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The Financial Value Added from Petroleum-Fund Mechanism

Navn: Heidi Dahle, Gine Andrea Granheim

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Gine Andrea Granheim

Heidi Dahle

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Espen Henriksen

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Abstract

In this thesis, we quantify the financial value added by reallocating petroleum revenues from oil reserves to the Government Pension Fund Global. By making a comparable scenario of a hypothetical oil reserve with lower extraction rate and no fund, we identified a value created from the risk reduction of investment diversification. The estimation of value added is sensitive to oil price volatility and investors' level of risk aversion. In addition, we perform a portfolio optimization between the combination of investing in a fund and storing below ground. We find evidence of improvements in mean return, standard deviation and Sharpe ratio, compared to the current strategy of high extraction rate and investing all petroleum revenues in the Government Pension Fund Global.

Acknowledgement

We would like to express our gratitude to our supervisor, Associate Professor Espen Henriksen at BI Norwegian Business School, for valuable input and support throughout the process of writing this thesis. Additionally, we thank Rune Hult for allowing us to access data on the petroleum production from the Norwegian petroleum directorate.

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1 Introduction

The objective of this thesis is to quantify the financial value added by reallocating wealth from oil and gas reserves to a global financial portfolio. This is often overlooked, but is arguably by far the largest financial value added from the Norwegian fund construction. Observers with limited financial insight sometimes focus solely on realized returns for a financial portfolio in isolation and confuse this with "value added". The Norwegian Government Pension Fund Global (GPFG), has been exposed to systematic risk, which is associated with a risk premium in expectation. During the last decade, we have experienced the longest bull market in history, which means the owners have also ex post been rewarded for being exposed to risk. Realized returns from systematic risk exposure is, however, not financial value added in any conventional sense of the term.

Petroleum activities are today Norway's largest industry measured in value creation, government revenue, investment and export value. After the discovery of Norway's large oil reserves, a high extraction rate was established and the Government Pension Fund Global was developed. The GPFG is invested in international listed equities, fixed income and unlisted real estate, with the purpose of having a well-diversified investment portfolio (NBIM, 2018). The fiscal spending rule was initiated as a political coordination mechanism to ensure that the Government's net cash flow from petroleum is transferred to the fund, and that only expected returns of the GPFG is available to cover the national budget deficit. The financial value added from the fund construction stems, by and large, from diversification of national wealth. As far as we are aware, nobody has, however, tried to give a quantitative estimate for this value added. Hence, we find a further investigation of the value added interesting.

To address our research question, we strive to quantify the value creation of reallocating oil- reserve wealth to a global financial fund. Specifically, we contrast the Norwegian strategy with a scenario of only extracting the amount of oil needed to cover national annual consumption, and keeping the excess oil reserves in the ground - a hypothetical oil reserve of oil. With this, we identify the equivalent amount a representative investor would have needed to be indifferent between a concentrated portfolio of oil reserves in the ground and the current globally diversified financial portfolio. In addition, we perform a portfolio optimization based on the diversification between investing in the GPFG and the oil portfolio to see whether it improves the mean return, standard deviation and Sharpe ratio.

The empirical results show evidence of a value creation by reallocating petroleum revenues from oil reserve to Government Pension Fund Global. The estimation of value added is, however, sensitive to the oil price volatility and investors' level of risk aversion. Also, when constructing the minimum variance portfolio, we managed to outperform the Government Pension Fund Global's mean return, standard deviation and Sharpe ratio by weighting the portfolio between investing in the GPFG and the oil portfolio.

The rest of the paper is organized as follows: Section 2 provides an overview of background information and literature on the oil production and Government Pension Fund Global. Section 3 provides theory and performance measurements, and section 4 data and assumptions used in our method. Section 5 provides the methodological approach. Section 6 gives the empirical results with discussion and limitation while section 7 provides a conclusion and suggestions for future research.

2 Background and Literature Review

To be able to answer our research question properly, it is crucial to understand the financial concepts of the Norwegian oil production and the Norwegian Pension Fund Global. The national oil production, net cash flow to Government and consumption will be important for estimating the size and value of our hypothetical oil reserve. The return of the GPFG and the Brent Spot price will be used when quantifying the value added. In addition to these, the MSCI World Index will be used in our portfolio optimization. Hence, in this section we outline the main aspects of the petroleum activities and the construction of the GPFG with key numbers and previous studies significant to our research.

2.1 The Norwegian Oil Production

According to Poplawski-Ribeiro, Villafuerte, Baunsgaard & Richmond (2012), nearly one quarter of the world's countries are dependent on non-renewable resources and for many oil exporting countries, crude oil or gas reserves are their single most important national asset. Any change in the value of reserves directly and materially affects these countries' wealth, and thus the wellbeing of their citizens (Gintschel and Scherer, 2008). Having recognized this, many of oil exporting countries have been depositing oil revenues in funds dedicated to future expenditure, which also applies for the Norwegian Government Pension Fund Global.

The new era of the Norwegian economy and welfare dates back to 1969, when oil was first found in the Northern sea (Regjeringen, 2018). From production started in 1971, over 7 billion standard cubic meters of oil equivalents (Sm^3) has been extracted. *Figure 1* graphically shows the total production on the Norwegian continental shelf, and one can see a steady increase every year as new reserves have been discovered. For the three last years, the annual production has been close to 1,5 billion barrels and according to the Norwegian petroleum directorate, it is still around 55 percent of the expected oil and gas resources that have not yet been retrieved on the Norwegian continental shelf (Dagens Næringsliv, 2018). According to Norwegian

Petroleum, this will only last for approximately 60 more years if the extraction continues in the same pace as today (Teknisk Ukeblad, 2016).

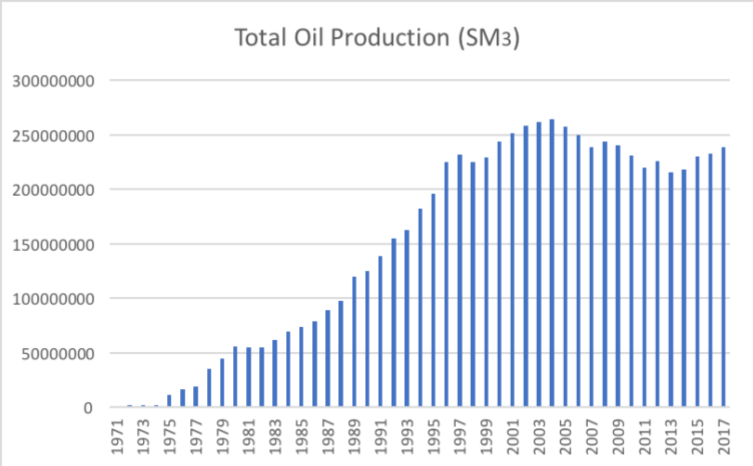

Figure 1: Total extraction of oil 1971-2017

2.1.1 Net Cash Flow to the Government

As the oil resources belong to the people of Norway, the Government gets a large share of the value created through taxation and the States Direct Financial Interest (SDFI) in the petroleum industry (Norsk Petroleum, 2018). The net cash flow to the Government consists of a large portion of tax income, cash flow from the SDFI system including fees and dividend from Equinor. Equinor (formerly Statoil) is Norway's largest petroleum company and 67% of the shares are owned by the Norwegian Government (Norsk Petroleum, 2018). *Figure 2* shows how the revenues from petroleum activities are distributed in the different income areas in the period 1971-2017.

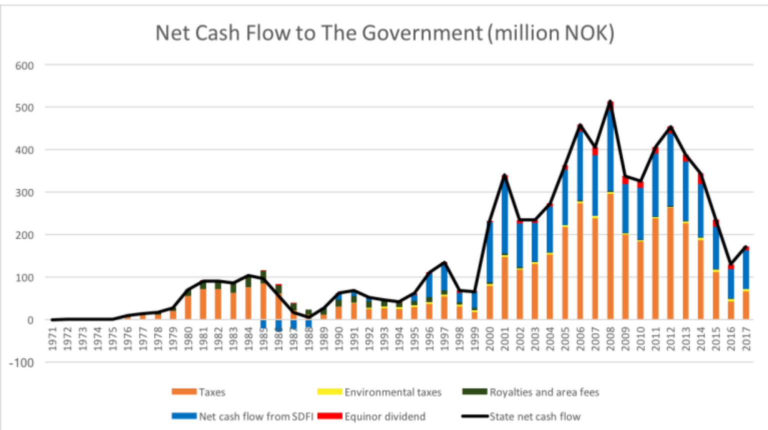

Figure 2: Net Cash Flow to the Government 1971-2017

2.1.2 Oil Consumption

In 2018, about every seventh krone used to cover the Government's budget comes from the Government Pension Fund Global (Norsk Petroleum, 2018). Through the action plan, transfers from the GPFG to the state budget allow us to finance public goods without reducing the fund's capital. Deficit in this budget will be covered with petroleum revenues, while the excess revenues are invested in the fund. *Figure 3* is a graphically representation of the annual oil corrected surplus which amount to approximately 2174 billion NOK. This will represent the oil actually consumed in the period 1980-2017.

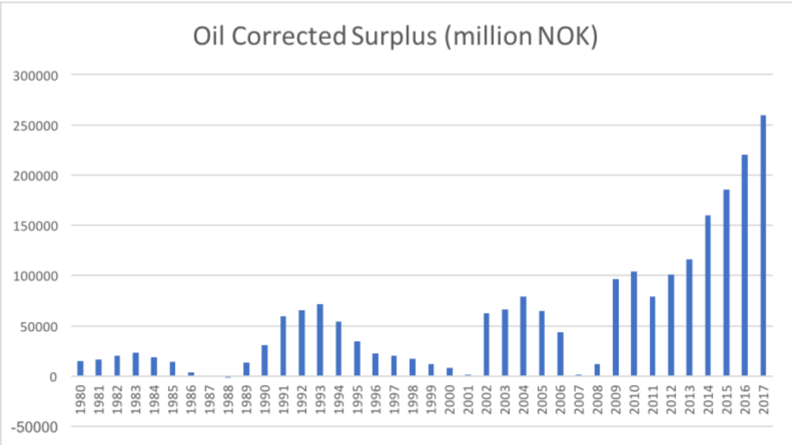

Figure 3: Oil Corrected Surplus 1980-2017

2.2. The Government Pension Fund Global

For the first 20 years of the oil adventure, Norway spent the oil revenues as they were received. The oil price shocks of the 1970s and 1980s created major fluctuations in the fiscal balance and when the oil prices collapsed in the late 1980s, people were tired of these fluctuations and started regretting not taking more advantage of the revenues from the oil production. This created a broad support for constructing a sovereign wealth fund and resulted in establishing the Government Pension Fund Global in 1990 (Wills, Senbet & Simbanegavi, 2016). Today, the Ministry of Finance is the formal owner of the GPFG on behalf of the Norwegian people, but it is the Norwegian Bank Investment Management (NBIM), that manages the GPFG on behalf of the Ministry of Finance. The fund is well diversified between different countries, currencies and markets for both listed equities, unlisted real-estate and fixed income (NBIM,2018).

2.2.1 Market Value

The first deposit to the GPFG was made in May 1996, and ever since, capital from the Government's petroleum revenues has regularly been transferred to the fund. The total market value of the GPFG as of 2017 was approximately 8488 billion NOK or 1020 billion USD. *Figure 4* shows a graphical representation of the development of the GPFG's market value in the period from 1996 to 2017.

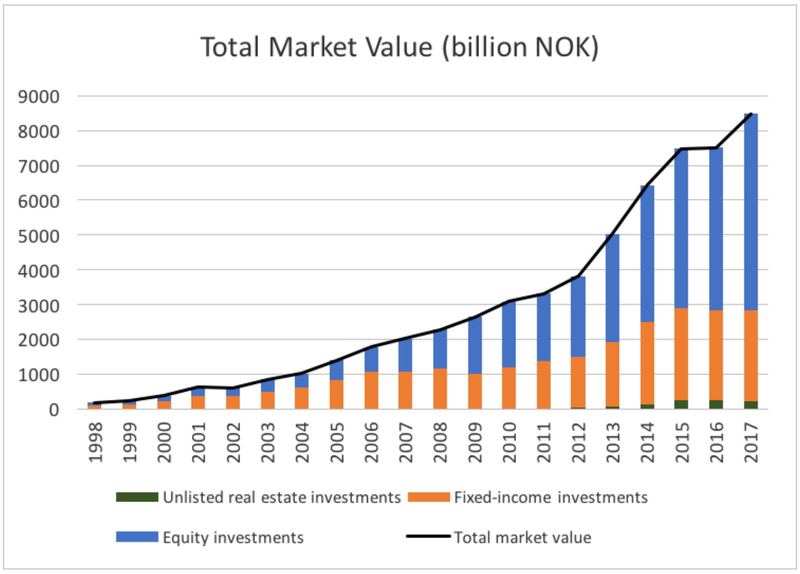

Figure 4: Market Value of the Government Pension Fund 1998-2017

2.2.2 Return

The accumulated annualized return of the GPFG in 2017 was 6,09%, while the annual return was 13,66% (NBIM, 2018). For the majority of the GPFG's lifetime, it has seen positive annual returns. However, the effects of the financial crisis are especially visible with a negative annual return of 23,31% in 2008 and positive return of 25,62% the following year. *Figure 5* graphically presents the annual return and accumulated annualized return.

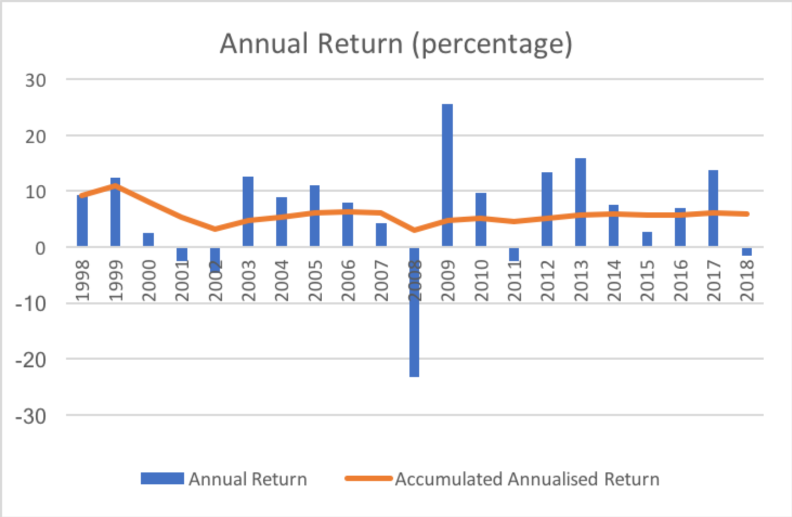

Figure 5: Annual Return of Government Pension Fund 1998-2017

2.3 Risk Factors and Proxies

To account for the risk of keeping the oil below ground in our hypothetical oil reserve scenario we will solely depend on the volatility of the Brent Spot price, while we will use the MSCI Global index as proxy for the GPFG's return in the period before the fund construction.

2.3.1 Brent Spot Price

There are several factors that could have been considered when evaluating the risk of storing the oil below ground. Examples are the risk of the oil becoming worthless or the risk of other nations exploiting the Norwegian reserves. Factors such as these are close to impossible to measure and will therefore be neglected in our research. Lin Gao, Steffen Hitzemann, Ivan Shaliastovich, and Lai Xu (2016) provides empirical evidence that oil price variance captures significant information concerning economic growth and asset prices. Their empirical findings show in periods of high uncertainty, oil producers tend to increase their inventories to alleviate the probability of stock-outs. This shows that the oil price has a considerable effect on the oil production and consequently the oil reserves.

The oil price volatility will therefore, be the main source of risk for our hypothetical oil reserve and we have more specifically chosen to use the Brent Crude Oil as this is the most common type of oil in Norway. The Brent Spot price is known to have periods of high volatility, which makes it hard to produce accurate forecasts. The uncertainty around future prices, makes it a significant risk factor for the Norwegian Government and the GPFG is a strategy for protecting against the price fluctuations. The annual spot prices with respective volatility is presented in *Figure 6*.

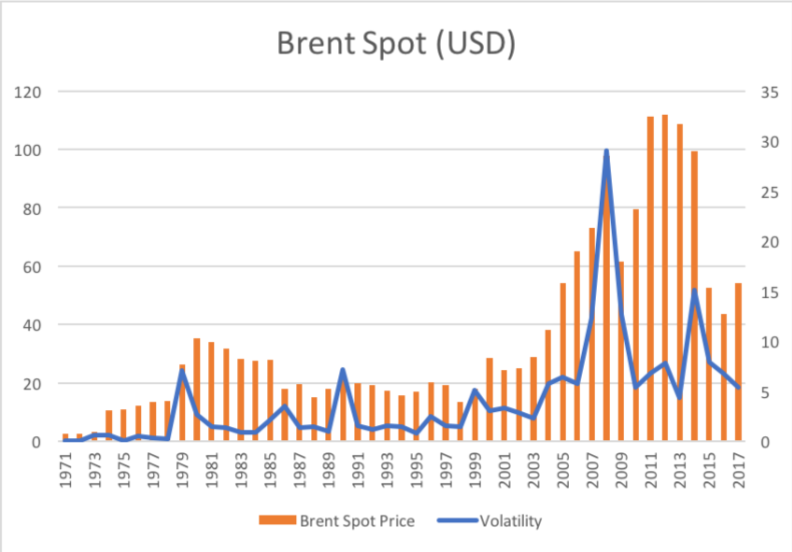

Figure 6: Brent Spot Price and Volatility 1971-2017

2.3.2 MSCI World Index

The GPFG's investments are measured against a benchmark index based on indices from FTSE Group and Bloomberg Barclays Indices (NBIM, 2018). However, since FTSE Group index did not exist in the period 1971-1997, and Barclays only contains bonds, we will use data on the MSCI World Index. MSCI stands for Morgan Stanley Capital International and was the first global market index created in 1968. It measures the performance of 4500 large and mid-cap companies, and is often used to describe the conditions of the world stock market (The Balance, 2018). The MSCI index will make it possible to look at the differences in the optimal portfolio for the period leading up to the creation of the fund. We will use it as a proxy for the GPFG in our analysis for the portfolio optimization in the period from 1971 to 1997.

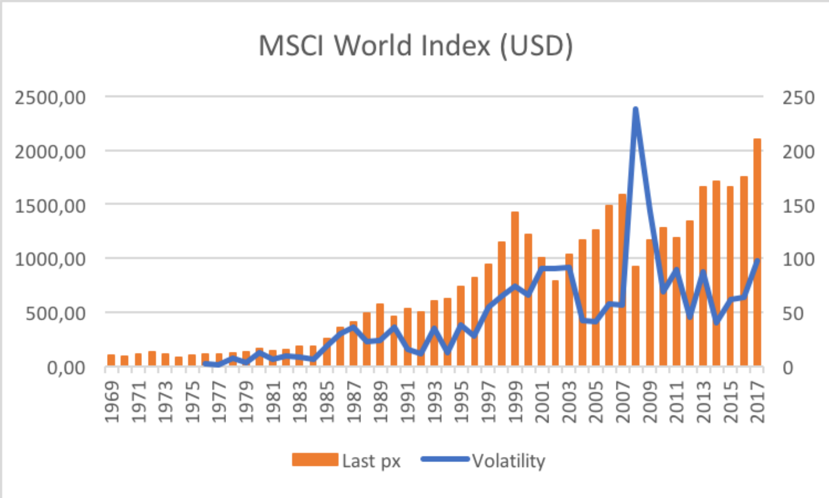

Figure 7: MSCI World Index 1969-2017

2.4 Previous studies

Most studies conducted on the topic has focused on the risk and return of the financial fund in isolation, but without considering the main source of value added from the fund construction itself. Existing theories of optimal oil extraction is also known for not considering the volatile financial markets. For especially oil exporters, this will be important factors as the prices are known to be highly volatile, which at times leads to periods where the below-ground assets are worth more than the above-ground fund (Van Den Bremer et al, 2016).

There have been several studies on valuing oil and gas reserves as well as checking different factors influencing the value and return. Clinch and Magliolo (1992) found that changes in reserves due to production dominated all other reserve information when measuring a firm's reserve stock. Contrary, Spear (1994) and later Cormier and Magnan (2002), find that discoveries are more important than production. Spear (1994) found that the individual components of reserve amounts changes (such as discoveries, production, purchases) improved the relationship with returns. More recent research by Boyer & Filion (2007) did however conclude with a negative relationship between changes in production and returns.

Miller and Upton (1985) presents an implication of the Hotelling model, called the Hotelling Valuation Principle. The Hotelling Valuation Principle is a disputed valuation method for the in-situ value of non-renewable resources. It states that the value of a unit is equal to current price less the cost of extraction. This follows from the fact that, given value maximization, the net price rise at the rate of interest which results in the present value of the net price will be the same regardless of when the resource is extracted. There are several assumptions required for this valuation principle, and many of them are violated especially in the oil and gas industry. Despite this, there exist numbers of mixed results from previous studies.

Further, Van den Bremer et al (2016) found that subsoil oil should alter a fund's portfolio through additional leverage and hedging, consumption should be a constant share of total wealth and any unhedged volatility must be managed by precautionary savings. They also suggested the rate of oil extraction should be higher if oil prices are volatile and positively correlated with financial markets, generating a higher rate of return on subsoil oil as compensation for the risk it is exposed to.

The Norwegian Bank Investment Managers continuously valuate the GPFG's market value and realized returns from systematic risk exposure and the Norwegian petroleum directorate regularly estimates the values of the remaining oil reserves. Despite this, the empirical literature has not uncovered the total financial value added by investment diversification. This is what we attempt to explore further in our analysis.

3 Theory

In this section, we present and explain the main theories used to support our research. To estimate the size and value of the hypothetical oil reserve, we use simple mathematics inspired by Hotelling Valuation Principle. To quantify the value added by reallocating petroleum revenues from hypothetical oil reserve to Government Pension Fund Global, we rely on Utility Valuation. Lastly, our portfolio optimization will be based upon Modern Portfolio Theory and evaluated by the performance measurements; mean return, standard deviation and Sharpe ratio.

3.1 Valuation Principle

To estimate a value for our hypothetical oil reserve we will use the Hotelling Valuation Principle, which states that the value of a unit is equal to current price less the cost of extraction.

The value of an exhaustible natural resource reserve can be calculated by taking the expected cash generated in each year into the future and discounting each year back at the appropriate rate (Shumlich and Wilson, n.d.),

$$V = \sum_{t=0}^{\infty} \frac{(p_t - c_t)q_t}{(1+r_t)^t} \tag{1}$$

V = current value
t = time of cash flow
p = selling price per unit
c =extraction cost per unit
q = quantity extracted and sold
r = fair discount rate (or cost of capital).

The value is constrained by the total quantity of the reserves, R,

$$\sum_{t=0}^{\infty} q_t \leq R \tag{2}$$

The Hotelling (1931) principle states that the present value of the net price at any future time should be equal to the current net price,

$$\frac{(p_t - c_t)}{(1 + r_t)^t} = p_0 - c_0 \tag{3}$$

By substituting this into equation (1), we get

$$V = (p_0 - c_0) \sum_{t=0}^{\infty} q_t \tag{4}$$

The constraint equation (2) will hold with equality by only considering economically viable reserves, which implies we get an even more simplified equation

$$V = (p_o - c_0)R \tag{5}$$

We now have that the value of a reserve depends on the current price, current cost and the size of the reserve.

3.2 Utility Valuation

Risk averse investors penalize the expected return of a risky portfolio to account for the risk involved. The different degrees of risk aversion are considered to be from 1 to 5, where 5 is the highest level of risk aversion. To quantify the rate at which investors are willing to trade off return for risk, we assume that each investor can assign a utility score to competing portfolios on the basis of expected return and risk of those portfolios.

Portfolios receive higher utility scores for higher expected returns and lower scores for higher volatility. We can interpret the utility score of risky portfolios as a certainty equivalent rate. The certainty equivalent is the rate that, if earned with certainty, would provide a utility score equivalent to that of a portfolio under consideration (Bodie, Kane and Marcus, 2014). To put it another way, the certainty equivalent is the guaranteed amount of cash that would yield the same exact expected utility as a given risky asset with absolute certainty, and represents the opportunity cost of risk.

The utility score is calculated as follows;

$$U = E(r) - 0{,}5A\sigma^2$$

U = the utility value
A = investors risk aversion
E(r) = expected return on portfolio
σ = variance of returns

This formula is consistent with the notion that utility is enhanced by expected returns and diminished by high risk.

3.3 Modern Portfolio Theory

We will use modern portfolio theory - a method introduced by Harry Markowitz in the 1950s - for constructing a portfolio that will maximize returns for a given level of risk. It is based on the correlation and variance of the assets and how each asset contributes to the risk and return of the portfolio. The theory assumes that investors are risk adverse, and that they require higher expected returns when taking on more risk. It allows investors to determine how to spread their investment in order to construct a portfolio which gives the best risk-return trade-off available (Jorion, 1992).

3.3.1 Efficient Frontier

The risk- expected return relationship of efficient portfolios is graphically represented by a curve known as the efficient frontier. The efficient frontier is the optimal correlation between risk and return in modern portfolio theory. All efficient portfolios, each represented by a point on the efficient frontier, are well-diversified and the optimal allocations at various levels of risk preference (Werne, 2015).

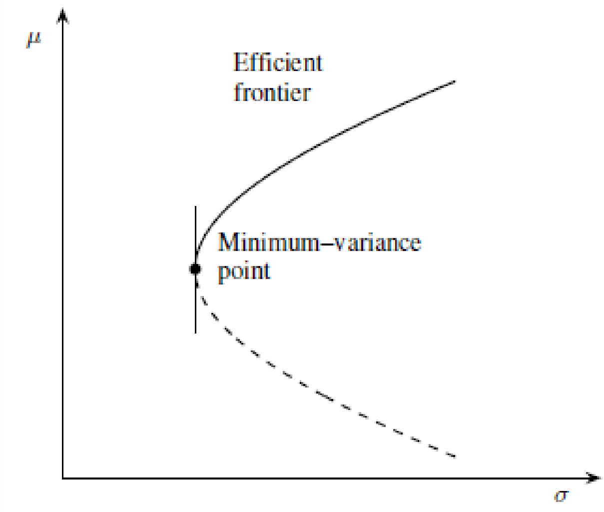

Figure 8: Efficient frontier

3.3.2 Covariance

In modern portfolio theory, the covariance is a statistical measurement used to reduce the overall risk for a portfolio. A positive covariance means that assets generally move in the same direction and negative covariance in opposite directions. Analysts use historical price data to determine the measure of covariance between different assets. This assumes that the same statistical relationship between the asset prices will continue into the future, which is not always the case. The goal is to include assets that show a negative covariance to minimize the risk of a portfolio. The covariance formula takes the daily return minus the mean return for each asset, multiplied by each other, divided by the amount of trading periods for the respective time frame.

$$\sigma_{x,y} = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

3.3.3 Correlation

According to modern portfolio theory, investors are able to diversify away the risk of investment loss by reducing the correlation between the returns from the selected securities in their portfolio. Investors should measure the correlation coefficients between the returns of different assets and strategically select assets that are less likely to lose value at the same time. The closer to 1, the more positively and perfectly correlated are the assets. A perfectly negative correlation (-1) implies that one asset's gain is proportionally matched by the other asset's loss. A zero correlation has no predictive relationship. The correlation formula takes the covariance divided by the standard deviation of asset x multiplied by asset y.

$$\rho_{x,y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}$$

3.4 Performance Measurements

The performance elements presented in the following section will be used to evaluate our results in section 6. We calculate the mean excess returns and standard deviation of each asset, and we find the portfolio performances by calculating the optimal weights of diversification, it expected return, standard deviations and Sharpe ratio.

3.4.1 Expected Return

Expected return is the profit or loss an investor anticipates on an investment that has known or expected rates of return. For a single asset, the expected return is simply the average return of the asset calculated by summing all the observations and dividing it by the number of observations. Expected return for a two-asset portfolio depends on the expected return for each asset with the corresponding weights invested in each.

$$\bar{R}_p = X_1 \bar{R}_1 + X_2 \bar{R}_2$$

3.4.2 Standard Deviation

Standard deviation is a statistic that measures the dispersion of a dataset relative to its mean. It is calculated as the square root of variance by determining the variation between each data point relative to the mean. If the data points are further from the mean, there is higher deviation within the data set; thus, the more spread in the data, the higher the volatility. The standard deviation of the portfolio is displayed in the formula below, where ρ is the correlation between the two risky assets.

$$\sigma_p = [X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho]^{1/2}$$

3.4.3 Sharpe Ratio

We measure the Sharpe ratio to test whether the combined portfolio have a higher return than the Government Pension Fund Global. Sharpe ratio was developed by Sharpe in 1966, and is one of the most common measurements of risk-adjusted performance. The ratio is calculated ex post as excess return of the portfolio divided by the standard deviation of the excess return. As for our portfolio performance, the risk-free rate is assumed zero.

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}$$

4 Data Description and Assumptions

Several different sources have been used to collect all necessary data for the different parts of our analysis. We will overall use monthly frequencies denoted in USD from June 1971, until December 2017, which gives us a span of 46 years and 7 months to analyze. A summary of all data conducted is listed in *appendix 8*. In the following section, we give a brief overview of the data used in valuing the hypothetical oil reserve and value added, data used in portfolio optimization, risk factors, proxies and the assumptions underlying our analysis.

4.1 Data Used to Calculate Size and Value of Oil Reserve

Below we have listed and the described the data used to calculate our hypothetical oil reserve and further the value added.

4.1.1 USD/NOK Exchange Rate

We obtained the monthly NOK/USD exchange rate from Bloomberg in the period 1971-2017, consisting of 559 observations. Since some data necessary in our research is denoted in Norwegian kroner and some in U.S. Dollars, we will use the USD/NOK exchange rate to convert all data denoted in NOK to USD. This means all results presented in section 6 will be expressed in USD.

4.1.2 Oil Extraction

We received data on the total national oil production from the Norwegian petroleum directorate. The full range of monthly extraction for the period June 1971 until December 2017 includes 559 unique observations denoted in oil equivalents (Sm^3).

4.1.3 Net Cash Flow to the Government

The Net cash flow to the Government is important for our research as it will represent the amount of physical oil available to the Government. Consequently, we will adjust the total amount of oil extracted, to the amount needed to cover total net cash flow to the Government in our research.
From Norwegian Petroleum's websites, we retrieved the Government's total net cash flow from petroleum activities in NOK. The annual data spans from 1971 until 2017 with 47 observations. However, since we use monthly observations denoted in USD in our analysis, we had to convert our annual data into monthly. The number of observations was adjusted to 559 and total cash flow to the Government is 7 290 billion NOK or USD 1 118 billion USD.

4.1.4 Oil Corrected Surplus

From the Norwegian Governments webpages, we extracted the historical numbers on yearly structural oil-corrected surplus which in this analysis will be used as yearly consumption of oil revenues. The data consist of 38 annual observation which was adjusted to monthly observations. Together with the data of extraction 1971-1980 our total observations are 559 and total consumption from the entire period is 2 249 billion NOK. This is equivalent to the NOK amount we would need from oil revenues to cover the deficit from the national budget.

4.1.5 Cost of Producing a Barrel of Oil

To realize the value of the below ground reserves, the cost of producing barrels of oil will accrue. Therefore, we assume the production costs have a substantial impact when calculating the value of the hypothetical oil reserve. Due to the last years' low oil prices, there has been increased focus on production and cost efficiency in the petroleum industry. According to E24 (2018), some companies, such as Equinor and Lundin have managed to shrink their production cost to below USD 5,00, but the average production cost per barrel in Norway is USD 11,00. However, according to Wallstreet Journal's Barrel Breakdown (2016), the average cost of producing one barrel in Norway is USD 21,31 and includes gross taxes, capital spending, production costs and administrative/transportation cost. We assume all these aspects are relevant for our research and choose to include the total cost presented in *table 1* when calculating the value of the hypothetical oil reserve and value added.

Table 1: Cost of Producing a Barrel of Oil

Gross taxes	**USD 0,19**
Capital spending	USD 13,76
Production costs	USD 4,24
Administrative/transportation costs	USD 3,12
Total	USD 21,31

4.2 Data Used to Calculate Value Added and Optimal Portfolio Diversification

We will perform portfolio optimization for two different periods, 1971 to 1997 using data on Brent Spot and MSCI Index and 1998 to 2017, using data on Brent Spot and return from the GPFG. The data will be used to construct the variance-covariance matrix and correlation matrix.

4.2.1 Return and Variance of Government Pension Fund Global

Data on monthly return was obtained from NBIM's webpages. The data consists of 240 observations from 1998-2017. We will use this data to find the average annual return and variance for the GPFG, which will be one of the assets in our portfolio optimization in the second period.

4.2.2 Brent Spot Price

We collected data on the monthly Brent Spot prices from June 1971 until December 2017 from Macrobond. The Brent Spot price will be converted into returns, and act as the returns of a concentrated portfolio of oil. This portfolio will be one of two assets available for the portfolio optimization in both periods.

4.2.3 MSCI World Index

The monthly data for the period of 1971 to 1998 was obtained from Thomsom Reuters Datastream. The MSCI World Index, described deeper in section 2.3.2, will act as a proxy for the GPFG in first period of our analysis.

4.3 Covariance and Correlation between the variables

4.3.1 Variance - Covariance Matrix

Table 2 presents the variance-covariance matrices for the two periods. The diagonal elements of the matrix contain the variances of the variables and the off-diagonal elements contain the covariance between all possible pairs of variables. The positive covariance between the GPFG and the reserve indicates that the two variables show similar behavior. The MSCI index has a negative covariance with the reserves, which indicates that if MSCI tends to increase, the reserves will decrease.

Table 2: Variance - Covariance matrix

Period 1: 1971-1997		
	MSCI	**Oil Port.**
MSCI	0,0225	-0,0023
Oil Port.	-0,0023	0,1742

Period 2: 1998-2017		
	GPFG	**Oil Port.**
GPFG	0,0107	0,0074
Oil Port.	0,0074	0,0951

4.3.2 Correlation Matrix

As seen in *table 3*, the correlation between the GPFG return and MSCI is approximately 0,784 which is a decent positive relationship and indicates that they move together and that MSCI is a relatively good proxy for the GPFG's return. The GPFG and MSCI`s correlation with the oil portfolio is 0,232 and 0,102 respectively. This is in line with modern portfolio theory, which stresses that investors should look for a consistently uncorrelated (near zero) pool of assets to limit risk.

Table 3: Correlation Matrix

	Oil Portfolio	**GPFG Return**	**MSCI**
Oil Portfolio	1.0000	0.2321	0.1020
GPFG Return	0.2321	1.0000	0.7844
MSCI	0.1020	0.7844	1.0000

4.4 Assumptions

As some relevant factors are difficult to measure or find, in addition to the desire to make the research as simple and feasible as possible, the following assumptions have been taken.

Assumption 1: Extraction of oil includes both oil and gas, where we use the extraction measured in Sm^3 of oil equivalents. Further, we do not account for the differences between the oil and gas prices, hence the values are calculated using the Brent Spot price.

Assumption 2: Mentioned in section 2, there is estimated to still be approximately 55% of petroleum below ground. We do not take into account the petroleum reserves not yet retrieved or discovered.

Assumption 3: As described in section 4.1.3, the net cash flow to the Government consists of both dividends, taxes and fees payed by petroleum companies, and does not reflect direct revenues from selling oil. This combination makes it hard to accurately estimate the quantity of barrels needed to cover the deficit in the national budget. The total amount of oil extracted is therefore considered to be the total net cash flow to the Government adjusted to barrels in the period 1971-2017.

Assumption 4: Since we do not have any data on consumption in the period 1971-1980, we assume that all oil extracted in this period was consumed.

Assumption 5: The valuation of the oil reserve will be done with the assumption that at end of 2017, all oil left in reserves will be extracted and sold to the fixed Brent Spot price in the same period, subtracted the average production cost.

Assumption 6: The volatility of the oil reserve is solely based upon the Brent Spot price.

Assumption 7: The MSCI index is a proxy for the Government Pension Fund's return in the period 1971-1998.

Assumption 8: All capital considered in our portfolio optimization is invested in risky assets. Therefore, capital allocations between the risky portfolio and the risk-free asset is not a subject of discussion in this thesis and the risk-free rate is assumed to be zero.

The purpose of the assumptions will be described closer in next section's methodology.

5 Methodology

In the first step of our method of research, we construct and value a hypothetical oil reserve as a comparable scenario to investing in the Government Pension Fund Global. Secondly, we will identify the equivalent amount a representative investor would have needed to be indifferent between a concentrated portfolio of oil reserves in the ground and the current globally diversified financial portfolio, given a set of preferences for expected return and variance. This will further quantify the value added from diversification. Third and last, we will do a portfolio optimization, based on modern portfolio theory on the diversification between investing in the fund and storing below ground to see whether it improves our performance measurements; standard deviation, mean and Sharpe ratio. In this section, we present a more thoroughly description of the three steps of our research.

5.1 Calculating the Value of Oil Reserve

First, we will construct a hypothetical oil reserve based on the Hotelling Valuation Principle, which will represent a scenario with lower extraction and where the GPFG was never established. Oil and gas reserves are usually classified in two main groups; proved and probable. In most cases, the recoverable volume is not certain, but in our case, we assume the size of the reserve is proved and certain. Adelman, Koehn and de Silva (1989) states that it is common to combine oil and gas to a value per barrel of oil equivalent when determining the value of in-ground reserves, which is what we will do. *(see assumption 1)*

The data on total extracted oil equivalent in standard cubic meter (Sm^3) will be used to estimate the total amount of oil barrels extracted in the period from 1971 until 2017. One barrel equals 6,289814 standard cubic meters of oil. Further, we will estimate a value of the barrels extracted each month in USD using the Brent Spot price *(see assumption 2)*. The total amount of oil extracted is considered to be the total net cash flow to the Government adjusted to barrels in the period 1971-2017 *(see assumption 3).* The consumption and net cash flow to the Government are stated in NOK, hence we must convert these into USD before converting the monetary value into barrels of oil.
In the period 1971-1980 we assume the number of barrels consumed to be equal to the number extracted *(See assumption 4)*. We will further subtract the monthly estimated number of barrels consumed from the monthly number of barrels needed to cover the Governments cash flow. This will provide an estimate of the number of barrels left in the reserve each year as well as the total number of barrels left in the reserve by the end of 2017. Ultimately, we will value the end of 2017 reserve using the end of 2017 Brent Spot Price adjusted for production costs *(see assumption 5).*

5.2 Calculating the Value Added

The second step of our research is based upon the theory presented in section 3.2. In order to compare the hypothetical oil reserve constructed in previous section to today's scenario with the Government Pension Fund Global, we must assign a utility score for our two portfolios; the GPFG portfolio and the oil portfolio. We will be dependent on the Brent Spot data and GPFG return in the period from the fund was constructed until 2017, to calculate the two portfolios expected return, standard deviation and Sharpe ratio *(see assumption 6)*. Further, we present utility values for the 1 to 5- range of levels of risk aversion, and use the estimated certainty equivalent to identify how much higher the expected return would have to be for an investor to be indifferent between the riskier portfolio of oil and the diversified portfolio of the GPFG.

The certainty equivalent is the guaranteed return that an investor would accept from investing 100% in the oil portfolio rather than investing 100% in the GPFG. The estimated certainty equivalents will be multiplied with the valuation of the hypothetical oil reserve which will obtain 5 different values, representative for the financial value added by reallocating petroleum revenues from oil reserve to GPFG. The extra value required for an investor to be indifferent between the two portfolios will therefore reflect the value added from reducing risk exposure through investment diversification.

Lastly, we present an alternative method to show the amounts of expected return and standard deviation an investor would require to attain the same Sharpe ratio for both scenarios and be indifferent between investing 100% in the GPFG or 100% in the hypothetical oil portfolio.

5.3 Calculating the Optimal Portfolio Weights

In the last step of research in this paper, we will check if we can improve today’s fund return and standard deviation by building an optimal portfolio diversification between investing in the fund and storing the oil. We will perform the test twice; First with data available for the Norwegian investment managers from 1971-1997, that is Brent spot and the MSCI index. The results will give an indication of what would be the optimal portfolio given only the data available at the time the GPFG was established. For the second test, we will use Brent spot and actual data on the return in the period 1998-2017. The se results will give an indication of whether the first test was a good forecast for the actual return in the second test. The resulting portfolio weights, return, standard deviation and Sharpe ratio will be evaluated.

For the portfolio optimization, we will assume the following two alternatives are the only assets possible to invest in:

Asset 1- GPFG

Investing in the GPFG - which again is well-diversified in the stock market - we use two different periods in our analysis. For the first period, we will use the MSCI Index as a proxy for the fund return with data from 1971 to 1997 (*see assumption 7).* The second period will consist of the realized return of the GPFG, with data from 1998 to 2017.

Asset 2 – Oil Portfolio:

We use the hypothetical reserve as an asset, constructed by assuming we store the oil below ground and only extracting what needed to cover national consumption. The Brent spot will be turned into returns and the standard deviation of the Brent spot, for the two periods, will be used as a measure of risk for this asset in our analysis *(see assumption 6)*.

We will build a model calculating the portfolio's expected return and standard deviation dependent on the weights invested in the two risky assets involved *(see assumption 8.)* Further, we find optimal weights given the constraints of maximum return and minimum standard deviation. This means we will solve the optimization problem by first finding the highest possible return ignoring standard deviation. Second, we will find the portfolio with lowest possible risk. Our final conclusion and comparison will be dependent on the lowest possible standard deviation achieved. In other words, the minimum variance portfolio.

6 Results and Analysis

In this section, we will present our results from section 5.1, 5.2 and 5.3 with respective performance evaluation and limitations to our research.

6.1 The Value of the Oil Reserve

For simplicity, the total amount of oil extracted was assumed to equal total net cash flow to the Government converted to barrels of oil equivalents for the period 1971-2017. This adjusts the total number of barrels available for the Governments disposal from 44 542 743 245 to 25 741 814 642. Based on this logic, the estimated size of the oil reserves at the end of 2017 will be 16 866 380 828 barrels of oil.

Our valuation of the hypothetical oil reserve, taking production costs into account, will be USD 726 266 358 454. *Table 4* presents a summary of our key numbers denoted in barrels for the extraction, net cash flow, consumption and hypothetical oil reserve, in addition to our final valuation of the oil reserve. This valuation is used in the next section for the value added. In *appendix 1 and 2*, is an annual presentation of the monthly data used in our calculations of *table 4*.

Table 4: Estimation of the Hypothetical Oil Reserve

Number of barrels extracted	44 542 743 245
Number of barrels as net cash flow to Government	25 741 814 642
Number of barrels consumed	8 875 433 814
Number of barrels in oil reserve end 2017	16 866 380 828
Value of oil reserve including production cost	**USD 726 266 358 454**

6.2 The Value Added

Table 5 presents the expected return, standard deviation and Sharpe ratio of our two portfolios. The expected return and standard deviation are further used to calculate the utility score for the respective portfolios. The utility score calculated from the GPFG's return and standard deviation will further present the basis for our certainty equivalent.

In contrast to the well-diversified GPFG with an expected return of 6,808% and standard deviation of 10,385%, we have the concentrated oil portfolio with an almost doubled return and tripled risk. The extra values required for an investor to be indifferent between the two portfolios will therefore reflect the value added from reducing risk exposure through investment diversification.

Table 5: Expected Return, Standard Deviation and Sharpe Ratio

	GPFG	**Oil Portfolio**
μ:	6,808 %	11,371 %
σ:	10,385 %	30,843 %
Sharpe:	0,656	0,369

In *table 6* we present the utility scores with respective values required at different levels of risk aversion. When the Norwegian investors have a risk aversion level of 4, the certainty equivalent must be 0,238, which is equivalent to requiring an extra value of USD 172 536 427 270 to be indifferent between investing 100% in the GPFG and 100% in the Oil portfolio. Similarly, at a risk aversion level of 5 the certainty equivalent is 0,280, requiring extra value of 203 309 480 667. These estimated values are interpreted as the value added by reducing risk exposure through investment diversification. *Appendix 3 and 4* is a presentation of how the value added changes when valuing the hypothetical oil reserve with different Brent spot prices and production costs when assuming the level of risk aversion is 4.

Table 6: Utility Score, Certainty Equivalent

Investor Risk Aversion	***Utility Score GPFG***	***Utility Score Oil Portfolio***	***Certainty Equivalent***	***Value Added***	***Total Value required***
1	0,063	0,066	0,110	80 217 267 080	806 483 625 534
2	0,057	0,018	0,153	110 990 320 477	837 256 678 931
3	0,052	-0,030	0,195	141 763 373 874	868 029 732 328
4	0,047	-0,077	0,238	172 536 427 270	898 802 785 725
5	0,041	-0,125	0,280	203 309 480 667	929 575 839 121

This table shows at given level of risk aversion, column 1, the utility score for the Government Pension Fund Global and the oil portfolio in column 2 and 3. The certainty equivalent and the equivalent extra value required to be indifferent to invest in the two portfolios in column 4 and 5. Column 6 is the total amount the oil portfolio must be valued at for an investor to be indifferent between the two portfolios.

In *table 7* we present another method to show the amounts of expected return and standard deviation an investor would require to attain the same Sharpe ratio for both scenarios and be indifferent between investing 100% in the GPFG or 100% in the hypothetical oil portfolio. The minimum expected returns an investment manager would require if investing 100 % in the GPFG when taking on the same risk as investing 100% in the hypothetical oil portfolio is 20,233%. The maximum standard deviation an investment manager would accept investing 100% in hypothetical oil reserve when requiring the same return as investing 100% in the GPFG is 17,334%.

Table 7: Sharpe Ratio as Indifference Measurement

	GPFG	**Oil Portfolio**
μ:	20,233 %	11,371 %
σ:	30,843 %	17,334 %
Sharpe:	0,656	0,656

6.3 The Optimal Portfolio Diversification Results

Table 8 shows a summary of the mean return, standard deviation and Sharpe ratio of the data used in the two portfolio optimizations performed. Looking at period 1 as a prediction for the investment managers in the years leading up to 1998, and period 2 as the actual return of our portfolio, we can check if period 1 is a good prediction for period 2.

We see the return of the hypothetical oil reserve (11,371%) is almost the doubled compared to the GPFG return (6,808%) for the same period. This is offset by the high standard deviation of the oil reserve (30.908%), which is approximately three times as high compared to the GPFG (10,385%). Due to this risk-return relationship, the Sharpe ratio indicates that the GPFG is overall performing better than the hypothetical oil reserve for period 2.

Table 8: Return, Standard Deviation and Sharpe Ratio of Data

	Period 1 1971-1997		***Period 2: 1998-2017***	
	MSCI	***Oil Portfolio***	**GPFG**	***Oil Portfolio***
μ:	13,081 %	12,819 %	6,808 %	11,371 %
σ:	15,016 %	41,811 %	10,385 %	30,843 %
Sharpe:	0,871	0,307	0,656	0,369

The results from our portfolio optimizations for the two periods are presented in *table 9*. Looking at the results from the constraint with maximum return, the assets are simply fully weighted on the asset with highest return regardless of how high the volatility is. Period 1 is fully weighted on the MSCI Index and with a relative high Sharpe ratio of 0,871. In period 2, the portfolio is fully weighted on the hypothetical oil reserve and obtains a considerably lower Sharpe ratio of 0,369. In both time periods, however, the Sharpe ratio of the minimum-variance portfolio are superior to the maximum return portfolios. Our further discussion will be based upon the minimum-variance portfolios.

Table 9: Results Portfolio Optimization

	Period 1: 1971-1997		***Period 2: 1998-2017***	
	Max μ	**Min σ**	**Max μ**	**Min σ**
MSCI/GPFG	100,00 %	87,65 %	0,00 %	96,35 %
Oil Portfolio	0,00 %	12,35 %	100,00 %	3,65 %
Sum weights	100 %	100 %	100 %	100 %
μ_P:	13,081 %	13,048 %	11,371%	6,975 %
σ_P:	15,016 %	13,956 %	30,843 %	10,305%
Sharpe:	0,871	0,935	0,369	0,677

This table presents the results obtained from the two portfolio optimizations performed. Column 1 and 2 presents period 1's maximum return and minimum standard deviation results with optimal weights between investing in MSCI Index and hypothetical oil reserve and respective optimal mean return, standard deviation and Sharpe ratio. Likewise, Column 3 and 4 present the optimization results with the Government Pension Fund Global for period 2.

When comparing the weight diversification of the two periods, we can see that the prediction, Period 1 of optimal diversification, shows a slightly higher optimal weight in the oil portfolio (12,35%) compared to the optimal weight of the oil portfolio in period 2 (3,65%). This is most likely explained by the negative covariance between the MSCI Index and oil portfolio in period 1 and indicates the predictive optimization would have suggested the investment managers to allocate a higher weight in the hypothetical oil reserve than what would be optimal in period 2.

Table 10 shows how the suggested optimal weights from period 1 would perform in period 2. The return and overall Sharpe ratio would perform better, but the standard deviation will not be minimized and thus it will not be the optimal allocation. The performance measurements are very close to the actual performance, but the weights are too far apart to conclude that the historical data prior to the GPFG construction could be a good predictive estimate of optimal allocation for the investment management.

Table 10: Performance of Predicted Allocation

	87,65%/12,35%	
	Period 1	**Period 2**
μ:	13,048 %	7,372 %
σ:	13,956 %	10,634 %
Sharpe:	0,935	0,693

6.3.1 Performance Evaluation of the Optimal Portfolio

In *table 11*, we have listed the final results from the minimum-variance portfolio optimizations together with the return, standard deviation and Sharpe ratio of the scenarios of the GPFG and hypothetical oil reserve. The efficient frontier for both period 1 and 2 can be found in *appendix 5 and 6* and is a graphically presentation of the connection between risk and return, given the weights invested between the GPFG and oil reserve.

Table 11: Summary of Return, Standard Deviation and Sharpe Ratio

	Optimal Portfolio	**GPFG**	**Oil Portfolio**
Weights:	96,35%/3,65%	100%	100%
$\boldsymbol{\mu_P}$**:**	6,975%	6,808 %	11,371 %
$\boldsymbol{\sigma_P}$**:**	10,305 %	10,385 %	30,843 %
Sharpe:	0,677	0,656	0,369

This table gives a summary of mean return, standard deviation and Sharpe ratio for the three portfolios possible to invest in the period between 1998-2017.

By evaluating the three scenarios in *table 11* by the performance of their Sharpe ratio, we see investing 100% of the assets in the hypothetical oil reserve has the weakest performance. The reserve has the highest return, but because of the considerably lowest standard deviation, it performs poorest out of the three. This means, investing in the oil portfolio would be the riskiest solution.

The optimal weights of period 2 shows an allocation of 96,35% in GPFG and 3,65% in hypothetical oil reserve, gives a return of 6,975 % which is slightly higher than the actual average return of the GPFG 6,808%. In addition to a higher expected return, the standard deviation of our optimal portfolio is 10,305% which is lower than the standard deviation of the GPFG, 10,385%. Likewise, our Sharpe ratio of 0,677 outperform the GPFG of 0,656. Therefore, according to our calculations, given that the risk factor of storing oil is solely based upon the Brent spots volatility, a 96,35/3,65 weighted portfolio would have outperformed both mean return, standard deviation and Sharpe ratio of GPFG.

Appendix 7 provides an overview of the benefits form diversification at differently weighted portfolios for period 2. For our optimal portfolio, we were able to improve the risk of the portfolio with 0,808% compared to the weighted average standard deviation.

6.4 Limitations

Due to lack of previous research, we had to make several assumptions in order to create a method. Based on these assumptions we were able to quantify a value creation by reallocating petroleum reserves to Government Pension Fund Global. Having said that, our valuation of the hypothetical oil reserve is based on a simple valuation principle, which makes it is highly sensitive to oil price volatility. This weakness will further affect our calculations of the value added from risk reduction by investment diversification. However, the choice of valuation method was made for simplicity.

It is also important to consider that the Brent Spot Price has risen rapidly and steadily through the time period 1971-2017 (except from the crisis in 2008) and that the barrels were in reality sold to a much lower price than the price we are evaluating the entire reserve at. As a matter of fact, 33 573 152 176 out of the 44 542 742 037 barrels extracted, approximately three quarters, has been sold below the end of 2017 Brent spot price of USD 64,37.

Furthermore, the assumption of making the net cash flow to Government the total extraction is a rough assumption which do not take into account the extra amount of barrels the companies need to extract to cover the production costs and meet the taxes and fees to the Government. In reality, it might be a substantially larger amount of oil extracted than the barrels considered to cover the net cash flow to the Government. This, again would leave us with a smaller hypothetical oil reserve than we have accounted for in our research.

The MSCI World index is a common benchmark for stock funds, but as the GPFG is invested in both stocks, bonds and real estate it may not optimally reflect the behavior. The benchmarks used by NBIM for the GPFG consist of two asset class indices, Bloomberg Barclays index for bonds and the FTSE Global Equity index. A combination of these indexes would give a more realistic picture of the GPFG, but the data for our chosen period was not available or optimal for our research.

7 Conclusion and Future Research

Traditional calculations of financial wealth fund's value creation tend to solely contemplate the expected premium received from taking on systematic risk associated with the fund's asset allocation, and fail to notice that this is not financial values added by the fund construction itself.

In our analysis, we have managed to quantify the financial value added by reallocating petroleum revenues from oil reserves to the Government Pension Fund Global. By making a comparable scenario with lower extraction rate and no financial fund, we were able to identify the value created by risk reduction from diversification of the petroleum assets. With a risk aversion level of 4, the certainty equivalent will be 0,238 which represents a value added of USD 172 536 427 270 created from investment diversification. Our results however, shows the value added is sensitive to the Brent Spot volatility and investors' level of risk aversion.

Additionally, we find evidence revealing that a 95,35%/3,65% combination of investing in the GPFG and the oil portfolio would perform a greater return and lower standard deviation than the strategy of extracting all and investing in the Government Pension Fund Global. This indicates, when evaluating with our performance measurements, a strategy with lower extraction rate could have been more optimal yielding an expected return of 6,975% and standard deviation of 10,305%. Further, our hypothesis of testing whether period 1 was a good prediction for period 2, shows that even if the performance measurements was relatively similar, the optimal weights of allocation was too far apart for the historical data to be concluded as a good prediction.

Our study contributes to the literature on the value creation of constructing a financial welfare fund. Specifically, revealing a separation between value created by realized returns from systematic risk exposure and value created by risk reduction from diversification. We hope the results provide valuable and new insight on ways to calculated and distinguish the value added from establishing the Government Pension Fund Global, and is in interest for readers concerned with oil production and its asset allocation.

For future research, we advise examining more thorough how other factors, besides the Brent Spot, is interacting with the risk and return of oil reserves. A weakness of our research is the simplified method of calculating the size of our hypothetical oil reserve. More variables should be considered to get a more accurate estimate on number of barrels left below ground. For an example examining the production contribution and the respective costs for the individual petroleum companies to identify how many barrels of oil is needed to be extracted by each company to be able to meet its costs and obligations to the Government. When it comes to the utility valuation method used to identify the certainty equivalent, we recommend future studies to do a more thoroughly research when assigning a more accurate level of risk aversion for investors. Other approaches could be to make less or other assumptions, such as splitting oil and gas equivalents, using other indexes as proxies, include the estimated oil reserve not yet retrieved in the analysis, or simply make the research more robust.

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Appendices

Appendix 1: Data Used Calculating Hypothetical Oil Reserve

Year	Oil equivalent	Barrels	Brent Spot	USD/NOK	Value extraction USD	Consumption value USD	Net cashflow USD
1971	357	2 246	2,6429	6,9389	5 936	5 936	16 070
1972	1 927	12 121	2,6950	6,5897	32 665	32 665	47 530
1973	1 870	11 759	3,3433	5,7545	39 314	39 314	82 916
1974	2 014	12 669	10,6750	5,5272	135 239	135 239	138 767
1975	10 995	69 159	10,7975	5,2268	746 740	746 740	225 897
1976	16 227	102 063	12,1392	5,4566	1 238 965	1 238 965	1 863 108
1977	19 300	121 392	13,4308	5,3235	1 630 397	1 630 397	2 788 055
1978	34 866	219 300	13,8825	5,2420	3 044 427	3 044 427	3 203 239
1979	44 319	278 760	26,2050	5,0644	7 304 895	7 304 895	5 436 928
1980	55 798	350 957	35,1950	4,9392	12 351 946	3 098 711	14 227 433
1981	54 652	343 753	34,0175	5,7388	11 693 614	2 885 425	15 837 369
1982	54 817	344 791	31,5942	6,4545	10 893 390	3 120 149	14 035 559
1983	61 979	389 837	28,2875	7,2960	11 027 510	3 180 380	11 804 239
1984	69 762	438 788	27,6833	8,1629	12 147 115	2 264 888	12 723 962
1985	73 987	465 363	27,7950	8,5972	12 934 758	1 672 397	11 158 310
1986	78 767	495 432	17,8425	7,3950	8 839 739	492 359	7 744 462
1987	89 281	561 562	19,6483	6,7375	11 033 749	-6 531	2 580 866
1988	97 946	616 060	14,9983	6,5170	9 239 868	-282 339	829 627
1989	119 672	752 712	18,0283	6,9045	13 570 150	1 957 721	4 013 470
1990	125 081	786 739	23,4350	6,2598	18 437 225	4 981 349	9 930 825
1991	138 492	871 086	19,8908	6,4822	17 326 634	9 134 494	10 696 737
1992	154 846	973 951	19,3692	6,2133	18 864 613	10 521 287	8 430 061
1993	162 639	1 022 972	17,2567	7,0942	17 653 087	10 134 512	6 633 097
1994	182 649	1 148 827	15,8192	7,0599	18 173 493	7 719 524	5 997 018
1995	195 709	1 230 975	16,9083	6,3353	20 813 729	5 435 597	9 817 134
1996	224 915	1 414 677	20,2183	6,4561	28 602 402	3 520 724	17 280 095
1997	232 308	1 461 173	19,3858	7,0729	28 326 055	2 837 292	19 123 540
1998	225 375	1 417 569	13,4808	7,5450	19 110 015	2 313 325	9 096 181
1999	229 665	1 444 550	17,9650	7,7991	25 951 349	1 547 095	8 512 646
2000	243 613	1 532 278	28,4617	8,8018	43 611 180	902 425	26 437 825
2001	251 510	1 581 954	24,4675	8,9913	38 706 455	182 400	37 869 944
2002	258 364	1 625 064	24,9517	7,9839	40 548 052	7 814 703	29 340 162
2003	261 705	1 646 077	28,8842	7,0800	47 545 562	9 343 220	33 144 715
2004	264 406	1 663 064	38,1583	6,7412	63 459 745	11 755 401	40 539 198
2005	257 737	1 621 117	54,2767	6,4426	87 988 824	10 052 334	56 532 104
2006	249 566	1 569 726	64,9933	6,4135	102 021 732	6 860 832	71 620 125
2007	238 341	1 499 119	72,9517	5,8610	109 363 211	228 973	69 245 888
2008	243 638	1 532 435	97,7733	5,6390	149 831 253	2 092 031	91 140 309
2009	240 598	1 513 314	61,5875	6,2898	93 201 239	15 351 978	53 837 123

2010	231 107	1 453 620	79,5375	6,0437	115 617 262	17 219 655	53 947 280
2011	219 620	1 381 369	111,1892	5,6059	153 593 278	14 163 534	72 533 154
2012	226 294	1 423 345	111,8017	5,8172	159 132 373	17 344 746	78 174 816
2013	215 395	1 354 792	108,7933	5,8753	147 392 355	19 820 805	66 161 669
2014	218 437	1 373 925	99,2433	6,3011	136 352 902	25 393 695	54 614 023
2015	230 041	1 446 914	52,4967	8,0637	75 958 148	22 981 156	29 270 797
2016	232 726	1 463 806	43,5767	8,4014	63 787 784	26 215 709	15 503 257
2017	238 415	1 499 584	54,2458	8,2712	81 346 189	31 374 555	20 800 205
Total	**7 081 727**	**44 542 743**			**2 050 626 562**	**329 805 087**	**1 114 987 735**

Appendix 2: Calculation of Number of Barrels

Year	Barrels	Consumption in barrels	Net cash flow in barrels	Barrels left in reserve
1972	12 121	12 121	17 636	25 280 235
1973	11 759	11 759	24 800	25 268 476
1974	12 669	12 669	12 999	25 255 807
1975	69 159	69 159	20 921	25 186 649
1976	102 063	102 063	153 479	25 084 585
1977	121 392	121 392	207 586	24 963 193
1978	219 300	219 300	230 739	24 743 894
1979	278 760	278 760	207 477	24 465 134
1980	350 957	88 044	404 246	24 377 090
1981	343 753	84 822	465 565	24 292 268
1982	344 791	98 757	444 245	24 193 511
1983	389 837	112 431	417 295	24 081 080
1984	438 788	81 814	459 625	23 999 266
1985	465 363	60 169	401 450	23 939 097
1986	495 432	27 595	434 046	23 911 503
1987	561 562	-332	131 353	23 911 835
1988	616 060	-18 825	55 315	23 930 660
1989	752 712	108 591	222 620	23 822 068
1990	786 739	212 560	423 760	23 609 508
1991	871 086	459 231	537 772	23 150 277
1992	973 951	543 198	435 231	22 607 079
1993	1 022 972	587 281	384 379	22 019 798
1994	1 148 827	487 985	379 098	21 531 813
1995	1 230 975	321 474	580 609	21 210 338
1996	1 414 677	174 135	854 675	21 036 203
1997	1 461 173	146 359	986 470	20 889 844
1998	1 417 569	171 601	674 749	20 718 243
1999	1 444 550	86 117	473 846	20 632 126
2000	1 532 278	31 707	928 892	20 600 419
2001	1 581 954	7 455	1 547 765	20 592 964
2002	1 625 064	313 194	1 175 880	20 279 771

2003	1 646 077	323 472	1 147 505	19 956 299
2004	1 663 064	308 069	1 062 394	19 648 230
2005	1 621 117	185 205	1 041 554	19 463 024
2006	1 569 726	105 562	1 101 961	19 357 462
2007	1 499 119	3 139	949 202	19 354 323
2008	1 532 435	21 397	932 159	19 332 927
2009	1 513 314	249 271	874 157	19 083 656
2010	1 453 620	216 497	678 262	18 867 158
2011	1 381 369	127 382	652 340	18 739 776
2012	1 423 345	155 139	699 228	18 584 637
2013	1 354 792	182 188	608 141	18 402 450
2014	1 373 925	255 873	550 304	18 146 577
2015	1 446 914	437 764	557 574	17 708 813
2016	1 463 806	601 600	355 770	17 107 213
2017	1 499 584	578 377	383 443	16 528 836
Total	**44 542 743**	**8 765 766**	**25 294 602**	

Appendix 3: Estimate of Value Added by Varying Brent Spot

Brent Spot	*Value of oil reserve*	*Value added*
20	-22 094 958 885	-5 258 600 215
21,31	0	0
30	146 568 849 395	34 883 386 156
35	230 900 753 536	54 954 379 341
40	315 232 657 676	75 025 372 527
45	399 564 561 816	95 096 365 712
50	483 896 465 956	115 167 358 897
55	568 228 370 096	135 238 352 083
60	652 560 274 236	155 309 345 268
64,37	726 266 358 454	172 851 393 312
70	821 224 082 516	195 451 331 639
75	905 555 986 656	215 522 324 824
80	989 887 890 796	235 593 318 009
85	1 074 219 794 936	255 664 311 195
90	1 158 551 699 076	275 735 304 380
95	1 242 883 603 216	295 806 297 565
100	1 327 215 507 356	315 877 290 751

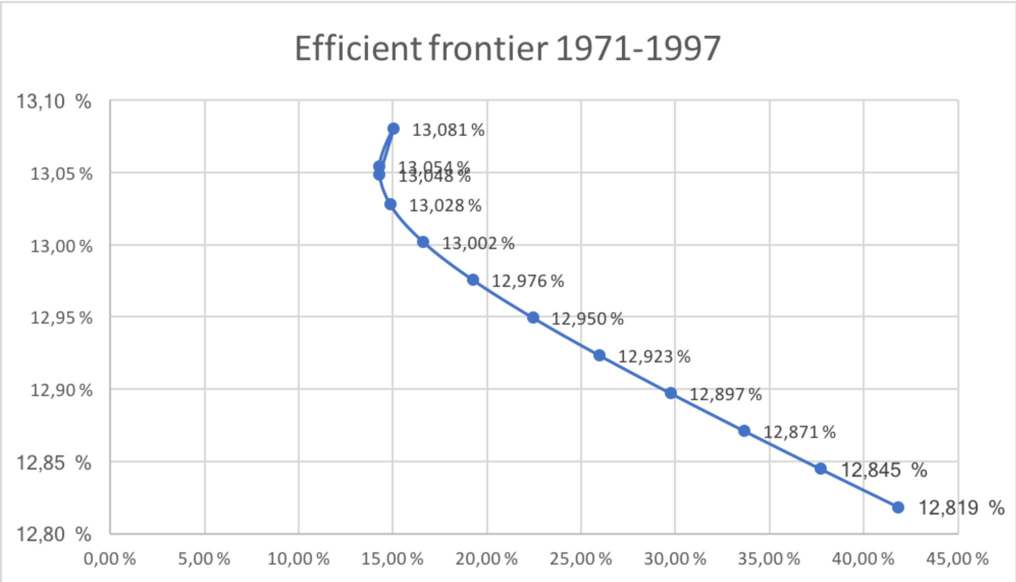
This table shows the how the value added fluctuate by varying the Brent Spot Price. The calculations are based on a constant production cost of USD 21,31, a risk aversion level of 4 and a certainty equivalent of 0,238.

Appendix 4: Estimate of Value Added by Varying Production Cost

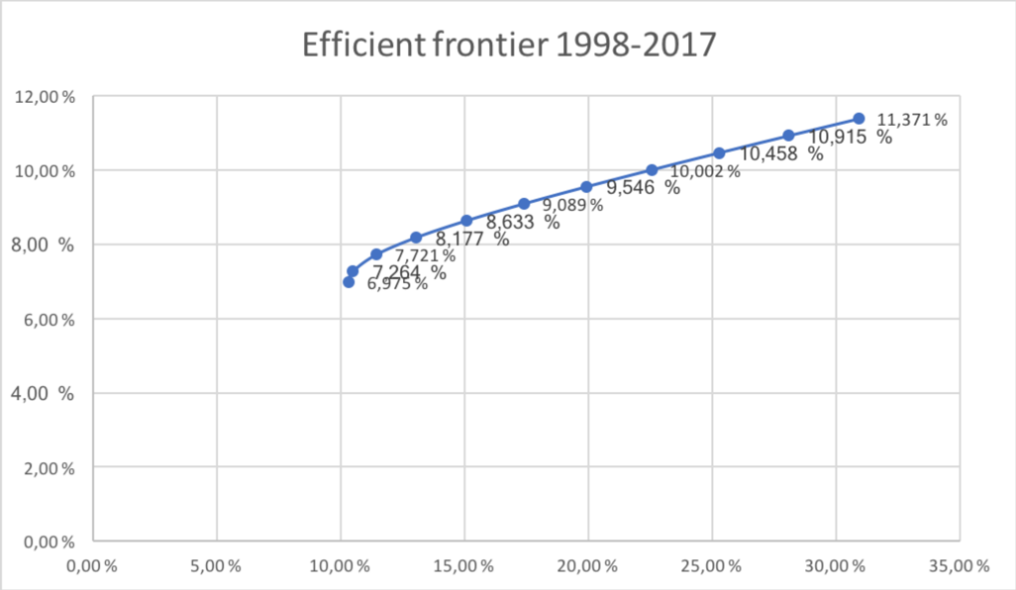
Production Cost	*Value of oil reserve*	*Value added*
0	1 085 688 933 899	258 393 966 268
5	1 001 357 029 759	238 322 973 083
10	917 025 125 619	218 251 979 897
15	832 693 221 479	198 180 986 712
21,31	726 266 358 454	172 851 393 312
25	664 029 413 199	158 039 000 341
30	579 697 509 059	137 968 007 156
35	495 365 604 919	117 897 013 971
40	411 033 700 779	97 826 020 785
45	326 701 796 639	77 755 027 600
50	242 369 892 499	57 684 034 415
55	158 037 988 358	37 613 041 229
60	73 706 084 218	17 542 048 044
64,37	0	0
70	-94 957 724 062	-22 599 938 327

This table shows the how the value added fluctuate by varying the production costs. The calculations are based on a constant Brent Spot price of USD 64,37, a risk aversion level of 4 and a certainty equivalent of 0,238.

Appendix 5: Efficient Frontier Period 1



Appendix 6: Efficient Frontier Period 2



Appendix 7: Benefits from Diversification

GPFG	***Oil Portfolio***	μ_P	σ_P	σ_{AVG}	***Diversification Benefit***
0 %	100 %	11,371 %	30,908 %	30,908 %	0,000 %
10 %	90 %	10,915 %	28,076 %	28,855 %	0,779 %
20 %	80 %	10,458 %	25,289 %	26,803 %	1,514 %
30 %	70 %	10,002 %	22,563 %	24,751 %	2,188 %
40 %	60 %	9,546 %	19,923 %	22,698 %	2,776 %
50 %	50 %	9,089 %	17,408 %	20,646 %	3,238 %
60 %	40 %	8,633 %	15,081 %	18,594 %	3,513 %
70 %	30 %	8,177 %	13,043 %	16,542 %	3,499 %
80 %	20 %	7,721 %	11,449 %	14,489 %	3,041 %
90 %	10 %	7,264 %	10,503 %	12,437 %	1,934 %
96,35 %	3,65 %	6,975 %	10,326 %	11,134 %	0,808 %
100 %	0 %	6,808 %	10,385 %	10,385 %	0,000 %

This table shows the approximate benefit of diversification of assets between investing in GPFG and storing the oil below ground, column 1 and 2 presents the weights invested in each asset, column 3 and 4 the portfolio's mean return and standard deviation. Column 5 is the average weighted standard deviation used as benchmark to identify the benefit of diversification in column 6.

Appendix 8: Summary of Data Conducted

Data	Source	Frequency	Period	
Brent Spot	Macrobond	Monthly	1971-2017	Last price
Market Value Fund	NBIM.no	Annual	1998-2017	
MSCI	Thomson Reuters Eikon, Bloomberg	Monthly, Annual	1976-2017, 1969-2017	Last price
Net cash flow from petroleum	Norskpetroleum.no	Annual	1971-2017	
Oil and gas production	Oljedirektoratet.no	Monthly	1971-2017	
Oil corrected surplus	Statsbudsjettet.no	Annual	1980-2017	
Return Fund	NBIM.no	Annual	1998-2017	