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Has the Inclusion of Unlisted Real Estate Improved the Risk-Return Trade-off of the Government Pension Fund- Global?

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"Has the Inclusion of Unlisted Real Estate Improved the Risk-Return Trade-off of the Government Pension Fund- Global?"

GRA 19502 – Master Thesis

Supervisor: Chunyu Yang

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Abstract

This paper examines whether the inclusion of unlisted real estate has improved the risk-return trade-off of the Norwegian Government Pension Fund - Global.

Firstly, we did a regression analysis with the fund returns as the dependent variable against the Fama and French market factors and bonds as explanatory variables. In the next part, we calculated the Sharpe ratio and Treynor's measure to analyze the performance measures of the fund. Lastly, we did a Markowitz optimization analysis to find what has been the optimal allocation towards unlisted real estate in the fund.

We found that the risk-return trade-off has been improved after the inclusion of unlisted real estate. In addition, the mean-variance optimizer suggested a higher allocation towards unlisted real estate. Based on these findings, we conclude that the inclusion of unlisted real estate into the GPFG has improved the risk-return trade-off

1. Introduction

In 1969, oil companies discovered Ekofisk, the largest oil field ever found. This was the start of the Norwegian oil adventure, and the establishment of the Government Pension Fund – Global (GPFG) in 1990. The fund was established to ensure a long-term management of the excess petroleum revenues. The first deposit was made in 1996, and the fund has grown to become one of the largest pension funds in the world. For a long time, unlike its peers, the fund did not include unlisted real estate into its portfolio and the fund was only invested in stocks and bonds. Norges Bank Investment Management (NBIM) made the first recommendation to the Ministry of Finance in 2006 to include unlisted real estate as an asset class. Five years later, NBIM made the first unlisted real estate investment in 2011, and has since aimed to increase their real estate investments by 1% each year.

1.1 NBIM and GPFG

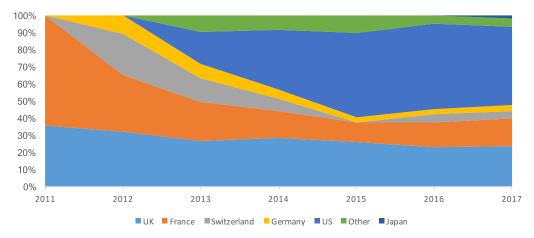
The formal framework of the GPFG was first established through the Norwegian Parliament's Government Pension Fund Act. NBIM is responsible of managing the fund, and their aim is to achieve the highest possible return within the investment mandate that is given by the Ministry of Finance. The fund is invested globally in international stocks, bonds and real estate.

1.2 NBIM's unlisted real estate investments

NBIM invests in unlisted real estate to create a more diversified portfolio and to reduce the overall risk of the portfolio. To manage this, the fund takes advantage of its limited liquidity requirements and their significant available capital to make real estate investments with a long-term horizon (NBIM, 2017). The fund's unlisted real estate investments are globally diversified across cities in 14 countries in Europe, US and Asia, which are expected to continue to play a key role in the global economy. The unlisted real estate investments are divided into office, retail and logistics, whereas the office sector is the most important. Since

the fund is invested globally, NBIM invests through partners to benefit from their local knowledge and expertise.

By the end of 2017 NBIM had invested 58.2% of its unlisted real estate portfolio in office properties (NBIM, 2017). It is also clear that, by looking at a timeline of their unlisted real estate investments, the US has become their most important region. In 2017, 46.2% of their unlisted real estate market value was invested in the US (NBIM, 2017).



Share of market value

Figure 1: Real estate investments by country. All numbers gathered from annual reports since the inclusion of unlisted real estate.

1.3 Motivation

When deciding on a research question, we wanted to combine our interest in real estate and portfolio management. Since NBIM started investing in unlisted real estate recently, the investment strategy has been highly debated. Their mission is to *"Safeguard and build financial wealth for future generations" (NBIM, 2017)*, and we believe it is important to be critical about how the fund is managed.

While working on our research question, Hoddevik and Priestly published a newspaper article in Dagens Næringsliv (Hoddevik & Priestley, 2017). The article criticized NBIM's inclusion of unlisted real estate, arguing that it has not improved the risk-adjusted return of the fund. This increased our motivation for providing a more in-depth analysis of NBIM's return on unlisted real estate, stocks and bonds.

Today the fund has allocated 2.5% of its investments in unlisted real estate, while NBIM has a mandate to allocate up to 7% of its funds. We will therefore expect to see a further increase in real estate investments in the following years. We are interested in investigating what contribution the inclusion of unlisted real estate has had to the GPFG. Considering NBIM's investment strategy, region allocation and type of real estate investments, our research question to be studied is:

"Has the Inclusion of Unlisted Real Estate Improved the Risk-Return Trade-Off of the Government Pension Fund – Global?"

To address this question, we will have to make a time series analysis of the GPFG's monthly returns. We will apply different conventional methods to measure the risk-adjusted return and risk-return trade-off after the inclusion of unlisted real estate.

1.4 Real Estate Investments

Most institutional investors have exposure towards real estate either directly or through companies which are exposed to changes in the real estate market. In the world portfolio, real estate consists of approximately 6.2% of total investments (Van Nieuwerburgh, Stanton, & de Bever, 2015). The global real estate market consists of the following sectors:

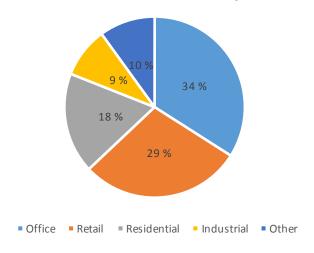




Figure 2: Real estate market by sector (Teuben & Clacy-Jones, 2014).

The global real estate market can be divided into residential and commercial real estate, whereas the latter covers office, retail, industrial and "other" in the graph above.

Investments in unlisted real estate are known to be illiquid, capital intensive and highly dependent on a stable cash flow. In this thesis, we will analyze NBIM's investments in unlisted real estate. As previously mentioned, NBIM started investing in real estate in 2011, while real estate has been common in other pension funds for a long time. The graph below shows that unlisted real estate has had a percentage share of total pension fund assets between 3.2% and 7% since 1990 until 2011.

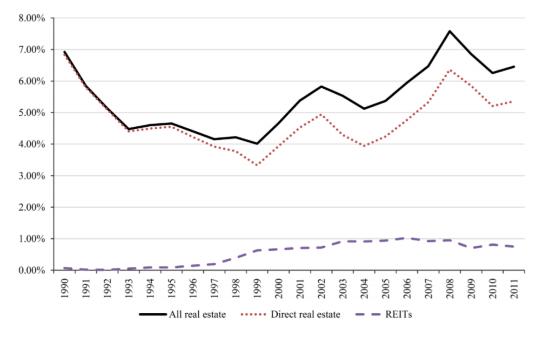


Figure 3: Real estate as a percentage of total pension fund assets. (Andonov, Eichholtz, & Kok, 2015).

2. Literature review

Jensen (1968) addressed the problem on how to evaluate the performance of mutual fund managers. By using the Capital Asset Pricing Model (CAPM), he argues that superior performance by managers can be captured by Alpha (Jensen, 1968). Jensen found that managers underperformed with an average value of negative alpha compared to a risk-adjusted benchmark. The research that followed Jensen suggested that there is a set of skilled managers that can outperform the market (Grinblatt & Titman, 1989). Hendricks (1993) found that it is possible to develop a trading strategy on past performance to generate economically significant returns.

In the article by Bond and Mitchell (2010) they investigate whether fund managers deliver superior risk-adjusted return in the direct real estate market. They do this by using a dataset containing annual fund performance from 1981 to 2006. They conclude that few managers have managed to generate excess riskadjusted returns. However, according to Lee and Ward (2001), there is evidence that returns on commercial real estate assets show persistence. To explain Lee and Wards findings, Bond and Mitchell look at two possible reasons. The first reason can be that it is possible to artificially smoothen property valuation, and therefore show persistence in the returns. The second reason is that the knowledge of the underlying persistence can lead to momentum trading that would achieve positive risk-adjusted performance. Bond and Mitchell also address that it is not clear that this persistence could be exploited because of high transaction costs and illiquidity which is required to trade in commercial property assets.

Goetzmann and Ibbotson (1990) found several reasons for institutional investors to include real estate as an asset class in the portfolio. They found that real estate was the only major asset class that had consistently exceeded inflation over the past 16 years, and thus provides a hedge against inflation to the institutional investors (Goetzmann & Ibbotson, 1990). The article also found that commercial real estate has little or no correlation to the stock market, which makes it an effective hedge against fluctuations in the financial markets. They also mention the US real estate market specifically, and how low correlations between the real estate markets across different regions can reduce risk for investors that can diversify regionally. Goetzmann and Ibbotson's study is supported by the findings of Giliberto (1990). He found that equity real estate investment trusts (EREITs) performance was heavily influenced by stock and bond market movements, while un-securitized real estate investments were barely affected (Giliberto, 1990). His study argues that institutional investors that invest in EREITs must accept a volatility similar of that of stocks. A study by Liu and Mei (2003) argues that there is no significant premium for investing in real estate. Since they did not find any risk premium associated with real estate investments, they argued that the only benefit would be a potential diversification effect against unsystematic risk, and they therefore conclude that the best asset allocation is to only include stocks and bonds.

It is also argued in the literature that institutional investors should invest 15-20 percent of the fund assets in real estate, however, institutional investors invest only 2-3 percent of their assets in real estate (Chun, Sa-Aadu, & Shilling, 2004). Rehring argues that the weight of real estate should increase with the investment horizon (Rehring, 2012). Therefore, according to research made by Chun et al., institutional investors are underinvested in real estate. Chun et al. argue that institutional investors are underinvested and should have a higher allocation towards real estate because of several reasons; Investments in real estate have relatively high average returns compared to the risk, real estate risk has moderated and real estate investments are informationally inefficient, hence skilled investors could exploit mispricing (Chun et al., 2004).

From the literature review we have different arguments regarding the inclusion of real estate as an asset class, where the majority argues for inclusion. To the best of our knowledge, no previous research has looked at NBIM's unlisted real estate investments, using the most recent data, and performed a regression analysis, Markowitz mean-variance optimization and a performance measure analysis to see if the inclusion of unlisted real estate has improved the risk- return trade-off.

3. Theory

In this part, relevant financial theories are presented that will help build our arguments when answering our thesis question "*Has the Inclusion of Unlisted Real Estate Improved the Risk-Return Trade-Off of the Norwegian Government Pension Fund – Global?*"

3.1 Risk-Return Trade-off

There is always risk associated with an investment. If an investor wants higher expected returns a price, in terms of accepting higher risks, must be paid. In the case where higher expected returns are achievable without bearing extra risk, all investors would buy the high-return asset and eventually increase the prices. The asset will be considered attractive by investors until its expected return is commensurate to its risk. This is an implication of the highly competitive financial markets and the no-free-lunch proposition (Bodie, Kane, & Marcus, 2014). As thousands of investors search for investments with high expected returns, it is rare or impossible to find any arbitrage opportunities. We will therefore assume that there is a risk-return trade-off in every asset included in a portfolio, where higher expected returns are expected to have higher risk and vice versa. A portfolio manager diversifies a portfolio with an aim to limit the exposure to each asset and to improve the risk-return trade-off. In a mixed portfolio with different asset classes it is necessary to not only consider the risk of an asset separately, one must also consider the interplay between all assets of the portfolio. Diversification will influence portfolio risk and how to measure the risk-return trade-off of a portfolio, and these implications are more closely elaborated in modern portfolio theory (MTP) that is developed by Harry Markowitz.

3.2 Modern portfolio theory

MTP was first introduced by Harry Markowitz in his paper *Portfolio selection* (1952). Markowitz argued that investors could maximize their expected returns with minimal risk through diversification by using the "expected returns – variance of returns rule" (perhaps better known as *mean-variance portfolio*). The

concept is that mean-variance portfolios can earn the same return as a single asset, but with lower risk. This is contrary to the no-free-lunch proposition. The different combinations of mean-variance portfolios that are available to an investor are summarized in the *minimum-variance frontier*. Given the assumptions that investors only focus on maximizing returns and reducing volatility, a rational investor would choose the portfolio with the lowest possible volatility for a given expected return, or the highest expected return for a given volatility. The efficient frontier that is figured below represents the available set of mean-variance portfolios. Any portfolio above the Global Minimum-Variance Portfolio is considered as efficient, because they earn a higher return than those portfolios that are below the Minimum-Variance Portfolio, but have the same variance. The dots in between the variance frontier represent inefficient investments because it is possible to achieve higher returns without increasing the variance/risk. An investor can apply the minimum-variance framework to choose the optimal weight of each asset class to create a minimum-variance portfolio

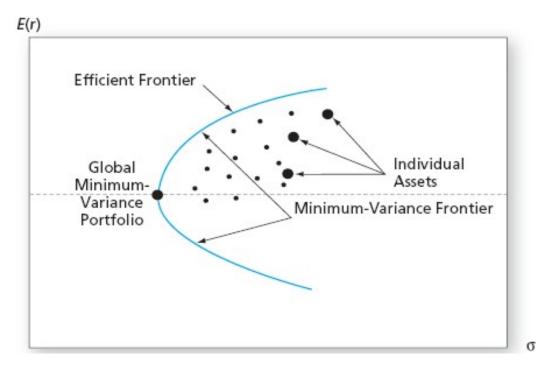


Figure 4: Efficient Frontier: (Bodie et al., 2014)

3.3 Liquidity preference

Given the same expected return, it will always be desirable for an investor to hold liquid assets that can quickly be converted in to cash. Liquid assets are less risky because they can easily be sold in times of contracting markets. If an investor holds an illiquid asset the portfolio risk will increase, and the risk must be awarded in terms of higher expected returns. This is the *liquidity preference theory*, which was first introduced by John Maynard Keynes (Keynes, 1937). The theory argues that investors require a liquidity premium for the increased risk of holding long-term assets such as real estate that reduce the investors liquidity.

3.4 Adjusting Returns for Risk

Financial risk is necessary to increase returns, and every asset has its own risk profile that needs to be considered before investing. Some of the most common financial risk factors are currency risk, equity risk, foreign investment risk, credit risk and liquidity risk. Portfolio performances must be adjusted for risk before they can be assessed appropriately. The easiest way to measure performance is to compare investment funds with similar risk profiles relative to the comparison universe. However, similar funds often have different strategic investment categories, so that they are not fully comparable (Bodie et al., 2014). It is therefore desirable to use more precise risk-adjustment approaches. A common approach is to apply the mean-variance criteria when adjusting for risk. When the CAPM was introduced, many academicians developed different approaches to measure risk-adjusted performance. Even though each approach has its limitations, they are widely used to measure performance. Some of the most common approaches will be elaborated in the following part:

3.4.1 Sharpe Ratio

The Sharpe Ratio (Sharpe, 1994) is the reward-to-volatility ratio, and is the most commonly used measure of risk-adjusted returns. The ratio measures the trade-off between reward and risk by dividing the assets risk premium by the standard deviation of excess returns. A shortcoming is that its numerical measure is not easy to interpret. The ratio does not compare rates of return, only ratios that are pure numbers and therefore difficult to interpret whether the difference is economically significant or not (Bodie et al., 2014).

Sharpe ratio =
$$\frac{(\bar{r}_p - \bar{r}_f)}{\sigma_p}$$

3.4.2 Treynor's measure

Treynor's measure also gives a measure of trade-off between reward and risk. It is like the Sharpe ratio, but it uses systematic risk in the denominator. The Treynor's measure is desirable to use when an asset is part of a large investment portfolio. In that case, one should measure the excess return against the systematic risk to evaluate the assets contribution to performance (Bodie et al., 2014).

Treynor's measure =
$$\frac{(\bar{r_p} - \bar{r_f})}{\beta_p}$$

4. Methodology

We have divided our analysis into three parts. In the first part of our analysis of whether unlisted real estate has improved the risk-return trade-off, we have conducted a regression analysis to analyze the performance of the GPFG both including and excluding unlisted real estate, compared against different market factors. In addition, we have extended the Fama and French analysis by including bonds. We have done this to see if unlisted real estate has improved the risk-adjusted return of the fund. A positive alpha would indicate that the GPFG has yielded a risk-adjusted return above its expected return given its risk.

In the second part, we have analyzed the descriptive statistics and computed different performance measures of the returns on unlisted real estate, stocks and bonds. The performance measures give us an indication of how each asset class contributes to the GPFG's return.

Lastly, we used the Markowitz mean-variance optimization to find the optimal allocation of the different asset classes in a well-diversified portfolio. We applied the mean-variance approach to see whether unlisted real estate should be included or not when optimizing the Sharpe ratio of the GPFG.

4.1 Regression analysis

We have employed two different models to measure NBIM's unlisted real estate investments risk-adjusted return: the Fama and French 3-factor model (E. F. Fama & French, 1993) and the Fama and French 5-factor model (F. E. Fama & French, 2015). The models have also been extended by adding bonds as an independent variable. The reason for this is that NBIM is invested in unlisted real estate, equity and bonds, and we wanted to analyze the funds exposure towards both stocks and bonds.

We ran an OLS time-series regression for each model specification. The analysis is divided into two parts. The first regression takes the GPFG's excess return including real estate above the risk-free rate as the dependent variable (GPFG^{excess}). Next, we ran a regression analysis of the GPFG's excess return

above the risk free rate, excluding their unlisted real estate investments. By running two different regression analysis, we analyze the effect on the riskadjusted return of including unlisted real estate into the portfolio.

The Fama and French 3-factor model explains GPFG^{excess} by including markets excess return above risk free rate, "small capitalization minus big capitalization" (SMB) portfolio returns and "High minus low book-to-value" (HML) portfolio returns as explanatory factors. The second regression analysis includes bonds.

$$GPFG^{excess} = R_f + \beta_1 (R_m - R_f) + \beta_s (SMB) + \beta_h (HML) + \alpha$$

$$GPFG^{excess} = R_f + \beta_1 (R_m - R_f) + \beta_s (SMB) + \beta_h (HML) + \beta_b (Bonds) + \alpha$$

To strengthen our analysis, we also applied the Fama and French 5-factor model. By adding two more explanatory variables our aim was to capture a higher explanatory power of GPFG^{excess}. Fama and French added the difference between the returns on diversified portfolios of stocks with robust and weak profitability (RMW), and the difference between the returns on diversified portfolios of the stocks of low and high investment firms (CMA) (F. E. Fama & French, 2015).

$$GPFG^{excess} = R_f + \beta_1 (R_m - R_f) + \beta_s (SMB) + \beta_h (HML) + \beta_r RMW + \beta_c CMA + \alpha$$

$$GPFG^{excess} = R_f + \beta_1 (R_m - R_f) + \beta_s (SMB) + \beta_h (HML) + \beta_r RMW + \beta_c CMA + \beta_b (Bonds) + \alpha$$

4.2 Performance measures

We have chosen two different performance measures in our analysis to decide whether unlisted real estate has improved the risk-return trade-off. These measures are the Sharpe ratio and Treynor's measure. Since the GPFG is a welldiversified portfolio, an important performance ratio is Treynor's measure. To analyze the diversification effect of including real estate, we have examined the relationship between equity, bonds and real estate by looking at the assets' correlation coefficients.

4.3 Markowitz mean-variance optimization

As part of our analysis, we used the mean-variance optimization to find the optimal asset allocation between equity, bonds and real estate. We used this method because we were interested to see what asset allocation maximizes the GPFG`s risk-adjusted return in terms of Sharpe ratio.

5. Data

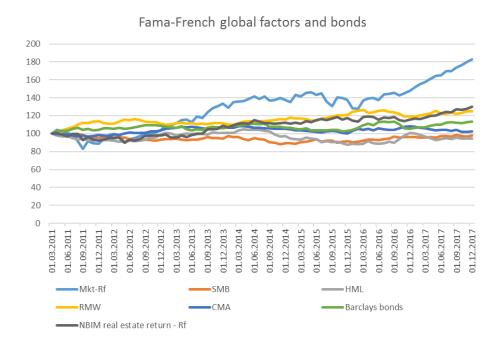
The data we use to analyze the GPFG's risk-return trade-off are monthly and cover the period 30.04.2011 - 31.12.2017, which gives us 81 observations. This is the time horizon since NBIM included unlisted real estate into their portfolio. We have also collected data that covers the period 31.05.2006 - 31.12.2017 to analyze how real estate performs over a longer time period. All returns are US denominated because most of NBIM's unlisted real estate, equity and bond investments are in US dollar (NBIM, 2017).

5.1 GPFG

The GPFG's monthly returns were collected from NBIM's homepage, and is the basis of our analysis. We have collected the monthly returns of the total portfolio and for each asset class separately. In addition, we have included the funds return both including and excluding unlisted real estate.

5.2 Fama and French Global Factors

The Fama and French's global 3 and 5-factors and the risk-free rate have been collected from the Dartmouth library (French, 2018) which covers 23 developed markets. We apply global factors because the GPFG is globally diversified.





5.3 Indices

5.3.1 MSCI World Stock Index: (MSCI WSI)

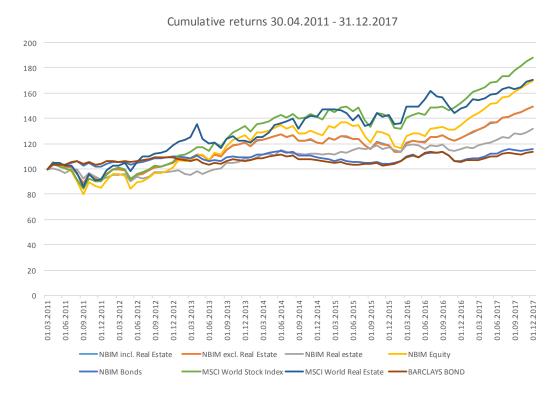
The index is a free float weighted equity index that captures large and mid-cap companies across 23 developed markets. It covers approximately 85% of the free float-adjusted market capitalization in each country. Since NBIM's equity investments are globally diversified we believe that the MSCI World Stock Index serves as a good benchmark for their equity investments (MSCI, 2018a).

5.3.2 MSCI Real Estate Index (MSCI WREI)

It is difficult to find comparable data for unlisted real estate investments, and we could not find a suitable index for unlisted real estate. A critical problem with unlisted real estate indices is the low degree of transparency. They are often based on private valuations, which makes the reported returns less reliable. However, previous research argues that listed and unlisted real estate are comparable in the long-term (Van Nieuwerburgh et al., 2015). Despite the fact that our data set covers a short period, we believe the MSCI Real Estate Index is the best benchmark for NBIM's unlisted real estate investments. The index consists of large and mid-cap real estate equities across 23 developed markets. It is free float-adjusted and the securities are classified in the Real Estate Sector according to the Global Industry Classification Standard (MSCI, 2018b).

5.3.3 Barclays US Aggregated Bond Index (Barclays Agg)

The index is a broad-based flagship benchmark that measures the investment grade, US dollar-denominated, fixed-rate taxable bond market in 24 developed and emerging markets. It includes Treasuries, government-related and corporate securities, mortgage backed securities, asset-backed securities and commercial mortgage-backed securities (Bloomberg, 2018).



The cumulative returns of our data are summarized in the graph below.

Figure 6: Cumulative monthly returns of indices and the NBIM's asset classes

5.4 Testing Classical linear regression model assumptions

Before conducting the regression analysis, we tested the data for classical linear regression (CLRM) assumptions. This was necessary to ensure that we obtain unbiased coefficients, which potentially could have lead us to misleading conclusions. We found evidence for non-normality in our data. A possible solution to this could have been to create dummy variables. However, given our relatively small data sample we believe that removing outlying residuals may have a critical effect on the regression output, creating an artificial good fit. We have therefore chosen to work with the original data set.

5.5 Descriptive statistics

The table below lists the descriptive statistics for the returns during the period 30.04.2011 - 31.12.2017. As expected, we can see that equity has had the highest return, followed by real estate and bonds. Since we are interested in the risk-return trade-off, the volatility is an important measure in our analysis. The world real

estate market has a volatility of 13.75%, which is higher than the world equity index of 11.74%. This is contrary to NBIM's investments where unlisted real estate has a lower volatility of 7.27% compared to 13.41% volatility in NBIM equity. However, volatility does not explain whether the variation in returns reflect downside or upside risk. The negative skewness in all asset classes implies that we have more downside risk. From the table we see that NBIM's unlisted real estate investments have the most negative skewness of -0.8, implying that the investments have had the largest downside risk.

| | NBIM RE | NBIM E | NBIM B | MSCI WSI | MSCI WREI | Barclays Agg |
|----------------|---------|-----------------|--------|----------|-----------|--------------|
| Mean | 4,07 % | 7 <i>,</i> 85 % | 2,19 % | 9,33 % | 7,92 % | 1,84 % |
| Std. Error | 0,03 | 0,05 | 0,02 | 0,05 | 0,05 | 0,02 |
| Std. Deviation | 7,27 % | 13,41 % | 5,03 % | 11,74 % | 13,75 % | 4,61 % |
| Variance | 0,53 % | 1,80 % | 0,25 % | 1,38 % | 1,89 % | 0,21 % |
| Kurtosis | 1,84 | 1,53 | 0,56 | 1,28 | 1,29 | 0,81 |
| Skewness | -0,80 | -0,62 | -0,39 | -0,51 | -0,32 | -0,55 |

5.5.1 NBIM and indices

Figure 7: Descriptive statistics; NBIM`s unlisted real estate, equity and bonds + indices. All data collected 07.03.2018.

In the table below, we have summarized the correlations between all asset classes and indices. We see that NBIM's unlisted real estate investments have a correlation of 0.68 and 0.58 towards equity and bonds respectively. As expected when comparing these results with the correlation between the indices, MSCI WREI has a higher correlation towards stocks. This is because MSCI WREI comprises listed real estate stocks. However, as previously mentioned, we expect that unlisted and listed real estate will have the same correlation towards stocks in the long term.

| | NBIM RE | NBIM E | NBIM B | MSCI WSI | MSCI WREI | Barclays Agg |
|--------------|---------|--------|--------|----------|-----------|--------------|
| NBIM RE | 1 | | | | | |
| NBIM E | 0,68 | 1 | | | | |
| NBIM B | 0,58 | 0,56 | 1 | | | |
| MSCI WSI | 0,65 | 0,99 | 0,51 | 1 | | |
| MSCI WREI | 0,58 | 0,77 | 0,72 | 0,76 | 1 | |
| Barclays Agg | 0,44 | 0,35 | 0,95 | 0,31 | 0,60 | 1 |

Figure 8: Correlation matrix; NBIM's unlisted real estate, equity and bonds + indices.

5.6 Descriptive statistics period 30.06.2006 – 31.12.2017

We have included a longer sample period to get an overview of real estate performance in the long term. Table 10 lists the descriptive statistics for the returns during the period 30.06.2006–31.12.2017.

When including the financial crisis, the average returns in all asset classes are lower, and the returns have a higher volatility compared to our sample period between 30.04.2011 - 31.12.017. As expected the downside risk has also increased which is explained by a more negative skewness.

| | MSCI WSI | MSCI WREI | Barclays Agg |
|----------------|----------|-----------|--------------|
| Mean | 6,59 % | 5,25 % | 3,70 % |
| Std. Error | 4,62 % | 6,00 % | 1,66 % |
| Std. Deviation | 15,73 % | 20,41 % | 5,63 % |
| Variance | 2,47 % | 4,16 % | 0,32 % |
| Kurtosis | 3,58 | 5,44 | 0,96 |
| Skewness | -1,14 | -1,28 | -0,24 |

5.6.1 Indices before-under-and after financial crisis

Figure 9: Descriptive statistics; Indices. All data collected 07.03.2018.

In the next table we have summarized the correlation coefficients. The high correlation between unlisted real estate and stocks of 0.88 indicates that there have been low diversification effects by including real estate as an asset class since 2006. Considering that the financial crisis was triggered by a housing bubble this is a possible explanation for the increased correlation.

| | MSCI WSI | MSCI WREI | Barclays Agg |
|--------------|----------|-----------|--------------|
| MSCI WSI | 1 | | |
| MSCI WREI | 0,88 | 1 | |
| Barclays Agg | 0,39 | 0,49 | 1 |

Figure 10: Correlation matrix of indices

5.7 Potential shortcomings

A shortcoming is that we do not have much data. NBIM made its first unlisted real estate investment in 2011, so we can only base our findings on 7 years of time

series data. This is not optimal considering that unlisted real estate investments have a long-term perspective. A critical implication is that we might not capture the liquidity premium in our sample period.

6. Results and Analysis

In the following part we will provide the results of our analysis of whether or not the inclusion of unlisted real estate has improved the risk-return trade-off of the GPFG.

6.1 Fama and French

The regression results of the Fama and French 3-factor model is summarized in the two tables below. The first table shows that the GPFG excluding unlisted real estate has insignificantly underperformed the market factors by -0.086%.

| 3-Factor | Coefficients | Standard Error | t Stat | P-value |
|-----------|--------------|----------------|--------|---------|
| Intercept | -0,00086 | 0,00 | -1,17 | 0,24 |
| Mkt-Rf | 0,76 | 0,02 | 36,60 | 0,00*** |
| SMB | 0,02 | 0,06 | 0,44 | 0,66 |
| HML | -0,04 | 0,04 | -0,88 | 0,38 |

Figure 11: Regression output; Fama and French 3-Factor.. Excluding unlisted real estate **** indicates significance on 1% level *** indicates significance on 5% level

In the second table, we get close to equal results when including unlisted real estate into the portfolio. However, the alpha has been improved, indicating that the inclusion of unlisted real estate has improved the risk-return trade-off of the GPFG.

| 3-Factor | Coefficients | Standard Error | t Stat | P-value |
|-----------|--------------|----------------|--------|---------|
| Intercept | -0,00083 | 0,00 | -1,14 | 0,26 |
| Mkt-Rf | 0,76 | 0,02 | 36,33 | 0,00*** |
| SMB | 0,03 | 0,06 | 0,54 | 0,59 |
| HML | -0,04 | 0,04 | -0,87 | 0,39 |

Figure 12: Regression output; Fama and French 3-Factor. Including unlisted real estate *** indicates significance on 1% level ** indicates significance on 5% level

In the following two tables, we have summarized the regression results when including bonds as a factor. The extended models help us analyze the GPFG's exposure towards both stocks and bonds. The first table shows that the GPFG excluding unlisted real estate has at a 1% significance level underperformed the market factors by -0.11%.

| 3-Factor+Bonds | Coefficients | Standard Error | t Stat | P-value |
|----------------|--------------|----------------|--------|---------|
| Intercept | -0,001098 | 0,00 | -3,17 | 0,00*** |
| Mkt-Rf | 0,70 | 0,01 | 67,22 | 0,00*** |
| SMB | -0,03 | 0,03 | -1,11 | 0,2724 |
| HML | 0,05 | 0,02 | 2,50 | 0,01** |
| Barclays Bonds | 0,46 | 0,03 | 16,33 | 0,00*** |

Figure 13: Regression output; Fama and French 3-Factor + bonds. Excluding unlisted real estate *** indicates significance on 1% level

** indicates significance on 5% level

Again, in the second table, we get close to equal results when including unlisted real estate into the portfolio. The alpha has been improved, which indicates that the inclusion of unlisted real estate has improved the risk-return trade-off of the GPFG.

| 3-Factor+Bonds | Coefficients | Standard Error | t Stat | P-value |
|----------------|--------------|----------------|--------|---------|
| Intercept | -0,001072 | 0,00 | -3,05 | 0,00*** |
| Mkt-Rf | 0,70 | 0,01 | 65,92 | 0,00*** |
| SMB | -0,02 | 0,03 | -0,89 | 0,38 |
| HML | 0,05 | 0,02 | 2,48 | 0,02** |
| Barclays Bonds | 0,46 | 0,03 | 16,08 | 0,00*** |

Figure 14: Regression output; Fama and French 3-Factor + bonds. Including unlisted real estate *** indicates significance on 1% level

** indicates significance on 5% level

From the Fama and French 5- factor model, we get somewhat similar results as the previous regressions, and it does not provide us with any additional information. We have therefore only included the regression output for the 5factor models in the appendix.

From the 3-factor models we can see that the regression alphas are statistically significant on a 5% significance level, when bonds are included in the regression. This means that statistically the fund has underperformed the market factors and bonds. We also found that the inclusion of unlisted real estate has improved the risk-adjusted return of the GPFG. Considering the possible diversification effect as well, the inclusion of unlisted real estate might improve the risk-return trade-off

of a portfolio consisting of stocks and bonds. This will be more closely analyzed in the following parts.

6.2 Performance measures

We have summarized what we believe are the most important performance measures in the table below. When calculating the beta coefficients, we have used the MSCI WSI as the market proxy.

By looking at NBIM's unlisted real estate investments, we have found that it has a standard deviation of 7.27% and a beta of 0.40, implying that it is less volatile than the market index MSCI WREI. A reason for the lower volatility can be that NBIM uses consultants when valuing their real estate investments, which may lead to smoothed valuations. Also, it is difficult to measure the real value until the real estate is sold, and the lower average returns might be because of conservative valuation methods from the consultancy firms.

Since we assume that the GPFG is a well-diversified portfolio with close to zero nonsystematic risk, we have included the Treynor's measure as a part of our analysis. Among the three asset classes, NBIM's unlisted real estate investments have the highest Treynor's measure of 0.10, implying that unlisted real estate has contributed with the highest return relative to the amount of the GPFG's systematic risk.

We also found that NBIM's unlisted real estate has a Sharpe ratio of 0.54, which is higher than the Sharpe ratio of bonds but lower than stocks. This implies that, if we were to choose a single asset portfolio, stocks would give us the best riskreturn trade-off during our sample period.

| | NBIM RE | NBIM E | NBIM B | MSCI WSI | MSCI WREI | Barclays Agg |
|-----------------|---------|---------|--------|----------|-----------|--------------|
| Mean | 4,07 % | 7,85 % | 2,19 % | 9,33 % | 7,92 % | 1,84 % |
| Std. Deviation | 7,27 % | 13,41 % | 5,03 % | 11,74 % | 13,75 % | 4,61 % |
| Beta | 0,40 | 1,13 | 0,22 | 1,00 | 0,89 | 0,12 |
| Sharpe ratio | 0,54 | 0,57 | 0,40 | 0,78 | 0,56 | 0,36 |
| Treynor`s Ratio | 0,10 | 0,07 | 0,09 | 0,09 | 0,09 | 0,14 |

Figure 15: Performance measures of historical returns: NBIM's unlisted real estate, equity and bonds + indices

Given the results from the performance measures, NBIM's unlisted real estate investments looks like an attractive asset class to include in the overall portfolio to improve the risk-return trade-off. The Sharpe ratio is relatively high compared to the other asset classes, and it has a very attractive Treynor's measure.

However, the performance measures on the separate asset classes do not give us any information about whether the inclusion on unlisted real estate has improved the overall portfolios risk-return trade-off. We have therefore compared the GPFG`s Sharpe ratio and Treynor`s measure when both including and excluding unlisted real estate into the portfolio. The results show that the fund achieves a better risk-return trade-off when including unlisted real estate.

| | NBIM excl. RE | NBIM incl. RE |
|--------------------|---------------|---------------|
| Sharpe ratio | 0,597 | 0,599 |
| Treynor `s measure | 0,073 | 0,073 |

Figure 16: Sharpe ratio and Treynor's measure

To further strengthen our analysis we have computed a variance-covariance matrix to conduct a Markowitz mean-variance optimization analysis in the following part.

6.3 Markowitz

We applied the Markowitz mean-variance optimizer to find the optimal portfolio mix for the GPFG between the period 30.04.2011-31.12.2017. Historically, NBIM should have increased their allocation towards unlisted real estate to 49% when maximizing the Sharpe ratio. This implies that unlisted real estate has had a diversification effect on the portfolio.

| Optimal Portfolio | o allocation |
|-------------------|--------------|
| Real Estate | 49 % |
| Equity | 38 % |
| Bonds | 12 % |
| Sum | 100 % |
| | |
| Expected return | 5,29 % |
| Std. Dev | 8,42 % |
| Sharpe ratio | 0,609 |

Figure 17: Markowitz mean-variance optimizer

The results from the optimizer are highly affected by the historically low bond yields and strong performance in stocks and unlisted real estate. In addition, the allocation does not comply with NBIM's investment strategy, which requires moderate risk and a certain allocation towards bonds. We have therefore added a constraint in the mean-variance optimizer of an allocation of 30% towards bonds.

When including the constraint of 30% allocation in bonds, we achieve a mixedasset portfolio with moderate risk. In this case, the mean-variance optimizer suggests an allocation of 37% towards unlisted real estate, which again proves that unlisted real estate has a positive contribution to the GPFG's Sharpe ratio. As expected, after adding the constraint, we can see that there is a small decrease in the Sharpe ratio of -0.003.

| Optimal Portfolio allocation | | | |
|------------------------------|--------|--|--|
| Real Estate | 37 % | | |
| Equity | 33 % | | |
| Bonds | 30 % | | |
| Sum | 100 % | | |
| | | | |
| Expected return | 4,75 % | | |
| Std. Dev | 7,58 % | | |
| Sharpe ratio | 0,606 | | |

Figure 18: Markowitz mean-variance optimizer with constraint

These results show that, according to mean-variance optimization, unlisted real estate has improved the risk-return trade-off of the GPFG. Considering that NBIM's unlisted real estate investments have a zero alpha against stocks and

bonds, a possible explanation to include unlisted real estate is that NBIM's asset allocation of stocks and bonds has not been mean-variance efficient.

6.4 Additional considerations

Our analysis is based on historical returns, and might therefore not be representative for the future. Our main focus has been to analyze the effect on the risk-return trade-off by including unlisted real estate since 2011, and we do not provide a suggestion for future allocation towards unlisted real estate in the GPFG.

We must also consider that we have not included the cost of buying, managing and selling unlisted real estate. This could possibly affect our findings since transaction costs of unlisted real estate are much greater than those of stocks and bonds.

7. Conclusion

This thesis has tested whether the inclusion of unlisted real estate has improved the risk-return trade-off of the Norwegian Pension Fund – Global.

The Fama and French 3 and 5-factor model regression analysis shows that the GPFG has had a higher risk-adjusted return when including unlisted real estate into the portfolio. These results suggest that unlisted real estate could be included into a portfolio consisting of stocks and bonds to improve the risk-return trade-off.

Analyzing the performance measures, we found that unlisted real estate has a relative high Sharpe ratio compared to stocks and bonds, and the highest Treynor's ratio. We also found that the GPFG has achieved a better risk-return trade-off when including unlisted real estate into the portfolio.

When maximizing the GPFG's Sharpe ratio since 2011, we found that the meanvariance optimization suggests a higher allocation towards unlisted real estate, than the current allocation. These results prove that unlisted real estate has improved the risk-return trade-off.

Based on these findings, our conclusion is that the inclusion of unlisted real estate as an asset class has improved the risk-return trade-off of the Government Pension Fund – Global, during the period 30.04.2011-31.12.2017.

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measure-risk-adjusted-returns/

| Regression St | atistics | | | |
|-------------------|--------------|----------------|----------|---------|
| Multiple R | 0,97245962 | | Excludi | ng |
| R Square | 0,9456777 | | real est | tate |
| Adjusted R Square | 0,94356125 | | | |
| Standard Error | 0,00641288 | | | |
| Observations | 81 | | | |
| | | | | |
| 3-Factor | Coefficients | Standard Error | t Stat | P-value |
| Intercept | -0,00086 | 0,00 | -1,17 | 0,24 |
| Mkt-Rf | 0,76 | 0,02 | 36,60 | 0,00*** |
| SMB | 0,02 | 0,06 | 0,44 | 0,66 |
| HML | -0,04 | 0,04 | -0,88 | 0,38 |

9. Appendices

Fama and French 3-Factor model. GPFG excluding unlisted real estate

| Regression St | atistics | | | |
|-------------------|--------------|----------------|----------|---------|
| Multiple R | 0,9720756 | | Includir | ng |
| R Square | 0,94493097 | | real est | tate |
| Adjusted R Square | 0,94278542 | | | |
| Standard Error | 0,00641733 | | | |
| Observations | 81 | _ | | |
| | | | | |
| 3-Factor | Coefficients | Standard Error | t Stat | P-value |
| Intercept | -0,00083 | 0,00 | -1,14 | 0,26 |
| Mkt-Rf | 0,76 | 0,02 | 36,33 | 0,00*** |
| SMB | 0,03 | 0,06 | 0,54 | 0,59 |
| HML | -0,04 | 0,04 | -0,87 | 0,39 |

Fama and French 3-Factor model. GPFG including unlisted real estate

| Regression Statistics | | _ | | |
|------------------------------|--------------|----------------|--------|---------|
| Multiple R | 0,99 | | | |
| R Square | 0,99 | | excl | |
| Adjusted R Square | 0,99 | | | |
| Standard Error | 0,00 | | | |
| Observations | 81 | | | |
| | | | | |
| 3-Factor+Bonds | Coefficients | Standard Error | t Stat | P-value |
| Intercept | -0,001098 | 0,00 | -3,17 | 0,00*** |
| Mkt-Rf | 0,70 | 0,01 | 67,22 | 0,00*** |
| SMB | -0,03 | 0,03 | -1,11 | 0,2724 |
| HML | 0,05 | 0,02 | 2,50 | 0,01** |
| Barclays Bonds | 0,46 | 0,03 | 16,33 | 0,00*** |

Fama and French 3-Factor model + bonds. GPFG excluding unlisted real estate

| Regression Statistics | | | | |
|------------------------------|--------------|----------------|--------|---------|
| Multiple R | 0,99 | | | |
| R Square | 0,99 | | incl | |
| Adjusted R Square | 0,99 | | | |
| Standard Error | 0,00 | | | |
| Observations | 81 | _ | | |
| | | | | |
| 3-Factor+Bonds | Coefficients | Standard Error | t Stat | P-value |
| Intercept | -0,001072 | 0,00 | -3,05 | 0,00*** |
| Mkt-Rf | 0,70 | 0,01 | 65,92 | 0,00*** |
| SMB | -0,02 | 0,03 | -0,89 | 0,38 |
| HML | 0,05 | 0,02 | 2,48 | 0,02** |
| Barclays Bonds | 0,46 | 0,03 | 16,08 | 0,00*** |

Fama and French 3-Factor model + bonds. GPFG including unlisted real estate

| Regression Statistics | | | | |
|------------------------------|----------------------------------|------------------------|--------------------------------|--------------------------------------|
| Multiple R | 0,98 | | | |
| R Square | 0,96 | | excludi | ng |
| Adjusted R Square | 0,95 | | real est | tate |
| Standard Error | 0,01 | | | |
| Observations | 81 | | | |
| | | | | |
| | | | | |
| 5-Factor | Coefficients | Standard Error | t Stat | P-value |
| 5-Factor Intercept | <i>Coefficients</i> | Standard Error 0,00 | | <i>P-value</i> 0,00*** |
| | ,,, | | -2,90 | |
| Intercept | -0,00213 | 0,00 | -2,90 34,59 | 0,00*** |
| Intercept Mkt-Rf | - 0,00213 0,81 | 0,00 0,02 | -2,90 34,59 2,66 | 0,00*** 0,00*** |
| Intercept Mkt-Rf SMB | - 0,00213 0,81 0,16 | 0,00 0,02 0,06 | -2,90 34,59 2,66 1,32 | 0,00*** 0,00*** 0,00*** |

Regression output; Fama and French 5-Factor. Excluding unlisted real estate *** indicates significance on 1% level

** indicates significance on 5% level

| Regression Statistics | | | | |
|----------------------------|----------------------------------|------------------------|--------------------------------|--------------------------------------|
| Multiple R | 0,98 | | | |
| R Square | 0,96 | | Includi | ng |
| Adjusted R Square | 0,95 | | real est | tate |
| Standard Error | 0,01 | | | |
| Observations | 81 | | | |
| | | | | |
| | | | | |
| 5-Factor | Coefficients | Standard Error | t Stat | P-value |
| 5-Factor Intercept | <i>Coefficients</i> | Standard Error 0,00 | | <i>P-value</i> 0,00 *** |
| | 1 | | -2,83 | |
| Intercept | -0,00209 | 0,00 | -2,83 34,21 | 0,00*** |
| Intercept Mkt-Rf | - 0,00209 0,81 | 0,00 0,02 | -2,83 34,21 | 0,00*** 0,00*** 0,00*** |
| Intercept Mkt-Rf SMB | - 0,00209 0,81 0,17 | 0,00 0,02 0,06 | -2,83 34,21 2,71 1,23 | 0,00*** 0,00*** 0,00*** |

Regression output; Fama and French 5-Factor. Including unlisted real estate *** indicates significance on 1% level

** indicates significance on 5% level

| Regression Statistics | | | | |
|------------------------------|--------------|----------------|--------|---------|
| Multiple R | 0,99 | | | |
| R Square | 0,99 | | excl | |
| Adjusted R Square | 0,99 | | | |
| Standard Error | 0,00 | | | |
| Observations | 81 | | | |
| | | | | |
| 5-Factor+Bonds | Coefficients | Standard Error | t Stat | P-value |
| Intercept | -0,000970 | 0,00 | -2,55 | 0,01** |
| Mkt-Rf | 0,69 | 0,01 | 47,82 | 0,00*** |
| SMB | -0,05 | 0,03 | -1,33 | 0,19 |
| HML | 0,10 | 0,03 | 3,22 | 0,00*** |
| RMW | 0,00 | 0,05 | 0,07 | 0,95 |
| СМА | -0,11 | 0,05 | -2,42 | 0,02** |
| Barclays bonds | 0,47 | 0,03 | 14,76 | 0,00*** |

Regression output; Fama and French 5-Factor + bonds. Excluding unlisted real estate *** indicates significance on 1% level

** indicates significance on 5% level

| Regression Statistics | | | | |
|------------------------------|--------------|----------------|--------|---------|
| Multiple R | 0,99 | | | |
| R Square | 0,99 | | incl | |
| Adjusted R Square | 0,99 | | | |
| Standard Error | 0,00 | | | |
| Observations | 81 | | | |
| | | | | |
| 5-Factor+Bonds | Coefficients | Standard Error | t Stat | P-value |
| Intercept | -0,000932 | 0,00 | -2,41 | 0,01** |
| Mkt-Rf | 0,69 | 0,01 | 46,59 | 0,00*** |
| SMB | -0,04 | 0,03 | -1,19 | 0,24 |
| HML | 0,10 | 0,03 | 3,01 | 0,00*** |
| RMW | 0,00 | 0,05 | -0,07 | 0,95 |
| СМА | -0,10 | 0,05 | -2,22 | 0,03** |
| Barclays bonds | 0,47 | 0,03 | 14,48 | 0,00*** |

Regression output; Fama and French 5-Factor + bonds. Including unlisted real estate *** indicates significance on 1% level

** indicates significance on 5% level

| Variance-Covariance matrix | | | | |
|----------------------------|-------------|----------|-------|--------|
| | Real estate | Equity | Bonds | |
| Real Estate | 0,0004 | 0,0006 | | 0,0002 |
| Equity | 0,0006 | 6 0,0015 | | 0,0003 |
| Bonds | 0,0002 | 0,0003 | | 0,0002 |

| Returns | |
|-------------|---------|
| Real Estate | 4,07 % |
| Equity | 7,85 % |
| Bonds | 2,19 % |
| | |
| Rf | 0,160 % |

| Optimal Portfolio allocation | |
|------------------------------|--------|
| Real Estate | 49 % |
| Equity | 38 % |
| Bonds | 12 % |
| Sum | 100 % |
| | |
| Expected return | 5,29 % |
| Std. Dev | 8,42 % |
| Sharpe ratio | 0,609 |

Average returns, variance-covariance matrix and the Markowitz optimizer