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Goal-Based Portfolios - A mean-variance optimization approach with subportfolios -

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MSc. in Business, major Finance

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Abstract

This thesis analysis the goal-based portfolio optimization approach and compares it to established theories of portfolio management. First, we review previous literature on the topic of portfolio optimization. Second, we identify the investor's problem and define the methodology. Further, we perform a quantitative analysis of the goal-based portfolio optimization approach. We use historical asset returns to simulate future portfolio outcomes and analyse the performance of an investment according to goal-based portfolio theory. We find that by dividing an investment into multiple subportfolios, and optimizing each subportfolio separately, decreases the portfolios probability of failure. We conclude that an investor, with specific goals beyond attaining highest possible return, is better off investing in subportfolios as opposed to a single portfolio.

Introduction

The role of an investment advisor can be summarized by three main steps; translating an investor's goals into the language of finance, determining an appropriate investment portfolio, and managing the investor's portfolio. Unfortunately, investors often misspecify their goals, which may cause difficulties for investment advisors. Markowitz's "portfolio selection" (1952) is considered the foundation of modern portfolio theory (MPT), and has been embraced by practitioners and theorists since its publication. Goal-based portfolio theory combines appealing components of Markowitz's mean-variance portfolio and the behavioral portfolio theory (BPT) of Shefrin and Statman (2000). Ultimately, goal-based portfolios combine investment strategies with clear and specific investor goals.

Following the financial crisis in 2008, goal-based portfolio theory has received increased attention. As investments turned out to be less liquid during the crisis, investors realized that their diversification strategies only worked under normal market conditions, and that a severe bear market could affect the fulfillment of personal goals. The financial crisis suggested a change in wealth management thinking. Das, Markowitz, Scheid and Statman (2010) argued that goal-based portfolio approaches were just as efficient as the mean-variance approach, when clients and wealth managers change their definitions of risk. By enabling clients to measure progress towards their goals, goal-based portfolios increase the clients' commitments to their lifecycle goal, and reduce negative behavioral bias, such as impulsive decision-making.

Investors tend to have multiple, sometimes conflicting goals, each with varying levels of risk tolerance. In this study, we investigate the goal-based portfolio optimization approach and compare it to the mean-variance portfolio optimization approach. Relying on historical asset returns we simulate future returns to find optimal portfolios for an investor.

Background and motivation

In his article “portfolio selection”, Markowitz (1952) proposed an approach to identify quantifiably set of portfolios that maximize return, and minimize variance. The set of portfolios with most attractive risk-return tradeoff is called the mean-variance frontier, and investors should optimally only consider investing in these portfolios. Markowitz introduced investor’s need of diversification, not only by increasing number of securities in the investor’s portfolio, but also by reducing the variance of returns. Even though most later research, in large, has been built on Markowitz’s portfolio selection theory, some researchers believes that individual investors are not merely concerned with attaining highest possible return at the lowest risk, but that they in fact attempt to reach specific goals with varying levels of risk tolerance.

The goal-based portfolio theory combines recognition of behavioral biases in investor’s investment decision and MPT. One of the early researchers to recognize the impact of investors behavioral bias is Thaler (1980), who states that individual investors are not concerned of the overall portfolio performance, rather they want to make investment decisions to meet specific goals. Thaler goes on describing that each goal has its own subportfolio with different risk levels.

Sortino and van der Meer (1991) introduced the Post-Modern Portfolio Theory, which redefines risk as the probability of not achieving the objectives. In recent years, contemporary portfolio theory has emerged, which seeks to protect investments against failure, even if it might reduce potential rewards.

The behavioural aspect of the goal-based portfolio theory is the main motivation for this study, and we hope to verify the appropriateness of this approach in constructing investor portfolios.

Problem formulation

Purpose

The purpose of this thesis is to analyse whether an investor, who seeks to achieve specific investment goals, is better off by separating her holdings and optimizing subportfolios, as opposed to investing her holding in a single optimized portfolio. We compare the goal-based portfolio optimization approach to the mean-variance portfolio optimization approach. We aim to analyse how several optimized subportfolios assigned to each goal perform in comparison to an optimization of one single portfolio.

Research Question

Is an investor better off by constructing subportfolios for each goal and optimizing them separately, as opposed to optimizing one single portfolio? *If so, at what risk tolerance level, if any, will an investor choose to optimize one single portfolio?*

Hypotheses

H_1 *Dividing an investment into multiple subportfolios enables the investor to increase accuracy when assessing the level of risk tolerance for each subportfolio. As the investor's level of risk tolerance may vary across different investment goals, a corresponding subportfolio will reduce the problem of risk tolerance misspecification.*

H_2 *By increasing accuracy in assessing the level of risk tolerance for each subportfolio, the likelihood of reaching the investor goals increase.*

These hypotheses are complementary to each other.

Literature Review

Mean-variance portfolios

In "Portfolio Selection" (1952), Harry Markowitz introduced the efficient frontier, helping investors select optimal portfolios by maximizing return and minimizing the variance of the return. Further, Markowitz discussed the importance of maximizing discounted value of future returns, while taking into account the potential associated risk. This mean-variance portfolio approach has since gained acceptance and is used by both private and institutional investors. Markowitz's contribution to corporate finance and financial economics has given him a status as a pioneer within the field, and his innovative work laid the foundation for what is now known as Modern Portfolio Theory (Mangram, 2013).

Das et al. (2010) introduced portfolio optimization with mental accounts (subportfolios), based on appealing features of Markowitz's mean variance portfolio theory and Shefrin and Statman's behavioural portfolio theory. Das et al. propose separating the investment into mental accounts before optimizing each account separately. The authors argue that behavioral biases cloud overall risk aversion. An investor's tolerance to risk will often vary depending on the specific investment goal. Hence optimizing several subportfolios enables the investor to assign different risk tolerance levels to different portfolios. Even though investors are attracted to the rational application of Markowitz´s mean-variance portfolio approach they also want their portfolios to satisfy specific personal goals, as opposed to simply maximizing return.

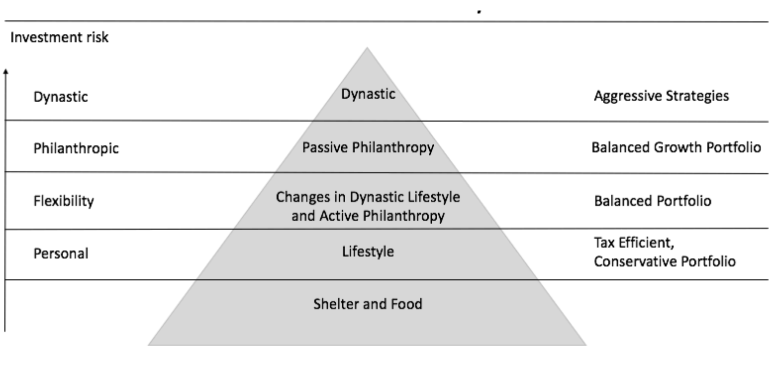
Das et al. (2010) show that optimizing subportfolios (mental accounting) are as optimal as a single portfolio optimization (mean-variance) when shortselling is allowed. However, when shortselling is not allowed, subportfolio optimization tends to lead to small losses in efficiency (Brunel, 2006). These efficiency losses are however smaller than those that arise from incorrectly specifying the investor risk aversion. Hence, correctly assessing an investor's risk aversion is crucial. Moreover, Das et al. (2010) find that efficiency losses decline as investors become more risk averse.

Goal-based portfolios

Several approaches of the goal-based portfolio theory have been proposed in the literature. Some researchers, such as Shefrin and Statman (2000), believe that the different theories emerge from the puzzle of Friedman and Savage (1948), the observation that people who buy insurance policies often buy lottery tickets as well. This puzzle is a contradiction to the assumption that an investor has a unique risk aversion level for all types of financial decisions. The puzzle may be used to explain the need for more sophisticated models for optimal investments, including achieving multiple investor goals. The authors suggest that investors tend to misspecify their risk aversion, since they often have different risk level for the different goals. Investors in goal-based portfolio theory face several optimization problems, one for each goal.

Investor goals can be grouped into three main categories; personal (lifestyle requirements), philanthropic (personal values) and dynastic (children's future) (Brunel, 2012). Shelter and food are considered basic human needs, and needs to be satisfied before the above categories are considered. The categories are ordered by importance to the investor, where the personal goals are prioritized over the philanthropic and so on, see Figure 1. Because of the different levels of importance, the risk tolerance levels will also differ.

Figure 1. The Behavioural Finance Portfolio Pyramid (Statman, 2004).



The portfolios are divided into different mental account layers (goals) of a portfolio pyramid, where each layer is associated with a different goal and outlook on risk. The investor aims to optimize each subportfolio separately as opposed to integrating the goals into one single portfolio with the same risk tolerance level and expected return. Das et al. (2010) introduce a framework of constructing subportfolios and optimizing them separately. The framework is based on the important assumption; that an investor are better at stating her goals and the corresponding risk tolerance level for a part of their holding (goal-based approach), than for one single portfolio. The authors argue that better problem specification gives superior portfolios.

Jean L.P. Brunel, the Chief Investment Officer of GenSpring Family Offices, wrote in "Goals-Based Wealth Management in Practice" (2012) about concrete examples of how to include investors' goals into optimal portfolios. Brunel relies on his experience as a wealth manager for insight into investor behavior. According to Brunel, he has met few investors who say; "Give me the highest possible return with the lowest possible risk, and all will be fine". According to Brunel, investors are in fact more concerned with reaching their goal, than only maximizing return given a risk tolerance level.

Dividing the investor's holding makes it easier for the investor to increase accuracy when assessing risk tolerance to the different subportfolios. Das, Markowitz, Scheid, & Statman (2011) argue that goal-based portfolios allow investors to formulate more coherently each goal, the horizon for each goal, and the attitude towards risk for each goal.

An investor who optimizes one single portfolio, with a long horizon and multiple goals, will likely make withdrawals during the investment period. Many investors, however, fail to realize the negative impact withdrawals can have on the performance of the portfolio. One reason for this might be the investor's ignorance to the difference between simple and compounding returns. After a withdrawal from a portfolio, the percentage gain required to break even is significantly higher than the initial loss or withdrawal (Feibleman & Takeda, 2013). Using subportfolios for each goal helps eliminate this problem.

Behavioral finance

Behavioral finance shows that investors are not risk averse but loss averse, suggesting measures based on the likelihood of loss (Nawrocki, 1999). Nevins (2004) argues for a different portfolio optimization approach due to the nature of human behavior. An investor does not need to hold a specific risky portfolio if there exists a less risky portfolio that will achieve the investor's goal, nor should an investor choose a more conservative portfolio if this portfolio is unlikely to achieve the goals, even though the investor is risk averse (Nevins, 2004).

Applying the concepts of behavioral finance, Shefrin and Statman (2010) suggest that an investor, in addition to having a variety of goals, also assigns different risk profiles to each of those goals. Some of these risk profiles may seem almost contradictory, yet they are not exclusionary, they merely reflect normal human behavior. The puzzle by Friedman and Savage (1948) is an example of this. Thus, investment advisors should develop investment strategies to match their clients' different goals and risk profiles.

In contrast to mean-variance investors, behavioral portfolio investors choose portfolios by considering among other things; expected wealth, desire for security, aspiration levels, and probabilities of achieving aspiration levels. BPT emphasizes the trade-off between thresholds and the probability of failing to reach them. In goal-based portfolio theory, investors maximize expected wealth subject to a maximum probability of failing to reach a threshold level of return, while in mean-variance theory investors either minimize variance, subject to a level of return, or visa versa.

Risk and diversification

Perhaps one of the most important steps of an investment process is the identification and integration of the investors risk tolerance level. By correctly identifying the risk level, the investment advisor is able to construct a portfolio that matches the clients risk profile (Janssen, Kramer, & Boender, 2013). Risk can be understood as the uncertainty of a future event that will affect welfare, and quantifies the possibility and size of potential losses. In Markowitz´s mean-variance portfolio, risk is defined as the standard deviation of the return, whilst in goal-based portfolio theory the definition of risk is changed. Brunel (2012) argues that risk should not be defined mathematically as a standard deviation of the return, but rather as the probability of not achieving the set goals, which is in fact the way most people naturally perceive risk. According to Das, Markowitz, Scheid and Statman (2010), in mean-variance theory investors are always risk averse, however, in behavioral portfolio theory investors might even be risk seeking. Nevertheless, risk itself, is not something to be avoided. In order to create wealth an investor needs to take on risk. By investing in risky assets and accepting short-term losses, an investor might gain potential long-term returns.

Diversification refers to the process of reducing overall portfolio risk by combining assets with different, not perfectly correlated risk profiles. By dividing the investment amount and assigning them to different investment goals, the investor might be able to further diversify the risk.

Historically the long-term performance of the stock market is positive and upward looking. In the short run, however, it is volatile and fluctuates. It is therefore essential to set the right risk tolerance level, so the investor is comfortable even when the market may be down. Unfortunately, many investors have a tendency to misspecify their tolerance for risk. When there is a bear market and the “fear” takes over, the investor may sell her position and stay out of the market until growth opportunities arise and the investor feels ”safe” again, consequently buying at an even higher level. When risk tolerance level is correctly specified, the investor can stay invested, avoid suboptimal reactions to market downturns, and let the long-term behaviour of the market get her where she wishes to be.

It is important to distinguish between how to measure risk, and activities to assess risk tolerance. The mean variance approach measures risk in terms of volatility of the stock return. When presented with different levels of volatility, assessing the comfortable level of risk tolerance might be difficult for an investor, which in turn may result in misspecification of risk tolerance. The use of volatility as the only measure of risk has become increasingly questioned. Volatility measures price fluctuations in stocks, bonds, portfolios etc. Both up and down fluctuations are inevitable, and come with the natural movements of the market. Risk should become a thorough conversation between the investor and her financial advisor, so that she may reach her goals in the best possible way (Kemp, 2015).

The process of assessing risk tolerance should be a thorough discussion between the investor and the investment advisor (Kemp, 2015). Many investment advisors base this discussion on two aspects; risk tolerance and the investment horizon. Using the logic of the long-run performance of the stock market, the longer horizon, the more risk the investor should take on. A different approach for the investment advisor is to present possible outcomes for the different risk levels; maximum loss, minimum loss and expected loss. Some investors do not bear to see their investment fall, and are therefore willing to accept a lower expected return. Others do accept short-run losses, in order to gain higher expect returns. Portfolio risk can be considered as a limited resource, hence the investor should budget the risk relative to her ability to accommodate losses.

One metric to be used to assess the investor's exposure to risk is Value at Risk (VaR). VaR measures the maximum amount that can be expected to be lost given a confidence level and a timeframe. While VaR represents "worst-case" losses associated with a probability over a given period, Conditional Value at Risk (CVaR) challenges the limitations of VaR, and measures the expected shortfall beyond the breakpoint of VaR. These risk metrics are complements to the traditional volatility measurement, and aims to provide a more intuitive approach to consider risk.

When the investment advisors define the risk profile, the difference between the willingness to take risk and ability to take risk should be clear (Janssen et al., 2013). The risk willingness refers to the emotional risk tolerance, and can be described as how much risk the client is willing to take on. This risk willingness can be measured with the help of questionnaires. The ability to take on risk, however, refers to how much risk the investor can tolerate, and is more of an economic question. This risk ability might be measured with the help of exploring plausible scenarios.

In short, correctly specifying the overall risk aversion level of the investor can be a difficult process. Breaking up the overall portfolio objective in concrete goals pertaining specific future outlays may help the investor to increase accuracy when assessing the risk tolerance level associated with each goal.

Empirical methodology

This section describes the methodology we use in this thesis. We construct a quantitative empirical analysis. We use historical financial time series data to construct optimal portfolios according to MPT and BPT. We compare the performance of two different investment approaches. After defining the portfolio problems and constraints, we use MatLab to simulate returns and optimal asset allocation. Each portfolio is unique and has different properties (investment horizon, risk level, expected return). The dataset we use is described in detail in a later section of this thesis.

The investor problem

For risk diversification purposes, the investor has the option of choosing between two investment strategies; 1) investing her total holding in a single diversified portfolio in accordance to MPT, or 2) investing her holding in three different subportfolios, each with a different risk level corresponding to a monetary goal. The investor has specific goals for the investment she wishes to achieve. To match the investor's investment goals, three different portfolios are constructed, and our aim is to compare the performance of the subportfolios against the single portfolio.

The specific goal definitions, e.g. saving for retirement, should have no relevance for this thesis, as we are comparing investment strategies. Instead, we focus on the monetary amount the investor needs to achieve her goals, hereby referred to as the goals. We assume, in 5 years the investor needs NOK 2.50m, in 10 years NOK 3.00m and in 15 years NOK 3.00m. The amount invested is the present value of the goals. We calculate the present values with the following formula;

$$PV = \frac{FV}{(1+r)^n} \tag{1}$$

where PV is the present value, FV is the monetary goal, r is the discount rate and n is the number of years.

When deciding on the appropriate discount rate, we evaluate a possible rate of return the investor might be able to achieve. As the purpose of the thesis is to compare the two investment strategies, the method of choosing discount rate is not crucial, however we find it necessary that both strategies uses the same rate. With a discount rate of 3,00% for the 5-year subportfolio (short-term portfolio), the investment is set to NOK 2,15m. For the 10-year subportfolio (mid-term portfolios) the discount rate is set to 5,00%, giving an investment of NOK 1,84m. The 15-year subportfolio (long-term portfolio) has a discount rate of 7,00%, hence an investment of NOK 1,08m. The total amount invested in the subportfolios is NOK 5,08m. As an alternative to the subportfolios, the investor could invest in a single portfolio over 15 years with withdrawals after 5 years and 10 years. These withdrawals match the short-term and mid-term goals.

For the short-term portfolio the investor has the lowest risk tolerance, for the mid-term portfolio she tolerates more risk and for the long-term portfolio she tolerates even more risk. Table 1 presents the portfolio properties.

Table 1. The table shows four different portfolios and their properties. The withdrawals from the single portfolio match the goals for the subportfolios. The investment amount for the single portfolio is equal to the total amount invested in the subportfolios. Risk aversion coefficient defines the investor's degree of risk aversion, where a higher tolerance to risk is represented by a lower coefficient.

	Portfolios			
	Subportfolios			**Single Portfolio**
	Short-term	**Mid-term**	**Long-term**	
Investment horizon	5 years	10 years	15 years	15 years
Risk tolerance	Low	Moderate	High	Moderate
Risk aversion coefficient	5	3	1	3,42
Monetary goal (NOK)	2 500 000	3 000 000	3 000 000	-
Discount rate	3,00 %	5,00 %	7,00 %	-
Investment amount (NOK)	2 157 000	1 842 000	1 088 000	5 087 000

Risk tolerance

In practice, the dialog between investor and advisor is important to correctly assess the investor's risk tolerance, and a common approach is to examine plausible scenarios and use questionnaires. However, for the purpose of the empirical part of this thesis, we are not concerned with correctly assessing the risk tolerance for the investor. The risk aversion coefficients for the investor are therefore fixed without any assessment of appropriateness.

As the investor has different risk tolerance for different parts of her holding, it is important that her risk tolerance for the different goals remain constant for the two investment strategies. The total investment amount is also the same for the two strategies. In order to find the appropriate risk aversion coefficient for the single portfolio, we use the following formula for a weighted risk aversion coefficient:

$$Overall\ RA = \frac{(PV_{ST.liab} * RA_{ST}) + (PV_{MT.liab} * RA_{MT}) + (PV_{LT.liab.} * RA_{LT})}{\Sigma PV_{Tot.liab.}} \qquad (2)$$

RA is the risk aversion coefficient. PV_{STliab} is the present value of the short-term liability, PV_{MTliab} is the present value of the mid-term liability and PV_{LTliab} is the present value of the long-term liability. $\Sigma PV_{Tot.liab.}$ is the sum of the present values.

An income-oriented investor seeks to secure his wealth with minimal risk, is comfortable with only modest long-term growth and has a short-to-mid range investment horizon. The investor can be categorized as conservative. For the 5-year subportfolio the risk level is low. A balance-oriented investor aims to reduce the potential risk by including income generating investments in the portfolio, accepts unassertive growth, allows for short-term price fluctuations and has a mid-to-long range investment horizon. The risk level is categorized as moderate. A growth-oriented investor (high risk tolerance) aims to maximize the long-term growth, allows for potentially large price fluctuations and has a long-term investment horizon. Generating current income is not a key objective for this investor.

Subportfolios

Each portfolio has a different amount of initial investment. Thus, we apply the initial investment to determine what the holding will be at end of the portfolio period, and whether the investor's goal has been met.

In case a portfolio fails to meet the goal of the investor, funds are taken from another portfolio at that time. E.g. if the short-term portfolio fails after 5 years, funds are taken from the mid-term portfolio at that time. Because the expected returns of the portfolios are higher than the investors required return, taking a small portion from one portfolio should in theory not significantly impact the portfolio.

In case of excess return (return that exceeds the goal) at the end of the investment period for the short-term and mid-term subportfolios, this excess return is reinvested in the subportfolio with shortest time to maturity, respectively in the mid-term and long-term subportfolio. Following the logic of the investor's tolerance to risk for a short-term portfolio, excess return from this portfolio should perhaps ideally be reinvested in a new short-term portfolio, as opposed to one with a different risk profile, however, for simplicity we avoid this.

The development of the investor's holdings throughout the portfolio period is displayed in the formula below.

$$w_t = (w_{t-1} + i_t)(1 + r_t) \tag{3}$$

$\boldsymbol{w_t}$ represents the investors holding at a given time (t, per month). $\boldsymbol{i_t}$ represents the investors investment at a given time (t, per month), which occurs at the beginning of the period and during the period depending on the portfolio in focus. $\boldsymbol{w_{t-1}}$ represents the investors holding in the previous month. $\boldsymbol{r_t}$ represents the monthly return that impacts the portfolio each month.

At the end of the portfolio period, the investor withdraws her desired amount with the purpose of satisfying her specific goal at that time. The excess return is then reinvested in another subportfolio and translated from w_{excess} to i_t and so on. g represents the goal. The excess return is represented by w_{excess} as below:

$$w_{excess} = w_t - g \quad (4)$$

Single portfolio

For the single portfolio, one initial investment is done in the beginning of the investment period, which is set to 15 years. The portfolio is adjusted along the way, as the investor makes withdrawals after 5 years and 10 years to meet her goals.

After specifying the risk tolerance, investment amounts and goals for the different portfolios, the mean-variance portfolio optimization approach is used to construct both the subportfolios and the single portfolio. In short, the same practical optimization method is used for all four portfolios. The methods we use to construct portfolios are explained in a later section. For simplicity, short selling is not available to the investor, there is no leverage to consider, and the investor's initial holding is assumed to be 100% invested at risk free rate in a bank account.

Data

Sample description

For the empirical section of this thesis, we use historical asset prices converted into total returns for 4 different asset classes. The assets are assumed to represent the world bond and equity market, and the data includes bond and stock indices. The data is annual returns in USD., during the period December 1986 to December 2017. Data are obtained from FactSet Research Systems Inc. and Bloomberg L.P., issued by Bank of America Merril Lynch (BofAML), Standard & Poor (S&P), Morgan Stanley Capital International (MSCI) and FTSE Russel. The following indices are included; ICE BofAML US Broad Market Index, FTSE Germany GBI USD, S&P 500 and MSCI World ex USA Index. All indices are capitalization-weighted. Each asset is weighted according to its market capitalization, hence assets with large capitalization receives a larger weighting in the index, which reflects the fact that large-cap companies have more impact on the economy.

Both Factset Research Systems Inc. and Bloomberg L.P. provide financial information and data for investment professionals. As the collected data covers a period of 31 years of historical prices, we find it satisfactory to use these databases for our analyses.

We use a total of 4 different assets in our analysis. The chosen sample period is 31.12.1986 to 31.12.2017. The sample period is assumed to provide a notable representation of historical stock and bond market behaviour. The sample period of 31 years covers periods with bear markets, such as the financial crisis of 2008 as well as strong bull markets. Annual data are regarded as sufficient for this research purpose.

All data are retrieved as last day prices, and converted to returns. A discrete approach is used, defining returns between two periods, *t* and *t-1* as:

$$Return_t = \frac{Price_t}{Price_{t-1}} - 1 \quad (5)$$

Asset classes

Bond indices:

ICE BofAML Broad Market Index

The ICE BofAML Broad Market Index tracks the performance of US denominated investment grade debt publicly issued in the US domestic market, including US Treasury, quasi-government, corporate, securitized and collateralized securities.

FTSE Germany Government Bond Index

The FTSE Germany Government Bond Index tracks the performance of government bonds issued by Germany.

Stock indices:

S&P 500 Index

Standard and Poor's 500 Index is a capitalization-weighted index of the 500 largest U.S. publicly traded companies by market value. The index measures the performance of the U.S. economy through changes in the aggregated market value.

MSCI World ex USA Index

The MSCI World ex USA index seeks to provide a broad stock measure of the world excluding US-based companies. The index captures large and mid-cap companies in developed markets countries, and were launched in March 1986. The index is based on the MSCI Global Investable Market Indexes (GIMI) methodology, which aims to provide exhaustive coverage of the relevant investment opportunity set with a strong emphasis on index liquidity, investability and replicability. The index is reviewed quarterly to reflect changes in the underlying equity markets.

Constructing optimal portfolios

Simulation method

Our simulation method takes 4 steps per portfolio. In **step 1**, we specify the available assets, which include MSCI World ex USA, S&P 500, US Treasury Bonds, and German Government Bonds.

In **step 2** we compute the mean, standard deviation and the correlation matrix of returns in order to generate a variance covariance matrix, see table 2, 3 and 4 below.

The formulas used to calculate the mean and covariance are listed below, where $\bar{X}$ is the mean, σ is the standard deviation and $cov(X,Y)$ is the covariance between two assets.

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (6)$$

$$\sigma = \sqrt{\frac{\sum(x-\bar{X})^2}{n-1}} \qquad (7$$

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} \qquad (8)$$

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{(n-1)} \qquad (9)$$

Table 2. This table reports historical average annual returns, standard deviations and VaR for each asset.

	World ex USA	S&P 500	US Treasury Bonds	German Government Bonds
Average returns	0,0598	0,0959	0,0247	0,0767
Std. Dev. Of returns	0,1838	0,1632	0,1119	0,1253
VaR				
90 %	0,24	0,17	0,16	0,13
95 %	0,30	0,22	0,19	0,17
99 %	0,41	0,32	0,26	0,25

Table 3. Correlation Matrix

	World ex USA	S&P 500	US Treasury Bonds	German Government Bonds
World ex USA	1	0,7484	-0,2792	-0,1403
S&P 500	0,7484	1	-0,2004	-0,0219
US Treasury Bonds	-0,2792	-0,2004	1	0,2662
German Government Bond	-0,1403	-0,0219	0,2662	1

Table 4. Variance-Covariance Matrix.

	World ex USA	S&P 500	US Treasury Bonds	German Government Bonds
World ex USA	0,0338	0,0224	-0,0057	-0,0032
S&P 500	0,0224	0,0266	-0,0037	-0,0004
US Treasury Bonds	-0,0057	-0,0037	0,0125	0,0037
German Government Bond	-0,0032	-0,0004	0,0037	0,0157

In **step 3** we generate the efficient frontier and the optimal portfolio. First, in order to establish a sufficient parameter, we set the number of portfolios to be considered along the efficient frontier to 30. We use mean of returns and the variance covariance matrix as core inputs. Second, we use the *portopt* function in MatLab to generate the efficient frontier, and the *portalloc* function to generate the optimal portfolio based on the risk-free rate, borrowing rate and the investor's risk aversion coefficient. Borrowing rate is set to not a number (NaN), as borrowing is not an option for the investor. As the function requires a risk-free rate, we set this to 2,00%. *Portalloc* function provides the optimal portfolio assuming the utility formula; *U = E(r) – 0,5*A*Var.* The optimal portfolio is the point where the investor's indifference curve is tangent to the efficient frontier. The optimal portfolio provides the asset allocation weights used for the simulation.

In **step 4** we run a Monte Carlo simulation of historical correlated asset returns. We use the expected method, which generates correlated asset returns where the sample mean and covariance are statistically equal to the input mean and covariance specifications. First, we define the forecasting period, depending on the horizon of the portfolio (5, 10 and 15 years). As the investor has different risk tolerance for the different portfolios, they have different asset allocation. We specify the number of simulations to 1 000, and we believe this will provide us with an appropriate expectation of future scenarios. The expected returns are generated by the *portsim* function with the following core inputs; mean of returns, variance covariance matrix, forecasting period, and number of simulations. At this point, the expected returns are a 5-by-4-by-1000 array. Next, we use the portfolio weights formed of the 4 assets obtained at the specific risk aversion level, and create arrays of portfolio returns. Each column represents a possible outcome for a portfolio, and corresponds to a sample path of the simulated returns. The portfolio array *PortRetExpected* is a 5-by-1000 matrix, and is used as input to a template we have constructed in excel.

Step 3 and 4 are repeated for each of the portfolios (short-term, mid-term, long-term and single portfolio), with different asset allocation weights corresponding to the investors risk aversion level.

The excel template

The excel template is constructed to analyse the portfolios with the simulated portfolio returns extracted from MatLab. The simulated returns, *PortRetExpected,* are imported into the template. The returns for the 1 000 simulated portfolios are multiplied with an investment amount in order to calculate real values during the investment period. After the investment period (5,10,15) we evaluate which of the portfolios have succeeded in reaching the goal, and which have not.

For the short-term portfolio, if any of the 1 000 simulated portfolios fails, necessary funds are taken from the corresponding mid-term portfolio to cover the missing amount. A consequence of this is that none of the 1 000 short-term portfolios will fail. The reasoning behind this is that we find it likely that the investor's goal that has the shortest horizon is of greater importance, hence it will be beneficial to cover shortfalls today at the expense of future goals. Also, as the expected return of the portfolios is higher than the required return, it is likely that an investor would prefer to cover shortfalls today at the expense of future goals. With a probability of failure equal to zero, the investor reaches her short-term goal. In case of excess return, these returns are reinvested in the mid-term portfolio at that point in time.

The mid-term subportfolio follows a similar procedure as the short-term portfolio, where failed portfolios are covered by funds from the long-term subportfolios, and any excess returns are reinvested in the long-term portfolio. Consequently, none of the mid-term portfolios will fail.

After 15 years, the investment period has ended. Any failed long-term portfolios will therefore not have the opportunity to be covered, and thus, they fail. For the strategy to be deemed a success, they should reach every goal. The short- and mid-term subportfolios are covered, but the long-term subportfolio is not. Probability of failure for the strategy with the subportfolios is calculated based on whether they reach their final monetary goal.

The single portfolio is constructed to match the monetary goals of the subportfolios, hence withdrawals, equal to the goals of the subportfolios, are made after 5, 10 and 15 years. Equivalent to the strategy with the subportfolios, for the single portfolio strategy to be deemed a success, every goal should be met. Some portfolios will fail, and some may even fail after 5 years. This strategy however is not able to cover potential shortfalls by funds from another portfolio. Probability of failure for the single portfolio strategy is calculated based on whether they reach the goals.

Finding the efficient frontier and the optimal risky portfolio

The efficient frontier is the set of optimal portfolios of the risky assets. The frontier provides the set of optimal portfolios with highest possible returns for a given level of risk (standard deviation). We compute the efficient frontier with MatLab using historical returns. After we have found the efficient frontier, we find the optimal asset allocation on the efficient frontier. When deciding on the optimal portfolio, the investor's degree of risk aversion and her indifference curve are important. The indifference curves provide the risk-return combination in which the investor gets constant utility. The indifference curves establish the required return given an increase in risk, and it follows that the investor is indifferent between all the points along the curve. The optimal asset allocation is the point where the investor's indifference curve is tangent to the efficient frontier.

After assigning risk aversion coefficients, MatLab calculates the risk-return utility score in order to find the investors indifference curves. To calculate the utility scores the MatLab cosed are based on the following formula:

$$U = E(r) - 0.5 * A * var \tag{10}$$

Where *U* is the investors utility value and *E(r)* is the expected return. *A* is the investors risk aversion coefficient and var is the variance of the return. *0.5* is a scaling convention in order to express the outputs in decimals.

The formula shows that the investor's utility increases as the expected return increases, and decreases when the variance increases. The level of risk aversion determines the relative magnitude of the changes in expected return and variance.

Empirical results

In this section we present the results obtained from the optimization of each portfolio, and display a constructive analysis of the portfolios. Further we provide a detailed analysis and comparison of the subportfolios against the single portfolio, in order to determine which investment approach is best suited for the

investor. The analysis and results are based on simulations of the portfolios, thus we expect that some of the portfolios do not meet the return requirement, and fail. This expectation proved to be correct. As some of the simulated subportfolios were not able to provide the return needed to reach the investor goals, they retrieved the funds from the subportfolio with longer time to maturity, which affected the performance of the remaining subportfolios.

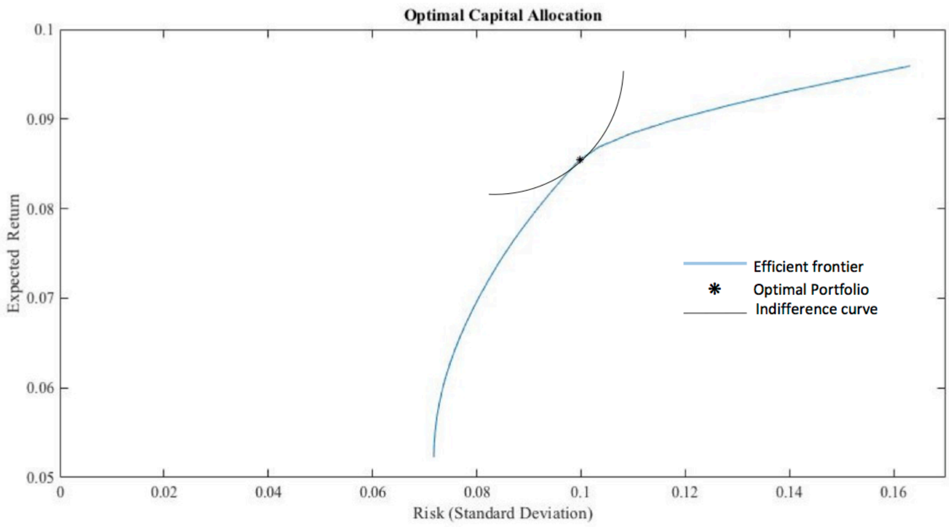
Efficient frontiers

Table 5. The table shows the expected return and standard deviation corresponding to the risk aversion coefficient for the portfolios. Standard deviation is the measurement for risk.

	Subportfolios			
	Short-term	**Mid-term**	**Long-term**	**Single portfolio**
Risk Aversion Coefficient	5	3	1	3,42
Expected Return	8,55 %	8,68 %	9,24 %	8,64 %
Risk (Std. Dev.)	9,98 %	10,32 %	13,48 %	10,21 %

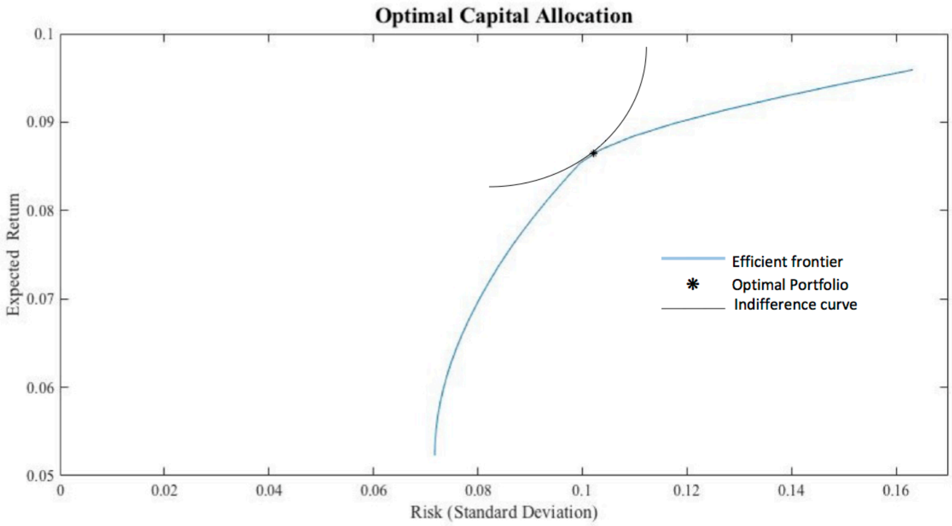
Above are the summary statistics for the efficient frontier, at different risk aversion levels, and following are the efficient frontiers displaying the optimal asset allocations corresponding to the different the risk aversion levels for the simulated portfolios.

Figure 2. The figure shows the efficient frontier (blue line), the investor's indifference curve (black line) and the optimal short-term portfolio (*).



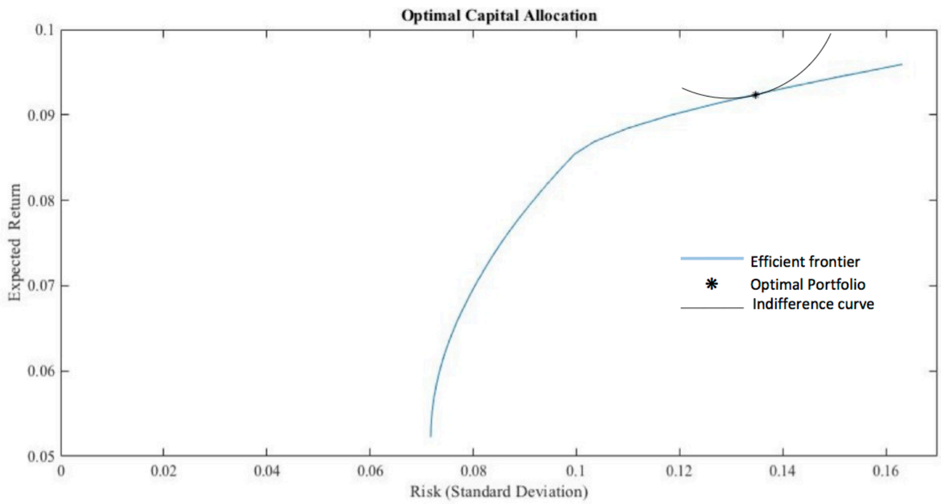
For the short-term portfolio, the point where the investor's indifference curve is tangent to the efficient frontier represents the optimal risky portfolio. The optimal short-term portfolio has an expected return of 8,55%, with an associated standard deviation of 9,98%, see figure 2 and table 5.

Figure 3. The figure shows the efficient frontier (blue line), the investor's indifference curve (black line) and the optimal mid-term portfolio (*).



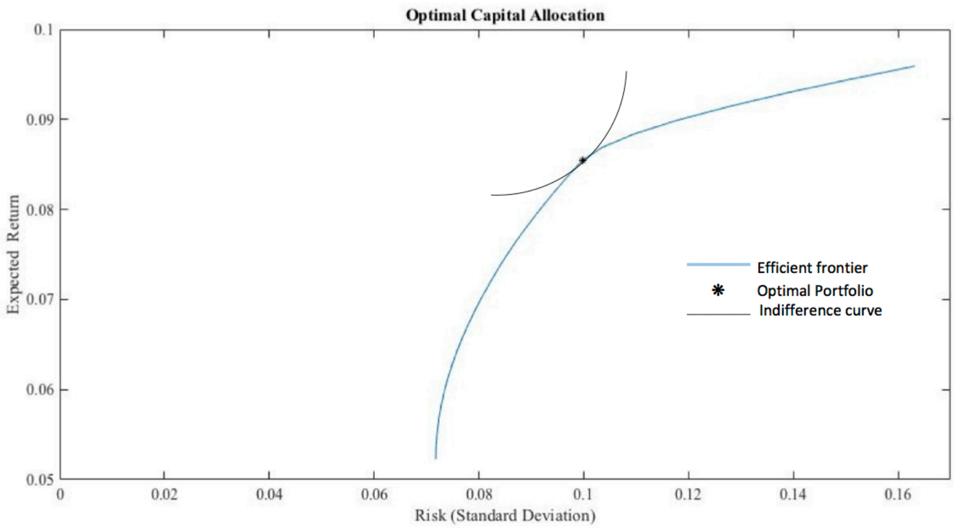
For the mid-term portfolio the investor's indifference curve is tangent to the efficient frontier at the optimal point associated with an expected return of 8,68% and a standard deviation of 10,32%, see figure 3 and table 5.

Figure 4. The figure shows the efficient frontier (blue line), the investor's indifference curve (black line) and the optimal long-term portfolio (*).



The optimal long-term portfolio has an expected return of 9,24% and a standard deviation of 13,48%, see figure 4 and table 5.

Figure 5. The figure shows the efficient frontier (blue line), the investor's indifference curve (black line) and the optimal single portfolio (*).



For the optimal single portfolio the expected return is 8,64% and standard deviation is 10,21%, see figure 5 and table 5. We observe that the shape of the efficient frontier has a breakpoint around standard deviation of 10%, where standard deviation levels up to 10% the efficient frontier is rather steep. Standard deviation above 10% the curve is less steep, meaning that the investor is less compensated per unit of risk taken on. Though none of the optimal portfolios above suggest investing at risk free rate, for an even more risk averse investor this might not be the case.

Optimal asset allocation weights

Table 6. The table shows the optimal asset allocation weights for the portfolios.

	Subportfolios			
Assets	**Short-term**	**Mid-term**	**Long-term**	**Single portfolio**
MSCI ex US	0,00 %	0,00 %	0,00 %	0,00 %
S&P 500	45,68 %	52,37 %	81,71 %	50,57 %
US Treasury Bonds	0,00 %	0,00 %	0,00 %	0,00 %
German Government Bonds	54,32 %	47,63 %	18,29 %	49,43 %

Table 6 displays the asset allocation weights for the optimal portfolios. We observe that none of these optimal portfolios invest in the indices MSCI ex US and US Treasury Bonds. However, for investors with less tolerance to risk, the optimal portfolios do include these indices, see appendix 2.

Optimally, an investor wants to invest in assets where she earns more per unit of risk taken on. Hence, an asset that historically offers high returns, and at the same time low risk would be optimal to invest in. It is reasonable to think that the largest companies in the US offer stable returns. Such large and well-established companies have lower risk than other growth-companies. Hence, investing in S&P 500 is reasonable for both risk averse investors as well as more risk willing investors, see appendix 2. To offset any risk taken on by investing in S&P 500, an investor can invest in low risk assets. Hence, investing in bonds is reasonable. For the optimal portfolios above, the risk-return tradeoff in indices S&P500 and German Government Bonds are higher than that of the other two indices.

Performance analysis

Annual returns

Table 7. The table shows portfolio performance statistics, including return measures, standard deviation and Sharpe ratio. All metrics are averages out of 1 000 simulated portfolios.

	Subportfolios			
	Short-term	**Mid-term**	**Long-term**	**Single portfolio**
Avg. annual return	8,54 %	8,60 %	9,38 %	8,72 %
Total return	42,72 %	86,05 %	140,67 %	130,86 %
Annual standard deviation	8,47 %	9,45 %	12,82 %	9,68 %
Annual Sharpe ratio	0,71	0,65	0,54	0,64

The average short-term portfolio, henceforth referred to as short-term portfolio, have an average annual return of 8,54%, which is the lowest return out of the 4 portfolios, see table 7. This result is expected as the portfolio allows for lower risk than the other portfolios. The average mid-term portfolio, henceforth referred to as mid-term portfolio, have an average annual return of 8,60%. Due to the longer investment period, asset allocation allows for more risk and growth-oriented perspectives. The average long-term portfolio, henceforth referred to as long-term portfolio, have an average annual return of 9,38%. The average single portfolio, henceforth referred to as single portfolio, has an average annual return is 8,72%. The annual standard deviation of the portfolios increase as average annual return increase, however, they do not increase in the same magnitude. This fact might indicate that the investor is not sufficiently compensated for the increase in risk. To assess this further we analyse the portfolios' Sharpe ratio. The Sharpe ratio is the average return earned, net risk free rate, per unit of risk. A higher Sharpe ratio represents more compensation per unit of risk taken on. The short-term portfolio's Sharpe ratio is 0,71, which is significantly higher than for the average mid-term portfolio. Nevertheless, assessing the superiority of the average subportfolios against the average single portfolio by analysing average annual returns and Sharpe ratios might not be sufficient, as it is difficult to combine the overall performance of the subportfolios against the single portfolio. Thus, we need to analyse other performance metrics.

Excess returns

Table 8. The table shows average excess return and probability of failure for each portfolio. All metrics are averages out of 1 000 simulated portfolios.

	Subportfolios			
	Short-term	**Mid-term**	**Long-term**	**Single portfolio**
Avg. excess return (NOK)	767 446	1 210 282	4 830 956	4 873 567
Probability of failure	-	-	0,07	0,16

The short-term portfolio had an average excess return of NOK 767 446 after 5 years, see table 8. This excess return was reinvested in the mid-term portfolio. The mid-term portfolio had an average excess return of NOK 1,21 million after 10 years, which was reinvested in the long-term portfolio. On average, after the investment period of 15 years, the investor had met her goals and gained excess return of approximately NOK 4,83 million. For the single portfolio, on average, the investor met her goals, and gained excess return after 15 years of approximately NOK 4,87 million. Only focusing on excess return might argue that investing in a single portfolio would be favorable to the investor.

Probability of failure

A key metric is the likelihood of the portfolios meeting their goal. To assess this, we calculate the probability of failure for the portfolios. This measurement is based on the likelihood of the investor reaching her goal, see table 8.

The subportfolios are linked as funds from one subportfolio can cover shortfalls in others. We therefore expect zero probability of failure in the short-term and mid-term portfolio, given the positive historical asset returns. We assume that a portfolio with a shortfall of 2% or less of its goal, is an acceptable outcome for the investor. Therefore, any portfolio with a shortfall of up to 2% is deemed a success.

As the probability of failure for the short-term and mid-term portfolio is expected to be zero, the key comparison is between the long-term portfolio and the single portfolio. The long-term portfolio has a probability of failure of 0.07. The single portfolio has a probability of failure of 0.16. Investing in subportfolios more than halves the probability of failure. Based on these results, the investor would be better off investing in subportfolios, as opposed to investing in one single portfolio.

Value at Risk (VaR)

Table 9. The table shows the Value at Risk at 90%, 95% and 99% for each portfolio.

	Subportfolios				
	Short-term	**Mid-term**	**Long-term**	**Sum**	**Single portfolio**
VaR 90%:					
in %	6,83 %	7,21 %	11,74 %		7,31 %
NOK	145 880	130 129	127 746	403 754	371 796
VaR 95%:					
in %	8,95 %	10,01 %	15,76 %		10,26 %
NOK	191 062	180 635	171 420	543 117	521 948
VaR 99%					
in %	13,67 %	15,74 %	23,65 %		16,20 %
NOK	291 925	284 114	257 308	833 347	824 343

VaR provides an anticipation of the riskiness of the investment, and is presumed to give an intuitive understanding of risk. As shortfalls in both the short-term and mid-term portfolio were covered by funds from other portfolios, this needed to be considered in calculating VaR and CVaR. We analysed the simulated returns of the portfolios to check how many of the short-term and mid-term portfolios that needed to be covered by funds from other portfolios. Among the simulated short-term portfolios, 12,2% failed to reach the goal. Among the simulated mid-term portfolios, 7,4% failed to reach the goal. These portfolios fell short with an average amount of NOK 24 095 and NOK 29 320 respectively. These potential shortfalls are not subject to a "worst-case" loss, as they are covered by funds from other portfolios. Thus, when calculating VaR and CVaR these shortfalls are subtracted from the initial investment.

With 99% probability, the investor will not lose more than NOK 833 347 when investing in subportfolios, and NOK 824 343 when investing in a single portfolio, see table 9. At all confidence levels the investor reduces her exposure to losses when investing single portfolios as opposed to investing in the subportfolios.

Conditional Value at Risk (CVaR)

Table 10. The table shows the Conditional Value at Risk at 90%, 95% and 99% for each portfolio.

	Subportfolios				
	Short-term	**Mid-term**	**Long-term**	**Sum**	**Single portfolio**
CVaR 90%:					
in %	6,85 %	7,30 %	11,96 %		10,71 %
NOK	146 239	131 820	130 145	408 204	544 843
CVaR 95%:					
in %	8,95 %	10,03 %	15,84 %		12,86 %
NOK	191 065	181 146	172 307	544 518	654 190
CVaR 99%					
in %	13,67 %	15,74 %	23,65 %		16,54 %
NOK	291 925	284 120	257 357	833 402	841 498

With a 99% probability the investor will not lose more than NOK 833 402 when investing in subportfolios, and NOK 841 498 when investing in a single portfolio, see table 10. Contrary to VaR, considering CVaR, the subportfolios are the preferred option at every confidence level.

Concluding remarks of the portfolios.

When deciding between investment strategies, we need to weigh the different performance metrics, and ultimately determine which metric is the most important.

The single portfolio generates an excess return of no more than NOK 42 000 more than the subportfolios. According to VaR the difference in worst-case losses for the two investment strategies is no more than approximately NOK 10 000 in favor of the single portfolio. Similarly, the differences in possible losses according to CVaR are small, however, in favor of the subportfolios.

As the goal-based portfolio investor is most concerned with reaching her set goals, we find it likely that the likelihood of achieving the goals should be most important. The probability of failure for the subportfolios is significantly lower than for the single portfolio. Hence, with the chosen risk aversion coefficients we use above, the investor would be better off investing in subportfolios, as opposed to a single portfolio.

Different risk aversion levels

The above analysis argues for investing in subportfolios as opposed to a single portfolio, given the level of risk tolerance for the investor specified above. In order to substantiate our findings, we further wish to explore whether this result holds for investors with other levels of risk tolerance, using different risk aversion coefficients. We expect, however, similar results when we expand the range of the risk aversion coefficients. We wish to answer the following; at what risk level, if any, would the single portfolio approach be the superior choice in terms of probability of failure?

We use the same methodology as described above, simulating optimal portfolios with different risk aversion coefficients. We analyse 20 different sets of risk aversion coefficients for the subportfolios, with risk aversion coefficients for the subportfolios ranging from 1-6 and consequently ranging from 2,21 to 5,21 for the single portfolio, see table 11.

Table 11. The table displays the probability of failure for the subportfolios (long-term portfolio) and the single portfolio calculated for different sets of risk aversion coefficients and overall risk aversion coefficients. * indicates that the subportfolios outperform the single portfolio in terms of lower probability of failure.

Risk Aversion Combinations	Overall Risk Aversion	Outcome	Probability of Failure	
			Subportfolios	Single Portfolio
3,2,1	2,21	*	7,00 %	14,80 %
4,2,1	2,63	*	6,40 %	15,00 %
4,3,1	3	*	7,00 %	15,20 %
5,2,1	3,06	*	6,30 %	15,30 %
4,3,2	3,21	*	5,40 %	15,50 %
5,3,1	3,42	*	7,00 %	15,80 %
6,2,1	3,48	*	6,40 %	15,80 %
5,3,2	3,63	*	5,30 %	15,90 %
5,4,1	3,78	*	7,10 %	15,90 %
6,3,1	3,84	*	7,00 %	16,00 %
5,4,2	4	*	5,40 %	16,10 %
6,3,2	4,06	*	5,30 %	16,10 %
6,4,1	4,21	*	6,80 %	15,80 %
5,4,3	4,21	*	5,20 %	15,80 %
6,4,2	4,42	*	5,40 %	15,80 %
6,5,1	4,57	*	6,80 %	15,80 %
6,4,3	4,63	*	5,30 %	15,80 %
6,5,2	4,78	*	5,40 %	15,70 %
6,5,3	5	*	5,40 %	15,80 %
6,5,4	5,21	*	5,30 %	15,50 %

The single portfolio

For the single portfolio, we find that probability of failure increases, as the investor becomes more risk averse, see table 11. As the investor becomes more risk averse, the optimal risky portfolio moves toward left on the efficient frontier, allowing for less risk (standard deviation), hence lower expected return. At the same time the investor goals are being held constant. With portfolios generating lower return, the ability to reach the goals becomes weaker. The trend of increasing probability of failure whilst risk aversion coefficients increase is as expected. Nevertheless, we observe that for risk aversion coefficients above approximately 4, the probability of failure decrease and stabilize, see table 11.

The subportfolios

We are unable to detect any trend for the probability of failure for the subportfolios. As the risk aversion coefficients become high it might seem that the probability of failure stabilizes. For all sets of risk aversion coefficients analysed the subportfolios have lower probability of failure than the single portfolio.

Concluding remarks

In terms of probability of failure the subportfolios outperform the single portfolio at all risk aversion coefficients analysed. It seems therefore that there do not exist a risk level where the single portfolio is the superior choice, hence this analysis substantiate our previous results where the investor is better off investing in subportfolios as opposed to a single portfolio when reaching the goals are the investors main concern.

Conclusion

This thesis provide a comparison between the goal-based portfolio optimization approach and the traditional mean-variance portfolio optimization approach, and aims to answer whether an investor is better off by investing in optimized subportfolios as opposed to investing in an optimized single portfolio. We use historical asset returns to simulate future portfolio outcomes and financial metrics to measure the performance of the optimized portfolios.

We find that the single portfolio generates higher excess returns than the subportfolios. The single portfolio decreases VaR, however, subportfolios decreases CVaR. The probability of failure for the subportfolios is more than half of that of the single portfolio. For the single portfolio we observe a clear trend of increasing probability of failure, as the investor becomes more risk averse. For the subportfolios we do not find a similar trend.

Given that constructing subportfolios subject to investor goals helps the investor to increase accuracy when assessing risk tolerance, the investor is more likely to achieve the goals as probability of failure is reduced with subportfolios. An investor with specific goals for her investment will be better off investing in multiple subportfolios, and optimizing each subportfolio separately, as opposed to investing in a single portfolio. This result holds for all risk aversion levels analysed.

Limitations of Study and Future Research

With a different set of data, other researchers might get a different result. Other researcher might also compare and weight the performance measures differently, hence a different conclusion can be expected. Deciding on the sufficient number of simulations might be a difficult task. Even with 1 000 simulations, we might have issues with simulation noise. We therefore suggest increased number of simulations for future studies.

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Appendix

Appendix 1 - MatLab Code

```
% Insertion of data

clear all;
clc;

% Define Start and End Date
start_date_monthly = datestr('1986-12-31');
end_date_monthly   = datestr('2017-12-29');

% Import 4 Asset Monthly Returns

[Data, ~, ~] = xlsread('NewDataInput.xlsx');
Data(:,1) = datenum(num2str(Data(:,1)), 'ddmmyy'); %convert to string date

dates_yearly  = Data(:,1);

% Create Values

Assets = {'MSCIxUS', 'S&P500', 'USBonds', 'GerBonds'};


% Specify and Determine Available Assets
MSCIxUS = Data(:,2);
SP500 = Data(:,3);
US_Bond = Data(:,4);
Ger_Bond = Data(:,5);


% Summary Statistics
% Means
m_MSCIxUS = mean(MSCIxUS);
m_SP500 = mean(SP500);
m_USBo = mean(US_Bond);
m_GerBo = mean(Ger_Bond);


AssetMean = [m_MSCIxUS m_SP500 m_USBo m_GerBo];

% Standard Deviations
s_MSCIxUS = std(MSCIxUS);
s_SP500 = std(SP500);
s_USBo = std(US_Bond);
s_GerBo = std(Ger_Bond);

Sigmas = [s_MSCIxUS, s_SP500, s_USBo, s_GerBo];
```

```
% Correlations
C = [MSCIxUS SP500 US_Bond Ger_Bond];
Correlations = corr(C);

% Covariance
AssetCovar = corr2cov(Sigmas, Correlations);

%% Portfolio Optimization w/Risk Aversion
 % First we generate the efficient frontier using the function portopt
NumPorts = 30; % 30 points along the efficient frontier

[PortRisk, PortReturn, PortWts] = portopt(AssetMean, ...
   AssetCovar, NumPorts);

 % Calling portop, while specifying output arguments, returns the arguments
 % and arrays representing the risk, return, and weights for each of the
 % portfolios along the efficient frontier. We use these as the first three
 % input arguments to the function portalloc.

%% Find the optimal risky portfolio and optimal allocation of funds
 % The function portalloc requires us to specify the risk-free rate, the
 % borrowing rate and the risk aversion in order to function properly.

RisklessRate = 0.025;
BorrowRate = NaN;
RiskAversion = 1; % Repeat for RA levels 3 and 5, and overall RA level of 3.42

 % Calling portalloc without specifying any output arguments gives a graph
 % displaying the critical points (i.e. the efficient frontier)

portalloc (PortRisk, PortReturn, PortWts, RisklessRate, ...
   BorrowRate, RiskAversion);


% Calling portalloc while specifying the output arguments returns the
% variance (RiskyRisk), the excpected return (RiskyReturn), and the weights
% (RiskyWts) allocated to the optimal portfolio)
[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction] = portalloc (PortRisk,
PortReturn, PortWts, ...
   RisklessRate, BorrowRate, RiskAversion);


% The proportion assigned to each of these two investment strategies is
% determined by the degree of risk aversion characterizing the investor.
```

```
%% Expected Portfolio Simulation
% Short-term subportfolio (5-year)

NumObs = 5; % NumObs: how far ahead we want to forecast.
NumSim = 1000;
RetIntervals = [];
NumAssets = 4;

rng(0);
RetExpected = portsim(AssetMean, AssetCovar, NumObs, ...
    RetIntervals, NumSim, 'Expected');

Weights = [0; 0.4568; 0; 0.5432]; %Insertion of portfolio weights from the
"RiskyWts" output generated from the risk aversion coefficient: 5.

PortRetExpected = zeros(NumObs, NumSim);

for i = 1:NumSim
    PortRetExpected(:,i) = RetExpected(:,:,i) * Weights;
end

%% Mid-term subportfolio (10-year)

NumObs = 10; % NumObs: how far ahead we want to forecast.
NumSim = 1000;
RetIntervals = [];
NumAssets = 4;

%rng(default);
rng(0);
RetExpected = portsim(AssetMean, AssetCovar, NumObs, ...
    RetIntervals, NumSim, 'Expected');

Weights = [0; 0.5237; 0; 0.4763]; % Insertion of portfolio weights from the
"RiskyWts" output generated from the risk aversion coefficient: 3.
```

```
PortRetExpected = zeros(NumObs, NumSim);

for i = 1:NumSim
   PortRetExpected(:,i) = RetExpected(:,:,i) * Weights;
end

%% Long-term subportfolio (15-year) and/or single portfolio (15-year)

NumObs = 15; % NumObs: how far ahead we want to forecast.
NumSim = 1000;
RetIntervals = [];
NumAssets = 4;

%rng(default);
rng(0);
RetExpected = portsim(AssetMean, AssetCovar, NumObs, ...
   RetIntervals, NumSim, 'Expected');

Weights = [0; 0.5057; 0; 0.4943]; % Insertion of portfolio weights from the
"RiskyWts" output generated from the risk aversion coefficient: 1 (long-term
portfolio) and 3.42 (single portfolio).

PortRetExpected = zeros(NumObs, NumSim);

for i = 1:NumSim
   PortRetExpected(:,i) = RetExpected(:,:,i) * Weights;
end
```

Appendix 2 – Input for efficient frontier

Appendix 2: Input for efficient frontier – risk and return levels with corresponding optimal asset allocation weights. Obtained from MatLab.

Higher Risk Aversion ↑

Lower Risk Aversion ↓

		Asset allocation weights			
Exp. Return	**Risk (Std. Dev.)**	**MSCI ex US**	**S&P 500**	**US Treasury Bonds**	**German Government Bonds**
5,23 %	7,18 %	18,52 %	10,34 %	44,86 %	26,28 %
5,38 %	7,18 %	17,20 %	12,25 %	43,10 %	27,45 %
5,53 %	7,20 %	15,89 %	14,15 %	41,33 %	28,63 %
5,68 %	7,24 %	14,57 %	16,06 %	39,57 %	29,80 %
5,83 %	7,28 %	13,26 %	17,96 %	37,80 %	30,97 %
5,98 %	7,34 %	11,94 %	19,87 %	36,04 %	32,15 %
6,13 %	7,41 %	10,63 %	21,77 %	34,28 %	33,32 %
6,28 %	7,49 %	9,31 %	23,68 %	32,51 %	34,49 %
6,43 %	7,58 %	8,00 %	25,58 %	30,75 %	35,67 %
6,58 %	7,69 %	6,68 %	27,49 %	28,98 %	36,84 %
6,73 %	7,80 %	5,37 %	29,39 %	27,22 %	38,02 %
6,88 %	7,93 %	4,05 %	31,30 %	25,46 %	39,19 %
7,03 %	8,06 %	2,74 %	33,21 %	23,69 %	40,36 %
7,18 %	8,21 %	1,42 %	35,11 %	21,93 %	41,54 %
7,33 %	8,36 %	0,11 %	37,02 %	20,16 %	42,71 %
7,48 %	8,52 %	0,00 %	38,00 %	17,67 %	44,33 %
7,64 %	8,70 %	0,00 %	38,91 %	15,11 %	45,98 %
7,79 %	8,88 %	0,00 %	39,81 %	12,55 %	47,64 %
7,94 %	9,08 %	0,00 %	40,72 %	9,98 %	49,30 %
8,09 %	9,29 %	0,00 %	41,62 %	7,42 %	50,96 %
8,24 %	9,50 %	0,00 %	42,53 %	4,86 %	52,61 %
8,39 %	9,73 %	0,00 %	43,43 %	2,30 %	54,27 %
8,54 %	9,96 %	0,00 %	45,04 %	0,00 %	54,96 %
8,69 %	10,35 %	0,00 %	52,89 %	0,00 %	47,11 %
8,84 %	10,97 %	0,00 %	60,75 %	0,00 %	39,25 %
8,99 %	11,78 %	0,00 %	68,60 %	0,00 %	31,40 %
9,14 %	12,76 %	0,00 %	76,45 %	0,00 %	23,55 %
9,29 %	13,85 %	0,00 %	84,30 %	0,00 %	15,70 %
9,44 %	15,05 %	0,00 %	92,15 %	0,00 %	7,85 %
9,59 %	16,32 %	0,00 %	100,00 %	0,00 %	0,00 %

Appendix 3 - Extracts from excel template

Appendix 3a: *E*xtract of the short-term portfolio. The extract demonstrates how we use the return distribution output from MatLab in excel to generate the real monetary values, and eventually the portfolios probability of failure. For a more details, see excel sheet.

Return distribution (Insert output in the blue area)		**1**	**2**	**3**	....	**999**	**1000**
	1	0,0862	0,1754	-0,1530	....	0,0384	-0,0059
	2	0,1185	0,1146	0,0068	....	0,0965	0,0850
	3	0,0090	0,2613	-0,0217	....	0,0909	0,2524
	4	0,2222	0,0789	0,1819	....	0,1030	0,1058
	5	0,1541	0,0481	0,0807	....	-0,0219	0,1133
Total return		**0,5900**	**0,6783**	**0,0947**	....	**0,3071**	**0,5507**
Av. Total return		**0,4272**					
Average annual return for each portsim		0,1180	0,1357	0,0189	....	0,0614	0,1101
Av. annual return for all		0,0854					
Std.dev annual return for each portsim		0,0708	0,0758	0,1111	....	0,0475	0,0829
Av. Std.dev annual return for all		0,0847					
Initial holding		**2 157 000,00**					
	0	2 157 000,00	2 157 000,00	2 157 000,00	....	2 157 000,00	2 157 000,00
	1	2 342 972,93	2 535 353,13	1 826 897,84	....	2 239 907,00	2 144 182,16
	2	2 620 708,74	2 825 795,15	1 839 253,85	....	2 456 117,60	2 326 541,04
	3	2 644 239,47	3 564 117,35	1 799 409,52	....	2 679 417,41	2 913 851,20
	4	3 231 791,06	3 845 353,99	2 126 655,76	....	2 955 466,03	3 222 226,67
	5	3 729 707,84	4 030 331,31	2 298 381,43	....	2 890 887,39	3 587 429,76
Require 2,5m		2 500 000,00	2 500 000,00	2 500 000,00	....	2 500 000,00	2 500 000,00
If holding below 2,5m take funds from mid-term subportfolio		-	-	201 618,57	....	-	-
Excess return a/withdrawal		1 229 707,8415	1 530 331,3085	-	....	390 887,3946	1 087 429,7614
if success = 1		1	1	1	....	1	1
Average holding a/5 years		767 446					
No. of times the mid-term portfolio fails		0					
No. of times the mid-term portfolio succeeds		1000					
Probability of failure		**0**					

Appendix 3b. Extract of the mid-term portfolio. The extract demonstrates how we use the return distribution output from MatLab in excel to generate the real monetary values, and eventually the portfolios probability of failure. For a more details, see excel sheet.

Return distribution (Insert output in the blue area)		**1**	**2**	**3**	....	**999**	**1000**
	1	0,0426	0,1563	0,1075	....	0,0900	0,2528
	2	0,0393	0,2043	0,0231	....	0,2923	-0,0439
	3	-0,0844	0,0697	0,0236	....	0,2042	0,2742
	4	0,1823	-0,0663	0,1528	....	-0,0837	0,0796
	5	-0,0257	0,0152	0,1200	....	0,0705	0,2093
	6	0,1983	0,2407	0,0553	....	0,1981	0,1619
	7	0,0247	0,0347	0,0154	....	0,0976	0,3209
	8	-0,0172	-0,1199	-0,1790	....	-0,0675	0,1802
	9	0,1035	-0,0061	-0,0153	....	0,1812	0,1359
	10	-0,0519	0,0653	0,1438	....	0,1833	-0,0490
Total return		**0,4115**	**0,5939**	**0,4472**	....	**1,1658**	**1,5219**
Av. Total return		**0,8605**					
Average annual return for each portsim		0,0411	0,0594	0,0447	....	0,1166	0,1522
Av. annual return for all		0,0860					
Std.dev annual return for each portsim		0,0901	0,1087	0,0931	....	0,1145	0,1192
Av. Std.dev annual return for all		0,0945					

Excess return from the short-term portfolio to be reinvested after 5 years		1 229 707,84	1 530 331,31	-	....	390 887,39	1 087 429,76
Initial holding		1 842 000,00					
	0	1 842 000,00	1 842 000,00	1 842 000,00	....	1 842 000,00	1 842 000,00
	1	1 920 513,08	2 129 961,83	2 039 992,23	....	2 007 698,20	2 307 649,53
	2	1 996 039,44	2 565 132,14	2 087 115,86	....	2 594 571,00	2 206 229,09
	3	1 827 524,65	2 743 840,93	2 136 457,68	....	3 124 320,86	2 811 202,86
	4	2 160 770,40	2 561 857,79	2 462 981,74	....	2 862 784,42	3 034 973,21
	5	2 105 184,41	2 600 719,06	2 758 467,78	....	3 064 505,97	3 670 068,25

If below target in 5-year portf., take from this portf.		2 105 184,41	2 600 719,06	2 556 849,20	....	3 064 505,97	3 670 068,25
plus excess return from 5 years		3 334 892,26	4 131 050,37	2 556 849,20	....	3 455 393,36	4 757 498,01
	6	3 996 221,50	5 125 243,29	2 698 177,89	....	4 139 744,18	5 527 669,33
	7	4 094 826,97	5 303 122,49	2 739 607,02	....	4 543 698,33	7 301 688,48
	8	4 024 273,32	4 667 429,94	2 249 115,11	....	4 236 958,10	8 617 474,35
	9	4 440 818,45	4 639 064,57	2 214 659,95	....	5 004 555,16	9 788 897,84
	10	4 210 281,91	4 942 064,18	2 533 234,47	....	5 921 794,17	9 309 415,84

Require 3m	3 000 000,00	3 000 000,00	3 000 000,00	....	3 000 000,00	3 000 000,00
If holding below 3m take funds from long-term subportfolio	-	-	466 765,53	....	-	-
Excess return a/withdrawal	1 210 281,9102	1 942 064,1814	-	....	2 921 794,1669	6 309 415,8365
If success = 1	1	1	1	....	1	1
Average holding a/10 years	2 346 496,53					

No. of times the mid-term portfolio fails	0
No. of times the mid-term portfolio succeeds	1000
Probability of failure	**0**

Appendix 1c: *E*xtract of the long-term portfolio. The extract demonstrates how we use the return distribution output from MatLab in excel to generate the real monetary values, and eventually the portfolios probability of failure. For a more details, see excel sheet.

Return distribution (Insert output in the blue area)	1	2	3	....	999	1000
1	0,0859	0,2726	0,2415	....	0,1290	0,2373
2	0,0177	0,2487	0,0380	....	-0,0428	0,2348
3	-0,0303	0,2543	-0,1864	....	-0,0709	0,1544
4	0,2572	0,1157	0,0753	....	0,1778	0,1552
5	0,0107	0,1814	0,2996	....	-0,1425	0,2231
6	0,0809	0,0431	0,1963	....	0,0779	-0,2149
7	0,0210	-0,1483	-0,0085	....	0,1383	0,1841
8	0,1355	-0,1543	0,2856	....	0,2296	0,1048
9	0,1656	0,0927	0,1763	....	0,0860	0,1432
10	0,3186	0,2946	-0,0281	....	0,4903	-0,1006
11	0,1719	0,0746	0,3003	....	0,0460	0,3376
12	0,1351	0,0075	0,1612	....	0,0769	-0,0019
13	0,1456	0,2206	0,0621	....	0,2910	0,3361
14	-0,0574	0,0481	-0,0590	....	0,1843	0,1623
15	-0,1326	0,0527	-0,0303	....	0,2882	-0,0399
Total return	**1,3253**	**1,6040**	**1,5237**	....	**1,9591**	**1,9156**
Av. Total return	**1,4067**					
Average annual return for each portsim	0,0884	0,1069	0,1016	....	0,1306	0,1277
Av. annual return for all	0,0938					
Std.dev annual return for each portsim	0,1160	0,1361	0,1441	....	0,1538	0,1508
Av. Std.dev annual return for all	0,1282					
Initial holding	1 088 000,00	1 088 000,00	1 088 000,00	....	1 088 000,00	1 088 000,00
Excess return from mid-term subportfolio	1 210 281,91	1 942 064,18	-	....	2 921 794,17	6 309 415,84
1	1 181 462,35	1 384 572,20	1 350 787,90	....	1 228 358,48	1 346 135,09
2	1 202 372,63	1 728 978,19	1 402 136,74	....	1 175 738,32	1 662 215,98
3	1 165 920,68	2 168 592,55	1 140 720,59	....	1 092 336,01	1 918 791,00
4	1 465 798,59	2 419 525,44	1 226 564,17	....	1 286 575,34	2 216 561,87
5	1 481 524,97	2 858 335,15	1 593 998,38	....	1 103 193,06	2 711 152,85
6	1 601 331,13	2 981 399,68	1 906 886,69	....	1 189 143,48	2 128 567,22
7	1 634 879,86	2 539 361,81	1 890 588,55	....	1 353 578,64	2 520 530,74
8	1 856 432,59	2 147 567,37	2 430 449,21	....	1 664 331,55	2 784 744,68
9	2 163 914,61	2 346 713,67	2 858 864,50	....	1 807 426,63	3 183 641,46
10	2 853 319,94	3 038 145,73	2 778 501,97	....	2 693 616,31	2 863 233,75
If below target in mid-term subportf., take funds from this portf.	2 853 319,94	3 038 145,73	2 311 736,44	....	2 693 616,31	2 863 233,75
plus excess return from mid-term subportfolio	4 063 601,85	4 980 209,91	2 311 736,44	....	5 615 410,47	9 172 649,59
11	4 762 127,18	5 351 605,69	3 006 051,74	....	5 873 993,43	12 269 357,88
12	5 405 304,23	5 391 977,54	3 490 497,87	....	6 325 858,34	12 246 354,18
13	6 192 162,49	6 581 277,86	3 707 331,43	....	8 166 677,53	16 361 935,94
14	5 836 898,31	6 898 149,76	3 488 585,60	....	9 671 871,66	19 017 158,23
15	5 062 828,18	7 261 457,83	3 382 775,08	....	12 459 310,40	18 258 256,35
Require 3m	3 000 000,00	3 000 000,00	3 000 000,00	....	3 000 000,00	3 000 000,00
Holding a/withdrawal	2 062 828,18	4 261 457,83	382 775,08	....	9 459 310,40	15 258 256,35
If success = 1	1	1	1	....	1	1
Average excess return a/15 years	4 830 955,90					
No. of times the long-term portfolio succeeds	930					
No. of times the long-term portfolio fails	70					
Probability of failure	**0,07**					

Appendix 3d. Extract of the single portfolio. The extract demonstrates how we use the return distribution output from MatLab in excel to generate the real monetary values, and eventually the portfolios probability of failure. For a more details, see excel sheet

Return distribution (Insert output in the blue area)		**1**	**2**	**3**	......	**999**	**1000**
	1	0,0763	0,2700	0,0562	......	0,1140	0,1245
	2	-0,0110	0,3012	0,0256	......	-0,0909	0,2693
	3	0,0041	0,1251	-0,0409	......	-0,0200	0,1591
	4	0,2020	0,0710	0,0614	......	0,1486	0,1936
	5	0,0014	0,1315	0,2207	......	-0,0544	0,1946
	6	0,0673	0,0110	0,1061	......	-0,1015	-0,1311
	7	-0,0186	-0,1614	0,0229	......	-0,0038	0,1757
	8	0,1187	-0,0561	0,1852	......	0,2320	0,1949
	9	0,1198	0,0640	0,1752	......	0,0544	0,1645
	10	0,1957	0,2280	0,0273	......	0,3508	0,0166
	11	0,1976	0,1252	0,1367	......	0,0547	0,2506
	12	0,1746	0,0515	0,2134	......	0,1604	0,0855
	13	0,1632	0,2389	0,0878	......	0,2040	0,2061
	14	-0,0795	0,0229	0,0577	......	0,1460	0,1908
	15	-0,0652	0,1242	0,0949	......	0,2050	0,0534
Total return		**1,1466**	**1,5470**	**1,4302**	......	**1,3994**	**2,1483**
Av. Total return		**1,3086**					
Average annual return for each portsim		0,0764	0,1031	0,0953	......	0,0933	0,1432
Av. annual return for all		0,0872					
Std.dev annual return for each portsim		0,0957	0,1208	0,0746	......	0,1265	0,0988
Av. Std.dev annual return for all		0,0968					
Initial holding		5 087 000,00	5 087 000,00	5 087 000,00	......	5 087 000,00	5 087 000,00
	1	5 475 191,48	6 460 575,54	5 373 089,49	......	5 666 905,64	5 720 500,02
	2	5 415 055,96	8 406 180,78	5 510 728,29	......	5 152 046,06	7 261 070,35
	3	5 437 192,05	9 457 680,69	5 285 104,91	......	5 049 261,15	8 416 282,03
	4	6 535 581,10	10 129 137,29	5 609 610,81	......	5 799 656,65	10 046 081,61
	5	6 544 578,72	11 460 750,43	6 847 635,44	......	5 483 977,68	12 001 365,35
Require 2,5m							
Withdrawal		2 500 000,00	2 500 000,00	2 500 000,00	......	2 500 000,00	2 500 000,00
Holding a/withdrawal		4 044 578,72	8 960 750,43	4 347 635,44	......	2 983 977,68	9 501 365,35
	6	4 316 821,71	9 059 536,62	4 809 081,24	......	2 681 064,85	8 255 936,30
	7	4 236 737,05	7 597 452,12	4 919 299,49	......	2 670 971,03	9 706 558,44
	8	4 739 798,86	7 171 340,81	5 830 502,04	......	3 290 672,96	11 597 881,96
	9	5 307 564,98	7 630 040,64	6 851 811,43	......	3 469 710,48	13 506 294,55
	10	6 346 413,72	9 369 709,28	7 038 816,19	......	4 686 755,14	13 730 570,34
Require 3m							
Withdrawal		3 000 000,00	3 000 000,00	3 000 000,00	......	3 000 000,00	3 000 000,00
Holding a/withdrawal		3 346 413,72	6 369 709,28	4 038 816,19	......	1 686 755,14	10 730 570,34
	11	4 007 762,67	7 167 429,65	4 590 935,51	......	1 779 002,87	13 419 620,24
	12	4 707 634,79	7 536 647,09	5 570 523,20	......	2 064 428,95	14 567 500,57
	13	5 476 017,22	9 337 008,86	6 059 388,58	......	2 485 578,26	17 569 946,86
	14	5 040 881,42	9 550 698,43	6 409 202,63	......	2 848 353,29	20 922 973,86
	15	4 712 117,78	10 737 152,77	7 017 200,58	......	3 432 363,73	22 039 741,64
		7 650 430,773					
Require 3m							
withdrawal		3 000 000,00	3 000 000,00	3 000 000,00	......	3 000 000,00	3 000 000,00
Holding a/withdrawal		1 712 117,78	7 737 152,77	4 017 200,58	......	432 363,73	19 039 741,64
If success = 1		1	1	1	......	1	1
Average holding a/15 years		4 873 566,66					
No. of times the single portfolio succeeds		842					
No. of times the single portfolio fails		158					
Probability of failure		**0,158**					

Appendix 4 – Preliminary Thesis

Preliminary Thesis

Sustainability of Goal-Based Portfolios

GRA 19502 – Master Thesis

MSc. in Business, major Finance

Supervisor: Dr. Bruno Gerard

Deadline:

15.01.2018

BI Oslo

Table of content

Introduction

Portfolio managers have in general two tasks; managing investor´s portfolios, and translating investors' goals into the language of finance. The latter may turn out to be a difficult task for managers, as investors have a tendency to specify their goals. Markowitz's "portfolio selection" from 1952 is considered the start of modern portfolio management theory, and has been embraced by practitioners and theorist since. Goal-based portfolio management theory combine appealing component of Markowitz´s mean-variance portfolio (1952) and the behavioral portfolio theory of Shefrin and Statman (2000). The purpose of goal-based portfolios is to tie investment strategies to certain investor goals.

After the financial crisis in 2008, goal-based portfolio management theory has gotten more attention. As investments turned out to be less liquid as expected after the crisis, investors realized that their diversification strategies only worked under normal market conditions, and that a severe bear market could influence the fulfillment of personal goals. The financial crisis indicated a need for a change in wealth management thinking. In 2010, Das, Markowitz, Scheid and Statman argued that goal-based processes are just as efficient as what the mean-variance processes, when clients and wealth managers change their definitions of risk. By enabling clients to measure progress towards their goals, goal-based portfolios increase the clients' commitments to their lifecycle goal, and reduce negative behavioral bias, such as impulsive decision-making.

Investors tend to have different goals they wish to achieve, and the tolerance of risk might differ for each goal. For simplicity, we will focus our study on wealth management, and define three categories of investor goals; personal, dynastic and philanthropic.

Background and motivation

Markowitz's article "portfolio selection" from 1952 is considered the start of modern portfolio management theory, and has been embraced by practitioners and theorist since. Markowitz laid out the mean-variance efficiency frontier, where investors choose the portfolio that maximizes return, and minimizes variance. Markowitz introduced investor's need of diversification, not only by increasing number of securities, but also by reducing the variance. Even though most later research, in large, has been built on Markowitz's portfolio selection theory, some researchers believes that individual investors are not merely concerned with attaining highest possible return, but that they in fact attempt to reach different goals. The investor goals differ from the desire to attain highest return in the sense that investors accept different risk levels for the different goals.

This goal-based portfolio theory introduces the behavioral aspect of investors, and one of the early researchers who challenged Markowitz was Thaler (1980) who stated that individual investors was not concerned of the overall portfolio performance, rather they wanted to make investment decisions to meet specific goals. Thaler goes on describing that each goal has its own subportfolio with different risk levels.

Sortino and van der Meer (1991) introduced the Post-Modern Portfolio Theory, which redefines risk as the probability of not achieving the objectives. In recent years, Contemporary Portfolio Theory (CPT) has emerged, which seeks to protect investments against failure, even if it might threaten potential reward.

Purpose

The purpose of this thesis is to analyse the goal-based portfolio theory, and compare it to the established mean-variance optimization theory laid out by Markowitz. We wish to explore existing gaps in the literature, and aim to answer whether goal-based portfolios help improve the performance of the portfolio as well as overall satisfaction of the client.

Current primary research question:

- Comparing the goal-based investment approach to traditional investment approach, has the overall return of the clients improved?

Additional subquestions:

- Does the goal-based approach require more insights and commitment from the portfolio manager, and or the investor?
- Does the goal-based approach require different investment strategies for different goals?
- Why some portfolio managers make the decisions they do, and how they design portfolios in a goal-based approach.

Literature review

Portfolio Optimization

The foundation of modern portfolio theory is based on the work by Markowitz (1952) who introduced the efficiency frontier, helping investors to maximize return and minimize the variance of the return. Das, Markowitz, Scheid, and Statman (2010) introduced portfolio optimization with mental accounts, their definition of goal-based portfolios. The authors suggest that investors tend to misspecify their risk aversion, since they often have different risk level for the different goals. Investors are attracted to Markowitz´s mean-variance portfolio by its rational and practical application. However, in goal-based portfolio theory, investors want their portfolios to satisfy underlying personal needs, rather than simply maximizing return.

Das, Markowitz, Scheid and Statman (2010) shows in their article that optimizing subportfolios (goal-based) are as optimal as overall portfolio optimization (mean-variance) when shortselling is allowed, however, when no shortselling, subportfolio optimization tend to give small losses. These losses are however far smaller than losses occurring because of the misspecification of an investor's risk aversion. Hence, the importance of correctly specifying an investor's risk aversion is crucial, and the efficiency loss declines, as investors become more risk averse.

Goal-based portfolios

There has been developed several refined goal-based portfolio theories, and some researchers, such as Shefrin and Statman (2000), believes that the different theories have emerged from the puzzle of Friedman and Savage (1948), the observation that people who buy insurance policies often buy lottery tickets as well. This puzzle is a contradiction to general beliefs of the rational of investment behavior, and does not coincide with Markowitz mean-variance theory of investment. The puzzle may be used to explain the need of more sophisticated models for optimal investments, including achievement of investor's goals. Investors in goal-based portfolio theory face several optimization problems, one for each goal.

Investor's goals typically fall into three categories; personal (lifestyle requirements), dynastic (children's future) and philanthropic (personal values etc.) (Brunel, 2012). The categories are ordered after importance for the investors, where the personal goals are prioritized over dynastic and so on, see Figure 1. Because of the different level of importance, the level of risk the investor is willing to tolerate will also differ.

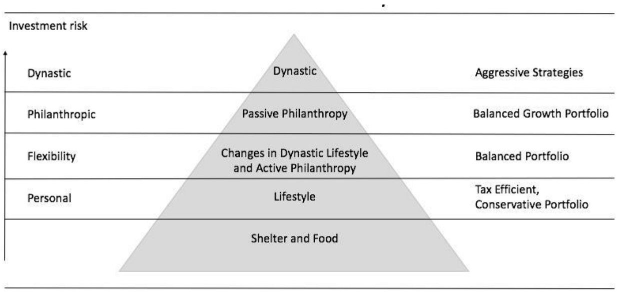

Figure 1: Behavioral Finance Portfolio Pyramid (Statman, 2004)

The portfolios are divided into different mental account layers (goals) of a portfolio pyramid, where each layer is associated with a different goal and outlook on risk. The investor aims to optimize each subportfolio separately as opposed to integrating all the goals into one aggregate portfolio. Das, Markowitz, Scheid and Statman (2010) introduced a framework of constructing subportfolios and optimizing them separately. The framework is based on two important assumptions, first, that investors are better at stating their goal thresholds and probabilities of reaching thresholds in mental accounts (goal-based approach), than their risk-aversion coefficients in mean-variance theory. Second, that investors are better able to state thresholds and probabilities for subportfolios. The authors represent simulations to show that better problem specification gives superior portfolios.

Jean L.P. Brunel's, the Chief Investment Officer of GenSpring Family Offices, article "Goals-Based Wealth Management in Practice" from 2012 also gives concrete examples of how to include investors' goals into optimal portfolios. Brunel's experience as wealth manager has given him significant insight in investors' behavior. According to Brunel, he has met few investors who say; "Give me the highest possible return with the lowest possible risk, and all will be fine".

Behavioral finance

Behavioral finance shows that investors are not risk averse but loss averse, suggesting measures based on the likelihood of loss (Nawrocki, 1999). Daniel Nevins (2004) is another author who describes the need of a different portfolio optimization theory because of the human behavior of investors. An investor does not need to hold a risky portfolio if there exists a less risky portfolio, which will achieve the investor's goal, nor should an investor choose a more conservative portfolio if the portfolio is unlikely to achieve the goals, even though the investor is risk averse Nevins (2004).

Applying the concepts of behavioral finance, Statman indicated that each investor has not only a variety of goals, but also different risk profiles to accompany each of those goals. Some of these risk profiles may seem almost contradictory, yet they are not exclusionary. They merely reflect normal human behavior, which the puzzle by Friedman and Savage is an example of. Thus, wealth managers must develop investment strategies to match their clients' different goals and risk profiles.

Behavioural portfolio theory emphasizes the trade-off between thresholds and the probability of failing to reach them. In goal-based portfolio theory, investors maximize expected wealth subject to a maximum probability of failing to reach a threshold level of return, while in mean-variance theory investors minimize variance, subject to a level of return.

Risk

Even though it is the assessment of the investor goals that is the key difference between mean-variance portfolio theory and goal-based portfolio theory, it might seem that the critical factor is how to define and integrate the risk level of the investor. The risk profile of the constructed portfolio should match the risk profile of the client (Janssen, Kramer and Boender, 2013).

Risk can be defined as the uncertainty of a future event. In Markowitz´s mean-variance portfolio, risk is defined as the standard deviation of the return, whilst in goal-based portfolio theory the definition of risk is changed. Brunel (2012) argues that risk should not be defined mathematically as a standard deviation of return, but rather as the probability of not achieving the set goals, which is in fact the way most people naturally describe risk. According to Das, Markowitz, Scheid and Statman (2010) in mean-variance theory, investors are always risk averse, however, in behavioral portfolio theory, investors might even be risk seeking. The authors have not included this fact in their analysis, however left it out for future research.

An investor's personal goal might have a low risk level, whilst the same investor can accept a higher level of risk for the goal of preserving a valuable inheritance for her children and grandchildren. Philanthropy, which entails the allocation of excess wealth for altruistic purposes, can have an even higher level of risk. This kind of mental accounting can however be difficult to include in the traditional framework that requires a single, overall risk tolerance. An alternative method could be to develop several subportfolios with different strategies and risk levels that can be integrated into an aggregate portfolio, such as Thaler's (1980) subportfolios.

Before an investment strategy can be devised for the clients, the wealth managers need to know the goals, liabilities and when the client wants to achieve them, as well as the clients risk profile. When the wealth managers define the risk profile, the difference between the willingness to take risk and ability to take risk should be clear (Janssen, Kramer & Boender, 2013). The risk willingness referrers to the emotional risk tolerance, and can be described as how much risk the client is willing to take on, and can be measured with the help of questionnaires. Risk ability refers to how much risk the clients can tolerate, and might be measured with the help of exploring plausible scenarios. These scenarios should include realistic real-world features that are significant to the client, and they may include:

- Means, volatilities and correlations that change with the investment horizon.
- Non-normal returns, fatter tails and skewed distribution
- Tail risk, where the probability distribution exhibits fatter tails due to correlations in the asset classes.

Research Methodology

The purpose of this section will be to gain insight into the field of goal-based portfolios, test the hypothesis and possibly come up with an appropriate answer to the research question. We will initiate the research by using MatLab, a programming language primarily intended for numerical computing.

Research Strategy

In this thesis, we will use quantitative research methods to test our hypothesis and research questions. Even though the goal-based portfolio approach introduces the behavioural aspect of investors, measuring the performance of portfolios is done with quantitative methods for obvious reasons. As it exists historical data on the performance of funds we find it unnecessary to include qualitative research methods.

Research Design

In our analysis, we´ll be working with time series data, which is data for a single entity (portfolio) collected over multiple time periods (Stock & Watson, 2015).

From our literature review we will choose on of the methods laid out, in order to construct goal-based portfolios (subportfolios). We will construct 3 subportfolios to accommodate the investor goals, and compare the performance of these portfolios with a benchmark portfolio. First we will specify 3 goals for the goal-oriented investor from the categories; personal, dynastic and philanthropic, e.g. retirement plan, children's education, and charity. After defining the goals, we will calculate the risk level the investor tolerates for each goal.

The benchmark portfolio will be a fixed list of securities to compare the performance of our subportfolios. We consider using the market value-weighted index, Standard and Poor's 500 (S&P 500). S&P 500 index is an index of 505 stocks issued by large US. firms, and is on of the common benchmarks of the US. stock market.

Our aim is to find a feasible method of integrating the risk levels. One option we consider is to use the mental account method laid out by Das, Markowitz, Scheid and Statman. The authors define risk as the probability of failing to reach a fixed threshold level of return for each subportfolio, which accommodate each investor goal. In this framework, the investor specifies that her goal is for her portfolio P to accumulate to the threshold dollar amount P_T after T years, implying a threshold return per year of {[Pt/P0]1/T − 1} ≡ H , and failing to meet this threshold level with probability α. Keeping threshold, H, fixed, and solving the problem for different α, gives corresponding maximized expected return levels.

Research Method

When initiating the analysis, it is necessary to include benchmark groups, i.e. industry combinations to use as comparison for which the constructed portfolios are evaluated. Since the constructed portfolios are not real, we need to simulate the performance of all the portfolios forward to make them comparable. We have decided to implement the Monte Carlo simulation, which takes account of historic data and we use the standard deviation, average return of past performance and other assumptions to determine probable future scenarios.

Before we start the analysis, we need to check whether the chosen dataset is stationary or not. If it is non-stationary, i.e. the data exhibits a trend, we need to make it a stationary process by removing the underlying trend. By making the process stationary, parameters such as mean and variance will remain constant over time, making it easier for us to perform analyses. Then we find the descriptive statistics for the benchmarks and use them as the theoretical values.

We start by performing an introductory test statistic (Z-test) to see how the constructed portfolios compare against the benchmarks. After this we can run a test for portfolio performance by finding the expected returns of the portfolio through the T-equation. If we compare the mean from the t-equation with the mean from the Z-equation, we may determine if there is a significant difference between the constructed portfolios and the benchmarks. A final test between the constructed portfolios and the benchmarks could be a paired mean comparison study.

A large part of the investing process is, understanding the sources of risk and reward related to the portfolios (Israel and Ross, 2014). Linear regression analysis is a common way to measure factor exposures to risk. A regression analysis can explain the relationship between the dependent variable (portfolio returns) and the explanatory variable (various factors). The regression analysis may include one or several different risk factors. However, investors should be careful when deciding on the factors, as not all of them may be applicable for the portfolios.

The CAPM may be used to assess the performance of portfolios. We would then compare the risk-adjusted returns of the portfolios against returns of the benchmark, and then use the ordinary least squares (OLS) regression to fit a straight line through the data points. The most common equation for this type of line is:

$$r - R_f = \beta x (R_m - R_f) + \alpha$$

where r = the portfolios return; R_f = Risk-free rate; β = systematic risk; α = the portfolios return compared to the overall market; and R_m = return of the market index. Using the R^2, we can measure the percentage of the portfolios performance that can be explained by the performance of the benchmark and other risk factors.

Progress until final thesis

Detailed plan:

February 15th	Chosen a method for constructing subportfolios, collected final literature.
March 1st	Chosen data, benchmark portfolio etc.
March 15th	Constructing portfolios
May 1st	Analyzing data
June 1st	Finalizing thesis
July 1st	Delivering of final thesis

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