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Optimizing Sports Scheduling:

Mathematical and Constraint Programming to Minimize Traveled Distance with Benchmark From The Norwegian Professional Volleyball League

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Summary

In this thesis, we present models to schedule round-robin tournaments. Based on attributes of the 2017/2018 schedule for the Norwegian top volleyball league, the Mizuno League, the models aim to (1) distribute the number of breaks more evenly among the teams in the league, (2) introduce a maximum number of consecutive home or away matches, (3) more evenly distribute the number of matches per round (4) reduce the total number of rounds from 15 to 14 (5) create a fairer and over-all better schedule and (6) minimize travel distance, and thereby costs.

The thesis describes five models to tackle the aims above:

- Model 1: A linear integer programming model to schedule a single round-robin.
- Model 2: A linear integer programming model to minimize breaks in a single round-robin tournament.
- Model 3: A constraint programming model to minimize breaks in a double round-robin tournament.
- Model 4: A linear integer programming model to minimize travel distance with predefined timetables in a double round-robin tournament.
- Model 5: A constraint programming model to minimize travel distance in a double round-robin tournament.

All models attempt to schedule a seasonal tournament based on the constraints and objective of the top Norwegian volleyball league, The Mizuno League. The Models are tested and benchmarked on the number of breaks and travel distance from the schedule of 2017/2018 season of the Mizuno League.

Model 2 to 5 all reduced the number of breaks compared to the Mizuno League. Model 5, a constraint programming model proved to reduce travel distance by 10% to 29%. Because of tight budgets, the most suitable model to schedule the Mizuno League is Model 5. Efforts have been taken in section 6 to alter the schedule from Model 5 to best fit the practical requirements of the Mizuno League.

Finally, we re-solve Model 5 where we change the objective of minimizing travel distance to minimize travel cost.

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1.0 Introduction

The choice of topic for our master thesis has been on our minds since we first started our Master of Science program back in 2016. During our degree we have tried to choose elective courses which combined our interests with real-world usefulness; ranging from data analysis through programming in Python to business optimization using mathematical modeling. With our elective courses, and our major in Logistics, Operations and Supply Chain Management, both students seem to have found their real interest in optimization using quantitative methods. We would like to thank our supervisor, Stéphane Dauzère-Pérès, for supporting the decision of topic and advising us throughout the process of modeling and writing. We would also like to thank our professor Atle Nordli not only for introducing the topic to us, but for helping us guide the way to what became our master thesis. With scheduling, a branch within operations research (OR), we saw the opportunity to put our interest to good use and see real-world usefulness using quantitative methods.

More specifically, we have chosen to study the scheduling of professional sports tournaments. Sports is big business, where nations battle to achieve the right to host major events like the Football World Cup or the Olympics. These significant events can create jobs, publicity and have economic opportunities if managed correctly by the host nation and the involved actors. It is not only the significant events in sports which involve massive opportunities. In the most popular of professional sports, the top leagues and tournaments can involve millions of fans and have massive financial investments in areas such as players, advertising, merchandise and broadcasting rights. How these events are scheduled and managed, have both financial and logistical impact on the actors, such as management, players, broadcasters, and media.

Tournaments come in different shapes and forms. For clarity, when we discuss tournaments, we will discuss only the annual/seasonal competition at the highest level for the respective sport. The schedule of a tournament is the decided match-ups, date and venue for the different matches throughout the season. The definition of an optimal schedule is dependent on the parties involved and their motivation. To be able to satisfy the many constraints a tournament has, and because of the computational difficulty, manual trial-and-error methods have traditionally been the way sports schedules have been created (Fry & Ohlmann, 2012).

An example of a good schedule from the host's perspective can be one which avoids the conflict of multiple attractive matches played at the same time slot, satisfies the different teams' wishes and have as many fair match-ups as possible throughout the tournament. Other actors might only be interested in a schedule which maximizes their revenue, minimizing the cost or minimize the traveling distances. Also, each sport, and its respective league, usually has its set of specific peculiarities.

1.1 Sports scheduling

The scheduling of sports tournaments is a multi-objective optimization problem which formulated on the specific requirements and objectives defined by the involved parties. The general problem in tournament scheduling is determining the date, the match-ups and the venue for a match to be played. How the general problem is to be extended or varied is dependent on the sport, its tournament format, the financial and geographical situation, and the researchers and practitioners involved.

Sports scheduling is said to be as much about developing an appropriate model, as it is about the solution methodology that is employed. Researchers can use previous work for inspiration and may use existing work for some parts of the problem, but solving a real-world problem is as much of an issue as the choice of solution methodology (Kendall, Knust, Ribeiro, & Urrutia, 2010).

With the increase of money invested in sports, new tools and knowledge, alongside the computational difficulty, scheduling sports tournaments has in recent years been subject to increased attention amongst researchers in the OR community, as well as organizers and practitioners in the area. Luckily, Knust (2018) has a website dedicated to classifying the extensive amount of literature on applications of OR techniques to sports scheduling. A closer look at the different techniques, methods, and models applied in real-world problems are to be discussed/explored later in the thesis.

In this thesis, we will present the development and application of normative mathematical and constraint programming models to schedule a sports tournament with the objective of minimizing distance traveled. The 2017/2018 season for the top league in Norwegian volleyball, Mizunoligaen (the Mizuno league), will be used for validation, and to benchmark and compare the models in this thesis.

1.2 Mizunoligaen (The Mizuno League)

According to numbers provided by FIFA, football as a sport has 270 million people actively engaged in it and approximately 1.3 billion fans around the world (FIFA, 2007). During the 2014 FIFA World Cup, the in-home television coverage reached a staggering 3.2 billion viewers (Kantar Media, 2014). With the amount of money and fans involved, it is no surprise that football is also the most covered sport in the research area of sports scheduling. At the time of writing, studies solely focusing on football cover nearly 40% of the 104 pieces of literature done on specific sports disciplines (Knust, 2018). During its preliminary phase, this thesis was also supposed to cover scheduling of tournaments in football. More precisely, Eliteserien, the top league in Norwegian football. However, the Norwegian research group SINTEF's division of applied mathematics has tried optimizing Eliteserien's schedule since 2006 (Jære, 2017). The practice currently used by SINTEF is similar to several of the existing studies on successful optimization of sports scheduling: Decompose the problem into first finding a home-away pattern (HAP), and secondly deciding on who should meet when, both parts solved under the necessary constraints (Jære, 2017).

Working with already optimized schedules can make for numerous exciting angles for a study. However, both authors find motivation in possibly having an impact on real-world processes. To strengthen the possibility of real-world impact, one of the least individually researched sports disciplines (less than 4% of literature), volleyball, was chosen for benchmark purposes (Knust, 2018).

Moreover, by choosing another discipline than football, we are challenged to look at how studies were done on the scheduling of different sports disciplines can be applied to the scheduling of volleyball.

Volleyball made its debut in the 1964 Olympics and is one of the most widely spread Olympic sports when counting national member federations (FIVB, 2018). Even though the sport is widely spread geographically, the number of players and the sport's popularity varies severely in the different nations.

Neither of the authors had any ties to or extensive knowledge about the Norwegian volleyball scene before the choice of sport. The amount of information publicly available on the Mizuno league is very limited. With invaluable help from the Norwegian Volleyball Federation's (NVBF) Director of Competition, David Cox, the authors were able to attain the necessary information about the Mizuno league.

The league has its name as a part of a larger sponsorship deal with the Japanese sporting goods brand Mizuno (Norges Volleyballforbund, 2017). It is divided into two separate leagues by gender, both operating under the same set of rules. The male and female version of the league both consist of 8 teams. Both leagues use a double round-robin schedule for its seasonal tournament, but with no clear mid-season. A matchup between team i and j can, therefore, take place in the same half of the schedule. The schedule is temporally relaxed, which means all teams do not necessarily play every round. Each team plays 14 matches in total. A team can play up to 2 matches per round. In its current form, the league consists of 15 rounds in total.

The previous season (2017/2018) started in early October of 2017 and concluded in early March of 2018. The work conducted in this thesis will use the 2017/2018 season for data input, information and benchmark purposes.

Today, the Mizuno league is scheduled in several steps. The first step the NVBF takes when scheduling the league is outsourcing a schedule which acts as a draft. The outsourced schedule was finished in May of 2017, five months prior to the first round of the season. The outsourced schedule is discussed later in the thesis and can be found in full in attachment 16. The schedule consisted of 14 rounds and had the objective to minimize distance traveled for the teams in the league. The final version of the schedule was finished in September of 2017, just prior to the first round of the season.

The NVBF has a large number of considerations to make when producing a schedule. One of the most substantial challenges is the availability of the venues where the matches are to be played. The teams in the league are also actively involved in the process of producing a schedule. The teams work with tight budgets where most of their expenses are related to traveling to and from matches. With this in mind, the NVBF strives to have a tournament schedule where the travel distance, and consequently expense, is kept to a minimum. The objective of minimizing travel distance, at seemingly any cost, results in a relaxed schedule where teams often find themselves playing two home or away matches during the same round (weekend). One team, for instance, played seven consecutive away matches in the 2017/2018 season.

The 2017/2018 Mizuno League schedule (found in section 4.2) reflects the objective of keeping costs as low as possible. The schedule would quickly be viewed as non-satisfactory in other professional sports where objectives like stadium attendance and fairness are vital factors.

The specific constraints and other considerations necessary for scheduling the Mizuno league is found in section 4.2.

1.3 Research question and problem statement

The following research question frames this master thesis:

"Scheduling in sports: How to model and optimize a sport schedule focusing on traveled distance, with validation on the Norwegian top volleyball league's constraints and objectives?"

The fundamental objective of the model is to use OR techniques to schedule a sports tournament. The variation of methods, constraints, and objectives can vary to a large extent. This thesis will present the development and application of normative integer linear programming and constraint programming models to schedule a sports tournament with the objective of balancing the tournament and minimizing travel distance. The Norwegian top volleyball league, the Mizuno League, have been used to both selecting the right constraints and objectives, and to test our results with a real case as a benchmark. Integer and constraint programming are exclusively used to limit the scope of the modeling phase.

The teams in the Mizuno league are all working with tight budgets. The budgets force the NVBF to find the silver lining between a feasible schedule and as low as possible costs. Today, the schedule is a time-consuming task where the scheduling is first outsourced before it is manually adjusted or altered in several steps. Even though the NVBF go through the complex process of finding the best schedule possible given their environment, it has some technical flaws. The number of breaks is unevenly distributed, with teams playing up to seven consecutive away matches. The rounds have high variation in the number of matches per round.

The development of the models presented in this thesis based on the Mizuno League. Additionally, in section 6.0 we tailor our results to fit the case of the Mizuno league. The goal of the models is to:

- Distribute the number of breaks more evenly among the teams in the league.
- Introduce a maximum number of consecutive home or away matches.
- More evenly distribute the number of matches per round.
- Reduce the total number of rounds from 15 to 14.
- Fairer and over-all better schedule.

- Minimize distance, and thereby costs.

2.0 Literature Review

Although sports scheduling has a general problem, there is no general solution. OR approaches have been applied by researchers in several sports throughout the years as specific requirements differ in different sports (Kostuk & Willoughby, 2012). Scheduling the Canadian football league (CFL) proved to be more complicated than scheduling the National Football League (NFL) in America. For example, NFL plays 15 of their 16 weekly matches on Sundays, while the CFL has their matches spread from Thursday till Monday.

The specific requirements when scheduling a tournament also has significant differences within the same sport. South American and European soccer is an attractive field in sports scheduling research. Della Croce and Oliveri (2006) scheduled a double round-robin tournament in Italy, where they balanced the requests from the soccer teams with the requests from the broadcaster. Durán et al. (2007) scheduled the professional soccer league in Chile using an integer linear programming model. The model was later implemented in Chile since the fans viewed the schedule as more attractive. R. V. Rasmussen (2008) illustrates the scheduling of the Danish football league's triple round-robin tournament by using integer programming. Bartsch, Drexl, and Kröger (2006) created an OR model for the professional leagues in Austria and Germany. Kendall (2008) analyzed travel efficiencies for soccer teams over the Christmas holiday in England. His approach managed to cut the total travel distance by 25 percent while satisfying all the restrictions set by the league. These are just some examples to emphasize that there is no standard as to which approach fits a sports scheduling problem. The problem differs with the environment such as its objectives, constraints, sport, geographical area and tournament type. Most of the research done on sports scheduling focus on the scheduling of temporally constrained tournaments. The research objectives can be classified into two broad groups: schedule balancing and travel distance minimization.

This literature review outlines the most typical approaches to sport scheduling problems and serves as the basis for the collection and analysis of data and modeling in the thesis.

Kendall et al. (2010) offer a complete annotated bibliography from over 160 journal articles, that is a collection of modeling approaches and literature, on the topic of sports scheduling up until 2010. R. V. Rasmussen and Trick (2008) examine the literature on round-robin tournament scheduling, and Ribeiro (2012) gives a review of problems and applications in sports scheduling. For graph-based models and resource-based models, Drexl and Knust (2007) surveyed the topic. These literature reviews serve as the base of our research, and in the section below we will go into more details about different aspect related to sports scheduling and findings in the more recent literature.

2.1 Tournaments

The problem of sports scheduling can be divided into two categories: temporally constrained and temporally relaxed (Nemhauser & Trick, 1998). The temporally constrained scheduling problems have an equal number of matches to the minimum required time slots. Tournaments with a temporally constrained problem are called compact tournaments. The temporally relaxed scheduling problems have a larger number of available time slots than the minimum required. This means that the teams in tournaments with temporally relaxed problems do not have to play every round (Kendall et al., 2010). The Mizuno League had in the 2017/2018 season a tournament with 15 rounds instead of the required 14 and are therefore temporally relaxed.

2.2 Round-robin tournaments

In a single round-robin tournament (SRR) or double round-robin tournament (DRR), we have an even number of teams n indexed by $i \in 1 \ldots$n. The teams must face each other team exactly once in SRR and twice in DRR, and correspondingly three times in triple round robin (3RR) and four times in quadruple (4RR). Denoted that each team needs to play every other team $\ell \geq 1$ times. All instances we consider in our research we set ℓ equal to two.

The number of rounds available in a temporally constrained tournament to schedule the matches is $\binom{n}{2}\ell = n(n-1)\,\ell/2$, and is equal to $(n-1)\,\ell$ (Kendall et al., 2010). The Mizuno League has 8 teams (n=8) and a double round-robin (ℓ=2), so the rounds available equals 14.

The home team *H* is the team that plays at their own stadium (home match) and an away team *A* is the team that visits the home team's stadium (away match). In double round-robin tournaments, the same team will both be a home team and an away team, when the same opponents play two matches against each other. When the second phase has the exact same sequence of matches as the first phase, but with opposite home teams, the literature refers to "mirrored" double round-robin tournaments. When the number of teams is odd, or the schedule is temporally relaxed, a team does not play in every round. A team not playing one round is in the literature referred to as a "bye" (Ribeiro, 2012). $n = \tilde{n} + 1$ denotes that a dummy team has been introduced for scheduling purposes. When a team is scheduled to face the dummy team, the team gets a bye.

Which teams and in what sequence they will play against their opposing teams and the corresponding venue is displayed sequentially in a HAP. Home matches, away matches and byes are presented in a sequence for every team, often in a vector, denoted with H, A, and B (bye). The HAP needs to satisfy specific constraints, for example, a fair pairing of teams. When analyzing a HAP problem to see if it is feasible is namely called the HAP Set Feasibility Problem (Briskorn, 2008).

Scheduling round-robin tournaments represent two main tasks:

(1) Determining which teams $i,j \in 1...n$ plays against each other in each round r = 1...(n-1) ℓ (i.e. timetable) (Kendall et al., 2010).
(2) The home-away pattern (HAP). These tasks have been solved in the literature sequentially (first the timetable, then the HAP) or the other way around, that is fitting a HAP into a given timetable (Ribeiro, 2012).

Most football leagues in Europe use the round-robin tournament format (Goossens & Spieksma, 2012). In Norway, as an example, both the professional football and volleyball league use the double round-robin format

2.3 Break minimization

Minimizing the number of breaks has been a topic explored by various researchers. Breaks are consecutively played home or away matches by the same team (de Werra, 1981, 1988). The objective has been to minimize break in most of the literature, to ensure fairness and attractiveness (Drexl & Knust, 2007). When solving the traveling tournament problem, however, Urrutia and Ribeiro (2006) showed that to minimize traveling distance, a large number of breaks can be preferable or even necessary.

2.4 The traveling tournament problem

The traveling tournament problem (TTP) has the objective of minimizing the total distance traveled (Bonomo, Cardemil, Durán, Marenco, & Sabán, 2012; Easton, Nemhauser, & Trick, 2001; Ribeiro, 2012; Ribeiro & Urrutia, 2007). Only minimizing breaks and ensuring a good HAP can be problematic for teams and players in areas where vast traveling distances are involved and where the individual teams' budgets are low. The TTP deals with a set of distances between each team's venue. Instead of minimizing matches, the models introduce constraints where it sets an upper limit *UB* and a lower limit *l* on breaks, consecutive home (or away) matches a team can play (Easton et al., 2001).

The TTP was first developed to help with the key issues in scheduling Major League Baseball, the top Baseball league in the United States. A league with dozens of requirements, but with the key issue of achieving a trade-off between home/away requirements and the distance traveled.

As previously mentioned, the computational difficulty has acted as motivation for researchers in the area. One could think that insight from the Traveling Salesman Problem (similar distance issue) or various complex sports scheduling problems with would make the TTP a relatively easy to solve. However, the combination of the distance issue and complex scheduling makes for a very challenging scheduling problem where the computational difficulty is specifically present (Easton et al., 2001). For many years, the most substantial problem solved to optimality with the TTP was a six-team instance. The first eight-team instance was solved to optimality by Irnich (2010). The boundaries regarding solving the TTP to optimality are continuously pushed as new methods, and the computational power progresses, alongside the internal competition amongst the relevant researchers. M. Trick (2018) administers an active website keeping track of any advancements within the TTP in sports.

Minimizing distance for the Mizuno League is an 8-team TTP instance. However, all the research where the TTP is solved to optimality had compact schedules: the number of rounds is equal to the number of matches played by each team. There are several leagues, Mizuno included, which share the same concerns as the TTP are looking to solve but utilize a relaxed schedule. Example of major leagues using a relaxed schedule is the National Basketball Association (NBA) and the National Hockey League (NHL) in the United States.

Tournaments beside the professional leagues need to be scheduled as well. An amateur table tennis tournament in Germany was scheduled by Schönberger, Mattfeld, and Kopfer (2004). The tournament was a double round-robin tournament. The constraints for the tournament included a balanced number of matches between teams, arena availabilities, and off-days. The problem was modeled as a constraint satisfaction problem. The model's variables were matches between the teams, with the values being the possible dates for the matches. When trying to solve the problem using constraint programming the results got poor when the number of teams got large. A generic algorithm was proposed, which was solved using a CSP model and local heuristic search with satisfactory results.

Bonomo et al. (2012) are the first to specifically research the TTP on a real-world sports league in Volleyball. The authors described an optimization process for the first division in Argentinian Volleyball. The 12 teams are geographically grouped into couples, with matches held on Thursday and Saturdays. Two teams from each couple play against two teams in a different couple. The teams do not return to their home location between matches. The authors solved two key issues: (1) how should the teams be coupled and (2) how to schedule the matches using integer programming and tabu search heuristics. Benchmarking their feasible schedules against two of the manually scheduled previous seasons, the reduction in the distance was from 15 to 22 percent.

2.5 Other common objectives

The *carry-over effect* is an effect a team has on its opponent, which carries over to the opponent's next match (Russell, 1980). The balancing of this effect has been explored in various problems to ensure fairness in the tournament (Kendall et al., 2010; Ribeiro, 2012).

For tournaments scheduled with more than one group, an objective can be to ensure that *weak and strong teams* do not play against each other consecutively to ensure fairness.

A group changing schedule is when no team plays against teams of the same group in two consecutive matches.

Some sports organizers, especially those in amateur leagues, have a desire that the objective function *minimizes costs*. All teams have a cost c_{ijt}. Here the objective is to minimize total cost or maximize total revenue or benefits. A version of this objective is when Durán, Guajardo, and Wolf-Yadlin (2012) among other things experimented with different tournament structures when scheduling to ensure Chile's Second Division Soccer League's profitability and public sentiment.

Balanced tournament design (BTD) is when no team plays more than two times on the same stadium (Kendall et al., 2010).

M. A. Trick, Yildiz, and Yunes (2012) created the *umpire scheduling problem*. Umpires are referees in baseball. The problem assigns teams of four umpires to a given schedule. A unique constraint in this problem is that the umpires have no home base and a sub-objective is to ensure that umpires are not assigned to the same team too many times. The primary objective in this problem is to minimize the total distance traveled by umpires with considerations to each of their travel distance, days off and the number of matches assigned.

2.6 Optimization approaches

Kendall et al. (2010) identified four common approaches to optimization problems in sports scheduling; *decomposition*, *metaheuristics*, *integer programming* and *constraint programming*.

The *decomposition* approach is often used in combination with integer programming approaches where the problem is divided into smaller problems that are solved sequentially. Subproblems are solved by either scheduling which teams play against each other first and then solving the HAP, or vice versa

Heuristic approaches are used to find suboptimal solutions with approximation algorithms because computing time and difficult formulations in integer programming can be very resource demanding for complex problems.

Integer programming utilizes mathematical optimization where some of the variables are restricted to integers and *constraint programming* utilized variables and constraints that are related to each other, and the constraint cannot be expressed in a linear fashion. In the scope of our research only integer- and constraint programming are utilized.

2.7 Constraints

Different sports tournament research works with different constraints. In our research, the primary objective will be to identify the best feasible scheduling model that satisfies requirements and constraints. Since constraints often are vaguely expressed by the sports organization, Kostuk and Willoughby (2012) prioritized constraints in categories ranging from "must-haves" to "nice to have" when scheduling the Canadian Football League.

R. V. Rasmussen and Trick (2008) presents the most typical constraints in sports scheduling, namely (1) *place constraints*, (2) *top-team and bottom-team constraints*, (3) *break constraints*, (4) *game constraints*, (5) *complementary constraints*, (6) *geographical constraints*, (7) *pattern constraints* and (8) *separation constraints*. We refer the reader to the paper by R. V. Rasmussen and Trick (2008) for references to specific literature with these constraints are listed.

Nurmi et al. (2010) give a more specific outline of typical constraints in the sports scheduling problem. The constraints are denoted below with the same number as in the original article, C[Number][Number]. When solving for a real-world sports organization, the constraints will differ with the organization's preferences. As a theoretical basis for modeling and collecting data a combination of the constraints from R. V. Rasmussen and Trick (2008) and Nurmi et al. (2010) is listed below:

(1) *Place constraints:*

When a stadium, for some reason, is unavailable or is especially attractive in a particular round, place constraints is required. Constraints like these ensure that a team's match is played home or away at a specific time.

C03. Each team plays at least *m1* and at most *m2* matches at home.
C04. Team *i* cannot play at home in round *r*.
C05. Team *i* cannot play away in round *r*.
C06. Team *i* cannot play at all in round *r*.
C07. There should be at least *m1* and at most *m2* home matches for teams *i1*, *i2*, on the same day.

(2) *Top-team and bottom-team constraints:*

Constraints that take special consideration for particularly strong or weak teams.

C28. Teams should not play more than *k* consecutive matches against opponents in the same strength group.
C29. Teams should not play more than *k* consecutive matches against opponents in the strength group *s*.
C30. At most m teams in strength group *s* should have a home match in round r.
C31. There should be at most m matches between the teams in strength group *s* between rounds *r1* and *r2*.
C32. Team i should play at least *m1* and at most *m2* home matches against opponents in strength group *s* between rounds *r1* and *r2*.

C33. Team i should play at least *m1* and at most *m2* matches against opponents in strength group s between rounds *r1* and *r2*.

(3) *Break constraints:*

Constraints to avoid breaks in specific rounds.

C12. A break cannot occur in round *r*.

C18. Every team must have exactly *k* number of breaks.

(4) *Match constraints:*

Constraints that decide specific time and rounds for matches. Media broadcasting of matches or stadium attendance is among the reasons for such constraints.

C10. Match *i*-team against *j*-team must be preassigned to round *r*.

C11. Match *i*-team against *j*-team must not be assigned to round *r*.

C24. Match *i*-team against *j*-team cannot be played before round *r*.

C25. Match *i*-team against *j*-team cannot be played after round *r*.

C34. Match *i*-team against *j*-team can only be carried out in a subset of rounds *r1*, *r2*, *r3*, …

(5) *Complementary constraints:*

If teams share stadiums, complementary constraints ensure that home matches are not played by both teams in a round.

C23. Team *i* wishes to play at least *m1* and at most *m2* home matches on weekday1, *m3* – *m4* on weekday2 and so on

(6) Geographical constraints:

Constraints that ensure that matches are scattered in a wide geographical area.

Nurmi et al. (2010) did not identify any specific constraints that fit this category

(7) *Pattern constraints:*

Pattern constraints ensure that match types like home matches, away matches, and byes have a maximum or minimum range that can be played sequentially by a team.

Upper and lower bound of breaks, sequences of home matches etc. are examples of pattern constraints.

C01. There are at most *r* rounds available for the tournament.

C02. A maximum of *m* matches can be assigned to round *r*.

C08. Team *i* cannot play at home on two consecutive calendar days.

C09. Team *i* wants to play at least *m1* and at most *m2* away tours on two consecutive calendar days.

C13. Teams cannot have more than *k* consecutive home matches.

C14. Teams cannot have more than *k* consecutive away matches.

C15. The total number of breaks must not be larger than *k*.

C16. The total number of breaks per team must not be larger than *k*.

C17. Every team must have an even number of breaks.

C22. Two teams play against each other at home and in turn away in 3RR or more.

C26. The difference between the number of played home and away matches for each team must not be larger than *k* in any stage of the tournament (a k-balanced schedule).

C27. The difference in the number of played matches between the teams must not be larger than *k* in any stage of the tournament (in a relaxed schedule).

C36. The carry-over effects value must not be larger than *c*.

(8) *Separation constraints:*

Separation constraints are a constraint that ensures that there is a lower limit of rounds between a match with the same pair of teams.

C19. There must be at least *k* rounds between two matches with the same opponents.

C20. There must be at most *k* rounds between two matches with the same opponents.

C21. There must be at least *k* rounds between two matches involving team *i1* and any team from the subset *t2*, *t3*, …

C35. A break of type A/H for team *i1* must occur between rounds *r1* and *r2*

3.0 Research Methodology

Research methodology is the process chosen to systematically solve a research problem (Bryman & Bell, 2015). This section will detail the research strategy, design, and method utilized to conduct the research. Choices regarding the method and approach to research will be angled to support the overall objective of our study. Detailing a systematic approach used to collect data, developing a model and ultimately answer our research question contributes to the overall quality of our research.

3.1 Research strategy

Research strategy is "...a general orientation to the conduct of business research" (Bryman & Bell, 2015). Quantitative research emphasizes quantification in the collection and analysis of data while qualitative research emphasizes words. In the modeling phase, we will only utilize quantitative methods. The collection of input data and the identification of constraints, requirements, and objectives will to some extent be qualitative, as some inputs might not be expressed quantitatively. Attachment 1 highlights some of the contrasts between qualitative and quantitative research data collection methods. Our research takes on a quantitative research method. In the relationship between theory and research, we will make use of a deductive approach where data collection and modeling will consistently be based on theory.

The objective of our thesis is to use existing theory and apply it to a new environment, namely sports. Normative analytical modeling is used "to produce a prescriptive result, typically through iteratively applying the analytical equations until some desirable value, usually of one dependent variable, is achieved" (Meredith, Raturi, Amoako-Gyampah, & Kaplan, 1989, p. 314).

In addition, with the models reviewed, we will impose an axiomatic research approach. When explaining axiomatic research Meredith et al. (1989) proposed that "...the primary concern of the researcher is to obtain solutions within the defined model and make sure that these solutions provide insights into the structure of the problem as defined within the model".

Pidd (1999, p. 121) considered six principles when modeling. These principles will advise us in our research strategy. The general idea is that we should not create something overly complicated and create many small models instead of one large and complicated model. All of his six principles can be found below:

(1) Model simple; think complicated.
   - We cannot simply make a single model that considers all real-world factors

(2) Be parsimonious; start small and add.
   - To schedule a sports tournament, we start small with a single round-robin and add complexities when needed

(3) Divide and conquer; avoid mega-models.
   - Overly general models are hard to explain, interpret and explain

(4) Use metaphors, analogies, and similarities.
   - We will refer to previous work in literature and develop on the shoulders of previous researches.

(5) Do not fall in love with data.
   - The need for data should be driven by modeling, and not the other way around

(6) Model building may feel like muddling through.

3.2 Research design

"A research design provides a framework for the collection and analysis of data" (Bryman & Bell, 2015, p. 49). The research in this thesis will employ a quantitative design. Constraints, requirements, and objectives of the case company cannot be identified by quantitative data collection alone, but the small extent of qualitative data does not impact the research design. Except for the modeling phase, a large part of the quantitative phase will be to convert some of the qualitative data into quants that can be used for modeling purposes.

A case study will focus on a bounded situation and are therefore distinguishable from other designs. It allows for a detailed analysis of a single problem. A case study design enables a combination of several research methods and reduces the reliance of a single method (Knights & McCabe, 1997), and Thomas (2011) argues that a pragmatic approach concerning methods when dealing with case studies is beneficial. Case study as a design choice is further encouraged by Normann (1970) for research where a broad conceptual framework is applied in complex systems. The Norwegian volleyball league, "Mizunoligaen", is referred to as a representative case where we "... explore a case that exemplifies an everyday situation or form of organization" (Bryman & Bell, 2015, p. 70) and will be used for benchmarking purposes.

The mixture of normative modeling and case study are the methods that best describe our research design. Indeed, we are (1) further developing existing models in OR and (2) provide insight into the structures of a real word problem.

3.3 Data Collection

Primary data is data collected by the researcher himself, whilst secondary data involves exploring existing materials (Bryman & Bell, 2015). Secondary data collection will take place prior to collecting primary data. This is the most appropriate approach to data collection when dealing with a single case study. The need for primary data can first be identified after collecting secondary data.

Secondary data is collected from research articles, reports, books, and web pages. The data consists of topics concerning sports scheduling in OR, modeling, optimization, sports characteristics and other data specific to the case at hand.

The collection of primary data consists of informal email and meetings to get clarification and to tailor constraints to real-world situations, but also to make a bridge between literature and reality. Since the case representative can express their constraints imposed on the sports schedule vaguely (Kostuk & Willoughby, 2012), follow-up question can be necessary to get a full understanding. Informal data collection gives us the flexibility to require all the necessary information to solve our problem.

3.4 Quality of research

Bryman and Bell (2015) identified the three most important criteria to ensure quality in the research: reliability, replication, and validity.

Reliability is related to the question of whether the study is repeatable or if the measurements are consistent. Three prominent factors for considering whether a result is reliable is *stability, internal reliability and inter-rater reliability* (Bryman & Bell, 2015). The development and use of any model, data or existing literature will be written in an explanatory matter. This ensures ease of troubleshooting, repeatability, and understanding for external readers. In addition, all results referred to in the thesis are produced by models which have been tested several times to ensure stability. The data used for input is historic and constant.

Replicability is similar to the concept of reliability. While reliability refers to the possibility of solving another study and getting the same results, replicability concerns the possibility of solving the study on a different occasion (Bryman & Bell, 2015). To ensure that our study is replicable, we have documented the necessary information and concerning methods for the study to the best of our ability. The documentation is done by presenting the models in an explanatory matter, as well as including the raw data and relevant code.

According to Bryman and Bell (2015), validity is concerned with the integrity of the conclusion that is generated from a piece of research. The primary issue with the research design in this thesis concerns external validity. Generalizability and that the phenomenon studied is represented by the result is reflected by external validity. Case studies have been subject to criticism for lack of generalizability (Ellram, 1996). The results of our study can be beneficial for other researchers or sports organizations facing problems with similar constraints. We decided to schedule a temporally constrained schedule, so the models will carry the most validity. Also, the way we collect and develop the variables and constraints in the model is a challenge where our described experiences can be of high value. This thesis' approach to a sports schedule problem will not apply to every problem of a similar nature. The results are derived in a specific context and will be preferred by readers with an aligned epistemological perspective.

4.0 The Models

4.1 Introduction

In the modeling phase, we started out with a basic model (Model 1) and gradually added constraints and variables. To tailor the model to better suited the Mizuno league we added an extension constraint in Model 2-5 to utilize their challenge with (un)available venues.

Section 4 presents the models and the underlying assumptions and thought process when developing the model. The technical results and schedules for each model will be discussed in section 5.

In section 4.2 we will first present some technical data surrounding the Mizuno league. The 2017/2018 schedule in the Mizuno league works as a basis for our modeling process. In section 4.3 we present some assumptions that are implemented in all our models. Model 1 is presented in section 4.5, where we simply display how we can create a basic timetable using linear integer programming. In Model 2 (section 4.6) we develop the model further by trying to create fair schedule and add a home-away pattern using integer linear programming. Section 4.7 contains Model 3, which minimizes breaks and schedules a double round-robin tournament using constraint programming. Model 4 minimizes travel distance using integer linear programming based on the timetable created by model 1,2 and 3 and can be found in section 4.8. Section 4.9 contains Model 5. Model 5 tries to minimize travel distance by creating a timetable and assigning HAP in one operation using constraint programming.

4.2 Description of the Mizuno league

An overview over the respective teams and a schedule for the 2017/2018 season in the Mizuno league can be found in table 1. The left-most column, T, displays the respective teams. The top row tells the respective rounds. If a cell is colored green, it indicates that the match is a home match for the team in column T. The cell is colored yellow for away matches. The numbers inside the cell tell which team play against the corresponding team in column T. Cells containing a "0" means there is no match for the team in column T in the corresponding round.

Team	Team #
BK Tromsø	Team 1
Førde Volleyballklubb	Team 2
Koll IL	Team 3
NTNUI Volleyball	Team 4
Randaberg IL	Team 5
Stod IL	Team 6
TIF Viking	Team 7
ToppVolley Norge	Team 8

ROUND

T	1		2		3		4		5		6		7		8		9		10		11		12		13		14		15	
1	3	0	2	0	0	0	7	0	0	0	4	6	5	8	0	0	3	0	0	0	5	0	6	0	4	0	8	0	7	2
2	4	6	1	3	4	0	0	0	0	0	7	0	3	0	5	8	0	0	0	0	6	0	7	0	5	0	0	0	8	1
3	1	0	7	2	0	0	4	0	0	0	5	8	2	7	0	0	1	0	4	6	8	0	0	0	0	0	6	0	5	0
4	2	0		0	2	7	3	0	8	0	1	0	6	0	7	0	5	0	3	0	0	0	5	8	1	0	0	0	6	0
5	7	0	6	0	8	0	0	0		0	3	0	1	0	2	0	4	6	0	0	1	0	4	0	2	7	8	0	3	0
6	2	0	5	8	0	0	0	0	8	0	1	0	4	0	7	0	5	0	3	0	2	7	1	0	0	0	3	0	4	0
7	5	8	3	0	4	0	1	0	0	0	2	0	3	0	4	6	0	0	0	0	6	0	2	0	5	0	0	0	1	8
8	7	0	6	0	5	0	0	0	4	6	3	0	1	0	2	0	0	0	0	0	3	0	4	0	0	0	1	5	2	7

Table 1 Schedule Mizuno league 2017-2018

As seen in table 1, the league does not seem to have any upper limit on number of breaks. For instance, team 3: Koll IL, plays seven consecutive away matches in round 6 to 10. This indicates that travel cost and available venues trump the need for a fair schedule or that the schedulers has not taken fairness into the consideration at all. The schedule is temporally relaxed. We assume the additional round is added to ease the scheduling process or because of the availability of venues.

To minimize travel distance, a distance matrix was created. A map was made in Google Maps. All the addresses for the different venues were plotted in to get accurate distances between them. The distance matrix is found in table 2.

Team	1	2	3	4	5	6	7	8
1	0	1675	1745	1150	2206	1030	1771	2121
2	1675	0	420	526	374	653	169	393
3	1745	420	0	498	464	625	457	379
4	1150	526	498	0	800	128	623	723
5	2206	374	464	800	0	927	206	103
6	1030	653	625	128	927	0	750	850
7	1771	169	457	623	206	750	0	233
8	2121	393	379	723	103	850	233	0

Table 2 Distance Matrix

Air travel is necessary for most of the matches. For distances less than 250 km, it is safe to assume the teams drive because of budgetary constraints. When a team has two consecutive away matches, and the distance from the opponent in round r to $r+1$ is less than 250 km, the team will rent cars for the second away match.

A brief overview of the means of travel given to us by our contact is presented below:

<u>BK Tromsø (Team 1)</u>:

Matches are played at Tromsøhallen. All teams fly to Tromsø for their matches here.

<u>Førde VBK (Team 2):</u>

Matches are played at Førdehuset. Teams fly to Førde. The plane tickets are quite expensive, some teams might choose to drive. Matches against Førde are often set to the same weekend as matches against Viking.

<u>Koll IL (Team 3)</u>:

Matches played at Kringsjåhallen. All teams fly to Oslo Airport.

NTNUI VB (Team 4):

Evert team fly to Værnes. Often combined with matches against Stod in Steinkjer.

Randaberg (Team 5):

Matches are played at Randaberghallen. Teams outside Stavanger area fly here. Often combined with matches against TVN.

Stod IL (Team 6):

Matches are played at Steinkjerhallen. Fly to Værnes, rent car and drive to Steinkjer. Often combined with matches against NTNU.

TIF Viking (Team 7):

Matches played at Stemmemyren. Fly to Bergen. Often combined with matches against Førde.

ToppVolley Norge (TVN) (Team 8):

Matches played at Suldalshallen – Fly to Stavanger, rent car and drive to venue. Often combined with matches against Randaberg.

4.3 Underlying assumptions

To reduce the complexity of quantitative modeling, we must simplify the case of the Mizuno league and make some assumptions:

- The additional 15th round in the 2017-2018 season of the Mizuno league will be discarded, and all modeling will attempt to make temporally constrained schedules (number of rounds = 14). Since most sports utilize temporarily constrained schedules, this assures the maximum external validity of our models.
- In a real-world scenario, every round is associated with specific dates; we will disregard the specific dates as they can be added at a later stage
- As we do not specify dates, we do not take the availability of venues into the model. However, Model 2-4 adds constraints that consider this.
- When we attempt to minimize travel distance, we do not consider the home-journey. When team *i* travels between round *r* and *r+1*, the models only measures distance when team *i* plays away. If team *i* has two consecutive away matches, it will in round *r* take the distance between *i*'s home venue and the opponent's venue. For the second away match, we measure the distance between the opponent's venue in round *r* and the opponent's venue in round *r+1*. For clarification purposes see figure 1:

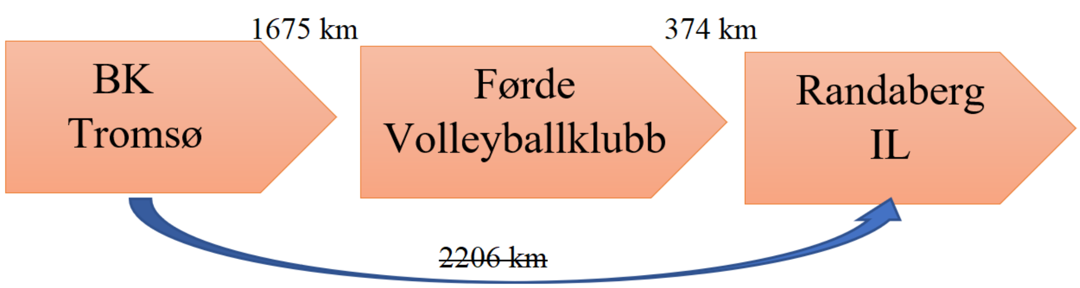

Figure 1 Distance calculation example

Figure 1 explanation:

- BK Tromsø have two consecutive away-matches with opponents: Førde Volleyballklubb in round one and Randaberg IL in round two. In the first round the model use the regular distance between BK Tromsø and Førde Volleyballklubb. In round two the measured distance BK Tromsø travels is the distance between the venues of Førde Volleyballklubb and Randaberg IL (=374 km).

- The model will not measure the home-journey (distance between the venue of Randaberg IL and BK Trømso)

These assumptions will be constant through all the described models.

4.4 Linear integer programming and constraint programming

The models were solved in IBM ILOG CPLEX 12.8 on a PC with an Intel Core i7 Quad core running at 1.8 GHz and 8 GB of RAM.

Integer programming is a technique for optimizing using constraints where the variables must be integers. We use the ILOG CPLEX search engine with the default branching strategy.

Constraint programming has been proven to be effective when solving scheduling problems and optimizing when the constraints are non-linear. In our case constraint programming is well-suited to find feasible schedules, but less suited to solve to optimality. No matter how well a schedule is solved for the Mizuno league, it would still need manual adjustment to some degree. One of the considerations the NVBF must make is consulting with all the teams involved regarding the schedule, as most of the players are not full-time athletes. It can, therefore, be argued that solving to optimality is less important in this case. We use the ILOG CP Optimizer with default propagation settings.

To display the performance of our models on the engines we use the running time in seconds. If a model cannot finish searching after 3600 seconds, the model will be terminated by limit. The best solution found will then be displayed as the result.

M. A. Trick (2003) compared the strengths of both IP and CP in round-robin scheduling. We decided to apply both methods, depending on how it best suited the relevant model(s). We decided to apply different methods for different models where it is most appropriate.

The default search method is used when solving both the linear and constraint programming models, see IBM ILOG CPLEX Optimization Studio OPL Language User's Manual for more info on algorithms and search methods.

4.5 Model 1: ILP Single Round-Robin

4.5.1 Creating a basic single round-robin schedule

The first model does not have a specific objective to solve. It is the first step towards a model which can solve the case of the Mizuno league.

With no specific objective at hand, the first step is to create a basic schedule. The task of creating a basic schedule where every team plays each other once, become increasingly difficult to do manually by hand when the number of teams increases. Since there is no specific objective yet, the solver will simply find the first feasible solution. A common technique in sports scheduling is to schedule a single round-robin tournament and mirror the result to create two identical half-seasons. We solve the problem using binary integer linear programming (LIP), where the decision variable is required to be 0 or 1. The relevant OPL-code for the model can be found in attachment 2. Section 4.5.2 contains the mathematical formulation and section 5.1.1 contains the results.

4.5.2 Mathematical formulation:

Parameters in the model:

n: number of teams = 8

T: Range of teams = 1..n

R: Range of rounds = 1..(n-1)

Variables:

$\forall\ i, j\ \in T, \forall\ r\ \in R$

$$x_{ijr} \begin{cases} 1, & if\ team\ i\ plays\ team\ j\ in\ round\ r \\ 0, & else \end{cases}$$

<u>Explanation of variables:</u>

We introduce the binary decision variable *x*. The variable has three indices i, j and r. Indices *i* and *j* represent each team 1..n, and *r* represent each round 1..n-1. With all possible combination of indices, it adds up to 448 binary variables, where 56 of them will take on the value of one (1) when all constraints are satisfied.

Constraints:

(1) $\sum_{r}^{n-1} x_{ijr} = 1$ $\forall\, i, j \in T,\ i \neq j$

(2) $x_{ijr} = x_{jir}$ $\forall\, i, j \in T\ \forall r \in R$

(3) $\sum_{j}^{n} x_{ijr} = 1$ $\forall\, i \in T\ \forall r \in R$

(4) $x_{iir} = 0$ $\forall\, i \in T\ \forall r \in R$

Explanation of the constraints:

The first constraint (1) requires all teams to play every other team only once. The second constraint (2) makes the order of the team-indices *i* and *j* are irrelevant. Constraint (3) requires all teams to play one match each round. The last constraint (4) makes sure that no team plays itself.

4.6 Model 2: ILP Mirrored Double Round-Robin

4.6.1 Adding HAP and objective

The simple model in the previous section is feasible from a mathematical point of view. To add a fairness criterion, Model 2 will consider breaks (consecutive home or away matches). Playing consecutively home or consecutively away can give unfair advantages or disadvantages to the affected teams. To make sure that no team ends up with an unfair number of breaks, we set an upper bound of two on consecutive home or away matches, *UB = 3*. Like Model 1, Model 2 is solved using LIP. The mathematical formulation is presented in section 4.6.2, the relevant OPL-code in attachment 3 and the results in section 5.1.2.

4.6.2 Mathematical formulation

Parameters in the model:

n: number of teams = 8

Ṙ: number of rounds = 7

QS: Mid-season round = $\frac{\dot{R}}{2}$ =4

UB: Upper bounds on breaks = 3

T: Range of teams = 1..n

R: Range of rounds = 1..(n-1)

Variables

$\forall\, i \neq j \in T, r \in R$

$$x_{ijr} = \begin{cases} 1, & \text{if team } i \text{ plays at home against team } j \text{ in round } r, \\ 0, & else \end{cases}$$

$\forall\, i \in T, r \; in \; 1..\dot{R} - 1$

$$h_{ir} = \begin{cases} 1 & \text{if team } i \text{ plays at home in round } r \; and \; r+1, \\ 0, & else \end{cases}$$

$$a_{ir} = \begin{cases} 1 & \text{if team } i \text{ plays away in round } r \; and \; r+1, \\ 0, & else \end{cases}$$

Explanation of variables:

The binary variable *x* has the same interpretations as in Model 1. To measure the number of breaks and to formulate the objective we include two binary variables, *h* and *a*. Both have two indices *i* in teams 1..8 and *r* in rounds 1..7.
The interpretation of the variables is that *h* (*a*) is 1 if team *i* plays home (away) in both rounds *r* and *r+1*.

Objective

Minimize breaks:

$$min \sum_{i}^{n} \sum_{r}^{\dot{R}-1} h_{ir} + a_{ir}$$

Explanation of objective:

By minimizing the sum of the value for variables *h* and *a*, we are minimizing the number of total breaks.

Constraints:

(1) $\sum_{r}^{\dot{R}} [x_{ijr} + x_{jir}] = 1 \qquad \forall\ i, j\ \in T,\ \ i \neq j$

(2) $\sum_{j}^{n} [x_{ijr} + x_{jir}] = 1 \qquad \forall\ i \in T, i \neq j\ \forall r \in R$

(3) $x_{iir} = 0 \qquad \forall i \in T\ \forall r \in R$

(4) $\sum_{j}^{n} \sum_{r}^{\dot{R}} x_{ijr}\ \leq QS + 1 \qquad \forall i \in T$

(5) $\sum_{j}^{n} \sum_{r}^{\dot{R}} x_{ijr}\ \geq QS \qquad \forall i \in T$

(6) $\sum_{k}^{\dot{R}-1} h_{ir} \leq UB \qquad \forall i \in T$

(7) $\sum_{k}^{\dot{R}-1} a_{ir} \leq UB \qquad \forall i \in T$

(8) $\sum_{j}^{n} [x_{jir} + x_{ji(r+1)}] \leq 1 + h_{ir} \qquad \forall i \in T\ \forall r \in R,\ \ r < \dot{R}$

(9) $\sum_{j}^{n} [x_{jir} + x_{ji(r+1)}] \leq 1 + a_{ir} \qquad \forall\ i\ \in T\ \forall r \in R, r < \dot{R}$

Explanation of constraints:

(1) Make sure that each team *i* plays every other team *j* once in the period of 7 rounds.

(2) In every round *r* and for every team *i*, team *j* plays either home or away.

(3) No team *i* play itself in any round *r*.

(4) – (5) Each team plays at least 7 and maximum 8 matches at home.

(6) Every team has a maximum of two consecutive home matches.

(7) Every team has a maximum of two consecutive away matches.

(8) Making the *h* variable dependent on the *x* variable.

(9) Making the *a* variable dependent on the *x* variable.

4.6.3 Venue extension

The 2017-2018 season for the Mizuno league (see *table 1*) had a significant number breaks. The number of matches per round had a high variation, and the season consisted of 15 rounds. This is the result of the NVBF manually adjusting a 14 round schedule which did not satisfy the necessary considerations for the Mizuno league.

In addition to keeping travel distance (cost) low, the schedule must also take into consideration: Unavailability of venues, specific input from teams, balancing fairness to a degree and other miscellaneous considerations.

To address the need for a more case tailored schedule, some constraints are added to the model for practical considerations. The case-specific constraints will reduce the need for manually adjusting a schedule produced through an optimization solver. For example:

1. If one team, *a*, must play home one specific round, *b*, because of an unavailable venue, we add a constraint to make sure team *a* plays home in round *b*.
2. If one team, *c*, has an unavailable home venue in one round *d*, we add a constraint to make sure team *c* plays away in round *d*.

Mathematical formulation of extension:

(Ex1): $\sum_j^n x_{ajb} = 1$, where a is the team that must play home in round b.

(Ex2): $\sum_j^n x_{cjd} = 0$, where c is the team that must play away in round d.

4.7 Model 3: CP Double Round-Robin

4.7.1 Scheduling a DRR using constraint programming

In section 4.6 we attempted to minimize breaks by using LIP. The model minimized breaks, but only in the first half of the season. The seam between the two half-seasons created additional breaks. It is necessary to minimize the breaks further by creating a schedule for both halves of the season in one model.

Because of difficulties modeling a double round-robin using linear programming alone, we utilize constraint programming instead. We can schedule a double round-robin by utilizing a property of constraint programming, an *alldifferent*-constraint. *Alldifferent*-constraints makes sure that all variables in an array have different values. In section 4.7.2 the mathematical formulation of Model 3 is written, in attachment 4 we present the relevant OPL-code and in section 5.1.3 we present the results.

4.7.2 Mathematical formulation

Parameters in the model:

n: number of teams = 8

Ṙ: number of rounds = 14

QS: Mid-season round $\frac{\dot{R}}{2}$ =7

UB: Upper bounds on breaks = 2

T: Range of teams 1..n

R: Range of rounds 1..Ṙ

Variables:

$\forall\, i \in T, r \in R$

$x_{ir} = \textit{the opponent of team } i \textit{ in round } r$

$h_{ir} = \begin{cases} 1 & \text{if team } i \text{ plays at home in round } r, \\ & 0\ else \end{cases}$

$a_{ir} = \begin{cases} 1 & \text{if team } i \text{ plays away in round } r, \\ & 0\ else \end{cases}$

$$b_{ir} = \begin{cases} 1 & \text{if team } i \text{ plays home or away in both round } r \; and \; r+1, \\ 0 & else \end{cases}$$

Explanation of variables:

We change the interpretation of the binary variable *h* and add the binary variable *a*. *h* (*a*) is one if team *i* plays home (away) in round *r*.

To count the number of breaks we need to add a variable *b***.** *b* is a binary variable that counts every time the *h* (or *a*) takes the value of one two consecutive rounds.

The interpretation of *x* is also changed in Model 3. *x* will now take on integer values which equals the opponent of team *i* in round *r*.

Objective

$$min \sum_{i=1}^{n} \sum_{r=1}^{\dot{R}} b_{ir}$$

Explanation of the objective:

By minimizing the sum of the value of the variable *b*, the model is minimizing breaks.

Constraints:

(1) $h_{ir} + h_{ir+1} \leq 1 + b_{ir}$ $\quad \forall\; i \in T \; \forall r \in 1..(\dot{R} - 1)$

(2) $a_{ir} + a_{ir+1} \leq 1 + b_{ir}$ $\quad \forall\; i \in T \; \forall r \in 1..(\dot{R} - 1)$

(3) $a_{ir} \neq h_{ir}$ $\quad \forall\; i \in T \; \forall r \in R$

(4) $\sum_{r=rr}^{rr+UB} h_{ir} \leq UB$ $\quad \forall i \in T \; \forall\; rr \in 1..(\dot{R} - UB)$

(5) $\sum_{r=rr}^{rr+UB} h_{ir} \geq 1$ $\quad \forall i \in T \; \forall\; rr \in 1..(\dot{R} - UB)$

(6) $x_{ir} = j \rightarrow x_{jr} = i$ $\quad \forall r \in R \; \forall\; i,j \in T, i \neq j$

(7) $x_{ir} > 0$ $\quad \forall r \in R \; \forall i \in T$

(8) $x_{ir} \neq i$ $\quad \forall r \in R \; \forall i \in T$

(9) $alldiffrent(x_{11}, \ldots, x_{nQS})$

(10) $alldiffrent(x_{1(QS+1)}, \ldots, x_{n\dot{R}})$

(11) $\sum_{i}^{n} h_{ir} = \frac{n}{2}$ $\quad \forall r \in R$

(12) $x_{i_1 r} = i_2 \rightarrow h_{i_1 r} + h_{i_2 r} = 1$ $\quad \forall\; i_1, i_2 \in T, i_1 < i_2 \; \forall\; r \in R$

(13) $x_{ir_1} = x_{ir_2} \rightarrow h_{ir_1} + h_{ir_2} = 1 \qquad \forall i \in T\ \forall r_1, r_2\ \in R, r_1 <\ r_2$

Explanation of variables:

(1) If one team plays home in both rounds *r* and *r+1, b* must take the value of one. Effectively *b* is counting home-breaks.

(2) If one team plays away in both rounds *r* and *r+1, b* must take the value of one. In addition to home-breaks *b* is counting away-breaks.

(3) Makes sure that *a* and *h* always have different values.

(4) – (5) Sets a lower and upper bound on breaks. Since UB is 3, the schedule will never schedule less than 1 and more than 3 consecutive home or away matches in a row.

(6) If the opponent of team *i* is team *j*, the opponent of team *j* must be *i.*

(7) Temporally constrained schedule. Every team plays every round.

(8) No team plays itself

(9) – (10) The *alldifferent* constraint makes sure that all variable in an array take on different values. In practice, this means that in the first half of the season team *i* must play all other teams. Constraint (10) has the same interpretation, but for the second half of the season.

(11) All rounds require n/2 = 4 home matches.

(12) In one match, one team plays home and the other team away

(13) In a DRR every matchup between two teams occurs twice in the season. The constraint makes sure that if team *i* plays home in the first matchup, team *j* must play home the second matchup.

4.7.3 Venue extension

The same extension we added to Model 2 in section 4.6.3 can be added to this model to account for venue times. Since the interpretation of *h* is different, this extension is simpler.

Mathematical formulation of extension:

(Ex1): $h_{ab} = 1 \quad , where\ a\ is\ the\ team\ that\ must\ play\ home\ in\ round\ b$

(Ex2): $a_{\boldsymbol{cd}} = 1 \quad , where\ c\ us\ the\ team\ that\ must\ play\ home\ in\ round\ d$

4.8 Model 4: LIP Minimize Travel Distance

4.8.1 Minimizing travel distance using LIP with predefined timetables*:*

Model 2 and 3 focused on creating a fairer and more balanced schedule. However, contact with the representative from the Mizuno League stated that a balanced schedule is not their top priority when creating a schedule. The budgets of the teams do not allow fairness to interfere with any possible cost savings in travel expenses. By using some of the mathematical formulation created by R. Rasmussen and Trick (2009), Model 4 attempts to minimize travel distance based on predefined timetables. We use the linear ILOG CPLEX solver to run our model. The model will try to minimize travel-distance by changing the home-away pattern from the three timetables we created in Model 1, 2 and 3 (see attachment 7, 8 and 9). We present the mathematical formulation in section 4.8.2, the relevant OPL-code in attachment 10,11 and 12, and the results in section 5.1.4.

4.8.2 Mathematical formulation:

Parameters in the model:

n: number of teams =8)

Ṙ: number of rounds =14

QS: Mid-season round $\frac{\dot{R}}{2}$ =7

UB: Upper bounds on breaks = 2

D_{ij}: The distance between team *i* and *j*. The distance from team *i* to *j* equals the distance from team *j* to *i*. See the distance matrix in table 2, section 4.2.

TT_{ir}: The opponent of team *i* in round *r.* See attachment 7, 8 and 9.

T: Range of teams 1..n

R: Range of rounds 1.. 2(n-1)

Variables

$\forall\ i \epsilon T\ \forall r \in 0..(2n-1)$

$$h_{ir} = \begin{cases} 1\ if\ team\ i\ plays\ home\ in\ round\ r \\ 0\ else \end{cases}$$

$\forall i \in T\ \forall r \in 0..\dot{R}$

$d_{ir} = \textit{the distance team } i \textit{ travels between round } r \textit{ and } r+1$

<u>Explanation of variables:</u>

The interpretation of *h* is the same as in Model 3.

To measure the distance, we introduce the variable *d*. *d* equals the distance team *i* travels between round *r* and *r+1*. *d* only measures distance when team *i* plays away. If team *i* has two consecutive away matches, it will in round *r* take the distance between *i*'s home venue and the opponent's venue. *d* will then take the distance between the opponent's venue in round *r* and the opponent's venue in round *r+1*.

Objective:

$$\min \sum_{i=1}^{n} \sum_{r=0}^{2(n-1)} d_{ir}$$

<u>Explanation of objective:</u>

d is the distance team *i* travels in round r. By minimizing the sum of *d*, we are effectively minimizing total travel distance

Constraints:

(1) $d_{ir} \geq (1 - h_{ir} - h_{ir+1}) D_{TT_{ir}TT_{ir+1}}$ $\quad \forall\, i \in T\ \forall r \in 0..\dot{R}$

(2) $d_{ir} \geq (h_{ir} - h_{ir+1}) D_{iTT_{ir+1}}$ $\quad \forall\, i \in T\ \forall r \in 0..\dot{R}$

(3) $d_{ir} \geq (-h_{ir} - h_{ir+1}) D_{TT_{ir}\, i}$ $\quad \forall i \in T\ \forall r \in 0..\dot{R}$

(4) $h_{i0} = 1$ $\quad \forall\, i \in T$

(5) $h_{i2n-1} = 1$ $\quad \forall\, i \in T$

(6) $\sum_{r=1}^{\dot{R}} h_{ir} = n - 1$ $\quad \forall i \in T$

(7) $h_{i_1 r} + h_{i_2 r} = 1$ $\quad \forall\, i_1, i_2 \in T, i_1 < i_2\ \forall r \in R, TT_{i_1 r} = i_2$

(8) $h_{ir_1} + h_{ir_2} = 1$ $\quad \forall i \in T\ \forall\, r_1, r_2 \in R, r_1 < r_2, TT_{ir_1} = TT_{ir_2}$

(9) $\sum_{r=rr}^{rr+UB} h_{is} \leq UB$ $\quad \forall\, i \in T\ \forall\, rr \in 1..(\dot{R} - UB)$

(10) $\sum_{r=rr}^{rr+UB} h_{is} \geq 1$ $\quad \forall\, i \in T\ \forall\, rr \in 1..(\dot{R} - UB)$

<u>Explanation of constraints:</u>

(1) – (3) Relates the variables and gives a lower bound on distance team i travels between round *r* and *r+1*.
(4) Every team starts at home, dummy rounds zero
(5) Every team ends at home, dummy round 2n-1
(6) Every team has n-1 home matches.
(7) In a matchup between i_1 and i_2, one team plays home and the other plays away
(8) In a matchup between i_1 and i_2, i_1 plays one home match and i_2 plays one home match
(9) – (10) The constraints gives a upper bound on breaks.

4.8.3 Venue extension:

The same extension we added to Model 2 and 3 in section 4.6.3, and 4.7.3 can be added to this model to account for venue times. Since we did not use the variable a, we simply utilize the fact that h = 0, is the same as $a=1$.

Mathematical formulation of extension:

(Ex1): $h_{ab} = 1 \quad , where\ a\ is\ the\ team\ that\ must\ play\ home\ in\ round\ b$
(Ex2): $h_{\boldsymbol{cd}} = 0 \quad , where\ c\ us\ the\ team\ that\ must\ play\ away\ in\ round\ d$

4.9 Model 5: CP Minimization Of Travel Distance

4.9.1 Minimize travel distance with constraint programming

In Model 4 we effectively minimized travel distance based on predefined timetables. The distances (objective value) will be discussed and benchmarked in section 5. The fundamental problem with Model 4 is that the most effective way to minimize travel distance is to create the timetable and HAP within the same model. Model 5 is our attempt to do exactly this. Model 5 is basically a merge of the four previous models. The model is solved using the ILOG CP Optimizer. The formal formulation of the constraint programming model used to solve the problem can be found in section 4.9.2. The constraint programming model in OPL modeling language can be found in attachment 9 and the results in section 5.1.5.

4.9.2 Mathematical formulation:

Parameters in the model:

n: number of teams = 8

Ṙ: number of rounds =14

UB: Upper bounds on breaks = 1,2,3 or 4

$\mathbf{D_{ij}}$: The distance between team *i* and *j*. The distance from team *i* to *j* equals the distance from team *j* to *i*. See the distance matrix in table 2, section 4.2.

m: middle of the season = $\frac{2(n-1)}{2}$ = 7

T: Range of teams 1..n

R: Range of rounds 1.. 2(n-1)

Variables:

$\forall i \in T \; \forall r \in R$

$$h_{ir} = \begin{cases} 1, if\ team\ i\ plays\ home\ in\ round\ r \\ \quad\quad 0, \quad\quad else \end{cases}$$

$d_{ir} =$ $the\ distance\ team\ i\ travels\ between\ round\ r\ and\ r+1$

$x_{ir} =$ $The\ opponent\ team\ of\ team\ i\ in\ round\ r$

<u>Explanation of variables:</u>

The decision variables *h* and *d* have the same interpretation as in Model 4. In addition we reintroduce the integer decision variable *x*. *x* equals the opponent of team *i* in round *r*.

Objective function

$$\min \sum_{i=1}^{n} \sum_{r=1}^{2(n-1)} d_{ir}$$

Explanation of objective:

d is the distance team *i* travels in round r. By minimizing the sum of *d*, we are effectively minimizing total travel distance

Constraints:

(1) $\sum_{r=rr}^{rr+UB} h_{is} \leq UB \qquad \forall\, i\ \in T\ \forall\, rr \in 1..(\dot{R} - UB)$

(2) $\sum_{r=rr}^{rr+UB} h_{is} \geq\ 1 \qquad \forall\, i\ \in T\ \forall\, rr \in 1..(\dot{R} - UB)$

(3) $x_{ir}\ = j\ \rightarrow x_{jr}\ = i \qquad \forall r \in R\ \forall i, j\ \in T, i \neq j$

(4) $x_{ir} > 0 \qquad \forall r \in R\ \forall i\ \in T$

(5) $x_{ir} \neq i \qquad \forall r \in R\ \forall i \in T$

(6) $alldiffrent(x_{11}, \dots, x_{nm})$

(7) $alldiffrent\left(x_{1(m+1)}, \dots, x_{n\dot{R}}\right)$

(8) $\sum_{i}^{n} h_{ir} = \frac{n}{2} \qquad \forall r \in R$

(9) $d_{ir} \geq (1 - h_{ir} - h_{ir+1}) D_{x_{ir} x_{ir+1}} \qquad \forall i \in T\ \forall r \in 0..2(\mathrm{n}-1)$

(10) $d_{ir} \geq (h_{ir} - h_{ir+1}) D_{i x_{ir+1}} \qquad \forall i \in\ T, \forall r 0..2(\mathrm{n}-1)$

(11) $d_{ir} \geq (-h_{ir} - h_{ir+1}) D_{x_{ir}\, i} \qquad \forall i \in T\ \forall r \in 0..2(\mathrm{n}-1)$

(12) $x_{i_1 r} =\ i_2\ \rightarrow h_{i_1 r} + h_{i_2 r} = 1 \qquad \forall\ i_1, i_2\ \in T, i_1 < i_{2,} \forall r \in R$

(13) $x_{ir_1} = x_{ir_2} \rightarrow h_{ir_1} + h_{ir_2} = 1 \qquad \forall\ i \in T\ \forall r_1, r_2\ \in R, r_1 <\ r_2$

(14) $h_{i0} = 1 \qquad \forall i \in T$

(15) $h_{i2n-1} = 1 \qquad \forall i \in T$

(16) $x_{i0} = 0 \qquad \forall i \in T$

(17) $x_{i(2n-1)} = 0 \qquad \forall i \in T$

Explanation of constraints:

(1) and (2) sets a lower and upper bound on consecutive home or away matches played by one team.

(3) Requires that if team *i* plays team *j* in round *r*, *j* must play *i* in round *r*.

(4) The constraint makes sure that every team plays every round.

(5) Makes sure that no team plays itself.

(6) and (7) requires that all teams plays every other teams both the first half and second half of the season.

(8) All rounds require $\frac{n}{2}$ home matches.

(9)-(11) measures distance travelled by away-teams.

(12) Makes sure that if team *i* plays home, the opponent *j* plays away.

(13) If team *i* plays home against team *j* one match, team *j* must play home the second match.

(14)-(15) Every team starts and end the season at home.

(16)-(17) makes sure that no matches are played in the dummy-round 0 and *2n-1*.

4.9.3 Venue extension:

The same extension we added to Model 4 in section 4.8.3 can be added to this model.

Mathematical formulation of extension:

(Ex1): $h_{ab} = 1 \quad , where\ a\ is\ the\ team\ that\ must\ play\ home\ in\ round\ b$

(Ex2): $h_{cd} = 0 \quad , where\ c\ us\ the\ team\ that\ must\ play\ away\ in\ round\ d$

5.0 Results and discussion

In section 5 the results from the models in section 4 will be discussed. When we refer to a schedule we want to recap that a green (yellow) cell means that the team in the left most column plays home (away) versus the team numbered in the cell.

Model:	**Solution method:**	**Solution time:**	**Objective:**	**Breaks:**	**Total travel distance:**	**Time-table:**
Mizuno League 2017/2018 (benchmark)	N/A	N/A	N/A	67	36555 km	Table 1 (Section 4.2)
Model 1	LIP	0.41 seconds	No objective	N/A	No objective	Attachment 7
Model 2	LIP	4.57 seconds	Minimize breaks	14		Attachment 8
Model 3	CP	3600 seconds (limit)	Minimize breaks	12		Attachment 9
Model 4	LIP	0.33 seconds (See section 5.1.5)	Minimize travel distance	30	38 505 km (lowest)	Attachment 10 (Timetable from Attachment 7)
Model 5	CP	3600 seconds (limit)	Minimize travel distance	UB=2 36 UB=3 40 UB=4 54	UB=2: 33062 km UB=3: 28.895km UB = 4: 26.131 km	Attachement 13, 14 and 15

Table 3 Summary of results

In table 3 we are showcasing the benchmark (Mizuno League) result measurements and the results from the five models in section 4. In 5.1 we will go through the results one by one.

5.1 Results

5.1.1 Results Model 1

The timetable produced by Model 1 was a single round-robin schedule which later was mirrored to create a double round-robin schedule. The timetable is found in attachment 7. Model 1 had no objective and was solved using linear integer programming.

The timetable satisfies the assumptions that every team must play each other team twice, every team plays one match each round and the season is split into two halves. The model did not consider breaks as it did not include a HAP. A HAP must be assigned manually with the assumption that every team must play n-1 (=7) matches at home. Model 1 acts as our base model and since it has no objective we cannot measure total travel distance or breaks. Model 4 minimizes travel distance using the timetable (schedule without HAP) model 1 created.

The problem was solved in 0.41 seconds with the ILOG CPLEX linear solver.

5.1.2 Results Model 2

The schedule from Model 2 can be found in attachment 8. Model 2 has the objective of minimizing breaks and was solved using integer linear programming.

The ILOG CPLEX linear solver solved the model in 4.57 seconds. The model is solving a single round-robin problem and the objective value is 6 (six breaks). However, after mirroring the schedule, we need to multiply the objective value with two and add the additional breaks occurring in the seam (round 7&8 (2 breaks)) between the two half seasons. In total the schedule has 14 breaks. Model 4 minimizes travel distance using the timetable model 2created.

5.1.3 Results Model 3

The schedule from Model 3 can be found in attachment 9. Model 3 has the objective of minimizing travel distance by both assigning matches and HAP at the same time. The model was solved using constraint programming.

The ILOG CP Optimizer got terminated by limit (3600 seconds). The solver found 19 solutions and the best objective value was 12 breaks.

5.1.4 Results Model 4

The schedules from Model 4 can be found in attachment 10, 11 and 12. Model 4 has the objective of minimizing travel distance by changing the HAP using predefined timetables. The model was solved using linear integer programming.

In table 4 we present the results when minimizing travel distance for the three timetables derived from Model 1, 2 and 3.

Timetable from model:	Solution method	Input timetable:	Solution time	Travel distance:	Output schedule:
1	LP	Attachment 7	0.33 sec	38505 km	Attachment 10
2	LP	Attachment 8	0.28 sec	39033 km	Attachment 11
3	LP	Attachment 9	0.12 sec	38954 km	Attachment 12

Table 4 Result Model 4

5.1.5 Results Model 5

The schedules from Model 5 can be found in attachment 13, 14 and 15. The objective of Model 5 is to minimize travel distance by assigning matches to a timetable and setting the home-away pattern. The model was solved using constraint programming.

The solutions with different upper bounds on consecutive home and away matches can be found in table 5. None of the instances got solved to optimality by the ILOG CP Optimizer and got terminated after one hour (3600s) of solving.

UB	Solution method	Solution time	Solutions found:	Best objective (Travel distance):	Output schedule:
1	CP	Terminated by limit (3600 s)	0	No solution	N/A
2	CP	Terminated by limit (3600 s)	169	33062 km	Attachment 13
3	CP	Terminated by limit (3600 s)	129	28.895 km	Attachment 14
4	CP	Terminated by limit (3600 s)	320	26.131 km	Attachment 15

Table 5 Result Model 5

5.2 Comparing the results with the benchmark

5.2.1 Schedule balance

In the 2017/2018 schedule for the Mizuno League (section 4.2, table 1) the schedule has a total of 67 breaks. Randaberg IL (Team 5) had the lowest number of breaks with six breaks in the season. Koll IL (Team 3) had the highest number of breaks with a total of 11 breaks in the season.

All our models outperform the Mizuno League in terms of schedule balance. Even the models which minimized travel distance had a more balanced schedule. All models have fewer breaks and less difference in the team with the highest break count and the team with the lowest break count.

If our primary concern would be schedule balance and fairness, Model 3 outperforms all the other models. Model 3 was terminated by limit and chose the best schedule out of 19 feasible solutions.

A summary of the number of breaks and teams with the lowest and highest break-count can be found in table 6.

Schedule:		**Number of breaks:**	**Lowest break-count:**	**Highest break-count:**
Mizuno 2017/2018		67	6 (Team 5)	11 (Team 3)
Model 2		14	1 (Team 1,6)	2 (Team 2,3,4,5,7,8)
Model 3		12	1 (Team 1,2,4,8)	2 (Team 3,5,6,7)
Model 4	Timetable from Model 1	30	3 (Team 1,7)	4 (Team 2,3,4,5,6,8)
	Timetable from Model 2	28	3 (Team 1,3,6,8)	4 (Team 2,4,5,7)
	Timetable from Model 3	24	1 (Team 2,4)	6 (Team 3)
Model 5	UB = 2	36	3 (Team 8)	6 (Team 1)
	UB = 3	40	4 (Team 2,5,6,7,8)	8 (Team 3)
	UB = 4	54	5 (Team 4,6)	9 (Team 1,7)

Table 6 Breaks comparison

5.2.2 Travel distance

In the 2017/2018 season of the Mizuno League, the total travel distance was 36555 km. When we are measuring the travel distance for the Mizuno League, we use the same method and distance matrix as in the models.

Model 5 with an upper bound of 4 consecutive home or away matches has the lowest total travel distance. 26.131 km is a 29 % reduction in travel distance. The drawback is the high upper bound, but it is still lower than the Mizuno 2017/2018 schedule. All three instances of Model 4 performed worse than the Mizuno 2017/2018 season in concerning travel distance. In table 7 we present a summary of travel distance in the different models and the potential reduction compared to the 2017/2018 schedule in the Mizuno Leauge.

Schedule:		**Travel distance:**	**Reduction in travel distance:**
Mizuno 2017/2018		36555 km	0%
Model 4	Timetable Model 1	38505 km	-5%
	Timetable Model 2	39033 km	.7%
	Timetable Model 3	38954 km	-7%
Model 5	UB = 2	33062 km	10%
	UB = 3	28.895 km	21%
	UB = 4	26.131 km	29%

Table 7 Travel distance comparison

5.3 Choosing the right model

The question of which model that is the most favorable is just as much a question of the requirement of the persons assessing them. In terms of balance and breaks, the schedule from Model 3 outperform the other models. Concerning travel distance, Model 5 with the different bounds (2-4), all reduce the travel distance significantly compared to the benchmark.

We mentioned earlier that the individual team’s budget is a concern in the Mizuno League. From the schedule in the 2017/2018 Mizuno Leauge season, we also see that little consideration of fairness and balance have been taken. We concluded that the top priority when scheduling the Mizuno League is to minimize cost. Model 5 performed best in terms of travel distance, and thereby cost. Based on these factors we conclude that Model 5 is the right model to schedule the Mizuno League. In section 6 we will discuss Model 5 further and display how it can be used in a real-world scenario.

6.0 Discussion and Conclusion

In section 5 we concluded that Model 5 is the most suitable model to tackle the mission of creating a schedule for the Mizuno League. A schedule perfectly suited for the Mizuno league is not produced within this thesis. We do not access the information regarding available venues or team preferences. However, this section will discuss how schedules produced by Model 5 can be adjusted to fit the real-world case of the Mizuno league.

With Model 5, schedules with feasible solutions from a mathematical point of view were achieved. A problem with the schedules produced is that they do not take the real-world scenario into account. All the schedules would, therefore, need manual adjustments to various degrees to be feasible in a real-world setting. When using the term real-world in this section, it refers exclusively to the case of the Mizuno League. When using the term alter or adjust related to a schedule, it merely refers to the act of manually placing a match in a different round.

6.1 How were the 2017/2018 Mizuno League scheduled?

Communication with our contact in the Mizuno League 2017/2018 gave us some insights into how the scheduling process is conducted.

The NVBF received a proposed schedule for the 2017/2018 season in May of 2017 (attachment 16) we will refer to this schedule as the *"outsourced schedule"*. The final schedule (Attachment 17) was not finished until days prior to the first round of the season.

No information was given on the reasoning behind the changes made from the outsourced schedule to the final schedule which we use as a benchmark. However, it is reasonable to assume that changes made were because of the availability of venues or simply that one team does not have the opportunity to play in a specific round. In our proposed models we added constraints to consider venue availabilities. However, the models are only able to utilize all requirements if they are known beforehand.

6.2 Is it real-world feasible to use Model 5 with UB>2?

With Model 5, a feasible schedule with the upper bound of three (3) and four (4) consecutive home or away matches (2 breaks and 3 breaks) was produced. The relevant OPL-code and schedule are found in attachment 6 and 14, respectively.

The Mizuno League 2017/2018 schedule shows that it is not a problem with a team having three consecutive home or away matches over the course of several rounds. In the 2017/2018 season, one team had seven consecutive away matches. The challenge is that the maximum amount of matches a team can play per round is two, as all the matches are set to either Saturday or Sunday.

Model 5 interprets the possible consecutive matches as a chance for team *i* to visit three different opponents, utilizing the travel distance between the opponent in round *r* to *r+1* and from *r+1* to *r+2*, before returning to the home location of team *i*. This would be feasible in a real-world setting if the Mizuno League had more than two days per week to play their matches. Attempts to manually adjust a model with UB=3 (or UB = 4) to make it feasible in a real-world setting proved to be too challenging, as the consecutive matches were set to separate rounds.
A comparison of the distance BK Tromsø would travel in reality and how it is measured by the model in round 2-4 is found in table 8.

Traveling team: BK Tromsø (Team 1)

Round:	**2**	**3**	**4**
Opponent team:	2	8	5
Theoretical distance:	1675	393	103
Real-world distance:	1675	393	**2206**

Table 8 Model 5 UB=3 for Tromsø BK in round 2-4

The teams in the Mizuno league are somewhat scattered, but a "cluster" of four teams (team 2, 5, 7 and 8) is found in the western coast of Norway, see figure 2. In terms of reducing travel distance, having three (or four) away matches without returning to their home location would be quite significant for the teams outside the cluster (team 1, 3, 4 and 6).

The idea of the Mizuno League reducing cost by introducing four consecutive away-matches to the western coast for teams 2, 5, 7 and 8, seems overly optimistic, and will not be discussed in this thesis.

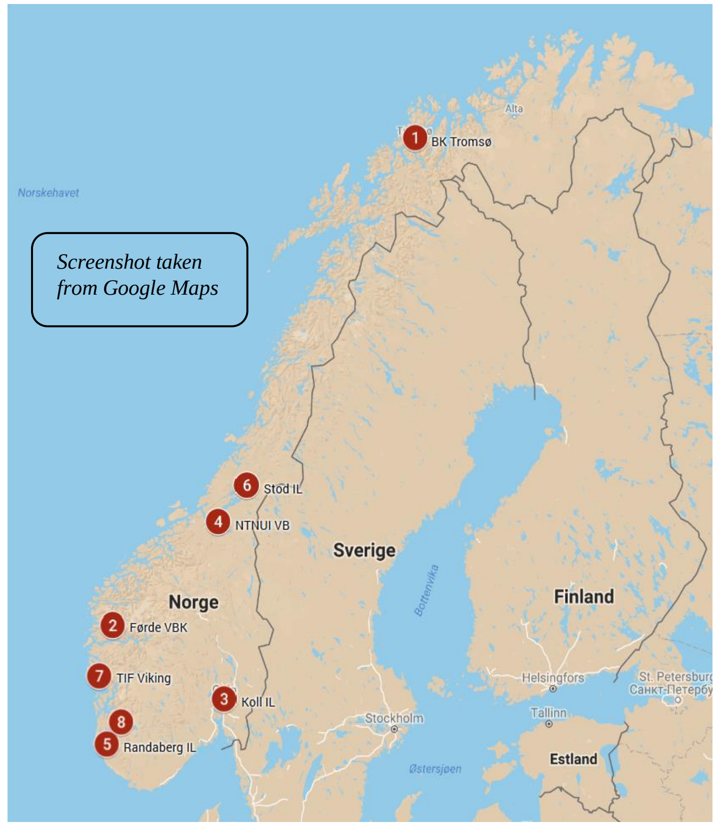

Figure 2 Map of team venues

6.3 How to schedule a feasible real-world schedule with UB=2?

With Model 5, a feasible schedule with the upper bound of two (2) consecutive home or away matches (1 break) was produced. The relevant OPL-code and schedule are found in attachment 6 and 13, respectively.

The schedule with UB=2 is not seen as feasible in a real-world setting. The schedule produced has consecutive away matches allocated to different rounds, when in practice, they are to be played in the same weekend. Consecutive home matches in separate rounds is not an issue, as home matches do not affect travel distance.

The solution to achieving feasibility in a real-world setting for the schedule with UB=2 is to alter the schedule manually. Simply allocating corresponding consecutive away matches to the same round is not enough. One must consider the fairness of the schedule and maintain the balance with regards to the number of matches per round and number of breaks per team.

To ensure the schedule is successful, a simple set of rules is followed when altering:

- There can be a maximum of two (2) matches per team in a single round
- There can be a minimum of three and a maximum of five matches played per round
- To the best of our ability, avoid teams traveling vast distances when playing both home and away on the same weekend to avoid time conflicts
- To the best of our ability, avoid more than two consecutive home or away matches
- No alteration should negatively change the outcome of the total distance traveled

An extract (round 1 to 5) of the schedule produced with Model 5 is found in table 9. The yellow cells indicate away matches for the team in column T, while the green cells indicate home matches. The team corresponding with the number inside the cell plays against the team in the column T.

Team	**T**	**1**		**2**		**3**		**4**		**5**	
BK Tromsø	**1**	2		7		3		5		8	
Førde Volleyballklubb	**2**	1		5		7		8		3	
Koll IL	**3**	7		4		1		6		2	
NTNUI Volleyball	**4**	8		3		6		7		5	
Randaberg IL	**5**	6		2		8		1		4	
Stod IL	**6**	5		8		4		3		7	
TIF Viking	**7**	3		1		2		4		6	
ToppVolley Norge	**8**	4		6		5		2		1	

Table 9 Extraction of schedule from Model 5 before manual adjustment

As previously mentioned and showed in table 9, the schedule produced with Model 5 does not allocate two matches per round for the teams. Every team plays exactly one match, meaning every round has four matches in total.

Placing corresponding consecutive away-matches (yellow cells) into the same round, is the first challenge when manually altering the schedule to be feasible in a real-world setting. The second challenge is to ensure that no teams are playing more than two matches in a single round. In addition, one single round cannot contain less than three, or more than five matches. Simultaneously we need to keep the balance of breaks maintained.

An extract of the proposed schedule which is feasible in a real-world setting is found in table 10.

Team	**T**	**1**		**2**		**3**		**4**		**5**	
BK Tromsø	**1**	2	7		3			8	5		
Førde Volleyballklubb	**2**	1		5		8	7			3	
Koll IL	**3**	7		4	1			6		2	5
NTNUI Volleyball	**4**			3	8	6		7		5	
Randaberg IL	**5**	6		2			8		1	4	3
Stod IL	**6**	5	8			4		3	7		
TIF Viking	**7**	3	1				2	4	6		
ToppVolley Norge	**8**		6		4	2	5	1			

Table 10 Extraction of Timetable from Model 5 after manual adjustment

A brief explanation of the actions taken in the first rounds when altering the schedule: (1) The match between team 1 and team 7 in round 2 is moved to round 1. (2) The match between team 6 and team 8 in round 2 is moved to round 1. Both these changes were made to have the corresponding consecutive away matches played in the same round. After these two alterations were made, round 1 now consisted of six matches, while round 2 consisted of two matches. To balance the number of matches per round, (3) the match between team 4 and team 8 in round 1 were moved to round 2.

Alterations done from round 2 till the very last round of the schedule is conducted in a similar fashion. The alterations are done in a round-by-round fashion, with continuously reviewing the rounds with consideration to the set of rules we previously described. With UB=2, and the corresponding consecutive away matches allocated to the same round, the total distance traveled will not change when making alterations. The schedule is therefore flexible when it comes to further alterations, e.g. input from teams regarding their preferences.

Figure 3 illustrates the travel route of BK Tromsø with the manually adjusted schedule. BK Tromsø has away-matches in round (weekend) 1, 4, 8 and 11. The team is only allowed to play two consecutive away-matches. Figure 3 is a display on how BK Tromsø can play all its away-matches effectively only using four rounds.

The full schedule after alterations is found in attachment 18.

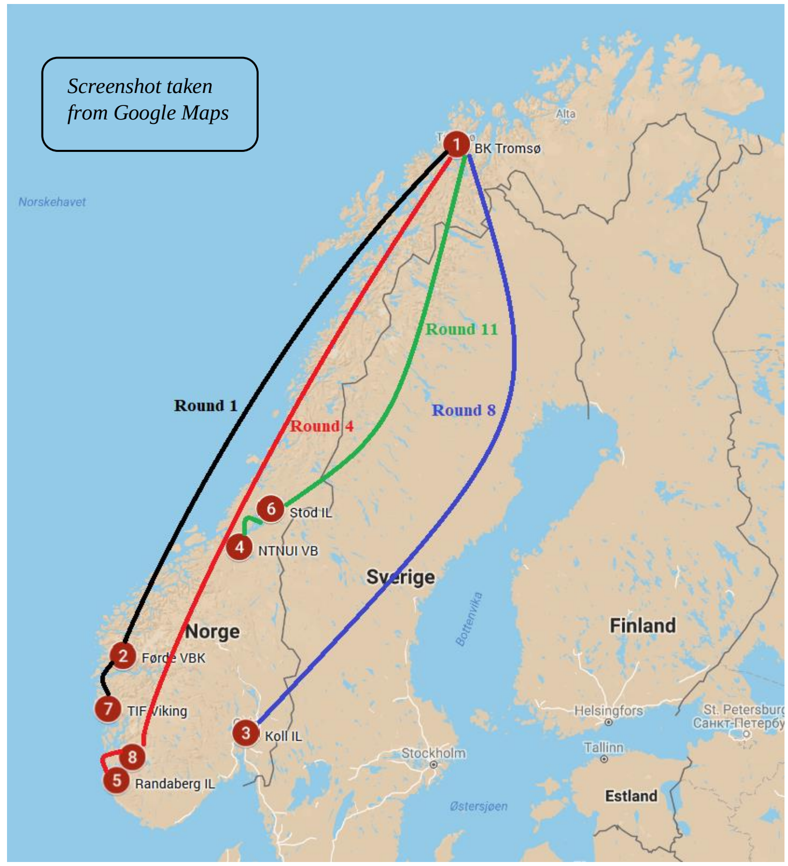

Figure 3 BK Tromsø travel route manually adjusted schedule

6.4 Utilizing Model 5 with the objective of minimizing cost

The schedules which have been discussed in previous sections have had the objective of minimizing the distance traveled. The work done on the model(s) and the alterations done on the produced schedule led us to look for additional improvements to be made.

The primary objective of the NVBF is cost reduction as the budget of the teams are tight. The reason minimizing travel distance has been the focus is because of travel-related cost being the teams most significant expense. Out of curiosity, a cost matrix was created. The idea is that utilizing a cost matrix to minimize cost is more fitted to the NVBF's goal of reducing cost, rather than trying to reduce cost by reducing travel distance.

The teams in the Mizuno league mainly use three means of travel when traveling to a match: (1) Travel by plane, (2) travel by private car and (3) travel by rental car. A descriptive list of transportation is found in section 4.2.

Travel by plane is the most expensive option and is necessary for the lion share of the matches. Travel by private car is both flexible and affordable, but only practical on shorter distances (<250 km) because of time constraints. Travel by rental car is utilized when a team has consecutive away matches. The team flies to a match on Saturday and drives a rental car to a nearby opponent on Sunday. The costs in the cost matrix are per person.

To obtain the prices of air travel in the cost matrix we used the travel section at Finn.no, which includes all available airlines from the selected airport. We assume the teams book their tickets at a reasonable time ahead of a match. We set the travel dates to Saturday and Sunday, 17^{th} and 18^{th} of November to ensure we did not experience any fluctuation in price during the study. The nearest airport was plotted in for every team where air travel is necessary. The price for Saturday and Sunday was divided by two to get an average price of a plane ticket for match days. Cost of traveling to and from the airport was not included.

The price of traveling by private cars uses a price per kilometer traveled. We assumed a price of 17 NOK per liter of gasoline, and consumption of 0.7 liters per 10 km. The cost per kilometer is slightly over 1 NOK per km. We assume four people are traveling per car. After adding some compensation for maintenance etc., we ended up setting a price per km of 1 NOK per person.

The price of traveling by rental car was set for the price of renting a feasible car for one day. In addition, we added the price of 1 NOK per km to cover fuel cost etc.

The price of a rental car varies from 400 NOK to several thousand per day. The only conditions we set for the car was that it could fit four players including luggage. Most of the rental companies had some form of deal with the airports for ease of travel. A price of 600 NOK per day was found for an average rental car. With four players per car, the final cost for travel by rental car per person was: 1 NOK per km + 150 NOK.

The cost of traveling by rental car is not included in the matrix. It is manually added when the schedule is produced, as there is no clear answer beforehand as to when it is needed.

The cost matrix is seen below in table 11.

Cost matrix

	Tromsø	**Førde**	**Koll**	**NTNU**	**Randaberg**	**Stod IL**	**TIF**	**TVN**
Tromsø	0	1250	494	561	867	561	609	867
Førde	1250	0	466	1226	1329	1226	169	1329
Koll	494	466	0	445	427	445	470	427
NTNU	561	1226	445	0	713	128	627	713
Randaberg	867	1329	427	713	0	713	206	103
Stod IL	561	1226	445	128	713	0	627	713
TIF	609	169	470	627	206	627	0	233
TVN	867	1329	427	713	103	713	233	0

Table 11 Cost matrix

The schedule produced which utilizes the cost matrix is found in attachment 19. It was manually adjusted in a similar fashion to the adjusted schedule in section 6.1.3. For comparison, we calculated the cost using the same matrix for the schedule in section 6.1 and for the 2017/2018 Mizuno League schedule. As for results, the schedule reduces cost by 15% compared to the 2017/2018 season of the Mizuno league. An overview of the total distances and costs is given in table 12. Details regarding distance and cost are given in attachment 16-19, and breaks in attachment 20-23.

	Distance, km	Cost, kr	Breaks
Outsourced schedule	37023	28452	67
The Mizuno league	36555	29971	67
Schedule, min. dist., UB=2	33062	28510	34
Schedule, min. cost., UB=2	34840	25511	39

Table 12 Total distance and cost for the manually adjusted schedules

The results presented in table 12 are purely theoretical as there are considerations we are not capable of implementing. Our contact at the NVBF mentioned that venue availability and team preferences made it hard to optimize a schedule, as the factors are prone to any changes.

What sparked our curiosity, and what can be derived from the results, is that distance may not be the optimal objective for the Mizuno league. The schedule utilizing the cost matrix had a 5% larger distance traveled, but a 10% reduced cost compared to the schedule utilizing the distance matrix. This is a result of the price of plane tickets not correlating directly with distance, but rather following the law of supply and demand. Travels to and from larger cities are relatively cheaper than traveling from the airport in for instance Førde.

6.5 Conclusion

Revisiting the research question can be useful when concluding. The research question was:

"Scheduling in sports: How to model and optimize a sport schedule focusing on traveled distance, with validation on the Norwegian top volleyball league's constraints and objectives?"

The thesis presented five different models to schedule sports. In section 5.0 we compared the results and found that every model outperformed the Mizuno League in terms of breaks. Additionally, Model 5 reduced the travel distance compared to the Mizuno league benchmark schedule with 10% to 29%.

To give Model 5 a more practical value we described how we could manually adjust the schedule in Model 5 to take practical requirements into account. The manually adjusted schedule reduced the travel distance by 10 % and breaks with 46%, it reduced the number of rounds to 14, and it is an overall fairer schedule.

Finally, we re-solved Model 5 with a cost matrix to reduce travel cost. This was motivated by the false assumption that travel distance has a one-to-one relationship with travel cost. The result increased travel distance by 5 % compared to the model that minimized distance; however, it reduced travel cost by 10 %.

6.6 Limitations

Finally, we want to highlight some limitations and challenges for another researcher(s).

Altering the schedule output from Model 5 is our attempt to make the schedule fit into a real-world scenario. Since we do not have access to the information on venue times and individual team's requirements, the manually adjusted schedule is still just a presentation of a theoretical method, not a final schedule that can be utilized by the Mizuno League organizers. Creating a model with particular requirements are beyond the reach of this thesis.

All information used to obtain the results have been disclosed in the thesis, but when using a case, such as The Mizuno League, there is a possibility that the authors lack some fundamental understanding about the internal processes within the case.

The communication with our contact within the league has been fruitful, but limited. The contact has been very helpful in helping us understand the schedule process and requirements. However, our results and models have not been approved or revised by the contact before the hand-in date of this thesis. We wish we had a thorough understanding of the underlying decisions and steps made when the NVBF adjusted the outsourced schedule (Attachment 16) from May till September 2017. Also, the understanding of which factors can force a change in the schedule mid-season and why.

The authors had minimal experience in linear and constraint programming before writing this thesis. The models are a display of what the authors, two business students, can learn in eight months by reading journals and user manuals. The model might lack some elegance in performance. However, the results give insight into the scheduling process even outside the world of mathematics. The results are by all means feasible and an improvement from the case's perspective but cannot be proven to be optimal from a mathematical point of view.

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8.0 Attachments:

Attachment 1 General differences between quantitative and qualitative research

Source:(Bryman & Bell, 2015).

Quantitative	Qualitative
Numbers	Words
Point of view of researcher	Point of view of participant
Researcher distant	Researcher close
Theory testing	Theory emergent
Static	Unstructured
Generalization	Contextual understanding
Hard data	Deep data

Attachment 2 OPL code Model 1

```
/*********************************************
 * OPL 12.8.0.0 Model
 * Author: krist
 * Creation Date: May 24, 2018 at 2:14:36 PM
 *********************************************/
//Setting the parameters
using CPLEX;
int n = 8;
range Teams = 1..n;
int nbRounds = n-1;
range Rounds = 1..nbRounds;
//variables
dvar    oolean x[Teams][Teams][Rounds]; // 1 if team I plays at home against team j
in round k
//objective
maximize sum (I in Teams, j in Teams, k in Rounds) x[i][j][k];
//constraints
 subject to {

  // (1) Every team plays every other team.
```

```
 Forall (I,j in Teams: I != j)
 con1:
  sum (k in Rounds) x[i][j][k] ==1;
// (2) every team play one match per week
forall (ordered I,j in Teams, k in Rounds)
  x[i][j][k] == x[j][i][k];
  con2:
//(3)
forall (I in Teams, k in Rounds)
  con3:
  sum (j in Teams) x[i][j][k] == 1;
//(4)  No team plays itself
forall (I in Teams, k in Rounds)
  con4:
 x[i][i][k] == 0;
}
```

Attachment 3 OPL code model 2

```
/*********************************************
 * OPL 12.8.0.0 Model
 * Author: krist
 * Creation Date: May 28, 2018 at 2:29:36 PM
 *********************************************/
//Setting the parameters
using CPLEX;
int n = 8;
range Teams = 1..n;
int nbRounds = n-1;
range Rounds = 1..nbRounds;
range Roundsy =1..(nbRounds-1);
int mid = nbRounds div 2;
int UB = 2;
//variables
dvar  oolean x[Teams][Teams][Rounds]; // 1 if team I plays home against
team j in round r
```

```
dvar  oolean h[Teams][Roundsy];       // 1 if team I plays home in round r
and r+1
dvar  oolean a[Teams][Roundsy];       // 1 if team I plays away in round r
and r+1
//objective
minimize sum (I in Teams, r in Roundsy)( h[i][r]+a[i][r]);
//constraints
 subject to {
 //(1)Every team plays every other team
  forall (I,j in Teams: i!= j)
 sum (r in Rounds) (x[i][j][r]+x[j][i][r]) ==1;
 //(2) Every team play one match per week
 forall (I in Teams, r in Rounds)
   sum (j in Teams: i!=j) (x[i][j][r] + x[j][i][r]) == 1;
 //(3) No team plays itself
  forall (I in Teams, r in Rounds)
  x[i][i][r] == 0;
 //(4) Every team plays a maximum of (mid+1) matches at home
forall (I in Teams)
sum(j in Teams, r in Rounds) x[i][j][r] <= mid +1;
//(5) Every team plays a minimum of min matches at home
forall (I in Teams)
sum(j in Teams, r in Rounds) x[i][j][r] >= mid;
//(6) Every team plays a maximum of UB consecutive home matches
 forall (I in Teams)
   sum(r in Roundsy) h[i][r] <= UB-1;
//(7)Every team plays a maximum of UB consecutive away matches
forall(I in Teams)
  sum(r in Roundsy) a[i][r] <= UB-1;
//(8) Making h dependent on x
forall(I in Teams, r in Roundsy)
  sum(j in Teams)(x[i][j][r]+x[i][j][r+1]) <= 1 + h[i][r];
//(9) Making a dependent on x
 forall(I in Teams, r in Roundsy)
   sum(j in Teams)(x[j][i][r]+x[j][i][r+1]) <= 1+a[i][r];

//(EX1) Team a must play home in round b
sum (j in Teams) x[a][j][b] == 1;
```

```
//(EX2) Team c must play away in round d
sum (j in Teams) x[c][j][d] == 0;
};
```

Attachment 4 OPL code model 3

```
/*********************************************
 * OPL 12.8.0.0 Model
 * Author: krist
 * Creation Date: Aug 02, 2018 at 9:38:26 PM
 *********************************************/
 using CP;
/****
SETTING THE PARAMETERS
*****/
//Number of teams
int n = 8;
//Number of rounds
int nbSlots = 2*(n-1);
//Range teams
range T = 1..n;
//Range rounds
range S = 1..nbSlots;
//Midseason
int mid = nbSlots div 2;
//Upper bound on breaks
int UB = 3;
/****
VARIABLES
*****/
 dvar    oolean h[T][S]; //h=1, if I plays home in round r
 dvar    oolean a[T][S]; //a=1, if I plays away in round r
 dvar    oolean b[T][S]; //Counts number of breaks
dvar int x[T][S] in T;  //x equals the opponent of team I in round r
```

```
/***
Solver parameters
*****/
execute {
    cp.param.timeLimit=3600;

    cp.param.logPeriod=10000;

    cp.param.DefaultInferenceLevel=”Extended”;
}

/*
OBJECTIVE: Minimize distance
*/
minimize sum(I in T, r in S) b[i][r];
/*******
CONSTRAINTS
*******/
subject to {
//(1) b counts home-breaks
forall(I in T, r in 1..nbSlots-1)
  con100:
  (h[i][r]+h[i][r+1]) <= 1 + b[i][r];
 //(2) b counts away-braks
forall(I in T, r in 1..nbSlots-1)
  con101:
  (a[i][r]+a[i][r+1]) <= 1 + b[i][r];
//(3) a is the opposite of h
forall(I in T, s in S)
a[i][s] != h[i][s];
//(4) Upperbound on number of breaks
```

```
 forall( I in T, ss in 1..11)
 sum(s in ss..ss+UB) h[i][s] <= UB;
//(5) lower bound on number of breaks
 forall( I in T, ss in 1..11)
 sum(s in ss..ss+UB) h[i][s]>= 1;
//(6) If team I play j in round r, j plays I in round r
forall(s in S, I,j in T: i!=j)
  x[i][s] == j => x[j][s] == I;
//(7) x Every team must play every round
forall(s in S, I in T)
  x[i][s] > 0;
//(8) No team plays itself
forall(I in T, s in S)
  con5:
 x[i][s] != I;
//(9) All teams plays eachother the first half of the season
 forall(I in T)
   con6:
   allDifferent(all (s1 in 1..mid) x[i][s1] );
//(10) All teams plays eachother the second half of the season
  forall(I in T)
    con7:
   allDifferent(all (s2 in mid+1..nbSlots) x[i][s2] );
//(11) All rounds required n/2 home matches
forall(s in S)
sum(I in T) h[i][s] ==n/2;
//(12) One team must play home and the otherone away
forall(i1, i2 in T, s in S: i1<i2 )
x[i1][s]==i2 => h[i1][s]+h[i2][s] ==1;
//(13) That every team plays one home match and one away match against each team
forall(I in T, s1,s2 in S: s1<s2)
 x[i][s1]==x[i][s2] => h[i][s1]+h[i][s2]==1;
```

```
};
```

//(Ex1):

```
h[a][b] ==1 //a is the team that must play home in round b
a[c][d] ==1 //c is the team that must play away in round d
```

Attachment 5 OPL code model 4

```
/*********************************************
 * OPL 12.8.0.0 Model
 * Author: krist
 * Creation Date: Jun 7, 2018 at 1:58:29 PM
 *********************************************/
 //LIP formulation source:
 //Rasmussen, R. and M. Trick (2009).
 //"The timetable constrained distance minimization problem." Annals of //Operations
Research 171(1): 45-59.
/* SETTING THE PARAMETERS*/
//Number of teams
int n = 8;
//Number of slots
int nbSlots = 2*(n-1);
//Range teams
range T = 1..n;
//Range teams to match array
range T2 = 0..n;
//Range slots
range S = 1..nbSlots;
//Range slots to match the array
range S0 = 0..nbSlots;
//Range slots with dummy slots to make sure every team starts and end at home
range SD = 0..(2*n-1);
// Upper bounds on breaks
```

```
int UB = 2;
//Distance matrix. Distance between standium of team I and j.
int D[T][T] = ...;
int D2[T2][T2] = ...;
//Timetable, when teams are supposed to play //should be given by our other model
int TT[T][S]=...;
int TT2[T][SD]=...;
int NULL[T][S]=...;

//variables
 //h equals 1 if team I plays home in slot s
 dvar   oolean h[T][SD];
 // The distance team I plays between slot s and slot s+1
 dvar int d[T][S0];

 //Check if our arrays are working
 execute{
        writeln(“The distance    oolean 1 and 6 is” + D[1][6])
        writeln(“In round 1 team 1 will play against “ + TT[1][1])
        writeln(“In round 0 team 1 will play against “ + TT2[1][0])
        writeln(“the travel distance of team 1 between slots 2 and 3 if team I plays
away in both slots” + D2[TT2[1][0]][TT2[1][1]])
}
//Objective: Minimize distance
minimize sum(I in T, s in S0)d[i][s];

//Constraints
subject to {
//(1) Relating the d variable to the other variables
forall (I in T, s in S0)
d[i][s] >= (1- (h[i][s]) – (h[i][s+1])) * D2[TT2[i][s]][TT2[i][s+1]];
//(2)
```

```
forall (I in T, s in S0)
d[i][s] >= ((h[i][s]) – (h[i][s+1])) * D2[i][TT2[i][s+1]];
//(3)
forall (I in T, s in S0)
d[i][s] >= (-h[i][s] -h[i][s+1])*D2[TT2[i][s]][i];
//(4) Every team starts home
forall (I in T)
h[i][0]==1;
//(5) Every team ends home
forall (I in T)
  h[i][15] == 1;
//(6)Every team has n.1 home matches
forall (I in T)
sum(s in S)h[i][s]== n-1;
//(7) In a matchup one team plays home, the otherone away
forall(i1, i2 in T, s in S: i1<i2 && TT[i1][s]==i2)
h[i1][s]+h[i2][s] ==1;
//(8) That every team plays one home match and one away match against each team
forall(I in T, s1,s2 in S: s1<s2 && TT[i][s1]==TT[i][s2])
 h[i][s1]+h[i][s2]==1;
//(9) Upperbound on number of breaks
 forall( I in T, ss in 1..(nbSlots-UB))
 sum(s in ss..ss+UB) h[i][s] <= UB;
//(10) Lower bound on number of breaks
 forall( I in T, ss in 1..(nbSlots-UB))
 sum(s in ss..ss+UB) h[i][s]>= 1;
};
```

Attachment 6 – OPL code model 5

```
using CP;
/****
SETTING THE PARAMETERS
*****/
//Number of teams
int n = 8;
//Number of rounds
int nbSlots = 2*(n-1);
//Range teams
range T = 1..n;
//Range teams to match array
range T2 = 0..n;
//Range rounds
range S = 1..nbSlots;
//Range rounds to match the array
range S0 = 0..nbSlots;
//Range Rounds with dummy rounds to make sure every team starts and end at home
range SD = 0..(2*n-1);
//Midseason
int mid = nbSlots div 2;
//Upper bound on breaks
int UB = 2;
//Distance matrix. Distance between standium of team I and j.
int D[T][T] = ...;
int D2[T2][T2] = ...;

/****
VARIABLES
*****/
//h equals 1 if team I plays home in round s
```

```
 dvar    oolean h[T2][SD];
// The distance team I plays between slot s and slot s+1
 dvar int d[T][S0];
// x equals the opponent of team I in round r
 dvar int x[T2][SD] in T2;
/***
Solver parameters
*****/
execute {

     cp.param.timeLimit=3600;

     cp.param.logPeriod=10000;

     cp.param.DefaultInferenceLevel=”Extended”;

 }

/*
OBJECTIVE: Minimize distance
*/
minimize sum(I in T, s in S0)d[i][s];

/*******
CONSTRAINTS
*******/
subject to {
//(1) Upperbound on number of breaks
 forall( I in T, ss in 1..12)
 sum(s in ss..ss+UB) h[i][s] <= UB;
//(2) Lower bound on number of breaks
 forall( I in T, ss in 1..12)
```

```
 sum(s in ss..ss+UB) h[i][s]>= 1;
//(3) If team I play j in round r, j plays I in round r
forall(s in S, I,j in T: i!=j)
  x[i][s] == j => x[j][s] == I;
//(4) x Every team must play every round
forall(s in S, I in T)
  x[i][s] > 0;
//(5) No team plays itself
forall(I in T, s in S)
  con5:
 x[i][s] != I;
//(6) All teams plays eachother the first half of the season
 forall(I in T)
   con6:
   allDifferent(all (s1 in 1..mid) x[i][s1] );
//(7) All teams plays eachother the second half of the season
  forall(I in T)
    con7:
   allDifferent(all (s2 in mid+1..nbSlots) x[i][s2] );
//(8) All rounds required n/2 home matches
forall(s in S)
sum(I in T) h[i][s] ==n/2;
//(9)
forall (I in T, s in S0)
  con09:
d[i][s] >= (1- (h[i][s]) – (h[i][s+1])) * D2[x[i][s]][x[i][s+1]];
//(10)
forall (I in T, s in S0)
  con10:
d[i][s] >= ((h[i][s]) – (h[i][s+1])) * D2[i][x[i][s+1]];
//(11)
forall (I in T, s in S0)
```

```
  con24:
d[i][s] >= (-h[i][s] -h[i][s+1])*D2[x[i][s]][i];
//(12) One team must play home and the otherone away
forall(i1, i2 in T, s in S: i1<i2 )
x[i1][s]==i2 => h[i1][s]+h[i2][s] ==1;
//(13) That every team plays one home match and one away match against each team
forall(I in T, s1,s2 in S: s1<s2)
 x[i][s1]==x[i][s2] => h[i][s1]+h[i][s2]==1;
//(14)
 forall(I in T)
h[i][0] == 1;
//(15)
forall(I in T)
h[i][2*n-1] == 1;
//(16)
forall(I in T)
x[i][0] == 0;
//(17)
forall(i in T)
x[i][2*n-1]==0;
};
```

Attachment 7 Timetable model 1

The output timetable from model 1, and the timetable input for model 4.

ROUND

Team:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	8	6	4	2	3	5	7	8	6	4	2	3	5	7
2	3	4	7	1	8	6	5	3	4	7	1	8	6	5
3	2	7	5	8	1	4	6	2	7	5	8	1	4	6
4	6	2	1	5	7	3	8	6	2	1	5	7	3	8
5	7	8	3	4	6	1	2	7	8	3	4	6	1	2
6	4	1	8	7	5	2	3	4	1	8	7	5	2	3
7	5	3	2	6	4	8	1	5	3	2	6	4	8	1
8	1	5	6	3	2	7	4	1	5	6	3	2	7	4

Attachment 8 Schedule model 2

The output schedule from model 2, and a timetable input for model 4. The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

ROUND

Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	7	3	4	6	2	8	5	7	3	4	6	2	8	5
2	6	4	7	8	1	5	3	6	4	7	8	1	5	3
3	5	1	8	7	4	6	2	5	1	8	7	4	6	2
4	8	2	1	5	3	7	6	8	2	1	5	3	7	6
5	3	7	6	4	8	2	1	3	7	6	4	8	2	1
6	2	8	5	1	7	3	4	2	8	5	1	7	3	4
7	1	5	2	3	6	4	8	1	5	2	3	6	4	8
8	4	6	3	2	5	1	7	4	6	3	2	5	1	7

Attachment 9 Schedule model 3

The output schedule from model 3, and input timetable for model 4. The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow.

ROUND

Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	4	2	7	6	8	3	5	5	6	8	4	2	3	7
2	7	1	8	3	5	6	4	4	7	3	5	1	8	6
3	5	6	4	2	7	1	8	8	4	2	7	6	1	5
4	1	7	3	8	6	5	2	2	3	7	1	5	6	8
5	3	8	6	7	2	4	1	1	8	6	2	4	7	3
6	8	3	5	1	4	2	7	7	1	5	8	3	4	2
7	2	4	1	5	3	8	6	6	2	4	3	8	5	1
8	6	5	2	4	1	7	3	3	5	1	6	7	2	4

Attachment 10: Schedule (with HAP) Model 4

Output schedule when minimizing travel distance. Predefined timetable derived from attachment 7 (model 1). The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

ROUND

Team:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	8	6	4	2	3	5	7	8	6	4	2	3	5	7
2	3	4	7	1	8	6	5	3	4	7	1	8	6	5
3	2	7	5	8	1	4	6	2	7	5	8	1	4	6
4	6	2	1	5	7	3	8	6	2	1	5	7	3	8
5	7	8	3	4	6	1	2	7	8	3	4	6	1	2
6	4	1	8	7	5	2	3	4	1	8	7	5	2	3
7	5	3	2	6	4	8	1	5	3	2	6	4	8	1
8	1	5	6	3	2	7	4	1	5	6	3	2	7	4

Attachment 11: Schedule (with HAP) model 4

Output schedule when minimizing travel distance. Predefined timetable derived from attachment 8 (model 2). The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

ROUND

Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	7	3	4	6	2	8	5	7	3	4	6	2	8	5
2	6	4	7	8	1	5	3	6	4	7	8	1	5	3
3	5	1	8	7	4	6	2	5	1	8	7	4	6	2
4	8	2	1	5	3	7	6	8	2	1	5	3	7	6
5	3	7	6	4	8	2	1	3	7	6	4	8	2	1
6	2	8	5	1	7	3	4	2	8	5	1	7	3	4
7	1	5	2	3	6	4	8	1	5	2	3	6	4	8
8	4	6	3	2	5	1	7	4	6	3	2	5	1	7

Attachment 12: Schedule (with HAP) model 4

Output schedule when minimizing travel distance. Predefined timetable derived from attachment 9 (model 3). The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

ROUND

Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	4	2	7	6	8	3	5	5	6	8	4	2	3	7
2	7	1	8	3	5	6	4	4	7	3	5	1	8	6
3	5	6	4	2	7	1	8	8	4	2	7	6	1	5
4	1	7	3	8	6	5	2	2	3	7	1	5	6	8
5	3	8	6	7	2	4	1	1	8	6	2	4	7	3
6	8	3	5	1	4	2	7	7	1	5	8	3	4	2
7	2	4	1	5	3	8	6	6	2	4	3	8	5	1
8	6	5	2	4	1	7	3	3	5	1	6	7	2	4

Attachment 13 Schedule Model 5 UB = 2

The output schedule from model 5 when we set UB=2. The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

ROUND

Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	7	3	5	8	4	6	3	5	8	6	4	2	7
2	1	5	7	8	3	6	4	4	3	5	7	6	1	8
3	7	4	1	6	2	5	8	1	2	7	5	8	4	6
4	8	3	6	7	5	1	2	2	7	6	8	1	3	5
5	6	2	8	1	4	3	7	6	1	2	3	7	8	4
6	5	8	4	3	7	2	1	5	8	4	1	2	7	3
7	3	1	2	4	6	8	5	8	4	3	2	5	6	1
8	4	6	5	2	1	7	3	7	6	1	4	3	5	2

Attachment 14 Schedule Model 5 UB = 3

The output schedule from model 5 when we set UB=3. The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

ROUND

Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	4	2	8	5	3	6	7	3	6	5	8	4	2	7
2	3	1	7	4	8	5	6	8	3	7	4	6	1	5
3	2	5	4	6	1	7	8	1	2	4	6	7	5	8
4	1	7	3	2	6	8	5	7	5	3	2	1	8	6
5	8	3	6	1	7	2	4	6	4	1	7	8	3	2
6	7	8	5	3	4	1	2	5	1	8	3	2	7	4
7	6	4	2	8	5	3	1	4	8	2	5	3	6	1
8	5	6	1	7	2	4	3	2	7	6	1	5	4	3

Attachment 15 Schedule model 5 UB = 4

The output schedule from model 5 when we set UB=4. The Team-column plays home versus the teams marked in green and away versus the teams marked in yellow

	ROUND													
Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	6	2	4	3	8	5	7	8	5	6	2	7	4	3
2	4	1	6	7	5	8	3	4	3	7	1	6	5	8
3	8	5	7	1	6	4	2	7	2	8	5	4	6	1
4	2	6	1	8	7	3	5	2	7	5	8	3	1	6
5	7	3	8	6	2	1	4	6	1	4	3	8	2	7
6	1	4	2	5	3	7	8	5	8	1	7	2	3	4
7	5	8	3	2	4	6	1	3	4	2	6	1	8	5
8	3	7	5	4	1	2	6	1	6	3	4	5	7	2

Attachment 16 – Outsourced schedule attained by NVBF

The schedule visualized with included cost and distance per match. Total cost and distance are calculated in the last row.

Round	Home	Away	Distance	Cost
1	Koll	BK Tromsø	1745	494
1	Stod	Førde	653	1226
1	NTNUI	Førde	128	278
1	Randaberg	TIF Viking	206	206
1	TVN	TIF Viking	103	103
2	Koll	TIF Viking	457	470
2	BK Tromsø	Førde	1675	1250
2	Koll	Førde	1745	494
2	Stod	TVN	850	713
2	NTNUI	TVN	128	278
3	TVN	Stod	850	713
3	Randaberg	Stod	103	253
3	TIF Viking	NTNUI	623	627
3	Førde	NTNUI	169	319
4	BK Tromsø	TIF Viking	1771	609
4	Koll	NTNUI	498	445
4	TVN	Randaberg	103	103

5	NTNUI	BK Tromsø	1150	561
5	Stod	BK Tromsø	128	278
5	Førde	TIF Viking	169	169
5	Randaberg	Koll	464	427
5	TVN	Koll	103	253
6	Stod	NTNUI	128	128
6	Førde	Koll	420	466
6	TIF Viking	Koll	169	319
6	Randaberg	BK Tromsø	2206	867
6	TVN	BK Tromsø	103	253
7	Randaberg	Førde	374	1329
7	TVN	Førde	103	253
7	Stod	TIF Viking	750	627
7	NTNUI	TIF Viking	128	278
8	Førde	TVN	393	1329
8	TIF Viking	TVN	169	319
8	Stod	Randaberg	927	713
8	NTNUI	Randaberg	128	278
8	BK Tromsø	Koll	1745	494
9	Stod	Koll	625	445
9	NTNUI	Koll	128	278
10	BK Tromsø	Randaberg	2206	867
10	Koll	TVN	379	427
10	TIF Viking	Stod	750	627
10	Førde	Stod	169	319
11	BK Tromsø	Stod	1030	561
11	TVN	NTNUI	723	713
11	Randaberg	NTNUI	103	253
11	TIF Viking	Førde	169	169
12	Førde	Randaberg	374	1329
12	TIF Viking	Randaberg	169	319
12	BK Tromsø	NTNUI	1150	561
13	BK Tromsø	TVN	2121	867
13	Randaberg	TVN	2206	867
13	Koll	Stod	625	445
14	NTNUI	Stod	128	128
14	Koll	Randaberg	464	427
14	TIF Viking	BK Tromsø	1771	609
14	Førde	BK Tromsø	169	319
		Total	**37023**	**28452**

Attachment 17 – The Mizuno league, season 2017/2018

The schedule visualized with included cost and distance per match. Total cost and distance are calculated in the last row.

Round	Home	Away	Distance	Cost
1	Koll IL	BK Tromsø	1745	494
1	Stod IL	Førde Volleyballklubb	653	1376
1	Toppvolley Norge	TIF Viking	233	233
1	NTNUI Volleyball	Førde Volleyballklubb	128	278
1	Randaberg IL	TIF Viking	103	103
2	Toppvolley Norge	Stod IL	850	863
2	Koll IL	TIF Viking	457	470
2	BK Tromsø	Førde Volleyballklubb	1675	1250
2	Koll IL	Førde Volleyballklubb	1745	494
2	Randaberg IL	Stod IL	103	253
3	Toppvolley Norge	Randaberg IL	103	103
3	TIF Viking	NTNUI Volleyball	623	627
3	Førde Volleyballklubb	NTNUI Volleyball	169	319
4	BK Tromsø	TIF Viking	1771	609
4	Koll IL	NTNUI Volleyball	498	445
5	NTNUI Volleyball	Toppvolley Norge	723	713
5	Stod IL	Toppvolley Norge	128	278
6	Stod IL	BK Tromsø	1030	711
6	Toppvolley Norge	Koll IL	379	577
6	NTNUI Volleyball	BK Tromsø	128	278
6	Førde Volleyballklubb	TIF Viking	169	169
6	Randaberg IL	Koll IL	103	253
7	Toppvolley Norge	BK Tromsø	2121	1017
7	Førde Volleyballklubb	Koll IL	420	466
7	NTNUI Volleyball	Stod IL	128	128
7	Randaberg IL	BK Tromsø	103	253
7	TIF Viking	Koll IL	169	319
8	Toppvolley Norge	Førde Volleyballklubb	393	1329
8	Stod IL	TIF Viking	750	627
8	NTNUI Volleyball	TIF Viking	128	128
8	Randaberg IL	Førde Volleyballklubb	103	253
9	Stod IL	Randaberg IL	927	863
9	BK Tromsø	Koll IL	1745	494
9	NTNUI Volleyball	Randaberg IL	128	278
10	Stod IL	Koll IL	625	445
10	NTNUI Volleyball	Koll IL	128	278

11	BK Tromsø	Randaberg IL	2206	867
11	Førde Volleyballklubb	Stod IL	653	1226
11	Koll IL	Toppvolley Norge	379	427
11	TIF Viking	Stod IL	169	319
12	Toppvolley Norge	NTNUI Volleyball	723	863
12	TIF Viking	Førde Volleyballklubb	169	169
12	BK Tromsø	Stod IL	1030	561
12	Randaberg IL	NTNUI Volleyball	103	253
13	Førde Volleyballklubb	Randaberg IL	374	1329
13	BK Tromsø	NTNUI Volleyball	1150	561
13	TIF Viking	Randaberg IL	169	319
14	BK Tromsø	Toppvolley Norge	2121	867
14	Koll IL	Stod IL	625	445
14	Randaberg IL	Toppvolley Norge	2206	867
15	Koll IL	Randaberg IL	464	427
15	Stod IL	NTNUI Volleyball	128	128
15	Førde Volleyballklubb	Toppvolley Norge	393	1329
15	TIF Viking	BK Tromsø	1771	609
15	Førde Volleyballklubb	BK Tromsø	169	319
15	TIF Viking	Toppvolley Norge	169	319
		Total	**36555**	**29971**

Attachment 18 – Schedule produced by Model 5 after manually adjusting. Minimizing distance, UB=2

The schedule visualized with included cost and distance per match. Total cost and distance are calculated in the last row.

Round	**Home**	**Away**	**Distance**	**Cost**
1	**Førde Volleyballklubb**	**BK Tromsø**	**1675**	**1250**
1	**TIF Viking**	**BK Tromsø**	**169**	**319**
1	**Randaberg IL**	**Stod IL**	**927**	**713**
1	**ToppVolley Norge**	**Stod IL**	**103**	**253**
1	Koll IL	TIF Viking	457	470
2	NTNUI Volleyball	ToppVolley Norge	723	713
2	Førde Volleyballklubb	Randaberg IL	374	1329
2	**NTNUI Volleyball**	**Koll IL**	**498**	**445**
2	**BK Tromsø**	**Koll IL**	**1150**	**561**
3	**TIF Viking**	**Førde Volleyballklubb**	**169**	**169**
3	**ToppVolley Norge**	**Førde Volleyballklubb**	**233**	**233**
3	Stod IL	NTNUI Volleyball	128	128
3	Randaberg IL	ToppVolley Norge	103	103
4	Koll IL	Stod IL	625	445
4	**NTNUI Volleyball**	**TIF Viking**	**623**	**627**
4	**Stod IL**	**TIF Viking**	**128**	**278**
4	**Randaberg IL**	**BK Tromsø**	**2206**	**867**
4	**ToppVolley Norge**	**BK Tromsø**	**103**	**253**
5	Førde Volleyballklubb	Koll IL	420	466
5	**NTNUI Volleyball**	**Randaberg IL**	**800**	**713**
5	**Koll IL**	**Randaberg IL**	**498**	**445**
6	BK Tromsø	NTNUI Volleyball	1150	561
6	**Stod IL**	**Førde Volleyballklubb**	**653**	**1226**
6	**NTNUI Volleyball**	**Førde Volleyballklubb**	**128**	**278**
6	TIF Viking	ToppVolley Norge	233	233
7	BK Tromsø	Stod IL	1030	561
7	ToppVolley Norge	Koll IL	379	427
7	**Randaberg IL**	**TIF Viking**	**206**	**206**
7	**ToppVolley Norge**	**TIF Viking**	**103**	**103**
8	Koll IL	BK Tromsø	1745	494
8	**Førde Volleyballklubb**	**NTNUI Volleyball**	**526**	**1226**
8	**TIF Viking**	**NTNUI Volleyball**	**169**	**319**
8	**Stod IL**	**Randaberg IL**	**927**	**713**
8	**BK Tromsø**	**Randaberg IL**	**1030**	**561**
9	**Koll IL**	**Førde Volleyballklubb**	**420**	**466**
9	**Randaberg IL**	**Førde Volleyballklubb**	**464**	**427**
9	**Stod IL**	**ToppVolley Norge**	**850**	**713**

9	**BK Tromsø**	**ToppVolley Norge**	**1030**	**561**
10	**TIF Viking**	**Koll IL**	**457**	**470**
10	**Randaberg IL**	**Koll IL**	**206**	**356**
10	NTNUI Volleyball	Stod IL	128	128
11	ToppVolley Norge	NTNUI Volleyball	723	713
11	**Stod IL**	**BK Tromsø**	**1030**	**561**
11	**NTNUI Volleyball**	**BK Tromsø**	**128**	**278**
11	Førde Volleyballklubb	TIF Viking	169	169
12	**Førde Volleyballklubb**	**Stod IL**	**653**	**1226**
12	**TIF Viking**	**Stod IL**	**169**	**319**
12	Koll IL	ToppVolley Norge	379	427
12	**TIF Viking**	**Randaberg IL**	**206**	**206**
12	**ToppVolley Norge**	**Randaberg IL**	**233**	**233**
13	BK Tromsø	Førde Volleyballklubb	1675	**1250**
13	**Koll IL**	**NTNUI Volleyball**	**498**	**445**
13	**Randaberg IL**	**NTNUI Volleyball**	**464**	**427**
14	BK Tromsø	TIF Viking	1771	609
14	Førde Volleyballklubb	ToppVolley Norge	393	393
14	Stod IL	Koll IL	625	445
		Total	**33062**	**28510**

Attachment 19 – Schedule produced by Model 5 after manually adjusting using cost matrix, UB=2

The schedule visualized with included cost and distance per match. Total cost and distance are calculated in the last row.

Round	Home	Away	Distance	Cost
1	**TIF Viking**	**BK Tromsø**	1771	**609**
1	**Førde Volleyballklubb**	**BK Tromsø**	169	**319**
1	Koll IL	ToppVolley Norge	379	427
1	**Randaberg IL**	**Stod IL**	927	**713**
1	**ToppVolley Norge**	**Stod IL**	103	**253**
2	**NTNUI Volleyball**	**TIF Viking**	623	**627**
2	**Stod IL**	**TIF Viking**	128	**278**
2	**NTNUI Volleyball**	**Førde Volleyballklubb**	526	**1226**
2	**Stod IL**	**Førde Volleyballklubb**	128	**278**
3	**Koll IL**	**Randaberg IL**	464	**427**
3	**BK Tromsø**	**Randaberg IL**	1745	**494**
3	BK Tromsø	ToppVolley Norge	2121	867
4	**Førde Volleyballklubb**	**Koll IL**	420	**466**
4	**TIF Viking**	**Koll IL**	169	**319**
4	**ToppVolley Norge**	**NTNUI Volleyball**	723	**713**
4	**Randaberg IL**	**NTNUI Volleyball**	103	**253**
5	ToppVolley Norge	Førde Volleyballklubb	393	393
5	Koll IL	Stod IL	625	445
5	**NTNUI Volleyball**	**BK Tromsø**	1150	**561**
5	**Stod IL**	**BK Tromsø**	128	**278**
6	**TIF Viking**	**Randaberg IL**	206	**206**
6	**Førde Volleyballklubb**	**Randaberg IL**	169	**169**
6	Koll IL	NTNUI Volleyball	498	445
6	ToppVolley Norge	TIF Viking	233	233
7	BK Tromsø	Koll IL	1745	494
7	TIF Viking	Førde Volleyballklubb	169	169
7	Stod IL	NTNUI Volleyball	128	445
7	Randaberg IL	ToppVolley Norge	103	103
8	Koll IL	BK Tromsø	1745	494
8	Førde Volleyballklubb	TIF Viking	169	169
8	NTNUI Volleyball	Stod IL	128	128
8	**ToppVolley Norge**	**Randaberg IL**	103	**103**
8	**Stod IL**	**Randaberg IL**	850	**713**
9	BK Tromsø	Førde Volleyballklubb	1675	1250
9	**TIF Viking**	**NTNUI Volleyball**	623	**627**
9	**Førde Volleyballklubb**	**NTNUI Volleyball**	169	**319**
10	**ToppVolley Norge**	**Koll IL**	379	**427**

10	**Randaberg IL**	**Koll IL**	103	**253**
10	**BK Tromsø**	**TIF Viking**	1771	**609**
10	**Koll IL**	**TIF Viking**	1745	**494**
11	**Stod IL**	**ToppVolley Norge**	850	**713**
11	**NTNUI Volleyball**	**ToppVolley Norge**	128	**278**
12	**Randaberg IL**	**BK Tromsø**	2206	**867**
12	**ToppVolley Norge**	**BK Tromsø**	103	**253**
12	**Førde Volleyballklubb**	**Stod IL**	653	**1226**
12	**TIF Viking**	**Stod IL**	169	**319**
13	NTNUI Volleyball	Randaberg IL	800	713
13	**Koll IL**	**Førde Volleyballklubb**	420	**466**
13	**Randaberg IL**	**Førde Volleyballklubb**	464	**427**
14	BK Tromsø	NTNUI Volleyball	1150	561
14	**Stod IL**	**Koll IL**	625	**445**
14	**NTNUI Volleyball**	**Koll IL**	128	**278**
15	BK Tromsø	Stod IL	1030	561
15	**TIF Viking**	**ToppVolley Norge**	233	**233**
15	**Førde Volleyballklubb**	**ToppVolley Norge**	169	**169**
15	Randaberg IL	TIF Viking	206	206
		Total	**34840**	**25511**

Attachment 20 – Breaks in the outsourced schedule (Attachment 16) - 67 breaks

Breaks occurring in the outsourced schedule. Breaks are categorized per team, and the total is calculated in the last row. Breaks are listed by number and consecutive home matches is denoted with H, away matches with A. Rounds with occurring breaks listed in parentheses.

Breaks	**#A/H - (Rounds)**	**Number of breaks per team**
BK Tromsø	1H (2,4) - 3A (5,6) - 4H (8,10,11,12,13) - 1A (14)	9
Førde	3A(1,2) - 2H(3,5,6) - 1A(7) - 1H(8,10) - 1H(12,14)	8
Koll IL	3H (1,2,4) - 6A(5,6,8,9) - 2H(10,13,14)	11
NTNUI	1H(1,2) - 2A(3,4) - 2H(7,8,9) - 2A(11,12)	7
Randaberg IL	1H(1,3) - 2H(5,6,7) - 2A(8,10) - 1A(12)	6
Stod IL	1H(1,2) - 1A(3) - 4H(5,6,7,8,9) - 4A(10,11,13,14)	10
TIF Viking	2A(1,2) - 1A(4,5) - 1A(7) - 4A(8,10,11,12,14)	8
TVN	1A(2) - 4H(3,4,5,6,7) - 2A(8,10) - 1A(13)	8
	Total:	**67**

Attachment 21 – Breaks in the 2017/2018 season of the Mizuno league (Attachment 17)

- 67 breaks

Breaks occurring in the actual 2017/2018 schedule for the Mizuno league. Breaks are categorized per team, and the total is calculated in the last row. Breaks are listed by number and consecutive home matches is denoted with H, away matches with A. Rounds with occurring breaks listed in parentheses.

Breaks	**#A/H - (Rounds)**	**Number of breaks per team**
BK Tromsø	1H(2,4) - 3A(6,7) - 4H(9,11,12,13,14) - 1A(15)	9
Førde	3A(1,2) - 2H(3,6,7) - 1A(8) - 2H(13,15)	8
Koll IL	3H(1,2,4) - 6A(6,7,9,10) - 2H(11,14,15)	11
NTNUI	2A(3,4) - 5H(5,6,7,8,9,10) - 3A(12,13,15)	10
Randaberg IL	1H(1,2) - 2H(6,7,8) - 2A(9,11) - 1A(13)	6
Stod IL	1A(2) - 1H(5,6) - 2H(8,9,10) - 3A(11,12,14)	7
TIF Viking	2A(1,2) - 1A(4,6) - 1A(8) - 4H(11,12,13,15)	8
TVN	2H(1,2,3) - 1A(5) - 2H(6,7,8) - 3A(14,15)	8
	Total:	**67**

Attachment 22 – Breaks in the schedule produced by Model 5, UB=2, minimizing travel distance (Attachment 18)

- 34 breaks

Breaks occurring in schedule produced by Model 5. Breaks are categorized per team, and the total is calculated in the last row. Breaks are listed by number and consecutive home matches is denoted with H, away matches with A. Rounds with occurring breaks listed in parentheses.

Breaks	**#A/H - (Rounds)**	**Number of breaks per team**
BK Tromsø	1A(1) - 1A(4) - 1H(6,7) - 1H(8,9) - 1A(11) - 1H(13,14)	6
Førde	1H(1,2) - 1A(3) - 1A(6) - 1A(9) - 1H(11,12)	5
Koll IL	1A(2) - 1H(8,9) - 1A(10) - 1H(12,13)	4
NTNUI	1H(2) - 1H(4,5) - 1A(8) - 1A(13)	4
Randaberg IL	1H(3,4) - 1A(5) - 1A(8) - 1H(9,10) - 1A(12)	5
Stod IL	1H(1) - 1H(4,6) - 1H(8,9) - 1A(12)	4
TIF Viking	1A(4) - 1A(7) - 1H(8,10) - 1H(12)	4
TVN	1H(7) - 1A(9)	2
	Total:	**34**

Attachment 23 – Breaks in the schedule produced by Model 5, UB=2, minimizing cost (Attachment 19)

- 39 breaks

Breaks occurring in the schedule produced with Model 5. Breaks are categorized per team, and the total is calculated in the last row. Breaks are listed by number and consecutive home matches is denoted with H, away matches with A. Rounds with occurring breaks listed in parentheses.

Breaks	**#A/H - (Rounds)**	**Number of breaks per team**
BK Tromsø	1A(1) - 1H(3) - 1A(5) - 1H(9,10) - 1A(12) - 1H(14,15)	6
Førde	1A(2) - 1H(10,12) - 1A(13)	3
Koll IL	1H(1,3) - 1A(4) - 1H(5,6) - 1A(9) - 1H(10,13) - 1A(14)	6
NTNUI	1H(2) - 1A(4) - 1A(6,7) - 1A(10) - 1H(11,12)	5
Randaberg IL	1A(3) - 1A(6) - 1A(8) - 1H(9,12) - 1H(13,15)	5
Stod IL	1A(1) - 1H(2) - 1H(5,7) - 1H(8,11) - 1A(12)	5
TIF Viking	1A(2) - 1H(4,6) - 1A(10) - 1H(12,15)	4
TVN	2H(4,5,6) - 1H(8,10) - 1A(11) - 1A(15)	5
	Total:	**39**