# Leaning Against the Credit Cycle* 

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#### Abstract

How should a central bank act to stabilize the debt-to-GDP ratio? We show how the persistent nature of household debt shapes the answer to this question. In environments where households repay mortgages gradually, surprise interest hikes only weakly influence household debt, and tend to increase debt-to-GDP in the short run while reducing it in the medium run. Interest rate rules with a positive weight on debt-to-GDP cause indeterminacy. Compared to inflation targeting, debt-to-GDP stabilization calls for a more expansionary policy when debt-to-GDP is high, so as to deflate the debt burden through inflation and output growth.


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## 1 Introduction

Household debt has received considerable attention in the debate on monetary policy after the 2007-2009 financial crisis. For instance, the Bank for International Settlements argues that central banks should pay attention to credit-to-GDP ratios, as well as their traditional objectives of inflation and output, because the former are associated with risks of financial crises and "balance sheet recessions" (see BIS (2014) and the references therein). The International Monetary Fund has sided with a less activist view, arguing that monetary policy should focus on its traditional objectives (IMF (2015)). The debate recently met practice in Sweden, where the Riksbank first faced heavy criticism for keeping its interest rate high in order to curtail households' indebtedness, and then later appeared to surrender that strategy by slashing its interest rate as inflation fell low. ${ }^{1}$ Unfortunately, the debate on how monetary policy should respond to debt has mainly been based on frameworks that do not account for the way in which household indebtedness typically fluctuates over time. First, credit tends to move in a gradual manner, as highlighted by several recent studies of the credit cycle, for instance Aikman et al. (2015) and Drehmann et al. (2012). Second, the historical evolution of household leverage ratios has largely been driven by variation in income growth, inflation and interest rates, rather than changes in new borrowing, as documented by Mason and Jayadev (2014). We therefore develop a model that accounts for the persistent nature of household debt, and by consequence ascribes key roles to inflation and output growth in driving the dynamics of real debt and the debt-toGDP ratio. Within this framework we address how a central bank should act to stabilize the ratio of household debt relative to GDP.

There are two specific stylized facts we want our model to capture before applying it to study monetary policy. First, debt is highly persistent. For instance, linearly detrended U.S. real household debt has a first-order autocorrelation coefficient above 0.99. Second, the contemporaneous cross-correlation between household debt and house prices is moderate, and debt lags house prices. Linearly detrended U.S. data reveals a contemporaneous correlation of 0.14 and a correlation between house prices and debt five quarters later of 0.22 . In contrast, the existing

[^0]models that are typically used to analyze monetary policy in the presence of housing and debt tend to miss these facts by assuming that households refinance all their debt each period, as in Iacoviello (2005). An implication of this setup is that the entire stock of debt responds swiftly to shocks and policy changes. This simplifying assumption might be useful and innocuous for many purposes, but cannot be relied upon in the current debate on how monetary policy should respond to households' indebtedness.

Our approach builds on the amortization framework proposed by Kydland et al. (2016). Here the speed with which debt is amortized follows a process calibrated to match the properties of a standard mortgage contract. Importantly, our framework implies a distinction between new borrowing and pre-existing debt. The latter is given from the past, and hence is not constrained by current swings in collateral value, but follows a gradual amortization process instead. Only current borrowing, due to house purchases or refinancing, is constrained by the current value of collateral. Hence, the evolution of household debt at the aggregate level, measured relative to GDP or relative to housing value, is decoupled from the evolution of new borrowing. We embed this debt specification into (i) a relatively simple calibrated DSGE-model with collateral constraints, akin to Monacelli (2009) and Iacoviello (2005), and (ii) the fully-fledged DSGEmodel on housing and the macroeconomy by Iacoviello and Neri (2010), which we re-estimate using data through 2014q1.

With gradual amortization, our models imply debt dynamics that are highly persistent. While there is feedback between debt and the macroeconomy via a collateral constraint on current borrowing, other (non-debt) macroeconomic variables move faster, and revert considerably earlier to steady state than debt after a shock. In this sense, we capture the coexistence of a low-frequency credit cycle together with a conventional business cycle, similar to that emphasized by recent empirical studies such as Drehmann et al. (2012). Moreover, our estimated model features dynamic cross-correlations between debt and house prices that are very similar to those in the data.

Because new loans constitute a small fraction of the total stock of debt, monetary policy shocks affect nominal debt only to a limited extent. Hence, inflation and output may potentially respond faster, so that real debt and debt-to-GDP rise immediately after a monetary policy tightening. Within our estimated model, this is exactly what occurs for debt-to-GDP.

However, as inflation and output eventually return to steady state, the debt-to-GDP ratio drops moderately below its steady-state level, and stays low for a considerable period. Notably, this medium-run decline is influenced by how debt is amortized. If mortgages are of the annuity loan type, the amortization rate increases over the lifetime of a mortgage. Hence, by reducing the share of new loans in the overall stock of debt, a monetary tightening raises the economy's average amortization rate. This effect propagates the extent to which a monetary policy shock reduces debt-to-GDP in the medium run.

The transient increase of debt-to-GDP after interest rate shocks is not in itself sufficient to conclude on how central banks should address debt-to-GDP movements. Instead, to address the role of monetary policy in stabilizing debt-to-GDP, systematic policy must be analyzed. We first consider simple interest rate rules. Here we find that it is detrimental to mechanically lean against households' debt burden by raising interest rates when debt-to-GDP is high. ${ }^{2}$ A positive response coefficient on debt-to-GDP (or the real debt level) in the policy rule induces equilibrium indeterminacy. In contrast, under 1-quarter debt the opposite conclusion follows. Hence, when debt is gradually amortized and therefore persistent, policy should not respond positively to it. When new loans constitute only a small fraction of aggregate debt, higher inflation expectations reduce the expected and via contemporaneous inflation current levels of real debt. Thus, a policy of systematically raising the interest rate when debt-to-GDP is high indirectly implies a negative interest rate response to inflation. Consequently, expectations of higher inflation more easily turn self-fulfilling. To curb this destabilizing influence, the interest rate rule must have a higher response coefficient on inflation in order to avoid indeterminacy. The persistent nature of debt means that the required coefficient on inflation in the interest rate rule increases sharply with the debt coefficient.

Notably, a policy of raising the interest rate when the debt-to-GDP ratio goes up, destabilizes not only inflation, but debt itself. A policy rule response to the debt-to-GDP ratio is stabilizing only if the response coefficient is negative. This result holds regardless of whether the interest rate reacts to current or expected future debt, and regardless of whether the interest rate reacts to past interest rates or output growth, in addition to debt and inflation. Moreover, a negative

[^1]response coefficient stabilizes debt both in the short and in the medium to long run. In contrast, for a policy rule with debt growth, a positive response coefficient will stabilize debt-to-GDP. Not surprisingly, such a policy increases inflation volatility somewhat. If we compare the option of reacting negatively to debt-to-GDP to the alternative of responding positively to debt growth, the former reduces debt-to-GDP volatility more powerfully, but also has a stronger destabilizing influence on inflation.

Finally, we contrast inflation targeting with debt-to-GDP targeting. Specifically, we allow the policymaker to freely set the interest rate under commitment in order to minimize a loss function with debt-to-GDP or inflation. A striking pattern emerges: whenever strict inflation targeting implies a reduction in debt-to-GDP, the optimal policy of a debt-to-GDP targeter is to engineer more economic contraction. Vice versa, if debt-to-GDP rises under inflation targeting (or under a simple policy rule), optimal stabilization of debt-to-GDP engineers more economic expansion. These results are the direct opposite of what is typically assumed in the debate on how monetary policy can contribute to containing households' debt burden.

The central mechanism behind all our results is that movements in real debt and debt-to-GDP are heavily influenced by inflation and output. Mason and Jayadev (2014) describe these forces as "Fisher dynamics". In particular, they show that the forces of inflation, nominal income growth and nominal interest rates have strongly influenced the dynamics of the U.S. debt-to-income ratio since 1929. In a nutshell, we find that Fisher dynamics are key for a central bank that aims to stabilize the debt-to-GDP ratio: when most of the existing debt stock is determined by decisions made in the past, the most effective strategy to reduce a high debt burden is to deflate it through inflation and income growth. While it might be controversial that debt-to-GDP stabilization calls for more expansionary policy when the debt-to-GDP ratio is high, the intuition is straightforward.

Our study is closely related to the arguments of Svensson (2013), who challenges the conventional view that tighter monetary policy reduces households' debt burden. He combines estimates of how inflation, output and house prices respond to monetary policy shocks with an accounting formula for debt dynamics. The formula's key ingredient is that mortgage contracts are refinanced infrequently. Our approach differs, as we study two models where all variables are jointly determined in equilibrium. This allows us to move beyond monetary policy shocks,
and study how systematic monetary policy affects debt-to-GDP movements. Notably, both Svensson's exercise and our models imply that a monetary policy shock is likely to raise households' debt-to-GDP ratio in the short run. Our results diverge from Svenssons's in the medium run, where our model implies that debt-to-GDP will fall. In a simultaneous paper to ours, Alpanda and Zubairy (2017) also distinguish between new loans and old debt. Consistent with our model, their impulse response functions indicate that monetary policy shocks increase the debt-to-GDP ratio in the short run, but without the medium-run decline that we find. This difference is most likely due to their assumption of a constant amortization rate.

On the empirical side, several recent papers have used vector auto regression (VAR) models to explore how monetary policy shocks affect debt-to-GDP. Bauer and Granziera (2016) study a panel of eighteen advanced countries, and find that the debt-to-GDP ratio rises on impact and then falls moderately in the medium run, consistent with our structural model. Robstad (2017) finds a similar pattern for Norway, utilizing a host of VAR models. In contrast, Laséen and Strid (2013) use a Bayesian VAR-model on Swedish data, and find that debt-to-GDP drops after a monetary policy shock. Importantly, our model's implications for systematic policy do not hinge on the short-run sign of the debt-to-GDP response. Rather, what matters is that debt is highly persistent, which is an indisputable feature of the data.

The distinction between new loans and existing debt is key in our analysis. Recent work supports the importance of this distinction for understanding the debt dynamics in the data. Justiniano et al. (2015a) highlight that in the recent boom-bust episode of the US housing market, the aggregate ratio of debt over real estate value peaked several quarters after house prices started falling. A standard model where all debt is continuously re-adjusted, can only explain this pattern as the consequence of lending standards being loosened at the onset of the financial crisis. Such an interpretation of the housing bust seems misguided. In contrast, Gelain et al. (2015) show that once one takes into account that current collateral constraints primarily matter for new loans, the behavior of the leverage ratio no longer is at odds with a credit contraction at the onset of the housing bust. The estimated DSGE model in this paper implies the same pattern.

While our paper uses the tools of Kydland et al. (2016) to model amortization dynamics, our contribution differs substantively, as we study monetary policy, while they analyze residential
investment. Closer to our topic, Calza et al. (2013), Garriga et al. (2017) and Rubio (2011) study the role of mortgage finance in the monetary transmission mechanism. All emphasize that monetary policy is likely to be less influential when fixed rate mortgages are prevalent. Here, we ignore the issue of fixed versus flexible rate mortgages, and focus instead on the distinction between how pre-existing debt and new loans are affected by the current value of collateral. This relates our paper to the work by Andrés et al. (2017), who study structural reforms when households are burdened with debt. The key assumption they make, similar to Justiniano et al. (2015a), is that households cannot be forced to deleverage faster than a given amortization rate, even if the collateral value of real estate falls faster than the amortization rate on existing debt. Ultimately, all these modeling approaches are reduced-form representations of households' liquidity management, aiming to avoid the curse of dimensionality that follows with a deeper modeling of housing choice. Iacoviello and Pavan (2013) make progress in modeling the lumpiness of housing purchases. Chen et al. (2013) incorporate the many details relevant for mortgage refinancing at a micro level, and succeed in accounting for the U.S. credit boom in the 2000s. However, these studies are conducted within partial equilibrium models, which treat house prices, inflation and GDP as exogenous, and therefore cannot address the questions we ask in this paper.

While our focus is on households, Gomes et al. (2014) offer a complementary view focusing on firms. They show how inflation movements affect investment and output if firms have longterm debt. The key mechanism is that inflation shocks propagate through real leverage. Hence, they share our focus on the Fisher dynamics of debt. In contrast to our study, they consider a model without nominal rigidities other than nominal debt, so that inflation responds positively to nominal interest rate hikes rather than vice versa. Moreover, they do not study policies that aim to stabilize debt and do not compare the empirical performance of their long-term debt model to the short-term debt alternative.

Section 2 develops a simple model that allows a transparent assessment of the interplay between household debt and monetary policy. Section 3 analyzes monetary policy in the simple model. In Section 4, we move to a fully-fledged DSGE model suitable for estimation. We embed gradual amortization into the framework of Iacoviello and Neri (2010), estimate it on U.S. data, and then explore if the monetary policy implications from the simple model hold here
too. Section 5 discusses likely implications for monetary policy responses to mortgage spreads instead of debt. Section 6 concludes.

## 2 A Simple Model

We consider a standard New Keynesian model with household debt and a collateral constraint. The novelty of our framework is to relax the assumption that all mortgages are repaid or refinanced within a quarter. Instead we allow for gradual amortization of outstanding debt, with the collateral constraint applying only to new mortgages for purchases or refinancing.

### 2.1 Households

There are two household types: patient lenders (indexed by $l$ ) and impatient borrowers (b), of mass $1-n$ and $n$, respectively. Both derive utility from a flow of consumption $c_{j, t}$ and services from housing $h_{j, t}, j=b, l$. They derive disutility from labor, $L_{j, t}$. Each household maximizes $E_{t} \sum_{t=0}^{\infty} \beta_{j}^{t} U\left(c_{j, t}, h_{j, t}, L_{j, t}\right)$, where

$$
\begin{equation*}
U\left(c_{j, t}, h_{j, t}, L_{j, t}\right)=\log \left(c_{j, t}-\gamma c_{j, t-1}\right)+\nu_{h} \log \left(h_{j, t}\right)-\nu_{L, j} \frac{L_{j, t}^{1+\varphi_{L}}}{1+\varphi_{L}} . \tag{1}
\end{equation*}
$$

Here $\gamma$ measures habit formation in utility, $\nu_{h}$ governs the utility from housing services, $\nu_{L, j}$ governs the disutility of labor, and $\varphi_{L}$ governs the elasticity of labor supply. ${ }^{3} \beta_{b}$ and $\beta_{l}$ are the households' discount factors, with $\beta_{b}<\beta_{l}$. The total housing stock is fixed at 1 , such that $(1-n) h_{l, t}+n h_{b, t}=1$ for all $t$.

## Impatient Households

Impatient households face the budget constraint

$$
\begin{equation*}
c_{b, t}+q_{t}\left(h_{b, t}-h_{b, t-1}\right)+\frac{1+r_{t-1}}{\pi_{t}} b_{b, t-1}=w_{b, t} L_{b, t}+b_{b, t} \tag{2}
\end{equation*}
$$

[^2]where $r_{t-1}$ is the net nominal interest rate at the end of period $t-1, \pi_{t} \equiv P_{t} / P_{t-1}$ is the gross inflation rate during period $t, w_{b, t}$ is the real wage, $q_{t}$ is the real price of housing, and $b_{b, t}$ is the borrowers' real debt at the end of period $t$.

Following Kiyotaki and Moore (1997) and Iacoviello (2005), debt accumulation is constrained by the value of borrowers' homes. However, in contrast to these studies, we do not assume that households adjust their entire debt stock each period. Instead, each quarter the household refinances only a fraction $\vartheta$ of their outstanding mortgage, collateralized by the same fraction $\vartheta$ of their housing wealth. The new amount borrowed cannot exceed a fraction $m$ of the expected discounted collateral value, $\vartheta E_{t}\left[q_{t+1} \pi_{t+1}\right] h_{b, t} / R_{t}$. The remaining debt from last period is amortized at a predetermined rate $\delta_{t-1}$, which evolves as if households hold annuity loan contracts, explained in more detail below. Given that the impatient households always borrow as much as possible, their total debt stock will satisfy

$$
\begin{equation*}
b_{b, t}=\vartheta m \frac{E_{t}\left(q_{t+1} \pi_{t+1}\right) h_{b, t}}{1+r_{t}}+(1-\vartheta)\left(1-\delta_{t-1}\right) \frac{b_{b, t-1}}{\pi_{t}} \tag{3}
\end{equation*}
$$

If all debt is refinanced or amortized each period $\left(\vartheta=1\right.$ or $\left.\delta_{t-1}=1\right)$, the constraint collapses to its conventional formulation where the current stock of debt is entirely determined by today's collateral value. ${ }^{4}$ In contrast, if only a share of existing debt is refinanced and amortization is gradual, $\vartheta<1$ and $\delta_{t}<1$, the current stock of debt is influenced by past borrowing as well as current collateral value.

For the amortization rate $\delta_{t}$, we follow Kydland et al. (2016) and specify a process that can be calibrated to match the profile of a typical annuity loan. The amortization rate evolves according to

$$
\begin{equation*}
\delta_{t}=\left(1-\frac{l_{b, t}}{b_{b, t}}\right) \delta_{t-1}^{\alpha}+\frac{l_{b, t}}{b_{b, t}}(1-\alpha)^{\kappa} \tag{4}
\end{equation*}
$$

where $\alpha \in[0,1)$ and $\kappa>0$ are parameters that govern amortization rate dynamics, and $l_{b, t}$ are the loans that are initiated in period $t$ and start a new amortization cycle. As in Kydland

[^3]et al. (2016), $l_{b, t}=b_{b, t}-\left(1-\delta_{t-1}\right) b_{b, t-1} / \pi_{t} .{ }^{5}$ When $\alpha=0$, equation (4) implies $\delta_{t}=1$ for all $t$, such that we recover a 1-period mortgage contract where all outstanding debt is refinanced each period. When $\alpha>0$, equation (4) captures how the amortization rate typically is low during the early years of a mortgage (when mortgage payments consist mainly of interest) and rises as principal is repaid. We later set $\alpha$ and $\kappa$ to approximate the amortization schedule of a typical 30-year mortgage contract.

Impatient households maximize lifetime utility subject to the law of motion for the amortization rate (4), the budget constraint (2), and the borrowing constraint (3). The first-order conditions for $c_{b, t}, L_{b, t}, h_{b, t}, b_{b, t}$, and $\delta_{t}$ are:

$$
\begin{align*}
\lambda_{t}= & \frac{1}{c_{b, t}-\gamma c_{b, t-1}}-\frac{\beta \gamma}{c_{b, t+1}-\gamma c_{b, t}},  \tag{5}\\
\nu_{l, b} L_{b, t}^{\varphi_{L}}= & \lambda_{t} w_{b, t}  \tag{6}\\
\lambda_{t} q_{t}= & \frac{\nu_{h}}{h_{b, t}}+\lambda_{t} \mu_{t} \vartheta m \frac{E_{t}\left(q_{t+1} \pi_{t+1}\right)}{1+r_{t}}+\beta_{b} E_{t}\left(\lambda_{t+1} q_{t+1}\right),  \tag{7}\\
\lambda_{t}= & \beta_{b} E_{t}\left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right)\left(1+r_{t}\right)+\lambda_{t} \mu_{t}-\beta_{b} E_{t}\left(\frac{\lambda_{t+1} \mu_{t+1}}{\pi_{t+1}}\right)(1-\vartheta)\left(1-\delta_{t}\right)- \\
& \lambda_{t} \eta_{t} \frac{b_{b, t-1}}{\pi_{t} b_{b, t}^{2}}\left(1-\delta_{t-1}\right)\left(\delta_{t-1}^{\alpha}-(1-\alpha)^{\kappa}\right)+  \tag{8}\\
& \beta_{b} E_{t}\left(\frac{\lambda_{t+1} \eta_{t+1}}{\pi_{t+1}}\right) \frac{\left(1-\delta_{t}\right)}{b_{b, t+1}}\left(\delta_{t}^{\alpha}-(1-\alpha)^{\kappa}\right) \\
\lambda_{t} \eta_{t}= & \beta_{b} E_{t}\left(\frac{\lambda_{t+1} \mu_{t+1}}{\pi_{t+1}}\right)(1-\vartheta) b_{b, t}+ \\
& \beta_{b} E_{t}\left(\frac{\lambda_{t+1} \eta_{t+1}}{\pi_{t+1} b_{b, t+1}}\right) b_{b, t}\left(\alpha \delta_{t}^{(\alpha-1)}\left(1-\delta_{t}\right)-\delta_{t}^{\alpha}+(1-\alpha)^{\kappa}\right) \tag{9}
\end{align*}
$$

where $\lambda_{t}, \mu_{t}$, and $\eta_{t}$ are the Lagrange multipliers associated with the budget constraint (2), the borrowing constraint (3) and the amortization rate process (4).

[^4]
## Patient Households

Patient households lend to the borrowers, and choose how much to consume, work, and invest in housing. From firms they receive profits through dividends, Divt. Their budget constraint is:

$$
\begin{equation*}
c_{l, t}+q_{t}\left(h_{l, t}-h_{l, t-1}\right)+\frac{b_{l, t-1}\left(1+r_{t-1}\right)}{\pi_{t}}=w_{l, t} L_{l, t}+b_{l, t}+\text { Div }_{t}, \tag{10}
\end{equation*}
$$

where $(1-n) b_{l, t-1}=-n b_{b, t-1}$.
Patient household's optimal choices are characterized by the first-order conditions:

$$
\begin{align*}
-U_{L_{l, t}} & =U_{c_{l, t}} w_{t}  \tag{11}\\
U_{c_{l, t}} & =\beta_{l} R_{t} E_{t}\left(\frac{U_{c_{l, t+1}}}{\pi_{t+1}}\right),  \tag{12}\\
U_{c_{l, t}} q_{t} & =U_{h l, t}+\beta_{l} E_{t}\left(U_{c_{l, t+1}} q_{t+1}\right), \tag{13}
\end{align*}
$$

### 2.2 Firms and Price Setting

Firms are owned by the patient households. A sector with perfect competition and flexible prices produce $y_{t}$ with the technology $y_{t}=\left[\int_{0}^{1} y_{t}(i)^{(\varepsilon-1) / \varepsilon} d i\right]^{\varepsilon /(\varepsilon-1)}$, where $i \in[0,1]$. The inputs are a continuum of intermediate goods $y_{t}(i)$ and $\varepsilon>1$ is the constant elasticity of substitution between these goods. Cost minimization implies that demand for each intermediate good is $y_{t}(i)=\left[P_{t}(i) / P_{t}\right]^{-\varepsilon} y_{t}$, where the price index for intermediates is $P_{t}=\left[\int_{0}^{1} P_{t}(i)^{1-\varepsilon} d i\right]^{1 /(1-\varepsilon)}$.

Intermediate goods firms are monopolistically competitive and use labor only:

$$
y_{t}(i)=\exp \left(z_{t}\right) L_{t}(i)^{1-\xi},
$$

where $z_{t}$ is an $\operatorname{AR}(1)$ productivity shock with autocorrelation coefficient $\rho_{z}$ and standard deviation $\sigma_{z}$. The labor input is given by $L_{t}(i)=\left(n L_{b, t}(i)\right)^{\infty}\left((1-n) L_{l, t}(i)\right)^{1-\varpi}$.

Each period an intermediate firm may reset its price with probability $1-\theta$, as in Calvo (1983). Otherwise the price is partially indexed to past inflation with the degree of indexation governed by $v \in(0,1)$, as in Smets and Wouters (2003). ${ }^{6}$ If $v=1$, prices are fully indexed

[^5]to past inflation, and if $v=0$, there is no indexation. With zero steady-state inflation, the log-linear Phillips curve is:
\[

$$
\begin{equation*}
\widehat{\pi}_{t}=\frac{\beta_{l}}{1+\beta_{l} v} E_{t} \widehat{\pi}_{t}+\frac{v}{1+\beta_{l} v} \widehat{\pi}_{t-1}+\frac{\left(1-\beta_{l} \theta\right)(1-\theta)}{\left(1+\beta_{l} v\right) \theta} \widehat{m c}_{t}, \tag{14}
\end{equation*}
$$

\]

where $\widehat{\pi}_{t}$ is inflation and $\widehat{m c}_{t}$ is marginal costs in log deviations from steady state.

### 2.3 Monetary Policy

As a baseline, we postulate that the central bank follows the simple policy rule

$$
\begin{equation*}
R_{t}=R_{t-1}^{\phi_{R}}\left(R^{s s} \pi_{t}^{\phi_{\pi}}\right)^{1-\phi_{R}} \varepsilon_{t}, \tag{15}
\end{equation*}
$$

where $R_{t}=1+r_{t}$ is the gross nominal interest rate, $R^{s s}=1 / \beta_{l}$ is the steady-state real interest rate, $\phi_{R}$ and $\phi_{\pi}$ are policy response coefficients, and $\varepsilon_{t}$ is an i.i.d. monetary policy shock.

### 2.4 Calibration

We choose the values of $n, \nu_{l, l}, \nu_{l, b}$ and $\varpi$ so that the model steady state matches key moments of the relatively stable 1990s' U.S. economy, as reported in Justiniano et al. (2015a) (JPT). The fraction of borrowers is set to $n=0.61$, the share of liquidity constrained households reported in JPT. The preference weights are $\nu_{L, l}=0.10$ and $\nu_{L, b}=0.23$ so that borrower households work 1.08 times more than lenders. The labor share parameter $\varpi$ is 0.5 . so that the ratio of borrowers' to lenders' labor income is 0.64 .

The housing preference weight, $\nu_{h}=0.075$, is set so that the model-implied ratio of housing wealth to yearly consumption is 2 , consistent with Iacoviello and Neri (2010). ${ }^{7}$ The parameters governing the amortization process, $\alpha$ and $\kappa$, are set so as to minimize the distance between the steady-state profiles of amortization and interest rate payments implied by equation (4), and the same profiles implied by an actual 30 -year, flexible interest rate, mortgage contract, as in Kydland et al. (2016). This procedure results in $\alpha=0.996$ and $\kappa=1.013$, which implies

[^6]a steady-state amortization rate of $\delta=0.019$. Later, when we vary the duration of the loan contract, we necessarily recalibrate these parameters accordingly. The loan-to-value ratio on new borrowing, $m$, is meant to represent the typical loan-to-value ratio of new mortgages, including both house purchases and refinancings. Typical values in the literature are between 0.75 and 0.85 . As a middle ground we set $m=0.8$, which Justiniano et al. (2015b) argue is consistent the Survey of Consumer Finances. The share of new mortgages, $\vartheta$, is set so that the steady-state ratio of debt relative to housing value, $\widetilde{m} \equiv R b_{b} /\left(q h_{b}\right)=m \vartheta /[\delta+\vartheta(1-\delta)]$, is equal to 0.5 , which is close to the average U.S. value after 1960. Under the 30 -year amortization schedule, it follows that $\vartheta=0.031$, which implies that a mortgage last 8 years on average. Whenever we vary the amortization schedule, $\vartheta$ is re-calibrated to maintain the same steady-state debt to real estate ratio, $\widetilde{m} .{ }^{8}$

The remaining parameters are simply set to typical values in the existing literature. As in Iacoviello and Neri (2010), patient households' discount factor is $\beta_{l}=0.9925$, consistent with a $3 \%$ annual interest rate, while $\beta_{b}=0.97$. The labor supply elasticity parameter $\varphi_{L}$ is set to 1 , while the labor elasticity of production, $1-\xi=0.67$. The price adjustment probability $\theta$ is 0.75 so that prices change once a year, on average. The elasticity of substitution between goods is $\varepsilon=6$, implying a $20 \%$ steady-state markup. In the monetary policy rule, the weights on the lagged interest rate and inflation are $\phi_{R}=0.75$ and $\phi_{\pi}=1.5$. The price indexation and habit parameters, $v$ and $\gamma$ are both set to 0.5 . Finally, the technology shock process has an AR-coefficient of $\rho_{z}=0.95$ and a standard deviation of $\sigma_{z}=0.0124$, chosen so that the model driven by technology shocks matches the standard deviation of GDP growth over the period 1960 to 2012.

Unless otherwise stated, we consider a first-order Taylor-approximation of the model, loglinearized around its steady state.

[^7]
## 3 Debt Dynamics and Monetary Policy

What does the persistent nature of household debt imply for a monetary policy authority concerned with the debt-to-GDP ratio? We address this question from three different angles. First, we explore how a monetary tightening is likely to affect debt-to-GDP. Second, we study the properties of simple policy rules with a weight on real debt or the debt-to-GDP ratio. Third, we characterize a monetary policy that targets debt-to-GDP stability and contrast it with strict inflation targeting.

### 3.1 Does an Interest Rate Hike Reduce the Debt Burden?

Figures 1 and 2 display the impulse response functions to a monetary policy shock in our model. We see that inflation, output and house prices are largely unaffected by the maturity of the mortgage contract. However, the dynamics of the debt burden, measured either as the stock of real debt or as debt-to-GDP, are starkly influenced. With 1-quarter debt, real debt and debt-to-GDP both fall markedly on impact and then return gradually to their steady-state levels. Qualitatively, this behavior is consistent with the conventional view that a policy tightening will reduce the debt burden. In contrast, with 30-year mortgage debt the effect of monetary policy is muted and the debt burden displays a hump-shaped response, first increasing and then declining, as best seen in Figure 2. On impact, debt-to-GDP hardly moves, but then rises to a peak response after about a year. It then falls gradually, reaches its steady-state level after approximately two years, and then drops further and stays low for an extended period. Figure 2 shows that real debt and debt-to-GDP stay moderately below their steady-state levels for approximately thirty years, reaching a trough about $0.4 \%$ below steady state after about ten years.

What explains the dynamics of debt under the 30-year amortization schedule? Here it is useful to consider the responses of inflation and GDP. With a 30-year amortization schedule, debt becomes highly persistent, as revealed by equation (3). Hence, on impact real debt and debt-to-GDP are both largely driven by the responses of inflation and output, respectively. The declines in these two variables serve to increases the debt burden. This is the "Fisher dynamics" highlighted by Mason and Jayadev (2014). However, since house prices decline as well, the flow of new annuity loans $\left(l_{b, t}\right)$ will drop, as shown in Figure 1. Because the initial drop
in house prices is relatively strong, this force counteracts the influence of reduced inflation and output. As house prices revert faster, the debt burden gradually builds up. The peak response of debt is reached when house prices are back to steady state. As inflation and output revert to steady state, the debt burden falls. However, after the effects from the other variables have died out, debt-to-GDP keeps falling. The reason is the initial drop in the flow of new loans. This drop, although modest, has long-lived effects due to the long-term nature of debt. Moreover the aggregate amortization rate, $\delta_{t}$, rises as new loans constitute a declining fraction of the total debt stock. Hence, when the other macroeconomic variables have returned to steady state, the initial drop in new loans causes a persistent fall in the aggregate debt burden.

How important are the dynamics of the amortization rate for these results? Figure 2 compares the benchmark model with an endogenous amortization rate, to one where $\delta$ is constant at the same steady-state level as the benchmark. We see that the amortization rate dynamics are unimportant for the initial increase in debt, but influence the subsequent decline. This is because the amortization rate is low for new mortgages with annuity loans. Hence, as the monetary tightening reduces the flow of new loans, the total stock of debt becomes "older," and the aggregate amortization rate, $\delta_{t}$ increases which brings the total debt stock down. With a constant amortization rate, the real debt stock and debt-to-GDP do not drop much below steady state in the medium run.

While not reported here, the shorter is the duration of debt, the weaker is the tendency for an initial increase in the debt-to GDP ratio, and the greater is the tendency for a subsequent decline. With all the other parameters left unchanged, debt-to-GDP responds negatively on impact for amortization schedules shorter than 5 years. ${ }^{9}$

With regard to the question of how monetary policy affects the aggregate debt burden, we see that the answer depends on the time horizon one has in mind. Consistent with the back-of-the envelope calculation of Svensson (2013), and in contrast with the conventional view, our model implies that tighter policy increases the debt burden in the short run. In the intermediate run, however, monetary policy tightening is likely to cause a mild, but prolonged reduction in

[^8]the debt burden, more in line with the conventional view. The extent to which this reduction in the debt burden comes about is determined by the tendency for amortization rates to increase with mortgage age. More generally, and more importantly, the effect of monetary policy on debt is muted, but persistent. Models that assume all debt is re-financed each period and subject to the prevailing value of collateral, will exaggerate the degree to which monetary policy can affect the debt burden.

### 3.2 Debt and Simple Policy Rules

The results above illuminate the transmission mechanism from monetary policy to household debt, but do not tell us how systematic monetary policy should be conducted to stabilize the debt-to-GDP ratio. We now turn to this question by studying the consequences of responding to debt through a simple interest rate rule of the form

$$
\begin{equation*}
R_{t}=R^{s s} \pi_{t}^{\phi_{\pi}}\left(\frac{b_{t} / y_{t}}{b / y}\right)^{\phi_{b / y}} \tag{16}
\end{equation*}
$$

where $\phi_{b / y}$ is a response coefficient and $b / y$ is the steady-state debt-to-GDP ratio.

## Reacting to the Debt Burden and Equilibrium Determinacy

A fundamental guide for systematic monetary policy is the "Taylor principle" (Woodford (2001)). The Taylor principle states that the nominal interest rate should react more than one-for-one to changes in inflation. If this principle is not satisfied, then expectations of higher inflation might turn self-fulfilling and induce belief-driven macroeconomic fluctuations, as increased inflation expectations lower the path of real interest rates and boost demand. Should monetary policy also respond to other variables, the critical cutoff value for the response coefficient on inflation might well be affected, so as to ensure that the ultimate policy response to inflation is greater than one. Several studies have explored how the responses to output and inflation together determine the scope for equilibrium indeterminacy, with Bullard and Mitra (2002) as a prominent example. In the current policy debate, it is natural to ask how a systematic response to debt-to-GDP, in addition to inflation, might affect the scope for equilibrium indeterminacy.

The upper-left panel in Figure 3 plots the determinacy region in the $\phi_{\pi}, \phi_{b / y}$-space when
debt is fully amortized every quarter $\left(\delta_{t}=1\right)$. When $\phi_{b / y}=0$, the critical value for the inflation coefficient is one, consistent with the Taylor principle. If policy begins to respond positively to the debt-to-GDP ratio, the required inflation coefficient falls moderately. Hence, in terms of ensuring equilibrium determinacy, responding to inflation and responding to the debt-to-GDP ratio are substitutes when $\delta_{t}=1$. To understand why, consider the effects of an increase in inflation expectations unjustified by fundamentals. All else equal, the real interest rate drops, while the forward-looking Phillips curve implies that current inflation increases. As reflected in the first term on the right-hand side of equation (3), higher inflation expectations boost the present discounted value of housing and thus stimulate new borrowing. With $\delta_{t}=1$, real debt dynamics are entirely driven by new loans, and the total stock of real debt increases. Hence, with one-period mortgages, the stock of real debt moves in the same direction as current inflation, and a positive coefficient on debt-to-GDP has similar stabilizing properties as a positive response to inflation.

The remaining three panels in Figure 3 plot the determinacy region if amortization is gradual, under 10-, 20- and 30 -year amortization schedules respectively. ${ }^{10}$ Now the relationship between the threshold inflation coefficient and the debt-to-GDP response coefficient is upward-sloping. This occurs because increased inflation expectations no longer move the real debt stock in the same direction as current inflation. While higher expected inflation still stimulates collateral value and thus new borrowing as seen in the first term of equation (3), higher current inflation deflates the real value of existing debt, as reflected in the second term. Because new mortgages constitute only a small fraction of total debt, the second effect dominates. Hence, if expected inflation rises without fundamental justification, then the inflationary pressure this generates will reduce the stock of real debt. A positive value of $\phi_{b / y}$ will then push the nominal interest rate down, making the real interest rate fall even further. To counteract this destabilizing force, the response to current inflation, $\phi_{\pi}$, must be greater than when $\phi_{b / y}=0$. The upwardsloping relationship between the critical values of $\phi_{\pi}$ and $\phi_{b / y}$ becomes steeper, the longer is the maturity of debt.

The upward-sloping relationships between $\phi_{b / y}$ and $\phi_{\pi}$ in Figure 3 are steep due to medium-

[^9]run debt dynamics. A moderate increase in the real interest rate will induce a reduction of debt in the future, as we saw in the analysis of monetary policy shocks in Figure 1. With a positive value of $\phi_{b / y}$, this implies a reduction of the future interest rate, which itself tends to support a sunspot induced increase of expected inflation. Thus, responding positively to the debt-to-GDP ratio is destabilizing for two reasons: (i) in the short run, the real debt level falls when inflation increases, and (ii) in the medium-run debt-to-GDP, falls if the current real interest rate increases. ${ }^{11}$

Reflecting the two forces at play here, the upper-right panel of Figure 3 shows that with 10 -year debt, there is a narrow intermediate region of $\phi_{\pi}, \phi_{b / y}$-combinations where the equilibrium is determinate. For instance, if $\phi_{b / y}=0.05$, the panel shows that $\phi_{\pi}=1.2$, yields a determinate equilibrium. But a slightly higher response to inflation, say $\phi_{\pi} \approx 1.3$, results in indeterminacy. The knife-edge region with determinacy is one where the inflation coefficient $\phi_{\pi}$ is barely big enough to compensate for the fact that a positive response to debt implies a negative contemporaneous response to inflation, and barely small enough to avoid causing a substantive medium-run decline in mortgage debt. This intermediate region is similar to that which can occur due to investment in productive capital, as emphasized by Benhabib and Eusepi (2005), Carlstrom and Fuerst (2005) and Sveen and Weinke (2005).

As emphasized above, the relative weight on future versus current inflation in equation (3) decides the slope of the determinacy frontier. In Figure 4, we dissect this mechanism further, by considering alternative values of $m$ and $\vartheta$. The solid lines refer to the baseline scenario with 30 -year debt. In the left panel, we see that a higher value of $m$ is associated with a steeper upward-sloping determinacy determinacy frontier. The reason is that $\vartheta$ is re-calibrated to maintain a constant ratio of debt to real estate value, $\vartheta=\delta /(m / \widetilde{m}-1+\delta)$, so that the weight $\vartheta m$ on collateral value in equation (3) is decreasing in $m$. In the right panel, we vary $\vartheta$ without re-calibrating the model. Hence, a greater value of $\vartheta$ is associated both with a higher weight on future versus current inflation in equation (3), and with a higher debt level. ${ }^{12}$ Consistent

[^10]with the intuition developed above, we see that the greater is $\vartheta$, the flatter is the determinacy frontier. When $\vartheta$ is assigned the same value as under the 1 -quarter debt specification studied above, i.e. 0.63 , the determinacy frontier is almost flat.

## Debt Volatility when Monetary Policy Reacts to Households' Debt Burden

Equilibrium determinacy aside, how should monetary policy respond to debt-to-GDP in order to stabilize this ratio? We address this question here by considering how the volatility of debt-to-GDP is affected by alternative response coefficients $\phi_{b / y}$ in (16), when our simple model economy is subject to productivity shocks. Throughout, $\phi_{\pi}$ is kept constant at 1.5. We also study a policy rule that reacts with a coefficient $\phi_{b}$ to the real debt level instead of its ratio to GDP.

The results from this exercise are displayed in Figure 5. In the upper left panel, we see that with 1-quarter debt, conventional wisdom applies, as debt-to-GDP exhibits a lower standard deviation when monetary policy responds positively to its movements. Negative responses induce volatility and eventually indeterminacy. In contrast, with 30 -year debt the pattern is exactly the opposite: The standard deviation of debt-to-GDP increases with the response coefficient on debt in the interest rate rule. Coefficients above zero induce indeterminacy, as we have seen earlier. If stability of debt-to-GDP is the objective, then the response coefficient on the debt-to-GDP ratio should actually be negative.

Notably, this conclusion is not caused by the scaling of debt by GDP. The dashed curves show that the same conclusion follows when policy responds to the real debt level rather than the debt-to-GDP ratio.

Based on the discussion of monetary policy shocks above, one might ask how a negative value for $\phi_{b / y}$ affects the medium-run dynamics of debt-to-GDP. Figure 5 therefore shows the responses of debt-to-GDP after a technology shock when monetary policy reacts negatively to debt-to-GDP with $\phi_{b / y}=-0.5$ or to the real debt level with $\phi_{b}=-0.5$. These responses are compared to the scenario where monetary policy reacts to inflation only. We see that the interest rate rules with negative $\phi_{b / y}$ and $\phi_{b}$ coefficients stabilize debt-to-GDP in the medium to long-run. We will return to this point later in a richer, estimated model that allows us to

[^11]focus on shocks that directly affect credit conditions.

### 3.3 Inflation vs Debt Stabilization

Rather than restricting monetary policy to follow a simple rule, an alternative is to specify the central bank's objectives and let it freely choose the interest rate path that best achieves these objectives, as advocated by Svensson (2003). We follow this approach here, and study monetary policy as the minimization of the following loss function

$$
\begin{equation*}
L_{t}=\sum_{j=0}^{\infty} \beta_{l}^{j}\left[(1-\Gamma)\left(\left(1-\lambda_{y}\right) \pi_{t+j}^{2}+\lambda_{y}\left(\frac{y_{t+j}}{y_{t+j}^{f}}\right)^{2}\right)+\Gamma\left(\frac{b_{b, t+j} / y_{t+j}}{b_{b} / y}\right)^{2}\right] \tag{17}
\end{equation*}
$$

where $\Gamma$ is the central bank's weight on stabilizing debt-to-GDP relative to the traditional objective of stabilizing inflation and the output gap, defined as deviations from the output level $y_{t}^{f}$ that would arise if prices were fully flexible.

We first compare the properties of a policy that stabilizes debt $(\Gamma=1)$ to a policy that targets inflation, holding concerns for output at zero $\left(\lambda_{y}=0\right)$. To this end, Figures 6 and 7 display impulse response functions to a technology shock when policy minimizes (17) under 1quarter and 30-year debt, respectively. Again we see that the implications for debt-stabilization policy are radically different under the two alternative debt specifications.

With 30-year debt, strict inflation targeting $\left(\Gamma=0, \lambda_{y}=0\right)$ is associated with a short term fall and a medium run increase in the debt-to-GDP ratio. To prevent these movements, the optimal policy to stabilize debt-to-GDP $\left(\Gamma=1, \lambda_{y}=0\right)$ contracts output and inflation more in the short run, and stimulates both variables more in the medium run, than under inflation targeting. In a nutshell, debt-to-GDP targeting implies a more contractive policy when debt-to-GDP would otherwise be low and a more expansionary policy when debt-toGDP would otherwise be high. This is directly opposite of what occurs under 1-quarter debt, where the debt-to-GDP targeting central bank engineers a lower path of inflation and output than an inflation targeter, in order to increase the debt-to-GDP ratio. Intuitively, the slower debt is amortized, the more a debt-to-GDP targeting central bank should rely on the "Fisher dynamics" from inflation and output to stabilize the debt-GDP-ratio. In contrast, for a strict inflation targeter, the policy prescription is essentially unchanged by the maturity of debt.

The top two panels of Figure 8 illustrate the degree to which a higher weight on debt-to-

GDP stability $(\Gamma)$ worsens performance in terms of the conventional monetary policy objective in (17), i.e., $L_{1}=\left(1-\lambda_{y}\right) \operatorname{var}(\pi)+\lambda_{y} \operatorname{var}\left(y_{t} / y^{f}\right)$. In each plot, three different scenarios are considered that differ by the weight $\lambda_{y}$ on the output gap in the loss function. ${ }^{13}$ In the top left plot, $\Gamma$ varies from 0.01 to 1 along the $x$-axis, whereas the top right plot considers the range from 0 to 1 . The purpose of this distinction follows from the plots themselves: if $\Gamma=0$, the debt-to-GDP ratio is extremely volatile, consistent with Figure 7, and hence only a slight preference for stabilizing this ratio implies massive reduction of its variance. More generally, the two plots show that moderate stabilization of the debt-to-GDP ratio comes at small costs in terms of output and inflation volatility, as the frontiers in the top left panel are relatively flat until the debt volatility reaches an intermediate level. When the volatility of debt-to-GDP is pushed toward zero, $L_{1}$ (measured on the vertical axes) increases sharply. Finally, we see that the more output is weighted in the loss function, the smaller is the $L_{1}$ associated with a given volatility of debt-to-GDP. Vice versa, debt-to-GDP is less volatile when $\lambda_{y}$ increases.

Finally, the bottom two panels of Figure 8 show the welfare consequences of increasing the emphasis on debt-to-GDP. The metric displayed is the percentage change in non-stochastic steady-state consumption that makes borrowers and lenders equally well off as in a regime with $\Gamma=0 .{ }^{14}$ The magnitudes here are moderate, as they typically are when assessing the merits of stabilization policy. For the lenders, we see that their welfare is monotonically decreasing in $\Gamma$. For borrowers, the relationship is slightly non-monotonic, as welfare first increases for very low values of $\Gamma$. This initial increase is related to the pattern in the upper right plot: When $\Gamma=0$, debt-to-GDP is highly volatile, and for borrowers this will be associated with volatile consumption too, due to their budget constraint. A slightly positive weight on debt-toGDP, however, reduces its volatility a great deal. Starting from a situation with less extreme volatility, the usual costs in terms of inefficient swings in activity and price dispersion due to

[^12]${ }^{14}$ Welfare is here computed by using a second-order approximation of the model, accounting for the inflation costs of inefficient price dispersion. The reported metric of welfare difference, is the value of $\lambda$ such that $\sum_{t=0}^{\infty} \beta_{j}^{t}\left[u\left(c_{j, t}^{\Gamma=0}, h_{j, t}^{\Gamma=0}, n_{j, t}^{\Gamma=0}\right)-u\left(c^{s s}, h^{s s}, n^{s s}\right)\right]=\sum_{t=0}^{\infty} \beta_{j}^{t}\left[u\left(c_{j, t}^{\Gamma=1}, h_{j, t}^{\Gamma=1}, n_{j, t}^{\Gamma=1}\right)-u\left((1+\lambda) c^{s s}, h^{s s}, n^{s s}\right)\right]$, where the superscripts $\Gamma=0$ and $\Gamma=1$ index outcomes under inflation and debt-to-GDP targeting respectively, and the superscript $s s$ denotes the non-stochastic steady state.
inflation dominate, and also borrower welfare declines with $\Gamma$. Hence, in this environment, a moderate concern for debt-to-GDP stability may be advisable on the grounds that it benefits households who are borrowing constrained. Yet, one should then keep in mind that such a concern implies more expansionary policy when debt-to-GDP is high, and less expansionary policy when debt-to-GDP is low, in contrast to what is typically advocated in policy debates.

## 4 Evidence From A Medium Scale Estimated Model

It is natural to ask whether the foregoing results are specific to our simple model and parameterization, and how quantitatively relevant are the key mechanisms. We therefore incorporate our long-term debt framework into a medium scale dynamic stochastic general equilibrium model, estimate it, and re-evaluate the main policy implications from above.

### 4.1 Model and Empirical Approach

We build on the framework developed and estimated by Iacoviello and Neri (2010) (IN, hereafter). This is a well-documented quantitative model on the linkages between housing and the macroeconomy, originally formulated with the assumption that the entire stock of debt is refinanced quarterly. Because our model and estimation procedure are direct extensions of IN, we limit ourselves to a brief outline of the features we have in common with IN, and highlight our extensions.

## The Model

As in our simple model, some households are patient and lend, while others are impatient and borrow subject to a collateral constraint tied to housing value. All households work, consume, and accumulate housing. In addition, patient households own the productive capital stock, and make loans to firms and impatient households. On the supply side there are two sectors: a housing sector that uses capital, labor and land to produce new homes, and a non-housing sector that uses capital and labor to produce consumption and new business capital. ${ }^{15}$

There are 10 structural shocks that affect intertemporal preferences, labor supply, housing

[^13]preferences, non-housing-sector productivity, housing-sector productivity, investment specific technology, final good costs, the inflation target, the monetary policy rate, and lending standards.

Relative to IN's original framework we introduce two extensions. First, we add long term debt, modeled in exactly the same way as in our simple model of Section 2. Second, for estimation purposes explained below, we introduce a stochastic process for the loan-to-value rate, $m_{t}$ :

$$
\begin{equation*}
\ln m_{t}=\left(1-\rho_{m}\right) \ln m+\rho_{m} \ln m_{t-1}+u_{t}^{m}, \tag{18}
\end{equation*}
$$

where $u_{t}^{m}$ is an independently and identically distributed innovation, with zero mean and standard deviation $\sigma_{m}$.

## Data and Empirical Strategy

The model is estimated with a Bayesian approach using 11 U.S. time series: real consumption, real residential investment, real business investment, real house prices, nominal interest rates, inflation, hours and wage inflation in the consumption sector, hours and wage inflation in the housing sector, and household debt. Data sources and transformations are reported in Appendix B, and graphically displayed in an online appendix.

Of the 11 observables, our only addition relative to IN is household debt. This additional observable motivates our introduction of the lending standard shock in equation (18).

We update IN's data set to the sample period 1965q1 to 2014q1, rather than ending in 2006q4. The new data have been revised, and alternative business and residential investment series have been combined to cover the full period, as explained in detail in Appendix B. Hence, while there are some minor differences, they do not affect the parameter estimates notably, as shown in online Appendix F.

A further difference from IN's approach is that we detrend non-stationary variables prior to estimation, each with its own linear trend. ${ }^{16}$ The alternative would be to formulate the theoretical model in stationary balanced growth terms and estimate trend parameters together with the other structural parameters of the model. Our strategy is motivated by the fact that

[^14]a theoretical balanced growth formulation of our model would assume that household debt, consumption, and income all grow at the same rate over the long-run, which clearly does not hold in the data for our sample period. This observation is beyond the scope of our paper to explain. Hence, we simply remove trends prior to estimation and abstract from growth in the model. Iacoviello (2015) adopts the same strategy for the same reason.

Several parameters are calibrated, and we impose priors on those that are estimated, following IN wherever possible. Calibrated parameter values are reported in Table 1 and the prior distributions of estimated parameters are reported in Tables 2 and 3. For the parameters that govern debt dynamics, namely $\vartheta, \alpha$, and $\kappa$, we necessarily depart from IN. Our approach here is to estimate the steady-state amortization rate $\delta$, while imposing that the steady-state ratio of debt over real estate value, $\widetilde{m} \equiv R b_{b} /\left(q h_{b}\right)=m \vartheta /[\delta+\vartheta(1-\delta)]$, equals 0.5 , the approximate mean value in the data for our sample period. In this way we estimate $\delta$ and thereby $\vartheta$ via the steady-state restriction. Note that the values of $\alpha$ and $\kappa$ that govern the mortgage amortization schedule, are then determined by the steady-state value of $\delta$. As with the other estimated parameters, we impose a prior distribution on $\delta$. Based on the aforementioned studies emphasizing that credit moves slowly, and the high observed persistence of debt in our data, we choose a normal prior distribution for $\delta$ with a mean of 0.1. ${ }^{17}$

### 4.2 Estimation Results and Model Properties

We estimate two versions of our model. The baseline version follows the approach outlined above and a comparison version imposes $\delta=1$.

The median of the posterior distributions of the estimated parameters for the two models and their $90 \%$ High Posterior Density (HPD) regions, are reported in Tables 2 and $3 .{ }^{18}$ Most

[^15]of the estimated parameter values are very similar in the two versions of the model and have overlapping HPD regions. ${ }^{19}$ An important difference, however, is that the estimated posterior median for $\delta$ is 0.03 in the model that allows for long-term debt. This implies a mortgage maturity of 73 quarters, approximately 18 years. The corresponding refinancing share, $\vartheta$, is then 0.042 which implies refinancing every 6 years on average. Another important difference between the two models is the autoregressive coefficient, $\rho_{m}$, in the process for lending standards. With long-term debt, the posterior median is 0.78 , while in the 1 -quarter debt model it is 0.98 . The HPD regions do not overlap. This result reflects the fact that the 1-quarter debt model lacks intrinsic debt persistence, in contrast to the framework with long-term debt.

Table 2 reports the log data density for both estimated models. The long-term debt model fits the data considerably better than the 1-quarter model: the $\log$ data density of the former is 6415.67 , while it is 6131.05 for the latter. These imply a logarithmic Bayes factor of 284.62 which is sufficiently high to conclude that there is decisive evidence in favor of the long-term debt model by the criteria of Kass and Raftery (1995). ${ }^{20}$

Why is the model fit substantially improved by the introduction of long-term debt? The answer lies in certain key moments of real household debt. The top panels of Figure 9 show the first to the fifth-order autocorrelations of linearly detrended real household debt in the data compared to the corresponding median, 5 th and 95 th percentile autocorrelations from model simulations. ${ }^{21}$ Figure 9 shows how the 73 -quarter debt model matches the autocorrelation

[^16]${ }^{21}$ Model simulations are computed using a random selection of 1000 draws from the posterior distribution. For each of them, 100 artificial time series of the main variables of length equal to that of the data are generated, giving a sample of 100,000 series. We apply to those series the same treatment that we apply to data (i.e. either linear detrending or demeaning) and for each simulation we compute moments of interest, obtaining distributions of the moments.
structure of real household debt in the data well, capturing the high persistence of credit, as emphasized by recent studies of the credit cycle noted in the introduction. In contrast, the 1-quarter debt model fails miserably along this dimension, implying very little persistence.

The middle panels of Figure 9 highlight another key dimension where the long-term debt model performs well. The panels show correlograms between house prices at time $k=0$ with debt at leads and lags up to 5 quarters away in the data and the model. The comovement between debt and house prices is interesting, as it reflects the contrasting mechanics of the models with immediate versus gradual amortization of mortgage debt. In the 1-quarter debt model, a house price increase that raises the collateral value also raises the entire stock of mortgage debt at the same time. But in the long-term debt model, only the flow of new loans responds to the increase in collateral value. Hence, the contemporaneous correlation at $k=0$ between debt and house prices is particularly high with 1-quarter debt, and declines markedly after $k=0$. This pattern does not hold in the data, where house prices instead lead debt, reflected by the positive slope of the correlogram after $k=0$. The long-term debt model captures this important pattern well. ${ }^{22}$

The bottom panel of Figure 9 displays the estimated series of lending standards shocks for both the 1 -quarter debt model and the long-term debt model. The established narrative of the recent U.S. housing market boom-bust episode is that lending standards were loosened in the boom phase and tightened thereafter (Mian and Sufi (2014)). The long-term debt model implies the same pattern. The 1-quarter debt model, in contrast, implies the exact opposite because it pegs debt to real estate value. Due to this peg, looser lending standards are needed for the 1quarter debt model to account for the slow deleveraging observed in the bust. In particular, the 1-quarter debt model interprets the leverage peak, which occurred due to collapsing house prices, as a massive relaxation of borrowing constraints. This is counterfactual, as shown by data from the Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS). The full shock decomposition in the online appendix further reveals that in accounting for the debt movements over this period, the 1-quarter debt model ascribes a greater role to housing

[^17]preference shocks, while the long-term debt model ascribes a large role to the same source that drives movements in GDP, namely labor supply shocks. The workings of the estimated model are further documented by its implied variance decomposition in Table $4 .{ }^{23}$

### 4.3 Impulse Response Functions

How does a monetary policy shock affect the debt burden in our estimated model? Figure 10 provides an answer, displaying the $90 \%$ probability bands of impulse response functions in the two estimated models. For the long-term debt model, the impact effect of a monetary policy shock on real household debt is no longer positive, but mildly negative, unlike in the simple model presented earlier. The medium run effects are very similar to those in the simple model. The debt-to-GDP ratio increases on impact and falls persistently in the medium run, similar to the simple model. There are two reasons why the real debt level no longer increases in the short run. First, debt is of shorter maturity in the estimated model. Second, the estimated degree of price stickiness is considerably higher (posterior median $\theta=0.89$ ) than in calibrated simple model $(\theta=0.75)$, which causes inflation and thereby the price level to drop less in response to the shock.

As in the simple model, we see that variables other than debt itself are relatively insensitive to the maturity of debt. But this pattern does not hold for all shocks, namely the three disturbances that most directly affect house prices and credit: the housing productivity shock, the housing preference shock and the lending standards shock. ${ }^{24}$ Figure 11 shows how the responses of debt and consumption differ with debt maturity for these three shocks. ${ }^{25}$

Starting with the housing preference shock in the top panel of Figure 11, we see that borrowers' consumption would rise on impact with 1-quarter debt, whereas it initially drops with long-term debt. This difference is due to the two forces at play here. All else equal, a higher

[^18][^19]preference for housing motivates all households to cut non-housing consumption in favor of housing. On the other hand, the resulting house price increase enables more borrowing. With 1-quarter debt, the latter effect is strong and borrowers' consumption rises. With long-term debt, the link from collateral value to debt is tempered in the short run, and consumption rises only after some time.

For housing productivity shocks in the middle panels of Figure 11, we have a similar scenario, but with the opposite sign. As housing costs are exogenously pushed down, house prices fall too, and borrowers' consumption follows the decline in collateral value. Long-term debt weakens this effect in the short run. Finally, we have the shock to lending standards in the bottom panels of Figure 11. When only a limited fraction of debt is immediately affected by the altered collateral value, the total stock of debt increases moderately on impact, but builds up over time. Hence, consumption will increase less in the short run and more in the long run, compared to the model with 1-quarter debt.

### 4.4 Monetary Policy

We now return to our main question: How should monetary policy be conducted to stabilize the debt-to-GDP ratio? In particular, we wish to assess if the policy implications from the stylized framework in Section 3 carry over to our estimated model with its larger set of shocks.

We start by considering simple rules. Holding the other coefficients in the interest rate rule constant at their estimated posterior medians ( $\phi_{\pi}=1.4, \phi_{y}=0.52, \phi_{R}=0.63$ ), we introduce a reaction to either the current debt-to-GDP ratio $\left(\phi_{(b / y)}\right)$, four-quarter debt growth $\left(\phi_{\Delta b}\right)$, the expected future debt-to-GDP ratio $\left(\phi_{E(b / y)}\right)$, or the newly initiated annuity loans $\left(\phi_{l}\right)$. The class of interest rate rules we consider is given by

$$
R_{t}=\left(R_{t-1}\right)^{\phi_{R}}\left[R^{s s} \pi_{t}^{\phi_{\pi}}\left(\frac{G D P_{t}}{G D P_{t-1}}\right)^{\phi_{y}} D_{j}^{\phi_{D_{j}}}\right]^{1-\phi_{R}}
$$

where $D_{j}$ is either $b_{b, t} / G D P_{t}, b_{b, t} / b_{b, t-4}, E_{t}\left(b_{b, t+i} / G D P_{t+i}\right)$ with $i=4,8$, or $l_{b, t}$.
Figure 12 displays the theoretical standard deviations of debt and inflation for alternative debt-responses when the economy is subject to the full set of shocks, except those stemming from
monetary policy itself (interest rate and inflation target shocks). ${ }^{26}$ The left-most panels show that a debt-to-GDP response ( $\phi_{b / y}$ ) has qualitatively similar effects to those in the simple model. First, responding positively to the debt-to-GDP ratio leads to equilibrium indeterminacy. With the estimated policy responses to inflation, output growth and the lagged interest rate, the threshold debt-to-GDP coefficient is given by the solid vertical line which lies marginally above zero. ${ }^{27}$ Second, a negative debt-to-GDP coefficient reduces the standard deviation of $b_{t} / y_{t}$. However, there is a distinct trade-off between debt-to-GDP and inflation stability, as negative values of $\phi_{b / y}$ make inflation considerably more volatile. In the left-most plots, the effect of responding to output growth is also displayed. As shown, the influence of $\phi_{b / y}$ is insensitive to the output response. ${ }^{28}$

When we consider a rule responding to debt growth rather than the debt-to-GDP ratio, the second column of panels in Figure 12 show that the conventional wisdom of raising the interest rate when debt grows more quickly has some merit. A moderate positive response to debt growth stabilizes the debt-to-GDP ratio. This effect is weaker than the influence of a negative $\phi_{b / y}$, but the costs in terms of inflation volatility are also smaller with the debt growth rule. Notably, the effect on debt-to-GDP volatility is non-monotonic. Beyond a moderate level (slightly below 1 here), raising the response coefficient further destabilizes both debt-to-GDP and inflation.

The third set of panels in Figure 12 shows that responding to expected, rather than current, debt-to-GDP, does not alter our main conclusions on the consequences of responding to debt-to-GDP. The main effect of increasing the debt-to-GDP forecast horizon is to raise the scope for equilibrium indeterminacy, as not only positive, but also negative response coefficients are associated with this phenomenon, as shown by the two vertical lines at negative values of $\phi_{E(b / y)}$.

[^20]Finally, the right-most panels in Figure 12 show results for policy rules that respond to newly initiated annuity loans, defined by $l_{b, t}=b_{b, t}-\left(1-\delta_{t-1}\right) b_{b, t-1} / \pi_{t}$. We see that negative responses to $l_{b, t}$ cause equilibrium indeterminacy, while responding positively increases the volatility of debt-to-GDP and inflation somewhat. ${ }^{29}$

Figure 13 further dissects the consequences of a negative $\phi_{b / y}$ in the interest rate rule. We focus on the three shocks that most directly impact housing and credit conditions. The solid curves display impulse responses under the estimated policy rule, which does not feature a response to debt, while the dashed curves follow when the policy rule has an additional negative coefficient on debt-to-GDP. For all three shocks, a negative value for $\phi_{b / y}$ causes debt-to-GDP to converge faster to steady state, similar to what Figure 5 showed for productivity shocks in the simple model. The stabilization is achieved both in the short and in the medium to long-run. Figure 13 also shows that if the policy rule instead contains a positive response coefficient on debt growth, debt swings are reduced relative to the estimated rule without any debt response at all, but to a lower degree than the rule responding negatively to the debt-to-GDP ratio. Intuitively, the debt growth policy destabilizes debt-to-GDP through Fisher dynamics when debt-to-GDP increases, but then after debt has peaked, there is a relatively long period with debt-contraction in which the debt growth rule is somewhat expansionary, bringing debt-to-GDP to target. This explains why a moderate coefficient on debt growth can stabilize debt-to-GDP somewhat, but not to the same extent as a negative $\phi_{b / y}$-coefficient.

Why does the negative $\phi_{b / y}$ curb the credit cycle? The reason is inflation and output dynamics. The three lower plots in Figure 13 make this point, by showing how the rule with a negative $\phi_{b / y}$ consistently leads to higher inflation when there is a credit expansion, or lower inflation when there is a credit contraction. Notably, these inflation movements are persistent, and the policy implies that inflation will deviate from steady state for a considerable period of time. This indicates that for those shocks that directly impact the housing market, there is a tradeoff between inflation and debt stability. This point re-emerges as we move to targeting policies.

[^21]As in Section 3.3, we define a targeting policy as one that minimizes a loss function of the type shown in equation (17) under commitment. Figures 14 and 15 characterize how outcomes under strict debt-to-GDP targeting deviate from strict inflation targeting and the estimated policy rule. The figures display responses to each of the eight non-policy shocks in the model. We see that for each shock that causes a fall in debt-to-GDP under inflation targeting, a monetary authority that targets the debt-to-GDP ratio engineers lower inflation and output. Likewise, if debt-to-GDP rises under inflation targeting (or the estimated policy rule), the debt-to-GDP targeter will boost inflation and generate a greater economic expansion. Hence, a central banker concerned with high leverage should aim to stimulate inflation and output so as to deflate the burden of existing household debt.

In short, the lessons from studying targeting policies in the simple model carry over to the richer estimated model: stabilization of the debt-to-GDP ratio calls for contractionary policy when the debt-to-GDP ratio is low, and expansionary policy when the debt-to-GDP ratio is high. This reflects that when the households' debt burden is largely determined by decisions made in the past, the most effective way to curb the credit cycle is to engineer a Fisherian debt deflation via higher inflation and output growth.

## 5 Other Environments

In the model environments we consider, the interest rates that households face are identical to the policy rate. Hence, we have abstracted from interest rate spreads. In practice, however, spreads are a potential variable that central banks might respond to instead of responding to a debt variable. What would the presence of long term debt imply for the way in which central banks should respond to interest rate spreads?

The answer would clearly depend on how credit spreads are determined. In the literature on spreads and macroeconomic dynamics, credit spreads are typically modeled as an increasing function of borrowers' leverage. For example, in the influential model of Bernanke and Gertler (1989), borrowers are entrepreneurs and the interest rate spread depends on the entrepreneur's debt relative to the value of his assets. The counterpart in our model would be that interest rate spreads are driven by household debt relative to real estate value, as in Forlati and Lambertini (2011). In this case, the logic we have laid out above is likely to reappear. With long-term debt,
an interest rate rule with a positive response coefficient on the interest rate spread would likely induce equilibrium indeterminacy through the same channels as our analysis has highlighted. The reason is that by reducing real interest rates, a non-fundamental increase in expected inflation will trigger house price appreciation. Under long-term debt, this appreciation reduces leverage in much the same way as inflation reduces real household debt and debt-to-GDP in our analysis above. Similar results are also likely to obtain if the interest rate spread is driven by the real debt stock alone, as in Curdia and Woodford (2010).

In the models we have considered, debt movements have little influence on output and inflation unless monetary policy reacts to them. However, in an environment where credit spreads increase with household debt, a long-term amortization schedule would imply that spreads could diverge from steady-state levels for extended periods of time. Hence, debt dynamics could affect output and inflation through the interest rates that households face, a channel that our study abstracts from. Under these circumstances, the crucial influence of the Fisher dynamics on household debt burdens that our analysis highlights would take on added importance. A precise investigation of the interplay between endogenous credit spreads and long-term debt is beyond the scope of our paper, but seem an interesting avenue for future research.

Another simplification we make, is to postulate that borrowing is constrained only by collateral value, and not household income. Here we follow the dominant approach in the literature. However, in reality loan-to-income constraints are also important. A monetary policy tightening may therefore have additional contractive effects on new borrowing, via household income, that we ignore. This caveat should be carried in mind when interpreting our results.

## 6 Conclusion

After the 2007-2009 financial crisis, household debt burdens have been high on the policy agenda. Unfortunately, discussions of this topic tend to implicitly assume that variation in aggregate debt-ratios reflect shifts in new borrowing and lending, which is misguided. By introducing a reasonable distinction between new loans and pre-existing debt, we develop a framework in which debt dynamics can match empirical regularities. This match is essential for any analysis that seeks to assess how monetary policy should respond to movements in debt burdens.

We show that the long-term nature of household debt matters for a central bank concerned
with the credit cycle, and that policy advice derived from models with 1-quarter debt should be treated with extreme caution. First, due to the gradual nature of households' debt accumulation, interest rate changes have a limited influence on aggregate debt measures. With long-term debt, tighter monetary policy is even likely to increase the debt-to-GDP ratio in the short run. Second, a policy that systematically increases the interest rate in response to a higher debt-toGDP ratio can be destabilizing. Not only can such a policy magnify the volatility of inflation, but it can destabilize debt itself. Moreover, by reacting positively to the debt-to-GDP level, monetary policy can open the door to equilibrium indeterminacy and belief driven fluctuations. Paradoxically, an interest rate rule with a negative response to the debt-to-GDP ratio serves to stabilize the households' debt burden. Compared to a strict inflation targeter, a central bank aiming to stabilize debt-to-GDP should act to stimulate the economy when a shock initiates a boom in the debt-to-GDP ratio, and restrain the economy when a shock initiates a decline in the debt-to-GDP ratio. Generally speaking, our results show that when household debt is mostly determined by decisions made in the past, a central banker concerned with a high debt-to-GDP ratio should aim to stimulate inflation and GDP rather than to tighten policy. Naturally, this implies a trade-off between curbing the credit cycle and stabilizing inflation.

This paper is part of a broader agenda to establish the principles behind "leaning against the wind" policies. It is notable that our findings go somewhat against the conventional wisdom in the area, but are consistent with the findings of Svensson (2013). Here there is a parallel to a result by Galí (2014), who shows how a monetary policy that systematically responds to rational asset price bubbles might paradoxically raise the volatility of the bubbles. Of course, these types of results may not necessarily be taken at face value in actual policy design, as they are derived from models that abstract from many of the aspects of real-world policy debates. For example, in our model there is no mechanism through which debt accumulation might trigger a financial crisis or hinder the speed of recovery. But we would argue that these potential mechanisms are exactly why it is important that the policy debate be based on frameworks that can account for the debt dynamics observed in the data.

Table 1: Calibrated Parameters in the Medium Scale Model

| Parameter | Description/Target | Value |
| :--- | :--- | :--- |
| $\beta_{l}$ | Steady-state annual real interest rate $3 \%$ | 0.9925 |
| $\beta_{b}$ | Impatient households' discount factor | 0.97 |
| $\nu_{h}$ | Ratio of housing wealth to GDP of $1.35 \%$ | 0.12 |
| $\xi$ | Capital share in goods production | 0.35 |
| $\mu_{h}$ | Capital share in housing production | 0.10 |
| $\mu_{l a}$ | Ratio of value of residential land to annual output of $50 \%$ | 0.10 |
| $\mu_{i b}$ | Ratio of business capital to annual GDP of 2.1\% | 0.10 |
| $\delta_{h}$ | Ratio of residential investments to total output of about $6 \%$ | 0.01 |
| $\delta_{k c}$ | Ratio of nonresidential investments to GDP of about 27\% | 0.025 |
| $\delta_{k h}$ | Ratio of nonresidential investments to GDP of about 27\% | 0.03 |
| $X, X_{w c}, X_{w h}$ | Steady-state mark-up of 15\% | 1.15 |
| $\widetilde{m}=R b_{b} / q h_{b}$ | Steady-state ratio of debt to real estate | 0.50 |
| $m$ | Loan-to-value ratio on new mortgages | 0.85 |
| $\rho_{s}$ | Annual autocorrelation of trend inflation around 0.9 | 0.975 |

Notes: All parameter values follow from Iacoviello and Neri (2010).

Table 2: Estimation: Prior and Posterior Distribution of the Structural Parameters

| Parameter | Prior distribution |  |  | Posterior distribution |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1-quarter debt model |  | Long-term debt model |  |
|  | Distribution | Mean | SD | Median | $90 \%$ HPD | Median | 90\% HPD |
| $\gamma_{l}$ | Beta | 0.5 | 0.075 | 0.29 | $0.22-0.36$ | 0.26 | 0.20-0.32 |
| $\gamma_{b}$ | Beta | 0.5 | 0.1 | 0.42 | $0.31-0.55$ | 0.51 | $0.41-0.62$ |
| $\varphi_{L, l}$ | Gamma | 0.5 | 0.1 | 0.39 | $0.27-0.53$ | 0.42 | $0.30-0.51$ |
| $\varphi_{L, b}$ | Gamma | 0.5 | 0.1 | 0.54 | 0.38-0.70 | 0.48 | 0.34-0.71 |
| $\mu_{l}$ | Normal | 1 | 0.1 | -0.05 | -0.08--0.02 | -0.05 | -0.08--0.03 |
| $\mu_{b}$ | Normal | 1 | 0.1 | 1.18 | $1.02-1.31$ | 1.12 | 0.96-1.31 |
| $\phi_{k, c}$ | Gamma | 10 | 2.5 | 20.14 | $17.09-23.29$ | 20.85 | $18.45-23.57$ |
| $\phi_{k, h}$ | Gamma | 10 | 2.5 | 10.60 | $6.76-15.02$ | 9.58 | $7.03-12.57$ |
| $\varpi$ | Beta | 0.65 | 0.05 | 0.65 | $0.57-0.73$ | 0.62 | $0.56-0.69$ |
| $\phi_{R}$ | Beta | 0.75 | 0.1 | 0.61 | 0.55-0.66 | 0.63 | $0.57-0.68$ |
| $\phi_{\pi}$ | Normal | 1.5 | 0.1 | 1.42 | $1.32-1.51$ | 1.40 | $1.31-1.50$ |
| $\phi_{y}$ | Normal | 0 | 0.1 | 0.56 | 0.46-0.65 | 0.52 | 0.44-0.68 |
| $\theta$ | Beta | 0.667 | 0.05 | 0.89 | 0.87-0.91 | 0.89 | 0.87-0.91 |
| $v$ | Beta | 0.5 | 0.2 | 0.52 | $0.41-0.65$ | 0.55 | $0.45-0.66$ |
| $\theta_{w, c}$ | Beta | 0.667 | 0.05 | 0.77 | $0.73-0.81$ | 0.76 | $0.72-0.80$ |
| $\iota_{w, c}$ | Beta | 0.5 | 0.2 | 0.08 | $0.02-0.15$ | 0.07 | $0.02-0.14$ |
| $\theta_{w, h}$ | Beta | 0.667 | 0.05 | 0.77 | $0.72-0.81$ | 0.75 | $0.72-0.81$ |
| $\iota_{w, h}$ | Beta | 0.5 | 0.2 | 0.40 | $0.21-0.60$ | 0.42 | 0.23-0.61 |
| $\zeta$ | Beta | 0.5 | 0.2 | 0.78 | 0.66-0.91 | 0.80 | 0.68-0.92 |
| $\delta$ | Normal* | 0.10 | 0.02 | 1 | - | 0.0307 | 0.0223-0.0412 |
| Log data density |  |  |  |  | 6131.05 |  | 6415.67 |

Notes: The median implied value of $\vartheta$ is 0.59 in the 1-quarter debt model, and 0.042 in the long-term debt model. * The prior distribution for $\delta$ refers only to the long-term debt model because $\delta=1$ with 1 -quarter debt. The sample is $1965 q 1$ to $2014 q 1$.

Table 3: Estimation: Prior and Posterior Distribution of the Shock Processes

| Lable 3: Estimation: Prior and Posterior Distribution of the Shock Processes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Prior distribution |  |  |  |  |  |  |  |  |  |  | Posterior distribution |  |  |  |
| Parameter | Distribution | Mean | SD | Median | $90 \%$ HPD | Median | Mong-term debt model |  |  |  |  |  |  |  |
| $\rho_{z}$ | Beta | 0.8 | 0.1 | 0.95 | $0.93-0.97$ | 0.96 | $0.94-0.98$ |  |  |  |  |  |  |  |
| $\rho_{A H}$ | Beta | 0.8 | 0.1 | 0.996 | $0.991-0.999$ | 0.996 | $0.992-0.999$ |  |  |  |  |  |  |  |
| $\rho_{A K}$ | Beta | 0.8 | 0.1 | 0.92 | $0.90-0.95$ | 0.93 | $0.90-0.95$ |  |  |  |  |  |  |  |
| $\rho_{v_{h}}$ | Beta | 0.8 | 0.1 | 0.97 | $0.95-0.99$ | 0.98 | $0.96-0.99$ |  |  |  |  |  |  |  |
| $\rho_{c}$ | Beta | 0.8 | 0.1 | 0.96 | $0.86-0.99$ | 0.96 | $0.95-0.99$ |  |  |  |  |  |  |  |
| $\rho_{\nu_{l}}$ | Beta | 0.8 | 0.1 | 0.97 | $0.95-0.99$ | 0.97 | $0.95-0.99$ |  |  |  |  |  |  |  |
| $\rho_{m}$ | Beta | 0.8 | 0.1 | 0.98 | $0.96-0.99$ | 0.78 | $0.68-0.87$ |  |  |  |  |  |  |  |
| $\sigma_{z}$ | Inv. Gamma | 0.001 | 0.01 | 0.0100 | $0.0091-0.0110$ | 0.0100 | $0.0091-0.0110$ |  |  |  |  |  |  |  |
| $\sigma_{A H}$ | Inv. Gamma | 0.001 | 0.01 | 0.0213 | $0.0195-0.0233$ | 0.0216 | $0.0198-0.0236$ |  |  |  |  |  |  |  |
| $\sigma_{A K}$ | Inv. Gamma | 0.001 | 0.01 | 0.0107 | $0.0089-0.0126$ | 0.0111 | $0.0096-0.0127$ |  |  |  |  |  |  |  |
| $\sigma_{\nu_{h}}$ | Inv. Gamma | 0.001 | 0.01 | 0.0382 | $0.0271-0.0508$ | 0.0335 | $0.0237-0.0452$ |  |  |  |  |  |  |  |
| $\sigma_{R}$ | Inv. Gamma | 0.001 | 0.01 | 0.0032 | $0.0027-0.0037$ | 0.0030 | $0.0027-0.0034$ |  |  |  |  |  |  |  |
| $\sigma_{c}$ | Inv. Gamma | 0.001 | 0.01 | 0.0123 | $0.0047-0.0288$ | 0.0122 | $0.0078-0.0185$ |  |  |  |  |  |  |  |
| $\sigma_{\nu_{l}}$ | Inv. Gamma | 0.001 | 0.01 | 0.0196 | $0.0161-0.0236$ | 0.0192 | $0.0157-0.0233$ |  |  |  |  |  |  |  |
| $\sigma_{p}$ | Inv. Gamma | 0.001 | 0.01 | 0.0039 | $0.0035-0.0044$ | 0.0039 | $0.0035-0.0044$ |  |  |  |  |  |  |  |
| $\sigma_{s}$ | Inv. Gamma | 0.001 | 0.01 | 0.0280 | $0.0211-0.0348$ | 0.0276 | $0.0216-0.0339$ |  |  |  |  |  |  |  |
| $\sigma_{m}$ | Inv. Gamma | 0.001 | 0.01 | 0.0180 | $0.0165-0.0196$ | 0.1069 | $0.0764-0.1368$ |  |  |  |  |  |  |  |
| $\sigma_{L, h}$ | Inv. Gamma | 0.001 | 0.01 | 0.1647 | $0.1511-0.1793$ | 0.1624 | $0.1495-0.1787$ |  |  |  |  |  |  |  |
| $\sigma_{\omega, h}$ | Inv. Gamma | 0.001 | 0.01 | 0.0051 | $0.0047-0.0056$ | 0.0050 | $0.0047-0.0056$ |  |  |  |  |  |  |  |

Notes: $\sigma_{L, h}$ and $\sigma_{\omega, h}$ are standard deviations for measurement errors in hours worked and wages in the housing sector. The sample is $1965 q 1$ to $2014 q 1$.

Table 4: Variance Decomposition

|  | Non- <br> house <br> prod. | Mon. <br> pol. | House <br> prod. | House <br> pref. | Inv. <br> spec. <br> prod. | Cost | Infl. <br> target | Labor <br> supply | Intert. <br> Pref. | Lend. <br> std. |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |
| GDP | 20.88 | 3.57 | 2.62 | 0.41 | 8.04 | 3.80 | 3.55 | 55.77 | 1.35 | 0.01 |
| Consumption | 24.39 | 2.76 | 0.15 | 0.19 | 3.50 | 3.52 | 3.34 | 59.55 | 2.59 | 0.01 |
| Inflation | 1.90 | 1.74 | 0.05 | 0.16 | 0.65 | 17.58 | 70.20 | 1.05 | 6.55 | 0.12 |
| Residential inv. | 0.28 | 0.78 | 67.32 | 18.49 | 0.04 | 0.14 | 0.19 | 10.31 | 2.43 | 0.01 |
| Business inv. | 14.63 | 4.11 | 0.05 | 0.04 | 33.05 | 4.52 | 4.08 | 30.39 | 9.04 | 0.09 |
| Hours cons. | 1.68 | 6.30 | 0.04 | 0.04 | 0.59 | 6.85 | 5.12 | 79.04 | 0.32 | 0.03 |
| Hours housing | 0.48 | 1.96 | 24.23 | 42.23 | 0.07 | 0.34 | 0.50 | 24.73 | 5.43 | 0.03 |
| House prices | 1.62 | 0.28 | 90.16 | 5.97 | 0.22 | 0.29 | 0.18 | 0.71 | 0.57 | 0.00 |
| Interest rate | 2.26 | 5.67 | 0.17 | 0.39 | 3.03 | 4.10 | 68.66 | 2.35 | 13.09 | 0.28 |
| Wages cons. | 1.77 | 6.60 | 0.08 | 0.17 | 1.52 | 1.38 | 68.46 | 12.98 | 6.93 | 0.13 |
| Wages housing | 1.97 | 5.97 | 0.05 | 0.17 | 1.45 | 2.18 | 66.53 | 14.23 | 7.29 | 0.15 |
| Househ. debt | 1.96 | 1.26 | 2.37 | 31.84 | 0.62 | 2.98 | 5.86 | 7.41 | 6.05 | 39.66 |

Notes: Long-run variance decomposition from the estimated model with long-term debt.

Figure 1: Contractionary Monetary Policy Shock


Notes: Responses to a 25 basis point increase of the quarterly policy interest rate. For the interest rate and inflation, percentage point deviations from steady state are displayed. For the other variables, percent deviations from steady state are displayed.

Figure 2: Contractionary Monetary Policy Shock - Debt Dynamics under Annuity Loans and Fixed Amortization Rate Loans


Note: Responses to a 25 basis point increase in the quarterly policy interest rate, percent deviations from steady state.

Figure 3: Equilibrium Determinacy with Debt-to-GDP in the Interest Rate Rule


Notes: Regions of equilibrium determinacy for alternative constellations of the interest rate response to debt-to$\operatorname{GDP}\left(\phi_{b / y}\right)$ and inflation ( $\phi_{\pi}$ ).

Figure 4: Equilibrium Determinacy with Debt-to-GDP in the Interest Rate Rule - The Influence of $m$ and $\vartheta$


Notes: Regions of equilibrium determinacy for alternative constellations of the interest rate response to debt-to$G D P\left(\phi_{b / y}\right)$ and inflation $\left(\phi_{\pi}\right)$. In the left panel, $m$ varies and $\vartheta$ is re-calibrated to maintain a constant ratio of total debt to real estate value. In the right panel, $\vartheta$ varies and the model is not re-calibrated.

Figure 5: Volatility and Impulse Responses under Simple Policy Rules


Notes: The upper plots shows standard deviations of debt-to-GDP for different coefficients on debt-to-GDP ( $\phi_{b / y}$ ) or real debt ( $\phi_{b}$ ) in the interest rate rule, under 1-quarter debt in the left plot and 30-year debt in the right plot. The bottom plot shows the response of debt-to-GDP after a positive productivity shock in the 30-year debt model with alternative policy rules.

Figure 6: Debt-to-GDP and Inflation Targeting with 30-Year Debt


Notes: Impulse responses under optimal policy aiming to stabilize inflation $(\Gamma=0)$ or debt-to-GDP $(\Gamma=1)$.

Figure 7: Optimal Debt-to-GDP and Inflation Targeting with 1-Quarter Debt


Notes: Impulse responses under optimal policy aiming to stabilize inflation $(\Gamma=0)$ or debt-to-GDP $(\Gamma=1)$.

Figure 8: Variance Frontiers and Welfare under Targeting Policies


Notes: The upper panels display the possibility frontiers when policy is set to minimize the loss function in equation (17). Along the frontiers, the weight on debt-to-GDP in the loss function ( $\Gamma$ ) varies. In the left panel, $\Gamma \in[0.01,1]$, in the right panel $\Gamma \in[0,1]$. The vertical axes display in terms of $L_{1}=\left(1-\lambda_{y}\right) \operatorname{var}(\pi)+\lambda_{y} \operatorname{var}(y)$. The bottom panels display the welfare gains of increasing $\Gamma$, for lender and borrower households separately. Welfare is measured as percentage share of steady-state consumption.

Figure 9: Model Fit


Notes: The $k$-th order autocorrelation of real household debt (top panels) and the cross correlation between house prices at time $t$ and real household debt at time $t+k$ (mid panels), in the data and in the estimated 1-quarter and long-term debt models. From both models, the median and $95 \%$ high posterior density region of simulated distributions are reported. The bottom panel plots the estimated sequence of lending standard shocks in the two versions of the model, with the vertical line marking the onset of the Great Recession.

Figure 10: Monetary Policy Shock - Impulse Responses


Notes: The $90 \%$ high posterior density regions of the responses to a monetary policy shock in the estimated models.

Figure 11: Credit and Housing Shocks - Impulse Responses of Debt and Consumption


Notes: The $90 \%$ high posterior density regions of the responses to housing preference, housing productivity, and lending standards shocks in the estimated models.

Figure 12: Debt-to-GDP and Inflation Volatility under Simple Policy Rules in the Estimated Model with Long-Term Debt


Notes: Standard deviations of debt-to-GDP and inflation when monetary policy responds to debt-to-GDP ( $\phi_{b / y}$ ), debt growth $\left(\phi_{\Delta b}\right)$, expected 1- and 2-year ahead debt-to-GDP $\left(\phi_{E(b / y)}\right)$, and new loans $\left(\phi_{l}\right)$. In all plots, the response coefficients on inflation, the lagged interest rate and output growth are set to their estimated values. The left-most plots also considers an alternative response coefficient on output growth. The vertical solid lines indicate threshold values for the response coefficients beyond which the equilibrium is indeterminate.

Figure 13: Impulse Responses Under Simple Policy Rules


Notes: Impulse response functions when monetary policy responds to debt-to-GDP ( $\phi_{b / y}$ ) or debt growth ( $\phi_{\Delta b}$ ). Policy coefficients on inflation, the lagged interest rate and output growth are always set to their estimated values.

Figure 14: Optimal Debt-to-GDP and Inflation Targeting in the Estimated Model with Long Term Debt


Notes: Impulse responses under optimal policy aiming to stabilize inflation $(\Gamma=0)$ or debt $(\Gamma=1)$, and when the interest rate follows the estimated simple rule.

Figure 15: Optimal Debt-to-GDP and Inflation Targeting in the Estimated Model with Long Term Debt


Notes: Impulse responses under optimal policy aiming to stabilize inflation ( $\Gamma=0$ ) or debt $(\Gamma=1)$, and when the interest rate follows the estimated simple rule.

## Appendix A: Estimated Model

Here we summarize the model from Iacoviello and Neri (2010) which we extend with long term debt.

Patient lender households maximize lifetime utility

$$
V_{l, t}=E_{0} \sum_{t=0}^{\infty} \beta_{l}^{t} \varepsilon_{c, t}\left[\frac{1-\gamma_{l}}{1-\beta_{l} \gamma_{l}} \ln \left(c_{l, t}-\gamma_{l} c_{l, t-1}\right)+\nu_{h, t} \ln \left(h_{l, t}\right)-\frac{\nu_{l, t}}{1+\varphi_{l}}\left(L_{l, c, t}^{1+\mu_{l}}+L_{l, h, t}^{1+\mu_{l}}\right)^{\frac{1+\varphi_{l}}{1+\mu_{l}}}\right],
$$

with respect to $c_{l, t}, h_{l, t}, L_{l, c, t}$ (hours in the consumption sector), and $L_{l, h, t}$ (hours in the housing sector), where $\mu_{l}$ measures the degree of labor mobility between the two sectors. Moreover, $\ln \varepsilon_{c, t}=\rho_{c} \ln \varepsilon_{c, t-1}+u_{t}^{c}, \ln \nu_{l, t}=\rho_{\nu_{l}} \ln \nu_{l, t-1}+u_{t}^{\nu_{l}}$, and $\ln \nu_{h, t}=\left(1-\rho_{\nu_{h}}\right) \ln \nu_{h}+\rho_{\nu_{h}} \ln \nu_{h, t-1}+u_{t}^{\nu_{h}}$ are processes for intertemporal preferences, labour supply and housing preference respectively, where $u_{t}^{c}, u_{t}^{\nu_{h}}$, and $u_{t}^{\nu_{l}}$ are independently and identically distributed innovations with standard deviations $\sigma_{c}, \sigma_{\nu_{h}}$, and $\sigma_{\nu_{l}}$. The patient household's budget constraint is:

$$
\begin{aligned}
& c_{l, t}+\frac{k_{c, t}}{A_{k, t}}+k_{h, t}+k_{i b, t}+q_{t} h_{l, t}+p_{l, t} l_{t}+\frac{R_{t-1} b_{l, t-1}}{\pi_{t}}+a\left(z_{h, t}\right) k_{h, t-1} \\
= & \frac{w_{l, c, t}}{X_{l, w c, t}} L_{l, c, t}+\frac{w_{l, h, t}}{X_{l, w h, t}} L_{l, h, t}-\phi_{t}+\left(R_{c, t} z_{c, t}+\frac{1-\delta_{k c}}{A_{k, t}}\right) k_{c, t-1}+\left(R_{h, t} z_{h, t}+1-\delta_{k h}\right) k_{h, t-1} \\
& +p_{i b, t} k_{i b, t}+\left(p_{l, t}+R_{l, t}\right) l_{t-1}+q_{t}\left(1-\delta_{h}\right) h_{l, t-1}+\operatorname{Div}_{l, t}-\frac{a\left(z_{c, t}\right)}{A_{k, t}} k_{c, t-1}+b_{l, t},
\end{aligned}
$$

where $k_{c, t}$ is capital in the consumption good sector, $k_{h, t}$ is capital in the housing sector, $k_{i b, t}$ is intermediate goods (priced at $p_{i b, t}$ ) rented to the housing sector, $l_{t}$ is land priced at $p_{l, t}$, $z_{c, t}$ and $z_{h, t}$ are capital utilization rates, and $b_{l, t}$ is debt. $A_{k, t}$ captures investment-specific technology, and evolves as $\ln A_{k, t}=\rho_{A K} \ln A_{k, t-1}+u_{t}^{K}$, where $u_{t}^{K}$ is independently and identically distributed with zero mean and standard deviation $\sigma_{A K}$. Loans are set in nominal terms and yield a nominally riskless return $R_{t}$. Real wages are denoted by $w_{l, c, t}$ and $w_{l, h, t}$, real rental rates by $R_{c, t}, R_{h, t}$, and $R_{l, t}$, and depreciation rates by $\delta_{k c}$ and $\delta_{k h} . X_{l, w c, t}$ and $X_{l, w h, t}$ denote wage markups, which arise due to monopolistic competition in the labor market and are collected by labor unions. $\operatorname{Div}_{l, t}=\left[\left(X_{t}-1\right) / X_{t}\right] Y_{t}+\left[\left(X_{l, w c, t}-1\right) / X_{l, w c, t}\right] w_{l, c, t} L_{l, c, t}+$ $\left[\left(X_{l, w h, t}-1\right) / X_{l, w h, t}\right] w_{l, h, t} L_{l, h, t}$ (with $X_{t}$ and $Y_{t}$ to be defined below) are lump-sum profits from final good firms and from labor unions. Finally, $\phi_{t}$ denotes adjustment costs for capital, and
$a(\cdot)$ are the costs of capital utilization:

$$
\begin{aligned}
& \phi_{t}=\frac{\phi_{k, c}}{2}\left(\frac{k_{c, t}}{k_{c, t-1}}-1\right)^{2} k_{c, t-1}+\frac{\phi_{k, h}}{2}\left(\frac{k_{h, t}}{k_{h, t-1}}-1\right)^{2} k_{h, t-1}, \\
& a\left(z_{c, t}\right)=R_{c}\left[\varrho \frac{z_{c, t}^{2}}{2}+(1-\varrho) z_{c, t}+\left(\frac{\varrho}{2}-1\right)\right], \\
& a\left(z_{h, t}\right)=R_{h}\left[\varrho \frac{z_{h, t}^{2}}{2}+(1-\varrho) z_{h, t}+\left(\frac{\varrho}{2}-1\right)\right],
\end{aligned}
$$

were $R_{c}$ and $R_{h}$ are the steady-state rental rates in sectors $c$ and $h$. In the estimation of the model, a prior is specified for the curvature of the capacity utilization function, $\zeta=\varrho /(1+\varrho)$.

Impatient borrower households choose $c_{b, t}, h_{b, t}, L_{b, c, t}$, and $L_{b, h, t}$ to maximize

$$
V_{b, t}=E_{0} \sum_{t=0}^{\infty} \beta_{b}^{t} \varepsilon_{c, t}\left[\frac{1-\gamma_{b}}{1-\beta_{b} \gamma_{b}} \ln \left(c_{b, t}-\gamma_{b} c_{b, t-1}\right)+\nu_{h, t} \ln \left(h_{b, t}\right)-\frac{\nu_{l, t}}{1+\varphi_{b}}\left(L_{b, c, t}^{1+\mu_{b}}+L_{b, h, t}^{1+\mu_{b}}\right)^{\frac{1+\varphi_{b}}{1+\mu_{b}}}\right],
$$

subject to the budget constraint

$$
c_{b, t}+q_{t} h_{b, t}=\frac{w_{b, c, t}}{X_{b, w c, t}} L_{b, c, t}+\frac{w_{b, h, t}}{X_{b, w h, t}} L_{b, h, t}+b_{b, t}-\frac{R_{t-1} b_{b, t-1}}{\pi_{t}}+q_{t}\left(1-\delta_{h}\right) h_{b, t-1}+\operatorname{Div}_{b, t}
$$

the collateral constraint in equation (3) with $m_{t}$ instead of $m$, and the law of motion for the amortization rate in equation (4). As stated in the main text, $m_{t}$ follows the process (18). $\operatorname{Div}_{b, t}=\left[\left(X_{b, w c, t}-1\right) / X_{b, w c, t}\right] w_{b, c, t} L_{b, c, t}+\left[\left(X_{b, w h, t}-1\right) / X_{b, w h, t}\right] w_{b, h, t} L_{b, h, t}$ are dividends from labor unions.

Wholesale firms hire labor and capital services and purchase intermediate goods to produce wholesale goods $Y_{t}$ and new houses $I H_{t}$. They maximize profits

$$
\max \frac{Y_{t}}{X_{t}}+q_{t} I H_{t}-\left(\sum_{i=c, h} w_{l, i, t} L_{l, i, t}+\sum_{i=c, h} w_{b, i, t} L_{b, i, t}+\sum_{i=c, h} R_{i, t} z_{i, t} k_{i, t-1}+R_{l, t} l_{t-1}+p_{i b, t} k_{i b, t}\right),
$$

where $X_{t}$ is the mark-up of final goods over wholesale goods prices. The production technologies are:

$$
\begin{aligned}
& Y_{t}=\left[z_{t}\left(L_{l, c, t}^{\varpi} L_{b, c, t}^{1-\varpi}\right)\right]^{1-\xi}\left(z_{c, t} k_{c, t-1}\right)^{\xi}, \\
& I H_{t}=\left[A_{h, t}\left(L_{l, h, t}^{\varpi} L_{b, h, t}^{1-\varpi}\right)\right]^{1-\mu_{h}-\mu_{i b}-\mu_{l a}}\left(z_{h, t} k_{h, t-1}\right)^{\mu_{h}} k_{i b, t}^{\mu_{i b}} \mu_{t-1}^{\mu_{l a}},
\end{aligned}
$$

where $\ln z_{t}=\rho_{z} \ln z_{t-1}+u_{t}^{z}$ and $\ln A_{h, t}=\rho_{A H} \ln A_{h, t-1}+u_{t}^{A H}$, with $u_{t}^{z}$ and $u_{t}^{A H}$ independently and identically distributed innovations with zero means and standard deviations $\sigma_{z}$ and $\sigma_{A H}$, respectively. The capital share in the goods production function is $\xi$, the capital share in the housing production function is $\mu_{h}$, the land share is $\mu_{l a}$, and the intermediate goods share is $\mu_{i b}$.

In the consumption sector, there is monopolistic competition and Calvo-type price rigidity at the retail level. Retailers buy goods $Y_{t}$ from wholesale firms in a competitive market with price $P_{t}^{w}$, differentiate the goods at no cost, and sell them at a markup $X_{t}=P_{t} / P_{t}^{w}$. Those goods are aggregated with a constant elasticity of substitution technology and converted into homogeneous consumption and investment goods by households. Each period a fraction of retailers set prices optimally, while remaining firms partially index prices to past inflation. The same Phillips curve for the consumption sector as in equation (14) then follows. Cost shocks are added to (14). These are independently and identically distributed with zero mean and standard deviation $\sigma_{p}$.

Patient and impatient households supply homogeneous labor services to unions, which differentiate them, set wages subject to a Calvo scheme and offer labor services to wholesale labor packers who reassemble the labor into the homogeneous composites $L_{l, c, t}, L_{b, c, t}, L_{l, h, t}, L_{b, h, t}$. Wholesale firms hire these composites. Nominal wage inflation $\left(\omega_{t}\right)$ is then given by:

$$
\begin{aligned}
& \ln \omega_{l, c, t}-\iota_{w, c} \ln \pi_{t-1}=\beta_{l}\left(E_{t}\left[\ln \omega_{l, c, t+1}\right]-\iota_{w, c} \ln \pi_{t}\right)-\frac{\left(1-\beta_{l} \theta_{w, c}\right)\left(1-\theta_{w, c}\right)}{\theta_{w, c}} \ln \left(X_{l, w c, t} / X_{l, w c}\right) \\
& \ln \omega_{b, c, t}-\iota_{w, c} \ln \pi_{t-1}=\beta_{b}\left(E_{t}\left[\ln \omega_{b, c, t+1}\right]-\iota_{w, c} \ln \pi_{t}\right)-\frac{\left(1-\beta_{b} \theta_{w, c}\right)\left(1-\theta_{w, c}\right)}{\theta_{w, c}} \ln \left(X_{b, w c, t} / X_{b, w c}\right) \\
& \ln \omega_{l, h, t}-\iota_{w, h} \ln \pi_{t-1}=\beta_{l}\left(E_{t}\left[\ln \omega_{l, h, t+1}\right]-\iota_{w, h} \ln \pi_{t}\right)-\frac{\left(1-\beta_{l} \theta_{w, h}\right)\left(1-\theta_{w, h}\right)}{\theta_{w, h}} \ln \left(X_{l, w h, t} / X_{l, w h}\right) \\
& \ln \omega_{b, h, t}-\iota_{w, h} \ln \pi_{t-1}=\beta_{b}\left(E_{t}\left[\ln \omega_{b, h, t+1}\right]-\iota_{w, h} \ln \pi_{t}\right)-\frac{\left(1-\beta_{b} \theta_{w, h}\right)\left(1-\theta_{w, h}\right)}{\theta_{w, h}} \ln \left(X_{b, w h, t} / X_{b, w h}\right) .
\end{aligned}
$$

The nominal interest rate is set according to the simple rule

$$
R_{t}=\left(R_{t-1}\right)^{\phi_{R}}\left[R \pi_{t}^{\phi_{\pi}}\left(\frac{G D P_{t}}{G D P_{t-1}}\right)^{\phi_{y}}\right]^{1-\phi_{R}} \frac{\varepsilon_{t}^{R}}{A_{t}^{s}}
$$

where $\varepsilon_{t}^{R}$ is an independently and identically distributed monetary policy shock with zero mean
and standard deviation $\sigma_{R}$, and $\ln A_{t}^{s}=\rho_{s} \ln A_{t-1}^{s}+u_{t}^{s}$ is an inflation target shock with independently and identically distributed innovations $u_{t}^{s}$ with zero mean and standard deviation $\sigma_{s}$. $G D P_{t} \equiv Y_{t}+\bar{q} I H_{t}+I K_{t}$.

The market clearing conditions are:

$$
\begin{aligned}
& C_{t}+\frac{I K_{c, t}}{A_{k, t}}+I K_{k, t}+k_{i b, t}=Y_{t}-\frac{\phi_{k, c}}{2}\left(\frac{k_{c, t}}{k_{c, t-1}}-1\right)^{2} k_{c, t-1}-\frac{\phi_{k, h}}{2}\left(\frac{k_{h, t}}{k_{h, t-1}}-1\right)^{2} k_{h, t-1}, \\
& h_{l, t}+h_{b, t}-\left(1-\delta_{h}\right)\left(h_{l, t-1}+h_{b, t-1}\right)=I H_{t}, \\
& b_{l, t}+b_{b, t}=0 .
\end{aligned}
$$

## Appendix B: Data used in the Estimation

Aggregate consumption: Real Personal Consumption Expenditure (seasonally adjusted, billions of chained 2009 dollars, table 1.1.6), divided by the Civilian Noninstitutional Population (CNP16OV, source: Bureau of Labor Statistics, retrieved from FRED). Source: Bureau of Economic Analysis (BEA). Log-linearly detrended.

Business fixed investment: Real Private Nonresidential Fixed Investment (seasonally adjusted, billions of chained 2009 dollars, table 1.1.6), divided by CNP16OV. Source: BEA. This series is available from 1999q1 only. To obtain values for 1965q1 to 1989q4, we first recovered the original Iacoviello and Neri (2010) (IN) series in levels, starting from the transformed series available at the AEJM website. The first value of the series has been imputed from vintages available in FRED2. Then we linearly regressed new data on IN's data for the period 1999q12006 q 4 . With the resulting regression coefficients, we extrapolated our data backwards to get the full new series. Log-linearly detrended.

Residential investment: Real Private Residential Fixed Investment (seasonally adjusted, billions of chained 2009 dollars, table 1.1.6.), divided by CNP16OV. Source: BEA. This series is available from 1999q1 only. We followed the same procedure as for Business Fixed Investment to get the full new series. Log-linearly detrended.

Inflation: Quarterly log differences in the implicit price deflator for the nonfarm business sector. Source: Bureau of Labor Statistics (BLS), retrieved from FRED: IPDNBS. Demeaned.

The nominal short-term interest rate: 3-month Treasury Bill Rate (Secondary Market Rate), expressed in quarterly units. Source: Board of Governors of the Federal Reserve System,
retrieved from FRED: TB3MS. Demeaned.
Real house prices: Census Bureau House Price Index (new one-family houses sold including value of lot) deflated with the implicit price deflator for the nonfarm business sector. Source: Census Bureau, https://www.census.gov/construction/cpi/. Log-linearly detrended.

Hours in the consumption sector: Total Nonfarm Payrolls (retrieved from FRED: PAYEMS) less all employees in the construction sector (retrieved from FRED: USCONS), times Average Weekly Hours of Production Workers (retrieved from FRED: AWHNONAG), divided by CNP16OV. Source: BLS. Demeaned.

Hours in the housing sector: All Employees in the Construction Sector (retrieved from FRED: USCONS), times Average Weekly Hours of Construction Workers (retrieved from FRED: CES2000000007), divided by CNP16OV. Source: BLS. Demeaned.

Wage Inflation in the consumption sector: The quarterly rate of change in Average Hourly Earnings of Production/Nonsupervisory Workers on Private Nonfarm Payrolls, Total Private (retrieved from FRED: AHETPI). Source: BLS. Demeaned.

Wage inflation in the housing sector: Quarterly changes in Average Hourly Earnings of Production/Nonsupervisory Workers in the Construction Industry (retrieved from FRED: CES2000000008). Source: BLS. Demeaned.

Household debt: Households and Nonprofit Organizations; Home Mortgages; Liability in billions of dollars (Source: Board of Governors of the Federal Reserve System, retrieved from FRED: HMLBSHNO), deflated with the implicit price deflator for the nonfarm business sector and divided by CNP16OV. Log-linearly detrended.

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[^0]:    ${ }^{1}$ For an overview of the Swedish debate, see the Financial Times, October 29, 2014: "Tactic of 'lean against the wind' has failed in Sweden," Financial Times, May 7, 2014: "Riksbank raises concern on household debt," Laséen and Strid (2013), and the multiple comments by Lars Svensson at http://larseosvensson.se.

[^1]:    ${ }^{2}$ Within our model, we study interest rate reactions to real debt or debt-to-GDP in deviation from steady state. The real-world counterpart would naturally be deviations from a trend.

[^2]:    ${ }^{3}$ Habits are included for the model's GDP movements to be reasonably sluggish, which in turn is important for debt-to-GDP dynamics. However, the policy conclusions drawn below do not hinge on the presence of habits.

[^3]:    ${ }^{4}$ Because $\beta_{b}<\beta_{l}$, the borrowing constraint binds in the non-stochastic steady state. In addition, we assume that the constraint holds always in the vicinity of the steady state. As is well-known, this assumption can be rationalized as long as the difference between $\beta_{b}$ and $\beta_{l}$ is sufficiently high relative to the volatility of the shocks considered. Beyond this, the gap between $\beta_{b}$ and $\beta_{l}$ has no substantial influence on our results.

[^4]:    ${ }^{5}$ Our definition of $l_{b, t}$ implies that it is the incremental debt beyond regular amortization which feeds into $l_{b, t}$. The natural alternative would be to assume all refinanced loans start a new amortization schedule, in which case $l_{b, t}=\vartheta m E_{t}\left(q_{t+1} \pi_{t+1}\right) h_{b, t} / R_{t}$. With this alternative assumption more debt is in the early stages of the amortization schedule and hence the average amortization rate across all loans is lower. Our results are qualitatively unaffected by the two alternative definitions of $l_{b, t}$. Quantitatively, the alternative assumption amplifies our results, as a lower amortization rate implies greater debt persistence. See online appendix for details.

[^5]:    ${ }^{6}$ Price indexation is included so that the model's inflation movements are reasonably sluggish. None of the monetary policy conclusions hinge on the existence of price indexation.

[^6]:    ${ }^{7}$ Both Iacoviello and Neri (2010) and Justiniano et al. (2015a) calibrate $\nu_{h}$ to match the housing wealth to output ratio. Because there is no capital accumulation in our model, we instead target the ratio of housing wealth relative to consumption, as in Campbell and Hercowitz (2005).

[^7]:    ${ }^{8}$ When later studying scenarios where debt is fully amortized each period $(\delta=1)$, we recalibrate the refinancing rate to $\vartheta=0.63$ to keep $\widetilde{m}=0.5$. In this case there is no distinction between pre-existing and refinanced debt.

[^8]:    ${ }^{9}$ Results for shorter amortization schedules, a nominal decomposition, and further details on how a monetary policy shock affects debt in our model, are provided in the working paper version of this article, available at http://www.norges-bank.no/en/Published/Papers/Working-Papers/2015/42015/.

[^9]:    ${ }^{10}$ We vary the amortization schedule by changing $\kappa, \alpha$ and hence $\delta$ to match the respective debt durations. As explained in the calibration section, $\vartheta$ is always re-calibrated to maintain the same debt-to-real estate ratio.

[^10]:    ${ }^{11}$ Consistent with the intuitions provided above, the effects of reacting to the level of real debt are similar to the effects of reacting to the ratio of debt relative to GDP. Results are available in the working paper version at http://www.norges-bank.no/en/Published/Papers/Working-Papers/2015/42015/
    ${ }^{12}$ With both high refinancing frequency and low amortization, the amount of household debt will necessarily

[^11]:    be high. In the model with $\vartheta=0.63$ in Figure 4 , the ratio of debt relative to real estate value is 0.79 .

[^12]:    ${ }^{13}$ As the weight on the output gap increases, $\lambda_{y}$ increases both in the loss function under which the model is solved, and in the computation of $L_{1}$.

[^13]:    ${ }^{15}$ The various agents' maximization problems are described in Appendix A.

[^14]:    ${ }^{16}$ We treat stationary variables in the same way as IN.

[^15]:    ${ }^{17}$ We also impose that debt maturities must be of an integer number of quarters. Hence, we consider only a discrete grid of 120 possible values for $\delta$ and $m$, each consistent with debt maturities of 1 to 120 quarters. To implement this discretization, any draw of $\delta$ from its continuous prior is translated into the closest value of $\delta$ in the discrete set.
    ${ }^{18}$ Draws from the posterior distribution of the parameters are obtained using the random walk Metropolis algorithm. We sampled $1,000,000$ draws and discarded half of them before computing posterior statistics. Details on prior and posterior distributions and convergence properties of our estimations are reported in online Appendix C.

[^16]:    ${ }^{19}$ The estimates are also highly similar to the parameters originally estimated by IN, as shown in the online Appendix E.
    ${ }^{20}$ Kass and Raftery (1995) suggest the following evaluation guidelines. Define $B_{10}$ as the Bayes factor for model 1 ( 73 -quarter debt model, in our case) compared to the alternative model 0 (1-quarter debt model). Values of $2 \log _{e}\left(B_{10}\right)>10,6<2 \log _{e}\left(B_{10}\right)<10$, or $2<2 \log _{e}\left(B_{10}\right)<6$ respectively suggest decisive, strong, or substantial evidence against the null hypothesis that model 0 is better than model 1 , and hence decisive, strong or substantial support for model 1.

[^17]:    ${ }^{22}$ We focus on the real debt level here as this is the series used in the estimation. But the correlation structure of debt-to-GDP exhibits similar properties, both in the data and in the model. Results are shown in Online appendix Section E.

[^18]:    ${ }^{23}$ Further documentation on the estimated long-term debt model is reported in the online appendix, together with the variance decomposition of the 1-quarter debt model. The latter is very similar to the decomposition from the long-term debt model, with the notable difference that housing preference shocks are much more important for debt dynamics in the 1-quarter debt model.

[^19]:    ${ }^{24}$ Table 4 reveals that these three shocks account for $74 \%$ of the long-run variance of real household debt.
    ${ }^{25}$ Impulse response functions for all shocks in the model are provided in the online appendix.

[^20]:    ${ }^{26}$ The standard deviations of all the shocks are as estimated, see Table 3.
    ${ }^{27}$ We do not plot the determinacy frontier in the $\phi_{b / y}, \phi_{\pi}$-space here. The resulting determinacy frontiers, both for responses to debt-to-GDP and to the real debt level, are highly similar to those in the simple model, and are reported in the online appendix.
    ${ }^{28}$ Increasing the output response $\left(\phi_{y}\right)$ alone, holding all the other response coefficients constant, has a nonmonotonic, but quantitatively small, effect on debt-to-GDP volatility. As $\phi_{y}$ increases from zero, debt-to-GDP volatility first falls, but later increases somewhat. For details, see the online appendix.

[^21]:    ${ }^{29}$ Because equation (3) implies $l_{b, t}=\vartheta\left[m E_{t}\left(q_{t+1} \pi_{t+1}\right) h_{b, t} / R_{t}-\left(1-\delta_{t-1}\right) b_{b, t-1} / \pi_{t}\right]$, a negative interest rate response to $l_{b, t}$ implies a negative response to current inflation, and thus generates indeterminacy through the same channel as responding positively to $b_{t} / y_{t}$ does.

