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Transportation Mode Selection in Inventory Models: A Literature Review

Erna Engebrethsen[a,*], Stéphane Dauzère-Pérès[a,b]

[a] *Department of Accounting, Auditing and Business Analytics,*
BI Norwegian Business School,
0484 Oslo, Norway
[b] *Mines Saint-Etienne, Univ. Clermont Auvergne*
CNRS, UMR 6158 LIMOS
CMP, Department of Manufacturing Sciences and Logistics
Gardanne, France

Abstract

Despite the significant share of transportation costs in logistics costs and the importance of considering transportation in inventory models, the majority of the existing models either neglect or simplify transportation costs and capacities, often assuming that only one transportation option is available. The complexity of modeling and choosing the optimal transportation mode or combination of modes has increased due to the increased variety of transportation options and pricing schedules after deregulation. In this paper, we review and classify inventory models with multiple transportation modes focusing on the freight cost functions, mode characteristics and the methods for modeling multiple modes. To our knowledge, no such review has previously been published. We discuss the benefits and weaknesses of each modeling method and, based on industrial practices, identify new areas for research.

Keywords: Transportation; Mode selection; Transportation Costs; Inventory Management; Literature review

1. Introduction

Shippers and carriers are the main actors in the procurement of transportation services. According to Friesz et al. (1986), shippers are those decision-making entities that want a particular commodity to reach a specific destination, while carriers are those decision-making entities that transport the commodities and thereby satisfy the shipper demand for profit. Companies that use their own transportation equipment for transporting their own goods are private carriers. Each carrier can offer various transportation modes, i.e. the means by which people and freight achieve mobility, depending on over what surface they travel: air, land (road, rail and pipelines), and water, including coastal and inland waterways (Chopra and Meindl, 2004). Within each of the physical modes (road, rail, sea, air), several options exist depending on the shipment size, such as parcel, Less than Truck Load (LTL) and Full Truck Load (FTL) for different container sizes, the type of service impacting the transportation lead time (emergency or regular) or the special cargo type, for example for frozen, oversized or bulk goods, and general carriers. A transportation carrier can differentiate the quality of its transportation service by varying the speed of movement, frequency of service, reliability of service, loss and damage rate, and accessibility of service (or spatial convenience) (Talley, 2006). Bausch et al. (1994) stress that even when an organization uses only a private fleet and the fleet's truck characteristics, capacities and costs are heterogeneous, it faces a multiple mode situation. Whenever the company decides to outsource the fleet, it is important to decide on the compensation format and the types of modes to be specified in the contract. These choices depend on the company's logistics strategy, including the

*Corresponding author

degree of involvement in detailed transportation planning, cost transparency and control, resulting in the applied Incoterms, i.e. the trading terms that specify if the product seller or the buyer is responsible for the transportation cost and transfer risk. Guélat et al. (1990) define a transportation mode as a means of transportation that has its own characteristics, such as vehicle type and capacity, as well as a specific cost function and a lead time. Even though the costs depend on the shipment size, in practice the shippers often rely on their personal judgment and experience to choose among different transportation modes and carriers, resulting in suboptimal decisions, instead of using analytical planning tools (Caputo et al., 2006). Transportation costs are important for companies to manage, as they can constitute up to 50% of the total logistics costs (Swenseth and Godfrey, 2002). Freight transportation is a significant part of the global economy. For example, in 2015, logistics costs in the USA represented almost 8% of GDP, whereas transportation costs stand for about 60% of the total logistics costs (Schulz, 2015). According to Ke et al. (2014), the transportation expense is often omitted or assumed fixed when the buyer decides replenishment quantities, and this inaccuracy can easily overwhelm any savings related to good inventory management. Transportation costs in supply chain models are frequently oversimplified by disregarding discount schedules, economies of scale and transportation capacity limits. According to Aissaoui et al. (2007) transportation capacities and costs are rarely explicitly considered in the research literature, and most papers implicitly consider transportation costs by including them in the purchasing price, or assume a simple and non-realistic linear transportation cost function expressed as the product of the unit transportation cost and the transported quantity. In the international purchasing context, savings gained from a shift to a low cost supplier can be offset by increased transportation costs. Many models assume that only a single transportation mode is available. However, in practice, shippers may choose among different transportation alternatives and switch from one to another as needed. In the literature on supplier selection in general, order splitting, when multiple suppliers can deliver a fraction of the total demand, typically when a single supplier is not dominating others according to various criteria (price, quality, delivery time) or cannot meet all demand alone, has been widely studied (Aissaoui et al., 2007). Although splitting shipments across multiple suppliers may increase total shipping costs as a result of diseconomies of scale (Perez and Geunes, 2014), less rebates and increased administration costs, it may offer certain benefits that can more than offset this increase. These benefits include reduction of inventory holding costs and risk related to the probability of stockout and single supplier dependency. On the opposite, the number of studies that consider multiple transportation modes and modal splitting, or combination of modes to ship parts of the same order, is rather limited. However, significant benefits, such as cost and risk reduction, can potentially be achieved in practice by explicitly considering multiple transportation modes in inventory models. When using different modes, with various capacities and cost functions, and mixing modes, total logistics costs can be lowered. The purpose of our literature review is to investigate the methods for modeling transportation mode selection decisions in inventory optimization models, to compare those to the industrial practices and propose directions for future research. We start with a review of the industrial practice and the most typical modes and price schedules, followed by a classification of inventory models in the literature that include transportation costs and modes. In these models, integrated transportation mode selection and inventory replenishment decisions need to be taken to minimize the sum of inventory and transportation related costs. As the main focus of this article is on transportation mode selection decisions, we narrowed down the search in the Web of Science database to include one of the following topic words: Transport mode, Modal split, Freight choice or Mode selection and Inventory, Replenishment, Order quantity or Lot siz*. The purpose was twofold:

1. To identify articles that illustrate different approaches for modeling transportation costs and modes in inventory models, and
2. To identify the relevant articles that consider more than one transportation mode in inventory models.

This search resulted in 720 articles published until 2017. In addition, we searched for articles containing the words Transport and Inventory in the title, resulting in 214 articles, most of those have also appeared in the previous search. Totally, we identified 811 relevant items, of which only 224 articles are from the following relevant literature categories: Operations Research, Management Science, Transportation, Transportation Science Technology, Management, Business, Engineering Industrial, Engineering Manufacturing, Engineering Multidisciplinary, Computer Science, Interdisciplinary Applications, Mathematics Applied, Mathematics Interdisciplinary Applications, Social Sciences, Mathematical Methods, Multidisciplinary Sciences. In addition, a number of recent review articles on inventory and lot sizing have been reviewed, such as Andriolo et al. (2014), Brahimi et al. (2006) and Brahimi et al. (2017), and new articles have been identified using a Snowball method by including relevant articles that are referred to in the initially selected articles. We therefore acknowledge that the list of articles may not be complete, as some relevant articles may be missing since authors might have used other keywords, making it difficult to identify all articles focusing on mode selection and inventory models. In total, 71 articles explicitly considering transportation mode selection decisions together with inventory replenishment decisions have been identified. 70 percent of the articles have been published during the last decade, indicating increased attention from the research community on this topic. As the main focus of this review is on inventory and lot-sizing models, the articles considering inventory and transportation costs in network design, and operational decisions such as multi-stop vehicle routing, packaging and scheduling models have been excluded from our review, as well as articles focusing on:

- Product supplier selection decisions (as we focused on only transportation provider selection),
- Behavioral research and transportation policies discussing modal splitting for industrial and passenger traffic, as inventory management decisions from a single company perspective is not considered.

The majority of the reviewed models are in a buying context where a company needs to decide on the order size from a supplier, considering inventory costs and inbound transportation costs for shipping goods from the supplier. In a few identified multistage models, such as for example Pazhani et al. (2016), both inbound transportation and outbound transportation are considered, while in some single vendor and multiple retailer problems, such as for example Gürler et al. (2014), the vendor is responsible for supplying customers.

This article is organized as follows. Typical transportation modes and cost functions used by companies that procure transportation services are first introduced in Section 2. This section is important to understand the limitations of transportation cost modeling in the research literature. Then, how transportation costs are considered in inventory models is surveyed in Section 3. The following classes for transportation costs are used to categorize papers: Constant unit costs, fixed charge function, FTL cost function, LTL cost function, approximation functions, carload discount schedule and combined replenishment mode function. Inventory models with multiple modes are reviewed and classified in Section 4. The motivations and the various ways of combining multiple modes are also discussed. Finally, conclusions and suggestions for further research are provided in Section 5.

2. Transportation modes and cost functions in practice

Following the deregulation of the transportation industry in the 1980s and 1990s, multiple transportation options are available for shippers due to the increased outsourcing of transportation and the development of containerization and palletization of goods. Logistics companies have become multimodal, offering more than one transportation mode to their clients, mainly because of extensive mergers and acquisition processes taking place in the industry (Dobie, 2005). The deregulation also

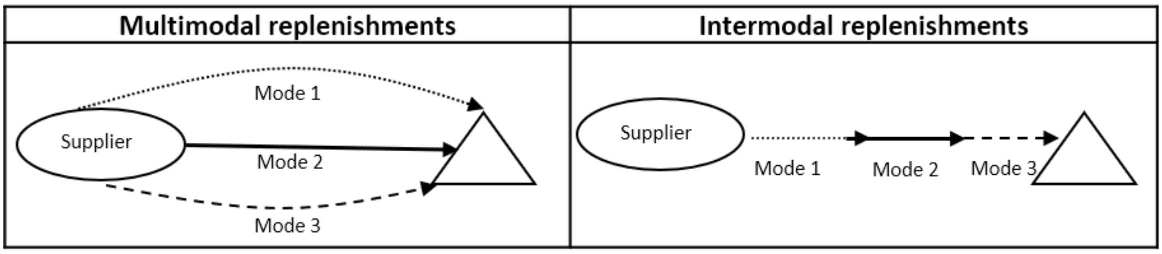

Figure 1: Illustration of multimodal transportation versus intermodal transportation

had an impact on the pricing of transportation services, when carriers started to compete with each other by offering discounts on the published base rates.

The freight rates depend on several factors such as means of transportation, shipping distance and weight, the shipment quantity, and the commodity class of the shipped items, which often depends on the volume, weigh, density and value of the shipment. Shipment quantity can be expressed in weight or volume units, in number of pallets, or in the units applied for the specific commodity class. According to Lapierre et al. (2004), most of the transportation companies usually offer parcels, LTL (less-than-truckload or LCL, less than container load, in sea transportation) or FTL (full-truckload or FCL, full container load, in sea transportation) shipment services, depending on the shipment quantity. Parcel carriers are usually used for quite small shipments (for example postal services, where various fixed costs are charged for small shipments up to a certain size or weight), LTL for medium-size shipments and FTL for large shipments. The size of the trucks or containers varies and, depending on the packaging option (for example the type of pallets), the maximum load capacity is different. In this article, it is assumed that transportation modes differ from each other by their capacity and cost function, regardless whether an intermodal or unimodal transport is used. Intermodal transportation reflects the combination of at least two modes of transport in a single transport chain, without a change of container for the goods (Macharis and Bontekoning, 2004). For example, most of the air or sea freight shipments are picked and delivered to the airport or port by trucks. The terms multimodal replenishments, modal splitting or combination of modes in this paper assume that a given order quantity can be split among several transportation modes, and each mode, which can be unimodal or intermodal, will deliver a fraction of the total order quantity. It corresponds to order splitting in procurement, when the total order quantity is divided between several suppliers. Figure 1 illustrates the difference between multimodal and intermodal replenishments.

Since a loading unit is often a standardized container, intermodal transportation is also referred to as containerized transportation (Demir et al., 2016). Different container types and capacities for each means of transport are available on the markets, and the goods can often be shipped on pallets to increase the efficiency of cargo handling. The pallet type depends on the characteristics of the product shipped, handling facilities and equipment capacities. Examples of different capacities for road transportation for different pallet types are illustrated in Table 1.

Characteristics	EUR-Pallet	Industrial Pallet	Asia Pallet
Size (mm)	800 x 1200	1000 x 1200	1100 x 1100
Load-bearing capacity per pallet (kg)	1500	1500	1300
Transportation capacities	Number of pallets per container		
Container 20” (2,33 m x 5,918 m)	11	9	10
Container 40” (2,33 m x 12,015 m)	25	21	21

Table 1: Capacities of a full truck or container for different pallet types

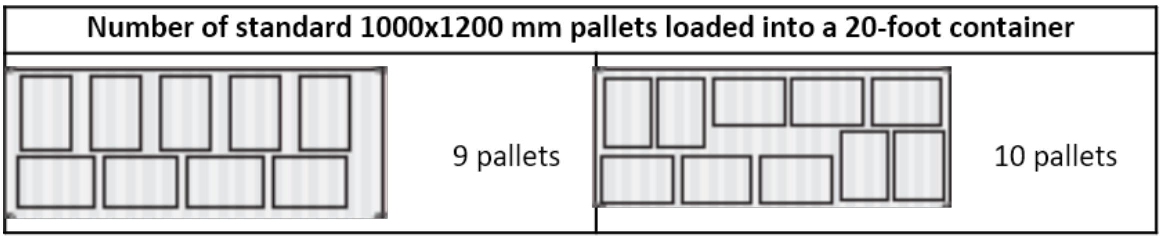

Figure 2: The maximum number of pallets depends on the loading configuration

The chosen loading configuration or the total weight limitation can also impact the maximum number of pallets that can be transported, for example in a standard container as shown in Figure 2. A shipper often needs to make a decision on the type of pallets, the loading configuration, the transportation mode and the carrier to be used to ensure efficient transportation of goods.

Therefore, the choice of transportation modes for the shipper may include:

- The choice among different modes (unimodal or intermodal) offered by the same carrier,
- The choice among different carriers offering similar modes, but different prices and/or capacities,
- The choice among different modes offered by different carriers.

In the next sections, we describe the characteristics of the main transportation modes: FTL and LTL.

2.1. FTL shipment costs

For the FTL mode, a fixed fee A is charged per container or per vehicle for shipping Q units up to a given capacity K, regardless of the filling rate. The total shipping costs $TC(Q)$ are expressed as:

$$TC(Q) = A \left\lceil \frac{Q}{K} \right\rceil$$

Figure 3 illustrates an FTL cost function. Note that a company may use less than the available capacity and transport this freight at the cost of a full load.

The capacity K is expressed in shipping units, which can be weight or volume units, or both, as well as number of pallets. This form of freight cost is known as the multiple-setup cost structure, and is commonly used for modeling FTL shipments (Toptal, 2012). The price schedule for parcels follows a similar cost structure. For example, Norwegian Post has the following fixed fees for small packages above 2 kg: 145 NOK for a package between 0 and 10 kg, 260 NOK for a package up to 25 kg, and 370 NOK for a package up to 35 kg. Figure 3 corresponds to the costs of shipping parcels, assuming that each parcel is within the same weight range with maximum weight of K, e.g. $A = 145$ NOK (respectively $A = 260$ NOK and $A = 370$ NOK) and $K = 10$ kg (respectively $K = 25$ kg and $K = 35$ kg) in the Norwegian Post example.

An example of prices for different container types and their loading capacity (in this case in pallets) for a specified origin-destination is presented in Table 2.

The following example shows that, when several FTL modes are available and for some shipment quantities, it can be beneficial to combine FTL modes. Consider the three FTL alternatives presented in Table 3.

For a shipment of 33 pallets, the costs of using only one type of FTL are shown in Table 4 for each of the three FTL modes and for the optimal combination of modes.

In this case, combining one FTL 1 and one FTL 2 leads to a saving of 800 NOK (14 %) compared with the best single-mode alternative. Hence, a variety of FTL modes exist with different capacities

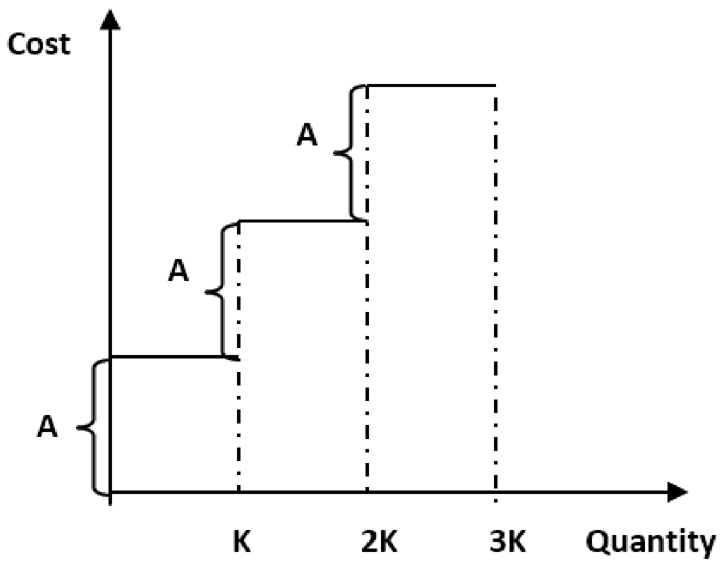

Figure 3: Example of an FTL (FCL) cost function

	20 ft. container	40 ft. container	40 ft. pallet-wide container	45 ft. pallet-wide container
Europallets per container	10	25	30	33
Price example, NOK	2596	3850	4080	4250

Table 2: Capacities and prices for shipping Europallets with various containers

	Pallets per container	Cost per container, NOK
FTL 1	11	1900
FTL 2	25	3000
FTL 3	30	3200

Table 3: Three FTL alternatives

Number of containers	Capacity	Total costs, NOK
3 × FTL 1	33 pallets	5700
2 × FTL 2	50 pallets	6000
2 × FTL 3	60 pallets	6400
1 × FTL 1 and 1 × FTL 2	36 pallets	4900

Table 4: Costs of shipping 33 pallets for each alternative

and loading configurations. The decision related to allocating the order quantity to various modes becomes more complex for the shipper as the number of palletizing, loading and mode (capacity) alternatives increases.

2.2. LTL shipment costs

When the freight size is not large enough to justify the cost of an FTL shipment, an LTL shipment is preferred. Freight rates for LTL shipments are expressed as cost per shipping unit. The transportation companies offering LTL shipments, also termed common carriers, consolidate shipments from different customers to increase the utilization of a truck or container. A realistic cost structure for LTL modes usually exhibits price breaks, where the unit price decreases for increased shipping quantity, resulting in a piecewise linear, all-unit discount function. A minimum shipment charge is imposed to discourage extremely small shipments at the LTL rate. In an all-unit discount schedule, if a certain quantity level is exceeded, a lower unit price applies to all units, not just those above the quantity break point, which is the case in an incremental discount schedule (Munson and Rosenblatt, 1998). An example of a realistic LTL price schedule is presented in Table 5.

Price-break intervals (prices in NOK), minimum charge = 400						
Number of pallets	1-6	7-11	12-17	18-23	23-30	30 (FTL)
Price per pallet	180	150	130	115	107	2 900 (total)

Table 5: Example of a realistic LTL price schedule in the retail industry

Özkaya et al. (2010) provide a detailed overview of the LTL market in the USA and similar cost structures for LTL, trying to identify the main cost drivers and factors for rate variability. They stressed that other factors than distance may impact the LTL carrier pricing, as for example the customer's negotiation power, freight desirability (i.e., whether the freight is stackable or palletized) or return freight balance, e.g. some long distance lanes can be priced lower than short distance lanes. Figure 4 shows a total transportation cost function for LTL shipments with 3 intervals, where a minimum cost C_{min} is charged for small shipments, and a unit rate α_i, where $\alpha_{i+1} < \alpha_i$, is charged for a quantity Q shipped within interval i, characterized by quantity limits M_i and M_{i+1}.

The transportation cost function $G(Q)$ can be written as follows:

$$G(Q) = \begin{cases} 0, & if \quad Q = 0 \\ C_{min} & if \quad 0 \leq Q < B_1 \\ \alpha_1 Q & if \quad B_1 \leq Q \leq M_2 \\ \alpha_i Q & if \quad M_i \leq Q \leq M_{i+1}, \ i = 2, 3, 4 \end{cases}$$

where B_1 is the quantity at which shipping costs are larger than C_{min}. However, as seen in Figure 4, for some quantities (belonging to the intervals $[B_i, M_i]$, for $i \geq 2$), the total costs are larger than the costs of shipping higher quantities at the next interval rate. This can be explained by the nature of an all-unit discount schedule that encourages higher volumes to be shipped to avoid the unreasonable differences in total costs. In practice, shippers tend to over-declare the LTL shipment size to obtain lower total costs. This means that, when the shipper is planning to ship Q units, and $M_i \leq Q < M_{i+1}$, the cost is calculated as $G(Q) = \min(\alpha_i Q, \alpha_{i+1} M_{i+1})$. Using the costs in the example of Table 5, shipping 6 pallets costs 1080 NOK, while shipping 7 pallets costs 1050 NOK. In this situation, the shipper pays the price of shipping 7 pallets (1050 NOK) while, in reality, he ships only 6 pallets to qualify for a discount between the two price breakpoints. Sethi (1984) was one of the first to focus on over-declaring practice in shipping, calling it "phantom" policy. In the industry, this is called "shipping Q but declaringM_{i+1}" (Chan et al., 2002) or the "bumping clause", whereby it is favorable that an actual weight is bumped into a higher-weight category (Çetinkaya and Bookbinder, 2003), or shipping of "phantom freight" (Ke et al., 2014).

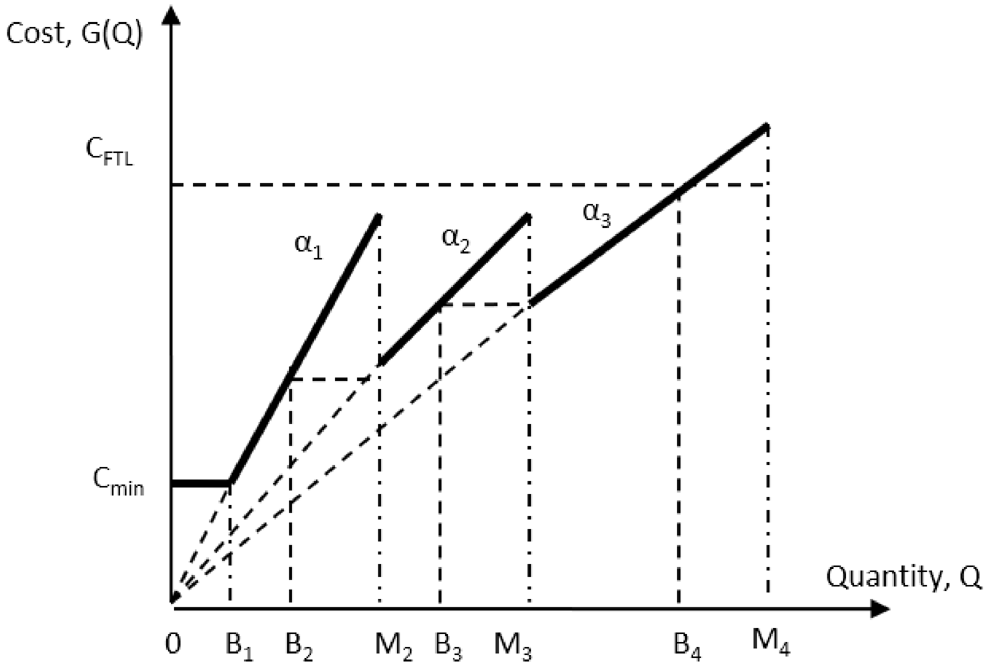

Figure 4: LTL-mode cost function with minimum charge and three intervals

Over-declaring shipments is reasonable when the actual shipping quantity falls within a range that lies between the rate breakpoint and a so-called indifference point, introduced by Russell and Krajewski (1991). The indifference point is defined as the shipment quantity which, when multiplied by its proper rate, yields the same total tariff that is charged at the next rate breakpoint. The indifference point B_i for interval $i \geq 2$ is expressed as $B_i = \frac{\alpha_i M_i}{\alpha_{i-1}}$, where α_i and M_i are, respectively, the unit freight rate and the lower quantity limit for the next interval i, and α_{i-1} is the unit freight rate for interval $i-1$. Indifference points are also used for calculating the quantity limits to use for the minimum charge or FTL rate. When both FTL and LTL modes are available, a shipper may over-declare a quantity that uses less than the full truck capacity and transport this freight at the cost of a full load. Hence, for the shipper to take his decisions, the price schedule in Figure 4 becomes the one in Figure 5.

The cost function in Figure 5 is obtained by chopping off the saw-teeth from the general all-unit discount schedule in Figure 4. Using the new cost function in a decision model, for quantities between B_i and M_i for $i \geq 2$, the shipper will over-declare by artificially announcing an inflated shipment quantity to a higher breakpoint M_i that results in a lower marginal tariff. However, the actual shipped quantity will be lower. When re-calculating the nominal LTL freight rate schedule, it is important to check if the indifference point is larger than the interval's lower limit. Occasionally, one may find that the indifference point $B_{i+1} \leq M_i$. In such cases, M_i is an anomalous or "fictive" break-point (Abad, 2007). One should drop the anomalous break-point and the corresponding freight rate from the schedule, since the freight will be over-declared anyway, as well as re-index the breakpoints that are larger than the anomalous break-point. Carter and Ferrin (1995) have examined the LTL rates of different carriers in USA, revealing the existence of so-called "anomalous" LTL rates, which usually exist when the discount between the lower and higher weight group rates are 50% or more. The anomalous rates stood for 30% of all rates in the examined database, indicating the need for more systematic pricing strategy and rate adjustments applied by carriers. The complexity of the "real" rate structure and potential disparities in LTL shipping costs require re-calculation of the rates to accurately estimate the shipping costs. In addition, many carriers operate with their own class

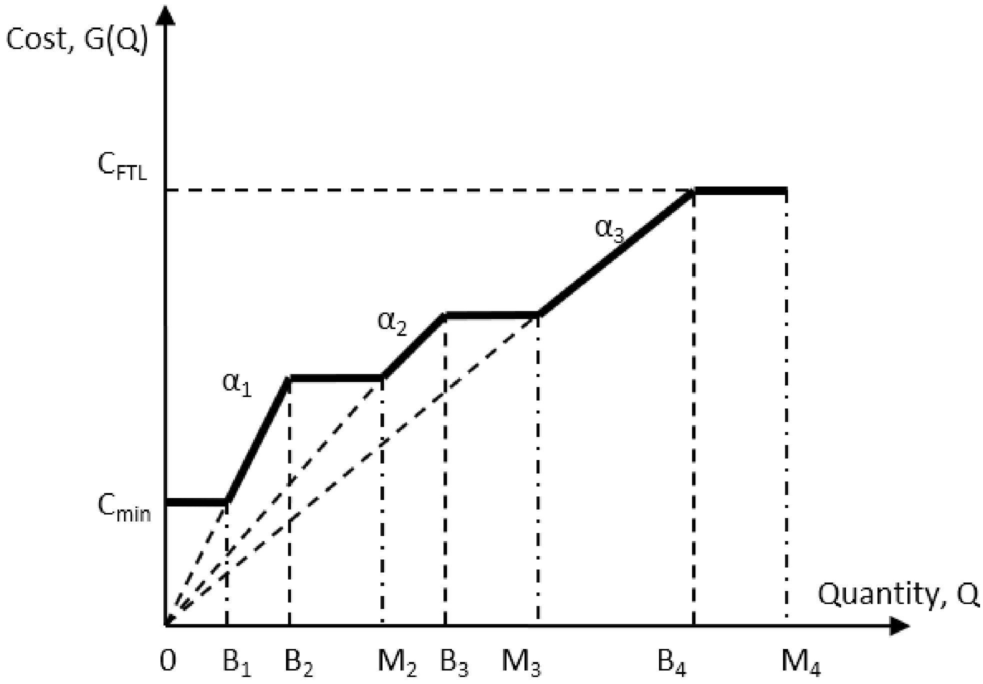

Figure 5: Modified LTL cost function with over-declaring

structures (based on value and density of goods) and different price-break intervals, making it difficult for the shipper to compare the proposed rates in a straightforward way. Hence, the main reasons for re-calculating the nominal LTL shipment rates, also to provide inputs to lot-sizing models, may include:

- Over-declaring and existence of anomalous or "fictive" price breaks. As mentioned above, some LTL price schedules are inconsistent and need to be carefully evaluated to estimate the correct transportation price.

- Shipping multiple items with various densities belonging to different shipment classes, for example by using an average density (Lapierre et al., 2004). Pricing schedules can be defined for a shipment class with a specified density, weight and height ranges, therefore additional calculations are needed to find the total cost when shipping non-homogeneous freight.

- Negotiated discounts on the published rates (up to 50 % -75 % according to Özkaya et al. (2010). After the deregulation of the transportation industry, carriers started to offer discounts from the published base prices, and these tariffs are now only a starting point for negotiations. Many carriers changed (mostly increased) their base prices so that they can offer better (higher) discounts to attract customers (Özkaya et al., 2010). Re-calculation of the published rates is therefore needed to establish the final rates.

3. Transportation costs in inventory models

3.1. Importance of transportation costs in inventory models and main shortcomings

We start this section by discussing the importance of transportation costs in inventory models and their main shortcomings with respect to the modeling of transportation costs:

- Ignoring or simplifying discount schedules,
- Considering transportation costs as a part of set-up/purchasing costs,
- Treating transportation discounts as purchasing discounts, and thus ignoring over-declaring.

Several authors stress that considering transportation costs in inventory models is important. Ertogral et al. (2007) study how explicit consideration of transportation costs when making inventory decisions impacts the total costs and the lot size, compared to the case where transportation costs are ignored in lot-sizing decisions. The authors demonstrate that, when incorporating transportation costs for the constant demand case, the shipment lot increases and the number of orders decreases because of economies of scale in transportation costs. In addition, it is shown that the total costs and the solutions with and without over-declaring become more similar as the transportation costs significantly increase compared to the holding costs. Yıldırmaz et al. (2009) in their inventory model with price-sensitive static demand and an FTL transportation mode conclude that neglecting the transportation cost leads to a 2.25% decrease in profit on average and, in some extreme cases, up to 60%. Even higher profit losses can be expected when the order placement cost is large, the sensitivity of demand to price is high, the truck cost is large, and truck capacities are small. Mendoza and Ventura (2013) analyzed the effect of not considering transportation costs and observed an increase of 14.7% of the average monthly logistics cost and 88.9% of the transportation cost. Pazhani et al. (2016) demonstrate in their multistage model with constant demand how transportation costs impact supplier selection decisions, and that up to 15% of logistics costs can be saved by integrating transportation and inventory decisions compared to considering them sequentially. Not only including transportation costs in inventory models, but also modeling those costs accurately and reflecting actual price schedules is important to obtain cost efficient replenishment plans. When transportation economies of scale are not explicitly taken into consideration in the vendor production plans and the buyer procurement plans, this leads to higher inventory costs and inefficient transportation plans (Rizk et al., 2006a). According to Archetti et al. (2014), the assumption that the transportation costs linearly depend on the shipped quantity usually makes the decision making models simple and efficiently solvable. However, while this may be a reasonable assumption in a tactical planning phase, where the detailed cost structure is not essential, the linearity assumption is too simplistic in an operational phase. Realistic freight prices as illustrated above have more complex piecewise linear (FTL) and non-linear cost structures (LTL). Swenseth and Godfrey (2002) show an example where, by considering discounts in transportation costs, up to 37% savings can be achieved compared to a case where constant unit cost without discounts are assumed. Hence, the carrier's freight rate structure can significantly affect the ordering policy and inventory levels. Authors like Toptal (2009) and Choudhary and Shankar (2014) stress that buyer's purchasing decisions are influenced by economies of scale in transportation costs in the presence of FTL transportation. The tendency to order more under quantity discounts from the supplier may result in increased transportation costs and the urge to fully utilize FTLs, increasing the purchasing and inventory costs. Ventura et al. (2013) show that LTL transportation cost approximations may lead to suboptimal solutions. In particular, power and quadratic functions lead to average transportation cost errors of 3.1% and 2.2% in relation to the actual transportation costs for their respective optimal solutions. Similarly, when the optimal solutions for the power and quadratic functions are compared with the optimal solution obtained with an MILP (Mixed Integer Linear Programming) model, average solution gaps of 1.2% and 0.5% are obtained. Consequently, when estimated functions are used for the transportation costs under an all-unit discount structure, the decision maker needs to be aware of the potential solution gaps. Many existing inventory models have treated freight breaks in the same way as price breakpoints in a purchasing discount schedule (see for example Hwang et al. (1990),Tersine and Barman (1991) and Burwell et al. (1997)). For example, Tersine and Barman (1991) propose a single-item EOQ model for a problem where all-unit and incremental types of discounts for both purchasing and transportation are available. They consider different combinations of discounts. The authors suggest to re-calculate the discount schedule for a combined (purchasing plus shipping) unit cost in each case. Burwell et al. (1997) extend the inventory model proposed by Tersine and Barman (1991) by assuming that the demand is price dependent. However, when freight and purchasing discounts are modeled similarly, the option of over-declaring the weight of shipments, which is available in practice, is ignored. Because of the over-declaring possibility, the shape of the transportation cost function that arises from the freight rate schedule differs from the shape of the cost function associated with the all-unit quantity discount schedule (Abad and Aggarwal, 2005). In addition, the minimum weight and maximum capacity limits are seldom imposed for purchasing discounts, while the freight discount schedule is usually applicable within the vehicle or container capacity. In contrast to price discounts, freight rate discounts are typically based on weight or standard shipping units (as for example pallets) rather than number of purchased units.

Despite the importance of transportation costs in supplier selection and order quantity allocation, existing inventory models have typically assumed that transportation costs are either managed by suppliers and, therefore, considered as part of the unit price, or managed by the buyer and, therefore, included in the setup cost (Mendoza and Ventura, 2013). These assumptions are not realistic for a number of trading terms (Incoterms) where the buyer is responsible for transportation and there are actual transportation price schedules for different modes.Konur and Schaefer (2014) note that most of the studies assume less-than-truckload (LTL) transportation for the shipment of the order with constant unit transportation costs, assuming that a single truck has sufficient capacity to ship any order size. Transportation costs are therefore often included within purchase costs. As pointed out by Jans and Degraeve (2008), replenishment modes can either be modeled as a part of set-up costs (see for example Jaruphongsa et al. (2005), who model different set-up cost structures for each mode), or as an extension of unit production or purchasing costs, either as a unit linear cost or with a discount scheme (as for example in Li et al. (2004)). In some models (Diaby and Martel (1993) and Chung et al. (1996)), transportation costs and the corresponding discounts are treated as a part of purchasing costs and other procurement related costs, and are included in the unit replenishment cost, which combines procurement and transportation costs.Toptal (2009) considers a joint replenishment function where procurement costs exhibit an all-unit discount schedule, while transportation costs have an FTL discount schedule. Absi et al. (2013) and Absi et al. (2016) propose a dynamic lot-sizing model with multiple supplying modes defined as a combination of a transportation mode (combining one or more types of vehicles) and a production facility. Each mode is characterized by its fixed and unit cost and carbon emission parameters. The problem consists in selecting the modes used in each period such that no carbon emission constraint is violated, and the cost of satisfying all the demands on a given time horizon is minimized. Higginson (1993) suggests that transportation costs are modeled either as a function of quantity, function of distance or function of both quantity and distance in the literature. The transportation cost functions with costs depending on the travel distance are mainly used when the shipper has the responsibility for the optimal utilization of the vehicle, for example in vehicle routing, inventory routing or network design models, where alternative routes can be selected. Ortolani et al. (2011) provide an overview of the internal, i.e. direct expenses for owning and operating vehicles, and external, i.e. generating burden for society such as environmental damage but not included in price, elements of transportation costs for various modes.

In the sections below, we list models of transportation costs found in the inventory and lot-sizing literature within the following groups:

- Constant unit costs,
- Fixed charge function,
- FTL cost function,
- LTL cost function,

- Cost approximation functions,
- Carload discount schedule,
- Combined replenishment mode function.

For a more general review of transportation cost function modeling in supply chain optimization models, the reader is referred to Bravo and Vidal (2013).

3.2. Constant unit costs

Transportation costs are charged per unit shipped, and examples are provided in Table 6.

Transportation costs	Authors	Demand	Items
Constant unit costs	Blauwens et al. (2006)	Stochastic	Single
Constant unit costs multiplied by a distance factor	Ab Rahman et al. (2016)	Constant	Single
Constant unit costs with unlimited vehicle capacity	Baumol and Vinod (1970)	Constant	Single
Constant unit costs with unlimited vehicle capacity	Buffa and Reynolds (1979)	Stochastic	Single

Table 6: Constant unit costs

3.3. Fixed charge cost function

Transportation costs consist of a fixed set-up cost and a constant unit cost per unit shipped, independently of the vehicle capacity. Examples are presented in Table 7.

Transportation costs	Authors	Demand	Items
Fixed charge function: Fixed and constant unit shipping cost	Van Hoesel et al. (2005)	Dynamic	Single
	Anily and Tzur (2005)		Multi
Fixed costs (for example for loading) and constant unit shipping costs	Larson (1988); Hall (1992)	Constant	Single
Fixed costs and variable costs paid per distance unit traveled	Tsao and Lu (2012)	Stochastic	Single

Table 7: Fixed charge cost function

A special type of fixed charge transportation cost functions is considered by van Norden and van de Velde (2005), who propose a dynamic multi-item lot-sizing model with a transportation capacity reservation contract, pointing out a difference between systematic and spot buying of transportation capacity. In systematic buying, the shipper can have a capacity reservation contract with transporters, which allows him to use any portion of the reserved fixed capacity for a guaranteed fixed price lower than the spot market price. For this type of contract, the exceeded capacity should be bought at the spot market at higher price if the actual volume is larger than the reserved capacity. The transportation cost function is:

$$f(r) = \begin{cases} c_0 + rc_1 \quad if \quad r \leq R \\ c_0 + Rc_1 + (r-R)c_2 \quad if \quad r > R \end{cases}$$

where $f(r)$ is the cost of transporting r pallets, c_0 is a fixed monthly fee irrespective of the volumes shipped, c_1 is the low (guaranteed) freight rate per pallet for the first R pallets, and c_2 is the high (spot market) freight rate per pallet. Note that this type of contract challenges the traditional assumption that freight rates decrease with transportation weights and/or volumes.

3.4. FTL cost function

A fixed cost is charged per vehicle or container regardless of how fully the capacity is utilized. Examples can be found in Table 8.

Transportation costs	Authors	Demand	Items
FTL cost: with a fixed cost per truck/container/vehicle	Ben-Khedher and Yano (1994); Ozdamar and Yazgac (1999)	Dynamic	Multi
	Aucamp (1982); Hall (1985); Sheffi et al. (1988)	Constant	Single
	Speranza and Ukovich (1994)	Constant	Multi
FTL, no partial fillings are allowed	Pantumsinchai and Knowles (1991); Dullaert et al. (2005)	Stochastic	Single
FTL, considering fast and slow modes	Kiesmüller et al. (2005)	Stochastic	Single
FTL with fixed set-up cost	Toptal (2009); Toptal (2012)	Constant	Single
FTL with unlimited capacity	Qu et al. (1999)	Stochastic	Multi
FTL with fixed set-up cost and discounts per FTL	Lee (1986)	Constant	Single

Table 8: FTL cost function

3.5. LTL cost function

Transportation costs are typically expressed as an all-unit discount schedule with decreasing unit rate for increased shipment quantities. Various examples can be found in Table 9.

Transportation costs	Authors	Demand	Items
LTL with minimum charge, multiple breaks and overdeclaring	Ke et al. (2014)	Constant	Single
LTL: modeled as all-unit discount schedule with multiple price breaks, with and without over-declaring, without minimum shipment price	Ertogral et al. (2007)	Constant	Single
LTL: All-unit discount schedules with multiple price breaks without over-declaring possibility and minimum shipment price	Vroblefski et al. (2000)	Constant	Single
LTL: All-unit or incremental freight discount without over-declaring and minimum shipment price, assuming unlimited vehicle capacity	Hwang et al. (1990); Tersine and Barman (1991); Burwell et al. (1997)	Constant	Single

Table 9: LTL cost function

Only few authors consider incremental discount schedules for LTL transportation costs. For example, Chan et al. (2002) consider a single-item dynamic lot-sizing model, where transportation costs include incremental discount. However, little empirical research is done to examine how often this type of discount schedules is applied in shippers' practice, compared to all-unit discount schedules.

3.6. Transportation cost approximation functions

Several authors simplify exact freight costs to avoid modeling complex transportation rates with price breaks. For example, the over-declaring possibility and the risk of discovering anomalous price breaks add complexity in modeling LTL costs. According to Swenseth and Godfrey (1996), prior to incorporating freight rates in the models, a logistics decision maker may choose between using the actual or approximated freight rates. The loss of accuracy, and the magnitude of potential errors from using approximate cost functions instead of modeling the true costs, must be assessed and weighted against the advantages derived from using simplified functions. In addition, finding the appropriate parameters for an approximation function requires additional work and time. Approximating actual transportation costs by assuming a constant unit charge function with a constant freight unit rate regardless of the weight shipped for a given distance, is for example applied by Baumol and Vinod (1970) in a so-called inventory-theoretical model developed for regional freight demand analysis. Langley (1980) suggests that, instead of using constant unit shipping costs, different types of functions to describe dependency of transportation costs on shipment size can be applied. These approximation functions are not total transportation functions that depend on the quantity, but unit freight rate functions, often based on historical cost data collected from the carriers for the purpose of creating approximation functions. If Q is the order quantity and r is the unit shipping cost, the following transportation rate functions $r(Q)$ are suggested:

- Proportional: $r(Q) = a - bQ$,
- Exponential: $r(Q) = a + bc^{Q}$, with $0 < c < 1$,
- Inverse: $r(Q) = a + \frac{b}{Q}$,
- Discrete function, where unit transportation costs are constant over specific ranges of Q, and decrease as certain minimum shipment volumes are reached

Comparisons of constant rate to proportional, exponential, inverse and discrete shipping cost functions have shown that the discrete step-wise declining cost function yields the lowest total costs (Langley, 1980). The exponential function has also been modeled by Buffa and Munn (1990) and Swenseth and Godfrey (1996). In their work, the shipping rate is expressed as an exponential function with a constant base K, where $0 \leq K \leq 1$ is a power variable corresponding to the shipping weight. More precisely, $F_y = F_x + (F_{y'} - F_x)K^{W_y}$, where F_x is the FTL freight rate per unit, $F_{y'}$ is the rate per unit at the lowest possible shipping weight $W_{y'}$ and thus corresponds to the highest rate per unit for a given distance. Hence, the unit shipping cost decreases as the shipping weight increases. The inverse function assumes that the shipper pays a fixed cost for any shipment, regardless of the actual shipment weight. For example, the shipper pays for an FTL, regardless of how full the truck or container is, and the unit shipping cost decreases as the shipping weight increases until it reaches the maximum weight capacity of FTL. The adjusted inverse function, as described by Swenseth and Buffa (1990), is a modification of the inverse function, where a new parameter is introduced, reflecting the premium cost paid for LTL shipments compared to an FTL shipment. This parameter is between 0 and 1, and it increases the freight rate as the shipping weight decreases.

Swenseth and Godfrey (1996) compare alternative continuous approximation functions for estimating actual freight rate functions that are simpler for computations. The alternative functions that are analyzed represent constant unit charge, proportional linear, exponential, adjusted inverse and inverse functions. The best approximation function for each type is obtained by finding the parameters that minimize the mean squared difference between the actual freight rates and those generated by each approximation function. When comparing the five functions, Swenseth and Godfrey

(1996) point out that the inverse function reflects the exact freight costs in situations where shipments are over-declared as FTL shipments. However, for lower weights, this function overstates the rates. The conclusion is that the proportional function performs the best, followed by the adjusted inverse, constant, exponential and inverse functions, respectively.

Swenseth and Godfrey (2002) suggest that, since it is difficult to incorporate exact transportation costs with discounts, in particular for multiple price breaks and over-declaring, it is better to apply an inverse function for FTL rates and an adjusted inverse function for LTL rates instead of using constant unit costs. For a given shipping weight W_y on a given route, the inverse function to determine the freight rate per pound F_y is as follows: $F_y = \frac{F_x W_x}{W_y}$, where F_x is the FTL rate per pound at maximum shipment capacity W_x. The adjusted inverse function takes the following form: $F_y = F_x + \alpha F \frac{W_x - W_y}{W_y}$, where α is a constant between 0 and 1. The authors show on an example that, by considering discounts in transportation costs, up to 37% savings can be achieved compared to a case where a constant unit cost without discounts are assumed.

Other authors have suggested to use the following functions for estimating LTL transportation costs $F(Q)$:

- $F(Q) = a + b(ln(Q))$, $b < 0$ (Arcelus and Rowcroft, 1991),
- $F(Q) = aQ^b$ power function, obtained using a curve fitting approach (Tyworth and Zeng, 1998).

In order to improve the fit of the functions proposed by Swenseth and Godfrey (1996), especially in the case of LTL, Tyworth and Ruiz-Torres (2000) and Tyworth and Zeng (1998) propose the use of a power function to model LTL freight rates.

Mendoza and Ventura (2009) address the issue of order quantity allocation in the supplier selection problem with multiple suppliers while considering inventory and transportation costs simultaneously. They use two continuous functions to estimate the actual LTL freight rates for an EOQ model:

- A proportional function proposed by Langley (1980) $F_y = A - a(Qw)$, and similar to the one proposed by Swenseth and Godfrey (2002) and
- A power function $F_y = a(Qw)^b$, where a and b are coefficients and Q is the shipped quantity, w is the weight under consideration proposed by Tyworth and Ruiz-Torres (2000).

The proposed approximation functions are recommended to use when the number of potential suppliers is large or when no specialized optimization software is available. The approximation models described above have been considering cycle inventory costs, based on the EOQ model for constant and stochastic demands and stochastic lead times. Tsai (2007) proposes various discount schedules for transportation and purchasing costs in a multi-item dynamic model, including linear, single break point, step, and multiple break point functions. By utilizing linearization techniques, the nonlinear models are approximated to a linear mixed 0-1 program solvable to obtain a global optimum. The author suggests conducting more research on linearization of nonlinear unit cost functions, as it is difficult to determine a global optimum when using exact transportation cost functions. Pazhani et al. (2016) suggest that the LTL cost structure can be approximated by defining different ranges for the order quantity and by assigning a fixed charge for each range. Even though several authors have tried to approximate transportation cost functions, little research has been done on examining the effect of using simplified functions instead of real rates on transportation costs and the conditions under which the approximation provides results that insignificantly deviate from the results obtained when using exact rates.

3.7. Carload discount schedule

This is a special type of cost function that combines the costs of LTL and FTL modes, assuming that both modes have the same capacity. It can be seen as a special case of all-unit discount schedule

Transportation costs	Authors	Demand	Items
Carload discount schedule: Combines FTL and LTL modes, where a constant unit LTL cost is charged up to a certain volume, from which a fixed charge for FTL is applied until the container is full.	Elhedhli and Benli (2005); Rieksts and Ventura (2008); Mendoza and Ventura (2008)	Constant	Single
Carload discount schedule	Li et al. (2004)	Dynamic	Single
Carload discount schedule. Only one full container can be ordered in each period	Van Eijs (1994)	Stochastic	Single
Carload discount schedule with multiple LTL price breaks, minimum price and over-declaring	Russell and Krajewski (1991); Abad (2007); Mendoza and Ventura (2013)	Constant	Single

Table 10: Carload Discount Cost Function

with a single price break, where a constant unit price is charged up to a break point, after which an FTL charge is paid until the container is fully filled. Examples are presented in Table 10.

Mendoza and Ventura (2013) stress that using such a price schedule is reasonable if the analysis is focused on the use of small shipment sizes (LTL) within one truckload.

3.8. Combined replenishment mode function

Transportation costs are considered as a part of replenishment costs jointly with production, purchasing or other procurement costs as in the examples in Table 11.

Transportation costs	Authors	Demand	Items
Multiple set-up structure	Jaruphongsa et al. (2005); Jaruphongsa et al. (2007)	Dynamic	Single
Part of unit procurement costs with discounts	Diaby and Martel (1993)	Dynamic	Single
Joint (transportation and production) fixed and unit supplying mode cost	Absi et al. (2013); Absi et al. (2016)	Dynamic	Single

Table 11: Combined Replenishment Mode Cost Function

In the first example, a cost structure with multiple setups is considered, i.e the cost of utilizing a replenishment mode consists of a fixed setup cost (e.g. cost of packaging and loading), a fixed cost per cargo, and a proportional delivery/procurement cost per unit. In the second example, the total procurement cost (i.e. ordering plus purchasing, plus transportation and reception) is a general piecewise linear function of the quantities shipped to and from the warehouse. In the third example, the mode corresponds to the combination of a production facility and a transportation mode with associated fixed and variable supplying costs.

4. Inventory models with multiple modes

The majority of inventory problems assume that items can only be purchased from a single supplier and/or delivered using a single transportation mode (Ekşioğlu, 2009). However, a number of studies relax these assumptions, showing that more cost effective solutions can be achieved when making inventory replenishment decisions while considering that several suppliers and transportation modes are available. The research stream considering multiple suppliers in lot-sizing decisions is rather large;

see for example the literature reviews by Minner (2003) or Aissaoui et al. (2007), compared to a limited number of studies considering multiple transportation modes. Many models also simplify the transportation costs, for example by assuming a constant cost per unit, while a realistic LTL cost structure has several breakpoints with discounts (Rieksts and Ventura, 2008).

The main focus in this section is to identify the methods for modeling transportation costs and types of transportation modes considered in the existing inventory models, including characteristics of the mode, cost and capacity, as well as to examine whether a modal splitting is allowed or not. The motivations to combine multiple modes are first discussed in Section 4.1, in particular related to the cost, speed and environmental impact of the transportation mode. Section 4.2 presents the assumptions made on the mode usage in the planning horizon. Then, the literature on papers modeling multiple transportation modes is classified in Section 4.3.

4.1. Motivations to combine transportation modes

The transportation mode can be changed more often than a product supplier with insignificant switching costs, therefore a mode selection decision can be taken together with the lot-sizing decision for each time period of the planning horizon. Only few studies have investigated the effect of using multiple modes instead of a single mode, as well as the conditions that impact multiple mode usage and savings.

4.1.1. Cost impact of combining transportation modes

Rieksts and Ventura (2010) propose that, for some systems, it may be optimal to use both FTL and LTL transportation modes simultaneously. The authors suggest that, if the inventory and setup costs are dominant, the order quantity may be a combination of full loads and a partial load. If the quantity of the partial load is not sufficient to justify another truckload using FTL transportation, it is optimal to use both modes of freight transportation. The dual-sourcing literature refers to inventory models where replenishment occurs through a regular channel and/or a more expensive but faster expedited channel (Boute and Van Mieghem, 2014). We only review the papers that consider different transportation modes explicitly as a part of expediting decision-making, and not general supply or delivery modes in a dual-sourcing context, as for example reviewed by Minner (2003). In some inventory models (for example, Kiesmüller et al. (2005)), faster transportation modes may impact the responsiveness of the supply chain and reduce the safety stocks and in-transit inventories compared to less expensive but slower modes, but they increase the transportation costs, and hence the total costs need to be evaluated in order to choose a mode.

Jain et al. (2010) and Jain et al. (2011) study the use of dual freight modes, express and regular, with fixed and variable costs and, based on examples, Jain et al. (2011) identify cost savings of more than 5% in average cost with the best (s, S) policy. The authors conclude that, when the fixed ordering cost is small relative to the fixed costs of freight modes, the freight costs dominate the savings in inventory costs, and the optimal decisions are similar to the single freight model. However, when the fixed cost of placing orders is large, the variable cost of express freight plays a more dominant role in determining the usage of each freight mode. The model with two freight modes offers a significantly higher cost savings over the best of the two single freight models when the mode cost difference is not too large. On the other hand, whenever one mode is clearly preferable to the other, the costs and policy parameters of the dual freight model are close to the best of the single freight models. The results suggest that, in the presence of large economies of scale in transportation costs when compared with ordering cost, it is advisable to primarily rely on the cheaper freight mode and use the other one only under extreme circumstances. Jain et al. (2010) conclude that, for small values of express variable costs, the use of express freight dominates; for large values, the use of regular freight dominates; and for intermediate values, significant fractions of the order quantity are shipped by each freight mode. The availability of two freight modes is most beneficial when the fixed costs of freight modes are small relative to the fixed ordering cost and the per unit cost of the express freight is not too large.

Transportation mode selection can also relate to the choice of a specific carrier or third party logistics service provider (3PL), offering different types of modes, FTL or LTL, and a specific type of transportation services, such as regular or emergency shipments that differ in terms of lead times and costs. Gürler et al. (2014) propose a model to support decision on contracting extra transportation capacity from a 3PL in addition to the internal fleet. Their findings indicate that, if the excess utilization charge is less than 25%, 3PL contracts become more beneficial even if the outsourcing cost is 25% more than in-house fleet costs under the selected parameter setting.

Geunes and Zeng (2001) investigate how backlogging arrangements can decrease the variability of transportation capacity and costs, considering expediting LTL and regular FTL modes, when compared with policies that expedite demand shortages. Perez and Geunes (2014) show that a supplier that offers different shipping mode options with different delivery costs and reliability levels may provide additional value to potential buyers by reducing the safety stock costs. Zahraei and Teo (2017) analyze the trade-offs between production smoothness, expediting through a faster transportation mode and safety stocks in a multistage supply chain. They show that the optimal solution tends to suggest deployment of more safety stock and freight expediting at the downstream smoothing stages. Nair and Closs (2006) study the effect of pricing policies and highlight the potential benefits of coordinating the operational policies of expediting by a faster air mode and replenishment with markdown policies in retail settings for products with short life cycles characterized by high demand variability. Zhao et al. (2012) investigate whether to use a cheaper means of transportation (usually ocean transportation) with larger variance in the lead time and therefore requiring a much higher capital investment, or a more expensive mode of transportation (air transportation), requiring lower capital investment. They show that a tight credit limit drives the decision maker to a much higher total expected operating cost, thus providing either a much smaller profit margin or inhibiting growth. Reiner et al. (2014) investigates the effect of using slow modes as temporary additional inventory capacity in case of low purchasing prices (speculative inventory).

4.1.2. Environmental impact of combining transportation modes

The environmental impact of transportation in inventory models and life cycle costing receive increased attention from the researchers as companies need to consider their carbon footprints when managing business decisions (Battini et al., 2014). As noted by Hoen et al. (2014), warehousing and transportation are the major drivers of carbon emissions in supply chains, and transportation mode selection problems need to quantify emissions and explicitly take them into account in decision making. The authors model transportation costs and emissions as a function of product characteristics, and determine which modes are preferred and for which range of the emission cost, given distance, cost and product characteristics. The “ownership” of the emissions for outsourced transportation is discussed and it is concluded that the shipper is responsible for the emissions resulting from transporting the items as he creates the demand for transport. However, it is in the best interest of the logistics provider to execute the transport as efficiently as possible, because emissions are aligned with fuel cost. They consider a variable emission factor, but no fixed emission factor per shipment, because transport is outsourced to a 3PL and the shipper has no control over the actual shipping. Absi et al. (2016) study a single-item green lot-sizing problem with dynamic demand and extend the work of Absi et al. (2013) to include a fixed carbon emission associated with each mode in addition to its unit carbon emission. Carbon emissions are restricted by an upper limit for each period. Palak et al. (2014) include carbon emission constraints in the mode selection decision, concluding that the optimal mode selection is impacted by the trade-offs between the total costs and carbon emissions in the supply chain. Assuming static demand, the authors consider shipping items from multiple suppliers using different modes, each characterized by costs, fixed and variable (per ton and km), and emissions, fixed and variable per mode. Several scenarios for emission constraints have been considered: emission constraints for the whole planning horizon, carbon taxes included in costs and carbon trade mechanism. Konur and Schaefer (2014) suggest that the preference for a LTL over a FTL carrier or vice versa also depends on the specifications of the carbon emission regulation policy in place. The authors determine the optimal economic order quantity with less-than-truckload (LTL) and full truckload (FTL) transportation under carbon cap, cap and trade, cap and offset, and taxing policies. Konur (2014) points out that different carriers available in the market offer trucks with different per truck costs and per truck capacities, fuel efficiency and varying emissions levels. He concludes that considering heterogeneous trucks for transportation not only decreases costs but also reduces the emissions as the carbon cap gets tighter. Battini et al. (2014) propose a "sustainable EOQ model" considering carbon emissions from warehousing and transportation activities for different modes for a constant demand case. This model is further developed by Andriolo et al. (2015) who introduce a haulage-sharing lot-sizing model in which two partners are cooperating in sharing transportation paths and handling units. A three-step methodology based on a bi-objective optimization approach (reducing both costs and emissions) is proposed, and the examples show that the haulage sharing is beneficial both for cost and emission reduction, compared to non-cooperative ordering activities. Arıkan and Jammernegg (2014) analyze the trade-off between the costs and emissions in a single period inventory control model where a newsvendor relies on dual suppliers or transportation modes, regular and emergency, including a constraint on the product carbon footprint. Arıkan et al. (2014) analyze the effect of transport lead time variability of emergency and regular modes through the replenishment policy on economic and environmental performances of supply chains with uncertain demand.

4.1.3. Other impacts of combining transportation modes

Fan et al. (2017) suggest using transportation modes with different lead times for risk mitigation to deal with supply chain disruptions. When choosing a low-speed mode, companies create a buffer time and flexibility to respond to a disruption and switch to a faster transportation mode to save time for adopting alternative plans and avoiding huge losses. The approach is similar to the concept of slow steaming, where the speed of transport is lower than the original operating speed. According to the authors, the best strategy to cope with short-term supply chain disruptions and disruptions at distribution centers is to save slack time during the transport of final products. In case of long-term supply chain catastrophes, the best strategy to cope with these disturbances is to save slack time during the transport of raw material as well as during the international transport of final products.

The impact of pricing decisions for optimal coordination has recently received more attention. For example, Toptal and Bingöl (2011) consider a model where the FTL carrier makes his pricing decision based on previous knowledge on the LTL carrier's price schedule and the retailer's ordering behavior. The retailer then determines his/her order quantity through an integrated model that explicitly considers the transportation alternatives and capacities. The numerical analysis shows that the FTL carrier may significantly increase his profit through better pricing, and there is further opportunity of savings if the truckload carrier and the retailer coordinate their decisions.

4.1.4. Examples of transportation mode combinations

The following examples of transportation mode choices in inventory models can be found in the literature:

- Choice between different FTLs with different capacities or shipping frequencies (Speranza and Ukovich (1994), Jaruphongsa et al. (2005)),
- Choice between FTL and LTL with the same or different maximum capacities (Diaby and Martel (1993), Rieksts and Ventura (2008)),
- Emergency versus regular modes, with different constant unit costs, lead times and emission rates (Kiesmüller et al. (2005), Jain et al. (2010), Jain et al. (2011), Fan et al. (2017)),
- Choice between different types of contracts: With or without reserved capacity with different unit costs (van Norden and van de Velde (2005), Gürler et al. (2014)),

- Choice among different container/packaging types loaded on the same truck (Ben-Khedher and Yano (1994)),
- Modes with different costs and emission rates (Absi et al. (2013), Palak et al. (2014), Konur and Schaefer (2014), Konur (2014), Absi et al. (2016)).

Some models allow only a single transportation mode to be selected and used, while the others allow several transportation modes to be combined and to be used simultaneously, each mode shipping a fraction of the demand. Combining transportation modes or mode mix corresponds to order splitting in procurement and supplier selection decisions, when the total order quantity is split among several suppliers. The reasons for order splitting in procurement can be a total cost or service improvement, reduction of dependency on a single supplier, transportation lead time reduction or if the total demand is larger than the supply capacity of a single supplier, etc. Order splitting, also termed as multiple sourcing, is often used in stochastic demand settings to reduce the risk of stock-out situations or the costs of safety stocks, typically combining regular and emergency shipments as, for example, in Thomas and Tyworth (2006). The benefits of combining transportation modes can be cost and emission savings, risk mitigation in case of disruptions, access to extra transportation capacity in addition to internal fleet.

4.2. Mode usage during the planning horizon

The majority of the existing studies on inventory planning consider that a single transportation mode is available and assume a simplified cost structure. The limited number of inventory models that take multiple modes into account can be divided into the following groups based on their assumptions on transportation mode usage:

1. Multiple transportation modes are available, but only one mode can be chosen for replenishments during the whole planning horizon. For example, Baumol and Vinod (1970) compare the total costs when using different modes with different unit transportation costs in an EOQ-type inventory model, and choosing the transportation mode that yields the lowest total costs. Other authors have studied the case where multiple locations need to be replenished, and each location can only be replenished using one transportation mode. However, these modes can be different for each customer. An example of such a model for stochastic demand, formulated as a mathematical programming model, can be found in Kutanoglu and Lohiya (2008).
2. Multiple transportation modes are available, but only one mode can be used for replenishments within each time period of the planning horizon. However, this mode can be different for each time period. Diaby and Martel (1993) proposed such a single-item dynamic lot-sizing model, assuming that only one mode can be used, but one can switch from one mode to another during the next time period.
3. Multiple transportation modes are available and can be used simultaneously, i.e. combined, and each mode can deliver a fraction of the total replenishment quantity during each period (see for example Jaruphongsa et al. (2005), Abad (2007) or Absi et al. (2013)).

4.3. Approaches to model multiple modes

When multiple transportation modes are available, the transportation costs, which depend on the shipment size, can be modeled in two ways:

1. By combining all modes into a single cost function, where each quantity corresponds to a pre-defined mode,
2. By modeling each mode with its own cost function, i.e. the decision variables reflect the quantity shipped by each mode.

When combining different modes into a single cost function, the quantity shipped by each mode is not modeled explicitly. Instead, one decision variable reflects the total shipping quantity, which corresponds implicitly to the pre-defined type of mode that is used. Two approaches can be used to combine transportation modes into one cost function: Pre-processing and use of a "car-load" discount schedule.

The pre-processing approach includes combining transportation cost functions for all modes and creating a new general cost function, where to each quantity is associated the cost of the mode with the lowest cost for this quantity. It is assumed that only one transportation mode can be used for each shipment quantity. Hence, the pre-processed models are valid if the modes from different transporters cannot be combined for the same order, if one particular supplier is always superior (cheaper and faster) to others, or if only one mode is available due to some restrictions for specific quantity intervals. The pre-processing calculation may become a complex task when the number of modes with various discount schedules increases and, subsequently, the number of price break-points increases. The graphical representation of such transportation function is provided by drawing the cost functions of all modes and marking the lowest costs among all modes for each quantity point. Diaby and Martel (1993) apply this pre-processing approach and derive an analytical expression for such a cost function that combines multiple modes (Figure 8, inspired from Diaby and Martel (1993) combines cost functions from Figure 6 and Figure 7), considering the problem of determining the optimal purchasing and shipping quantities over a finite planning horizon for a multi-echelon distribution system with dynamic demand. The procurement cost, which includes transportation costs, is a general (i.e. not necessarily concave or continuous) piecewise-linear function of the shipped quantities. The general cost function is denoted $G(Q)$ for the shipped quantity Q (measured in cwt, tons, pallets, etc.), where Q_{max} is the upper bound of the available capacity in a given time period. Figure 8 shows the cost structure that incorporates different discounts from suppliers and transporters. It is applied in situations where different transportation modes have similar lead times, and for constraints imposed by suppliers or carriers to qualify for certain types of discounts.

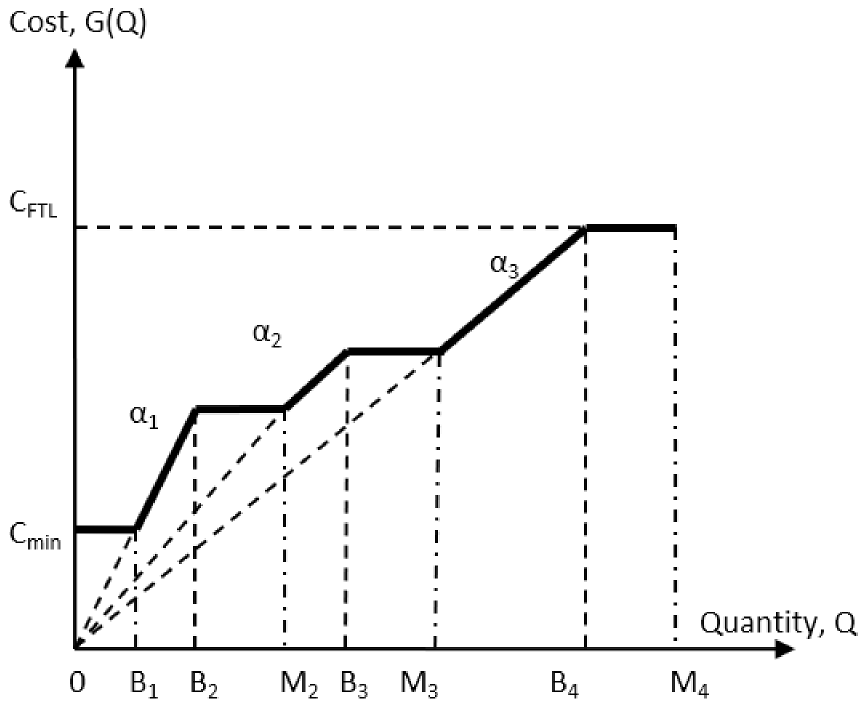

Figure 6: All-unit discount cost function

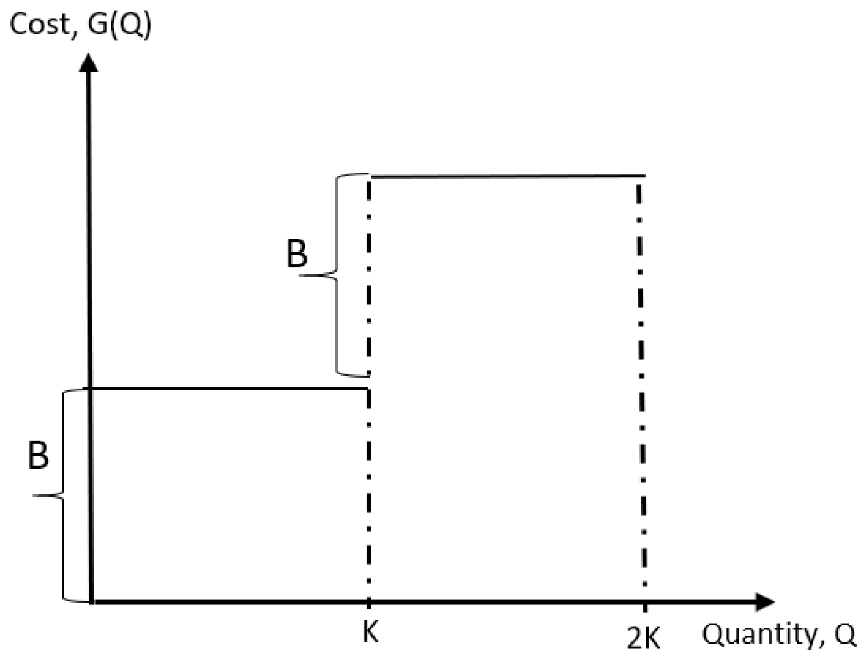

Figure 7: FTL cost function

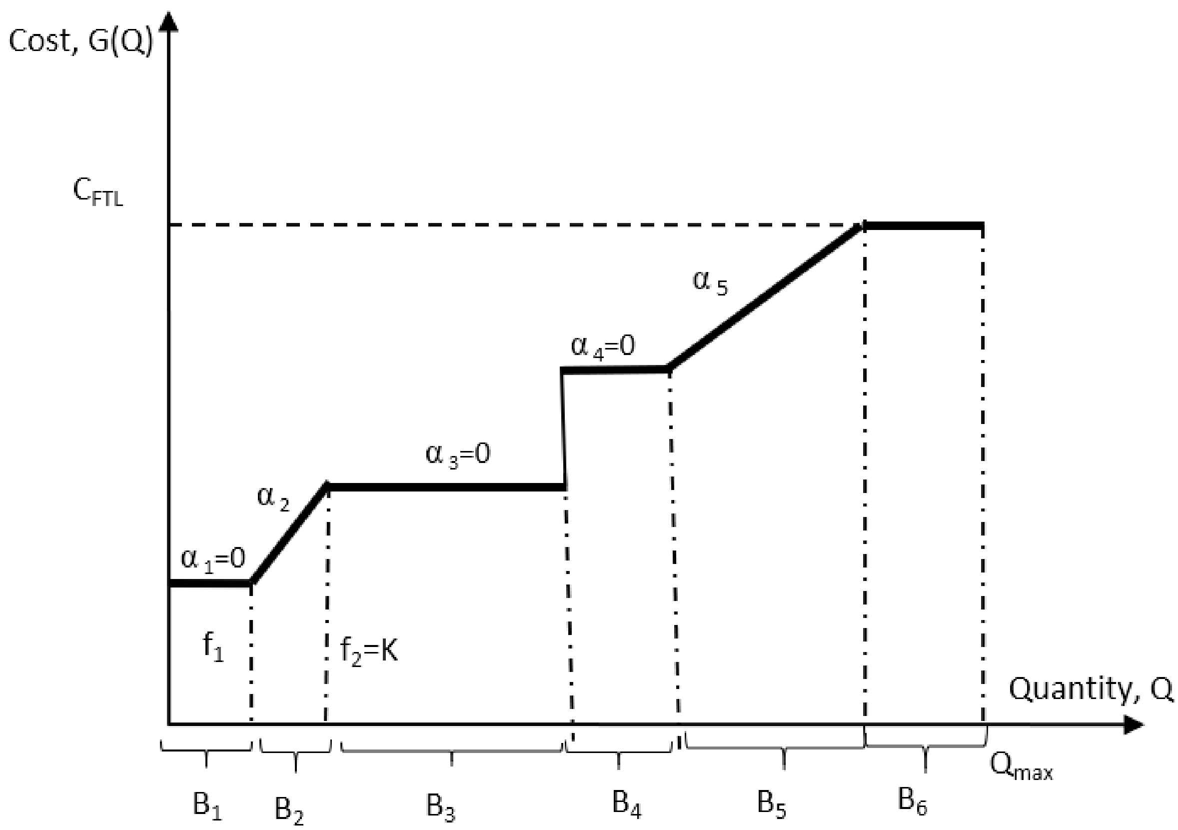

Figure 8: General cost function

The general cost function is generated by analyzing and comparing the costs of all alternative supply/transportation modes and selecting only the one with the least cost for each interval. The cost function is defined as follows: $G(Q) = \sum_{j=1}^{J}(f_j\delta_j + \alpha_j q_j)$, where j is the interval length, f is the fixed charge for each interval j (δ_j is 0 or 1), α is the unit procurement/shipment cost and q is the shipped quantity. The price breaks for each interval, the size of the interval and the lowest unit variable cost within the interval by comparing different modes need to be found a priori, by pre-processing the alternative cost functions for each mode and creating a general cost function, which is an input of the model. However, it is not applicable for a situation where the order can be split among different modes, or when determining the best optimal mode for each quantity interval is a non-trivial and time-consuming task. It can also be difficult to determine the maximum shipment quantity for which the cost needs to be pre-processed.

Another possibility to combine different modes in the same function is to use the so-called "car-load" discount schedule. This method can only be used for modeling two modes, FTL and LTL, with the same cargo limits in the same cost function. These two modes are treated as one mode, where a switch to FTL mode is based on shipped or over-declared quantity, as in Abad (2007) or Li et al. (2004). Up to one cargo capacity, the same principle as in pre-processing is applied where, for each quantity, the mode providing the lowest cost, FTL or LTL, is chosen. After the cargo capacity is reached, combining FTL and LTL for each shipment quantity is allowed, whenever the total costs are the lowest. The carload discount schedule is described by Nahmias (1989): "A carload consists of M units. The supplier charges a constant rate c per unit until you have paid the cost of a full carload, at which point there is no charge for the remaining units in that carload. Once the first carload is full, you again pay c per unit until the second carload is full and so forth". However, all the reviewed models with car-load discount schedule, except Abad (2007) and Mendoza and Ventura (2013), assume only one price-break interval, i.e. a constant unit cost for the LTL mode. A single-item problem for optimal lot size determination under carload discount schedule with constant demand rate is considered by Elhedhli and Benli (2005). A solution procedure leading to a comparison of at most three candidate points for optimality is proposed, showing that the optimal lot size for the carload discount schedule in the constant demand case is rather simple to determine. Van Eijs (1994) also considers a multi-item inventory system with a "car-load discount schedule" for shipping costs, where a constant unit LTL cost is charged up to a certain volume, from which a fixed charge for FTL is applied until the container is full. Only one full container can be ordered at each period. In contrast to the pre-processing method, this function allows modes to be combined when it is economical, for example shipping one full FTL with some LTL quantity. The pre-processing method allows either LTL or FTL to be used, i.e. no combinations are allowed.

As it has been shown, multiple modes can be modeled using a single cost function either by pre-processing or applying a carload discount schedule. In these methods, the type of mode used for each quantity is pre-determined by the shipment size. The drawback of a pre-processing method is that it does not assume that multiple modes with different capacities can be combined for the same order. Figure 9 illustrates the example of cost functions for the three FTL modes described in Section 3.5.

Using the pre-processing method, a single cost function could be obtained by choosing the lowest cost among the three modes for each quantity (i.e. lying either on the blue, orange or purple lines), and the obtained cost for some cases can be larger than when combining modes (the green dashed line). Several authors have considered that several FTL or multiple set-up modes are available and that it is possible to combine them for the same shipment. Knowles and Pantumsinchai (1988) assume a constant demand case; Pantumsinchai and Knowles (1991) and Dullaert et al. (2005) assume stochastic demand, while Jaruphongsa et al. (2005), Jaruphongsa et al. (2007), Ekşioğlu (2009) or Absi et al. (2013) consider a dynamic deterministic demand. The main benefit from allowing modes to be combined compared to a pre-processing approach is that lower costs can be obtained for some quantities. Some other benefits can also be mentioned:

- The reality is modeled and pre-processing is not required, in particular the published rates can

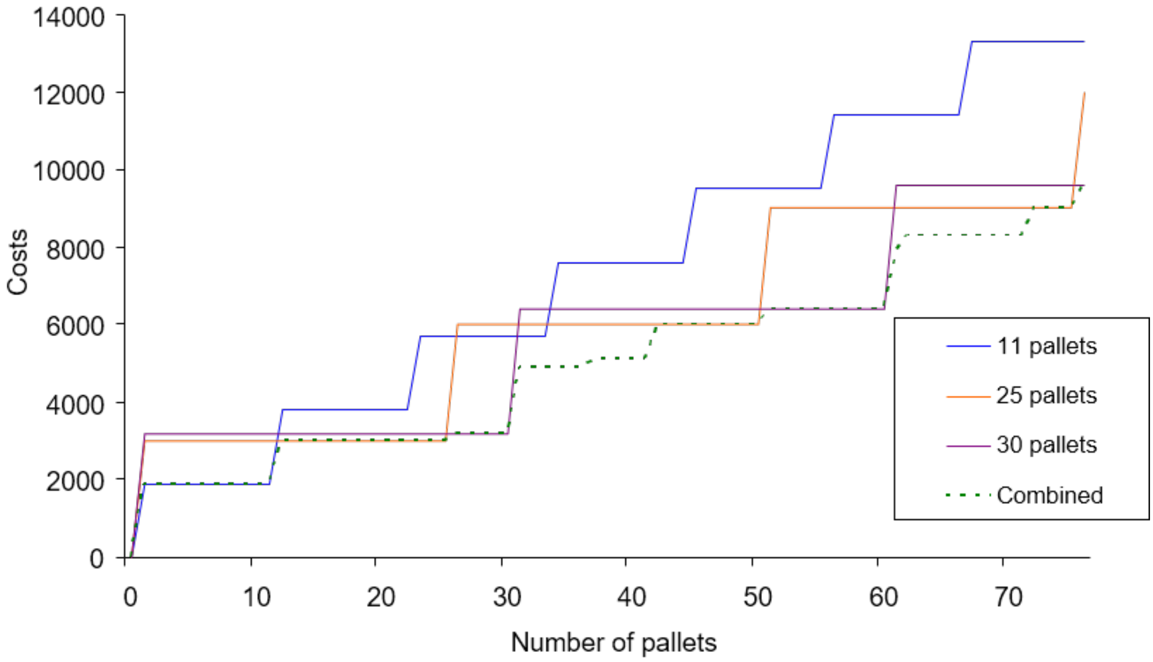

Figure 9: Multiple FTL modes are available (11', 25' and 30' containers)

be used straight away,

- It is easier to apply restrictions on each mode, and if needed to constrain the combinations that are allowed for each period (with pre-processing, the cost functions would need to be re-calculated for each period with new limitations).

Table 12 summarizes the methods used for modeling multiple modes, and the classification parameters of the reviewed models are listed in Table 13. Tables 14 through 20 review the existing inventory models that consider transportation mode selection decisions, classified according to the nature of the demand rate: Constant, stochastic and dynamic. Tables 14 through 20 are using the classification parameters defined in Table 13 for Columns "Cost type", "Mode usage" and "Modeling modes". We identified 19 papers with constant demand, where the models are mainly solved using EOQ-based heuristic algorithms, and half of the papers do not allow mode combinations. The reviewed models with constant demand mainly assume multiple FTL modes, few papers considering LTL mode use approximation or carload discount modeling. With 32 articles, the largest group of articles is the one with stochastic demand, where 30% of the articles assuming a dual sourcing context, i.e. using fast and regular shipment modes. Discrete event simulations, Markov process models as well as analytically derived heuristic policies and EOQ-heuristics represent various solution methods for such models. Few models integrate supplier selection together with mode choice decisions, as well as emission rates when considering various modes in addition to transportation lead times. LTL mode costs have been typically modeled without considering discount. Out the 20 articles with dynamic demand, a third also consider supplier selection decisions. Other considerations included energy and emission costs and fixed costs of owning fleet. To solve the mixed integer mathematical models, dynamic programming algorithms and various heuristics, as well as standard solvers, have been used.

5. Conclusions and suggestions for future research

During the deregulation of the transportation industry in the 1980s and 1990s, the majority of the published inventory models focused on transportation mode selection problems with constant demand

Method characteristics	Multiple modes with a single cost function		Each mode with its own cost function
	Method 1 (car-load discount)	Method 2 (pre-processing)	Method 3
Decision variable	Total shipment size	Total shipment size	Quantity shipped by each mode
Examples	Elhedhli and Benli (2005)	Diaby and Martel (1993)	Jaruphongsa et al. (2005)
Advantages	• Easy to model • No pre-processing needed • Freight rates provided by carriers can be directly used in the model	• The general cost function can describe different discount schedules • Applicable if it is easy to determine which rate is superior	• Multiple FTL modes can be combined • Published rates can be used straight away (no need for pre-processing) • It is easier to apply restrictions on each mode, and if needed to restrict the combinations that are allowed to be used
Drawbacks	Only two modes (FTL and LTL with single quantity interval) with identical maximum can be modeled	• Can provide suboptimal (worse compared to other methods) solutions since modal splitting is not allowed • Pre-processing procedure for provided freight rates is needed • Maximum shipment quantity to pre-process needs to be known in advance • Number of break-points can be very large • Difficult to define constraints on availability of specific mode (cost function needs to be processed again)	• The existing models consider that only FTL modes are available • Number of decision variables increases when the number of modes increases
Combination	Allowed	Not allowed	Allowed

Table 12: Summary of methods for modeling multiple modes

	Cost type	Mode usage	Modeling modes
1	Constant unit	Single mode in the whole horizon	Pre-processing
2	Fixed charge	Single shifting	Car-load
3	FTL	Combination	Own cost function
4	LTL		
5	Approximation		
6	Carload schedule		
7	Combined replenishment mode		

Table 13: Classification parameters

or stochastic demand, typically modeling constant unit transportation costs, or various FTL/container sizes, with transportation lead time as one of the main differentiation criteria for modes. Since the 90s, more papers appeared considering LTL and various discount schedules, as well as dynamic demand, allowing mode combinations. During the last decade, a large number of articles appeared that recognize the importance of considering transportation costs and order replenishment decisions. They also include other factors such as emission rates and constraints, realistic discount schedules and multiple modes, availability of external carriers, consideration of multiple supply chain stages and supplier selection decisions. Multi-objective models, and models combining various solution methods, for example AHP (Analytical Hierarchy Process) and Mixed Integer Programming approaches, have appeared. Due to the computational complexity of modeling transportation cost functions with discounts and mode selection, the researchers propose various solution algorithms and heuristics to cope with the computational limitations of standard solvers. Various methods for modeling transportation costs and multiple modes have been proposed in the inventory management literature. Although realistic transportation costs exhibit different discount schedules, the majority of the existing studies on inventory planning simplify the transportation costs by assuming linear unit transportation costs or a fixed charge cost structure without considering mode capacity constraints. Some authors, when modeling LTL transportation costs, simplify the cost function by omitting the minimum quantity charge (as for example Tsai (2007)), the over-declaring possibility (Vroblefski et al. (2000)) or the maximum mode capacity (Qu et al. (1999)). As pointed out by Rieksts and Ventura (2008), a realistic LTL cost structure has several breakpoints with discounts on the cost per unit. However, many authors simplify the LTL cost by assuming a constant cost per unit and a single price break. Mendoza and Ventura (2013) stress the difficulty of modeling the actual transportation cost structure due to discount schedules. But the impact of using approximation functions instead of real costs and linearization techniques should be investigated to a larger extend.

When reviewing inventory models with multiple modes, we classified the methods where (1) A preprocessing is performed and (2) The cost function of each mode is modeled. Some models assume a single mode is selected and used in all periods of the planning horizon (as, for example, in the inventory-theoretical models). Other models allow different modes to be used in each period but without combination opportunity, typically applying a pre-processed general transportation cost function that implicitly determines the mode for each quantity (Diaby and Martel (1993)).

Only a limited number of articles have considered a multi-mode delivery option, allowing several transportation modes to be combined for the same order, by either modeling a car-load discount schedule or modeling the exact cost for each mode. Most of the articles allowing modal splitting are developed for a single item and equal capacities for LTL and FTL modes, modeled as a carload-discount schedule, where the LTL mode has no minimum charge for extremely small shipments, only one price interval and therefore constant unit costs. Compared to supplier selection models, where

Table 14: Inventory models with multiple modes and constant demand (1/2)

Authors	**Nb. items**	**Modes**	**Costs**	**Cost types**	**Other mode characteristics**	**Modal splitting**	**Capacity**	**Mode usage**	**modeling modes**	**Solution methods**	**Nb. stages**	**Network type**
Ab Rahman et al. (2016)	Single	FTL and LTL	Approximation cost function	5 (3, 4)	Distance factor	Not allowed	Limited	1	3 (approximation)	EOQ-based heuristic algoirthm	1	One to one
Abad (2007)	Single	FTL and LTL	Carload discount with multiple price-breaks for LTL and over-declaring	4, 6	Capacity	Allowed	Limited (same for LTL and FTL)	3	2	EOQ-based model & search procedure	1	One to one
Abdelwahab and Sargious (1990)	Single	FTL, LTL	Fixed for FTL, fixed and variable per LTL	2, 5	Capacity, transit time	Not allowed	Limited	1	3	EOQ-model	1	One to one
Andriolo et al. (2015)	Multi	Container/vehicle types, rail, road, ship	Internal and external per mode	3	Capacity	Allowed	Limited	3	1	EOQ- based bi-objective optimization algorithm	1	Many to many
Axsäter and Grubbström (1979)	Single	Multi	Multiple modes	5	Velocity, transit time	Not allowed	Unlimited	1	3	EOQ-based analytical procedure	1	Many to many
Battini et al. (2014)	Single	Container/vehicle types, rail, road, ship	Internal and external per mode	2	Capacity	Allowed	Limited	3	1	EOQ- based algorithm	1	One to one
Bertazzi and Speranza (1999)	Multi	Trucks at different frequencies	Fixed per trip (FTL)	3	Frequency, capacity	Not allowed	Limited	1	3	MILP, dynamic programming heuristic	Multi	Many to many
Bouchery et al. (2016)	Multi	FTLs, LTL	Fixed cost per FTL and unit cost per LTL	1, 3	Fixed and variable emission rates, lead time, capacity	Not allowed	Limited	1	3	Multiobjective optimization, EOQ-based procedure	1	One to one
Hall (1985)	Single	FTL, LTL	Fixed for FTL, fixed and variable per LTL	2, 6	Capacity, transit time	Not allowed	Limited	1	3	EOQ-based algorithm	1	One to one
Knowles and Pantumsinchai (1988)	Single	Multiple containers	FTL costs	3	Capacity, no partial fillings	Allowed	Limited (integer multiple of each other)	3	3	Dynamic programming algorithm	1	One to one

Table 15: Inventory models with multiple modes and constant demand (2/2)

Authors	**Nb. items**	**Modes**	**Costs**	**Cost types**	**Other mode characteristics**	**Modal splitting**	**Capacity**	**Mode usage**	**modeling modes**	**Solution methods**	**Nb. stages**	**Network type**
Konur (2014)	Single	FTL with different types of trucks	Per truck	3	Emissions (per carried unit and empty truck)	Allowed	Limited	3	3	MIP and FTL search heuristic	1	One to one
Konur and Schaefer (2014)	Single	FTL and LTL	FTL per truck and LTL per unit	1, 3	Carbon emission per unit for LTL and emissions per empty truck and the loads for FTL.	Not allowed	Limited (FTL)	1	3	EOQ-based models	1	One to one
Mendoza and Ventura (2013)	Single	FTL and LTL	Carload discount with over-declaring	4, 6	Capacity	Not allowed	Limited (FTL)	1	2	MIP with standard solver	1	Many to one
Palak et al. (2014)	Single	Rail, track barge	Combined replenishment with fixed order cost and variable unit cost	2, 7	Fixed and variable emission per mode	Allowed	Only specific modes available over certain distance	3	3	MIP and dynamic programming algorithm	1	Many to one
Pazhani et al. (2016)	Single	Multiple FTL modes	Fixed per truck	3	Capacity	Allowed	Limited	3	1	MIP and standard solver	Multi	Many to many
Rieksts and Ventura (2008); Rieksts and Ventura (2010)	Single	FTL and LTL	Carload discount	1, 6	Capacity	Allowed	Limited	3	2	Exact algorithms and power-of -two heuristics	1	One to one
Sheffi et al. (1988)	Single	Multiple	Fixed cost per truck	3	Capacity, transit time	Not allowed	Limited	1	3	EOQ-based algorithm	1	One to one
Speranza and Ukovich (1994)	Multi	Trucks at different frequencies	Fixed per trip (FTL)	3	Frequency, capacity	Not allowed	Limited	1	3	MIP models	1	One to one

Table 16: Inventory models with multiple modes and stochastic demand (1/3)

Authors	Nb. items	Modes	Costs	Cost types	Other mode characteristics	Modal splitting	Capacity	Mode usage	modeling modes	Solution methods	Nb. stages	Network type
Arıkan and Jammernegg (2014)	Single	Two: express and regular	Per unit cost and emission	1	Emission upper bound constrain	Allowed	Unlimited	3	3	Analytically derived optimisation procedure	1	One to one
Arıkan et al. (2014)	Single	Regular and emergency	Per unit cost and emission rate, fixed per regular order	1, 2	Transportation lead time and emission	Allowed	Unlimited	3	3	Descrete event simulation	1	One to one
Baumol and Vinod (1970)	Single	Multiple	Constant unit shipping cost	1	Transit time	Not allowed	Unlimited	1	3	EOQ-based model	1	One to one
Blauwens et al. (2006)*	Single	Multiple	Constant unit costs	1	Transit time and its deviation, capacities	Not allowed	Limited	1	3	EOQ-based model	1	One to one
Bregman et al. (1990)	Multi	Variable and fixed cost options	Fixed and varable	1, 3	Lead time, total capacity	Allowed	Limited	2	3	Heuristic procedure	Multi	One to many
Chiang (2010)	Single	Regular and emergency	Fixed cost per emergency shipment	1, 3	Transportation lead time	Allowed	Unlimited	3	3	Analytically derived heuristic policy	1	One to one
Constable and Whybark (1978)*; Buffa and Reynolds (1979)*	Single	Multiple	Constant unit costs	1	Transit time and its deviation	Not allowed	Unlimited	1	3	EOQ-based procedure	1	One to one
Dullaert et al. (2005)	Single	Multiple	FTL costs	3	Lead time, different capacities, no partial filling is allowed	Allowed	Limited, no partial filling is allowed	3	3	Evolutionary algorithm heuristic	1	One to one
Feki et al. (2016)	Multi	3 modes: internal, external and spot	Unit costs	1	Availability rates per mode	Allowed	Total fleet	3	3	Dynamic programming formulation, simulations	1	One to one
Geunes and Zeng (2001)	Single	Inhouse fleet and LTL expediting	Fixed per shipment and variabel unit cost	1, 2	Transportation lead time	Allowed	Limited	3	1	Analytically derived heuristic policy	1	One to one

Table 17: Inventory models with multiple modes and stochastic demand (2/3)

Authors	**Nb. items**	**Modes**	**Costs**	**Cost types**	**Other mode characteristics**	**Modal splitting**	**Capacity**	**Mode usage**	**modeling modes**	**Solution methods**	**Nb. stages**	**Network type**
Gumasta et al. (2012)	Multi	Own fleet and external carriers	Fixed and variable costs	2	Various capacities of external carriers	Allowed	Limited	3	3	Analytically derived heuristic policy	1	One to many
Gürler et al. (2014)	Single	Inhouse fleet and outsourcing (with reserved/optional capacity)	Fixed cost for shipment and for maintaining/ reserving a fleet, excess usage penalties	2	Reserved or excess usage types	Allowed	Limited	3	3	Analytically derived policy parameters	Multi	One to many
Hoen et al. (2014)	Single	Multiple	Constant unit cost based on volumetric weight, cost factor per kilogram of product shipped over 1 km, and the distance	1	Lead time, unit emissions per mode	Not allowed	Limited per order	1	3	Optimization procedure	1	One to one
Jain et al. (2010); Jain et al. (2011)	Single	Two: express and regular	Fixed for regular and fixed and variable for express	2, 3	Lead time	Allowed	Limited	3	3	Heuristic decomposition approach	1	One to one
Kiesmüller et al. (2005)	Single	Two modes: fast and slow	FTL	3	Lead time, same capacity no partial filling is allowed	Allowed	Limited, no partial filling is allowed	3	3	Optimization algorithm (base-stock policy)	1	One to one
Nair and Closs (2006)	Multi	Ocean and air	Unit costs	1	Lead time	Allowed	Unlimited	3	3	Simulation tool	1	One to one
Pantumsinchai and Knowles (1991)	Single	Multiple containers	FTL	3	No partial filling is allowed	Allowed	Limited (integer multiple of each other)	3	3	Optimization heuristic	1	One to one
Perez and Geunes (2014)	Single	Two: reliable and unreliable	Fixed and variable cost, lead time	2	Lead time reliability	Allowed	Unlimited	3	3	Analythical and optimization procedure	1	One to one
Schaefer and Konur (2015)	Single	FTL and LTL	Fixed and variable costs and emissions, lead time	1, 2	Lead time	Not allowed	Limited	1	3	Heuristic bi-objective procedure	1	One to one

Table 18: Inventory models with multiple modes and stochastic demand (3/3)

Authors	Nb. items	Modes	Costs	Cost types	Other mode characteristics	Modal splitting	Capacity	Mode usage	modeling modes	Solution methods	Nb. stages	Network type
Tempelmeier and Bantel (2015)	Single	Inhouse fleet and external	Fixed per shipment and variabel unit cost, per order for external	1, 3	Transportation lead time	Allowed	Limited	3	1	Analytically derived heuristic policy	1	One to many
Toptal and Bingöl (2011)	Single	FTL carrier, LTL carrier	Both carriers announce their pricing schedules before the retailer makes his replenishment decision.	1, 3	FTL carrier makes pricing decision based on previous knowledge on LTL carrier's price schedule and the retailer's ordering behavior.	Allowed	Limited (same for LTL and FTL)	3	3	Analytically derived optimisation procedure	1	One to one
Tyworth (1991)*; Tyworth and Zeng (1998)*; Brown (1995)	Single	Multiple	Various approximation functions	5	Transit time and its deviation, capacities	Not allowed	Limited	1	3	EOQ-based procedure	1	One to one
Vernimmen et al. (2008)	Single	Multiple modes	Variable unit costs	1	Lead time, capacity	Not allowed	Limited	1	3	Analythically derived procedure	1	One to one
Wu et al. (2011)	Multi	Multiple	Unit costs	1	Lead time	Not allowed	Unlimited	1	3	Tabu algorithm	Multi	One to many
Zahraei and Teo (2017)	Single	Regular and extraordinary freight expediting	Unit costs	1	Lead time, total expediting capacity per stage	Not allowed	Limited	1	3	Analythically derived optimization procedure	Multi	One to one
hong Zhao et al. (2010)	Multi	Own and public vessels	Fixed and unit per trip, only fixed for own vessels	2	Lead time, capacity	Not allowed	Limited	1	3	Markov decision model	1	One to many
Zhao et al. (2012)	Single	Two modes: fast and slow	Unit costs, capital constraint	1	Lead time - deterministic for fast, stochastic for slow	Allowed	Unlimited	3	3	Simulation tool	1	One to one
Zhou et al. (2009)	Single	Two: reliable and unreliable 3PL	Costs, lead time variability	3	Capacity, availability	Not allowed	Limited	2	3	Analythical and Markov process simulations	1	One to one

Table 19: Inventory models with multiple modes and dynamic deterministic demand (1/2)

Authors	Nb. items	Modes	Costs	Cost types	Other mode characteristics	Modal splitting	Capacity	Mode usage	modeling modes	Solution methods	Nb. stages	Network type
Absi et al. (2013)	Single	Multiple replenishment modes	Fixed set up and unit cost	2, 7	Unit carbon emission with various carbon constraints	Allowed	Unlimited	3	3	MIP and dynamic programming algorithm	1	Many to one
Absi et al. (2016)	Single	Multiple replenishment modes	Fixed set up and unit cost	2, 7	Fixed and unit carbon emission for each mode. Emission cap per period	Allowed	Unlimited, carbon cap per period	3	3	MIP and dynamic programming algorithm	1	Many to one
Akbalik and Rapine (2017)	Single	Multiple replenishment modes	Fixed ordering and fixed per batch	3, 7	Divisable batch sizes	Allowed	FTL batches	3	3	MIP and approximation algorithm	1	Many to one
Bertazzi and Zappa (2012)	Multi	Sea and plane	Fixed and variable unit costs	1	Lead time, capacity	Allowed	Unlimited	3	3	MIP and standard solver	1	One to one
Bilgen and Günther (2010)	Multi	FTL and LTL	Fixed per truck and unit cost for LTL	1, 3	Predefined delivery points for FTL	Allowed	Limited per truck	3	3	MIP and standard solver	Multi	Many to many
Choudhary and Shankar (2014);	Single	Multiple carriers with FTL fleets	FTL	3	Carriers FTL-fleet capacity per carrier per period. One carrier per suppler.	Not allowed	Lot size not more than 1 FTL size.	2	3	Goal programming, standard solver	1	Many to one
Choudhary and Shankar (2013)	Single	Multiple carriers with FTL fleets	FTL	3	Carriers FTL-fleet capacity per carrier per period. One carrier per suppler.	Not allowed	Lot size less than 1 FTL size.	2	3	MIP, standard solver	1	Many to one
Diaby and Martel (1993)	Single	Multiple modes	All modes (FTL, LTL with various discounts) combined in a general cost function	1, 2, 3, 4	Capacities	Not allowed	Limited	2	1	MIP and Lagrangian relaxation-based algorithm	Multi	One to many
Ekşioğlu (2009)	Single	Multiple FTL modes	Multiple set-ups, FTL-like, and fixed set-up with constant unit costs, structures	2, 7	Capacities	Allowed	Limited	3	3	MIP and primal-dual algorithm	1	One to one

Table 20: Inventory models with multiple modes and dynamic deterministic demand (2/2)

Authors	Nb. items	Modes	Costs	Cost types	Other mode characteristics	Modal splitting	Capacity	Mode usage	modeling modes	Solution methods	Nb. stages	Network type
Hammami et al. (2012)	Multi	Multiple modes	Unit costs	1	Delivery frequency, lead time uncertainty	Not allowed	Limited	2	3	AHP an MIP using standard solver	1	Many to many
Jaruphongsa et al. (2005); Jaruphongsa et al. (2007)	Single	Two modes	Multiple setups, FTL-like, and fixed set-up with constant unit costs	2, 7	Capacity is integer of each other	Allowed	Limited (integer multiple of each other)	3	3	MIP and dynamic programming algorithms	1	One to one
Kopanos et al. (2012)	Multi	Own and 3PL fleet	FTL for 3 PL and unit cost for own	1, 3	Min and max capacity per truck	Allowed	Limited	3	3	MIP and standard solver	Multi	Many to many
Li et al. (2004)	Single	FTL and LTL	FTL cost and LTL with constant unit rate (Carload discount, single price break)	1, 6		Allowed	Limited (same for LTL and FTL)	3	2	MIP and dynamic programming algorithm	1	One to one
Mogale et al. (2017)	Multi	Multi	Unit and Fixed per trip	2	Capacity	Allowed	Limited	3	3	MINLP, , ant colony optimization Meta heuristic	Multi	Many to many
Mokhtari and Hasani (2017)	Multi	Multiple modes	Unit transportation and energy costs	1	Energy costs	Not allowed	Unlimited	1	3	Fuzzy goal programming, heuristic	1	Many to many
Reiner et al. (2014)	Single	Road and water	Unit costs	1	Lead time, capacity	Allowed	Limited	3	3	Simulations, dynamic process analysis	1	One to one
Rizk et al. (2006b); Rizk et al. (2006a)	Multi	Multiple modes	All modes (FTL, LTL with various discounts) combined in a general cost function	1, 2, 3, 4	Capacities	Not allowed	Limited	2	1	MIP and Lagrangian relaxation based algorithm	1	One to one
Toptal et al. (2014)	Single	own and external	Fixed per vehicle and holding cost per own vehicle	3	Lead time, capacity	Not allowed	Limited	2	3	MIP and tabu-search heuristic	1	One to many

order splitting is allowed in most of the studies, modal splitting, i.e. the simultaneously use of multiple transportation modes, seems to be an underdeveloped research area. The majority of the reviewed articles with multiple modes propose heuristic solution methods, and those using standard solver tools acknowledge the need for developing more efficient solution algorithms for larger problem instances to decrease the solution time. Akbalik and Rapine (2017) provide an overview of various solution methods for multi-mode replenishment, and propose to focus more on combining modes with different types of discount schedules.

Several papers consider modes with different capacities (Jaruphongsa et al. (2005), Dullaert et al. (2005), Jaruphongsa et al. (2007) and Ekşioğlu (2009)). However, in these studies, only multiple FTL modes are modeled, while LTL modes are not considered. In addition, some of the reviewed models (e.g. Dullaert et al. (2005) and Kiesmüller et al. (2005)) assume bulk shipments and that the modes always fully utilize the capacity. However, this assumption is not always realistic in practice for other types of shipments. No model has been found that combines realistic LTL modes with several price breaks and FTL modes with various capacities.

Other considered mode characteristics include transportation lead time (Kiesmüller et al. (2005)), carbon emission rates (Absi et al. (2013) and Konur (2014)), availability (Feki et al. (2016)) and frequencies (Speranza and Ukovich (1994)), carrier's total fleet capacity (Choudhary and Shankar (2014)) or mode type (internal or outsourced as in Gürler et al. (2014)). Based on our extensive literature review, we conclude that the majority of the inventory models:

- Simplify or omit transport costs,
- Disregard the re-calculation of the discount schedules to obtain true costs for LTL (over-declaring),
- Consider that a single mode is available with no opportunity to change or combine modes.

During the last decade, new research streams have appeared including:

- The consideration of environmental aspects such as carbon emission rate as selection criteria under different carbon cap mechanisms and policies (Absi et al. (2013) and Konur (2014)),
- Cost implication investigations of omitting, simplifying and approximating when modeling freight discounts (Mendoza and Ventura (2013)),
- Managerial implications in terms of costs and parameter analysis of conditions when using more than one mode (Jain et al. (2011)),
- Capacity and contract constraints when a shipper considers combining its internal fleet with outsourcing at reserved and spot rates (Gürler et al. (2014) and Feki et al. (2016)),
- Consideration of different modes for supply chain risk and disruption mitigation (Fan et al. (2017)),
- Optimal coordination of transportation and quantity discounts from the perspectives of the parties who offer them, rather than of those who take them (Ke et al. (2014)), cost sharing contracts and pricing mechanisms for better coordination (Yıldırmaz et al. (2009)).

Hence the significance of transportation costs and mode selection in inventory models has increased due to:

- Cost optimization opportunities, not only for the buyer but for the whole supply chain,
- Consideration of carbon emission criteria and policy constraints,
- Impact on operational and disruption risk mitigation when using modes with different lead times,

- Impact on contract design with suppliers and 3PLs with respect of pricing, capacity estimation.

Based on the performed review, we propose several directions for future research:

- Development of methods to increase computational efficiency for solving models with complex realistic discounts and multiple modes with different capacities. The computational complexity of finding the optimal replenishment plan increases with a larger number of transportation modes with different cost schedules and time periods. Due to the computational complexity of modeling real transportation costs, in particular when several modes are considered, most of the models focus on finding efficient heuristic procedures, rather than exact solutions. More work on investigating efficient approximation cost functions for transportation is also needed to help reduce computational times. Therefore, there is a need to focus on reducing computational times to provide timely decision support for managers.

- Investigation of the conditions and parameters for achieving savings by changing or combining modes to provide managerial recommendations. There is a need to define some guidelines for managers to indicate when they should consider using multiple modes and combining modes. Also, the decision to invest in a decision support tool should be motivated by substantial cost decrease and by a significant problem complexity to handle.

- Development of novel models that support Incoterms or trading terms and transportation mode selection decision for the buyer to decide whether he should manage and pay for transport or let the seller include transportation at additional costs. Depending on the volume transported, each of the parties may have various transportation options and costs, and each of these options should be evaluated together with inventory and lot-sizing decisions. Depending on the payment terms of the contract, if the payment should be done at the time the goods are picked up, and the selected trading terms Incoterms specify the buyer's responsibility for transportation (for example "Free on Board"), he will incur additional inventory in-transit costs.

- Development of models and approaches supporting optimal loading, palletizing and transportation mode combination decisions. As palletized goods have different configurations and container utilization rates depending on the pallet type, it can be beneficial to consider mode, pallet type and loading configuration decisions together. As stated by Battini et al. (2014), different emission rates can be associated with various container types and handling operations, and therefore should also be considered together with costs.

- Inclusion of other factors when selecting modes such as emission footprint, external transportation costs, supplier selection criteria, emission policy and capital constraints, risk, etc.

- Development of decision tools for re-calculating and comparing LTL rates and supporting inventory planning in an effective way. Decision makers most often rely on experience and simple spreadsheets to take complex decisions. To our knowledge, no paper in the literature report reports the actual implementation of its models or approaches.

- Development of models and approaches that support multimodal replenishment decisions, in particular for dynamic deterministic demand and multiple items, where multiple FTL and LTL modes with several LTL price-break intervals and different capacities are available.

- Investigation of optimal pricing, contract design, commitment volumes and coordination conditions between buyers, sellers and carriers.

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