Low Volatility	27/08/2014	4.7141	[4.6956, 4.7326]
Index	OMXS30	28/08/2014	10.1254	[10.0877, 10.163]
Index	S&P 350 Europe Low Volatility	03/12/2014	4.5564	[4.5383, 4.5745]
Index	OMXS30	03/12/2014	9.8910	[9.8535, 9.9284]
Basket	Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/2012	17.1458	[17.0324, 17.2526]
Basket	DAX, Hang Seng, OMXS30 & S&P 500	08/07/2013	17.7561	[17.6371, 17.8723]
Basket	MSCI Singapore & Taiwan Stock Exchange Index	09/04/2014	11.4737	[11.4067, 11.5407]
Basket	MSCI Singapore & MSCI Taiwan	10/09/2015	11.2333	[11.1625, 11.2935]
Basket	MSCI Singapore & MSCI Taiwan	08/10/2015	11.2603	[11.1896, 11.3209]

5.1.3 Zero-Coupon Bonds Prices and Disposable Investment Capital

The zero-coupon element in the principal protected note is discounted with the issuing banks borrowing cost at the time. The assumption made is that the issuing bank uses its own debt cost to create the bond leg of the principal protected note.

Table 5.4: Zero-Coupon Bonds Prices and Disposable Investment Capital. Shows the zero-coupon bond price, the bond yield which the zero-coupon bond is discounted with and the disposable investment capital. In column 1 we observe the underlying asset, in column 2the notional amount in SEK, in column 3 the zero-coupon yield, which is the yield the zero-coupon bond is discounted with. In column 4 the zero-coupon bond price for each of the products and in column 5 the investment capital disposal which is the capital an investor has available to buy options for.

Index	Notional amount (SEK)	Zero-Bond Yield	Zero-Bond Price	Disp. Investment Capital
OMXS30	100	3.84%	82.5472	17.4528
OMXS30	110	3.84%	82.5472	27.4528
OMXS30	100	1.72%	91.9049	8.0951
OMXS30	110	1.72%	91.9049	18.0951
OMXS30	100	1.96%	90.7385	9.2615
OMXS30	110	1.96%	90.7385	19.2615
S&P Nordic Low Volatility	100	2.33%	89.0823	10.9177
S&P Nordic Low Volatility	110	2.33%	89.0823	20.9177
Hang Seng	100	2.12%	90.0309	9.9691
Hang Seng	110	2.12%	90.0309	19.9691
OMXS30	100	1.62%	93.7400	6.2600
OMXS30	110	1.62%	93.7400	16.2600
OMXS30	102	0.89%	96.5260	5.4740
OMXS30	110	0.89%	96.5260	13.4740
S&P 350 Europe Low Volatility	102	0.63%	97.5281	4.4719
S&P 350 Europe Low Volatility	110	0.63%	97.5281	12.4719
S&P 500	100	2.75%	92.1359	7.8641
S&P 500	110	2.75%	92.1359	17.8641
S&P 500 Low Volatility	100	2.03%	90.3436	9.6564
S&P 500 Low Volatility	110	2.03%	90.3436	19.6564
S&P 500	100	2.13%	89.9480	10.0520
S&P 500	110	2.13%	89.9480	20.0520
S&P 350 Europe Low Volatility	100	2.23%	89.4536	10.5464
S&P 350 Europe Low Volatility	110	2.23%	89.4536	20.5464
S&P 350 Europe Low Volatility	100	2.08%	90.1360	9.8640
S&P 350 Europe Low Volatility	110	2.08%	90.1360	19.8640
OMXS30	100	1.47%	94.2744	5.7256
OMXS30	110	1.47%	94.2744	15.7256
S&P 350 Europe Low Volatility	102	0.92%	96.4012	5.5988
S&P 350 Europe Low Volatility	110	0.92%	96.4012	13.5988
OMXS30	102	0.63%	97.5281	4.4719
OMXS30	110	0.63%	97.5281	12.4719
Hang Seng, MSCI Singapore & MSCI Taiwan	100	1.42%	94.4859	5.5141
Hang Seng, MSCI Singapore & MSCI Taiwan	110	1.42%	94.4859	15.5141
MSCI Singapore & Taiwan Stock Exchange Index	102	1.35%	96.0591	5.9409
MSCI Singapore & Taiwan Stock Exchange Index	110	1.35%	96.0591	13.9409
DAX, Hang Seng, OMXS30 & S&P 500	100	2.47%	88.4830	11.5170
DAX, Hang Seng, OMXS30 & S&P 500	110	2.47%	88.4830	21.5170
MSCI Singapore & MSCI Taiwan	110	0.15%	99.5655	10.4345
MSCI Singapore & MSCI Taiwan	110	0.33%	99.0220	10.9780

Table 5.4 depicts the zero-coupon price, equal for both the safe and risky alternatives of the principal protected notes. The largest difference is the hypothetical capital available to the investor to buy call options on the underlying index/indices (pre-invest expected interest return on the bond ignoring the time-value of money). The investor will have more capital at his disposal if he chooses the risky alternative instead of the safe one. The risky PPN is not 100% capital

protected and the investor can experience the possibility of losing a part of his investment. But the gain will be an enhanced upside, since he will end up with a larger stake of options, i.e., participation rate. We can claim that the risky alternative is more suited for a retail investor willing to take more risk, who is looking for a superior yield while limiting his downside to some degree.

For the safe principal protected note the disposable investment capital is relatively low, indicating that the investor might not be able to buy a large portion of a call option, thereby reducing its possible participation rate. This will reduce the upside the investor can experience, but his investment will still be completely capital protected and will not experience any loss related to a market downturn. Which is appropriate for a risk adverse retail investor looking for these kind of features in an investment. The result from pricing the zero-coupon bond has tendencies of following the characteristics shown in the prospectus of the issuing banks. That investors investing in the safe alternative have a participation rate lower than 100%, and hence can only do a fractional investment in a call option.

5.2 Comparing the Participation Rates

The participation rate offered by the bank is the rate at which the upside of the underlying will be multiplied with. From the participation rate of the replicated PPN is possible to determine if the security shows tendencies of overpricing. This by comparing the replicated participation rate with the one issued by the bank.

5.2.1 Participation rate PPN with index options

The replicated PPN with index option does offer in many cases a larger participation rate than the one from the from the issuing bank (table 5.5). This indicates that the PPNs in the Swedish market are overpriced at issuance. The larger participation rate obtained shows that a retail investor could be better off, by replicating a PPN payoff by himself.

The source of difference in participation is a combination of the return on the zero-coupon bond and the cost of the call options on the index. Assuming that the issuing banks used their own cost of debt to determine the return on the bond, and used a pricing methodology comparable to the one we used, it is not possible to explain the large differences between participation rates that most, if not all of the products that we priced present.

During the sample period the interest rate environment was predominantly low, and the volatility levels moderately stable. Nonetheless, the yield spread between risky debt and risk-free was still important, giving room to a combination of relatively cheap call option prices and sufficient interest return from the risky bonds to finance the PPN strategy, and achieve meaningful participation rates. This scenario presents a favourable situation for the issuing banks, since they can offer attractive participation rates to the investors, even when these are below fair value.

The difference in participation rates presents a positively skewed distribution with a mean difference of 40% and a median difference of 17%. The overpricing in PPN with index options goes in line with previous research in this area. The overpricing can come from additional margin taken by the bank to cover transaction costs or else.

Table 5.5: Participation rate for safe and risky principal protected notes. Shows the participation rates generated by the pricing model in a comparison with the participation rate offered by the issuing bank. In column 1, we observe the type of PPN, in column 2 the underlying asset, in column 3 the issue date, in column 4 the investment capital at disposal, in column 5 the option price of the replicated embedded option, in column 6 the participation rate offered by the issuing bank, in column 7 the participation rate we obtain from the replication of the principal protected note, in column 8 the difference between the participation rate offered by the bank and the participation rate we obtained from the replication.

Type	Index	Issue Date	Disp. Invest.	Option Price Model	Bank Part. Rate	Rep. Part. Rate	Diff. in PR
Safe	OMXS30	17/02/11	17.4528	17.7700	80%	98.21%	18.21%
Safe	S&P 500	26/05/11	7.8641	11.1790	68%	70.35%	2.35%
Safe	OMXS30	20/12/12	8.0951	12.1425	50%	66.67%	16.67%
Safe	S&P Nordic Low Volatility	07/02/13	9.6564	7.5944	90%	127.15%	37.15%
Safe	OMXS30	07/05/13	9.2615	12.7547	60%	72.61%	12.61%
Safe	S&P 500	03/06/13	10.0520	13.3336	60%	75.39%	15.39%
Safe	S&P Nordic Low Volatility	10/01/14	10.9177	8.9525	70%	121.95%	51.95%
Safe	S&P 350 Europe Low Volatility	05/02/14	10.5464	5.8355	86%	180.73%	94.73%
Safe	Hang Seng	03/03/14	9.9691	14.9233	60%	66.80%	6.80%
Safe	S&P 350 Europe Low Volatility	05/03/14	9.8640	5.6951	87%	173.20%	86.20%
Safe	OMXS30	09/04/14	6.2600	11.4028	44%	54.90%	10.90%
Safe	OMXS30	07/05/14	5.7256	11.2050	37%	51.10%	14.10%
Safe	S&P 350 Europe Low Volatility	27/08/14	5.5988	4.7141	50%	118.77%	68.77%
Safe	OMXS30	28/08/14	5.4740	10.1254	50%	54.06%	4.06%
Safe	S&P 350 Europe Low Volatility	03/12/14	4.4719	4.5564	50%	98.15%	48.15%
Safe	OMXS30	03/12/14	4.4719	9.8910	50%	45.21%	-4.79%
Risky	OMXS30	17/02/11	27.4528	17.7700	131%	154.49%	23.49%
Risky	S&P 500	26/05/11	17.8641	11.1790	147%	159.80%	12.80%
Risky	OMXS30	20/12/12	18.0951	12.1425	105%	149.02%	44.02%
Risky	S&P Nordic Low Volatility	07/02/13	19.6564	7.5944	195%	258.83%	63.83%
Risky	OMXS30	07/05/13	19.2615	12.7547	145%	151.01%	6.01%
Risky	S&P 500	03/06/13	20.0520	13.3336	130%	150.39%	20.39%
Risky	S&P Nordic Low Volatility	10/01/14	20.9177	8.9525	155%	233.65%	78.65%
Risky	S&P 350 Europe Low Volatility	05/02/14	20.5464	5.8355	204%	352.09%	148.09%
Risky	Hang Seng	03/03/14	19.9691	14.9233	140%	133.81%	-6.19%
Risky	S&P 350 Europe Low Volatility	05/03/14	19.8640	5.6951	202%	348.79%	146.79%
Risky	OMXS30	09/04/14	16.2600	11.4028	149%	142.60%	-6.40%
Risky	OMXS30	07/05/14	15.7256	11.2050	144%	140.34%	-3.66%
Risky	S&P 350 Europe Low Volatility	27/08/14	13.5988	4.7141	160%	288.47%	128.47%
Risky	OMXS30	28/08/14	13.4740	10.1254	122%	133.07%	11.07%
Risky	S&P 350 Europe Low Volatility	03/12/14	12.4719	4.5564	140%	273.72%	133.72%
Risky	OMXS30	03/12/14	12.4719	9.8910	120%	126.09%	6.09%

5.2.2 Participation rate PPN with basket options

We can detect a pattern of under-pricing for the basket options (table 5.6). The participation rate obtained from replicating the PPN is lower than the stated rate by the bank. Yet, the sample tested is not large enough to draw a conclusion of fair pricing by the issuing bank.

Table 5.6: Participation rate for principal protected notes with basket options. Shows the participation rates generated by the pricing model in a comparison with the participation rate offered by the issuing bank. In column 1 we observe the type of option displayed. In column 2, we observe the underlying asset, in column 3 the issue date, in column 4 the investment capital disposal, in column 5 the option price of the replicated embedded option, in column 6 the participation rate offered by the issuing bank, in column 7 the participation rate we obtain from the replication of the principal protected note, in column 8 the difference between the participation rate offered by the bank and the participation rate we obtained.

Type	Index	Issue Date	Disp. Invest.	Option Price Model	Bank Part. Rate	Rep. Part. Rate	Diff. in PR
Safe	Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/12	5.5141	17.1458	60%	32.16%	-27.84%
Safe	DAX, Hang Seng, OMXS30 & S&P 500	08/07/13	11.5170	17.7561	60%	64.86%	4.86%
Safe	MSCI Singapore & Taiwan Stock Exchange Index	09/04/14	5.9409	11.4737	55%	51.78%	-3.22%
Risky	Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/12	15.5141	17.1458	165%	90.48%	-74.52%
Risky	DAX, Hang Seng, OMXS30 & S&P 500	08/07/13	21.5170	17.7561	150%	121.18%	-28.82%
Risky	MSCI Singapore & Taiwan Stock Exchange Index	09/04/14	13.9409	11.4737	186%	121.50%	-64.50%
Risky	MSCI Singapore & MSCI Taiwan	10/09/15	10.4345	11.2333	100%	92.89%	-7.11%
Risky	MSCI Singapore & MSCI Taiwan	08/10/15	10.9780	11.2603	107%	97.49%	-9.51%

We believe that the difference in participation rates arise from variations in the correlation and volatility coefficients of the underlying assets. Our model retrieves these factors from historical data, which are likely different to what a multivariate GARCH model would estimate. The aforementioned parameters are essential in the Cholesky decomposition process, and geometric Brownian motion employed to price the basket options. Additionally, the model for the basket options has only 250,000 iterations in the Monte Carlo simulation (in contrast to the 1,000,000 employed for the index options), resulting in lower accuracy.

The results from this part of the analysis indicate that investors interested in investing in index PPNs, with unrestricted access to derivative products and with sufficient bargain power, or/and the ability to replicate the desired derivatives, should overweight the possibility to mimic the PPNs by themselves.

5.3 Investor Holding Period Return

5.3.1 Credit value adjustment

The PPN investment is either fully or partly capital protected. But the investment is also connected with risk factors such as liquidity, currency and credit risk. The investor has the opportunity to sell the investment in the secondary market during the time to maturity. But, the secondary market for these products is not very liquid, hence the investor of a PPN experiences a large liquidity risk.

There is also the small possibility that the issuing bank of the PPN will default on the investment, which must be reflected in the price of the note. Hence a retail investor experiences credit risk when buying PPN from a dealer. To incorporate the credit risk in the price of the note, we included a credit value adjustment (CVA) to reflect this risk in the price of the PPN. The CVA is added into the total cost of the PPN together with the brokerage fee. The total amount is then used to determine the actual annualized holding period return (HPR).

Table 5.7: Credit value adjustment to incorporate the credit risk of the issuing bank. Shows the CVA and the parameters included to estimate the credit risk of the issuing bank. In column 1 we observe the type of option displayed. In column 2 the underlying asset , in columns 3 the issue date, in column 4 the recovery rate, in column 5 the average yearly hazard rate calculated from the CDS-spread and the recovery rate, in column 6 the implied default probability calculated from the hazard rate and time to maturity of the product, and column 7 the CVA calculated from the default probability and the recovery rate.

Type	Index	Issue Date	Recovery Rate	Average t year hazard rate	Implied Default Probability	CVA
Index Safe and Risky	OMXS30	17/02/11	40%	0.97%	4.75%	2.85
Index Safe and Risky	S&P 500	26/05/11	40%	0.62%	1.86%	1.11
Index Safe and Risky	OMXS30	20/12/12	40%	1.26%	6.10%	3.66
Index Safe and Risky	S&P 500 Low Volatility	07/02/13	40%	1.16%	5.61%	3.37
Index Safe and Risky	OMXS30	07/05/13	40%	1.11%	5.40%	3.24
Index Safe and Risky	S&P 500	03/06/13	40%	1.08%	5.26%	3.15
Index Safe and Risky	S&P Nordic Low Volatility	10/01/14	40%	0.66%	2.60%	1.56
Index Safe and Risky	S&P 350 Europe Low Volatility	05/02/14	40%	1.34%	6.48%	1.94
Index Safe and Risky	Hang Seng	03/03/14	40%	0.89%	4.34%	2.60
Index Safe and Risky	S&P 350 Europe Low Volatility	05/03/14	40%	1.26%	6.09%	1.83
Index Safe and Risky	OMXS30	09/04/14	40%	0.94%	3.67%	1.10
Index Safe and Risky	OMXS30	07/05/14	40%	0.89%	3.48%	1.04
Index Safe and Risky	S&P 350 Europe Low Volatility	27/08/14	40%	0.58%	2.29%	1.37
Index Safe and Risky	OMXS30	28/08/14	40%	0.58%	2.29%	1.37
Index Safe and Risky	S&P 350 Europe Low Volatility	03/12/14	40%	0.56%	2.22%	1.33
Index Safe and Risky	OMXS30	03/12/14	40%	0.56%	2.22%	1.33
Basket Safe and Risky	Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/12	40%	1.10%	4.30%	2.58
Basket Safe and Risky	DAX, Hang Seng, OMXS30 & S&P 500	08/07/13	40%	1.25%	6.06%	3.64
Basket Safe and Risky	MSCI Singapore & Taiwan Stock Exchange Index	09/04/14	40%	0.94%	3.67%	1.10
Basket Risky	MSCI Singapore & MSCI Taiwan	10/09/15	40%	0.56%	1.67%	1.00
Basket Risky	MSCI Singapore & MSCI Taiwan	08/10/15	40%	0.62%	1.84%	1.10

The CVA calculation from table 5.7 are ranging from an add-on of 0.96 SEK to 3.66 SEK. For this we assumed a constant recovery rate of 40%. The adjustment will depend on the riskiness of the bank, at that particular time, extracted from the CDS-spread, and on the tenor of the PPN. The table depicts that the products issued around 2012-2013 have the highest CVA adjustment in our sample; hence, this was a riskier time to buy PPNs from the issuing banks. In 2014 as shown in the table the CVA was lower, this could indicate that at the time it was safer for a retail investor to invest in PPNs when considering the credit risk.

5.3.2 Comparing investors returns with different benchmarks

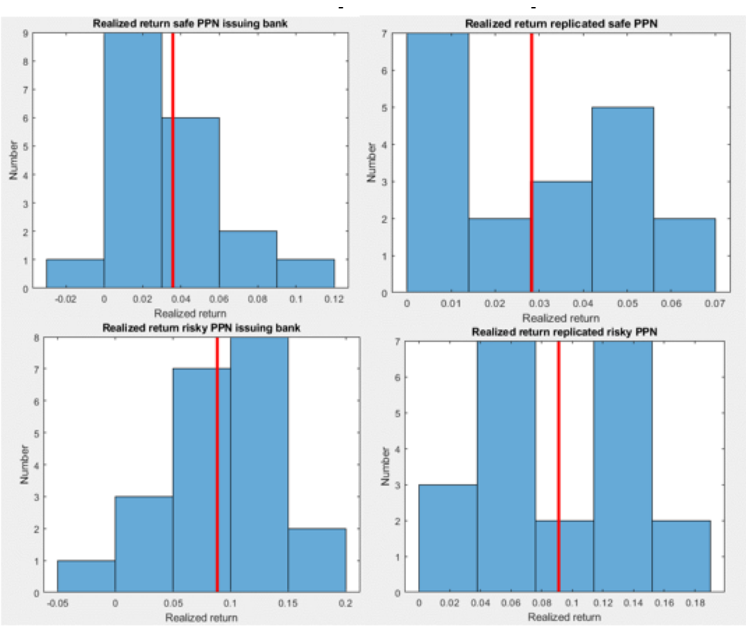
It has been determined already that standard-risk performance measures are not a suitable benchmark to assess PPNs efficiency. We have therefore chosen to compare the PPN annualized holding period returns (HPR), with other benchmarks composed by risk-free debt, riskier debt and the underlying equity asset, with these, we can to some extent determine the opportunity cost and therefore, the efficiency of these products.

We chose 7 different asset allocations with the following characteristics:

1. PPN from the issuing banks
2. PPN mimic strategy
3. Risk free Swedish government bond
4. Investment grade bonds from the issuing banks
5. Long position in the equity underlying
6. Equally weighted portfolio with risk free debt, risky debt and equity
7. Equally weighted portfolio with risky debt and equity

These portfolios with different characteristics illustrate different alternatives that a retail investor may consider together with the investment in the PPNs.

Figure 5.1: HPR from replicated PPN and issuing banks. The histograms above show the annualized HPR from the replicated safe PPN and from the issuing banks. The histograms below show the annualized HPR from the replicated risky PPN and from the issuing banks. The red line in the histograms show the mean of the HPR.



The returns obtained from the replication of the PPN and the banks are depicted in figure 5.1 in the form of histograms. All the four histograms are skewed to the right in line with the characteristic of the PPN that is capital protected. The left-hand side histograms depict the net of fees annualized holding period returns, realized from the notes issued by the banks. From the same table, we can also notice that as expected the safe notes yielded lower returns than their riskier counterparts because of their lower exposure to the underlying appreciation.

5.3.2.1 Return Comparison for Safe Index PPNs

In this case, the asset allocation that generated the highest HPR was the investment on the underlying index (table 5.8), which is equal to a 100% participation. This type of investment generally produces a higher annualized HPR than a safe PPN, since the participation rate in the safe PPN is usually below 100%. The trade-off is that this strategy is riskier, since the PPN is capital protected and hedges the downside for the investor. Excluding the sole long position on the index, the PPN prevails as the most profitable alternative compared to the other portfolios. If a retail investor would have chosen to invest in the balanced portfolios of debt and equity, he could be worse off in terms of returns.

Table 5.8: HPR and alternative investment sources to safe PPN. Shows the annualized holding period return for safe PPN and alternative investment sources. In column 1, we observe the underlying asset, in column 2 the issue date, in column 3 the HPR obtained from the PPN, in column 4 the HPR obtained from the replication, in column 5 the HPR from an investment in a Swedish zero-coupon bond, in column 6 the HPR obtained from investment in a zero-coupon bond form the issuing bank, in column 7 the HPR obtained from an investment in the underlying equity index, in column 8 the HPR obtained from investment in an equally weighted portfolio of the three alternative investment sources and in column 9 the HPR obtained from an investment in 50 % risky debt and 50% in the equity index.

Index	Issue Date	Bank Annual R	Rep. Annual R	Alt. Port. 1 Risk Free	Alt. Port. 2 Low Risk	Alt. Port. 3 Equity Index	Alt. Port. 4 Balanced	Alt. Port. 5 Debt + Index
OMXS30	17/02/11	3.48%	4.55%	0.63%	0.77%	4.75%	2.05%	2.76%
S&P 500	26/05/11	1.63%	0.87%	0.78%	0.92%	6.52%	2.74%	3.72%
OMXS30	20/12/12	2.81%	3.23%	0.21%	0.35%	9.34%	3.30%	4.84%
S&P 500 Low Volatility	07/02/13	9.44%	6.79%	0.27%	0.41%	14.46%	5.05%	7.44%
OMXS30	07/05/13	2.57%	3.02%	0.26%	0.39%	6.33%	2.33%	3.36%
S&P 500	03/06/13	5.88%	4.55%	0.30%	0.43%	19.87%	6.87%	10.15%
S&P Nordic Low Volatility	10/01/14	5.88%	5.40%	0.34%	0.47%	8.38%	3.06%	4.43%
S&P 350 Europe Low Volatility	05/02/14	6.38%	4.29%	0.31%	0.45%	9.40%	3.38%	4.92%
Hang Seng	03/03/14	7.13%	6.74%	0.29%	0.43%	15.96%	5.56%	8.19%
S&P 350 Europe Low Volatility	05/03/14	5.60%	4.00%	0.30%	0.42%	8.74%	3.15%	4.58%
OMXS30	09/04/14	1.16%	1.65%	0.29%	0.40%	2.93%	1.21%	1.67%
OMXS30	07/05/14	1.07%	1.69%	0.26%	0.37%	4.74%	1.79%	2.56%
S&P 350 Europe Low Volatility	27/08/14	2.89%	0.66%	0.12%	0.23%	8.23%	2.86%	4.23%
OMXS30	28/08/14	0.93%	0.62%	0.11%	0.23%	3.84%	1.39%	2.03%
S&P 350 Europe Low Volatility	03/12/14	2.06%	0.21%	0.04%	0.16%	4.85%	1.68%	2.50%
OMXS30	03/12/14	0.23%	0.13%	0.04%	0.16%	0.23%	0.14%	0.20%

In our sample period the Swedish government bonds and the bonds from the issuing bank yielded low returns, because of the low interest rates at the time. An investor knowing about these low returns ex-ante can alternatively choose to participate in equity derivatives and take advantage of the low call options prices that result from the low interest rates and volatility in the market at the time. This indicates that an investment in a safe PPN could be a good option for a risk adverse retail investor.

5.3.2.2 Return Comparison for Risky Index PPNs

Depicted in table 5.9 are the holding period returns comparison with the risky PPNs. In this case the PPNs with risky characteristic generated the highest HPR. The risky PPN generates a higher return than a long position in the underlying, given that the risky PPN has a participation rate larger than 100%, coming from the exposure level of the derivative entrenched. This alternative additionally, includes some downside risk protection, where in the case of a large market sell-off the investor could still recover a large part of his invested capital.

Table 5.9: HPR and alternative investment sources to risky PPN. Shows the annualized HPR for risky PPN and alternative investment sources. In column 1, we observe the underlying asset, in column 2 the issue date, in column 3 the HPR obtained from the PPN, in column 4 the HPR obtained from the replication, in column 5 the HPR from an investment in a Swedish zero-coupon bond, in column 6 the HPR obtained from investment in a zero-coupon bond form the issuing bank, in column 7 the HPR obtained from an investment in the underlying equity index, in column 8 the HPR obtained from investment in an equally weighted portfolio of the three alternative investment sources and in column 9 the HPR obtained from an investment in 50% risky debt and 50% in the equity index.

Index	Issue Date	Bank Annual R	Rep. Annual R	Alt. Port. 1 Risk Free	Alt. Port. 2 Low Risk	Alt. Port. 3 Equity Index	Alt. Port. 4 Balanced	Alt. Port. 5 Debt + Index
OMXS30	17/02/11	5.99%	7.26%	0.63%	0.77%	4.75%	2.05%	2.76%
S&P 500	26/05/11	4.56%	4.28%	0.78%	0.92%	6.52%	2.74%	3.72%
OMXS30	20/12/12	6.58%	8.80%	0.21%	0.35%	9.34%	3.30%	4.84%
S&P 500 Low Volatility	07/02/13	18.09%	18.55%	0.27%	0.41%	14.46%	5.05%	7.44%
OMXS30	07/05/13	6.98%	7.18%	0.26%	0.39%	6.33%	2.33%	3.36%
S&P 500	03/06/13	12.20%	11.81%	0.30%	0.43%	19.87%	6.87%	10.15%
S&P Nordic Low Volatility	10/01/14	11.94%	13.74%	0.34%	0.47%	8.38%	3.06%	4.43%
S&P 350 Europe Low Volatility	05/02/14	13.79%	15.41%	0.31%	0.45%	9.40%	3.38%	4.92%
Hang Seng	03/03/14	15.13%	13.90%	0.29%	0.43%	15.96%	5.56%	8.19%
S&P 350 Europe Low Volatility	05/03/14	12.12%	14.12%	0.30%	0.42%	8.74%	3.15%	4.58%
OMXS30	09/04/14	5.10%	4.99%	0.29%	0.40%	2.93%	1.21%	1.67%
OMXS30	07/05/14	5.55%	5.47%	0.26%	0.37%	4.74%	1.79%	2.56%
S&P 350 Europe Low Volatility	27/08/14	9.91%	14.15%	0.12%	0.23%	8.23%	2.86%	4.23%
OMXS30	28/08/14	3.33%	3.96%	0.11%	0.23%	3.84%	1.39%	2.03%
S&P 350 Europe Low Volatility	03/12/14	6.71%	11.37%	0.04%	0.16%	4.85%	1.68%	2.50%
OMXS30	03/12/14	1.65%	2.19%	0.04%	0.16%	0.23%	0.14%	0.20%

5.3.2.3 Return Comparison for Safe and Risky Basket PPNs

The results from the basket options are more mixed than the presented by their index counterparts, however they still show a tendency in the same direction (table 5.10) The basket PPNs are in average more profitable than a mixed portfolio of equity and debt, Additionally, investing in a PPN with a basket option can be a way for an investor to invest in multiple indices or equities at a lower cost and hedge its position. Which makes this alternative attractive for a type of investors seeking for an investment with these special features.

Table 5.10: HPR and alternative investment sources to basket PPN. Shows the annualized HPR for basket PPN and alternative investment sources. In column 1 we observe the type of option displayed. In column 2, we observe the underlying asset, in column 3 the issue date, in column 4 the HPR obtained from the PPN, in column 5 the HPR obtained from the replication, in column 6 the HPR from an investment in a Swedish zero-coupon bond, in column 7 the HPR obtained from investment in a zero-coupon bond form the issuing bank, in column 8 the HPR obtained from an investment in the underlying equity index, in column 9 the HPR obtained from investment in an equally weighted portfolio of the three alternative investment sources and in column 10 the HPR obtained from an investment in 50 % risky debt and 50% in the equity index

Type	Index	Issue Date	Bank Annual R	Rep. Annual R	Alt. Port. 1 Risk Free	Alt. Port. 2 Low Risk	Alt. Port. 3 Equity Index	Alt. Port. 4 Balanced	Alt. Port. 5 Debt + Index
Basket Safe	Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/12	4.03%	1.12%	0.23%	0.35%	10.09%	3.56%	5.22%
Basket Safe	DAX, Hang Seng, OMXS30 & S&P 500	08/07/13	5.78%	4.20%	0.33%	0.50%	20.50%	7.11%	10.50%
Basket Safe	MSCI Singapore & Taiwan Stock Exchange Index	09/04/14	-0.96%	0.00%	0.32%	0.46%	10.91%	3.90%	5.68%
Basket Risky	Hang Seng, MSCI Singapore & MSCI Taiwan	23/11/12	11.55%	6.02%	0.23%	0.35%	10.09%	3.56%	5.22%
Basket Risky	DAX, Hang Seng, OMXS30 & S&P 500	08/07/13	13.65%	11.62%	0.33%	0.50%	20.50%	7.11%	10.50%
Basket Risky	MSCI Singapore & Taiwan Stock Exchange Index	09/04/14	-0.94%	0.00%	0.32%	0.46%	10.91%	3.90%	5.68%
Basket Risky	MSCI Singapore & MSCI Taiwan	10/09/15	10.78%	13.26%	-0.10%	0.05%	11.79%	3.91%	5.92%
Basket Risky	MSCI Singapore & MSCI Taiwan	08/10/15	11.17%	3.30%	-0.10%	0.11%	10.80%	3.60%	5.45%

An important consideration is the composition of the investor’s combined portfolio. If he complements his personal portfolio with other assets providing liquidity, and to some extent lowly correlated returns to the PPN; he will be affected to a lesser extent to the disadvantages of the PPNs. Such asset classes might come in the form of money market products, equity investments, and fixed income products. Other types of alternative investments could also provide diversification benefits, nevertheless this would be less appropriate given their correlated risks and investment characteristics.

Another important consideration to weight in the investing decision process for PPNs is the possible tax benefit, inherent from bundling up the bond with the equity product. Since the profit from investing in a bond and an equity index separately would cost the investor a higher tax payment (the investor would need to pay tax on the interest received from the bond in addition to the tax on the equity index capital gains). With the PPN structure the investor can save on the tax payment for the bond interest and use that capital to further benefit on the equity investment. Allowing for a higher after-tax return in the overall investment portfolio.

The suitability of PPN by a retail investor will depend in large part on the market conditions, the risk profile, market view, liquidity needs, taxable status, access level to derivative markets hedging opportunities, and current existing portfolio. A retail investor under certain circumstances could benefit from the use of structure products as an investment vehicle.

We define an ideal retail investor profile with the following characteristics and under the following market conditions:

- Knowledgeable about the financial markets and products
- Positive market view in the underlying
- Limited access to derivative products, and desire to hedge/enhance return
- Scarce (or none) sources of competitive low rate borrowing
- No liquidity needs tied to the structured product
- Investment horizon from 3 to 5 years
- Taxable status that may benefit from the structure of the instrument
- Diversified portfolio, with other highly liquid securities and low correlation with the underlying asset of the structured product

This indicates that when the market conditions are right, certain type of retail investors with a specific market view (time horizon and asset class), and restricted access to derivative products, may benefit from an investment strategy involving principal protected notes.

6 Conclusion

6.1 Conclusion of the analysis

There are different types of investors in the market, with different needs and characteristics, and a group of them can benefit from investing in structured products. Such as retail investors with diversified portfolios, who satisfy their liquidity needs from alternative sources, and with restricted access to levered market positions and direct hedging. These kind of investors can successfully incorporate PPNs to their portfolios, and benefit in a way they could hardly do without them.

In a market environment with low interest rates, and moderate volatility, -and consequently low hedging costs-, an investor with the previously mentioned characteristics can benefit from the use of these hybrid notes and take advantage of the benefits of low hedging costs if that’s his wish. Since, the potential enhancing yields and/or low hedging costs that can be achieved with the use of PPN would have been very difficult to achieve with a mixed portfolio of bonds and equities.

Our analysis presents economic tendencies of overpricing for index PPNs, which indicates that retail investors with the ability to replicate the index options, or access to derivative products with low transaction costs, should overweight the possibility of mimicking the PPN themselves instead of buying it directly from a dealer.

The investment in PPN is exposed to credit, currency and liquidity risk. From our analysis of credit risk, we can determine that retail investors in the years 2012-2013 were more exposed to this specific risk factor. Nevertheless, given to the high credit rating of the issuing institutions, this is not an important undermining factor for the investment in these products. A retail investor investing in PPN with the underlying issued in a foreign currency will additionally experience a currency risk exposure, which will also determine the final payoff of the PPN. Furthermore, liquidity risk will be a part of the risks associated with the PPNs, since the products are often mid to long term investment vehicles and the secondary market for these is not very liquid. Depending on the circumstances and motivation of the retail investor these risk factors can act to the detriment of his investment.

Investing in the Swedish PPNs market in the period from 2011-2015, generated higher returns than other alternative portfolio allocations combining riskless debt, investment grade debt, and the desired underlying equity assets. Yet additionally, the PPN offered a benefit of capital protection for the holder. Moreover, by bundling together a taxable zero-coupon bond and the equity instrument, the retail investor can benefit from higher after-tax returns than if they would invest in both securities separately, enhancing his after-tax return.

When market conditions are adequate PPNs can be efficient investment vehicles for certain types of retail investors. The PPNs in the sample period generated returns higher or comparable to that of alternative benchmarks, which emphasises the claim that structured equity products can be efficient investing vehicles under certain conditions. Because of this, retail investors should not neglect the opportunities arose from investing in hybrid products with derivatives when assessing investment alternatives.

6.2 Recommended further research on the topic

There are several types of structured products in the Swedish market, with the largest group consisting of principal protected notes. For further research we recommend using a larger sample size collected for several years to obtain more robust results, since the amount of product issued every year are very limited.

Hence, our analysis might be exposed to some degree of time-period bias. Additionally, we recommend the development of a multivariate GARCH, and/or the use of the implied volatilities at the time of issuance, to assess the volatility parameter in the basket options. Our sample is collected among expired products, an alternative could be instead to use newly issued notes, and calculate the expected return for those. Our research might also have experienced problems of sample-selection bias where particular attributes of the products could have been systematically excluded due to lack of sufficient data availability. This problem could be even greater when performing research on active PPNs. An alternative approach could be to perform research on the variety of different structured products that exist on the Swedish market, to analyse if there are common attributes among these products that could be assessed further.

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8 Appendix

8.1 MATLAB code pricing Principal protected notes

8.1.1 MATLAB code pricing Principal protected notes with index option

Load Data

```
clear;
close all;
clc;
% Load data and set Inputs
load Underlying_Data_1
load TS_Data1
load FX_Data
load DY_Data1
```

Inputs

```
Underlying = OMX; % Index or Stock
CCY = 'SEK'; % Currency of the underlying (SEK, USD, EUR, JPY & HKD)
Iterations = 1000000; % 1,000,000 iterations
S0 = 100;  % Spot Price
% Dates
Start_Date = datetime('07-May-2013', 'InputFormat', 'd-MMM-y');
Maturity_Date = datetime('23-Apr-2018', 'InputFormat', 'd-MMM-y');
Obs_Dates = {'23-Apr-2017';'23-May-2017';'23-Jun-2017';'23-Jul-2017';'23-Aug-
2017';'23-Sep-2017';'23-Oct-2017';'23-Nov-2017';'23-Dec-2017';'23-Jan-2018';'23-
Feb-2018';'23-Mar-2017';'23-Apr-2017'}; % Observation Dates

idx_SD = find(Data_Date==Start_Date); % Date Index for S0
TMY = (days(Maturity_Date - Start_Date)+1)/365; % Time in natural years
Dif_Start_Mat = days252bus(Start_Date, Maturity_Date); %Number of sim to Maturity
Dif_Start_Obs = days252bus(Start_Date, Obs_Dates)+1; %Number of sim to Obs
```

FX, DY and RF

```
idx_FX_TS = find(FX_Dates==Start_Date); % FX rate routine
if strcmp(CCY,'USD') == 1
    FX = FX_USD(idx_FX_TS,:);
elseif strcmp(CCY,'HKD') == 1
```

```
    FX = FX_HKD(idx_FX_TS,:);
elseif strcmp(CCY,'JPY') == 1
    FX = FX_JPY(idx_FX_TS,:);
elseif strcmp(CCY,'EUR') == 1
    FX = FX_EUR(idx_FX_TS,:);
else
    FX = 1;
end

idx_SD_TS = find(TS_Dates==Start_Date); % Risk free rate routine
if strcmp(CCY,'USD') == 1
    rf = TS_USA(idx_SD_TS,round(days(Maturity_Date - Start_Date)/365));
elseif strcmp(CCY,'HKD') == 1
    rf = TS_HongKong(idx_SD_TS,round(days(Maturity_Date - Start_Date)/365));
elseif strcmp(CCY,'JPY') == 1
    rf = TS_Japan(idx_SD_TS,round(days(Maturity_Date - Start_Date)/365));
elseif strcmp(CCY,'EUR') == 1
    rf = TS_Germany(idx_SD_TS,round(days(Maturity_Date - Start_Date)/365));
else
    rf = TS_Sweden(idx_SD_TS,round(days(Maturity_Date - Start_Date)/365));
end

idx_DY = find(year(Start_Date)==DY_Years); % Annual historical dividend Yield
Routine
if Underlying(1,1) == OMX(1,1)
    DY = mean(DY_OMX(1:idx_DY-1,:));
elseif Underlying(1,1) == SP500(1,1)
    DY = mean(DY_SP500(1:idx_DY-1,:));
elseif Underlying(1,1) == HANG_SENG(1,1)
    DY = mean(DY_HS(1:idx_DY-1,:));
elseif Underlying(1,1) == NIKKEI(1,1)
    DY = mean(DY_NIKK(1:idx_DY-1,:));
elseif Underlying(1,1) == DAX(1,1)
    DY = mean(DY_DAX(1:idx_DY-1,:));
elseif Underlying(1,1) == CAC40(1,1)
    DY = mean(DY_CAC40(1:idx_DY-1,:));
elseif Underlying(1,1) == FTSE(1,1)
    DY = mean(DY_FTSE(1:idx_DY-1,:));
elseif Underlying(1,1) == EUROSTOXX(1,1)
    DY = mean(DY_EURO(1:idx_DY-1,:));
elseif Underlying(1,1) == MSCI_WORLD(1,1)
    DY = mean(DY_WORLD(1:idx_DY-1,:));
else
    DY = 0;
end
clear ans

clearvars -except Underlying CCY S0 TMY Iterations Start_Date Maturity_Date...
    AvgDate idx_SD FX rf Dif_Start_Mat Dif_Start_Obs DY
```

Statistics

```
Hist_Und = Underlying(1:idx_SD-1,1); % Historical Prices Underlying
Hist_Und = rmmissing(Hist_Und); % Remove missing data
Ret_Hist = diff(log(Hist_Und)); % Historical Returns
Mean_R = mean(Ret_Hist); % Mean historical daily Return
Mean_R_Y = Mean_R*(252^(1/2)); % Mean historical annual Return
Vol_Hist = std(Ret_Hist); % Index Historical Daily Volatility
Vol_Hist_Y = Vol_Hist*(252^(1/2)); % Annualized volatility
```

```
Z = norminv(0.95);
clear Hist_Und Underlying ans
```

Volatility Modelling Econ App 1

```
econometricModeler % We first test for heteroscedasticity, then we model with
                   % GARCH(1,1), EGARCH(1,1) and GJR(1,1), compare them and
                   % select the one with the lowest AIC and BIC.
```

Volatility Modelling Econ App 2

```
if  exist('GJR_Ret_Hist1','var')
        vol_mdl=GJR_Ret_Hist1;
    elseif exist('GARCH_Ret_Hist1','var')
        vol_mdl=GARCH_Ret_Hist1;
    else ,exist('EGARCH_Ret_Hist1','var')
        vol_mdl=EGARCH_Ret_Hist1;
end
Vol_Inf = infer(vol_mdl,Ret_Hist); % Infer conditional variance
[V,Y] = simulate(vol_mdl,252,'NumPaths',Iterations, 'E0', Ret_Hist, 'V0',
Vol_Inf);
F_Vol_D = mean(std(Y)); % Forecasted Daily Volatility
F_Vol_Y = mean(std(Y))*(252^(1/2)); % Forecasted Anual Volatility
```

Geometric Brownian Motion Simulation

```
Vol = F_Vol_Y; % Volatility model
r = rf-DY; % Daily Return for pricing %Mean_R;
deltaT = TMY/Dif_Start_Mat; %Time step
K = S0; % Strike price
Mu = r; % r
Qty_Und = 1;
Steps = Dif_Start_Mat;
Sim_Prices = zeros(Steps+1,Iterations);
for i=(1:Iterations)
    Rand_Draw = randn(Steps,Qty_Und); % Random draw of a normal value
    Eps = Rand_Draw; % Random number adjusted by cov
    Sim_Prices(:,i) = [ones(1,1);cumprod(exp(repmat((Mu...
        -Vol.*Vol/2)*deltaT,Steps,1)+Eps...
        *Vol*sqrt(deltaT)))]*S0; %GBM Paths
end
```

Option Pricing

```
Sim_Obs = Sim_Prices(Dif_Start_Obs,:);                  % Retrieve Data from Obs
Dates
Sim_ObsT = max([mean(Sim_Obs)-K;zeros(1,Iterations)]);  % Option Pay-off per node
MC1_Payoff_Call = mean(Sim_ObsT);                       % Option Pay-off
Call_Price_MonteCarlo = MC1_Payoff_Call*exp(-rf*TMY);   % Price Computation
Conf_Int1 = (MC1_Payoff_Call - Z*(std(Sim_ObsT)/sqrt(Iterations)))*exp(-rf*TMY);
Conf_Int2 = (MC1_Payoff_Call + Z*(std(Sim_ObsT)/sqrt(Iterations)))*exp(-rf*TMY);
```

Monte Carlo Plain Vanilla European Asian Option

```
Mean_Sim_Prices = max([mean(Sim_Prices)-K;zeros(1,Iterations)]);
MCA_PO = mean(Mean_Sim_Prices);
MCA_Pr = MCA_PO*exp(-rf*TMY);
```

PPN Pricing

```
Zero_Bond = 100*exp(-rf*TMY); Dif_100_Zero = 100-Zero_Bond; % Bond Pricing
Qty_Opt = Dif_100_Zero/Call_Price_MonteCarlo; % Participation Rate
```

Other models

```
RateSpec = intenvset('ValuationDate', Start_Date, 'StartDates', Start_Date, ...
'EndDates', Maturity_Date,'Rates', rf,  'Compounding', -1, 'Basis', 1);
StockSpec = stockspec(Vol, S0, 'continuous', DY);

    % Black and Scholes
[Call_BS] = blsprice(S0,K,rf,TMY,Vol,DY); % European Call with Black and Scholes


    % European geometric Average Price for the Asian option using the Kemna-Vorst
model
Call_KV = asiansensbykv(RateSpec, StockSpec, 'call', K, Start_Date,
Maturity_Date);

    % European arithmetic average price for the Asian option using the Levy model
Call_Levy = asianbylevy(RateSpec, StockSpec, 'call', K, Start_Date,
Maturity_Date);

    % Monte Carlo Model 2

OptSpec = 'call';                    % Call option
OptionGBM = gbm(r, Vol, 'StartState', 1); % Geometric Brownian Motion
[Paths, Times] = simBySolution(OptionGBM, Dif_Start_Mat, ...
'NTRIALS',Iterations, 'DeltaTime',deltaT,'Antithetic',true);
simPrice0 = squeeze(Paths);
TimesonAvgDate = Times(Dif_Start_Obs,:);

Times =  [0; TimesonAvgDate(end)];
simPriceOnAvgDate = simPrice0(Dif_Start_Obs,:); % Prices on average dates

% Pre-calculate premium of options
simPrice1 = [ones(1,Iterations); mean(simPriceOnAvgDate)];
Call_MC2 = optpricebysim(RateSpec, simPrice1 * S0, Times, OptSpec, K, TMY);
Qty_Opt2 = Dif_100_Zero/Call_MC2; % Participation Rate
```

```
clearvars -except Call_MC2 Call_Levy Call_KV Call_BS Call_Price_MonteCarlo...
    MCA_Pr
```

8.1.2 MATLAB code pricing Principal protected notes with basket option

Load Data

```
clear;
close all;
clc;
```

```
% Load data and set Inputs
load Underlying_Data_1
load TS_Data1
load FX_Data
load DY_Data1
```

Inputs

```
Underlying1 = SP500; % Index or Stock
Underlying2 = OMX;   % Index or Stock
Underlying3 = DAX;   % Index or Stock
Underlying4 = HANG_SENG; % Index or Stock

CCY1 = 'USD'; % Currency of the underlying (SEK, USD, EUR, JPY & HKD)
CCY2 = 'SEK'; % Currency of the underlying (SEK, USD, EUR, JPY & HKD)
CCY3 = 'EUR'; % Currency of the underlying (SEK, USD, EUR, JPY & HKD)
CCY4 = 'HKD'; % Currency of the underlying (SEK, USD, EUR, JPY & HKD)

Iterations = 250000; % 250,000 iterations
Qty_Und = 4; % Number of Underlying assets
S0 = 100;  % Underlying(idx_SD); % Spot Price
K = S0; % Strike price
% Dates
Start_Date = datetime('08-Jul-2013', 'InputFormat', 'd-MMM-y'); %Settlement Date
Maturity_Date = datetime('25-Jun-2018', 'InputFormat', 'd-MMM-y'); %Maturity Date
Obs_Dates = {'25-Jun-2017';'25-Jul-2017';'25-Aug-2017';'25-Sep-2017';...
    '25-Oct-2017';'25-Nov-2017';'25-Dec-2017';'25-Jan-2018';'25-Feb-2018';...
    '25-Mar-2018';'25-Apr-2018';'25-May-2018';'25-Jun-2018'}; % Observation Dates
idx_SD = find(Data_Date==Start_Date); % Date Index for S0
TMY = (days(Maturity_Date - Start_Date)+1)/365; % Time in natural years
Dif_Start_Mat = days252bus(Start_Date, Maturity_Date); %Number of sim to Maturity
Dif_Start_Obs = days252bus(Start_Date, Obs_Dates)+1; %Number of sim to Obs
```

FX, DY and RF

```
% RF - Risk Free Rate
idx_SD_TS = find(TS_Dates==Start_Date); % Risk free rate routine

if strcmp(CCY1,'USD') == 1
    rf1 = TS_USA(idx_SD_TS,round(TMY));
elseif strcmp(CCY1,'HKD') == 1
    rf1 = TS_HongKong(idx_SD_TS,round(TMY));
elseif strcmp(CCY1,'JPY') == 1
    rf1 = TS_Japan(idx_SD_TS,round(TMY));
elseif strcmp(CCY1,'EUR') == 1
    rf1 = TS_Germany(idx_SD_TS,round(TMY));
else
    rf1 = TS_Sweden(idx_SD_TS,round(TMY));
end

if strcmp(CCY2,'USD') == 1
    rf2 = TS_USA(idx_SD_TS,round(TMY));
elseif strcmp(CCY2,'HKD') == 1
    rf2 = TS_HongKong(idx_SD_TS,round(TMY));
elseif strcmp(CCY2,'JPY') == 1
    rf2 = TS_Japan(idx_SD_TS,round(TMY));
elseif strcmp(CCY2,'EUR') == 1
    rf2 = TS_Germany(idx_SD_TS,round(TMY));
```

```
else
    rf2 = TS_Sweden(idx_SD_TS,round(TMY));
end

if strcmp(CCY3,'USD') == 1
    rf3 = TS_USA(idx_SD_TS,round(TMY));
elseif strcmp(CCY3,'HKD') == 1
    rf3 = TS_HongKong(idx_SD_TS,round(TMY));
elseif strcmp(CCY3,'JPY') == 1
    rf3 = TS_Japan(idx_SD_TS,round(TMY));
elseif strcmp(CCY3,'EUR') == 1
    rf3 = TS_Germany(idx_SD_TS,round(TMY));
else
    rf3 = TS_Sweden(idx_SD_TS,round(TMY));
end

if strcmp(CCY4,'USD') == 1
    rf4 = TS_USA(idx_SD_TS,round(TMY));
elseif strcmp(CCY4,'HKD') == 1
    rf4 = TS_HongKong(idx_SD_TS,round(TMY));
elseif strcmp(CCY4,'JPY') == 1
    rf4 = TS_Japan(idx_SD_TS,round(TMY));
elseif strcmp(CCY4,'EUR') == 1
    rf4 = TS_Germany(idx_SD_TS,round(TMY));
else
    rf4 = TS_Sweden(idx_SD_TS,round(TMY));
end

% DY - Dividend Yield
idx_DY = find(year(Start_Date)==DY_Years); % Annual historical dividend Yield
Routine

if Underlying1(1,1) == OMX(1,1)
    DY1 = mean(DY_OMX(1:idx_DY-1,:));
elseif Underlying1(1,1) == SP500(1,1)
    DY1 = mean(DY_SP500(1:idx_DY-1,:));
elseif Underlying1(1,1) == HANG_SENG(1,1)
    DY1 = mean(DY_HS(1:idx_DY-1,:));
elseif Underlying1(1,1) == NIKKEI(1,1)
    DY1 = mean(DY_NIKK(1:idx_DY-1,:));
elseif Underlying1(1,1) == DAX(1,1)
    DY1 = mean(DY_DAX(1:idx_DY-1,:));
elseif Underlying1(1,1) == CAC40(1,1)
    DY1 = mean(DY_CAC40(1:idx_DY-1,:));
elseif Underlying1(1,1) == FTSE(1,1)
    DY1 = mean(DY_FTSE(1:idx_DY-1,:));
elseif Underlying1(1,1) == EUROSTOXX(1,1)
    DY1 = mean(DY_EURO(1:idx_DY-1,:));
elseif Underlying1(1,1) == MSCI_WORLD(1,1)
    DY1 = mean(DY_WORLD(1:idx_DY-1,:));
else
    DY1 = 0;
end

if Underlying2(1,1) == OMX(1,1)
    DY2 = mean(DY_OMX(1:idx_DY-1,:));
elseif Underlying2(1,1) == SP500(1,1)
    DY2 = mean(DY_SP500(1:idx_DY-1,:));
elseif Underlying2(1,1) == HANG_SENG(1,1)
    DY2 = mean(DY_HS(1:idx_DY-1,:));
```

```
elseif Underlying2(1,1) == NIKKEI(1,1)
    DY2 = mean(DY_NIKK(1:idx_DY-1,:));
elseif Underlying2(1,1) == DAX(1,1)
    DY2 = mean(DY_DAX(1:idx_DY-1,:));
elseif Underlying2(1,1) == CAC40(1,1)
    DY2 = mean(DY_CAC40(1:idx_DY-1,:));
elseif Underlying2(1,1) == FTSE(1,1)
    DY2 = mean(DY_FTSE(1:idx_DY-1,:));
elseif Underlying2(1,1) == EUROSTOXX(1,1)
    DY2 = mean(DY_EURO(1:idx_DY-1,:));
elseif Underlying2(1,1) == MSCI_WORLD(1,1)
    DY2 = mean(DY_WORLD(1:idx_DY-1,:));
else
    DY2 = 0;
end

if Underlying3(1,1) == OMX(1,1)
    DY3 = mean(DY_OMX(1:idx_DY-1,:));
elseif Underlying3(1,1) == SP500(1,1)
    DY3 = mean(DY_SP500(1:idx_DY-1,:));
elseif Underlying3(1,1) == HANG_SENG(1,1)
    DY3 = mean(DY_HS(1:idx_DY-1,:));
elseif Underlying3(1,1) == NIKKEI(1,1)
    DY3 = mean(DY_NIKK(1:idx_DY-1,:));
elseif Underlying3(1,1) == DAX(1,1)
    DY3 = mean(DY_DAX(1:idx_DY-1,:));
elseif Underlying3(1,1) == CAC40(1,1)
    DY3 = mean(DY_CAC40(1:idx_DY-1,:));
elseif Underlying3(1,1) == FTSE(1,1)
    DY3 = mean(DY_FTSE(1:idx_DY-1,:));
elseif Underlying3(1,1) == EUROSTOXX(1,1)
    DY3 = mean(DY_EURO(1:idx_DY-1,:));
elseif Underlying3(1,1) == MSCI_WORLD(1,1)
    DY3 = mean(DY_WORLD(1:idx_DY-1,:));
else
    DY3 = 0;
end

if Underlying4(1,1) == OMX(1,1)
    DY4 = mean(DY_OMX(1:idx_DY-1,:));
elseif Underlying4(1,1) == SP500(1,1)
    DY4 = mean(DY_SP500(1:idx_DY-1,:));
elseif Underlying4(1,1) == HANG_SENG(1,1)
    DY4 = mean(DY_HS(1:idx_DY-1,:));
elseif Underlying4(1,1) == NIKKEI(1,1)
    DY4 = mean(DY_NIKK(1:idx_DY-1,:));
elseif Underlying4(1,1) == DAX(1,1)
    DY4 = mean(DY_DAX(1:idx_DY-1,:));
elseif Underlying4(1,1) == CAC40(1,1)
    DY4 = mean(DY_CAC40(1:idx_DY-1,:));
elseif Underlying4(1,1) == FTSE(1,1)
    DY4 = mean(DY_FTSE(1:idx_DY-1,:));
elseif Underlying4(1,1) == EUROSTOXX(1,1)
    DY4 = mean(DY_EURO(1:idx_DY-1,:));
elseif Underlying4(1,1) == MSCI_WORLD(1,1)
    DY4 = mean(DY_WORLD(1:idx_DY-1,:));
else
    DY4 = 0;
end
```

Statistics / Cholesky and Var-Cov Matrix

```
% Calculate the daily log returns and the statistics

% Underlying1
S0_1 =Underlying1(idx_SD);
Hist_Und1 = Underlying1(1:idx_SD-1,1); % Historical Prices Underlying
Hist_Returns1 = diff(log(Hist_Und1)); % Historical Returns
Hist_Returns1(isnan(Hist_Returns1))=0; % Change NaN for 0
Mean_R1 = mean(Hist_Returns1); % Mean historical daily Return
Vol1 = std(Hist_Returns1); % Index Historical Volatility

% Underlying2
S0_2 =Underlying2(idx_SD);
Hist_Und2 = Underlying2(1:idx_SD-1,1);
Hist_Returns2 = diff(log(Hist_Und2)); % Historical Returns
Hist_Returns2(isnan(Hist_Returns2))=0; % Change NaN for 0
Mean_R2 = mean(Hist_Returns2); % Mean historical daily Return
Vol2 = std(Hist_Returns2); % Index Historical Volatility

% Underlying3
S0_3 =Underlying3(idx_SD);
Hist_Und3 = Underlying3(1:idx_SD-1,1);
Hist_Returns3 = diff(log(Hist_Und3)); % Historical Returns
Hist_Returns3(isnan(Hist_Returns3))=0; % Change NaN for 0
Mean_R3 = mean(Hist_Returns3); % Mean historical daily Return
Vol3 = std(Hist_Returns3); % Index Historical Volatility

% Underlying4
S0_4 =Underlying4(idx_SD);
Hist_Und4 = Underlying4(1:idx_SD-1,1);
Hist_Returns4 = diff(log(Hist_Und4)); % Historical Returns
Hist_Returns4(isnan(Hist_Returns4))=0; % Change NaN for 0
Mean_R4 = mean(Hist_Returns4); % Mean historical daily Return
Vol4 = std(Hist_Returns4); % Index Historical Volatility

Mat_Hist_R = [Hist_Returns1 Hist_Returns2 Hist_Returns3 Hist_Returns4];
Mat_Mean_R = mean(Mat_Hist_R)*(252^(1/2)); % Mean Annualized return Matrix
Mat_Mu = ([rf1-DY1,rf2-DY2,rf3-DY3,rf4-DY4])/(252^(1/2)); % Negative annual Mu
Mat_Vol_R = std(Mat_Hist_R)*(252^(1/2)); %Annualized Volatility Matrix
Mat_Corr = corrcoef(Mat_Hist_R); %Correlation Matrix
Mat_Chol = chol(Mat_Corr); %Cholesky decomposition from correlation Matrix

Mat_S0 = ones(1,Qty_Und)*S0; %S0 Matrix, for underlying assets
% Mat_S0 = [S0_1,S0_2,S0_3,S0_4]; "Alternative"
deltaT = TMY/Dif_Start_Mat; %Time step
Z = norminv(0.95);
```

Geometric Brownian Motion

```
Sim_Prices = zeros(Dif_Start_Mat+1,Iterations,Qty_Und);
for i=(1:Iterations)
    Rand_Draw = randn(Dif_Start_Mat,Qty_Und); % Random draw of a normal value
    Rand_Cov_Adj = Rand_Draw * Mat_Chol; % Random number adjusted by cov
    Sim_Prices(:,i,:) = [ones(1,Qty_Und);cumprod(exp(repmat((Mat_Mu...
        -Mat_Vol_R.*Mat_Vol_R/2)*deltaT,Dif_Start_Mat,1)+Rand_Cov_Adj...
```

```
        *diag(Mat_Vol_R)*sqrt(deltaT)))]*diag(Mat_S0); %GBM Paths
end

Sim_Und1 = Sim_Prices(:,:,1); % Simulation Underlying 1
Sim_Und2 = Sim_Prices(:,:,2); % Simulation Underlying 2
Sim_Und3 = Sim_Prices(:,:,3); % Simulation Underlying 3
Sim_Und4 = Sim_Prices(:,:,4); % Simulation Underlying 4
```

Option Pricing

```
Und1 = max([mean(Sim_Und1(Dif_Start_Obs,:))-K;zeros(1,Iterations)]); %Call Pay-off
per node
Und1_PayOff = mean(Und1); % Call Option pay-off for underlying 1
Und1_Price = Und1_PayOff*exp(-rf1*TMY); % Call option 1 at the money price

% Underlying 2
Und2 = max([mean(Sim_Und2(Dif_Start_Obs,:))-K;zeros(1,Iterations)]); %Call Pay-off
per node
Und2_PayOff = mean(Und2); % Call Option pay-off for underlying 1
Und2_Price = Und2_PayOff*exp(-rf2*TMY); % Call option 1 at the money price

% Underlying 3
Und3 = max([mean(Sim_Und3(Dif_Start_Obs,:))-K;zeros(1,Iterations)]); %Call Pay-off
per node
Und3_PayOff = mean(Und3); % Call Option pay-off for underlying 1
Und3_Price = Und3_PayOff*exp(-rf3*TMY); % Call option 1 at the money price

% Underlying 4
Und4 = max([mean(Sim_Und4(Dif_Start_Obs,:))-K;zeros(1,Iterations)]); %Call Pay-off
per node
Und4_PayOff = mean(Und4); % Call Option pay-off for underlying 1
Und4_Price = Und4_PayOff*exp(-rf4*TMY); % Call option 1 at the money price

% Basket Option Price
Call_Basket_Option = mean([Und1_Price,Und2_Price,Und3_Price,Und4_Price]);
```

PPN Pricing

```
rf = mean([rf1,rf2,rf3,rf4]); % Weighted averaged risk free rate
Zero_Bond = 100*exp(-rf*TMY); Dif_100_Zero = 100-Zero_Bond; % Bond Pricing
Qty_Opt = Dif_100_Zero/Call_Basket_Option; % Participation Rate

% Show results
disp("Call Basket Option Price Monte Carlo"); disp(Call_Basket_Option);
disp("Participation Rate MC"); disp(Qty_Opt);
```

Asian Option

```
Und1_A = max([mean(Sim_Und1)-K;zeros(1,Iterations)]); %Call Pay-off per node
Und1_PayOff_A = mean(Und1_A); % Call Option pay-off for underlying 1
Und1_Price_A = Und1_PayOff_A*exp(-rf1*TMY); % Call option 1 at the money price
Und1_Conf_Int1 = (Und1_PayOff_A - Z*(std(Und1_A)/sqrt(Iterations)))*exp(-rf*TMY);
Und1_Conf_Int2 = (Und1_PayOff_A + Z*(std(Und1_A)/sqrt(Iterations)))*exp(-rf*TMY);
```

```
% Underlying 2
Und2_A = max([mean(Sim_Und2)-K;zeros(1,Iterations)]); %Call Pay-off per node
Und2_PayOff_A = mean(Und2_A); % Call Option pay-off for underlying 1
Und2_Price_A = Und2_PayOff_A*exp(-rf2*TMY); % Call option 1 at the money price
Und2_Conf_Int1 = (Und2_PayOff_A - Z*(std(Und2_A)/sqrt(Iterations)))*exp(-rf*TMY);
Und2_Conf_Int2 = (Und2_PayOff_A + Z*(std(Und2_A)/sqrt(Iterations)))*exp(-rf*TMY);

% Underlying 3
Und3_A = max([mean(Sim_Und3)-K;zeros(1,Iterations)]); %Call Pay-off per node
Und3_PayOff_A = mean(Und3_A); % Call Option pay-off for underlying 1
Und3_Price_A = Und3_PayOff_A*exp(-rf3*TMY); % Call option 1 at the money price
Und3_Conf_Int1 = (Und3_PayOff_A - Z*(std(Und3_A)/sqrt(Iterations)))*exp(-rf*TMY);
Und3_Conf_Int2 = (Und3_PayOff_A + Z*(std(Und3_A)/sqrt(Iterations)))*exp(-rf*TMY);

% Underlying 4
Und4_A = max([mean(Sim_Und4)-K;zeros(1,Iterations)]); %Call Pay-off per node
Und4_PayOff_A = mean(Und4_A); % Call Option pay-off for underlying 1
Und4_Price_A = Und4_PayOff_A*exp(-rf4*TMY); % Call option 1 at the money price
Und4_Conf_Int1 = (Und4_PayOff_A - Z*(std(Und4_A)/sqrt(Iterations)))*exp(-rf*TMY);
Und4_Conf_Int2 = (Und4_PayOff_A + Z*(std(Und4_A)/sqrt(Iterations)))*exp(-rf*TMY);

% Basket Option Price
Call_Basket_Option_Asian =
mean([Und1_Price_A,Und2_Price_A,Und3_Price_A,Und4_Price_A]);
Conf_Int1 = mean([Und1_Conf_Int1,Und2_Conf_Int1,Und3_Conf_Int1,Und4_Conf_Int1]);
Conf_Int2 = mean([Und1_Conf_Int2,Und2_Conf_Int2,Und3_Conf_Int2,Und4_Conf_Int2]);
disp("Call Basket Option Asian"); disp(Call_Basket_Option_Asian);
```

Longstaff-Schwartz model for Basket Options

```
RateSpec = intenvset('ValuationDate', Start_Date, 'StartDates',...
Start_Date, 'EndDates', Maturity_Date, 'Rates', rf, 'Compounding', -1);
BasketStockSpec = basketstockspec(Mat_Vol_R, Mat_S0,ones(1,Qty_Und)/Qty_Und,
Mat_Corr);
[Price,Paths,Times,Z] = basketbyls(RateSpec,BasketStockSpec,"call",K,...
    Start_Date,Maturity_Date,'AmericanOpt',0,'NumTrials',Iterations);

disp("Call Basket Option Price Longstaff-Schwartz"); disp(Price);
disp("Participation Rate LS"); disp(Qty_Opt);

Paths1 = Paths(:,1,:); Paths1 = permute(Paths1,[1 3 2]);
Paths2 = Paths(:,2,:); Paths2 = permute(Paths2,[1 3 2]);
Paths3 = Paths(:,3,:); Paths3 = permute(Paths3,[1 3 2]);
Paths4 = Paths(:,4,:); Paths4 = permute(Paths4,[1 3 2]);

Mean_Sim_Prices1 = max([mean(Paths1)-K;zeros(1,Iterations)]);
MCA_PO1 = mean(Mean_Sim_Prices1);
MCA_Pr1 = MCA_PO1*exp(-rf*TMY);

Mean_Sim_Prices2 = max([mean(Paths2)-K;zeros(1,Iterations)]);
MCA_PO2 = mean(Mean_Sim_Prices2);
MCA_Pr2 = MCA_PO2*exp(-rf*TMY);

Mean_Sim_Prices3 = max([mean(Paths3)-K;zeros(1,Iterations)]);
MCA_PO3 = mean(Mean_Sim_Prices3);
MCA_Pr3 = MCA_PO3*exp(-rf*TMY);

Mean_Sim_Prices4 = max([mean(Paths4)-K;zeros(1,Iterations)]);
```

```
MCA_PO4 = mean(Mean_Sim_Prices4);
MCA_Pr4 = MCA_PO4*exp(-rf*TMY);

Call_Basket_Option_Asian2 = mean([MCA_Pr1,MCA_Pr2,MCA_Pr3,MCA_Pr4]);
disp("Call Basket Option Asian 2"); disp(Call_Basket_Option_Asian2);
```

8.2 Volatility modelling

8.2.1 Results Principal protected note with index option

Table 8.1: Statistical test and volatility for PPNs with index option. Part one of the table shows the p-values from the statistical test performed to fit the return data to the GARCH-models. Part two of the table contain the goodness of fit result connected with the EGARCH (1,1) model. This model had the greatest fit for all the return series, and thus its result is displayed. The third part of the table shows the yearly forecasted volatility from the EGARCH (1,1) model and the yearly historical volatility.

	P-value			EGARCH (1,1)			
ISIN	ADF-test	Ljung-Box Q	Engle ARCH	AIC	BIC	Forcasted Volatility	Historical Volatility
SE0005095585 and SE000509559	0,001	8,40E-04	1,11E-15	-12979	-12956	22,94%	23,97%
SE0005505591and SE000550560	0,001	4,22E-04	0,000	-15566	-15543	17,44%	17,74%
SE0005133071 and SE000513308	0,001	3,25E-12	0,000	-14147	-14125	20,36%	21,33%
SE0005506086 and SE000550609	0,001	0,003	0,000	-14244	-14221	24,99%	25,42%
SE0005796570 and SE000579658	0,001	0,002	0,000	-14755	14731	22,45%	23,04%
SE0005677002 and SE000567701	0,001	9,99E-09	0,000	-16816	-16793	14,14%	14,65%
SE0005768215 and SE000576822	0,001	0,0022	0,000	-14620	14597	22,54%	23,10%
SE0005562451 and SE000556246	0,001	7,83E-09	0,000	-16678	-16655	14,46%	14,67%
SE0006027504 and SE000602751	0,001	0,002	0,000	-15326	-15302	21,85%	22,76%
SE0006257838 and SE000625784	0,001	3,04E-08	0,000	-18267	-18243	13,52%	14,37%
SE0006257887 and SE000625789	0,001	0,0017	0,000	-15781	-15757	21,89%	22,59%
SE0006027454 and SE000602746	0,001	1,14E-08	0,000	-17774	-17750	13,69%	14,41%
SE0004898955 and SE000489896	0,001	0,005	2,00E-15	-12281	-12258	22,57%	24,40%
SE0003916931 and SE000391694	0,001	4,11E-11	2,22E-16	-10685	-10664	20,68%	22,22%
SE0004950491 and SE000495050	0,001	5,09E-14	0,000	-14825	-14802	14,07%	15,65%
SE0003722198 and SE000372220	0,001	0,0119	3,00E-14	-9478,7	-9457,2	24,77%	24,48%

8.2.2 Result statistical models ISIN: SE0005095585

Formula ARMA(2,2) process:

$$(1-\varphi_1 L-\varphi_2 L^2)=\mu+(1+\theta_1 L+\theta_2 L^2)\varepsilon_t \tag{8.1}$$

Table 8.2: Result from statistical tests. This table shows the result from Augmented Dickey-Fuller test, Ljung-Box Q test and Engle ARCH for the PPN with ISIN: SE0005095585. The fourth column show the Ljung-Box Q test for an ARMA (2,2) process.

ISIN: SE0005095585

	ADF-test	Ljung-Box Q	Engle ARCH	ARMA(2,2) Ljung-Box Q
P-Value	1,00E-03	8,41E-04	1,11E-15	0,1543
Test Statistic	-47,7607	45,8662	64,2423	26,3571
Critical Value	-1,9416	31,4104	3,8415	31,4104
Reject/ Do not reject null	**Reject null**	**Reject null**	**Reject null**	**Do not reject null**

8.2.3 Results Principal protected notes with basket option

Table 8.3: Historical volatility PPNs with basket option. This table contain the historical yearly volatility for PPNs which contain basket option. The volatility measure used in the pricing model is an average of the historical volatility for each of the indices.

ISIN	Historical volatility index 1	Historical volatility index 2	Historical volatility index 3	Historical volatility index 4	Average historical volatilty
SE0005190865 and SE0005190873	21,27%	23,83%	22,74%	26,08%	23,48%
SE0007184429	18,84%	20,13%	-	-	19,49%
SE0007184320	18,89%	20,16%	-	-	19,53%
SE0005678000 and SE0005678018	19,80%	19,70%	-	-	19,75%
SE0004870079 and SE0004870087	26,76%	21,06%	21,93%	-	23,25%

8.3 Parameters in the model

Table 8.4: Parameter input PPN with index option. Shows the most important input parameters which has been used to replicate the principal protected note with index option. In column 1, we observe the ISIN of the PPN. In column 2, we observe the underlying asset, in column 3 the issue date, in column 4 the yearly risk-free rate added to the pricing model, in column 5 the forecasted yearly volatility and in column 6 the yearly dividend yield.

ISIN	Index	Issue Date	Risk-free rate	Volatility	Dividend yield
SE0003722198 and SE0003722206	OMXS30	17.02.11	3,14%	24,48%	3,26%
SE0003916931 and SE0003916949	S&P 500 Index	26.05.11	0,83%	22,22%	2,11%
SE0004898955 and SE0004898963	OMXS30	20.12.12	1,02%	24,40%	3,45%
SE0004950491 and SE0004950509	SP500 Low Volatility Index	07.02.13	0,83%	15,65%	2,65%
SE0005095585 and SE0005095593	OMXS30	07.05.13	1,31%	23,97%	3,51%
SE0005133071 and SE0005133089	S&P 500 Index	03.06.13	1,03%	21,33%	2,13%
SE0005505591 and SE0005505609	S&P Nordic Low Volatility	10.01.14	1,03%	17,74%	3,23%
SE0005562451 and SE0005562469	S&P 350 Europe Low Volatility Index	05.02.14	0,58%	14,67%	3,54%
SE0005506086 and SE0005506094	Hang Seng Index	03.03.14	1,52%	25,42%	3,21%
SE0005677002 and SE0005677010	S&P 350 Europe Low Volatility Index	05.03.14	0,61%	14,65%	3,54%
SE0005768215 and SE0005768223	OMXS30	09.04.14	1,17%	23,10%	3,54%
SE0005796570 and SE0005796588	OMXS30	07.05.14	1,05%	23,04%	3,54%
SE0006027454 and SE0006027462	S&P 350 Europe Low Volatility Index	27.08.14	0,10%	14,41%	3,54%
SE0006027504 and SE0006027512	OMXS30	28.08.14	0,44%	22,76%	3,54%
SE0006257838 and SE0006257846	S&P 350 Europe Low Volatility Index	03.12.14	0,04%	14,37%	3,54%
SE0006257887 and SE0006257895	OMXS30	03.12.14	0,15%	22,59%	3,54%

Table 8.5: Parameter input PPN with basket option. Shows the most important input parameters which has been used to replicate the principal protected note with basket option. In column 1, we observe the ISIN of the PPN. In column 2, we observe the underlying assets, in column 3 the issue date, in column 4 the yearly risk-free rate added to the pricing model, in column 5 the forecasted yearly volatility and in column 6 the yearly dividend yield.

ISIN	Index	Issue Date	Risk-free rate	Volatility	Dividend yield
SE0004870079 and SE0004870087	Hang Seng, MSCI Singapore, MSCI Taiwan	23.11.12	0,53%	23,25%	3,85%
SE0005190865 and SE0005190873	DAX, Hang Seng, OMXS30 & S&P 500	08.07.13	1,27%	23,48%	3,01%
SE0005678000 and SE0005678018	Msci Singapore, Taiwan Stock Exchange Index	09.04.14	0,80%	19,75%	3,98%
SE0007184320	Msci Singapore, Msci Taiwan	10.09.15	1,23%	19,08%	3,89%
SE0007184429	Msci Singapore, Msci Taiwan	08.10.15	1,02%	19,49%	3,89%