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Simple rules or optimization for a dollar-neutral investor?

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Abstract

In this thesis, we study portfolio construction and asset allocation for a long/short investor. We construct equally weighted portfolios based on known firm characteristics and compare these to mean-variance optimization models in two different datasets. We find that high turnover and estimation error diminish the effects of optimization after transaction costs. Simple median-based 1/N strategies are not necessarily optimal, but all strategies manage to outperform mean-variance models in the sample consisting of a larger number of assets. Further, the median-based 1/N strategies we consider could be used as potential benchmarks for active characteristic-based strategies.

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1 Introduction

Recent research shows that the simple 1/N asset allocation rule outperforms more sophisticated optimization models out-of-sample, because of large estimation errors in forecasting asset returns. In this thesis, we want to examine different methods of portfolio construction for a long-short investor. We are interested in finding whether a simple approach can outperform mean-variance optimized portfolios. By long-short investor, we specifically mean a dollar-neutral investor.

In the recent years, attention within modern portfolio theory has tilted towards stock characteristics rather than moments of asset returns to allocate wealth amongst assets. Some characteristics have shown a positive relationship with subsequent stock returns (Lewellen 2015), and we believe using characteristics rather than moments of asset returns can manage to reduce estimation error. Typical ranking schemes are popular because of their simplicity, and in many cases, we do not need to calculate covariance matrices. Former studies do not consider a dollar-neutral investor, which is why we want to examine if such schemes manage to outperform more sophisticated models for an investor facing these constraints. Therefore, our research question is: "Optimization or simple rules for a dollar-neutral investor?"

We will use models based on the theory presented in literature such as Uppal et al. (2009), Markowitz (1952) and Lewellen (2015) to compare different allocation strategies. Two different datasets are considered in our analysis; a sample of S&P 500 stocks and Fama & French Industry portfolios. Furthermore, we will produce results using Matlab, showing out-of-sample performance with relevant performance measures.

We find that when number of investable assets increases, high turnover and estimation error makes it hard for mean-variance models to outperform simple allocation strategies net of transaction costs. The characteristic based portfolios produce varying results in terms of sharpe ratio for the two datasets, but we observe that "slow movers" e.g. book-to-market and size have the lowest turnover and transaction costs. We conclude that the simple approaches we consider do not show outstanding performance compared to sophisticated models, but can be used as benchmarks for active characteristic based strategies.

In the next section, we will present recent studies related to our research question. In section three we discuss relevant theory. Section four and five describe the data and methodology that we have used to answer the question. In section six we present our results. In section seven, we draw conclusions based on our findings, and finally, in section eight we present suggestions for future research.

2 Literature review

The issue of portfolio optimization has long been a topic of interest in the financial world. The underlying economic theory of optimal portfolio construction was pioneered by Markowitz (1952), Merton (1971), Merton and Samuelson (1969) and Fama (1970), with Markowitz perhaps the most influential of these.

2.1 Markowitz - portfolio selection

The most common formulation of portfolio choice problems is the mean-variance paradigm presented by Markowitz in 1952 in his article "Portfolio Selection". The idea of the mean-variance paradigm is to choose portfolio weights that optimize the overall risk-return trade-off (Sharpe Ratio). Harry Markowitz' work was revolutionary for two reasons. Before him, finance literature barely considered the relationship between risk and return of assets. Markowitz presented a framework where he considered risk and return jointly by the assets' return and their covariance. The second was that he formulated an optimization problem, which assists managers in their financial decision making. The framework has two intuitive points. First, it shows that imperfectly correlated assets can be combined into portfolios with the preferred expected return/risk characteristics. Second, the paradigm states that once a portfolio is fully diversified, the investor must take on more risk (greater allocations) to achieve higher expected returns (Brandt, 2009).

There are some problems regarding this theory, firstly Markowitz assumes quadratic utility only. Secondly, the paradigm ignores any preferences towards higher-order return moments (i.e. Skewness and Kurtosis). Thirdly, the mean-variance problem works best for single periods, while most investments have longer horizons which means that the portfolio needs continuously rebalancing (Brandt, 2009).

Researchers are well aware of the issues with the mean-variance model, and many have tried to improve the estimation of return, variance and covariance by for example shrinkage of the estimates, constraining portfolio weights, conditions on the utility function or return distribution of assets. Researchers such as Lee (1977) and Kraus and Litzenberger (1976) have included higher orders moments such as skewness. Much time has been devoted to improving the mean-variance optimizer, but many of the different methods require tremendous resources and tools to obtain "decent" results (Brandt, Santa-Clara, Volkanov, 2009). Despite much work having been done to improve on the work of Markowitz, the mean-variance theory has remained a central part of financial theory. Edwin and Gruber (1997) claim there are two reasons for the original mean-variance staying relevant. The mean-variance theory places a large data requirement on the investor, and there is no evidence that adding additional moments improves the desirability of the portfolio. Second, the implications of the mean-variance portfolio are well developed, widely known and have considerable intuitive appeal.

2.2 Naive strategy

Uppal et al. (2009) found that the 1/N asset allocation rule performs quite well versus more complex mathematical models based on Markowitz' approach, and extensions of it. The 1/N rule is an equally weighted portfolio, where we consider N risky assets and allocate wj=1/N to each of the N risky assets. This method is simple to use and is therefore favoured by many investors. Furthermore, it is described as naive because of its roughness and a common-sense construction of a portfolio, using a logical approach without applying sophisticated mathematical models.

In their study, they compared the naive 1/N rule with 14 different asset allocation strategies. Based on three performance measures; Sharpe Ratio, certainty-equivalent value and turnover rate, they show that the 1/N rule performs quite well out-of-sample. Also, the estimation window necessary to outperform the 1/N strategy may be very large. Moreover, the researchers conclude that their main finding is that the large error in forecasting may diminish the gains from optimization.

2.3 Anomalies

An anomaly is typically a deviation from a common rule, and in asset pricing, it is mainly referred to average stock returns that are not explained by asset pricing models. In our thesis, we use anomalies because we want to identify characteristics that allow us to distinguish between which assets to go long/short without forecasting returns. We focus on common known anomalies as Momentum, Book-to-market, and Size.

2.3.1 Momentum - past winners and losers

Momentum strategies have shown to give impressive out of sample performance in different sets of data and time periods. Jegadeesh and Titman (1993) documented that strategies that buy stocks with high returns during the past 3 to 12 months and sell stocks with poor returns over the same period earn profits of about one percent per month for the following year. Furthermore, Jegadeesh and Titman (2002) confirmed their findings by testing momentum strategies for a new sample over the period of 1990 to 1998 and results are still persistent and profits are about the same magnitude. There is no clear way to understand the reason for this anomaly. Some argue that gains from momentum strategies arise because of inherent biases in the way that investors interpret information or because of delayed information (Conrad and Kaul 1998), others mean that the return comes from the compensation of risk. Conrad and Kaul (1998) argue that profitability could be entirely because of the cross-sectional variations in stock returns, and under their hypothesis, momentum strategies yield positive returns on average even if the expected returns on stocks are constant over time.

2.3.2 Book-to-Market ratio - btm

Many studies find that portfolio strategies based on going long in high book-to-market stocks and short low book to market stocks can predict returns over the next three to five years. There are mainly two competing explanations for this. Firstly, B/M based portfolio strategies represent compensation for risk. Secondly, the return on B/M-based portfolio strategies results from systematic risk pricing of extreme B/M securities (Ali, Hwang, and Trombley, 2003). It is reasonable to think that arbitrageurs would quickly eliminate the effect of mispricing, but the argument that the volatility of arbitrage returns deters arbitrage activity is likely to be an essential reason why the B/M effect exists. Ali, Hwang, and Trombley (2003) find that the B/M effect is more significant for stocks with higher volatility, consistent with the mispricing explanation.

2.3.3 Size - Market capitalization

It is well documented that simple portfolio strategies based on market capitalization can be profitable. Banz (1981) and Reinganum (1981) concluded that small firms on average earned higher rates of returns than large firms. Reinganum (1983) finds that Size was an excellent indicator for long-run rates of return. Since Banz (1981) work, there has been a disagreement over whether the size effect is still present, some have even declared it as dead after the 1980s. However, there is evidence that the size premium in the US and international equity markets has been positive and large in recent years (Van Dijk, 2009).

2.4 Cross-sectional forecasting

Since Markowitz, a lot of research has been done on the subject of portfolio construction, but no single model has managed to establish itself as a clear winner. However, the field of cross-section of stock returns and its patterns stands out. Fama and French (1993) started this movement by showing that the market beta is barely related to the cross-section of average stock returns and introduced an overall market factor, firm size and book-to-market value as more appropriate factors. Fama and French (1996) found that size, book-to-market and lagged returns robustly describe the cross-section of expected returns. Chan, Karceski, and Lakonishok (1998) showed that these are also related to variances and covariances of returns. For these reasons, many have followed up on this line of work with different characterizing parameters and patterns. However, there is not always a clear way to determine the relationship between characteristics and risk factors. Fama and French (1993) argue that the association between these characteristics and returns happen because the characteristics are proxies for non-diversifiable factor risk. However, Daniel and Titman (1997) argue that the return premia is not because of correlation benefits and co-movements, but rather the characteristics themselves that appear to explain the cross-sectional variation in stock returns. Kozak, Nagel, and Santosh (2018) argue that there is no standard interpretation, as there is no clear distinction between factor pricing and behavioral asset pricing.

Lewellen (2015) found out that many of the documented patterns are highly significant and seem almost certainly to be real, and not due to randomness or data-tinkering. The literature shows that many of the firm characteristics are correlated with subsequent stock returns, and Lewellen examines the characteristics to find whether the estimates of cross-sectional properties line up with true expected returns. Lewellen (2015) studied cross-sectional properties of return forecasts derived from Fama-MacBeth regressions, and how these could be used by investors in real time to construct portfolios. Lewellen showed that a combination of up to 15 characteristics could be used to estimate a stock's expected return.

Although there is an ongoing discussion on determining the relationship between the characteristic and the risk factor, most conclude that characteristics seem to have explanatory power in predicting subsequent stock return. Moreover, combining characteristics that have these explanatory powers to produce portfolios, could be a promising way to pursue.

2.5 Motivation

In the prior research, most articles consider a constrained case and unconstrained case. The constrained case is a typical long-only portfolio, while the unconstrained case gives the ability for the investor to hold short positions. Jacobs, Levy, and Starer (1998) state that many hedge fund practitioners seek to constrain their portfolios to be neutral with respect to some factor. In particular, they seek to be dollar-neutral by committing the same amount of capital to their long holdings as their short holdings and achieving a net position of zero. In some cases, fund managers are required to operate within these boundaries (required by clients, or taxation, accounting or behavioral reasons). Recent literature suggests that simple strategies outperform sophisticated models and that firm characteristics are correlated with future returns. We want to investigate if simple strategies will outperform the well known mean-variance approach proposed by Markowitz under a dollar-neutral condition.

3 Theory

Now we will examine relevant theory regarding our research question, namely finding applicable models for a dollar neutral investor. We will discuss topics which emphasize the importance of our work, describing quantitative portfolio management, the mean-variance optimization model, forecasting returns with stock characteristics and common practice in hedge funds.

"*Modern portfolio theory has one, and really only one, central theme: In constructing their portfolios investors need to look at the expected return of each investment in relation to the impact that it has on the risk of the overall portfolio*". Litterman, B. (2004).

3.1 Quantitative portfolio management

On a general level, there are two basic approaches to managing equity portfolios; the traditional approach and the quantitative approach. This paper focuses on a quantitative approach. "*Quants"* use statistical models to make forecasts of each stock's return, risk, and cost of trading based on measurable factors. Because these processes usually are computerized, they can evaluate a vast amount of securities and can be updated more frequently. These models tend to be unbiased as they are based on historical data, but a downside of such strategy is that it cannot uncover misleading or unrepresentative data the way a traditional analyst can. Quantitative strategies often spread the risk across many small bets, which can add value with only slightly favourable odds (Alford, Jones and Lim, 2003).

3.2 Mean-variance optimization

Markowitz presented the efficient frontier in 1952, as the set of portfolios which offer the highest possible return for any given level risk. The idea is that all rational investors will invest somewhere along the efficient risk-return spectrum given their risk preferences. The mean-variance framework can be applied in several ways. The optimization problem can be formulated to maximize expected return, given a level of risk, or by minimizing variance given a level of expected return. This process eventually leads to the efficient frontier (Edwin and Gruber, 1997).

In the original methodology proposed by Markowitz in 1952, we have a portfolio of n different assets where asset *i* will give the return R_i. Let μ_i and σ_i^2 be mean and variance of the assets and let $\sigma_{i,j}$ be the covariance between R_i and R_j. The amount allocated in asset *i* is w_i and $\tilde{r}_p$ is the realized return of the portfolio. When R is the return of the portfolio then we have:

$$\mu = E[R] = \sum_{i=1}^{n} \mu_i w_i \tag{1}$$

$$\sigma^2 = Var[R] = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} \sigma_{i,j} w_i w_j \tag{2}$$

$$Cov(\tilde{r}_a, \tilde{r}_b) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{i,j} \tag{3}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{4}$$

$$\tilde{r}_p = \sum_{i=1}^{N} w_i \tilde{r}_i \tag{5}$$

The idea is that for different choices of $w_{1\ldots.}, w_i$, the investor will obtain different combinations of μ and σ^2. All possible combinations of return and variance is called the attainable set. Further, we find what Markowitz calls the efficient frontier where the minimum σ^2 for a certain μ and the maximum μ of a given σ^2. Markowitz defines the investor as risk averse, which preferably want the greatest return with the smallest amount of risk, the optimal combination found on the efficiency set.

To find the optimal portfolio, which will give the investor the highest risk-return tradeoff, Markowitz defined that $w_i(\equiv w_i^p)$ is the weight on the fraction of portfolio *p* which is invested in asset i. For standard portfolios, we have that:

The equation for the weights of two assets are both given by solving the following equation:

$$min_{w_i,j=1,\dots,N}\ (\sigma_p^2) = \ min_{w_i,i=1,\dots,N}\ [\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\ ] \qquad (6)$$

To get the portfolio with the highest $\frac{E(r_{p)}}{\sigma_p}$, we have to solve equation (6) subject to equation (7), where r is any constant.

$$\sum_{i=1}^{N} w_i E(\tilde{r}_i) = \tilde{r} \qquad (7)$$

For the Optimal Portfolio (k_{opt}), where w is a vector of portfolio weights, $\sum$ is the variance-covariance matrix of returns, E(R-r_f) is the vector of asset excess returns, and 1 is a vector of ones where all are of dimension n, which is the number of assets in the portfolio.

We have that:

$$w_{OPT} = k_{OPT} * \Sigma^{-1} E(R - r_f) \qquad (8)$$

The former is the standard outline of the mean-variance optimization tool. This framework is formulated in a way that gives investors the option to impose constraints and objectives into the optimization problem and find the efficient portfolio under different circumstances. For the specific purpose of this paper, we will aim to optimize the portfolio weights with regards to sharpe ratio under conditions for a dollar neutral investor.

3.3 Critique of the mean-variance optimization model

One of the main issues with mean-variance portfolios is that the large error in forecasting moments of asset returns significantly impacts the portfolio weights and resulting portfolio weights are also not necessarily well diversified. The mean-variance approach is dependent on more or less correct estimates of expected returns. Some quantities can be over- or under-estimated, which will lead to higher weights to assets with over-estimated expected returns and under-estimated risk, and vice-versa. This will magnify the problem, and for this reason, many refer to such optimizers as “error maximizers”. The impact of estimation error can be significant, and it often happens when assets are close substitutes for another (Michaud 1998). Weights can in many cases take extreme values because the model is susceptible to small changes in expected return. Because of this, it is common practice to impose constraints on asset weights.

One of the assumptions behind the mean-variance approach is the positive definiteness of the covariance matrix. Brandt, Santa-Clara, and Valkanov (2009) were critical to this assumption as there is no way of guaranteeing it. The positive definiteness of the covariance matrix means that no single asset can be replicated through a combination of other available assets.

3.4 Forecasting returns with stock characteristics

Cross-sectional estimation was pioneered by Fama and MacBeth (1973). Their model built on the traditional CAPM, they estimated betas from the first-pass regression, then they performed a second-pass cross-sectional regression for each month in the estimation period. Elton (2009) argues that their paper is one of the most influential articles ever written in this field and that virtually every subsequent paper uses one or more of the elements they introduced. Many empirical studies use the idea of time series data to identify risk exposure while cross-sectional differences identify risk premia. The intuition behind their work has been the building blocks of many of the forecasting methods available.

Fama and French (1992) showed that firm characteristics are correlated with subsequent returns and that one could use those characteristics instead of focusing on moments of asset returns to get a reasonable picture of firm’s expected return. They showed that a firm’s size and book-to-market values were significantly related to expected returns building on the work of Fama and MacBeth(1973). There are essentially three types of factors that one can use; factors based on economic theory, based on empirical work, or directly extracted from the returns using statistical procedures (Brandt, 2009).

3.5 Hedge funds

Hedge funds are attractive for two reasons. Hedge funds are appealing because they provide the potential to increase expected portfolio return at the expense of little or no change in expected risk (Winkelmann et al. 2003). There are some fundamental characteristics that set hedge funds apart from their active manager counterparts. They are not faced with the same constraints as traditional managers. An active manager is usually constrained from making short sales and is limited to investing in assets that are included in the benchmark. Hedge funds can go both long and short and can take advantage of both positive/high and negative/low returns in the market. They are also not limited to any benchmark and have access to a broader investment universe.

The characteristics of a hedge fund present several appealing opportunities. A hedge fund manager has more opportunities and a higher chance of finding good trades, but it also makes for a complicated portfolio optimization process. Many managers make the mistake of handling the long/short portfolio as two portfolios. This leads to optimizing a long-only portfolio, and a short-only portfolio, and combine these. This process neglects the positive gains that can be achieved by optimizing a portfolio where relationships between all stocks are considered (Jacobs, Levy, Starer, 1998).

3.6 Hypothesis

This paper aims to compare the model of Markowitz (1952) to simple ranking schemes based on firm characteristics. We will do so under specific constraints facing a hedge fund manager, namely a dollar neutral investor. Optimization models are often computationally heavy and require a lot of work, and often quite inaccurate. We will investigate whether imposing dollar-neutral constraints will have an effect on the optimizer’s predictive ability.

We are also interested to see if simple rules manage to outperform more sophisticated models. We suspect that simple strategies based on asset characteristics such as Book-to-Market ratio, Momentum and Size can produce better results out of sample regarding return, volatility and sharpe ratio compared to the traditional mean-variance approach. Finally, since turnover and transaction costs are of great concern for any investor, we want to compare the performance of the portfolios net of transaction costs out-of-sample.

4 Data

4.1 Data

For this thesis, we will use 48 Industry Portfolios from Kenneth French’s library, with book equity and market capitalization from the same source. The data contains monthly observations from January 1988 to December 2017. Indexes with missing values for book equity have been excluded, and the final dataset contains 40 industries.

There are some issues with the portfolios from Kenneth French’s library as they are composed of a varying number of stocks in each industry. The idea behind using portfolios instead of single stocks is that portfolios are already diversified to some degree. In this case, however, the portfolios are not necessarily well diversified. The smallest industry portfolio contains three companies at the end of our sample period, while the largest contains 417 companies. Another issue with these portfolios is that they are not investible. One might find ETFs that closely replicate the largest portfolios in this sample, and one can construct the smallest ones without too much effort. Nevertheless, an investor will not be able to produce these results exactly.

We will analyze a second dataset using US stocks from the CRSP/Compustat databases for the same time period as the industry portfolios. We will get stock prices and the number of shares outstanding from CRSP monthly files. All stock prices are adjusted for dividends and stock issues using the adjustment factor provided by the CRSP/Compustat database. We have used the adjusted prices to calculate the monthly log return of each asset. Accounting data which will be used for book equity valuation is downloaded from the Compustat annual file. Any stock with missing data for price, total assets or total liabilities have been omitted.

Due to limited computational power, we have had to reduce the number of assets included in the optimization process. All assets that do not have book-to-market values have been excluded. From the assets that met all the criterions, we picked 100 random stocks. Note that changing those with some other random sample from the S&P 500 could give different results. We use the first 60 months as the estimation window, and the forecasting window starts in January 1993 for all of the strategies. All portfolios are rebalanced monthly, and all the results we present in this paper are out of sample.

We will use the 1-month treasury bill rates as a proxy for the risk-free rate, the data is retrieved from Kenneth French's library but is sourced to Ibbotson Associates.

4.2 Construction of characteristic portfolios

For our high-minus-low strategies, we have chosen three characteristics that Fama (2008) and Lewellen (2015) found to be highly correlated with subsequent stock returns. We will take a simple approach to construct portfolios based on characteristics. The three characteristics we use are the standardized values of book-to-market (*btm*), market capitalization (*size*) and momentum (*mom*).

4.2.1 Market capitalization (size):

Kenneth French provides monthly data for the size of the industry portfolios. We have used the log values of the data. For the S&P500 stocks, we have defined *size* as the log of the market value of each asset. Market value is calculated by multiplying the price by outstanding shares at the end of each month.

4.2.2 Book-to-market (btm):

For the industry portfolios, the source provides annual book-to-market values. In order to get monthly data, we first had to extract book equity. Kenneth French has used the book equity of fiscal year t-1 and size from December of t-1. We define book equity as $be_{t-1} = btm_t \times size_{t-1}$, where size is the observation at December of each year. Finally, *btm* is calculated the same way as Brandt, et.al (2009), as one plus log of book equity divided by size.

For the second dataset, we followed the methodology of Brandt et al. (2009) and calculated the book equity as total assets minus liabilities plus balance sheet deferred taxes and investment tax credit minus preferred stock value. We calculate *btm* in the same manner as for the industry portfolios.

4.2.3 Momentum (mom):

In calculating momentum, we follow the recipe of Brandt, et al. (2009) again and define *mom* as the monthly compounded return between months t-13 and t-2.

4.2.4 Combined portfolios:

Since all factors are standardized with mean zero and standard deviation of one, they are on the same "scale". The combined portfolios are defined as:

$$bsm_{d,n} = [btm_{d,n} + mom_{d,n} + (-size_{d,n})] \qquad (9)$$

$$bs_{d,n} = [btm_{d,n} + (-size_{d,n})] \qquad (10)$$

Where *bsm* is a $d \times n$ matrix of the combined factors based on all three characteristics. *bs* is a $d \times n$ matrix of the combined factors based on only *btm* and *size*. *btm*, *mom* and *size* are all $d \times n$ matrices with the standardized variables. n is the number of assets, d equals the number of months.

The reason why we use $(-size_{i,j})$is that we want to short large companies, so assets with lowest *size* values are the ones we want to buy. This way, a stock with the desired characteristics i.e. high btm and mom, and low size will get a high "score". Similarly a stock with high size value, low btm and mom will get ranked lower on the new combined scale. A stock with high btm, low momentum and medium size will likely end up somewhere in the middle. Using this method results in a new variable where assets are ranked based on their combined characteristics and overall desirability.

5 Methodology

5.1 Mean-variance optimization for a dollar-neutral investor

In this thesis, we will use the standard interpretation of the Markowitz optimization tool, we will allow short selling and also add constraints facing a dollar neutral investor. The short side will fund the long side, so basically there is no money invested. We need to re-modify some of the conditions so that the optimizer will meet the requirement for a dollar-neutral investor. Hence we have:

$$\sum w_{portfolio} = 0 \quad (11)$$

$$\sum |w_{portfolio}| = 2 \quad (12)$$

Equation (11) states that the sum of the portfolio weights must equal zero and equation (12) says that the absolute sum of weights must always equal two. In this way, we can ensure that the portfolio holds a dollar-neutral condition, with equal positions for the long and short side of the portfolio.

Through programming in Matlab, we added a constraint to set our budget equal to zero, i.e. $\sum_{i=1}^{N} w_i = 0$. It is shown that an unconstrained mean-variance portfolio sometimes results in extreme weights (Uppal et. al., 2009), and we want to avoid this without limiting the portfolio too much. Therefore, we have imposed a weight constraint in the optimization to limit positions in individual assets to a maximum of 1 and a minimum of -1. All mean-variance portfolios are optimized to maximize sharpe ratio, regardless of the level of risk aversion.

We run two specifications of the mean-variance portfolio, one where the optimization does not take transaction cost into account, and one where it does. It is interesting to find whether optimizing while accounting for the transaction cost will affect the optimal portfolio. We use a simple built-in function in Matlab to optimize with transaction cost. The function works as a boundary. We only make a trade when the net return of a new position is *expected* to be greater than the return on current holdings. However, it is important to bear in mind that this is only one of many methods to reduce the number of trades.

We use a 60-month rolling window based on historical log returns to estimate means and covariances of all assets. All means and covariances are estimated each month, and we use the previous month's information to decide positions at time t. We forecast 300 months out of sample.

5.2 Simple rules: "ranking schemes"

We have used the intuition behind a naive equally weighted portfolio, and the goal is to construct simple allocation strategies for a dollar-neutral investor. We utilize high-minus-low strategies based on the median value of each characteristic to take equally weighted positions in each asset, and the same amount in total short and total long positions to get a dollar-neutral portfolio. For *mom* and *btm*, we take long positions in all assets above the median value, and short positions in all assets below the median value. Since size is negatively correlated with returns (Banz, 1981), we take the opposite position in *size* portfolio, i.e. long below the median value and short above the median value. The same approach is used for the combined portfolios, except we sort based on a new variable (combined characteristic), rather than a single characteristic. For every month, we use rankings of month t-1 to decide which positions to take at month t.

5.3 Limitations

Margin account requirements are of great concern in the real world, but we have made a simplifying assumption and considered an investor who does not face such requirement. Regulation T governs the cash accounts and amount of credit brokers and dealers may offer the clients for purchase of securities. This limits the ability for a long-short investor to increase leverage without increasing the cash balance Jacobs, Levy & Markowitz (2006). Of course, this can lead to lower net returns for a dollar-neutral investor and make some of the strategies we present unprofitable. Another cost of a dollar-neutral strategy not considered in this thesis is the cost of borrowing shares.

5.4 Key performance indicators

To analyze and compare the different models, we have to consider relevant performance measures. The chosen measures are sharpe ratio, asset turnover, and transaction costs. These three measures are in line with measurements used in recent literature regarding this topic such as Uppal et al. (2009) and Brandt et al. (2007). A description of each measurement is presented below.

5.4.1 Sharpe ratio

The Sharpe Ratio is a common formula to measure the trade-off between risk and return. It simply divides the portfolio excess return by its standard deviation. The traditional sharpe ratio applies to a long-only portfolio to measure return in excess of the investible risk-free rate per unit of standard deviation. We define sharpe ratio as Winkelman et al. (2003) for a long-short hedge fund:

$$SR = \frac{\mu - r_f + r_f}{\sigma} \tag{13}$$

5.4.2 Asset turnover

The asset turnover represents the absolute change in weights from one period to another. Turnover is of particular interest in this paper since a dollar neutral investor must trade both long and short, and it is an essential consideration for whether a strategy is useful in practice.

$$Turnover = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}(|w_{j,t} - w_{j,t-1}|) \tag{14}$$

5.4.3 Transaction cost

Brandt et al. (2009) created a model with time-varying transaction cost based historical data and found that the average transaction cost for US stocks has been 0.5%. We take a simple approach to this and assume a constant transaction cost of 0.5%. The average transaction cost of a strategy is defined as:

$$avg.trs.cost = \frac{1}{N}\left(\sum_{j=1}^{N} 0.5\% \times |w_{j,t} - w_{j,t-1}|\right) \tag{15}$$

And the return net of transaction cost is:

$$r_p = \sum_{j=1}^{N} r_j w_j - 0.5\% \times |w_{j,t} - w_{j,t-1}| \tag{16}$$

5.5 Benchmarks

Evaluation of performance is one of the difficulties when it comes to hedge funds. It is more difficult to measure a portfolio which consists of all sorts of assets with different risk characteristics in both long and short positions. Winkelmann et al. (2003) explain that there are several challenges in determining the correct benchmark for hedge funds. Hedge fund returns are more driven by skill than a traditional long-only fund, and returns are unique because the underlying strategies are different in each fund. Several major index providers have created indexes for hedge funds, but these will typically not pass the tests that would be required to be considered as benchmarks. Winkelmann et al. (2003) further state that hedge funds are typically measured relative to cash.

Much of our work is inspired by the paper of Uppal et al. (2009) in showing how the 1/N strategy outperforms mean-variance optimized portfolios. However, the traditional 1/N for a long-only investor is not applicable as a benchmark in our case since the portfolios operate under different constraints. We will, therefore, use cash as our benchmark, and we will compare the mean-variance optimized portfolios to simple equal weighted portfolios based on firm/industry characteristics.

6 Empirical results and analysis

This section presents the analysis of the Markowitz optimization model, individual characteristic portfolios, and combined characteristics portfolios. We divide our results into six sections. In each section, we present results and then discuss our findings in relation to relevant theory and methodology.

In the first section, we present a visual inspection of the portfolio's returns during the out-of-sample forecast. The second section examines a “base case” where we have the standard Markowitz approach for a dollar-neutral investor (mvop) compared with simple allocation strategies based on individual characteristics which are book-to-market (btm), market capitalization (size) and momentum (mom). In the third section we take our analysis one step further, by combining the characteristics to produce portfolios, and once again compare with the standard

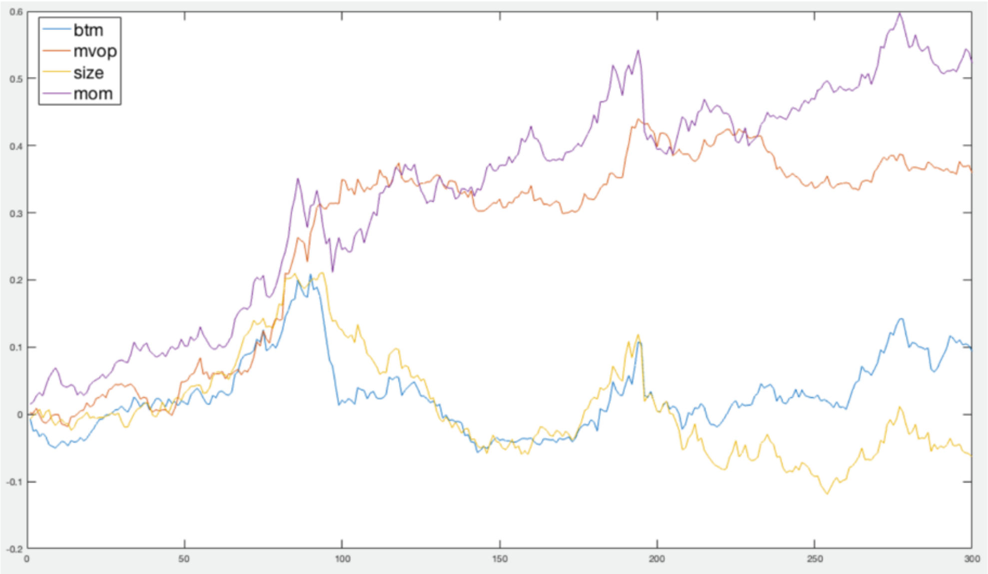
Markowitz optimization model. The fourth section discusses the relevance of turnover and transaction costs and presents a Markowitz optimization model which takes into account the effect of transaction cost when optimizing the allocation of assets. The fifth section focuses on results net of transaction costs for all portfolios. Finally, we discuss the main findings and compare models across datasets.

Our data spans from 1988 to 2017, but we show results out-of-sample from January 1993- December 2017 because some variables require at least a five-year estimation window.

6.1 Visual inspection

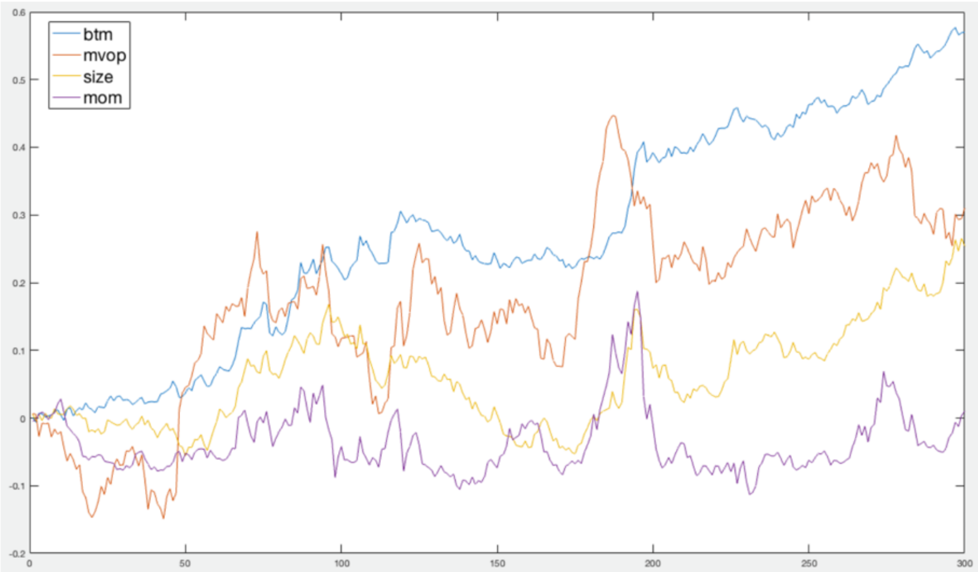
Graph 1: Cumulative returns 1993-2017, dataset: Industry Portfolios

The graph displays cumulative return over the out-of-sample period for book-to-market (btm), size, momentum (mom), mean-variance optimization model (mvop) portfolio. The characteristics are equally weighted above and below the median. The x-axis displays 300 months, with a 50-month interval, while the y-axis displays cumulative returns up to 0,6, with an interval of 0,1.



Graph 2: Cumulative returns 1993-2017, dataset: S&P 500

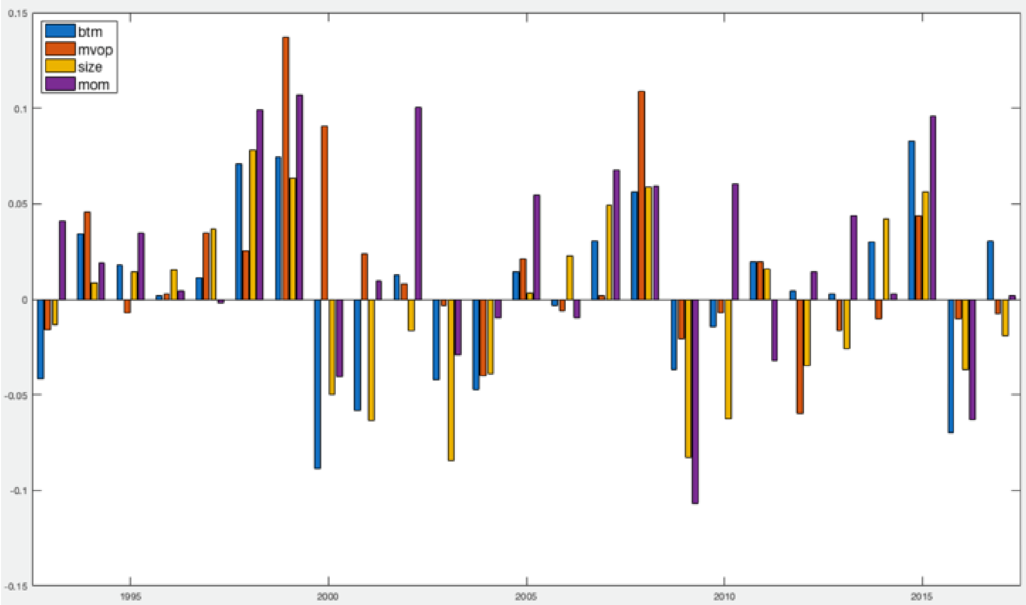
The graph displays cumulative return over the out-of-sample period for book-to-market (btm), size, momentum (mom) and mean-variance optimization model (mvop) portfolios. The characteristics are equally weighted above and below the median. The x-axis displays 300 months, with a 50-month interval, while the y-axis displays cumulative returns up to 0,6, with an interval of 0,1.



Graph one and two visualize the cumulative return for four portfolios; mvop, mom, size, and btm. Mom achieves the highest cumulative return for the industry portfolios and btm for the second dataset. Performance of size in terms of cumulative return improves quite a lot in the second data set compared to the industry portfolios. The mvop portfolio is more volatile for stocks, which can be explained, partly because the second dataset has a larger number of assets and it consists of individual companies. The industry dataset already has some diversification because each industry can consist of up to 450 companies, making each asset less volatile.

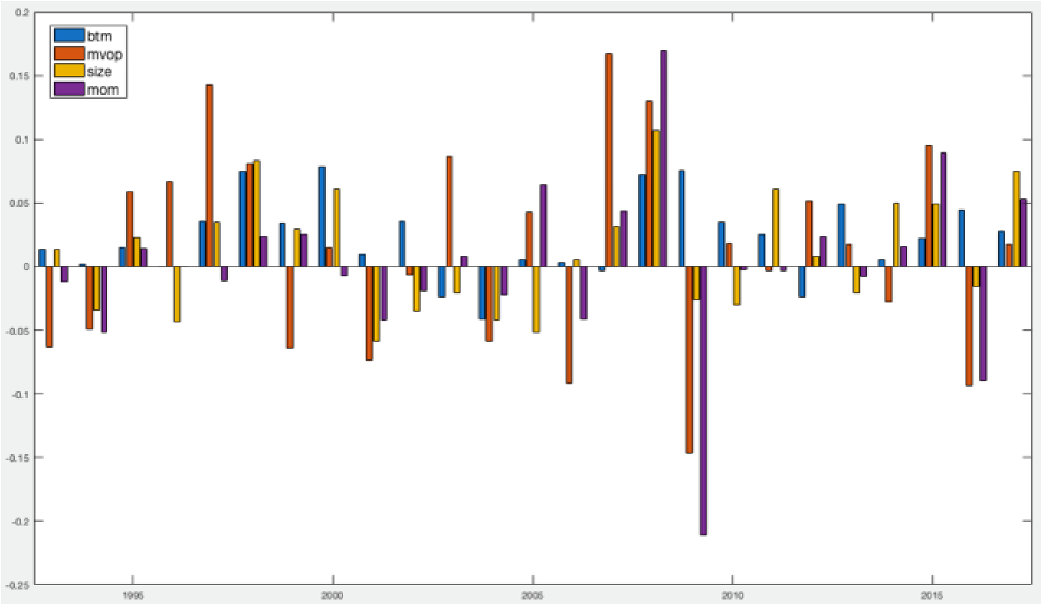
Graph 3: Yearly returns 1993-2017, dataset: Industry portfolios

The graph displays yearly returns throughout the whole out-of-sample period. The portfolios considered are mean-variance optimization model, size, momentum and book-to-market. The characteristics are equally weighted above and below the median. The x-axis displays the full out-of-sample period (1993 -2018), with a five-year interval, while the y-axis displays yearly returns up to 0,15, with an interval of 0,05.



Graph 4: Yearly returns 1993-2017, dataset: S&P 500

The graph displays yearly returns throughout the whole out-of-sample period. The portfolios considered are mean-variance optimization model, size, momentum and book-to-market. The characteristics are equally weighted above and below the median. The x-axis displays the full out-of-sample period (1993 -2018), with a five-year interval, while the y-axis displays yearly returns up to 0,2, with an interval of 0,05.



Graph three and four show the yearly returns for the same four portfolios. The two graphs give valuable information about how the strategies behave during different time periods. Our data sample contains of three notable crisis', United States savings and loans crisis in 1989-91, the dot-com bubble in 2001 and the global financial crisis in 2008-09. The first crisis happens before the out-of-sample estimation window. The dot-com crash is most apparent in the industry dataset, where all portfolios except for the mvop experience losses. Momentum suffered the most from the global financial crisis and it lost almost 20% of its value in the S&P 500 dataset. Such a strategy has one big drawback which is referred to as "momentum crashes". These crashes tend to happen after a crisis, where past losers typically outperform the past winners substantially (Daniel & Moskowitz, 2016).

Research shows that common characteristics, which we consider in this paper, have all been historically cyclical and their periods of underperformance have not corresponded (Bender et. al. 2013). We see similar evidence in our sample looking at graph 1-4. One way to address the problem of cyclicality can be by combining characteristics to produce portfolios. The idea of combining characteristics for a dollar-neutral investor is something we will discuss further in this paper.

6.2 Single characteristic portfolios

Table 1: Single characteristic portfolio

The table displays the annual average return, volatility, sharpe ratio, turnover, average number of trades, max and minimum weights for the risk-free rate (1-month T-bill), mean-variance optimal portfolio (mvop), book-to-market (btm), market capitalization (size) and momentum (mom) strategy. All figures are out-of-sample measurements. Panel A displays results from the industry dataset, while Panel B shows results from the S&P 500 dataset.

Strategy	rf	mvop	btm	size	mom
Panel A: Industries					
Average return	2,40 %	1,44 %	0,38 %	-0,25 %	2,10 %
Volatility	0,6 %	4 %	4 %	4 %	5 %
Sharpe Ratio	0	0,4	0,1	-0,06	0,43
Turnover	--	900 %	47 %	51 %	311 %
Trades	--	40,00	0,88	0,85	5,19
Max Weights	--	0,239	0,025	0,025	0,025
Min Weights	--	-0,217	-0,025	-0,025	-0,025
Panel B: S&P 500					
Average return	2,40 %	1,24 %	2,28 %	1,02 %	0,04 %
Volatility	0,6 %	8 %	4 %	4 %	6 %
Sharpe Ratio	0	0,15	0,66	0,23	0,07
Turnover	--	2962 %	83 %	45 %	350 %
Trades	--	100	3,47	1,89	14,59
Max Weights	--	0,31	0,01	0,01	0,01
Min Weights	--	-0,22	-0,01	-0,01	-0,01

Table 1 shows the results of the mean-variance optimized dollar neutral portfolio compared to dollar-neutral portfolios constructed with characteristics. In the base case, we have used high minus low strategies on stocks below and above the median. Regarding sharpe ratio, the mvop does quite well in Panel A, achieving 0,4. Furthermore, the mvop portfolio achieves a rather low volatility, which demonstrates the strategy's utilization of the covariance matrix to reduce volatility. We also see that the largest/smallest position taken with that strategy are quite reasonable, which means constraining the optimization gave the result we were aiming for in avoiding unreasonable weights. In terms of returns, mom achieved the highest return in Panel A, while btm is the clear superior in Panel B.

The size characteristic gives somewhat inconsistent results when looking at the returns from the two datasets. It is important to notice that this inconsistency is much due to the fact that size from panel A is derived from the market capitalization of industries rather than individual stocks. The results can be misleading because one industry can include, e.g. 20 companies while another as many as 400 companies. Furthermore, some companies that we would ideally go long can be placed in large industries, and consequently be shorted.

The mvop and momentum have the highest turnover and number of trades of all strategies considered. Mvop does 40 trades on average with a turnover of almost 900% in Panel A, and 100 trades on average with turnover three times as high for Panel B. One reason for the increase in mvop's turnover is the larger number of assets in Panel B. Another reason is, as previously pointed out, that the mvop tends to find close substitutes and executes a trade, leading to a considerably high turnover. The momentum strategy also has a rather high turnover as past winners, and past losers change rapidly. Btm and size are quite impressive in terms of turnover as they seem to be relatively stable. Lewellen (2015) describes these characteristics as level variables that change slowly, which suggests that predictability in monthly returns is likely to extend to longer horizons.

6.4 Combined characteristic portfolios

Table 2: Correlation of returns for the single characteristic portfolios

The table displays the correlation between the returns for the single characteristics portfolios in both datasets.

Panel A: Correlation matrix for industries			
	btm	size	mom
btm	1,00	0,64	0,48
size		1,00	0,34
mom			1,00
Panel B: Correlation matrix for stocks			
	btm	size	mom
btm	1,00	0,44	0,22
size		1,00	0,38
mom			1,00

Lewellen (2015) was particularly interested in the relationship between stock characteristics, and how one can construct portfolios by combining them. Table three shows the correlation of returns between the characteristics portfolios. We see that characteristics portfolios overall are not perfectly positively correlated with each other and that there could be benefits in combining the characteristics.

Table 3: Combined characteristics

The table displays the annual average return, volatility, sharpe ratio, turnover, average number of trades, max and minimum weights for the two datasets considered. We present results from two portfolios that are equally weighted above and below the median based on different combination of characteristics. The bsm is constructed as a combination of all three characteristics, book-to-market, size and momentum, while bs is a combination of book-to-market and size.

Strategy	bsm	bs
Panel A: Industries		
Average return	1,42 %	0,45 %
Volatility	5 %	4 %
Sharpe Ratio	0,30	0,10
Turnover	179 %	48 %
Trades	2,99	0,79
Max Weights	0,025	0,025
Min Weights	-0,025	-0,025
Panel B: S&P 500		
Average return	0,63 %	1,24 %
Volatility	5 %	4 %
Sharpe Ratio	0,13	0,31
Turnover	199 %	68 %
Trades	8,30	2,84
Max Weights	0,01	0,01
Min Weights	-0,01	-0,01

Combining the characteristics yields interesting results. For the two different datasets, the results considered in table four are somewhat contradictory. The bsm portfolio is better than bs in terms of sharpe ratio for Panel A, while it is the opposite for Panel B. The strategies are not more volatile with stocks, and the difference in sharpe ratio lies mainly in the difference in returns.

The combined portfolios show some benefits in finding a middle ground between the characteristics in terms of return and turnover. However, all portfolios have similar levels of volatility so the combinations result in lower sharpe ratios than the best single characteristics portfolios in table 1. We see that combining certain characteristics helps to decrease the level of turnover. This can, in fact, be preferable for an investor who wants to be exposed to momentum but sees that high turnover from a pure momentum strategy might diminish the realized return.

Our approach to combining characteristics is untraditional and simplified relative to others that use regressions e.g Lewellen (2015). In contrast to a regression analysis, where each characteristic is weighted differently, we treat them as equally informative. We believe that the inconclusive results we achieve from the combined portfolios can be partly attributed to the way the combinations are constructed.

6.5 Transaction costs

Table 4: Mean-variance optimization with and without transaction costs

The table displays the same performance criterions as the past tables with addition to average return net of transaction costs, sharpe ratio net of transaction costs and average transaction costs. The two strategies considered are the original mean-variance optimization model (mvop) and a mean-variance optimization model with transaction costs (mvopt). The transaction cost considered is 0,5% for all assets.

Strategy	mvop	mvopt
Panel A: Industries		
Average return	1,44 %	6,37 %
Volatility	4 %	11 %
Sharpe ratio	0,4	0,58
Avg. net of transaction costs	-3 %	2 %
SR net of transaction costs	-0,83	0,16
Avg. Transaction costs	4,50 %	4,50 %
Turnover	900 %	910 %
Trades	40	40
Max Weights	0,24	0,70
Min Weights	-0,22	-0,85
Panel B: S&P 500		
Average return	1,24 %	1,48 %
Volatility	8 %	8 %
Sharpe ratio	0,15	0,19
Avg. net of transaction costs	-13,56 %	-6,12 %
SR net of transaction costs	-1,65	-0,77
Avg. Transaction costs	15,00 %	8,00 %
Turnover	2962 %	1521 %
Trades	100	100
Max Weights	0,31	0,23
Min Weights	-0,22	-0,25

All investors are subject to transaction costs in practice, and it is essential to consider the returns net of transaction costs. The mvop has a large turnover and a considerably higher number of trades than other strategies. This reflects much of the problem previously stated with this optimization model. Table 5 shows that the mvop had an average annual return net of transaction costs of -3% for panel A and -13.56% for panel B. The second portfolio (mvopt) in table five is the result of an optimization where we included transaction costs in the process. We managed to achieve a positive return and sharpe ratio net of transaction costs, which seems to be a result of more concentrated portfolios as seen from max/min weights. The results are also improved for Panel B considering the return net of transaction costs and turnover, but the return net of transaction cost is still negative.

There are some notable differences between the strategies in Panel A and B. In Panel A, we see a large increase in return and volatility and more concentrated weights in mvopt. In Panel B however, there is no increase in volatility, while the return and the weights do not change much. This means that the mvopt is still well diversified in Panel B.

6.6 Results net of transaction costs

Table 5: Summary of results net of transaction costs

The table displays a summary of results for eight different portfolios. btm, size, mom, bsm and bs are all portfolios based on equally weighting above and below the median. We focus on results net of transaction costs.

Strategy	rf	mvop	mvopt	btm	size	mom	bsm	bs
Panel A: Industries								
Average net return	2,40 %	-3,06 %	1,82 %	0,13 %	-0,51 %	0,53 %	0,52 %	0,21 %
Volatility	0,6 %	4 %	11 %	4 %	4 %	5 %	5 %	4 %
Sharpe Ratio net of tra.	0,00	-0,83	0,16	0,03	-0,12	0,11	0,11	0,05
Avg. transaction cost	--	4,5 %	4,5 %	0,2 %	0,3 %	1,5 %	0,9 %	0,2 %
Turnover	--	900 %	910 %	47 %	51 %	311 %	179 %	48 %
Panel B: S&P 500								
Average net return	2,40 %	-13,56 %	-6,12 %	1,86 %	0,78 %	-1,71 %	-0,36 %	0,91 %
Volatility	0,6 %	8 %	8 %	4 %	4 %	6 %	5 %	4 %
Sharpe Ratio net of tra.	0,00	-1,65	-0,77	0,54	0,18	-0,28	-0,07	0,23
Avg. transaction cost	--	15,0 %	8,0 %	0,4 %	0,2 %	1,7 %	0,9 %	0,3 %
Turnover	--	2962 %	1521 %	83 %	45 %	350 %	199 %	68 %

Table six show summary of results net of transaction costs. We examine that for panel A, mvop and size yield negative return on average over the sample period. The high turnover for the mvop diminishes the positive return of the strategy. A weak specification for the size characteristic in Panel A is one reason for its poor return given the market capitalization is based on the size of the industry rather than individual companies. In Panel A, the mvopt portfolio achieves the highest sharpe ratio of all portfolios, followed by mom and bsm. In Panel B, none of the mean-variance portfolios manage to deliver a positive return net of transaction cost and btm is a clear superior in terms of sharpe ratio.

Overall, we find that a high turnover is the most important reason for a low return and sharpe ratio net of transaction costs. The transaction costs that are a result of the high turnover are quite significant in some cases. For example, mvop in Panel B has an average transaction cost of 15%, which means that the portfolio needs to generate 15% per year just to break even. A long-short investor will generally make a higher number of trades than a long-only investor, so a certain degree of turnover is expected. However, the 15% return that is required in the mvop is unrealistic to achieve under the dollar neutral condition. Characteristics that are "slow movers" tend to yield the best results as they rarely trigger a trade and keep the transaction costs low. For instance, btm makes less than one trade per month on average with the industry portfolios and has an average annual transaction cost of 0.2%.

6.7 Comparing results from the two datasets

Table 6: Ranking based on performance indicators

The table displays ranking of all seven portfolios based on three criteria, cumulative return before transaction cost, Sharpe ratio before transaction cost and average transaction cost. It is important to note that this procedure does not necessarily tell which strategy is better, but rather a tool to analyze the portfolios based on important criteria.

Strategy	Cumulative return	Sharpe Ratio	Avg. transaction cost	Overall score	Overall rank
Panel A: Industries					
mvop	3	2	7	12	3
mvopt	1	4	6	11	2
btm	6	6	1	13	4
mom	2	1	5	8	1
size	7	7	3	17	5
bs	5	5	2	12	3
bsm	4	3	4	11	2
Panel B: S&P 500					
mvop	4	4	7	15	4
mvopt	2	3	6	11	3
btm	1	1	3	5	1
mom	7	7	5	19	6
size	5	5	1	11	3
bs	3	2	2	7	2
bsm	6	6	4	16	5

We show one way to use the intuition behind the 1/N asset allocation rule for a long-short investor, by using known asset characteristics to determine the distribution of wealth amongst assets. Further, our analysis of two distinct datasets give results that are somewhat contradictory. For the industry dataset, we find that a mean-variance optimization model which takes into account transaction costs when deciding asset allocation yields the highest sharpe ratio net of transaction costs. For the 100 random stocks in the S&P 500, we find that simple rules manage to outperform both of the optimization models in terms of sharpe ratio net of transaction cost.

The results from our analysis show that a dollar-neutral investor faces tough constraints and it is hard to find an easy-to-use method of allocating assets. The dollar-neutral condition, in addition to being equally weighted, makes it challenging for the investor to achieve better results when using characteristics to produce portfolios. Most of the assets in the characteristic portfolios are concentrated around the median and are likely to have similar returns. It is difficult to make a profit when all assets around the median are included and equally weighted. By sorting the assets into e.g. quantiles one might avoid this issue and would more easily manage to distinguish and invest in only the top and bottom performers.

The issues of the mean-variance optimization (e.g. turnover, transaction cost and estimation error) are more apparent in the larger and more volatile dataset with S&P 500 stocks. As the number of assets increases, the degree of estimation error also seems to become more severe. The model manages to more easily find close substitutes, which in turn increases turnover and transaction cost from one dataset to another. This is in contrast to the 1/N strategies as they are relatively stable in this regard, and there is not much change in turnover and transaction cost.

Simple rules based on firm characteristics show promising results for two reasons: First of all, we are able to obtain a rather low turnover and transaction cost, especially for characteristics that are slow movers, e.g. Size and Book-to-Market ratio. Secondly, the characteristics yield the best result in the S&P 500 dataset, which is the most realistic scenario as all these assets are investable and easy to trade.

7 Conclusion

In this thesis, we ask whether simple rules can outperform sophisticated mathematical models for a dollar-neutral investor. Recent literature finds an interesting relationship between firm characteristics and subsequent stock returns, which enables investors to consider intuitive anomalies rather than moments of asset returns to determine allocation amongst assets. The dollar-neutral constraint makes it hard for all strategies considered to achieve noteworthy impressive sharpe ratios net of transaction costs. We show that simple median-based 1/N strategies are not necessarily optimal for a long-short investor, but outperform mean-variance strategies in the S&P 500 dataset. Instead of using the median as the determining value of long-short positions, we propose the use of e.g. quantiles, to more easily distinguish between top and bottom performers. It is difficult to determine if any of the characteristics are superior to the others, but slow-movers are efficient regarding turnover and transaction cost.

We found, using two distinct sets of data that many of the known issues with the mean-variance model persist under the conditions of a dollar neutral investor. Extreme weights were an issue when we added more constraints and conditions to the model with few assets, but were not a concern when we used a larger investment universe. Brandt et al. (2009) mention that the mean-variance optimization is best suited for single periods, which manifests itself in the high turnover we observe in our analysis. This is perhaps one of the main reasons why the model is seldom used by practitioners.

An advantage with the equally weighted characteristic based portfolios is that it allows investors to easily express their asset characteristic beliefs. Also, the simple strategies we consider in this paper could be used as potential benchmarks, especially for active characteristic-based strategies.

8 Further research

Combining characteristics to utilize underlying correlation and diversification benefits can, in some cases give better results than the single characteristic portfolios. Combining characteristics is also beneficial in mitigating the cyclicality of the characteristics. This observation is an interesting topic for future research. Brandt et al. (2009) proposed a simple approach of optimizing portfolios with a large number of assets. They do this through parametrizing the portfolio weight of each stock as a function of the stock's characteristics and estimate the coefficients by maximizing the average utility of the investor. In their paper, they focus on Value, Size and Momentum anomalies. They only consider a long-only and an unconstrained case. By taking their approach, considering a long-short investor, could be a promising way to pursue for future research.

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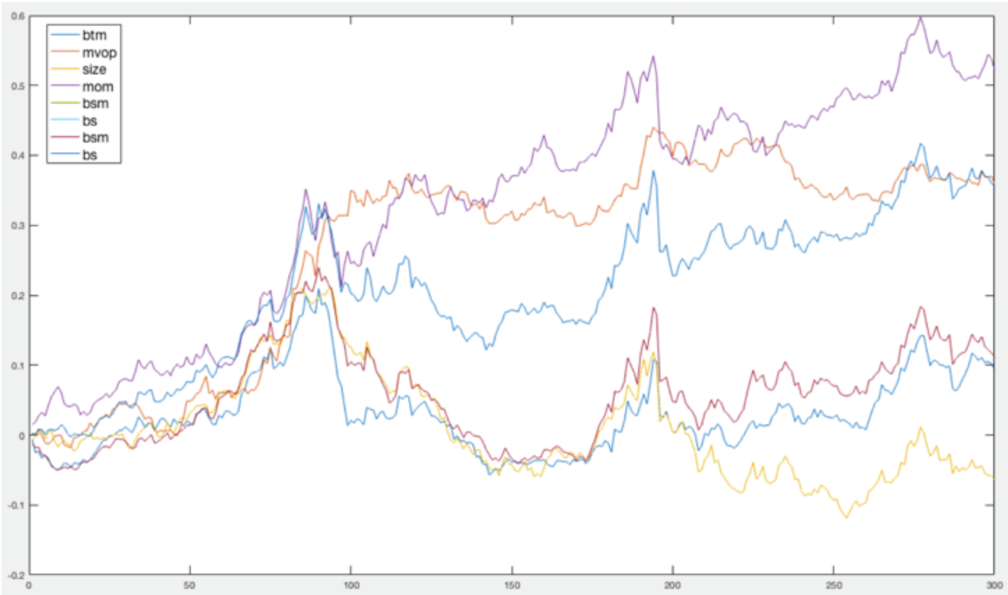
10 Appendix

10.1 Graphs

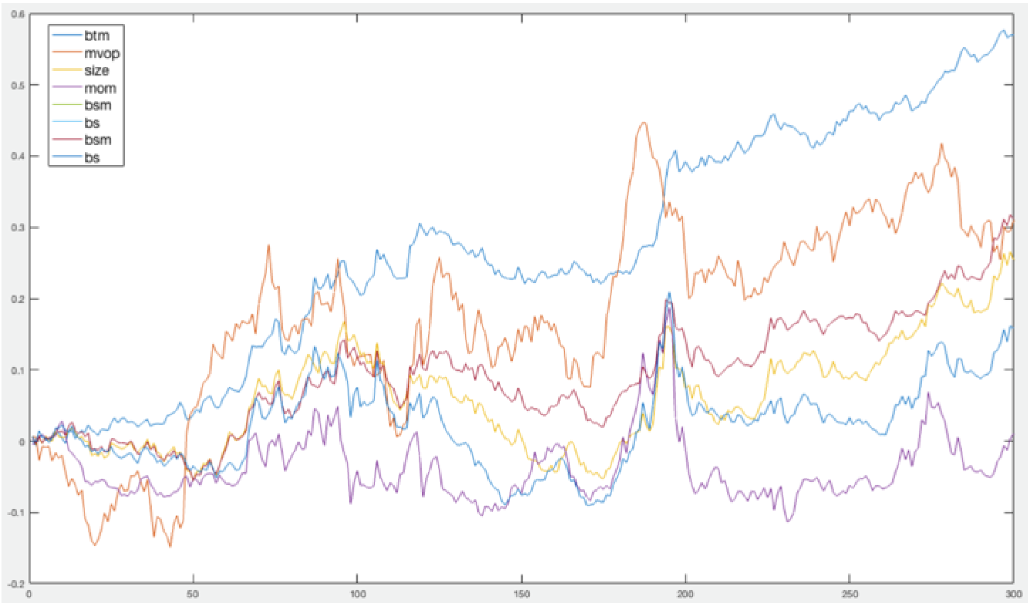
Graph 5: Cumulative return of all portfolios 1993-2017, dataset: Industry portfolios

The graph displays cumulative returns throughout the out-of-sample period for all portfolios. The x-axis displays 300 months, with a 50-month interval, while the y-axis displays cumulative returns up to 0,6, with an interval of 0,1.



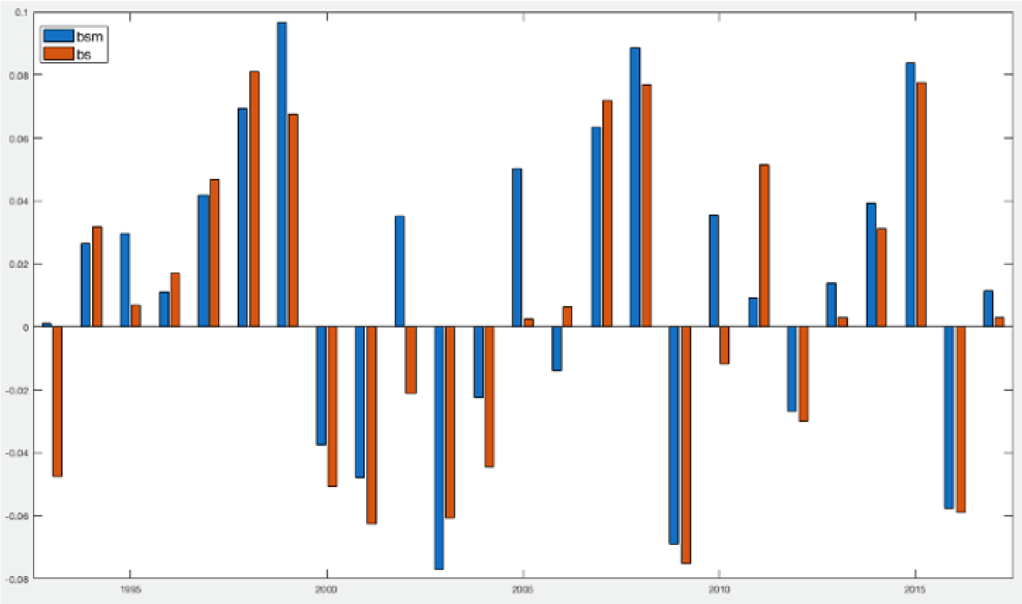
Graph 6: Cumulative return of all portfolios 1993-2017, dataset: S&P 500 stocks

The graph displays cumulative returns throughout the out-of-sample period for all portfolios. The x-axis displays 300 months, with a 50-month interval, while the y-axis displays cumulative returns up to 0,6, with an interval of 0,1.



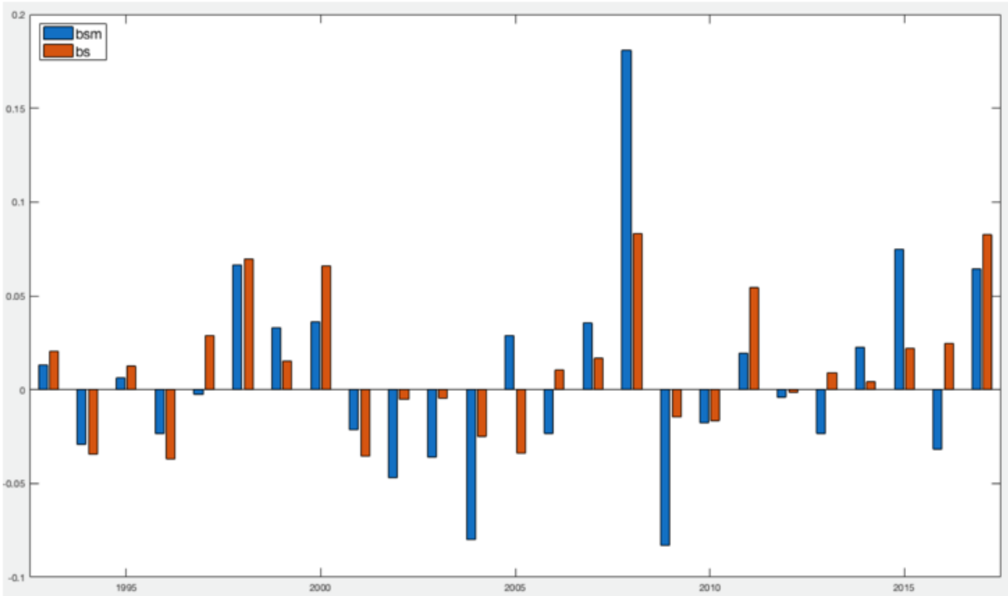
Graph 7: Yearly returns for bsm and bs 1993-2017, dataset: Industry portfolios

The graph displays yearly returns throughout the out-of-sample period for the combined characteristic portfolios, bsm and bs. The x-axis displays the full out-of-sample period (1993-2018), with a five-year interval, while the y-axis displays yearly returns up to 0,1, with an interval of 0,02.



Graph 8: Yearly returns for bsm and bs 1993-2017, dataset: S&P 500

The graph displays yearly returns throughout the out-of-sample period for the combined characteristic portfolios, bsm and bs. The x-axis displays the full out-of-sample period (1993-2018), with a five-year interval, while the y-axis displays yearly returns up to 0,2, with an interval of 0,05.



10.2 MatLab Code

```
%% Analysing Kenneth French Portfolios
%% Import data

ret = readtable('Book1.csv');

assets=ret.Properties.VariableNames(2:end);
rets = ret(739:end,2:end);
rets = table2array(rets)/100;
rf = 0;

[d, n] = size(rets);
date = ret(799:end,1);
%% Dollar Neutral Portfolio
DN = Portfolio('AssetList', assets, 'RiskFreeRate', rf);
DN = setBounds(DN, -1, 1);
DN = setBudget(DN, 0, 0);
DN = setOneWayTurnover(DN, 1, 1, 0);

% Out of Sample looping
wgtDN = ones(n,(d-60));
for i = 61:d
    DN = estimateAssetMoments(DN, rets(i-60:i-1,:));
    K = estimateMaxSharpeRatio(DN);
    wgtDN(:,i-60) = wgtDN(:,i-60) .* K;
    disp(i)
end
clear K i

wgtDN = transpose(wgtDN);
% Calculate Portfolio Stats
[StatsDN, retDN, cumDN] = ptfstats(wgtDN, rets);
clear DN
%% DN with transaction cost
DN = Portfolio('AssetList', assets);
DN = setBounds(DN, -1, 1);
DN = setBudget(DN, 0, 0);
DN = setOneWayTurnover(DN, 1, 1, 0);
Buycost = 0.005;
Sellcost = 0.005;
DN = setCosts(DN, Buycost, Sellcost);

% Out of Sample looping
wgtDNt = ones(n,(d-60));
for i = 61:d
    DN = estimateAssetMoments(DN, rets(i-60:i-1,:));
    K = estimateMaxSharpeRatio(DN);
    wgtDNt(:,i-60) = wgtDNt(:,i-60) .* K;
    disp(i)
end
clear K i rf

wgtDNt = transpose(wgtDNt);
% Calculate Portfolio Stats
[StatsDNt, retDNt, cumDNt] = ptfstats(wgtDNt, rets);

%% Equal weighted
wgtEW = ones(d-60,n) / n;
[StatsEW, retEW, cumEW] = ptfstats(wgtEW, rets);

%% Create variables
% Volatility
volmat = ones(d-12,n);
for i = 13:d
    volmat(i-12,:) = volmat(i-12,:) .* std(rets(i-12:i-1,:),0,1);
end
clear i

% Momentum
mom = ones(d-12,n);
cumBASE = cumsum(rets,1);

for i=14:d
    mom(i-12,:) = mom(i-12,:) .* (cumBASE(i-2,:) - cumBASE(i-13,:));
end
```

```
clear i cumBASE

% MKT CAP
mktcap = readtable('KFSize.csv');
mktcap = table2array(mktcap(739:end-3,2:end));

mktcap = log(mktcap(60:end-1,:)); % Correct the size,log & lag the variable

% BTM
btm = readtable('KFbtm.csv');
btm = table2array(btm(:,2:end));
ME = ones(25,40);
% Loop to find the mktcap at m12 of each year.
for i=1:25
    ME(i,:) = ME(i,:) .* mktcap(i*12,:);
end
clear i

BookEquity = ME .* btm(92-24:end,:); %Find Book Equity for the last 25 years
btm = ones(312,n);
for i=1:25
    for j=1:12
        btm(i*12+j,:) = btm(i*12+j,:) .* BookEquity(i,:);
    end
end
clear i j avgsize BE

btm = log(1 + (btm(13:end,:) ./ mktcap)) ;

%% Standardize Variables
volmat=zscore(volmat(49:end,:),0,2);
mom = zscore(mom(49:end,:), 0, 2);
mktcap = zscore(mktcap, 0, 2);
btm = zscore(btm, 0, 2);

%% Momentum Strategy
[mwMOM] = HMLmed(mom);
[StatsMOM, retMOM, cumMOM] = ptfstats(mwMOM, rets);

%% Market Cap Strategy
[mwMKT] = HMLmed(mktcap);
[StatsMKT, retMKT, cumMKT] = ptfstats(mwMKT, rets);

%% Book-to-Market Strategy
[mwBTM] = LMHmed(btm);
[StatsBTM, retBTM, cumBTM] = ptfstats(mwBTM, rets);

%% Combined Strategy
COMB1 = mom + mktcap + (btm * -1);
COMB2 = mktcap + (btm * -1);

%Median
[mwCOMB1] = HMLmed(COMB1);
[StatsCOMB1, retCOMB1, cumCOMB1] = ptfstats(mwCOMB1, rets);

[mwCOMB2] = HMLmed(COMB2);
[StatsCOMB2, retCOMB2, cumCOMB2] = ptfstats(mwCOMB2, rets);

%% Correlation matrix
CorrNames = {'BTM','MKT','MOM'};

corrmat = [retBTM retMKT retMOM];
corrmat = array2table(corrcoef(corrmat3),'RowNames', CorrNames, 'VariableNames',
CorrNames);

%% Create a table for results
Names = {'EW','BTM','MKT','MOM','COMB1', 'COMB2'};

Stats = vertcat(StatsEW, StatsBTM, StatsMKT, StatsMOM,...
    StatsCOMB1, StatsCOMB2);
Stats.Properties.RowNames=Names


%% Analysing S&P 500 Stocks
%% Import data
price = readtable('Stocks.csv');
BookEquity = readtable('BookEquity.csv');
```

```
%Calculate correct book value and merge with pricedata
at = table2array(BookEquity(:,11:14));
at(:,2:3)=at(:,2:3)*-1;
at = array2table(nansum(at,2));
BookEquity = BookEquity(:,1:2);
BookEquity(:,3) = at; clear at

%% Clean Data
price = outerjoin(price, BookEquity); clear BookEquity
price(:,10:11) = [];

% Fill in the blanks in number of shares
ns = table2array(price(:,8));
for i = 1:size(ns)
    if isnan(ns(i,1))
        ns(i,1) = ns(i+1,1);
        if isnan(ns(i+1,1))
            ns(i,1)=ns(i+2,1);
        end
    end
end
price(:,8)=array2table(ns); %Put back in pricedata
clear i ns
%Fill blanks in book value
price(:,10) = fillmissing(price(:,10),'Next');
price(:,4:5) = [];
dates = table2array(price(:,3));
dates = (datetime(dates,'InputFormat','yyyy/MM/dd'));

price(:,3) = [];

%Remove stocks with iid>2
price = table2array(price);
clean = (price(:,2)>1);
price(clean,:) = [];
dates(clean,:) = [];
clear clean

%% Create variable matrices
% Returns
assets = price(:,1);
prices = price(:,4) ./ price(:,3);
rets = table(assets, dates, prices);
rets = (unstack(rets, 'prices', 'assets'));
rets = rets(1:end-1,2:end);

%Book to market
me = price(:,5) .* 1000 .* prices;
bv = price(:,7) .* 1000 ./ me;
btm = table(assets, dates, bv);
btm = (unstack(btm, 'bv', 'assets'));
btm = btm(:,2:end);
%Market cap
mktcap = table(assets, dates, me);
mktcap = (unstack(mktcap, 'me', 'assets'));
mktcap = mktcap(:,2:end);

clear bv me prices
% Clear missing data
btm = rmmissing(btm,2);
mktcap = rmmissing(mktcap,2);

vars = mktcap.Properties.VariableNames;
rets = rets(:,vars); clear vars

% Finalise tables
rets = table2array(rets);
btm = table2array(btm);
mktcap = table2array(mktcap);
```

```
for i = 2:383
    ret(i,:) = log(rets(i,:) ./ rets(i-1,:));
end

rets = ret; clear ret i
mktcap = log(mktcap);
%%
mktcap = mktcap(:,1:100);
btm = btm(:,1:100);
rets = rets(:,1:100);
[d,n] = size(btm);

% Momentum
mom = ones(d-12,n);
cumBASE = cumsum(rets,1);

for i=14:d
    mom(i-12,:) = mom(i-12,:) .* (cumBASE(i-2,:) - cumBASE(i-13,:));
end
clear i cumBASE
%% Standardize Variables
mom = zscore(mom(73:end,:), 0, 2);
mktcap = zscore(mktcap(85:end,:), 0, 2);
btm = zscore(btm(85:end,:), 0, 2);
rets = rets(24:end,:);


%% Dollar Neutral Portfolio
NA = 100;
DN = Portfolio('NumAssets', NA);
DN = setBounds(DN, -1, 1);
DN = setBudget(DN, 0, 0);
DN = setOneWayTurnover(DN, 1, 1, 0);

[d,n]=size(rets);
% Out of Sample looping
wgtDN = ones(n,(d-60));
for i = 61:d
    DN = estimateAssetMoments(DN, rets(i-60:i-1,:));
    K = estimateMaxSharpeRatio(DN);
    wgtDN(:,i-60) = wgtDN(:,i-60) .* K;
    disp(i)
end
clear K i

wgtDN = transpose(wgtDN);
% Calculate Portfolio Stats
[StatsDN, retDN, cumDN] = ptfstats(wgtDN, rets);
clear DN

%% DN with transaction cost
DN = Portfolio('NumAssets', NA);
DN = setBounds(DN, -1, 1);
DN = setBudget(DN, 0, 0);
DN = setOneWayTurnover(DN, 1, 1, 0);
Buycost = 0.005;
Sellcost = 0.005;
DN = setCosts(DN, Buycost, Sellcost);

% Out of Sample looping
wgtDNt = ones(n,(d-60));
for i = 61:d
    DN = estimateAssetMoments(DN, rets(i-60:i-1,:));
    K = estimateMaxSharpeRatio(DN);
    wgtDNt(:,i-60) = wgtDNt(:,i-60) .* K;
    disp(i)
end
clear K i rf

wgtDNt = transpose(wgtDNt);
% Calculate Portfolio Stats
[StatsDNt, retDNt, cumDNt] = ptfstats(wgtDNt, rets);
```

```
%% Equal weights
wgtEW = ones(300,n) / n;
[StatsEW, retEW, cumEW] = ptfstats(wgtEW, rets);

%% Momentum Strategy
[mwMOM] = HMLmed(mom);
[StatsMOM, retMOM, cumMOM] = ptfstats(mwMOM, rets);

%% Market Cap Strategy
[mwMKT] = HMLmed(mktcap);
[StatsMKT, retMKT, cumMKT] = ptfstats(mwMKT, rets);

%% Book-to-Market Strategy
[mwBTM] = LMHmed(btm);
[StatsBTM, retBTM, cumBTM] = ptfstats(mwBTM, rets);

%% Combined Strategy
COMB1 = mom + mktcap + (btm * -1);
COMB2 = mktcap + (btm * -1);

%Median
[mwCOMB1] = HMLmed(COMB1);
[StatsCOMB1, retCOMB1, cumCOMB1] = ptfstats(mwCOMB1, rets);

[mwCOMB2] = HMLmed(COMB2);
[StatsCOMB2, retCOMB2, cumCOMB2] = ptfstats(mwCOMB2, rets);


%% Correlation matrix
CorrNames = {'BTM','MKT','MOM'};

corrmat = [retBTM retMKT retMOM];
corrmat = array2table(corrcoef(corrmat3),'RowNames', CorrNames, 'VariableNames',
CorrNames);

%% Create a table for results
Names = {'EW','BTM','MKT','MOM','COMB1', 'COMB2'};

Stats = vertcat(StatsEW, StatsBTM, StatsMKT, StatsMOM,...
    StatsCOMB1, StatsCOMB2);
Stats.Properties.RowNames=Names
```

Functions:

Calculate portfolio stats, returns and cumulative return

```
function [stats, retSeries, cumulative] = ptfstats(Portfolioweights, Assetreturns)

retSeries = sum((Portfolioweights .* Assetreturns(end-299:end,:)),2);
cumulative = cumsum(retSeries);
average = mean(retSeries)*12;
trcost = 0.005 .* sum(abs(diff(Portfolioweights, 1, 1)),2);
avgtrc = mean(trcost)*12;
avgnet = average - avgtrc;
vol = std(retSeries)*sqrt(12);
sr = average / vol;
volnet = std(retSeries(2:end,:) - trcost)*sqrt(12);
srnet = avgnet / volnet;
turnover = mean(sum(abs(diff(Portfolioweights, 1, 1)),2))*1200;
trades = mean(sum(abs(diff(Portfolioweights,1,1))>0,2));
minw = min(min(Portfolioweights));
maxw = max(max(Portfolioweights));

stats = table(average, vol, sr, avgnet, srnet, avgtrc, turnover, trades, maxw, minw);
end
```

Determine long/short positions

```
Low-minus-high
function [weights] = LMHmed(Factormatrix)
n = size(Factormatrix,2);
Q = median(Factormatrix, 2);
Long = Factormatrix <= Q;
```

```
Long = Long .* (1/(n));
Short = Factormatrix >= Q;
Short = Short .* (-1/(n));

weights = Long + Short;
end
```

High-minus-low

```
function [weights] = HMLmed(Factormatrix)
n = size(Factormatrix,2);
Q = median(Factormatrix, 2);
Long = Factormatrix >= Q;
Long = Long .* (1/(n));
Short = Factormatrix <= Q;
Short = Short .* (-1/(n));

weights = Long + Short;
end
```

10.3 Preliminary thesis

Faraz Seyedi
Haakon Gromstad

BI Norwegian Business School
Master Thesis

- Portfolio optimization for a long/short investor -

Supervisor:
Chunyu Yang

Hand-In date:
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GRA 19502 – Preliminary thesis report

Programme:
Master of Science in Business – Major in Finance

Table of Contents

I Introduction

In this thesis we want to examine optimal portfolio construction for a long-short investor. By long-short investor, we specifically mean a dollar neutral investor. Recent litterature show that the simple 1/N asset allocation rule outperforms more sophisticated optimization models out-of-sample. The reason for this is much due to the large estimation errors in forecasting asset returns. Furthermore, studies suggests a parametric approach, that focuses on stock characteristics rather than moments of asset returns to allocate wealth amongst assets, which manages to substantially reduce the large estimation error. However, the portfolio strategies examined do not consider a dollar neutral investor. We suspect that a dollar neutral investor is affected differently by the estimation error. This investor also face constraints in some form, which can influence i.e. the asset turnover. Therefore our research question is: “What is the most robust portfolio construction considering a dollar neutral investor?”.

We will use models presented in recent literature by Brandt, Santa-Clara and Valkanov (2009), Lewellen (2015) and Uppal. et. al (2009) in order to compare different optimization strategies. Furthermore, produce results using MatLab, showing in-and out-of-sample performance with relevant measurements. We will draw conclusions based on results given from the different strategies.

In the next sections, we will present recent studies in the field of portfolio optimization. In section III we discuss relevant theory regarding our research question. Section IV describes the data and the methodology that we will use to answer the research question. The last section will describe our further progress throughout the semester.

II Litterature Review

The issue of portfolio optimization has long been a topic of interest in the financial world. The underlying economic theory of optimal portfolio construction was pioneered by Markowitz(1952), Merton(1969, 1971), Samuelson(1969) and Fama(1970), with Markowitz perhaps the most influential of these.

Markowitz - Portfolio Selection

The most common formulation of portfolio choice problems is the mean-variance paradigm presented by Markowitz in 1952 in his article "Portfolio Selection". The idea of the mean-variance paradigm is to choose portfolio weights that optimizes the overall risk-return trade-off (Sharpe Ratio). First of all, it shows that imperfectly correlated assets can be combined into portfolios with the preferred expected return/risk characteristics. Secondly, the paradigm states that once a portfolio is fully diversified, the investor must take on more risk (greater allocations) to achieve higher expected returns. There are some problems regarding this theory, firstly Markowitz assumes quadratic utility only. Secondly, the paradigm ignores any preferences towards higher-order return moments (ie. Skewness and Kurtosis). Thirdly, the mean-variance problem works best for single periods, where most investments have longer horizons which means that the portfolio needs continuously rebalancing. (Brandt, 2009).

Uppal et al. (2009) found that the 1/N asset allocation rule performs quite well versus more complex mathematical models based on Markowitz' approach, and extensions of it. The 1/N rule is an equally weighted portfolio, where we consider N risky assets and allocate 1/N to each of the N risky assets. The weight in each asset is set to equal to wj=1/N. This method is simple to use and is therefore favoured by many investors. Furthermore, it is described as naive because of its roughness and a common sense construction of a portfolio, using a logical approach without taking into account sophisticated mathematical models.

In their study, they compared the naive 1/N rule with 14 different asset allocation strategies. Based on three performance measures; Sharpe Ratio, certainty-equivalent value and turnover rate, they show that the 1/N rule performs quite well out-of-sample. Also, the estimation window necessary to outperform the 1/N strategy may be very large. Moreover, the researchers conclude that their main finding is that the large error in forecasting may overwhelm the gains from optimization.

Cross-Sectional Forecasting

Since Markowitz, a lot of research has been done on the subject of portfolio construction but no single model has managed to stand out as a clear superior. However, the field of cross-section of stock returns and its patterns stands out. Fama and French (1993) start this movement by showing that the market beta is barely related to the cross-section of average stock returns, and introduce an overall market factor, firm size and book-to-market value as more appropriate factors. Fama and French (1996) found that size, book-to-market and lagged returns robustly describe the cross section of expected returns. Chan, Karceski and Lakonishok (1998) showed that these are also related to variances and covariances of returns. For these reasons, many have followed up on this line of work with different characterizing parameters and patterns. Lewellen (2015) argues that many of the documented patterns are highly significant and seem almost certainly to be real, and not due to randomness or data-tinkering. The literature shows that many of the firm characteristics are correlated with subsequent stock returns, and Lewellen examines the factors to find whether the estimates of cross-sectional properties line up with true expected returns.

Lewellen (2015) studied cross-sectional properties of return forecasts derived from Fama-MacBeth regressions, and how these could be used by investors in real time to construct portfolios. Lewellen showed that combination of up to 15 characteristics could be used to estimate a stock’s expected return in real time. The strategy is *reasonably* accurate over the next month, but gets noisier for longer periods of time. He also recommends a shrinkage of 20%-30% of the monthly estimates, and up to 50% for annual estimates. Another potentially problematic aspect of his work is that there is no clear way of optimizing the weights in the individual assets.

Parametric Portfolio Construction

It has been known that stock characteristics are related to stock’s expected return, variance and covariance with other stocks. Many have tried to exploit this fact, including Brandt, Santa-Clara and Valkanov (2009). They propose a simpler approach to optimizing portfolios with large number of assets. They do this through parametrizing the portfolio weight of each stock as a function of the stock’s characteristics and estimate the coefficients by maximizing the average utility of the investor.

The proposed model has several major advantages. First, it avoids the step of modelling the joint distribution of returns and characteristics, and focuses directly on the portfolio weights. Second, parametrizing the portfolio policy results in a reduction in dimensionality. The difficulties of the Markowitz approach were briefly discussed earlier. Using Markowitz' approach for N stocks requires modelling N first and (N^2 + N)/2 second moments of returns, which makes a relatively small number of stocks of 100 unmanageable with over 300,000 third moments. The parametric policy involves only N portfolio weights regardless of the joint distribution of asset returns and investor preferences. The third point is that their approach captures the relation between the characteristics and expected returns, variances, covariances and even higher order moments of returns.

The model is highly appealing because of its simplicity. The traditional mean-variance method presents a formidable econometric problem, and requires substantial resources to handle even with different fixes. The idea of this new model is to reduce the estimation error and avoid problems of over-fitting.These challenges result in optimization based on characteristics to be rarely used by practitioners. The parametric portfolio policy is simple to implement from a practical perspective, and the authors state that the model is easily modified and extended.

The strategies compared to the equal weighted portfolio strategy in Uppal et al. (2007) are limited to models that consider moments of asset returns, but not other characteristics of the assets. In 2009 Uppal et al. made an extended version of their article where they included testing the naive portfolio strategy against the more recent model created by Brandt, Santa- Clara and Valkanov, which focuses on other characteristics of equity returns.They proposed the idea of modeling the portfolio weights in firm i as a benchmark plus a linear function of the firm i's characteristics. By doing this one manages to reduce the estimation error and avoid problems of over-fitting. Because Brandt, Santa Clara and Valkanov use asset specific characteristics, only two data sets considered in uppal et al (2009) were applicable for testing the strategy against the equal weight portfolio. Uppal et al. concludes that this strategy does lead to an improvement, but is still beaten by the naive strategy because of its substantially lower turnover rate. The method of Brandt, Santa-Clara and Valkanov is not applicable for all asset classes such as international stock indexes, where it is not clear what characteristics that explain returns on country indexes. However, more energy needs to be devoted to improving the estimation of parameters for the moments of asset returns and further research maybe include more variables in constructing portfolios (Uppal et.al 2009).

The model proposed by Brandt, Santa-Clara and Volkanov (2009) was tested by Uppal et. al.(2009), and it was shown that the model did not present a significant improvement compared to 1/N model, mostly due to high turnover. The question of how transaction costs would impact the performance of this model was unanswered. This led to the introduction of a method which includes adding trade costs and a no-trade boundary. The authors state that this method will reduce turnover without damaging the out-of-sample returns achieved by the model.

Motivation

In the prior research we examine that all articles consider a constrained case and unconstrained cases. Whereas the constrained case is a typical Long-only portfolio, while the unconstrained case gives the ability for the investor to hold short positions. Jacobs, Levy and Starer (1998) state that many hedge fund practitioners seek to constrain their portfolios to be neutral with respect to some factor. In particular, they seek to be dollar-neutral by committing the same amount of capital to their long holdings as their short holdings, and achieving a net market exposure of zero. They stress that an unconstrained portfolio is more likely to be optimal compared to a dollar-neutral portfolio, but sometimes fund managers are required to operate within these boundaries(required by clients, or taxation, accounting or behavioral reasons). Whatever the reason is, it is generally not a pressing financial reason for doing this. Nevertheless, it is common practice and therefore relevant. Jacobs, Levy and Starer (1999) define a dollar neutral portfolio as one where the net holding of risky securities is zero. In a more formal way we classify the weight constraints for a dollar neutral investor as follows:

$$\sum w_{long} = -\sum w_{short}$$

$$\sum w_{portfolio} = 0$$

$$\sum |w_{portfolio}| = 2$$

This is common practise amongst hedge funds, where they go short in assets with low expected returns and long in assets with high expected returns. We suspect that this will affect the error in forecasting differently so that it is possible to produce results that can outperform what we consider as a benchmark, the naive 1/N strategy. One thing that Uppal et. al. (2009) criticized this model for was the high turnover, which we believe might be reduced by imposing these constraints and therefore achieve better results.

We would also like to see how the model performs with other types of assets. Uppal et. al.(2009) mention that the proposed model is difficult to implement on other asset classes, and that it would be unclear which parameters are most significant. We will conduct our tests on country indexes from Datastream MSCI to check whether Uppal et. al. are correct, and if the model performs well on other types of assets.

III Theory

Now we will examine relevant theory regarding our research question, namely finding the most applicable model for a dollar neutral investor. We will discuss topics which emphasize the importance of our work, describing quantitative portfolio management, common practice in hedge funds and some background on the work that has been done to reduce estimation error and forecasting quality by other researchers.

> "*Modern portfolio theory has one, and really only one, central theme: In constructing their portfolios investors need to look at the expected return of each investment in relation to the impact that it has on the risk of the overall portfolio*".Litterman, B. (2004).

Quantitative Portfolio Management

On a general level, there are two basic approaches to managing equity portfolios; the traditional approach and the quantitative approach. This paper is focused on a quantitative approach. "*Quants"* use statistical models to make forecasts of each stock's return, risk, and cost of trading based on measurable factors. Because these processes usually are computerized, they can evaluate a vast amount of securities and can be updated more frequently. These models tend to be unbiased as they are based on historical data, but a downside of such strategy is that it cannot uncover misleading or unrepresentative data the way a traditional analyst can. Quantitative strategies often spread the risk across many small bets, which can add value with only slightly favourable odds (Alford, Jones and Lim, 2003).

Constraints, weights and estimation error

It is well known in the standard mean-variance portfolio optimization that optimal portfolio weights are very sensitive to small changes in expected excess return. A main problem is that the weights can in some cases take extreme values, especially in a unconstrained case where the investor can hold short positions. Because of this, the common practice in portfolio optimizations is to have many constraints on asset weights (Litterman, 2004). However, the "Black Litterman approach" addresses this sensitivity without adding constraints. Where he instead assumes that there are two ways to capture information about future excess returns: investor views and market equilibrium. The estimates of the expected returns are combined by both sources of information. By doing this the model provides more reasonable weights in the view portfolio (investors own expectations), because there is always uncertainty related to that view. In respect to the total portfolio risk, the model balances the contributions to expected return on the view portfolio and the market portfolio. This will result in more realistic portfolio weights (Litterman, 2004). However, the estimation error is still present as in the Markowitz optimization paradigm, and there is no way of guaranteeing the positive definiteness of the covariance matrix of estimation errors (Brandt, Santa-Clara and Valkanov 2009).

Factor Models

A popular approach to reducing the statistical error of an estimation model is to impose a factor structure on the covariance matrix. This has several benefits as it reduces the number of free parameters in the matrix, and it is a way of ensuring the positive definiteness of the covariance matrix. This method dramatically simplifies the Markowitz method. For instance, with five factors, there are 3515 coefficients to estimate if the factors are correlated, compared to 125,000 in the case without factors. The difficulty of such model arises with the choice of common factors. There are essentially three types of factors that one can use; factors based on economic theory, based on empirical work, or directly extracted from the returns using statistical procedures.

Cross-Sectional Portfolio

Cross sectional estimation was pioneered by Fama and MacBeth (1973). Their model built on the traditional CAPM, they estimated betas from first-pass regression, then they performed a second-pass cross-sectional regression for each month in the estimation period. The equation they tested was:

$$\widetilde{R}_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\beta_i - \hat{\gamma}_{2t}\beta_i^2 + \hat{\gamma}_{3t}S_{ei} + \eta$$

With this equation, they were able to study how parameters change over time, and could test a series of hypotheses regarding the CAPM. Based on this they concluded that neither beta squared nor residual risk has an influence on returns.

Elton (2009) argues that their paper is one of the most influential papers ever written in this field and that virtually every subsequent paper uses one or more of the elements they introduced. A lot of empirical studies use the idea of time series data to identify risk exposure while cross-sectional differences identify risk premia. The intuition behind their work has been the building blocks of many of the forecasting methods available. For example, Fama and French (1992) showed that a firm’s size and book-to-market values were significantly related to expected returns building on the work of Fama and MacBeth(1973). This is a topic that is highly relevant to the objective of this paper and the models that will be tested are directly related to the early work of Fama and MacBeth (1973).

Hedge Funds

Hedge funds are attractive for two reasons. They offer the opportunity to increase expected returns, while they are believed to diversify total portfolio risk. Hedge funds are appealing because they offer the potential to increase expected portfolio return at the expense of little or no change in expected risk (Winkelmann, Clark, Rosengarten and Tyagi, 2003). There are some fundamental characteristics that set hedge funds apart from their active manager counterparts. They are not faced with the same constraints as traditional managers. An active manager is usually constrained from making short sales, and is limited to investing assets that are included in the benchmark they are compared to. Hedge funds can go both long and short, and can take advantage of both positive/high and negative/low returns in the market. They are also not limited to their any benchmark and have access to a larger investment universe.

One of the difficulties for investors when it comes to hedge funds is evaluation. It is more difficult to measure a portfolio which consists of all sorts of assets with different risk characteristics in both long and short positions. Hedge funds are most commonly compared to cash, while a traditional active manager who holds long positions only is compared to an index.

The characteristics of a hedge fund present several interesting opportunities. A hedge fund manager has more opportunities and higher chance of finding good trades, but also makes for a complicated portfolio optimization process. Many managers make the mistake of handling the long/short portfolio as two portfolios. This leads to optimizing a long only portfolio, and a short only portfolio, and combine these. This process neglects the positive gains that can be achieved through optimizing a portfolio where relationships between all stocks are considered (Jacobs, Levy, Starer, 1998).

Hypothesis

The aim of this paper is to compare the model of Brandt, Santa-Clara and Valkanov (2009) to several benchmarks, including 1/N, Lewellen (2015) and a value weighted model. We will do so under specific constraints facing a hedge fund manager, namely a dollar neutral portfolio. Furthermore, we will examine how the model performs using other asset classes by testing it on indexes as well as stocks.

IV Methodology and data

Data

As we have previously mentioned, we are interested in finding whether the model of Brandt, Santa-Clara and Valkanov (2009) can be applied to different asset classes. For this purpose, we will get World Country Indexes from MSCI, which contains around 50 countries, including several emerging market countries. We will get data on the factors to be tested such as book-to-market value, momentum etc. from the same source.

We also want to see the impact of the dollar neutral portfolio on estimation error and turnover. For this purpose, we will be using US stocks from the CRSP/Compustat databases. Depending on which parameters we decide to base our forecasts on, we will get stock data from CRSP monthly files and accounting data from the Compustat annual file.

In both cases, we want our model to be as solid as possible, and we will consider data for as many years as possible for both the set of stocks, and indexes.

Model description

Brandt, Santa-Clara and Valkanov

Our main focus will be on the proposed approach by Brandt, Santa-Clara and Valkanov (2009). The simple linear specification of the weights is as follows:

$$w_{i,t} = \overline{w}_{i,t} + \frac{1}{N_t}\theta^T \hat{x}_{i,t}$$

with the aim of maximizing the conditional expected utility:

$$\max\ E_t\left[u\left(r_{p,t+1}\right)\right] = E_t\left[u\left(\sum_{i=1}^{N_t} w_{i,t} r_{i,t+1}\right)\right]$$

where theta is a vector of coefficients to be estimated, $\hat{x}_{i,t}$ is the vector of characteristics of stock i (standardized cross-sectionally with zero mean and unit std.dev) and $\bar{w}_{i,t}$ is the weight of the stock i in a benchmark portfolio. The model estimates weights as a single function of characteristics that applies to all stocks over time, rather than estimating one weight for each stock at each point in time.

The intercept is represented by the stocks weight in the benchmark portfolio, and the term $\theta^T \hat{x}_{i,t}$ is the deviation of optimal weight from the benchmark portfolio. The characteristics are standardized, so that $\hat{x}_{i,t}$ is stationary and that the average of $\theta^T \hat{x}_{i,t}$ is zero. The latter means that the sum of deviations of optimal weights from the benchmark portfolio is zero.

In their article they express the most important manner of their parameterization is that the coefficients are constant across assets and through time. This involves that the portfolio weight in each stock depends only on the stock's expected characteristics and not on the stocks' expected return. For example if two stocks have similar characteristics related with expected returns and risk should have more or less the the same weights in the portfolio, even if their sample returns are very different. (Brandt, Santa-Clara and Volkanov. 2007).

The return of the portfolio policy is written as:

$$r_{p,t+1} = \sum_{i=1}^{N_t} \bar{w}_{i,t} r_{i,t+1} + \sum_{i=1}^{N_t} \left(\frac{1}{N_t} \theta^T \hat{x}_{i,t}\right) r_{i,t+1} = r_{m,t+1} + r_{h,t+1}$$

The formulation of this return function is interesting for our case. *h* is described as a long-short hedge fund with weights $\frac{1}{N_t} \theta^T \hat{x}_{i,t}$ that add up to zero. So the return on the dollar neutral portfolio that we are going to analyze will be the following:

$$r_{p,t+1} = \sum_{i=1}^{N_t} \left(\frac{1}{N_t} \theta^T \hat{x}_{i,t} \right) r_{i,t+1} = r_{h,t+1}$$

Furthermore, we follow the authors in assuming that the investors have constant relative risk aversion (CRRA) preferences, with utility function

$$u(r_{p,t+1}) = \frac{(1 + r_{p,t+1})^{1-\gamma}}{1 - \gamma}$$

For the case of a long only portfolio, we need to renormalize the weights for the sum of the optimal portfolio weights to be one. We do this according to presented method by Brandt, Santa-Clara and Valkanov (2009):

$$w_{i,t}^{+} = \frac{\max[0, w_{i,t}]}{\sum_{j=1}^{N_t} \max[0, w_{i,t}]}$$

For comparison, we will use a value weighted portfolio (market portfolio), an equally weighted portfolio (1/N strategy) and the method used in Lewellen (2015) as benchmarks. Lewellen presents a method of using the cross-sectional properties of returns in order to create a portfolio, and is an interesting comparison in this case. We are also interested in the comparison with the 1/N strategy as Uppal et. al. (2009) showed that the 1/N consistently outperformed 14 extensions of the sample-based mean-variance model.

Lewellen (2015) utilises the predictive power of the cross-sectional properties of returns, and derives forecasts from Fama-MacBeth regressions of stock returns on the lagged firm characteristics. He uses up to 15 characteristics to forecast returns, and creates high-minus-low (H-L) strategies with reasonable success.

Key performance indicators

In order to analyse and compare the different models we have to consider relevant performance measures. The chosen measures are sharpe ratio, certainty equivalent return and Asset Turnover. These three measures are in line with measurements used in recent literature regarding this topic such as Uppal et.al (2009) and Brandt, Santa-Clara and Valkanov (2007). A description of each measurement is now presented:

Sharpe Ratio

The Sharpe Ratio is a common formula to measure the trade off between risk and return. It simply divides the portfolio excess return by its standard deviation.

$$SR = \frac{\mu - r_f}{\sigma}$$

Certainty Equivalent

The certainty equivalent measures the risk free rate that the investor is willing to accept instead of investing in a risky portfolio policy.

$$CEQ = (\mu - r_f) - \frac{\gamma}{2}\sigma^2$$

Asset Turnover

The asset turnover represents the absolute change in weights from one period to another. Since we do not consider transaction costs in our paper, we consider this as an important measure because rapidly changing positions in stocks can give sufficient costs which affects the actual return in the end of the period.

$$Turnover = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|w_{j,t+1} - w_{j,t}\right|\right)$$

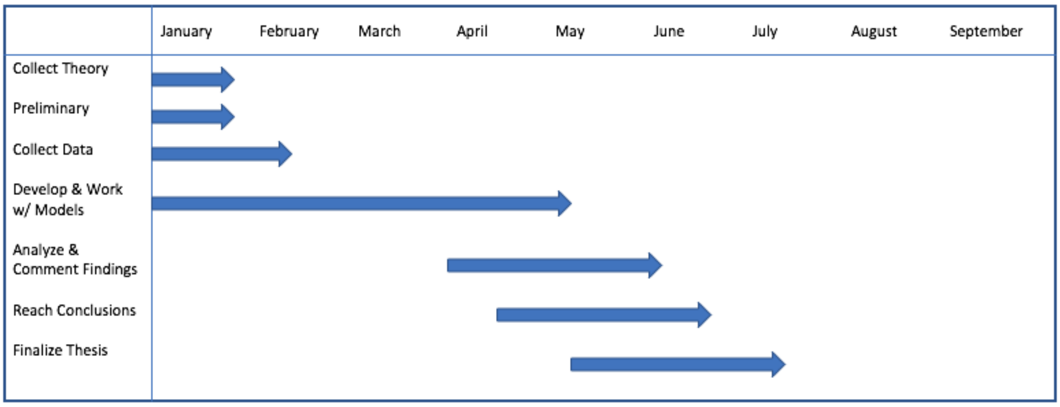
In addition to these three measurements of performance we want to calculate the return-loss for each strategy tested against the equally weighted portfolio. We will use the same measurement as in the article of uppal et al. (2009). Return-loss defines the additional return needed for strategy k to perform as well as the 1/N strategy in terms of sharpe ratio.

Return-loss:

$$return - loss_k = \frac{\mu_{ew}}{\sigma_{ew}} \times \sigma_k - \mu_k$$

V Further Progress:

To get a better understanding of what needs to be completed throughout the semester, it is practical to visualize the progress.



We expect to get all the needed data before the start of february. We have quite a lot to do and learn when it comes to programming and working in Matlab, so we will start working in Matlab to get familiar with the program before we have all the necessary data. All of our results and conclusions will come from the work we do in Matlab, and we expect that the stage of developing and working with the models discussed will be the most time consuming part of this thesis. We plan to finish the tests by march, so we can start analyzing and commenting upon our findings. There will almost certainly be need of going back to the models while commenting the results, but we hope to be completely finished with everything related to Matlab by mid-April. From March, we will start writing our comments, and continually work out a conclusion and work on finalizing the thesis. Our plan is to work consistently with the thesis every week, but there will also be times when projects and exams in other subjects will hinder our progress to a degree. The deadline is in september, but we aim to be finished sometime in June. This way, we will be ahead of schedule in case of any unexpected setbacks.

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