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The Low Volatility Anomaly: An empirical study of Oslo Stock Exchange

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The Low Volatility Anomaly: An empirical study of Oslo Stock Exchange

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Abstract

In this paper we study the relation between the cross-sections of idiosyncratic volatility and returns on the Norwegian stock market. Our timeframe ranges from 1980 through 2015 and all companies registered on the Oslo Stock Exchange in this period is included in our study.

The methodology used in this thesis is inspired by Ang et al. (2006), Bali and Cakici (2008) and Baker and Haugen (2012). The methodology consists of sorting stocks into quintile portfolios based on their lagged level of idiosyncratic volatility. The portfolios follow the L/M/N strategy outlined by Ang et al. (2006).

In addition, we run robustness analysis where we test our analysis on several factors in order to gather information as to what affects our results, and if our results hold regardless of these factors.

We found that stocks with high idiosyncratic volatility performs poorly relative to stock with low idiosyncratic volatility. In addition, we find that this performance can be explained by return reversals, where high volatility portfolios return tends to have a reversal after a period of high fluctuations. We also find that large differences in size have an effect on the results.

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1. Introduction

In this thesis, we study the low volatility anomaly on the Norwegian market, and test for possible explanations for the anomaly by conducting a comprehensive study. We follow the methodology outlined by Ang et al. (2006). In addition, we perform robustness tests to account for some of the criticism the article has met. The analysis is conducted on value- and equal weighted portfolios with both daily- and monthly data frequency. We use the same portfolio formation strategy as Ang et al. (2006) and begin by measuring idiosyncratic volatility using a rolling window on daily data. Furthermore, we sort the portfolios on volatility into quintile portfolios, where the portfolios are rebalanced every month.

In finance, the assumption of a positive relation between risk and expected return is widely accepted. In 1952 Harry Markowitz introduced his paper on Portfolio Selection, which lay the foundation for portfolio management. Based on his work, the Capital Asset Pricing Model (CAPM) was introduced independently by Sharpe (1964), Lintner (1965) and Mossin (1966). The CAPM predicts that the expected return increases by risk, and thus, all investors should invest in the security that gives the highest expected return relative to its risk. A commonly used measure of returns, given the exposure of risk, is the Sharpe ratio (Sharpe, 1966).

Although the theory of CAPM and the work of Markowitz (1952) have been commonly accepted in the world of finance, recent studies have found empirical evidence for the positive relation not to hold. Ang et al. (2006) studies the relationship between the cross-section of volatility and expected returns, and find that US firms with high idiosyncratic volatility, defined relative to the Fama French (1993) model, tend to perform poorly in comparison to low volatility securities. Extending their research, Ang et al. (2009) find that the same relation persists on the international market. Ang et al. (2006) paper has been subject to criticism as well. Researchers such as Bali and Cakici (2008) and Fu (2009) find that the results of Ang et al. (2006) are only due to small illiquid firms. In addition, Bali and Cakici (2008) criticizes the use of daily data as they find monthly return data to provide a more accurate proxy for the expected future volatility than daily data.

Although the international-and the US market has been extensively researched, there exists few empirical studies on this subject for the Norwegian market. Given the increase of focus on the matter and the lack of empirical research on the Norwegian market, we replicate Ang et al. (2006) results on the Oslo Stock Exchange.

In addition to Ang et al. (2006) use of daily data, we expand our study by including the use of monthly data. This expansion is supported by Bali and Cakici (2008) who discovered that the anomaly disappeared when using a monthly data frequency. Our results reveal, regardless of volatility measure and data frequency, that the low volatility anomaly persists in the Norwegian market for value weighted portfolios. The anomaly is also found for equal weighted portfolios on monthly measures of volatility.

We investigate for possible explanations for the low volatility anomaly on the Norwegian market. Amongst possible theories, we study size, time, liquidity and momentum factors. Our findings show that the low volatility anomaly seems to be driven by differences in market capitalizations, liquidity and return reversals.

We begin with the literature review, where we try to gather a sound picture of the current studies and literature for the anomaly. The structure of the thesis following the literature review will be: Explanation of our dataset in chapter 3, the methodology used in chapter 4, results in chapter 5 and we end with our conclusion in chapter 6.

2. Literature Review

In recent years, several researchers have challenged the traditional financial theory of CAPM, where the expected return should reflect the relative risk one undertakes. One of these is the popular paper by Ang et al. (2006), who find a negative relation between the cross-sections of idiosyncratic volatility and expected return. However, other researchers such as Malkiel and Xu (2002) and Bali and Cakici (2008) find the opposite result using different methodologies. The purpose of this literature review is to shed light on the most important findings in regard to this topic. The literature review is structured as following: First we discuss some of the research conducted on the low volatility anomaly, second, we look closer at some of the methodical differences used when examining the anomalous relation. Further, we will look at some of the explanations in regard to the presence of the anomaly.

2.1 The Low Volatility Anomaly

Ang et al. (2006) examines the cross-sectional relationship between volatility and expected returns in the U.S stock market from 1963 to 2000, where the idiosyncratic volatility is defined relative to the standard Fama French (1993) three-factor model. Ang et al. (2006) discover that stocks with high idiosyncratic volatility have abnormal low average returns, and that there exists a strong significant difference of -1.06% per month between the highest and lowest value weighted quintile portfolio. Additionally, Ang et al. (2006) discover that the Fama French (1993) three-factor model is unable to price these portfolios as the difference in Fama-French alphas between the portfolios is -1.31% per month and statistically significant.

Ang et al. (2006) looks at the stocks properties in the quintile portfolios for an explanation for the anomalously low returns the high volatility portfolio attracts. Although the high volatility portfolio contains 20% of the securities, it has only 1.9% of the total market value, which is a very small proportion of the total market. Ang et al. (2006) finds that the stocks with high (low) volatility, are generally small (large) stocks with high (low) book-to-market ratios. They control for a large number of cross-sectional effects such as size, volume, liquidity, turnover, leverage, coskewness, bid-ask spreads, momentum effect, book-to-market, and dispersion in analysis' forecasts, but none can explain the anomalously low returns.

Extending their research, Ang et al. (2009) conduct a more detailed analysis of the U.S market and examine if the anomalous relation can be identified in the international markets. The study is executed to uncover if the discovered anomaly could have some underlying economic source, or if the anomaly is just due to a small data sample. With the detailed analysis, Ang et al. (2009) is able rule out market frictions, information dissemination, and option pricing as reasons for the discovered relation between high idiosyncratic volatility and low average returns in the U.S market. They also discovered that the same negative relation between idiosyncratic volatility and future average returns observed in the U.S market, could be observed across a large sample of international developed markets. However, Ang et al. (2009) does not report the results from the Norwegian market.

Although additional researchers such as Boyer et al. (2010), Chen and Petkova. (2012) and Stambaugh et al. (2015) find a similar anomalous relation as Ang et al. (2006), Malkiel and Xu (2002) finds a significant positive relation between idiosyncratic risk and the cross section of expected return at the firm level. However, Malkiel and Xu (2002) use the idiosyncratic volatility of one of the 200 beta/size portfolios to which that stock is held by each month, and their results are therefore not based on the individual stocks idiosyncratic volatility as that of Ang et al. (2006).

Bali and Cakici (2008) also examines the relation between the cross-section of idiosyncratic volatility and expected stock returns. In addition to further examine the existence and significance of the relation between idiosyncratic risk and expected return, Bali and Cakici (2008) examines the methodological differences between some of the previous studies. They identify several factors such as data frequency used to estimate the idiosyncratic volatility, weighting scheme and the exclusion process, as factors that could contribute to difference in empirical results. When controlling for the identified factors, Bali and Cakici (2008) could not find a robust evidence for the negative relation between risk and return.

In more recent studies Fu (2009) argues that the idiosyncratic volatility is time-varying and, in order to capture this property, that one should use the exponential

GARCH model and out-of-sample data to estimate the expected idiosyncratic volatility. Using the EGARCH, Fu (2009) uncovers a strong positive relation between conditional idiosyncratic volatility and expected returns. Fu (2009) argues that Ang et al. (2006) results could be explained by the return reversal of a subset of small stocks with high idiosyncratic volatility. However, Guo et al. (2014) reviles that Fu (2009) results are unreliable due to the look-ahead bias created by Fu (2009) EGARCH idiosyncratic volatility methodology.

2.2 Volatility Measure

Volatility cannot be observed, and thus has to be measured. The total volatility is the securities combined idiosyncratic and systematic volatility. Due to the property of the total volatility, securities with high idiosyncratic volatility tend to have high total volatility and visa verse. Baker and Haugen (2012) sort securities into quintile portfolios according to the estimated total volatility for each security over the previous 24 months. They find evidence for a negative relation between taking relative risk and the expected return. The paper is of particular interest as the study is conducted on 21 developed countries, including the Norwegian market, and 12 emerging markets. This makes, to our knowledge, Baker and Haugen (2012) the only researchers who explicitly reports their results of a similar anomaly to that of Ang et al (2009), for the Norwegian market.

2.3 Explanations for the Low Volatility Anomaly

As the anomaly discovered by Ang et al. (2006) is somewhat of a puzzle, there has been devoted several research papers to identify the reason behind this negative cross-sectional relation between idiosyncratic volatility and expected returns.

2.3.1 Data Frequency in Volatility Measure

Ang et al. (2006) estimates the idiosyncratic volatility using daily data. In the paper by Bali and Cakici (2008) the researchers test for the effect different data frequency used to estimate idiosyncratic volatility, with respect to the Fama French (1993) three-factor model, could have. Bali and Cakici (2008) find that a wide variety of statistical tests support the use of monthly return data when estimating idiosyncratic volatility, as it proves to be a more accurate proxy for expected future volatility. When sorting stocks by idiosyncratic volatility, measured on monthly data, they find a flat or sometimes positive, but insignificant, cross-sectional relation between the idiosyncratic volatility and expected return. Thus, according to Bali and Cakici (2008), the idiosyncratic anomaly found by Ang et al. (2006) could be subjected to microstructure problems, such as the bid-ask bounce, as they use daily return data to estimate the idiosyncratic volatility.

2.3.2 Arbitrage Asymmetry

Miller (1997) finds that when short sale is restricted, the spread between the security valuation is likely to increase with risk, and that the security price tends to reflect the optimistic valuation. This optimistic valuation lead to the possibility of experiencing lower expected return on risky securities. Stambaugh et al. (2015) looks closer at the relation between arbitrage risk, which is represented by idiosyncratic volatility, and arbitrage asymmetry to answer Ang et al. (2006) idiosyncratic volatility anomaly. The arbitrage asymmetry is defined by Stambaugh et al. (2015) as the investors incapability, or reluctance to short a security they see as overpriced but would buy if they see it as underpriced. They find that securities with high (low) idiosyncratic volatility, have a higher degree of overpricing (underpricing). Combining their results with the arbitrage asymmetry they find that the negative idiosyncratic effect for overpricing is stronger than the positive idiosyncratic effect created by underpricing. Thus, Stambaugh et al. (2015) argues that the discovered anomaly is due to the combination of mispricing and arbitrage asymmetry in the market.

3. Data

The data used in our analysis is gathered from the Oslo Børs Information (OBI), obtained through Bernt Arne Ødegaards website and the BI library. The data used reaches from the period of 1981 to 2015. In this section we go through all the data gathered and used, in our analysis. All data has been gathered for daily and monthly frequency, unless otherwise stated.

3.1 Return

Ødegaard (2017) describes return measure as following: "Returns are generated using the following algorithm for calculating the price: If close (trade) price is available, use that. Otherwise, if both bid and ask(offer) is available, use the average. If only bid or ask is available, use that." (Ødegaard, 2017). In periods where a security has not been traded, the security is dropped. The returns are raw returns adjusted for dividends and other corporate events such as stock dividends, stock splits and such (Ødegaard, 2017). The returns are not annualized, and is measured as

$$\mathrm{R_i} = \frac{\mathrm{P_i(t+1) - P_i(t)}}{\mathrm{P_i(t)}}$$

Formula 1: Stock Returns

Where P(t) is the price at time t, and P(t+1) is the price at time t+1.

We choose to trim our dataset to remove any extreme return outliers. As we do not wish to cut too large parts of our data, we set the trimming at the 0.01^{st} percentile, and the 99.9^{th} percentile. We find that setting these levels remove the most extreme returns.

In addition, we choose to add filters to remove securities which might skew our results. Adding filters to data is a very common method used to remove outliers, see Fama French (1992) for example. Ødegaard (2017) recommends a set of filters to use. We take inspiration from these filters, however, we find some too restrictive for our use. Ødegaard (2017) recommends removing stocks who are seldom traded, and thus imposing a requirement of at least 20 trading days in a year. We agree with this requirement, as seldom traded securities may skew our volatility measures. Thus, if a security has been traded less than 20 days in a year, we drop the given security for the given year. Another requirement is set in regard to the price of a security. Ødegaard recommend dropping securities whose stock value fall below NOK 10 in a given year, to get rid of so called “penny-stocks”. However, removing these stocks would remove 37% of our dataset, and thus remove more than merely outliers. After some testing we deemed NOK 1 to be fitting as it will only remove 6.7% of our dataset. Lastly, we remove low valued stocks, per Ødegaards recommendation. Ødegaard recommends removing securities whose value drop below NOK 1 million any given year.

3.2 Price Factors

The price factors are gathered through Ødegaards website as well. In total there are five pricing factors, three of whom are the Norwegian replication of Fama French (1998), called SMB, HML and UMD. In addition, we gather the market returns, which is needed to perform the Fama French (1993) 3-factor regression. Furthermore, we have the liquidity factor made by Naes et al. (2009), named LIQ.

3.3 Risk Free Interest Rate

In order to calculate excess returns we must include the risk-free interest rate to our dataset. We use the risk-free rate found on Ødegaards website. Before 1982 the interest rate used is the shortest possible bond yield for treasuries (Ødegaard, 2017). From 1982-1986 we have to use some imperfect proxies for the overnight NIBOR rate as the period was somewhat “messy” in regard to interest rates (Ødegaard, 2017). From 1986 the interest rate used is the interbank rate, NIBOR, monthly.

4. Methodology

In this chapter we go through the methodology used in order to uncover the low volatility anomaly on the Norwegian market and the factors behind the anomaly. The methodology is based and inspired by Ang et al. (2006), Bali & Caciki (2008), and Baker & Haugen (2012). We begin by defining the various volatility measures used in our analysis, then describe how we measure volatility using portfolios. Next, we discuss how we evaluate the portfolios, and lastly, we go into how we test for our results.

4.1 Defining Volatility

We focus solely on the cross-section of volatility on Oslo Stock Exchange, and thus our volatility measures will be measured on a firm level. We do this as it facilitates controlling for firm specific features.

We choose to primarily focus on two measures of volatility, the idiosyncratic- and the total volatility. The measure for idiosyncratic volatility is gathered from Ang et al. (2006) and is measured relative to the Fama French (1993) three-factor model:

$$r_{i,t} - r_{ft} = \alpha_i + \beta^i_{MKT} MKT_t + \beta^i_{SMB} SMB_t + \beta^i_{HML} HML_t + \varepsilon_{i,t}$$

Formula 2: Fama French (1993) 3-factor model.

Where $r_{i,t}$ is the return of firm i in month t, r_{ft} is the risk-free interest rate, MKT_t is the excess market return, SMB_t and HML_t is the Fama French (1993) small-minus-big and high-minus-low factor respectively. $\varepsilon_{i,t}$ is the error term of the regression, and what we use to define idiosyncratic volatility. To estimate the volatility, we use a rolling window regression, measuring for L months. The application of the volatility measurements will be extensively discussed in the following subchapters.

Idiosyncratic volatility is defined as the standard deviation of the residual of the regression described above, and thus the following is the measurement of idiosyncratic volatility:

$$IVOL_{t+1} = \sqrt{\sum_{i=1}^{N-1} \frac{\varepsilon_{i,t} - \overline{\varepsilon_{\imath,t}}}{n-1}}$$

Formula 3: Idiosyncratic Volatility measured relative to Fama French (1993) 3-factor model.

Where $\overline{\varepsilon_{i,t}}$ is the average of the error term for the given estimation period, n is number of observations in the given estimations period and $\varepsilon_{i,t}$ is the daily error terms.

Another measurement we choose to focus on is the total volatility, also used by Ang et al. (2006) and Baker and Haugen (2012). Total volatility is defined as the standard deviation of a firm's excess return for a given time period. We use a rolling window when computing total volatility as it corresponds to our portfolio strategy, described in the following subchapter.

$$\mathrm{TVOL} = \sqrt{\sum_{i=1}^{N-1} \frac{R_i - \overline{R_i}}{n-1}}$$

Formula 4: Total Volatility

Where R_i is the excess return of stock i, n is the number of observations and $\overline{R_i}$ is the average excess return of stock i for a given time period. When run on daily data, R_i is the daily returns. For monthly data, R_i, is measured monthly

We apply the measurement of idiosyncratic- and total volatility to both daily- and monthly data using a window of L = 1 months and L = 24 months, respectively. The first window (L = 1) corresponds to the method of Ang et al. (2006), and L = 24 months corresponds to that of Baker and Haugen (2012) and Bali and Cakici (2008).

4.2 Constructing Portfolios

We begin by estimating the idiosyncratic volatility for L months using rolling regressions on the cross-sectional returns. The securities are then sorted into quintile portfolios based on their level of volatility. Then we set a waiting period of M months before we begin investing. After the waiting period we invest and hold the investments for N months. The strategy is called the $L/M/N$ strategy, which is equivalent to the strategy that Ang et al. (2006) uses.

Ang et al. (2006) use of daily data has met some criticism, most notably by Bali and Caciki (2008). Bali and Caciki (2008) find that there is strong evidence for the use of monthly data as it provides a more accurate proxy for the expected future volatility than daily data. We therefore, in addition to the 1/0/1 strategy, apply a 24/0/1 strategy using monthly data. That is, for daily data, the estimation- and holding period is set to one month each, and the portfolios are rebalanced monthly, whereas the estimation period is 24 months for monthly data.

We construct five portfolios based on this strategy. The securities are sorted based on their volatility, into quintiles. Portfolio 1 (P1) is the portfolio with the lowest volatility securities, and Portfolio 5 (P5) is the portfolio with highest volatility securities. We also construct a portfolio called P1-5, which corresponds to a long position in portfolio P1 and short in portfolio P5. This portfolio will be able to tell us whether we are experiencing the anomaly when evaluating the portfolios.

4.3 Evaluating Portfolios

The portfolios are evaluated by looking at the portfolios monthly average total returns and their corresponding standard deviation. In addition, we use the Sharpe Ratio to sort the portfolios based on their risk adjusted performance. The monthly excess returns for the portfolios are also regressed according to the Fama French (1993) three-factor model. The alphas for the given portfolios along with the coefficient of the independent variables and t statistics are reported. The alphas will indicate whether an investor could gain additional value by constructing these portfolios. The interpretation of our results will therefore rely on the differences in returns, sharpe ratios and alphas, and the significance of returns and alphas.

4.4 Portfolio Properties

We regress our portfolios excess return against a five-factor model. This is done to evaluate whether the different portfolio returns are due to specific factors. The five-factor regression include the market excess return, a size, book-to-market, momentum and illiquidity factor.

4.5 Filtering

We will subject our analysis to a set of filters to explore how filters affect our results and the sensitivity of our results. The filters used in our thesis are inspired by Ødegaard (2017). Previous studies such as Bali and Cakici (2008) found that the anomaly could be explained by small illiquid firms. Thus, we expect that any evidence for the anomaly will perish as the filters becomes more restrictive.

We subject our analysis to five set of filters. Our main results include the base filters. When testing for liquidity, we change the required number of trading days from 20 days in our base filter, to a quarter of a year[1]. To test for size, we use the filter of Ang et al. (2009), which is eliminating firms with the 5% lowest capitalization, for the entire period. This is done to exclude firms with very small market capitalization to test whether the results are due to smaller firms. Next, we test the exact filters Ødegaards (2017), which we described in chapter 3, i.e. increasing the minimum price of a stock from NOK 1 in the base filter to NOK 10 within a year.

Our last filtering will be somewhat restrictive where we apply all the mentioned filters at once. We do realize this will exclude large parts of our dataset, but we find it interesting to see if a mix of increased requirements on value, price and liquidity will yield a different result.

We also perform our analysis on subsamples, dividing our time period into before and after the boom of internet and technology, thus our first subsample is from 1980-1999 and the second from 2000-2015. The purpose of this is to investigate whether there is a time factor or a singular event affecting our results.

4.6 Double Sorting

As previously discussed, we perform a five-factor regression to uncover whether the portfolios indicate different characteristics. In addition to this regression we perform a double sorting similar to the sorting methods used by Ang et al. (2006), using market capitalization.

In order to double sort, the securities are first sorted into quintiles based on one of the firm factors, and within each quintile we sort the portfolios into quintiles based on the volatility measure. This allows us to investigate whether the low volatility exists on subsets of securities based on their firm specific features. We report within

[1] 62.5 days, assuming a trading year is 250 days.

each factor quintile, the portfolio containing the lowest volatility securities as well as the highest volatility securities. The double sorted portfolios are subscripted with "C" for conditional.

In addition to the double sorting, we present a UCMC (Unconditional minus conditional) portfolio for the P1-5 portfolio, in a similar fashion as done by Boyer et al. (2010). The UCMC allow us to test statistically whether our results are driven by firm specific factors. The factor we test is the size factor. The UCMC portfolio is constructed by doing a double sort and averaging the P1-5 portfolio across the firm factors. From there, we take a long position in the unconditional portfolio P1-5, and short position in the conditional portfolio P1-5^{C}.

4.7 Recursive Volatility

The strategy outlined in chapter 4.2 uses rolling window regression, where the estimation period of volatility is set to a defined time period. We suspect that by using this strategy, we might sort a security into a quintile based on short-term events, and not on the actual riskiness of the security. Thus, we perform a recursive regression where the starting point of estimation remains the same throughout the time period, i.e. extending the estimation period every month.

4.8 Return Reversals

Fu (2009), found evidence for high volatility securities being subjected to return reversals. Using simple measures for volatility, such as idiosyncratic- and total volatility, will not uncover whether the results could be explained by these reversals. As we do not go into the sophisticated volatility measure that is EGARCH, we control for this factor by adding a "look-ahead" bias to our portfolio construction, similar to what Fu (2009) was criticized for by Guo et al. (2014). Thus, instead of investing one month ahead, we invest in the same month as the volatility is measured to test for reversals[2].

Testing for return reversals can show whether the anomaly is driven by our strategy, i.e. the estimation and holding period, or whether the anomaly is independent of strategies.

[2] This strategy, for obvious reasons, is not a tradable strategy.

5. Results

We will in this chapter use the methodology outlined in chapter 4 and present the results accordingly. We begin by replicating Ang et al. (2006) main results using daily estimates of idiosyncratic- and total volatility. Further, due to the concerns raised by Bali and Cakici (2008) regarding the data frequency, we explore if the use of monthly data changes our results. In the following subchapters we will test the robustness of our results as well as find possible explanations for our results.

5.1 Daily Estimates of Volatility

In this section we replicate Ang et al. (2006) main results using the same strategy and measures on volatility. Our findings give evidence in support of the low volatility anomaly.

Table 1 presents the portfolio performance for portfolios sorted on idiosyncratic volatility estimated using daily data. Panel A consists of value weighted portfolios whereas panel B consist of equal weighted portfolios.

Table 1
Portfolios sorted on Idiosyncratic Volatility using daily data

Panel A: Value Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.18***	6.14	0.10	1.09	0.903***	-0.146***	0.067**	-0.005***
	[3.89]				[36.592]	[-5.761]	[2.254]	[-6.264]
P2	1.15***	6.83	0.08	1.72	0.989***	-0.014	-0.005	-0.007***
	[3.40]				[29.461]	[-0.292]	[-0.146]	[-4.422]
P3	1.19***	7.68	0.08	2.33	1.139***	0.236***	0.005	-0.011***
	[3.12]				[25.425]	[3.934]	[0.109]	[-5.614]
P4	0.72	8.86	0.02	3.28	1.232***	0.414***	-0.092	-0.018***
	[1.64]				[20.515]	[5.465]	[-1.340]	[-6.648]
P5	0.60	9.43	0.00	6.14	1.218***	0.530***	-0.037	-0.020***
	[1.29]				[18.063]	[5.563]	[-0.407]	[-6.138]
P1-5	0.58	7.56	0.07		-0.315***	-0.676***	0.103	0.015***
	[1.55]				[-4.144]	[-6.503]	[1.071]	[4.315]
Panel B: Equal Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.26***	4.82	0.14	1.09	0.744***	0.152***	0.078***	-0.005***
	[5.30]				[33.117]	[4.553]	[2.862]	[-4.843]
P2	1.43***	5.97	0.14	1.72	0.928***	0.249***	0.03	-0.006***
	[4.87]				[31.257]	[6.022]	[1.052]	[-4.294]
P3	1.41***	6.69	0.12	2.33	1.062***	0.474***	0.038	-0.010***
	[4.27]				[32.973]	[10.403]	[1.123]	[-6.444]
P4	1.28***	6.96	0.10	3.28	1.058***	0.582***	-0.004	-0.012***

	[3.71]				[26.908]	[10.469]	[-0.079]	[-6.611]
P5	1.71***	8.07	0.14	6.14	1.055***	0.685***	0.048	-0.008***
	[4.29]				[15.949]	[6.471]	[0.589]	[-3.229]
P1-5	-0.45	6.09	-0.08		-0.311***	-0.533***	0.03	0.004
	[-1.50]				[-4.559]	[-5.127]	[0.349]	[1.449]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Even though there is no clear pattern across portfolio returns, our findings show that for value weighted portfolios, generally, return decreases by volatility, from 1.18% on portfolio P1 in panel A, to 0.6% in portfolio P5. We also find that the returns of portfolio P4, P5 and P1-5[3] are not statistically significantly different from zero, and thus, we see that there is an anomaly in regard to returns, although the return of portfolio P1-5 are insignificant.

On equal weighted portfolios however, we find the opposite relation, where returns tend to increase by volatility, from 1.26% in portfolio P1 to 1.71% in portfolio P5. Additionally, all returns are highly significant except for portfolio P1-5. Therefore, by solely looking at return there is no evidence of the low volatility anomaly.

Looking at the factor loading on SMB relative to the Fama French (1993) 3-factor model, we believe the different results, given the different weighting scheme, might be explained by the companies which the portfolios contain. From the factor loading on SMB, it would seem that there is a large difference in the market capitalization between the different portfolios and that the market capitalization rapidly decreases by volatility. Thus, when using equal weights, the companies with small market capitalization receives a higher weighting than the value weighted portfolios yield. Therefore, due to variations in market capitalization, to compare the equal weighted portfolios may give false conclusions as they do not reflect the real return of the securities within their portfolio.

[3] Portfolio P1-5 consist of going long in portfolio P1 and short in portfolio P5.

When looking at the Sharpe Ratio in Table 1 we find that in panel A the Sharpe ratio rapidly decreases with volatility, which is clear evidence of the low volatility anomaly. Using equal weighted portfolios, we find that the Sharpe ratio decreases from P1 to P4, but that portfolio P5 has a Sharpe Ratio equal to that of portfolio P1.

We find the alphas of our regressions to be highly significant across almost all portfolios and decreasing by volatility. On value weighted portfolios, as shown in panel A, portfolio P1 has an alpha of -0.005, whereas portfolio P5 is -0.02. More interestingly, the alpha of portfolio P1-5 is significant and positive at 0.015. The same is true for equal weighted portfolios, although with a smaller spread, where P1 has an alpha of -0.005, P5 has one of -0.008 while portfolio P1-5 has an alpha of 0.004, although insignificant.

Our results of Table 1 provide some evidence for the low volatility anomaly. We see that the patterns of returns across portfolios are somewhat unclear, and the return of portfolio P1-5 are not significant for value- or equal weighted portfolios. However, we do find clear results that the alphas move monotonously negative relative to volatility, and thus support evidence for the low volatility anomaly.

The results in Table 1 shows some consistency with Ang et al. (2006) main results. Although our findings are not as strong as Ang et al (2006) when looking at the portfolio returns, we do have a positive, highly significant, alpha for the P1-5 portfolio. Additionally, the portfolio properties seem to follow the same characteristics as that of Ang et al (2006) such as the rapidly decrease in the SMB factor loading with respect to the portfolios volatility.

Table 2
Portfolios sorted on Total Volatility using daily data.

Panel A: Value Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	TVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.27***	5.90	0.12	1.35	0.858***	-0.014	0.086***	-0.005***
	[4.40]				[33.264]	[-0.358]	[2.645]	[-3.949]
P2	1.26***	6.69	0.10	2.08	0.966***	-0.031	0.004	-0.006***
	[3.83]				[29.292]	[-0.669]	[0.103]	[-3.917]
P3	1.11***	7.47	0.07	2.77	1.074***	0.134**	0.013	-0.010***
	[3.03]				[27.464]	[2.244]	[0.248]	[-5.176]
P4	0.77*	8.95	0.02	3.79	1.291***	0.304***	-0.091	-0.018***
	[1.76]				[26.475]	[4.286]	[-1.612]	[-7.001]

P5	1.05**	10.75	0.04	6.95	1.290***	0.525***	-0.016	-0.018***
	[1.99]				[13.460]	[3.775]	[-0.135]	[-5.240]
P1-5	0.22	9.13	0.02		-0.432***	-0.539***	0.102	0.013***
	[0.49]				[-3.995]	[-3.470]	[0.814]	[3.463]

Panel B: Equal Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	TVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.26***	4.57	0.15	1.35	0.691***	0.199***	0.081***	-0.005***
	[5.61]				[30.521]	[5.430]	[2.809]	[-4.504]
P2	1.60***	5.93	0.17	2.08	0.930***	0.299***	0.047*	-0.005***
	[5.50]				[37.478]	[9.279]	[1.946]	[-4.291]
P3	1.33***	6.52	0.11	2.77	1.020***	0.366***	0.022	-0.010***
	[4.17]				[32.774]	[9.840]	[0.646]	[-6.590]
P4	1.36***	7.29	0.11	3.79	1.122***	0.581***	0.004	-0.012***
	[3.81]				[27.541]	[9.212]	[0.099]	[-6.510]
P5	1.85***	8.45	0.15	6.95	1.078***	0.690***	0.029	-0.009***
	[4.44]				[15.255]	[6.142]	[0.342]	[-3.304]
P1-5	-0.59*	6.43	-0.09		-0.387***	-0.491***	0.052	0.004
	[-1.86]				[-5.394]	[-4.514]	[0.590]	[1.486]

Panel A show value weighted portfolios sorted on total volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and TVOL are reported in percentages. TVOL is the estimated total volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Table 2 provides similar results when portfolios are sorted on total volatility. In panel A we find that low volatility portfolios perform better than high volatility portfolios, where returns move from 1.27% in P1 to 1.05% in P5. However, using total volatility we now see that portfolio P4 and P5 are significant on the 10% and 5% level, respectively. Moving to panel B we find the same patterns here as in Table 1. The returns of portfolio P1 through P5 are all highly significant, where P1 has an average return of 1.26% and P5 has an average return of 1.85%. while P1-5 has significant negative returns on the 10% level.

Looking at the alphas in Table 2 we find that they are all highly significant in panel A, where they decrease by volatility, and P1-5 is positive at 0.013. In panel B, the alpha of portfolio P1 is -0.005 and of portfolio P5 is -0.009, and we thus see a decrease by volatility here as well. However, the alpha of the equal weighted portfolio P1-5 is still insignificant.

From the factor loadings relative to the Fama French (1993) 3-factor model, we find some interesting results. As previously discussed we see a clear pattern in the SMB factor loading in Table 1 and Table 2, which indicates that the high volatility portfolios generally include smaller firms than the low volatility portfolios. We also see that the market exposure increases by volatility, which is as expected due the nature of their riskiness. Although the HML factors are mostly insignificant, we do see some pattern from the low volatility portfolios to the high volatility portfolios. We see that the HML factor is significant, and positive for the low volatility portfolios, and that the factor decreases by volatility, although mainly insignificant. This could indicate that the low volatility portfolios are generally underpriced whereas the high volatility portfolios are overpriced.

From Table 1 and Table 2, we find that regardless of whether portfolios are sorted on idiosyncratic- or total volatility, we find evidence in support of the low volatility anomaly using value weighted portfolios. Furthermore, when using equal weighted portfolios, the presence of the anomaly is unclear. Although the difference in alphas from portfolio P1 to P5 grows negatively, the alpha of portfolio P1-5 is insignificant. In addition, we see that the returns increase by volatility. Using idiosyncratic volatility, both return and alpha is insignificant for portfolio P1-5. However, when sorted on total volatility, the return of portfolio P1-5 is significant at the 10% level. Thus, we cannot conclude that there is evidence in favor of the anomaly using equal weighted portfolios

Table 3
Firm Characteristics

	Panel A: Idiosyncratic Volatility			Panel B: Total Volatility		
	Market Share	Market Cap	Trading days	Market Share	Market Cap	Trading days
P1	0.56	40	195	0.40	27	184
P2	0.21	14	195	0.28	21	193
P3	0.13	8	185	0.17	12	186
P4	0.07	5	163	0.10	6	166
P5	0.04	2	124	0.05	3	126

Panel A and B display the monthly average market share and market cap of each individual portfolio sorted on idiosyncratic and total volatility, respectively. Trading days refer to monthly average annualized trading days.

In Table 3 we report the firm features of our portfolios and find the market capitalization to decrease rapidly relative to volatility. This is consistent with our discussion regarding the SMB factor loadings. We see clear patterns in firm features, where the low volatility portfolios have much higher market capitalization and have a large proportion of the total market share. This is similar to Ang et al. (2006) results, where portfolio 5 only held 1.9% of the total market value, on average. Also, when interpreting the average number of trading days, we find indications for a higher illiquidity in the high volatility portfolios than the low volatility portfolios. We will discuss these results more thoroughly in the following subchapters. Due to the marginal differences in results using idiosyncratic- and total volatility, we decide to move on using only idiosyncratic volatility for the rest of our analysis.

5.2 Monthly Returns

Following the discussion of Bali and Cakici (2008), we provide an analysis using monthly frequency of returns instead of daily. Table 4 show our findings of this analysis. The strategy used is the 24/0/1 strategy.

Table 4
Portfolios sorted on Idiosyncratic Volatility using monthly data.

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
Panel A: Value Weighted Portfolios								
P1	1.11***	5.84	0.09	4.60	0.833***	-0.117***	0.046	-0.005***
	[3.85]				[29.225]	[-3.714]	[1.560]	[-4.552]
P2	1.28***	7.06	0.10	6.65	1.036***	0.040	0.067	-0.008***
	[3.65]				[27.734]	[0.906]	[1.467]	[-4.665]
P3	0.87**	7.53	0.04	8.56	1.068***	0.088	-0.033	-0.012***
	[2.33]				[26.892]	[1.639]	[-0.634]	[-5.704]
P4	1.35***	9.11	0.09	11.15	1.289***	0.220***	-0.008	-0.012***
	[2.99]				[21.052]	[3.009]	[-0.130]	[-4.418]
P5	0.59	10.05	0.00	17.38	1.412***	0.469***	-0.110	-0.023***
	[1.80]				[19.626]	[5.438]	[-1.383]	[-7.206]
P1-5	0.53	7.59	0.07		-0.579***	-0.587***	0.156*	0.017***
	[0.12]				[-7.148]	[-6.168]	[1.827]	[4.896]
Panel B: Equal Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.10***	4.54	0.12	4.60	0.691***	0.199***	0.064**	-0.006***
	[4.91]				[27.174]	[5.677]	[2.397]	[-5.548]
P2	1.11***	5.60	0.10	6.65	0.875***	0.315***	0.102***	-0.010***
	[4.00]				[29.936]	[7.949]	[2.939]	[-7.382]
P3	1.13***	6.30	0.09	8.56	0.976***	0.367***	0.025	-0.011***
	[3.60]				[32.106]	[8.703]	[0.623]	[-6.767]
P4	1.59***	7.39	0.14	11.15	1.158***	0.562***	0.001	-0.010***
	[4.33]				[31.454]	[11.154]	[0.022]	[-5.604]
P5	1.56***	8.59	0.12	17.38	1.255***	0.670***	-0.081	-0.012***
	[3.66]				[25.088]	[9.996]	[-1.208]	[-4.841]

P1-5	-0.46	6.16	-0.08	-0.564***	-0.470***	0.145**	0.006**
	[-1.50]			[-9.588]	[-6.028]	[2.138]	[2.390]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

In panel A, the returns of portfolio P1 for value weighted portfolios are 1.12%, whereas portfolio P5 has a return of 0.59%, whereas in panel B, the return of P1 is 1.10% and for P5 is 1.56%. Interestingly, contrary to Bali and Cakici (2008), we find no changes in our results and the patterns of returns are the same, using monthly and daily return data, both for value- and equal weighted portfolios. Additionally, we find the same differences in alphas, and that for both value- and equal weighted portfolios, portfolio P1-5 has positive significant alphas.

The Fama French (1993) factor coefficients still provide similar results as previously, although the significance of the coefficients is somewhat different. Both the market coefficient and the SMB factors has a positive relation to volatility, whereas the HML factor show a negative, although insignificant, relation to volatility.

We believe that our results, which is in contradiction to that of Bali and Cakici (2008), might be due to the shorter timeframe of our analysis as well as the fewer number of listed firms on Oslo Stock Exchange, thus, reducing the number of data points used. However, our results can be supported if we consider Baker and Haugen (2012), which reviled a negative relationship between volatility and return for the Norwegian market.

These results are clear and show that the low volatility anomaly persists in Norway using both daily- and monthly return measures on value weighted portfolios, when applying the methodology of Ang et al. (2006). However, when using equal weighted portfolios, we find unclear evidence for the anomaly using daily data, while a more prominent evidence is found when applying a monthly data frequency.

Our analysis provides, so far, evidence for the persistence of the low volatility anomaly regardless of frequency of return data or volatility measure when using value weighted portfolios. When using equal weighted portfolios sorted on total volatility, the conclusion becomes more unclear. As previously discussed, we could use more sophisticated measures of volatility, however, this would be more time-consuming than necessary as the focus of this thesis is not to find the best measure of volatility, but rather exploring the possibility of a low volatility anomaly in the Norwegian market. The focus of the remainder of this thesis will be to test the robustness of our results and look at the portfolio properties to examine and provide an explanation for the anomaly.

5.3 Pricing Factors

As previously discussed we found that high volatility portfolios generally have a higher exposure to market returns and the SMB factor, which indicates that the high volatility portfolios are riskier and includes smaller firms. In this section we will go through the pricing factors from the Fama French 5-factor model for our 1/0/1 and 24/0/1 strategy for both value and equal weighted portfolios, sorted on idiosyncratic volatility.

In the table below, we present the beta coefficient for five factors run against the monthly excess returns of the different portfolios. Beta MKT, SMB and HML are the same factors as we presented in Table 1 and Table 2. Beta UMD is an extension of the Fama French 3-factor model, and measures momentum. If the UMD coefficient is zero it stands to reason that the returns are relatively stable. If the coefficient is negative or positive, it means that the portfolios experiences return reversals. Evidence for return reversals could reveal that the low volatility anomaly is created by securities which has experienced an abnormal positive return and thus are experiencing an abnormal low return the following month. Thus, making the anomaly viable only for our strategy.

Table 5
Portfolio Coefficients relative to the Fama French (1998) 5 – factor model.

	Alpha	Beta MKT	Beta SMB	Beta HML	Beta UMD	Beta LIQ
Panel A: Value Weighted						
1/0/1 Strategy						
P1	-0.005***	0.916***	-0.170***	0.063**	0.040**	0.039
	[-6.676]	[29.783]	[-5.990]	[2.136]	[2.135]	[0.993]
P5	-0.019***	1.311***	0.423***	-0.075	-0.210***	0.320***
	[-5.794]	[17.265]	[4.039]	[-0.926]	[-2.990]	[2.645]
P1-5	0.014***	-0.394***	-0.593***	0.139	0.250***	-0.281**
	[3.883]	[-4.464]	[-5.293]	[1.553]	[3.394]	[-2.080]
24/0/1 Strategy						
P1	-0.005***	0.845***	-0.136***	0.043	0.014	0.037
	[-4.765]	[23.806]	[-3.520]	[1.448]	[0.607]	[0.806]
P5	-0.021***	1.360***	0.564***	-0.102	-0.172***	-0.152
	[-6.818]	[17.015]	[6.376]	[-1.306]	[-3.010]	[-1.476]
P1-5	0.016***	-0.515***	-0.700***	0.145*	0.186***	0.189*
	[4.545]	[-5.754]	[-6.960]	[1.751]	[2.876]	[1.671]
Panel B: Equal Weighted						
1/0/1 Strategy						
P1	-0.004***	0.710***	0.200***	0.090***	-0.000	-0.109***
	[-4.706]	[28.199]	[5.655]	[3.347]	[-0.016]	[-3.218]
P5	-0.008***	1.202***	0.496***	-0.008	-0.184***	0.493***
	[-3.140]	[15.246]	[5.366]	[-0.115]	[-3.058]	[4.485]
P1-5	0.003	-0.491***	-0.296***	0.098	0.183***	-0.601***

	[1.429]	[-6.081]	[-3.433]	[1.419]	[3.162]	[-5.404]
24/0/1 Strategy						
P1	-0.006***	0.693***	0.200***	0.062**	-0.032	0.01
	[-5.300]	[22.718]	[5.206]	[2.338]	[-1.455]	[0.228]
P5	-0.012***	1.273***	0.658***	-0.095	-0.138**	0.072
	[-4.484]	[23.211]	[8.358]	[-1.451]	[-2.487]	[0.738]
P1-5	0.006**	-0.580***	-0.458***	0.156**	0.106*	-0.062
	[2.129]	[-8.876]	[-5.050]	[2.362]	[1.779]	[-0.569]

The table report the coefficients of the Fama French (1998) model for the portfolios reported in Table 1 & Table 3. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

The alpha of the portfolios is similar as to what we found previously. The alphas decrease by volatility, and the alpha of portfolio P1-5 is in general significant and positive for both strategies in both panel A and B. Market exposure and exposure to SMB also increases by volatility and is significant across all portfolios. The betas for the HML factor are less clear, but for the 1/0/1 we find them to be significantly positive, thus indicating that they might be underpriced. For portfolio P5 they are insignificantly negative for both strategies in both panel A and B. This could imply that they are overpriced, but we cannot tell for sure due to the insignificance. These findings indicate the same as we found in Table 1 and Table 2, i.e. that the low volatility portfolios generally consist of larger firms that have low exposure to the market and are underpriced, whereas the high volatility portfolios are generally small, high risk with indications of overpricing

The patterns in the momentum factor show that for all four different measures, the factor loadings on portfolio P1 are insignificant and very close to zero, whereas, portfolio P5 has highly significant negative factor loadings for both equal- and value weighted portfolios and for daily and monthly returns. This suggests that the high volatility portfolios experience return reversals.

When it comes to the liquidity factor, LIQ, we find that the 24/0/1 strategies all have insignificant factor loading. We believe this is due to the nature of how the returns are calculated as explained in chapter 3.1, as the effect of the bid-ask spread, which is a measure of liquidity, on daily returns are larger than for monthly returns. For the 1/0/1 strategies, we find that the factor loading on liquidity for low volatility portfolios are insignificant using value weighted portfolios, and highly significantly negative for equal weighted portfolios. The high volatility portfolios are however highly significantly positive for both value- and equal weighted portfolios, thus implying that the securities are illiquid (See Naes et al. (2012)).

Summarizing this subchapter, we find evidence supporting that the low volatility portfolios consist of larger, underpriced firms that are highly liquid, whereas the high volatility portfolios consist of smaller, illiquid firms that could be overpriced and subjected to return reversals.

5.5 Filters

As discussed in chapter 3, we will subject our analysis to several filters. Through this chapter, our analysis has been subjected to filters inspired by Ødegaard (2017), as previous explained we found one of his filters to be quite restrictive, as it removed large parts of our dataset. In our previous analysis we have found evidence for the low volatility anomaly when following Ang et al (2006) methodology. Furthermore, we have found the portfolios has a large difference in the SMB factor loading, and that the high volatility portfolio tends to include smaller, more illiquid stocks. In this subchapter, we will test for different filters on our daily data to see how robust the results are in regard to filtering.

To test for liquidity, we increase the requirement for number of trading days within a given year from 20 days to 62.5 days in our base filter. This means a security must at least have been actively traded during a quarter of a year[4]. The results are shown in Table 6 and reveals no significant changes for our value weighted portfolios. However, shifting our focus towards the equal weighted portfolios we see that the alpha of the P1-5 portfolio has become highly significant and amount to 0.008. The results indicate that the volatility anomaly is not solely driven by illiquid stocks as we find a more prominent evidence for the anomalies existence when increasing the required trading frequency.

We apply the filtering method used by Ang et al (2006) where we exclude the companies with the lowest 5% value. The results are found in Table 7 and show a similar relation as our previous analysis for the value weighted portfolios. Moving to panel B we find the alpha to be positive and highly significant. With the applied

[4] Assuming a trading year is 250 days.

filter we also see the SMB factor loading in panel B has increased from 0.582 to 0.719 for portfolio P5, while P1 has decreased from 0.152 to 0.108. Thus, the filter seems to have increased the difference between the small stocks and large stocks within the portfolios.

By imposing the filters used by Ødegaard (2017), i.e. increasing the minimum price of a stock from NOK 1 to NOK 10, as can be seen in Table 8, we make our filters more restrictive than original. The results show no change for the value weighted portfolios, however, the returns on the equal weighted portfolios has become significant negative for the P1-5 portfolio, while its alpha is still insignificant but negative.

The results of filtering show that liquidity and penny stock isolated cannot explain the low volatility anomaly. Further, we test whether they combined might explain the anomaly. In Table 9 we report the result of requiring securities to be at least NOK 10 per stock per year, excluding the 5% lowest value securities and increasing the number of trading days required to a quarter of a trading year. Using these filters, we find that, for both value- and equal weighted portfolios, the evidence for the anomaly perishes. We find that the alphas for portfolio P1-5 in both panel A and B are insignificant, and the returns are both negative, although only significant in panel B.

5.6 Subsampling

We test for whether our results are driven by time factors by running our original analysis on two subsamples. The first subsample ranges from 1981 to 1999, and the second ranges from 2000-2015. In Table 10, which reports the first subsample period, we cannot find any significant changes in our results for the value weighted portfolios. However, for the equal weighted portfolios, the negative return for the P1-5 portfolio in panel A has become significant on the 10% level while the alpha remains insignificant. Further the alpha for the equal weighed portfolio using monthly data has become insignificant.

For the 2000-2015 sample period shown in Table 11, we find more prominent evidence for the volatility anomaly. Although P1-5 returns are insignificant, except for panel A (1), the alphas are positive and significant, independent of weighting scheme and data frequency used.

The notable difference in results between the two subperiods might be explanation by the difference in total market capitalization between P1 and P5, as shown in Table 12. The P1 portfolio for value (equal) weighted portfolios increases from 9 (7) to 76 (68) while the corresponding market share only increases by 8% (8%). This severe increase in average market capitalization and modest increase in market share indicates that the Norwegian market was not comprised of several low volatile companies with high market capitalization before the period of 2000. This notion is in accordance with the Norwegian market, where the market capitalization ratio between the four largest companies, becomes more balanced over time (Ødegaard (2017)).

5.7 Double Sorting

Following our previous discussions, we perform a double sort similar to Ang et al. (2006) on size factor. The method is described in chapter 4.6. We choose to only report portfolio P1-5, as this is the portfolio of interest. Table 13 reports the average monthly returns, standard deviation, Sharpe Ratio and the alphas relative to the Fama French (1993) 3 factor model. We find that, for both value- and equal weighted portfolios, regardless of data frequency, that the anomaly is present for the medium large and large portfolios by looking at the alphas relative to Fama French (1993). In addition, the alphas for the average portfolios are highly significantly positive for all measures. This indicates that, in general, the anomaly is present for value- and equal weighted portfolios for both frequency measures.

The results are somewhat striking, as they could indicate that the anomaly is due to securities with a large market capitalization, which would be in contradiction to Ang et al. (2006) and Bali and Cakici (2008).

However, in Table 14, when we look at the market capitalization across all double sorted portfolios, we find that the difference in market capitalization from low to high volatility portfolios increase rapidly by size, i.e. the difference in market capitalization from P1 to P5 for the smallest firms is almost non-existent, while for the largest firms, portfolio P1 is almost eight times as large as P5, on average. The results indicate clearly that difference in size can be an explanation for the anomaly.

Following the discussion of Boyer et al. (2010), we construct the UCMC (Unconditional-Minus-Conditional) portfolio as described in chapter 4.6. The results found in Table 15 show significant alphas for value weighted portfolios for both daily- and monthly frequencies. This supports that size is an explanatory factor for the anomaly. For equal weighted portfolios, we find negative alphas which is only significant on daily frequencies. This is in accordance to previous discussions regarding the effect the weighting schemes has on our portfolios.

5.8 Return Reversal & Recursive Volatility

We previously found indications that the high volatility portfolios may witness return reversals, and thus we decide to investigate this further. In addition, we also test our analysis using recursive volatility, i.e. we use a recursive rolling window regression, where we first estimation period for volatility is one month and increases monthly. This will correct for special occurrences in returns, which in turn places securities into portfolios in which their properties do not belong.

When testing for recursive volatility, we find in Table 16 no evidence of this. The results show that for value weighted portfolios that the anomaly exists with positive significant alpha in portfolio P1-5. In addition, we see that the return on low volatility portfolios, in general, are higher that high volatility portfolios. For equal weighted portfolios, we find the same results are we did previously. The alphas for portfolio P1-5 are positive, albeit insignificant. The return of P1-5 is negative and significant at the 10% level. The return patterns show that return increases by volatility, however, the Sharpe ratio of portfolio P1 is equal to that of portfolio P5. The results from Table 16 does not indicate that our results are unchanged recursive volatility, thus we can with fair confidence say that the securities' volatility from previous month are not subjected to a significant adjustment in the following month.

A more interesting and clear result is shown when testing for return reversals in Table 17. In panel A, the returns are highly significant and increases with volatility, where P1-5 has a highly significant return of -1.94%. A similar pattern is revealed when using equal weighted portfolios as shown in panel B. In panel B, we find highly significant returns that increases by volatility, where the return of portfolio P1-5 is -3.03%.

Comparing the alphas, we see that it decreases by volatility and that P1-5 in panel A has become negative and insignificant at -0.001, while the alpha of the P1-5 in panel B has become highly significant with at -0.013. Thus, the results are in high contrast to what our original results show in Table 1.

The results from the Table 17 is clear and show that the low volatility anomaly can be explained by return reversals. Thus, the results are in line with the pricing factor analysis conducted in chapter 5.3, which indicated that the highly volatile portfolio could be subjected to a negative return reversal.

6. Conclusion

Traditional asset pricing theory suggests that the relationship between risk and expected return should be positive as a rational investor is unwilling to take on more risk for a smaller reward than he could have received by taking on less risk. Ang et al. (2006) studied the relationship between the cross-section of volatility and expected return and found that the relation in fact is negative. In the aftermath of their study several other researchers have taken upon themselves to study the anomaly even further. However, little research has been conducted on the Norwegian market, and thus we decided to investigate the anomaly.

Using data from the Oslo Stock Exchange ranging back to 1980, we used the methodology of Ang et al. (2006) and measured idiosyncratic volatility using rolling regression. We sorted stocks into quintile portfolios sorted on the idiosyncratic volatility and used a portfolio formation strategy to evaluate the performance of the different levels of volatility. We found that stocks with high idiosyncratic volatility performs poorly relative to stock with low idiosyncratic volatility. In addition, when researching what the possible explanations for the anomaly could be, we found that the high volatility stocks incur return reversals, and that the strategy presented by Ang et al. (2006) is unable to uncover this reversal. Our findings show that on the short term, the high volatility stocks tend to have high returns when incurring high volatility, whilst after a period of high volatility their returns will revert back to normal.

Our findings also show that low volatility stocks are on general more liquid and have a bigger market capitalization. This is in accordance with previous studies such as Bali and Cakici (2008). When using the double sorting technique by Ang et al. (2006), we uncover that the difference in market capitalization is marginal for the low value portfolios and that the difference increases rapidly. The analysis also show that the anomaly only persists for the large market capitalization portfolios which incurs large spread in market capitalization, and thus the anomaly can be explained by difference in sizes. Using the difference-in-difference method by Boyer et al. (2010), we find that size is an explanatory factor for the anomaly.

Throughout the thesis we have reported our results using different data frequencies and volatility measures. We found that our results persist regardless of measure and data frequency for the value weighted portfolios. As our focus was to investigate the low volatility anomaly, and not to find an optimal method of volatility measure, we found our measures suitable for its purpose.

The discovered return reversal might be due to overpricing and arbitrage asymmetry as investors might be reluctant or unable to take a short position on the Norwegian market. However, our results cannot with certainty indicate that high volatile securities are subjected to overpricing. Thus, a more comprehensive research on this matter must be conducted.

In addition, further research on this subject should investigate whether the anomaly found, using the strategy detailed in chapter 4, actually can be exploited when incorporating market frictions and transaction costs.

As previously discussed, the results found in this study is subjected to both the methodology and filters used. The intention of this study is not to find a definitive answer for the existence of the anomaly, nor to convey our results as the unconditional truth. However, we wish to shed light on this particular topic for the Norwegian market and to seek answers for our thesis question.

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6. Appendix

Exhibit 1 – Filtering

Table 6
Portfolios sorted on daily Idiosyncratic Volatility – Filter at least 62.5 days

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
Panel A: Value Weighted Portfolios								
P1	1.16***	6.19	0.09	1.09	0.901***	-0.157***	0.065**	-0.005***
	[3.79]				[34.017]	[-5.878]	[2.126]	[-5.883]
P2	1.26***	6.90	0.10	1.65	0.989***	-0.037	-0.02	-0.006***
	[3.71]				[28.335]	[-0.734]	[-0.476]	[-3.640]
P3	1.09***	7.95	0.06	2.20	1.160***	0.180***	0.037	-0.012***
	[2.76]				[27.795]	[3.276]	[0.817]	[-5.728]
P4	0.84*	8.69	0.03	2.99	1.257***	0.421***	-0.05	-0.017***
	[1.95]				[25.183]	[5.656]	[-0.765]	[-6.867]
P5	0.67	10.74	0.01	5.31	1.376***	0.492***	-0.082	-0.021***
	[1.26]				[15.619]	[4.097]	[-0.839]	[-5.891]
P1-5	0.49	8.72	0.06		-0.475***	-0.649***	0.147	0.016***
	[1.14]				[-4.699]	[-4.891]	[1.399]	[4.101]
Panel B: Equal Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.21***	5.08	0.12	1.09	0.775***	0.105***	0.057**	-0.005***
	[4.84]				[33.772]	[3.541]	[2.107]	[-4.906]
P2	1.45***	6.15	0.14	1.65	0.964***	0.236***	0.011	-0.006***
	[4.77]				[33.965]	[5.571]	[0.379]	[-4.522]
P3	1.34***	7.17	0.10	2.20	1.134***	0.448***	0.064*	-0.011***
	[3.78]				[34.581]	[9.154]	[1.874]	[-6.943]
P4	1.23***	7.42	0.09	2.99	1.168***	0.592***	-0.02	-0.014***
	[3.34]				[29.968]	[11.621]	[-0.452]	[-7.694]
P5	1.43***	8.70	0.10	5.31	1.206***	0.695***	-0.029	-0.013***
	[3.31]				[19.142]	[6.586]	[-0.350]	[-5.091]
P1-5	-0.21	6.39	-0.03		-0.431***	-0.590***	0.086	0.008***
	[-0.67]				[-6.469]	[-5.605]	[1.002]	[3.099]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Table 7
Portfolios sorted on daily Idiosyncratic Volatility – Ang et al. (2009) Filter.

Panel A: Value Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.22***	6.17	0.10	1.08	0.901***	-0.138***	0.067**	-0.005***
	[4.00]				[33.085]	[-4.964]	[2.122]	[-5.253]
P2	1.19***	6.77	0.09	1.67	0.976***	-0.039	-0.006	-0.007***
	[3.56]				[29.823]	[-0.898]	[-0.154]	[-4.000]
P3	1.14***	7.86	0.07	2.23	1.160***	0.224***	0.017	-0.012***
	[2.94]				[27.015]	[3.900]	[0.356]	[-5.722]
P4	0.79*	8.51	0.03	3.07	1.190***	0.325***	-0.037	-0.016***
	[1.89]				[24.057]	[5.086]	[-0.609]	[-6.425]
P5	0.49	9.41	-0.01	5.49	1.233***	0.485***	-0.076	-0.021***
	[1.06]				[17.163]	[5.500]	[-0.919]	[-6.446]
P1-5	0.73*	7.58	0.09		-0.332***	-0.623***	0.143	0.016***
	[1.93]				[-3.868]	[-6.199]	[1.571]	[4.585]

Panel B: Equal Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.16***	5.06	0.11	1.07	0.774***	0.108***	0.065**	-0.006***
	[4.67]				[34.219]	[3.560]	[2.354]	[-5.454]
P2	1.48***	6.18	0.14	1.62	0.960***	0.212***	0.016	-0.006***
	[4.87]				[33.723]	[4.817]	[0.516]	[-3.978]
P3	1.42***	6.89	0.12	2.12	1.075***	0.369***	0.042	-0.009***
	[4.17]				[32.387]	[8.612]	[1.224]	[-5.626]
P4	1.29***	7.33	0.10	2.84	1.159***	0.560***	0	-0.013***
	[3.56]				[30.894]	[11.202]	[-0.006]	[-7.261]
P5	1.37***	8.35	0.09	4.79	1.214***	0.719***	-0.041	-0.014***
	[3.31]				[22.790]	[10.610]	[-0.614]	[-6.052]
P1-5	-0.20	5.94	-0.03		-0.440***	-0.610***	0.106	0.008***
	[-0.69]				[-7.617]	[-8.717]	[1.521]	[3.464]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 8
Portfolios sorted on daily Idiosyncratic Volatility – Ødegaard filter.

Panel A: Value Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.15***	6.09	0.09	1.03	0.884***	-0.141***	0.070**	-0.005***
	[3.82]				[33.036]	[-5.358]	[2.528]	[-5.235]
P2	1.31***	6.75	0.11	1.59	0.951***	-0.067	-0.005	-0.005***
	[3.93]				[24.602]	[-1.399]	[-0.114]	[-2.895]
P3	1.47***	7.77	0.11	2.10	1.143***	0.176***	0.008	-0.008***
	[3.83]				[25.773]	[3.148]	[0.181]	[-3.772]
P4	0.96**	8.05	0.05	2.88	1.146***	0.337***	-0.099*	-0.014***
	[2.41]				[22.339]	[5.450]	[-1.848]	[-5.841]
P5	1.02**	9.08	0.05	5.24	1.097***	0.374***	-0.025	-0.013***
	[2.27]				[15.096]	[3.938]	[-0.305]	[-4.050]
P1-5	0.13	7.59	0.02		-0.213***	-0.515***	0.095	0.008**
	[0.35]				[-2.594]	[-4.935]	[1.080]	[2.184]

Panel B: Equal Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.33***	4.74	0.16	1.03	0.718***	0.136***	0.087***	-0.003***
	[5.69]				[31.596]	[4.101]	[3.230]	[-3.335]
P2	1.39***	5.60	0.14	1.59	0.872***	0.218***	0.060**	-0.006***
	[5.03]				[32.751]	[6.139]	[2.201]	[-4.465]
P3	1.59***	6.35	0.16	2.10	1.005***	0.407***	0.071**	-0.007***
	[5.08]				[33.507]	[9.027]	[2.083]	[-4.788]
P4	1.40***	6.14	0.13	2.88	0.964***	0.494***	-0.046	-0.008***
	[4.62]				[32.137]	[11.855]	[-1.401]	[-5.836]
P5	2.06***	6.42	0.23	5.24	0.873***	0.573***	0.068	-0.002
	[6.45]				[18.810]	[8.768]	[1.163]	[-0.894]
P1-5	-0.72***	4.86	-0.15		-0.155***	-0.438***	0.02	-0.002
	[-3.00]				[-2.906]	[-6.164]	[0.308]	[-0.836]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Table 9
Portfolios sorted on daily Idiosyncratic Volatility – All filters.

Panel A: Value Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.11***	6.28	0.08	1.01	0.898***	-0.155***	0.060*	-0.006***
	[3.59]				[31.234]	[-5.011]	[1.895]	[-5.118]
P2	1.39***	6.79	0.12	1.50	0.932***	-0.107**	0.032	-0.004**
	[4.16]				[24.638]	[-2.253]	[0.703]	[-2.032]
P3	1.49***	7.51	0.12	1.92	1.099***	0.143**	-0.025	-0.007***
	[4.04]				[27.662]	[2.330]	[-0.633]	[-3.280]
P4	1.10***	8.09	0.06	2.52	1.200***	0.276***	-0.033	-0.013***
	[2.74]				[28.859]	[4.577]	[-0.672]	[-6.075]
P5	1.30***	9.63	0.07	4.13	1.223***	0.348***	-0.091	-0.011***
	[2.73]				[16.607]	[3.486]	[-1.091]	[-3.450]
P1-5	-0.18	7.93	-0.02		-0.325***	-0.503***	0.152*	0.006
	[-0.47]				[-3.724]	[-4.461]	[1.650]	[1.568]

Panel B: Equal Weighted Portfolios

Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.32***	4.72	0.16	1.02	0.712***	0.128***	0.089***	-0.004***
	[5.64]				[31.499]	[4.092]	[3.336]	[-3.375]
P2	1.31***	5.58	0.13	1.55	0.875***	0.201***	0.049*	-0.006***
	[4.77]				[34.765]	[5.003]	[1.844]	[-5.177]
P3	1.59***	6.33	0.16	2.04	0.997***	0.381***	0.062*	-0.007***
	[5.10]				[34.629]	[8.424]	[1.923]	[-4.560]
P4	1.41***	6.05	0.14	2.75	0.966***	0.471***	-0.014	-0.008***
	[4.70]				[34.863]	[11.914]	[-0.422]	[-6.056]
P5	1.73***	6.41	0.18	4.91	0.919***	0.614***	0.008	-0.006***
	[5.44]				[18.934]	[10.949]	[0.163]	[-3.240]
P1-5	-0.41*	4.86	-0.09		-0.207***	-0.487***	0.082	0.002
	[-1.70]				[-3.586]	[-7.148]	[1.596]	[1.114]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.

*** p<0.01, ** p<0.05, * p<0.1

Exhibit 2 - Subsamples

Table 10
Portfolios sorted Idiosyncratic Volatility – Sample period 1981-1999.

	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
Panel A: Daily Data Frequency								
(1) Value Weighted Portfolios								
P1	1.38***	6.59	0.08	1.07	0.908***	-0.168***	0.031	-0.005***
	[3.08]				[23.444]	[-6.214]	[0.634]	[-3.989]
P5	1.14*	9.65	0.03	6.28	1.141***	0.589***	0.061	-0.019***
	[1.74]				[13.293]	[4.496]	[0.539]	[-4.329]
P1-5	0.24	8.09	0.03		-0.233**	-0.757***	-0.030	0.014***
	[0.44]				[-2.271]	[-5.254]	[-0.247]	[2.828]
(2) Equal Weighted Portfolios								
P1	1.56***	5.29	0.14	1.07	0.750***	0.161***	0.079*	-0.005***
	[4.35]				[24.901]	[3.819]	[1.966]	[-3.512]
P5	2.47***	9.08	0.18	6.28	1.022***	0.784***	0.157	-0.007*
	[4.02]				[11.888]	[5.166]	[1.341]	[-1.738]
P1-5	-0.91*	7.16	-0.13		-0.272***	-0.624***	-0.078	0.002
	[-1.88]				[-2.930]	[-4.143]	[-0.602]	[0.415]
Panel B: Monthly Data Frequency								
	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
(1) Value Weighted Portfolios								
P1	1.16***	6.14	0.05	4.74	0.831***	-0.117***	-0.017	-0.007***
	[2.75]				[21.520]	[-3.176]	[-0.444]	[-4.361]
P5	0.62	9,83	-0.02	17.06	1.302***	0.421***	-0.023	-0.025***
	[0.91]				[15.908]	[4.171]	[-0.205]	[-5.959]
P1-5	0.54	7.28	0.08		-0.471***	-0.537***	0.005	0.018***
	[1.08]				[-4.872]	[-4.777]	[0.047]	[3.656]
(2) Equal Weighted Portfolios								
P1	1.33***	5.05	0.10	4.74	0.715***	0.210***	0.045	-0.008***
	[3.86]				[22.229]	[4.656]	[1.211]	[-4.920]
P5	1.94***	8.51	0.13	17.06	1.137***	0.689***	0.046	-0.012***
	[3.32]				[19.075]	[8.106]	[0.494]	[-3.784]
P1-5	-0.61	6.04	-0.10		-0.423***	-0.480***	-0.000	0.005
	[-1.46]				[-5.342]	[-4.466]	[-0.003]	[1.339]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 1999. Numbers in brackets show the Newey-West robust t-statistics.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 11
Portfolios sorted Idiosyncratic Volatility – Sample period 2000-2015.

Panel A: Daily Data Frequency

	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
	(1) Value Weighted Portfolios							
P1	0.97**	5.61	0.12	1.12	0.920***	-0.101*	0.120***	-0.005***
	[2.41]				[28.309]	[-1.815]	[3.227]	[-4.590]
P5	-0.05	9.19	-0.04	5.96	1.291***	0.449***	-0.130	-0.022***
	[-0.08]				[13.810]	[3.619]	[-0.922]	[-4.599]
P1-5	1.02**	6.87	0.15		-0.371***	-0.551***	0.250*	0.017***
	[2.05]				[-3.820]	[-4.365]	[1.678]	[3.551]
	(2) Equal Weighted Portfolios							
P1	0.94***	4.24	0.15	1.12	0.733***	0.145**	0.071*	-0.004***
	[3.07]				[20.835]	[2.356]	[1.843]	[-3.099]
P5	0.79	6.68	0.08	5.96	1.021***	0.479***	-0.091	-0.010***
	[1.64]				[14.100]	[4.330]	[-0.946]	[-3.259]
P1-5	0.14	4.44	0.03		-0.288***	-0.334***	0.162*	0.007**
	[0.44]				[-4.334]	[-4.063]	[1.811]	[2.100]

Panel B: Monthly Data Frequency

	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
	(1) Value Weighted Portfolios							
P1	1.03***	5.58	0.13	4.30	0.905***	-0.065	0.089***	-0.004***
	[2.49]				[24.668]	[-1.416]	[2.673]	[-2.625]
P5	0.83	9.17	0.06	17.26	1.467***	0.597***	-0.137	-0.016***
	[1.22]				[15.707]	[4.216]	[-1.055]	[-3.657]
P1-5	0.20	6.55	0.03		-0.562***	-0.662***	0.226	0.012**
	[0.40]				[-5.269]	[-4.333]	[1.617]	[2.440]
	(2) Equal Weighted Portfolios							
P1	0.82***	3.97	0.14	4.3	0.680***	0.218***	0.037	-0.004***
	[2.82]				[15.293]	[3.760]	[0.988]	[-2.627]
P5	1.19*	8.46	0.11	17.26	1.383***	0.728***	-0.193*	-0.011***
	[1.91]				[14.686]	[5.309]	[-1.744]	[-3.040]
P1-5	-0.37	5.91	-0.06		-0.703***	-0.510***	0.231**	0.007**
	[-0.85]				[-8.385]	[-4.380]	[2.092]	[2.044]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 2000 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Table 12
Firm Characteristics for portfolio P1 & P5 for subsamples

Panel A: Daily data

	1980-1999		2000-2015	
	Market Share	Market Value	Market Share	Market Value
P1	0.52	9	0.6	76
P5	0.04	1	0.03	4

Panel B: Monthly data

	1980-1999		2000-2015	
	Market Share	Market Value	Market Share	Market Value
P1	0.52	7	0.6	68
P5	0.04	1	0.03	6

Panel A and B display the monthly average market share and market cap of portfolio 1 and 5, sorted on idiosyncratic volatility for each subsample period.

Exhibit 3 – Double Sorting

Table 13
Portfolios Double Sorted on Size and Idiosyncratic Volatility

Panel A: Daily Frequency

(1) Value Weighted Portfolios

	Small	Medium Small	Medium	Medium Large	Large	Average
	Portfolio P1-5					
Return	-1.06	0.16	1.11***	0.98**	0.70	0.38
	[-1.63]	[0.28]	[2.73]	[2.19]	[1.58]	[1.34]
SD	13.16	11.85	8.23	9.07	8.99	5.75
SR	-0.08	0.01	0.13	0.11	0.08	0.06
FF-3 Alpha	-0.009	0.007	0.015***	0.019***	0.014***	0.009***
	[-1.38]	[1.07]	[3.62]	[4.17]	[3.34]	[3.63]

(2) Equal Weighted Portfolios

	Small	Medium Small	Medium	Medium Large	Large	Average
	Portfolio P1-5					
Return	-1.64**	-0.27	1.02**	0.92*	0.52	11.03
	[-2.58]	[-0.44]	[2.54]	[1.93]	[1.28]	[0.38]
SD	12.82	12.59	8.13	9.66	8.22	5.88
SR	-0.13	-0.02	0.12	0.09	0.06	0.02
FF-3 Alpha	-.0108*	0.005	0.014***	0.018***	0.013***	0.008***
	[-1.77]	[0.73]	[3.49]	[3.57]	[3.31]	[3.01]

Panel B: Monthly Frequency

(1) Value Weighted Portfolios

	Small	Medium Small	Medium	Medium Large	Large	Average
	Portfolio P1-5					
Return	0.27	0.96*	-0.17	1.54**	1.06**	0.74**
	[0.52]	[1.66]	[-0.32]	[2.55]	[2.23]	[2.17]
SD	10.69 %	11.73 %	10.82 %	12.26 %	9.65 %	6.68 %
SR	0.02	0.08	-0.02	0.13	0.11	0.11
FF-3 Alpha	0.010*	0.018**	0.009	0.028***	0.022***	0.017***
	[1.66]	[2.58]	[1.32]	[3.56]	[4.09]	[4.70]

(2) Equal Weighted Portfolios

	Small	Medium Small	Medium	Medium Large	Large	Average
	Portfolio P1-5					
Return	-0.65	-0.31	-0.82*	0.19	0.06	-0.31
	[-1.18]	[-0.57]	[-1.90]	[0.45]	[0.15]	[-1.04]
SD	11.04	10.98	8.68	8.43	8.28	5.92
SR	-0.06	-0.03	-0.09	0.02	0.01	-0.05
FF-3 Alpha	0.004	0.007	0.003	0.011***	0.011***	0.007***
	[0.71]	[1.43]	[0.74]	[2.83]	[2.70]	[2.96]

Panel A show value weighted portfolios double sorted on size and idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period

is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Table 14
Size Characteristics within each quintile portfolio of double sorted portfolios

	Small	Medium Small	Medium	Medium Large	Large
P1	12	37	91	253	5910
P2	11	34	84	240	2060
P3	13	35	76	232	1360
P4	14	37	83	224	1060
P5	14	36	92	216	769

The table report the market capitalization for the double sorted portfolio in table 10.

Table 15
Alphas Relative to Fama French (1993) on UCMC Portfolios

Panel A: Daily Frequency		
	$UCMC^{VW}$	$UCMC^{EW}$
FF-3 Alpha	0.0056*	-0.00403**
	[1.86]	[-2.39]
Panel B: Monthly Frequency		
	$UCMC^{VW}$	$UCMC^{EW}$
FF-3 Alpha	0.0046***	-0.0034
	[2.71]	[-0.92]

The table report the alphas relative to Fama French (1993) for the value- and equal weighted UCMC portfolios on daily- and monthly frequency and

Exhibit 4 – Return Reversals

Table 16
Portfolios sorted on daily Recursive Idiosyncratic Volatility

Panel A: Value Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.15***	6.22	0.09	1.26	0.910***	-0.149***	0.060**	-0.005***
	[3.75]				[35.533]	[-4.772]	[2.043]	[-6.381]
P2	1.18***	6.62	0.09	1.93	0.972***	0.053	0.004	-0.007***
	[3.62]				[26.562]	[1.277]	[0.103]	[-4.539]
P3	1.28***	7.85	0.09	2.60	1.155***	0.190***	0.016	-0.010***
	[3.29]				[26.179]	[3.359]	[0.348]	[-4.863]
P4	0.53	8.63	-0.01	3.63	1.215***	0.404***	-0.116*	-0.019***
	[1.24]				[23.778]	[5.584]	[-1.729]	[-7.339]
P5	0.80*	9.71	0.02	6.78	1.218***	0.580***	0.052	-0.019***
	[1.66]				[14.871]	[5.156]	[0.513]	[-5.865]
P1-5	0.35	8.00	0.04		-0.307***	-0.729***	0.008	0.013***
	[0.89]				[-3.295]	[-5.993]	[0.072]	[3.995]
Panel B: Equal Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.26***	4.85	0.14	1.26	0.747***	0.137***	0.058**	-0.004***
	[5.28]				[34.253]	[4.247]	[2.123]	[-4.395]
P2	1.43***	5.58	0.14	1.93	0.917***	0.277***	0.054**	-0.006***
	[4.95]				[30.242]	[6.628]	[2.005]	[-4.704]
P3	1.47***	6.69	0.13	2.60	1.058***	0.432***	0.030	-0.009***
	[4.44]				[33.956]	[9.678]	[0.884]	[-5.816]
P4	1.17***	7.01	0.08	3.63	1.069***	0.612***	-0.019	-0.013***
	[3.39]				[25.284]	[9.098]	[-0.413]	[-7.310]
P5	1.77***	8.12	0.15	6.78	1.048***	0.673***	0.063	-0.008***
	[4.40]				[15.025]	[6.242]	[0.765]	[-2.920]
P1-5	-0.51*	6.17	-0.08		-0.301***	-0.537***	-0.006	0.003
	[-1.67]				[-4.203]	[-4.991]	[-0.067]	[1.218]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured using recursive estimation, and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1

Table 17
Portfolios sorted on daily Idiosyncratic Volatility – Controlling for Return Reversals

Panel A: Value Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	1.10***	5.69	0.09	1.09	0.806***	-0.181***	0.061***	-0.005***
	[3.91]				[37.789]	[-7.113]	[2.757]	[-5.187]
P2	1.17***	6.89	0.08	1.72	1.001***	0.046	0.002	-0.008***
	[3.45]				[26.715]	[1.096]	[0.040]	[-5.039]
P3	1.37***	8.31	0.09	2.34	1.218***	0.131**	0.016	-0.010***
	[3.33]				[29.007]	[2.285]	[0.301]	[-4.828]
P4	0.95*	10.00	0.04	3.29	1.429***	0.423***	-0.051	-0.019***
	[1.92]				[24.305]	[6.016]	[-0.810]	[-6.208]
P5	3.04***	13.23	0.19	6.17	1.640***	0.877***	-0.242**	-0.004
	[4.66]				[17.021]	[6.542]	[-2.010]	[-0.793]
P1-5	-1.94***	11.52	-0.17		-0.834***	-1.059***	0.303**	-0.001
	[-3.41]				[-7.636]	[-7.221]	[2.373]	[-0.197]

Panel A: Equal Weighted Portfolios								
Portfolio	Return	Standard Dev.	SR	IVOL	FF-3 MKT	FF-3 SMB	FF-3 HML	FF-3 Alpha
P1	0.67***	3.87	0.02	1.09	0.570***	0.045**	0.065***	-0.007***
	[3.51]				[30.693]	[2.109]	[3.202]	[-8.584]
P2	0.75***	5.25	0.03	1.72	0.826***	0.227***	0.064**	-0.012***
	[2.91]				[33.150]	[6.709]	[2.462]	[-10.646]
P3	0.89***	6.47	0.05	2.34	1.031***	0.406***	0.049	-0.014***
	[2.79]				[35.003]	[9.876]	[1.407]	[-10.500]
P4	1.23***	7.34	0.09	3.29	1.138***	0.566***	0.027	-0.014***
	[3.39]				[33.061]	[12.506]	[0.580]	[-7.666]
P5	3.71***	9.67	0.32	6.17	1.315***	0.926***	0.004	0.006*
	[7.78]				[18.746]	[8.731]	[0.051]	[1.921]
P1-5	-3.03***	7.90	-0.38		-0.745***	-0.880***	0.06	-0.013***
	[-7.77]				[-9.866]	[-7.994]	[0.667]	[-4.219]

Panel A show value weighted portfolios sorted on idiosyncratic volatility relative to the Fama French (1993) 3-factor model, whereas panel B show equal weighted portfolios. The volatility is measured monthly and the portfolios are rebalanced monthly as well. The return, standard deviation and SR (Sharpe ratio) are estimated by monthly averages. The return, standard deviation and IVOL are reported in percentages. IVOL is the estimated idiosyncratic volatility of the portfolios. The FF-3 variables show the coefficients of the Fama French (1993) 3-factor regression on excess returns. MKT, SMB, HML and Alpha is the excess market return, the small-minus-big, high-minus-low and constant in the regression, respectively. Sample period is 1981 to 2015. Numbers in brackets show the Newey-West robust t-statistics.
*** p<0.01, ** p<0.05, * p<0.1