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Hedging House Price Risk in Norway

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Content

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Abstract

In this paper, we empirically examine the effectiveness of housing futures for homeowners in Oslo and try to answer the questions of whether housing futures should be introduced in Norway. If an individual buys a house today at time *t* for a price *S(i,t)*, he will be exposed to fluctuations in the value for that house. By following the methodology described in Bertus et al. (2008) and Schorno et al. (2014), we want to see if we can hedge the risk of house price fluctuations for homeowners by introducing futures on a housing index in Oslo. Since there are no actual housing futures available in Oslo, we have used three different housing indices as a proxy for future returns and as an underlying for hedging. The results of our analysis show that housing futures, on the one hand, fail to decrease the variability of homeowners' returns, and on the other hand, are quite successful in increasing the actual returns of hedgers. This shows us that housing futures, if introduced, can attract speculators but not people who actually need housing derivatives to give away the risk. Therefore, we conclude that housing futures should not be introduced in Norway for now, and future research is needed in this area.

Key words: housing futures, house price risk, arithmetic repeat-sales house price index, geometric repeat-sales house price index.

1. Introduction

During the last years, housing prices in Norway have been steadily increasing, which makes the homeowners wonder: what is the risk that the prices can go down, significantly reducing the value of owner-occupied houses? In the light of house price risk, homeowners may be willing to hedge themselves against price fluctuations.

According to TradingEconomics.com, the homeownership rate in Norway in the beginning of 2017 was 82.7%, which is one of the highest in the world. This means that more than 80% of Norwegian residential real estate is owner-occupied. Homeowners buy houses for many reasons, mostly to have a place to live that corresponds to their tastes and needs (Englund, Hwang, & Quigley, 2002) or to protect themselves from rent fluctuations (Sinai & Souleles, 2005). No matter what reasons for the house purchase are, the result for homeowners is that the house begins to constitute a large portion of their wealth. When the price of housing falls, the value of houseowners' portfolio falls as well. Therefore, to secure homeowners from housing price fluctuations various instruments can be used, and in the following sections we will look at different studies those examined the effectiveness of housing derivatives.

Introduction of housing derivatives

According to Jud & Winkler (2009), there were several attempts to start trading derivatives on real estate. In November 1990, the Chicago Board of Trade (CBOT) together with economists Case, Shiller, and Weiss evaluated the possibility to launch home-price futures. However, after finding out that people were more likely to sell such futures rather than to buy them, the CBOT decided not to start the project. In 1991, the London Futures and Options Exchange (FOX) began trading real estate futures. But the market for housing futures was shut down in October 1991 due to low demand for trading. Before 2006 there were also minor attempts to launch real estate futures; however, they all failed due to the same reasons: low trading volumes.

In 2006, Chicago Mercantile Exchange (CME) launched housing futures and options based on the S&P/Case-Shiller Home Price Index. The Index was first created in 1980s by Karl E. Case and Robert J. Shiller. Economists invented the Index for the purpose of measuring the average change in home prices for single-family housing. According to S&P CoreLogic Case-Shiller Home Price Indices Methodology (2017), the Index measures price changes given the constant level of housing quality and uses 'repeat sales method' (described in more details in Section 3) of index calculations, i.e. only houses those were sold at least twice are included into Index calculation sample.

Betrus et al. (2008) and Schorno et al. (2014) examined the effectiveness of established housing futures for Las Vegas and concluded that CME futures were not that successful in mitigating house price risk. We decided to perform the similar analysis as in Betrus et al. (2008) and Schorno et al. (2014) but for Oslo. The question we want to investigate is whether it is possible to effectively hedge house price risk in Oslo using housing futures, and, following that, whether housing derivatives should be introduced in Norway.

Contribution to the literature

To our best knowledge, this is the first paper written in English that examines the question of hedging house price risk in Norway and the first paper which investigates the effectiveness of housing futures in Norway. In addition, we develop the existing literature on housing indices by comparing how different house price indices are constructed and how they can be used as the underlying for housing derivatives.

Structure

The thesis is structured in the following way: After the introduction in Section 1, in Section 2, we present the overview of the existing studies about hedging house price risk using different hedging instruments and the problems associated with them. Section 3 consists of the methodology part and shows how we are going to construct house price indices and then use them to estimate the effectiveness of hedging with housing futures by implementing four different hedging strategies. Section 4 presents the description of the data that will be used in the research. Finally, Section 5 presents the results of our analysis and concludes.

2. Literature review

Hedging house price risk with CME futures

After the introduction of CME housing futures in 2006, many scientists decided to investigate the question of effective hedging with the newly available derivatives. Among the first ones to discuss this issue were Bertus, Hollans, & Swidler (2008). They consider hedging from the point of view of mortgage portfolio investors, real estate developers, and individual homeowners. The authors compare the effectiveness of two hedging strategies: naïve one with hedging ratio equal to 1 and the minimum variance hedge strategy. For the period before the introduction of CME futures Bertus et al. (2008) use returns on S&P/Case-Shiller Home Price Index as a proxy for futures returns. The results show that investors could reduce portfolio variance by 89%. In addition, Bertus et al. (2008) indicate that for individual homeowner there is a one to one exposure; therefore, hedging with the naïve strategy could be quite successful as opposed to a dynamic strategy where the exposure for the homeowner changes over time. However, authors indicate a couple of limitations of their analysis. The first limitation is the data since it was taken for the period from 1994 to mid-2006. High effectiveness of hedging strategies should also be verified with the data from the crisis of 2007-2009. The second limitation concerns the type of used strategies and the hedging horizon. According to the authors' findings, optimal hedging position changes over time, which indicates that the analysis should focus on not only static but also dynamic strategies. A dynamic strategy means the homeowner will rebalance his hedging position based on his current exposure to the housing market so the hedge always will be the most optimal at that given time. Moreover, the authors consider the hedging horizon of one quarter; however, homeowners are exposed to price risk for much longer terms (e.g. 5-7 years).

Another article "Hedging house price risk with futures contracts after the bubble burst" by Schorno, Swidler & Wittry (2014) extends the existing literature in managing house price risk and covers the period 2006-2011. While the earlier paper by Bertus et al. (2008) considers only naïve and static hedging strategies, Schorno et al. (2014) analyzes the hedging effectiveness of the CME futures using forward-looking and conditional hedging strategies, which rely on market information to update a quarterly hedge ratio. Focusing on the Las Vegas metropolitan area, they examine whether CME contracts based on the S&P/Case-Shiller Las Vegas Real Estate Index (LVRX) could be used to mitigate house price risk. Schorno et al. (2014) use the quarterly percentage change in the value of the S&P/Case-Shiller LVRX as a proxy for the return on the futures contract. They use this as the underlying to see if individual homeowners could reduce their exposure by hedging.

The forward-looking strategies they test are rollover minimum variance and rollover conditional OLS strategies using a five years of data sample from just prior to the hedge horizon to construct the minimum variance hedge ratio. Following Bertus et al. (2008), the authors use returns on S&P/Case-Shiller Home Price Index as a proxy for returns on futures for the period before CME housing futures were available (before 2006). After 2006, Schorno et al. (2014) use directly futures returns, which is a big improvement compared to previous study since these futures are what the homeowners actually use for hedging. The authors test in total four different strategies with different results. The strategies tested are two strategies from Bertus et al. (2008), which are a simple naïve strategy and a static minimum variance strategy, and two forward-looking strategies. Having used house price index as a proxy for futures returns authors conclude that the best hedging strategy is the rollover minimum variance, while the worst is the static minimum variance strategy. However, when the realized futures returns replace the index returns, the performance of all strategies is quite poor, which is likely due to illiquid market of CME housing futures. When the market for CME housing futures is illiquid, prices are not efficient meaning that the price for futures will fall because of less demand. Because of this, we see different results when we use CME housing futures directly and when we use the house price index as a proxy for futures returns in hedging.

Interestingly, Schorno et al. (2014) find that the naïve strategy may be the best approach to manage systematic risk given the difficulty of implementing the other strategies (i.e. homeowners need to monitor constantly the change in housing price to adjust their position in housing futures) combined with their low hedging effectiveness.

Other hedging instruments

In addition to the hedging strategies proposed by Bertus et al. (2008) and Schorno et al. (2014), other articles have also looked at other possible ways to hedge housing prices. Alternatives could be index-linked mortgages (Syz, Vanini, & Salvi, 2008), structured swaps (Fabozzi, Schiller, & Tunaru, 2009) or commodity futures that have high correlation with housing market (Hinkelmann & Swidler, 2008).

Fabozzi et al. (2009) look at three different types of structured swaps used in real estate market in the United Kingdom in relation to managing housing risk. These are balance guaranteed swaps, a cross-currency balance guaranteed swap and a balance guaranteed LIBOR-base rate. Unfortunately, there are many problems related to the design of all these swaps. In balance guaranteed swaps, the collateral coupon leg is paid at the end of the period and mortgage payments are collected every day in the period on a continuous basis. This creates a prepayment risk for the writer of the swap since payments are not done at the same time and it could be a big problem if interest rates are fluctuating. Because of this prepayment risk, balance guaranteed swaps are often extremely expensive and very rare in practice. Also since the reference floating rate is three-month LIBOR, there is a basis between the reference three-month LIBOR collected monthly from the swap and the same reference three-month LIBOR paid quarterly to the note holders that funded the securitization. This basis risk will not be large as long as interest rates are stable but could create uncertainty for the home owners since the interest rates for the paid and collected amount will not be exactly the same. This means we have an imperfect hedge. Lastly, these types of products can become very complicated and difficult to understand for a private consumer with little or no experience in financial markets.

Index linked mortgages as proposed by Syz et al. (2008) is a much easier way to hedge for individuals than swaps and they are tailor made for retail consumers. Syz et al. (2008) use data from 1985-2005 and 5-year index linked mortgages. The basic idea of this type of hedge is to link the mortgage to a house price index so that the interest payments and/or the principal are linked to the underlying index movements. The mortgage is therefore no longer an interest rate but a house price derivative. If the index drops, you will pay lower interest or price decrease is directly subtracted from the mortgage's principal at maturity. Either way the volatility is reduced. Therefore, this type of property derivative reduces the homeowner's exposure to house price risk while reducing the credit risk exposure of the bank through asset-liability immunization.

Lastly, other articles have also looked at existing commodity futures and found a commodity that correlates with the house index (Hinkelman & Swidler, 2008). They used commodity future prices from 1983q2 to 2005q4 to examine whether existing futures contracts can effectively be used to offset volatility in national house prices. For this hedging strategy to work there needs to be a high correlation between the house prices and a portfolio of futures prices. Examples of futures could be currency, metal, energy, interest rate and grain to mention some. In Hinkelman & Swidler (2008), they tested 31 different futures and found only the British Pound and Platinum to be statistically significant for hedging house prices in the US market.

General problems with housing derivatives

Some scientists wonder whether it is optimal to use S&P/Case Shiller Home Price Index as the underlying for housing derivatives. Nagaraja, Brown, & Wachter (2010) point at some disadvantages of using repeat sales methodology in Case-Shiller Index. Firstly, only small amount of houses was sold more than once; therefore, according to authors, repeat sales indices are constructed based on very small and unrepresentative sample. Secondly, all houses need renovation over time; therefore, there is actually no repeat sale of the same house, which violates one of the basic assumptions of the Index methodology (the constant level of house quality). In addition, Dröes & Hassink (2013) state that house price indices cannot be used to measure house price risk due to the fact that the indices underestimate the idiosyncratic volatility of home prices. They perform the analysis for the Netherlands and show that the idiosyncratic variation in house prices is more than 85%. Therefore, according to authors, housing futures which use house price indices as the underlying provide good hedge only for the market risk of house prices, while the idiosyncratic component of the risk remains too high.

One major overall concern is the liquidity of the market for housing derivatives. In the US, the initial response to CME housing futures has been moderate and daily volume has been small. Cao & Wei (2010) argue this was because of absence of sufficient valuation models. Although real-estate derivatives should be preferred to insurance-type contracts because of direct settlement, the liquidity of housing derivatives is key to their use by individuals and professional asset managers. The problem is that liquidity can only be established after financial institutions decide to be more active in the housing derivative markets, as suggested by Case, Shiller, & Weiss (1993). The success of the housing futures depends upon whether they serve the needs of hedgers as well as speculators, and according to Hinkelman & Swidler´s (2008) analysis, hedgers may not be able to effectively manage their risk unless their geographic portfolio weights largely replicate those in the futures index that they are using since house prices vary a lot even within the same city. This means that if we own a house in a specific part of the city, we must ensure that our housing futures replicate this area and not an average of the city. This creates a problem both for the creation of futures as we would need specific futures for every district and a liquidity problem since the number of buyers will be drastically reduced compared to city level or national futures.

According to De Jong, Driessen, & van Hemert (2007), hedging with CME futures have little benefit for homeowners. Mainly this is due to large idiosyncratic variation in house prices. This is because CME futures use S&P/Case-Shiller house price index as the underlying; however, this index is a city-level index; therefore, CME futures cannot fully hedge the risk of individual home price change. This is called basis risk and arises when there is imperfect correlation between two investments and this creates the potential for excess gains or losses in a hedging strategy, thus adding risk to the position (Investopedia.com, 2018).

There is also another basis risk for hedgers since there is no simple adjustment factor to housing futures prices. All these factors imply ineffective hedging and investors will not use the housing derivatives to manage housing risk. Therefore, it appears that the success of home price futures contracts hinges upon whether there is significant hedging activity, which, in turn, is dependent upon whether the derivative contracts can be used to effectively hedge house price risk.

3. Methodology

By following the methodology described in Bertus et al. (2008) and Schorno et al. (2014) we want to see if we can hedge the risk for homeowners by introducing futures on a housing index in Oslo. If an individual buys a house today at time t for a price *S(i,t)*, he will be exposed to fluctuations in the value for that house. Even if the homeowner does not plan to sell the house, collateral requirement on mortgages are still relevant and the owner would want to reduce the risk of big changes in value for the house.

One way to do this is to introduce a housing index, *X(t)*. This index can be used to sell and buy housing futures and the home owner can reduce the exposure to the housing market by selling futures since this is the opposite position as owing a house. If we use a 5-year hedging horizon, the financial situation for the home owner will be the following:

$$S(i, t + 5) - Q[X(t + 5) - F(t, t + 5)], \qquad \text{(Eq.1)}$$

where *F(t,t+5)* is the 5-year futures at time t and *Q* is the NOK amount per index point.

For this hedge to minimize the variance, Q needs to be equal to the conditional β of *S(i,t+5)* on *X(t+5)*, i.e. the conditional covariance divided by the conditional variance of *X(t+5)*. Since we do not have these futures available, we will instead use the four different hedging strategies described by Schorno et al. (2014). We will also introduce different price groups to see if there are any differences in hedging based on the value of the house.

We will evaluate the effectiveness of hedging house price risk with housing index that is constructed as the arithmetic repeat-sales price index. The methodology of constructing such an index is described in Shiller (1991). In addition, we will construct a simpler geometric repeat-sales price index following the procedure described in Bailey, Muth, & Nourse (1963). The difference between arithmetic and geometric indices is that the first one is value-weighted (i.e. this index is affected by the change in value of the most expensive houses), while the second one treats all houses equally.

Finally, we will examine the effectiveness using median house price index published at SSB to see whether it is possible to effectively hedge house price risk without using complicated procedures for construction of repeat-sales index.

From Ambita we got an Excel sheet with sales data for over 400.000 house sales that included price, the date of the sale and a unique identification number for each house.

Before starting the construction of repeat-sales indices, we performed basic data cleaning procedures:

1) firstly, we dropped all observations for houses that was sold for 0 NOK;
2) secondly, we dropped too frequent selling observations and left only 1 sale for each house per quarter;
3) thirdly, we dropped 1st and 100th percentile of observations based on the selling price in order to exclude too cheap houses (which are most likely to be sold within one family) and the most expensive house (which have rather unique housing characteristics);
4) finally, we dropped all observations for houses that was sold only once during 1993-2017.

After the data cleaning, we ended up with 336 244 observations with 116 577 houses sold more than once.

Geometric Repeat Sales House Price Index

Bailey, Muth, & Nourse (1963) were the first ones to introduce the repeat-sales methodology for house price index estimation. The biggest problem with construction of housing index is the high cross-time variation inside the sample of the sold houses (e.g. not all houses sold in 1993 are also sold in 1994, which means that the index estimated for 1994 does not show accurate price change of real-estate property within a year). Therefore, to see how house prices actually change over time, Bailey et al. (1963) decided to look just at the first and the second sale prices of the same houses (assuming that the quality of the house does not change between sales).

First, we define the matrix of independent variables Z, where Z_{it} equals

a) -1 if house *i* was first sold at time *t;*

b) 1 if house *i* was sold for the second time at time *t*;

c) zero otherwise.

If houses were sold more than twice, we have multiple sales pairs for the same house, e.g. if the house is sold three times, we have first and second sales, and then second and third sales as two separate pairs.

Then we define the vector of dependent variable Y, where

$$Y_{it} = \ln(P_{it^*}) - \ln(P_{it}), \qquad \text{(Eq.2)}$$

where P_{it} is the price of house *i* at time *t*. *t* denotes the time of first sale and t^* is the time of the second sale.

Next, we define vector

$$\gamma = (Z'Z)^{-1}Z'Y, \qquad \text{(Eq.3)}$$

and each element of the vector γ is equal to the logarithm of house price index at time *t*:

$$\gamma_t = \ln(Index_t) \qquad \text{(Eq.4)}$$

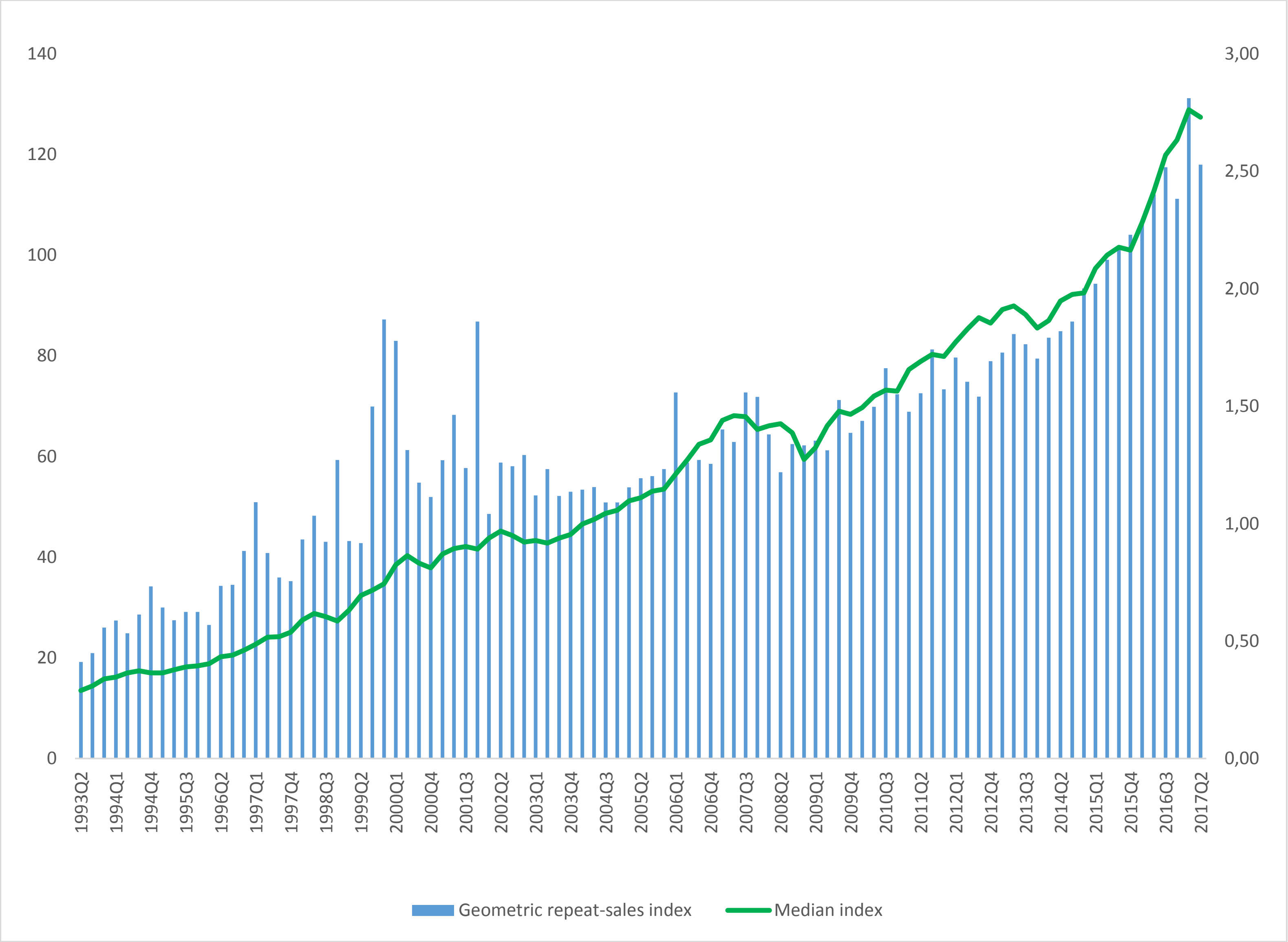
Finally, we calculate the house price index with the formula

$$Index_t = \exp(\gamma_t) \qquad \text{(Eq.5)}$$

Because we take the exponential of the estimated index, the index proposed by Bailey et al. (1963) is called the geometric repeat-sales house price index.

After implementing the above-mentioned procedures, we have calculated the geometric repeat-sales index (see Figure 1, on the right-hand side axis it is shown the evolution of geometric index over time, while left-hand side axis is used to compare geometric index with median index taken from SSB).

Figure 1 Geometric Repeat-Sales Index



Arithmetic Repeat Sales House Price Index

In geometric repeat-sales index, we do not use the prices of the houses directly (they only appear in matrix Y in the form of the change in price from period *t* to period t^*). Therefore, geometric index is an equally-weighted index (i.e. it treats all the houses equally). An alternative interpretation of the index is that it represents the value of the portfolio that consists of houses with equal weights invested in each house. Shiller (1991) decided to construct a value-weighted housing index, which will "represent" the portfolio of houses with more expensive house receiving more money. Shiller (1991) also does not use logarithms of house prices, which is why his index is called an arithmetic repeat-sales house price index. These differences between the arithmetic and geometric repeat sales house price index is also the reason for the big difference in how the indexes look and how they differ from median house price index.

The methodology of constructing such an index is as follows. First, we define the matrix of instrumental variable Z, where Z_{it} equals

a) -1 if house *i* was first sold at time *t;*

b) 1 if house *i* was sold for the second time at time *t*;

c) zero otherwise.

If houses were sold more than twice, we have multiple sales pairs for the same house. If it is sold three times, we have first and second sales, and second and third sales as two separate pairs.

Then we define matrix of independent variables X and the vector of dependent variable Y, where

$$X_{it} = P_{it}Z_{it}\ for\ t = 1,2, \dots, n \quad \text{(Eq.6)}$$

$$Y_{it} = P_{it}Z_{it}\ for\ t = 0, \quad \text{(Eq.7)}$$

where P_{it} is the price of house *i* at time *t*.

Next, we define vector

$$\gamma = (Z'X)^{-1}Z'Y, \quad \text{(Eq.8)}$$

and each element of the vector γ is equal to the reciprocal house price index at time *t*:

$$\gamma_t = \frac{P_0}{P_t} \qquad \text{(Eq.9)}$$

Finally, we calculate the house price index with the formula

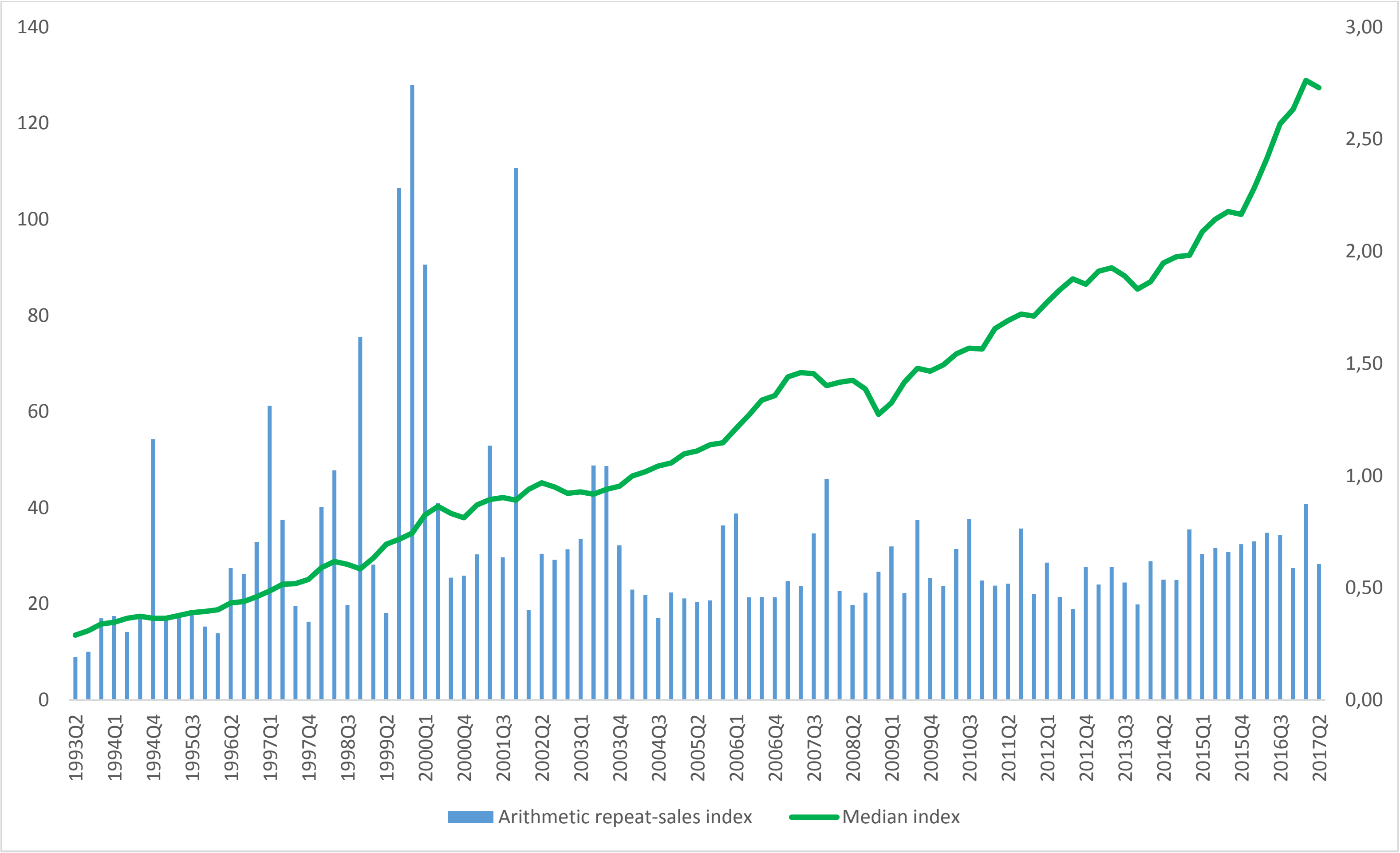
$$Index_t = \frac{1}{\gamma_t} \qquad \text{(Eq.10)}$$

Figure 2 shows how arithmetic repeat-sales index evolves over time (right-hand side axis). As we can see, the arithmetic house price index shows that house prices spiked in 1990s and the beginning of 2000s, but the increase was mostly driven by spike in prices for expensive houses. Nowadays, however, the prices for expensive houses remain approximately the same as in 1993, which causes arithmetic house price index to stay at almost the same level, while median index increases over time.

A short example of how to construct both geometric and arithmetic indices is given in Appendix.

Finally, the median index is the average price change of all houses sold in that period. The index uses a hedonic method to adjust for size, location, type of house and age of the house to make sure it compares the same type of housing portfolio from period to period.

Figure 2 Arithmetic Repeat-Sales Index



Hedging Strategies

To estimate the hedging effectiveness, we will start by running a fixed-effects panel data linear regression. The aim of this regression is to estimate the hedging position (beta in the regression) that should be taken by the houseowner to effectively hedge his house price risk. The estimated beta will be later used for all of our 4 hedging strategies (discussed below) with β measured in different ways, and we will do this individually for all four price groups of houses (see Table 1). The regression we use is:

$$S_{it} = \alpha + \beta F_t + \varepsilon_t, \qquad \text{(Eq.11)}$$

where S_t is the return earned by the houseowner through the sale of the house *i* at quarter *t* (see Equation 12 below),

F_t is the return on house price index (either median, arithmetic or geometric) at quarter *t*.

α is the constant regression parameter,

β is the regression slope coefficient for the risk minimizing hedge,

ε_t is the error term.

We use returns on three different indices (median, arithmetic, and geometric) to see which index performs the best as the underlying for housing futures.

On the left-hand side, we use actual returns on the house earned by each homeowner in our dataset. We calculate actual returns with the formula:

$$S_t = \ Return_{it^*} = \left(\left(\frac{Sale\ \ price_{it^*}}{Sale\ price_{it}} \right)^{\overline{Number\ of\ quarters\ between\ the\ sales}^{\,1}} \right) - 1, \qquad \text{(Eq.12)}$$

where *t* represents the quarter of the first sale of house *i* and t^* represents the quarter of the second sale of house *i*.

F_t is calculated by comparing the value of indices (either median, arithmetic, or geometric) in the consecutive quarters:

$$F_t = \frac{Index_t}{Index_{t-1}} - 1 \qquad \text{(Eq.13)}$$

Table 1 Price groups' range

Price group	***Price range, NOK***	***Number of observations***
Cheap	[200 000; 2 000 000]	147 136
Medium	(2 000 000; 5 000 000]	140 954
Nice	(5 000 000; 10 000 000]	30 598
Expensive	(10 000 000; 100 000 000]	17 556
Total		*336 244*

Since we do not have housing futures in Oslo, we will assume, similar to Bertus et al. (2008) and Schorno et al. (2014), that returns on housing index are the good proxy for housing futures returns.

Based on equation 11, we will estimate the effectiveness of four hedging strategies for all four price groups:

1) a simple naïve strategy with β equals to 1 during the entire life of the hedge. We assume that the hedger has long position on his house; therefore, he should take the opposite (short) position on futures contracts;
2) a static minimum variance hedge with β as the position in the housing index and this position does not change during the hedge horizon;
3) rollover minimum variance hedge;
4) a rollover conditional OLS strategy.

Since the average period between house sales is 6 years, first, we will take the hedging horizon of 5 years (as in Bertus et al. (2008)), and then we will expand the hedging horizon to 6 years[1].

We will use the estimate of hedging effectiveness equal to

$$\frac{return_h}{return_u} - 1, \qquad \text{(Eq.14)}$$

$$return_h = \sum_{t=1}^{n=20}(\overline{S}_t - (\frac{Index_t}{Index_{t-1}} - 1)) \text{ for strategy 1,} \qquad \text{(Eq.15)}$$

[1] the results obtained with the 6-year hedge horizons show the same pattern as the results presented in Section 5. Therefore, we decided to exclude the analysis for 6-year hedge periods from the text of this paper

$$return_h = \sum_{t=1}^{n=20}(\overline{S_t} + \beta(\frac{Index_t}{Index_{t-1}} - 1)) \text{ for strategy 2,} \quad \text{(Eq.16)}$$

$$return_h = \sum_{t=1}^{n=20}(\overline{S_t} + \beta_t(\frac{Index_t}{Index_{t-1}} - 1)) \text{ for strategy 3,} \quad \text{(Eq.17)}$$

$$return_h = \sum_{t=1}^{n=20}(\overline{S_t} + \beta_t^*(\frac{Index_t}{Index_{t-1}} - 1)) \text{ for strategy 4,} \quad \text{(Eq.18)}$$

$$return_u = \sum_{t=1}^{n=20}(\overline{S_t}), \quad \text{(Eq.19)}$$

where $return_h$ is the return of the hedged portfolio,

$return_u$ is the return of the unhedged portfolio,

$\overline{S_t}$ is the average of returns for all houses sold in quarter *t* (as defined in Equation 12),

$Index_t$ is house price index (either median, arithmetic, or geometric) value in quarter *t*,

$t = 1$ is the first quarter of the 5-year hedging period,

$n = 20$ is the last quarter of the hedging period,

β is estimated using Equation 11,

β^* is estimated using Equation 27 (see below).

Our unhedged portfolio consists of the house, and the hedged portfolio has house plus futures contracts. We decided to include this measure to see whether hedging can increase returns and attract speculators to trading with housing futures.

In addition, we will examine whether hedging reduces the volatility of the returns. To do so, we will also calculate the measure of hedging effectiveness that was also used by Schorno et al. (2014):

$$1 - \frac{\sigma_h^2}{\sigma_u^2}, \quad \text{(Eq.20)}$$

$$\sigma_h^2 = Var(\overline{S_t} - \left(\frac{Index_t}{Index_{t-1}} - 1\right)) \text{ for strategy 1,} \quad \text{(Eq.21)}$$

$$\sigma_h^2 = Var(\overline{S_t} + \beta\left(\frac{Index_t}{Index_{t-1}} - 1\right)) \text{ for strategy 2,} \quad \text{(Eq.22)}$$

$$\sigma_h^2 = Var(\overline{S_t} + \beta_t\left(\frac{Index_t}{Index_{t-1}} - 1\right)) \text{ for strategy 3,} \quad \text{(Eq.23)}$$

$$\sigma_h^2 = Var(\overline{S_t} + \beta_t^* \left(\frac{Index_t}{Index_{t-1}} - 1\right)) \text{ for strategy 4,} \quad \text{(Eq.24)}$$

$$\sigma_u^2 = Var(\overline{S_t}), \quad \text{(Eq.25)}$$

where σ_h^2 is the variance of the hedged portfolio,

σ_u^2 is the variance of the unhedged portfolio,

$\overline{S_t}$ is the average of returns for all houses sold in quarter *t* (as defined in Equation 12),

$Index_t$ is house price index (either median, arithmetic, or geometric) value in quarter t,

$t \in [1; 20]$ covers 20 quarters of the 5-year hedge horizon,

β is estimated using Equation 11,

β^* is estimated using Equation 27 (see below).

Our unhedged portfolio consists of the house, and the hedged portfolio has house plus futures contracts.

We define measures of hedging effectiveness (Equations 14 and 20) in the way that positive values are associated with successful hedge, while negative measures indicate the failure of hedging at the particular hedge period.

Strategy 1: a simple naïve strategy

We use the panel regression introduced earlier (equation 11) and make some simple assumptions. Our regression is:

$$S_{it} = \alpha + \beta F_t + \varepsilon_t,$$

In this strategy we have the following assumptions:

a) the hedge horizon is equal to 5 years. We estimate hedging effectiveness first for period 1993Q3-1998Q2 and then for each subsequent hedge period we move one quarter ahead (e.g. the second period is 1993Q4-1998Q3);
b) we set β equal to 1 for the whole 5 years.

For each hedge horizon, we calculate the total return on median, arithmetic, and geometric indices, i.e. the total return equals to the sum of all returns on the corresponding index during the hedge horizon of 5 years. The return of the unhedged portfolio is the average actual return for the corresponding period.

Then, we calculate the return on hedged portfolio (we subtract the return on median, arithmetic or geometric indices from the unhedged return (see equation 15). Since the hedger owns the house, he wants to give away the risk of price decline. Therefore, the hedger shorts housing futures, meaning that the position on the contracts becomes negative).

For each hedge horizon, we calculate the hedged returns and variance of returns inside the horizon (since we have a 5-year hedging horizon, we have 20 quarters with hedged returns). After that, we estimate the hedging effectiveness.

Strategy 2: a static minimum variance hedge

Assumptions for this strategy are:

a) Hedging horizon is 5 years;
b) We still use the same regression as before (equation 11) but now β is calculated using 5-year out-of-sample data, and then we use this estimated beta for subsequent 5-year hedge horizon (i.e. we estimate beta using data from 1993Q3 to 1998Q2, and then use this beta as the position in futures for next 5-year period from 1998Q3 to 2003Q2);
c) For each hedge horizon, we calculate the total return on median, arithmetic, and geometric indices (return on indices is multiplied by the estimated positions, see equation 16). The return of the unhedged portfolio is the average actual return for the corresponding period;
d) Then, we calculate the return on hedged portfolio (we add the return on median, arithmetic or geometric indices to the unhedged return);
e) For each hedge horizon, we calculate the hedged returns and variance of returns inside the horizon. After that, we estimate the hedging effectiveness.

Strategy 3: rollover minimum variance strategy

Similar to the static minimum variance strategy the rollover minimum variance strategy also uses five years of data from just prior to the hedge horizon to construct the minimum variance hedge ratio to be used in hedging. However, the rollover strategy uses this hedge ratio only for the first quarter of the hedge and then rolls forward to the next successive window to estimate the hedge ratio for the second quarter of the hedge. As an example, if we want to look at hedging in the period 2011q1 to 2015q4, we first use data from 2010q4 back to 2006q1 to find the hedge ratio for 2011q1 then roll over and find hedge ratio for 2011q2 using the period 2011q1 back to 2006q2 and so on. Following a rollover strategy like this means you constantly rebalance and should maintain a more optimal hedge ratio throughout the hedging period. One additional risk with a rollover strategy is roll-over risk, the risk of rolling over at an unfavorable price.

Strategy 4: rollover conditional OLS strategy

We follow Miffre (2004) when implementing the conditional OLS strategy and like them assume a linear relationship between β_t and a set of mean zero information variables V_{t-1} which are available at time *t-1*. We have the following specification for β_t:

$$(\beta_t|V_{t-1}) = \beta_0 + \beta_0 V_{t-1}, \qquad \text{(Eq.26)}$$

where β_0 is the mean hedge ratio and $\beta_1 V_{t-1}$ is the deviation from β_0 as new information is known in the market, measured through the information variables. If we substitute the time dependent $(\beta_t | V_{t-1})$ into equation 11 we get the following formula:

$$S_{it} = \alpha + \beta_0 F_t + \beta_1 V_{t-1} F_t + \varepsilon_t, \qquad \text{(Eq.27)}$$

where S_{it} and F_t are identical to those in Equation 11. We then see that if there is no new or meaningful information in the market at time *t*, the vector of parameters β_1 is jointly equal to zero and the conditional OLS reduces to the traditional OLS model we see in equation 11.

Our set of information variables are based on Jacobsen & Naug (2004), where they find (1) interest rates, (2) unemployment, (3) household wages, and (4) new housing built to be the most important factors in Norway to drive housing prices.

4. Data Review

In order to follow the methodology from "Hedging house price risk with futures contracts after the bubble burst" by Schorno, Swidler, & Wittry (2014) we have collected house price data from Norwegian Statistical Bureau (Statistisk Sentralbyrå, 2017) and Ambita AS. The Norwegian Statistical Bureau is the national statistical institute of Norway and the main producer of official statistics. They are responsible for collecting, producing and communicating statistics related to the economy, population and society at national, regional, and local level. Ambita AS is a Norwegian technology company specialized in housing data and fully owned by The Ministry of Trade, Industry and Fisheries.

From SSB we got the house price index for Oslo (Statistisk Sentralbyrå, 2017) and this is the median index in our analysis, and from Ambita we got an Excel sheet with over 400.000 house sales price, the date of the sale and a unique identification number for each house so we could construct our repeat sales indices following Bailey et. al. (1963) and Shiller (1991). The returns on these indices (median index received from SSB and arithmetic and geometric indices constructed by us using data from Ambita) will then be the F_t in our regression (equation 11).

The house price index from SSB is a quarterly index with data going back to 1992q1 and the dataset from Ambita is 1993-2017. The fact that we have data going back 24 years gives us a better chance to see how the hedging effectiveness will be over time and especially in times of recession like the financial crises 2007-2009.

The quality of the data is also an important part of the analysis. Both "The Norwegian Statistical Bureau" and "Ambita AS" are fully owned by the government and provide official housing prices that are reported after a sale. This makes our results more trustworthy knowing that the underlying data is of high quality.

In the Rollover Conditional OLS Strategy we also need data about (1) interest rates, (2) unemployment, (3) household wages and (4) new housing built (information variables in vector V_{t-1} in Equations 26 and 27) since these are the most important factors to determine housing prices according to Jacobsen (2004). Data about interest rates in Norway is obtained from "Norges Bank" (Norges Bank, 2018) and goes back to 1991, updated monthly. The unemployment rate is downloaded from SSB (Statistik

Sentralbyrå, 2018) and is updated quarterly since 1997q1. Household wages are also downloaded from SSB, but there is only yearly statistic available in the period 1990-2016 (Statistisk sentralbyrå, 2018). Lastly statistic about new housing built is also found on SSB on a yearly basis from 2006-2017 (Statistik Sentralbyrå, 2018). The summary of all data sources we use in the paper and the application of the data is shown in Table 2.

Table 2 Data Sources

Data source	Application
SSB - Statistisk Sentralbyrå	• Median house price index, return on which is used as F_t in Equation 11 • Unemployment rate, which is used as an information variable in Equation 27 • Household wages, which are used as an information variable in Equation 27 • New housing built, which is used as an information variable in Equation 27
Ambita AS	• House prices necessary for construction of geometric and arithmetic house price index, returns on which are used as F_t in Equation 11
Norges Bank	• Interest rates, which are used as an information variable in Equation 27

In order to make our data comparable we need to convert all the data into the same time unit, and we have chosen the quarterly one. We already have the median house price index and unemployment rate on a quarterly basis but we need to convert interest rates, household wages, and housing built from yearly data to quarterly data. Since we have yearly nominal interest updated monthly from "Norges Bank" we will use this to go from yearly to quarterly. This is done by taking an average of the yearly rates for every month in the quarter, and then raising one plus this average rate to the power of ¼. For example, if interest rates in January, February, and March are 1.3%, 1.2%, and 1.5% respectively, the historic quarterly rate would be:

$$(1 + ((0{,}013 + 0{,}012 + 0{,}015)/3))^{0{,}25} = 1{,}0033 = 0{,}33\%.$$

For the household wages and housing build it is reasonable to believe that they, on average, will be linear through the year and we can therefore take the yearly figure and divide by four to go from yearly to quarterly data.

The descriptive statistics for house price indices, interest rates, unemployment rates, household after-tax income, and new housing built is presented in Tables 3 and 4. In addition, Figure 3 shows how house sales (the data we have obtained from Ambita) are distributed among years.

Table 3 Actual returns for different price groups

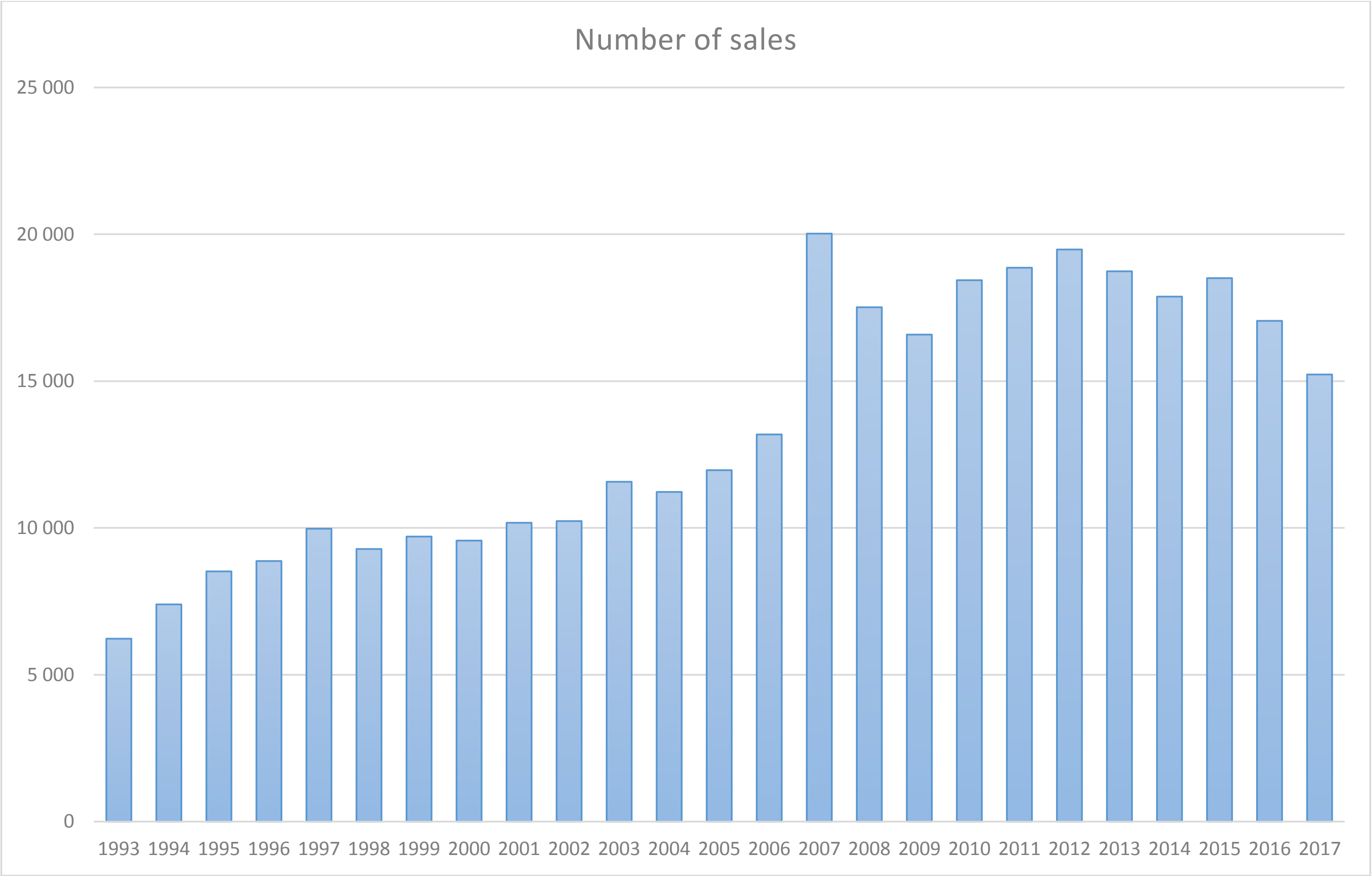
	Price group 1	Price group 2	Price group 3	Price group 4
Mean	0.008	0.10	0.10	0.24
Standard error	0.004	0.04	0.015	0.06
Median	0.002	0.04	0.06	0.09
Standard deviation	0.04	0.36	0.13	0.61
Kurtosis	6.33	75.69	42.47	32.10
Skewness	1.65	8.35	5.78	5.51
Minimum	-0.08	-0.02	-0.05	0.00
Maximum	0.19	3.35	1.13	4.08
Count	96	95	86	88

Note. Actual returns mean the average of returns for all houses sold in quarter t (as defined in Equation 12).

Table 4 Descriptive Statistics

	Return on median index	Return on geometric index	Return on arithmetic index	Interest rate, %	Unemployment rate, %	After-tax income, NOK	New housing built
Mean	0.02	0.03	0.16	3.95	3.59	391778	311776
Standard error	0.003	0.015	0.08	0.14	0.07	15148	3470
Median	0.02	0.02	0.001	3.50	3.50	373200	311214
Standard deviation	0.03	0.15	0.76	2.53	0.63	78709	12020
Kurtosis	0.43	3.49	17.70	-0.76	-0.73	-1.56	-1.12
Skewness	-0.07	0.78	3.62	0.50	0.10	0.17	-0.09
Minimum	-0.08	-0.44	-0.83	0.50	2.40	289500	292414
Maximum	0.11	0.63	4.87	10.87	4.90	508800	329358
Count	96	96	96	323	83	27	12

Figure 3 House sales distribution over years



5. Results of the Hedging

The Table 5 below shows for how many hedge horizons (measured as the percentage of total number of hedge periods) we obtain positive hedging effectiveness (measured with increased hedged returns and decreased variance of hedged returns) if we implement the hedge with housing futures written on either median, arithmetic or geometric indices. The presented measures of hedging effectiveness are calculated for 4 price groups of houses and for 4 hedging strategies.

Next, we will look in more details at the results of each hedging strategy.

Strategy 1: a simple naïve hedging

The graphs presented below (Figures 4-7) show how hedging effectiveness changes throughout different hedge horizons (put on x-axis, but not shown here). We separate the results for price groups and measures of hedging effectiveness. So, if we look at Figure 4, we can see that hedging reduces returns on the portfolio consisting of the cheapest houses (price group 1) and housing futures, and increases the variability of those returns, which taken together clearly indicates that our strategy fails to provide house owners with successful hedge. In addition, the hedging effectiveness (measured both with increased returns and decreased variance) reaches extremely low values for cheap houses (e.g. the value of "effectiveness variance" of -500 means that our hedge increases the variability of returns by 500% compared to the variability of returns without hedge).

The same pattern (but with less negative magnitude) is observed among other price groups: hedging fails to give more stable returns to the homeowners or, at least, increase the existing returns.

However, such bad results for Strategy 1 are expected, since naïve hedge does not account for correlation between S_t and F_t. What we can conclude now is that even though Strategy 1 is very simple and does not require sophisticated analysis or additional data for implementation, it is successful only in approximately 25% of cases (see Table 5) and only if the value of the house exceeds 5 millions NOK.

Table 5 Summary of hedging results

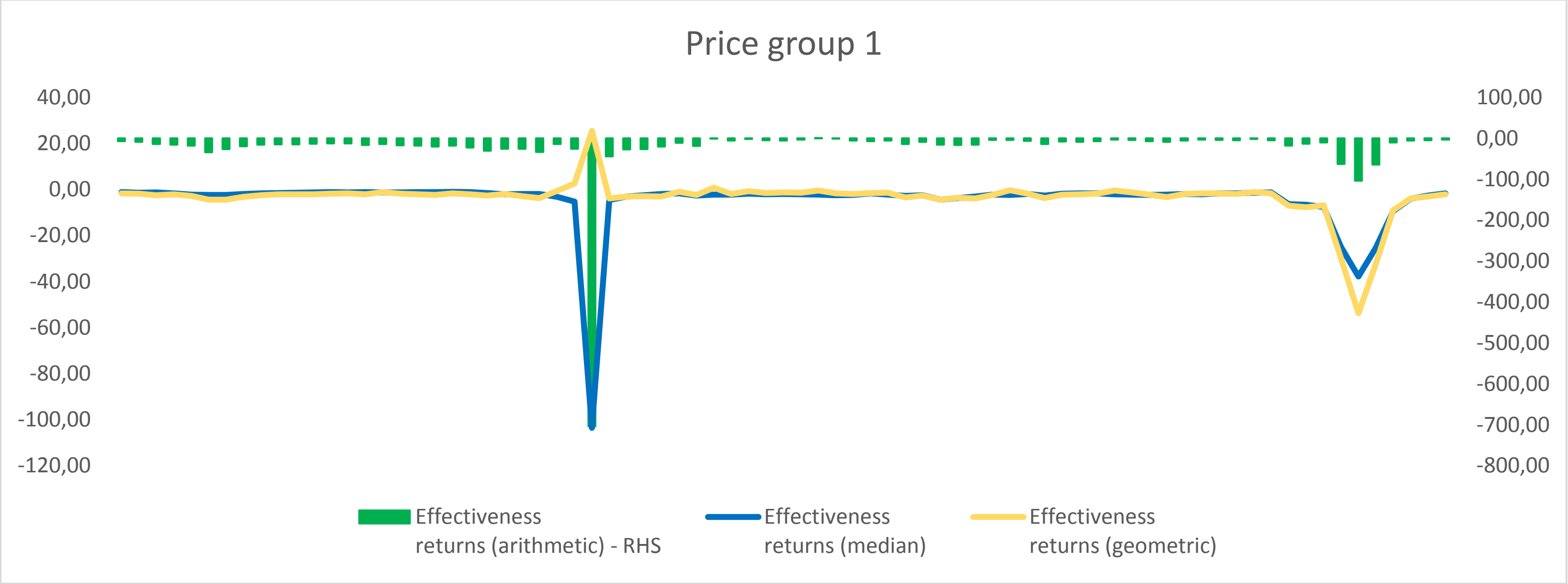
House price group	Median index		Arithmetic index		Geometric index	
	Effectiveness returns	Effectiveness variance	Effectiveness returns	Effectiveness variance	Effectivenes s returns	Effectiveness variance
Strategy 1. Number of hedge periods: 69						
Cheap	0,00%	1,30%	1,30%	0,00%	3,90%	0,00%
Medium	0,00%	1,32%	1,32%	0,00%	3,95%	1,32%
Nice	0,00%	23,88%	1,49%	0,00%	4,48%	1,49%
Expensive	0,00%	21,74%	1,45%	2,90%	4,35%	10,14%
Strategy 2. Number of hedge periods: 49						
Cheap	100,00%	14,04%	77,19%	31,58%	89,47%	3,51%
Medium	33,93%	33,93%	39,29%	37,50%	53,57%	37,50%
Nice	23,40%	17,02%	63,83%	14,89%	48,94%	6,38%
Expensive	8,16%	14,29%	81,63%	42,86%	44,90%	20,41%
Strategy 3. Number of hedge periods: 49						
Cheap	94,74%	0,00%	68,42%	29,82%	87,72%	0,00%
Medium	53,57%	23,21%	53,57%	7,14%	71,43%	3,57%
Nice	12,77%	0,00%	51,06%	2,13%	42,55%	0,00%
Expensive	4,08%	10,20%	71,43%	24,49%	28,57%	2,04%

Table 5 – continued

Strategy 4. Number of hedge periods: 5						
Cheap	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Medium	100,00%	0,00%	100,00%	0,00%	100,00%	0,00%
Nice	0,00%	0,00%	80,00%	0,00%	0,00%	20,00%
Expensive	100,00%	0,00%	0,00%	0,00%	40,00%	0,00%

Notes. “Effectiveness returns” is measured using Equation 14. “Effectiveness variance” is measured using Equation 20.

Figure 4 Hedging results of Strategy 1 for Price group 1



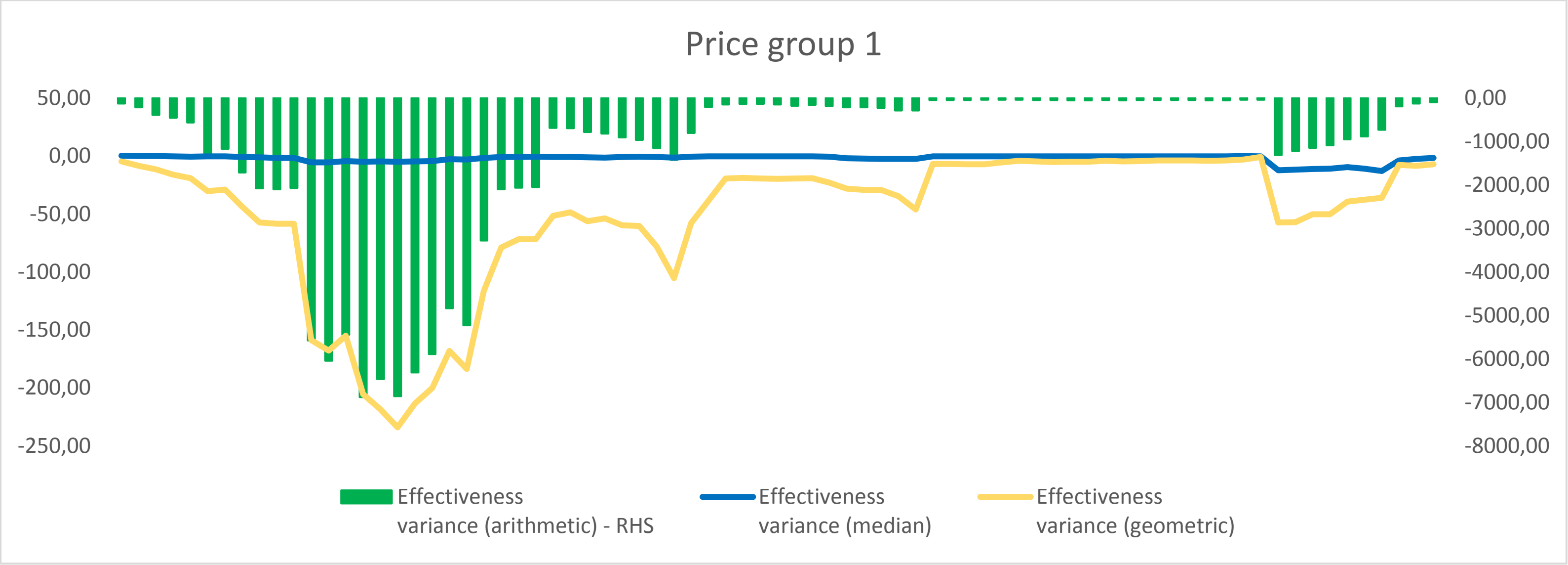
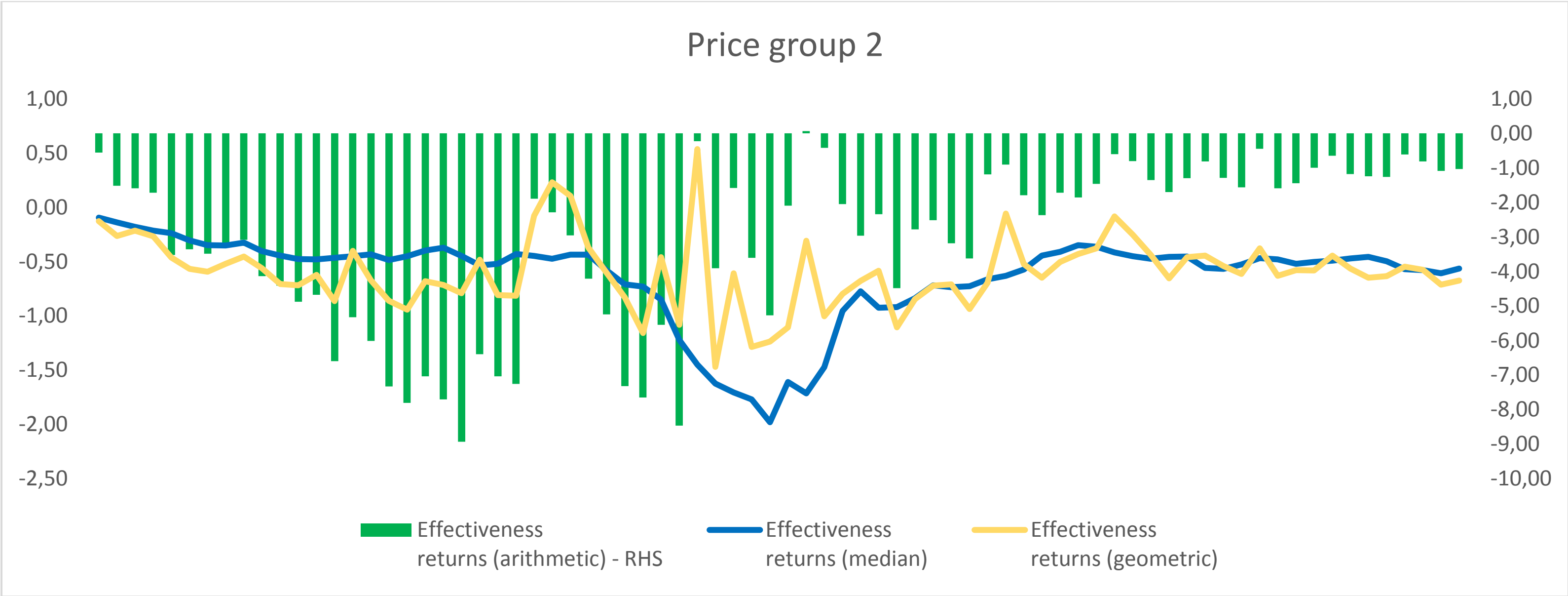


Figure 5 Hedging results of Strategy 1 for Price group 2



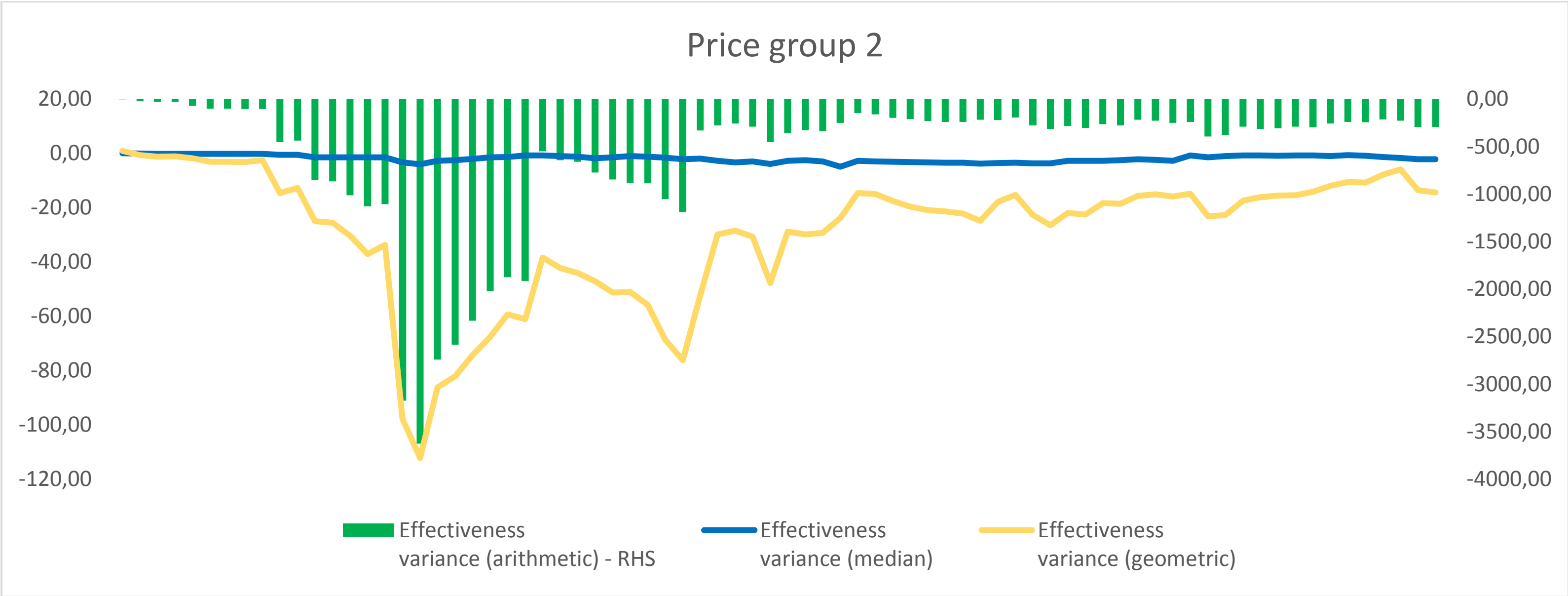
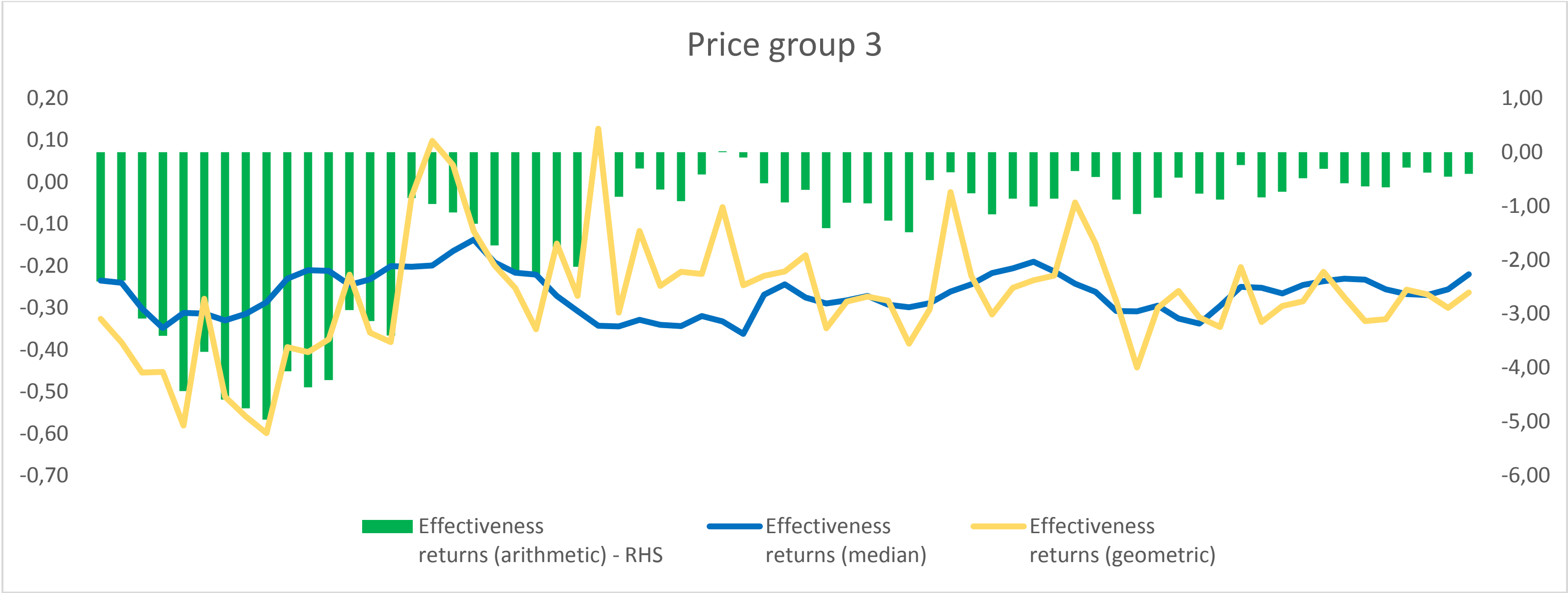


Figure 6 Hedging results of Strategy 1 for Price group 3



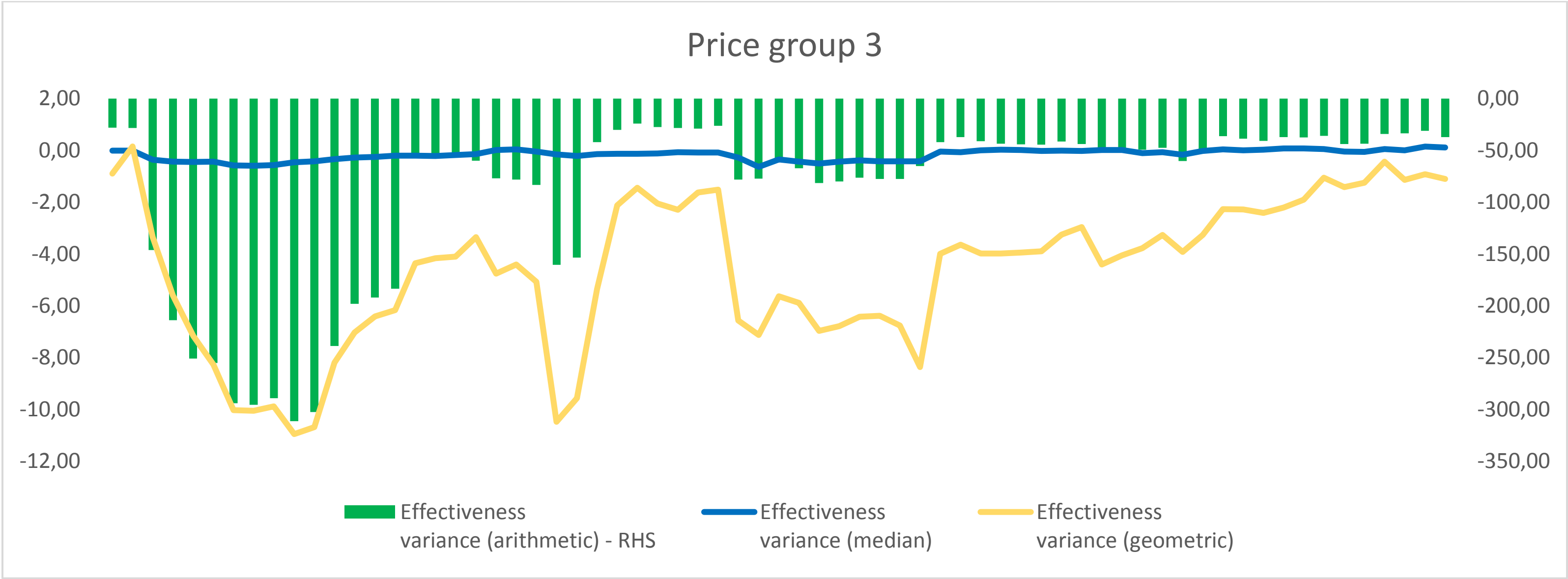
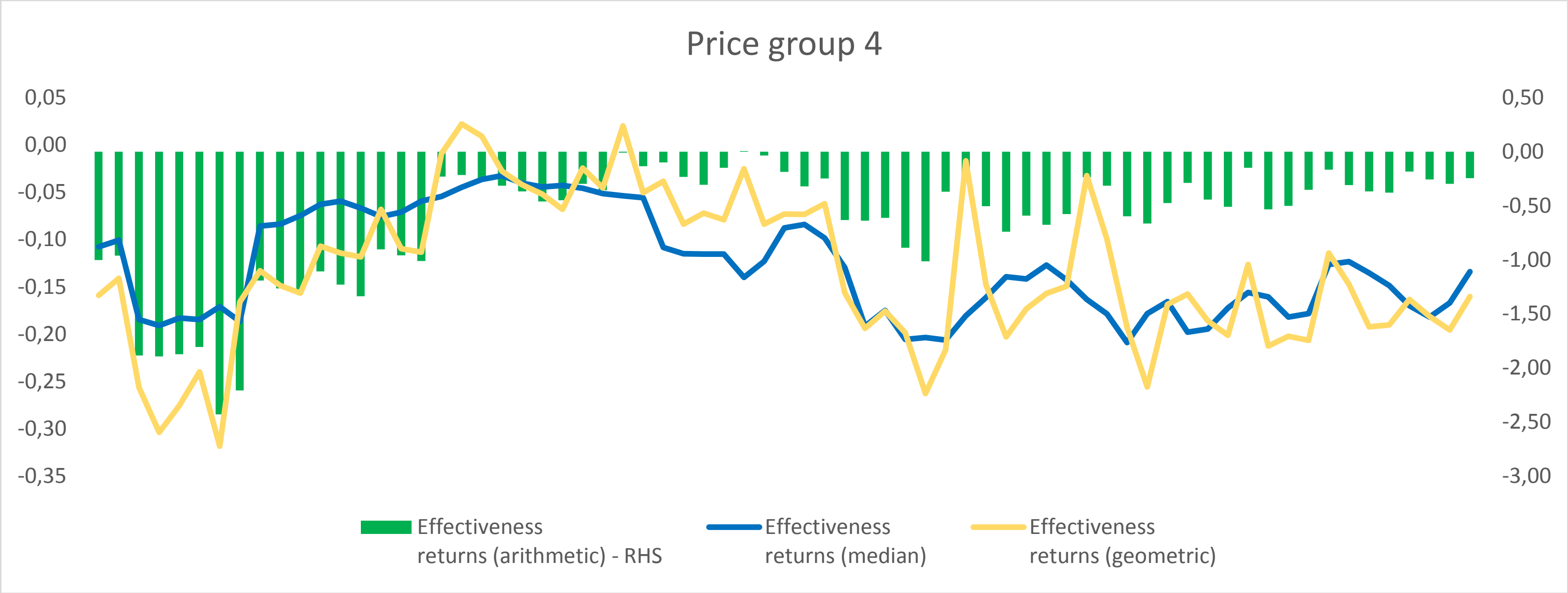
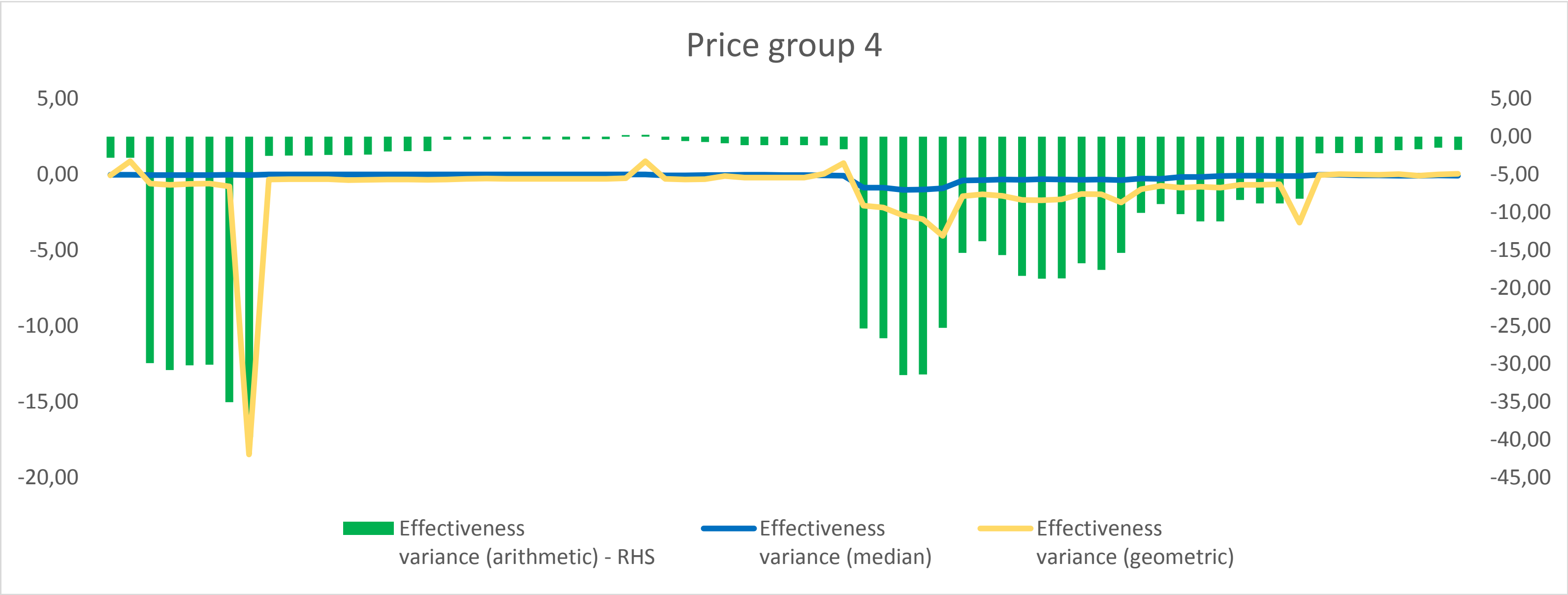


Figure 7 Hedging results of Strategy 1 for Price group 4





Strategy 2: a static minimum variance hedge

A static minimum variance strategy performs better than the naïve one (see Table 5 and Figures 8-11 below): hedging (with any index) increases the returns in more than 75% of cases for cheap houses and it is possible to increase returns in more than 50% of cases with at least one index used as the underlying for remaining house price groups. It should also be mentioned that hedging with arithmetic index is more effective for expensive houses compared to hedging with median or geometric index.

If we, however, look at how effective hedging is in reducing the variance of returns, the results, despite showing improvement compared to strategy 1, are still poor: the best we can achieve is the reduction of variance in approximately 43% of cases when hedging returns on expensive houses with arithmetic index.

The results of strategy 2 are now more consistent with famous risk-return trade-off: higher returns are normally associated with higher risk and variability of returns, which is what we observe now. However, a trade-off is clearly seen only for portfolios consisting of cheap houses and futures, while for portfolios with more expensive houses high volatility does not always come together with increased return. Therefore, now we will proceed to more sophisticated strategies.

Figure 8 Hedging results of Strategy 2 for Price group 1

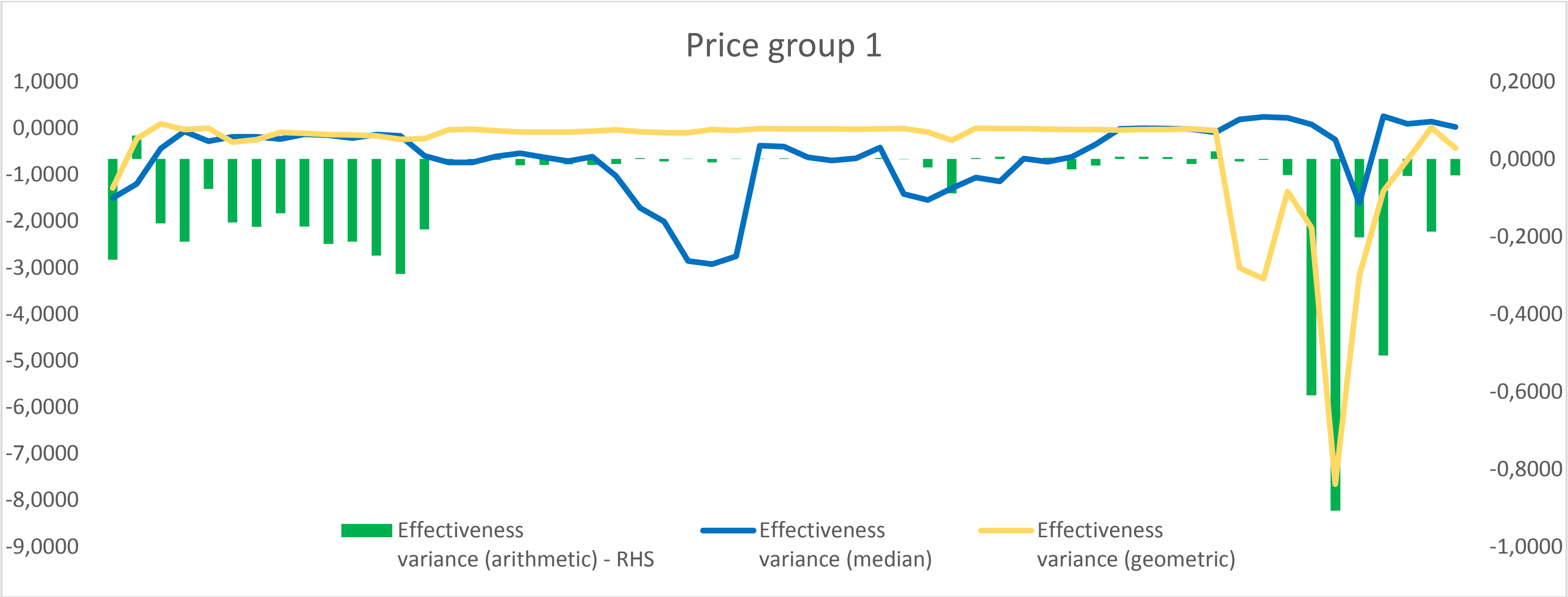
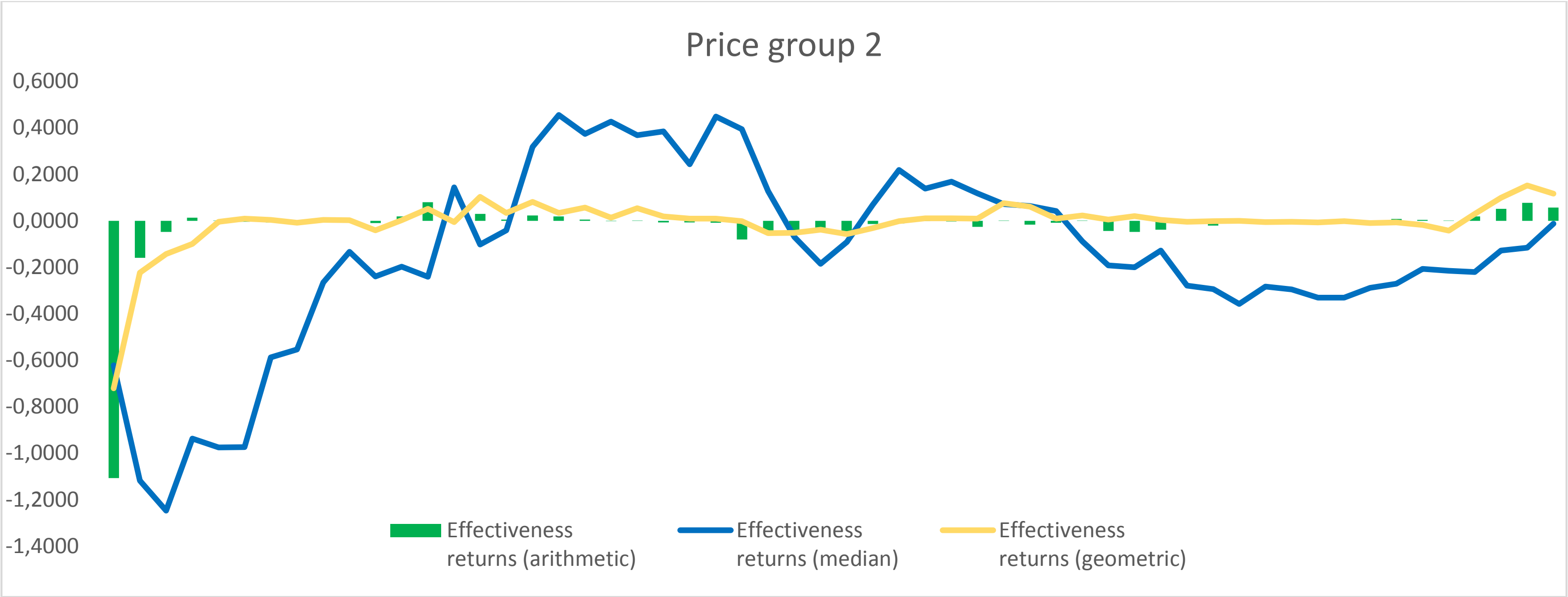


Figure 9 Hedging results of Strategy 2 for Price group 2



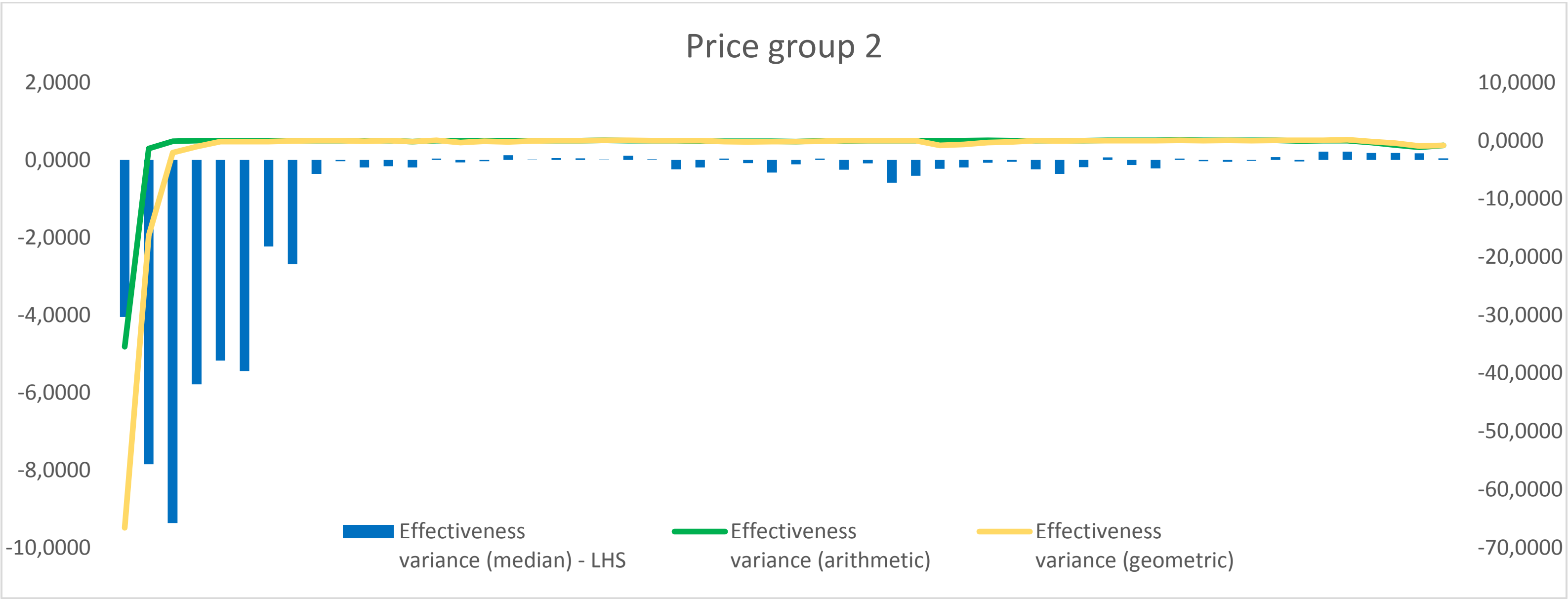
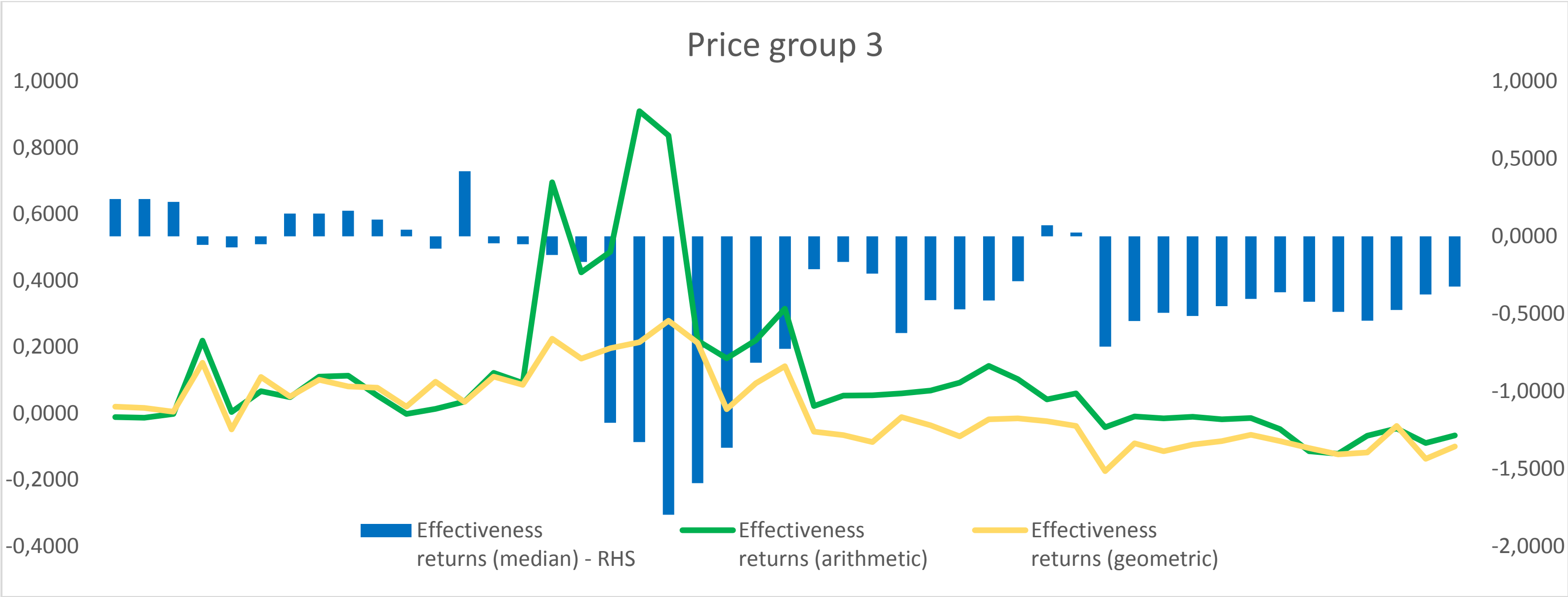


Figure 10 Hedging results of Strategy 2 for Price group 3



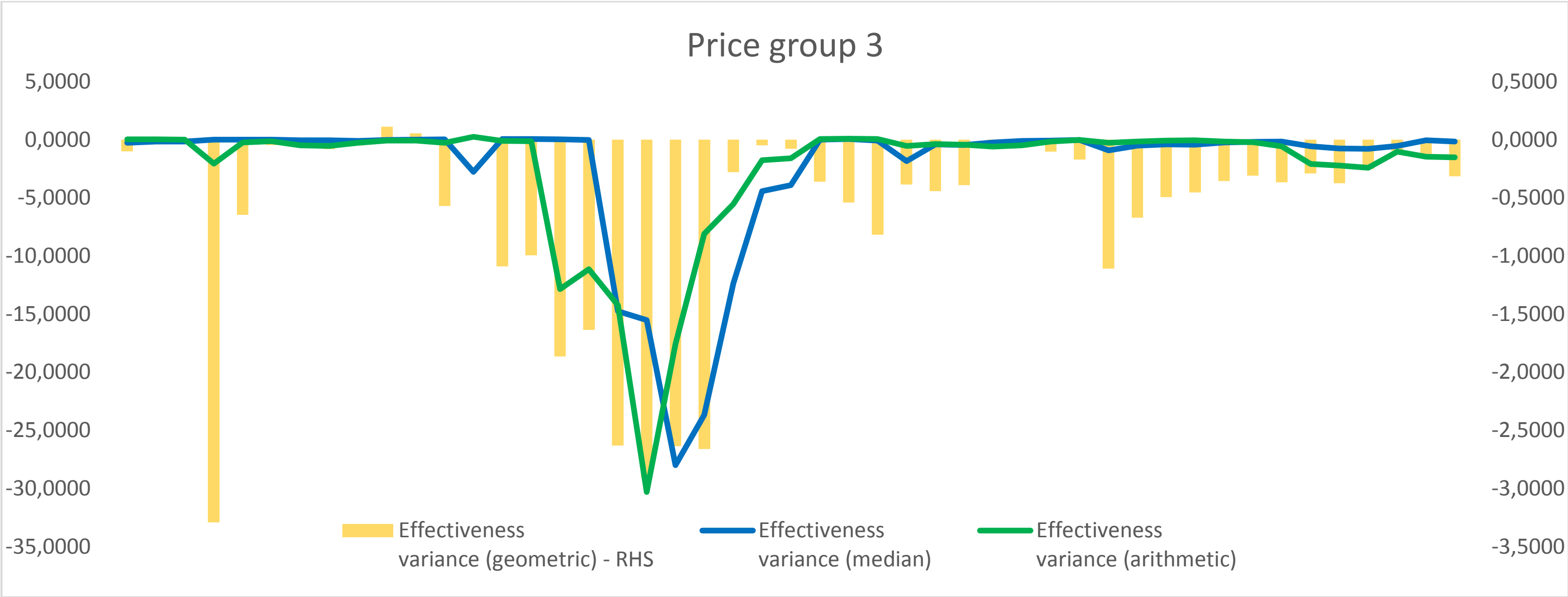
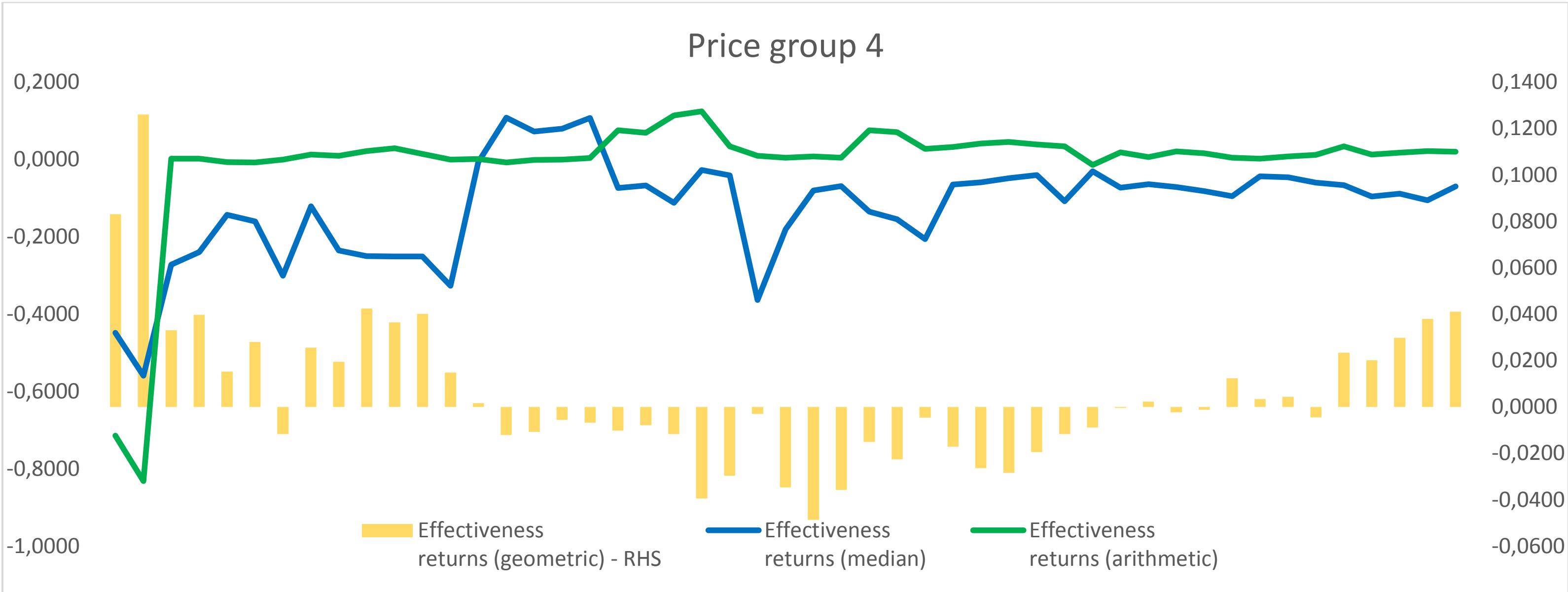
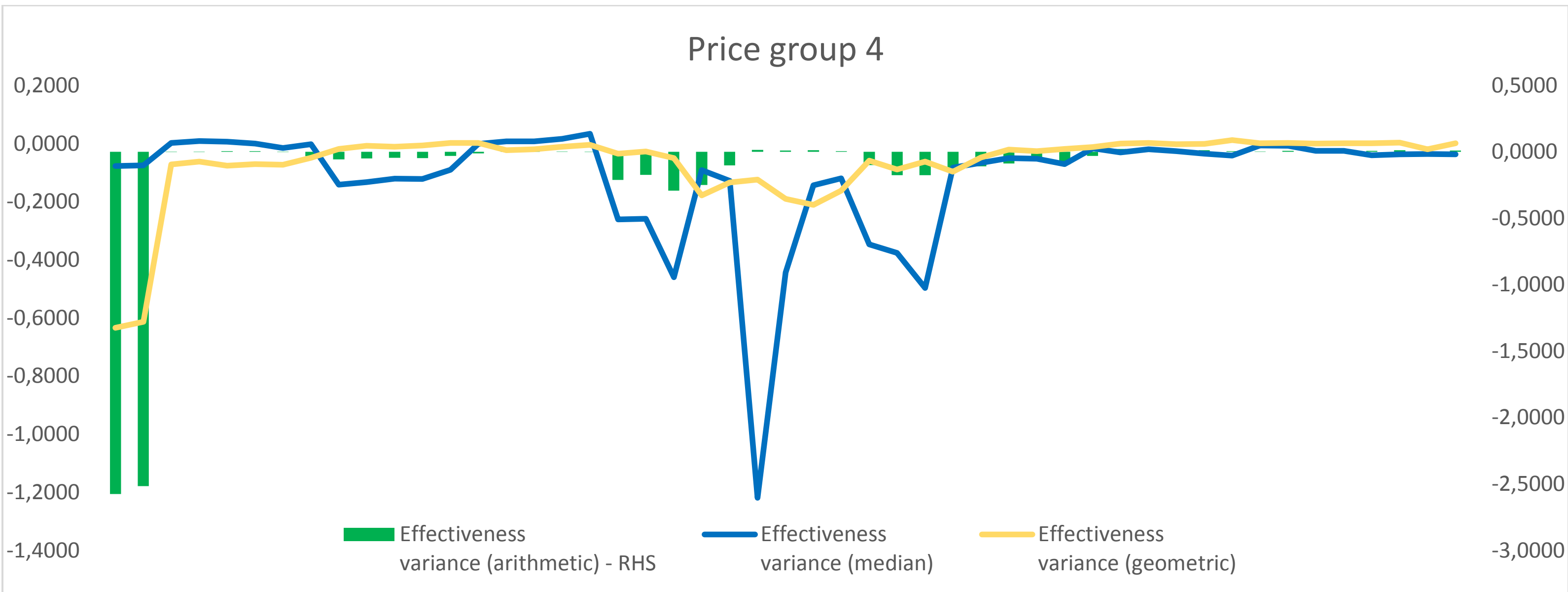


Figure 11 Hedging results of Strategy 2 for Price group 4





Strategy 3: rollover minimum variance strategy

In rollover minimum variance strategy, the hedging effectiveness (measured by increase in returns) is positive in more than 50% of cases (see Table 5 and Figures 12-15 below), which suggests that hedging increases the returns on portfolio and protects homeowners from the risk of negative returns. Hedging with arithmetic index gives more stable positive result compared to hedging with other indices.

When we look at hedging effectiveness measured by the reduction in variance, the results are worse than the results in strategy 2. This suggests that more sophisticated strategies of hedging do not guarantee a more stable return on the hedged portfolio.

An interesting pattern that we observe is that the performance of the median index is the worst among all three indices: hedging with median index extremely increases the volatility of returns without proportionally increasing the returns for the homeowners (except the cases when hedged portfolio includes cheap houses). The same pattern could be observed if we look at the results of Strategy 2; now, however, it is clearer.

In addition to bad performance of median house price index, we can also point at a sudden drop in hedging effectiveness (both measures) during last hedging horizons. The decline in hedging effectiveness takes place when we start to include 2006-2007 years into out hedging periods. This suggests that despite the dynamics and flexibility of rollover strategy, it fails to protect homeowners against sudden and significant drop in house prices (as happened during crisis 2007-2009). For house owners, who normally need hedge for long-term horizons, the constant change in hedging positions results in significant losses. During crisis times, situation on the housing market changes so fast that beta (which we calculate using past data) fails to measure the most optimal hedging position. Therefore, we can conclude that for house owners as for long-term hedgers the static strategy can be the most optimal among all strategies we have discussed so far.

Now we will move to the final strategy, which incorporates information variables in beta calculation. However, since we have less data available for those information variables, we can only estimate hedging effectiveness for 5 periods. The results we will present below should, therefore, be used for getting the overall picture of hedging performance, but not to be the solid basis for final conclusions.

Figure 12 Hedging results of Strategy 3 for Price group 1

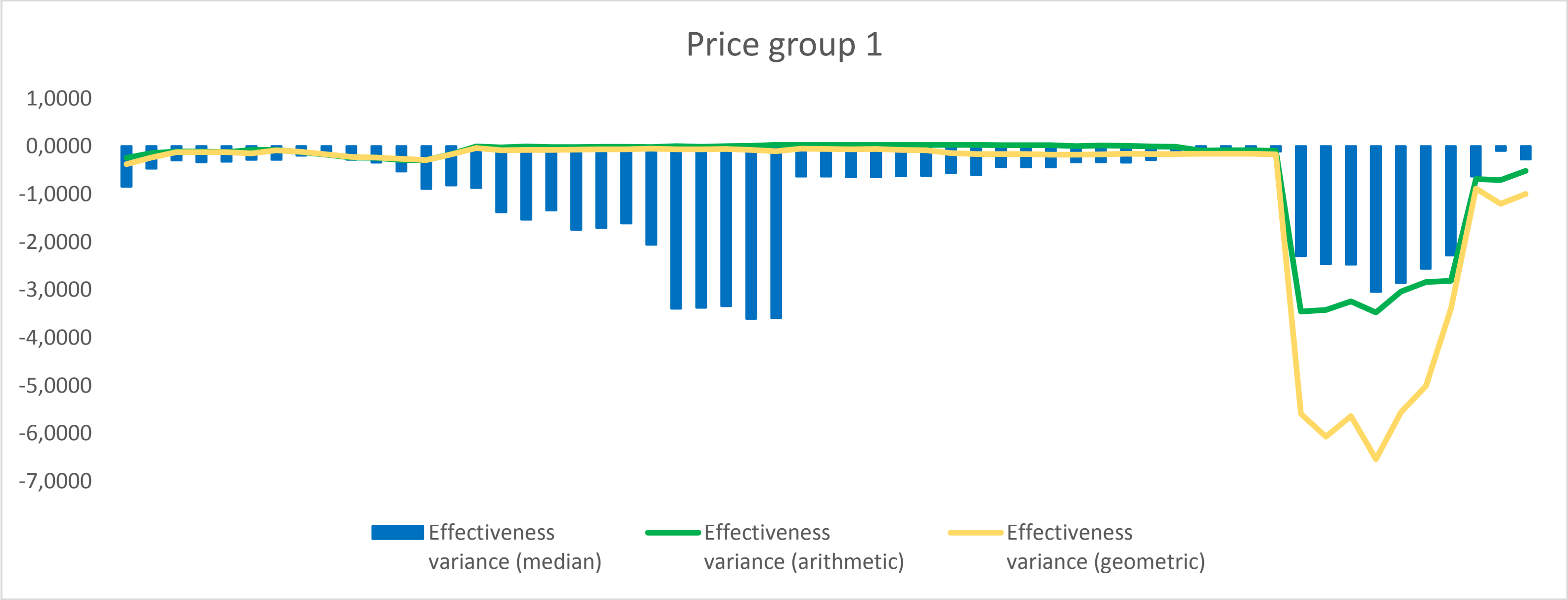
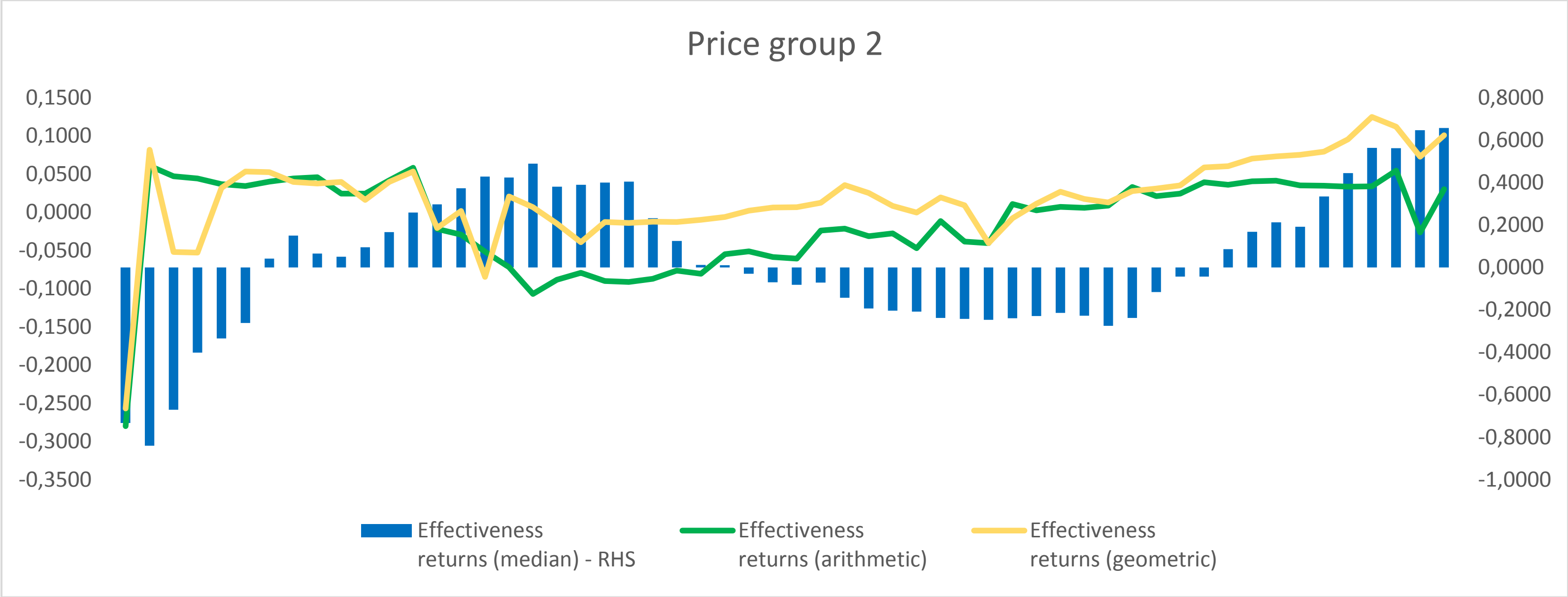


Figure 13 Hedging results of Strategy 3 for Price group 2



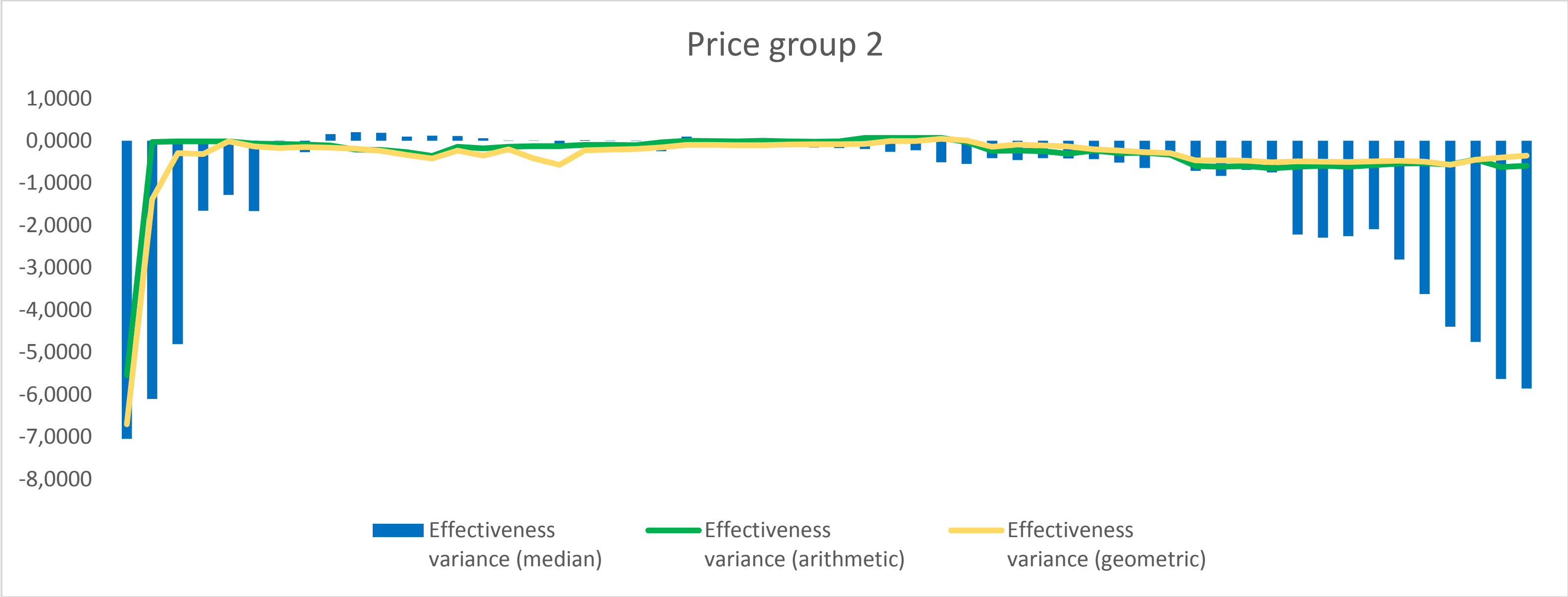
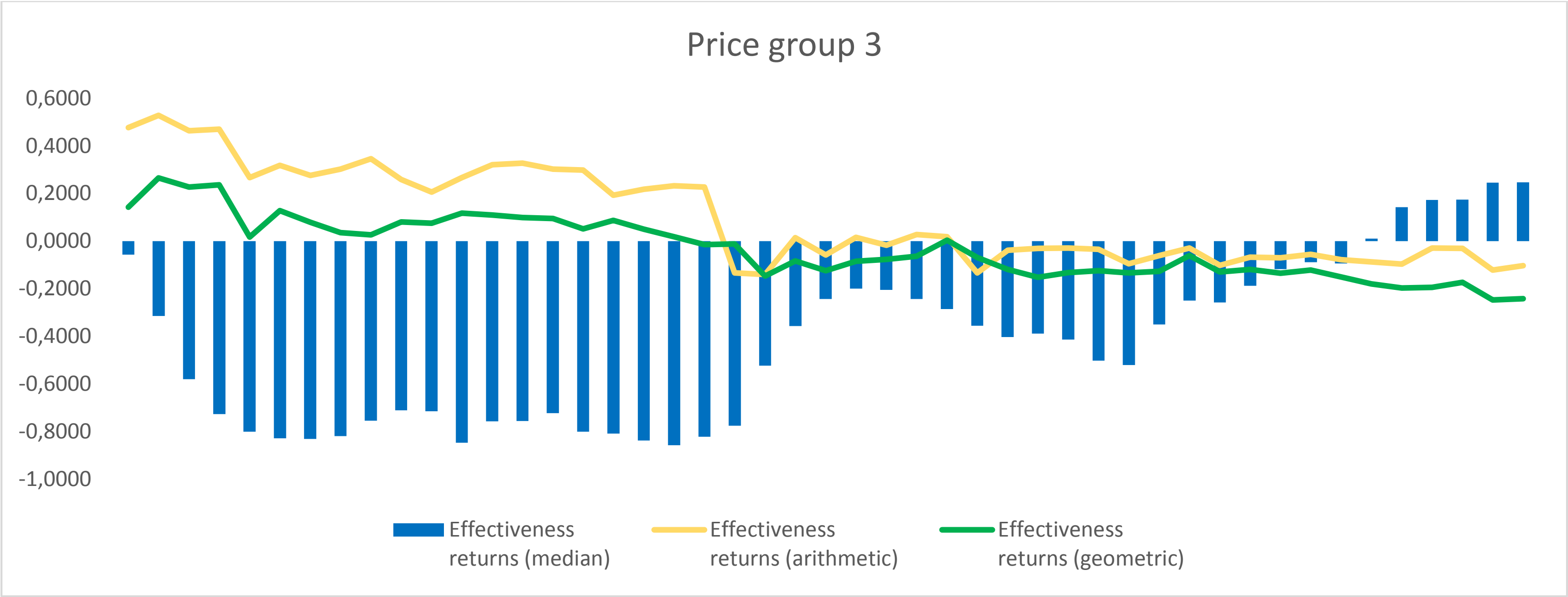


Figure 14 Hedging results of Strategy 3 for Price group 3



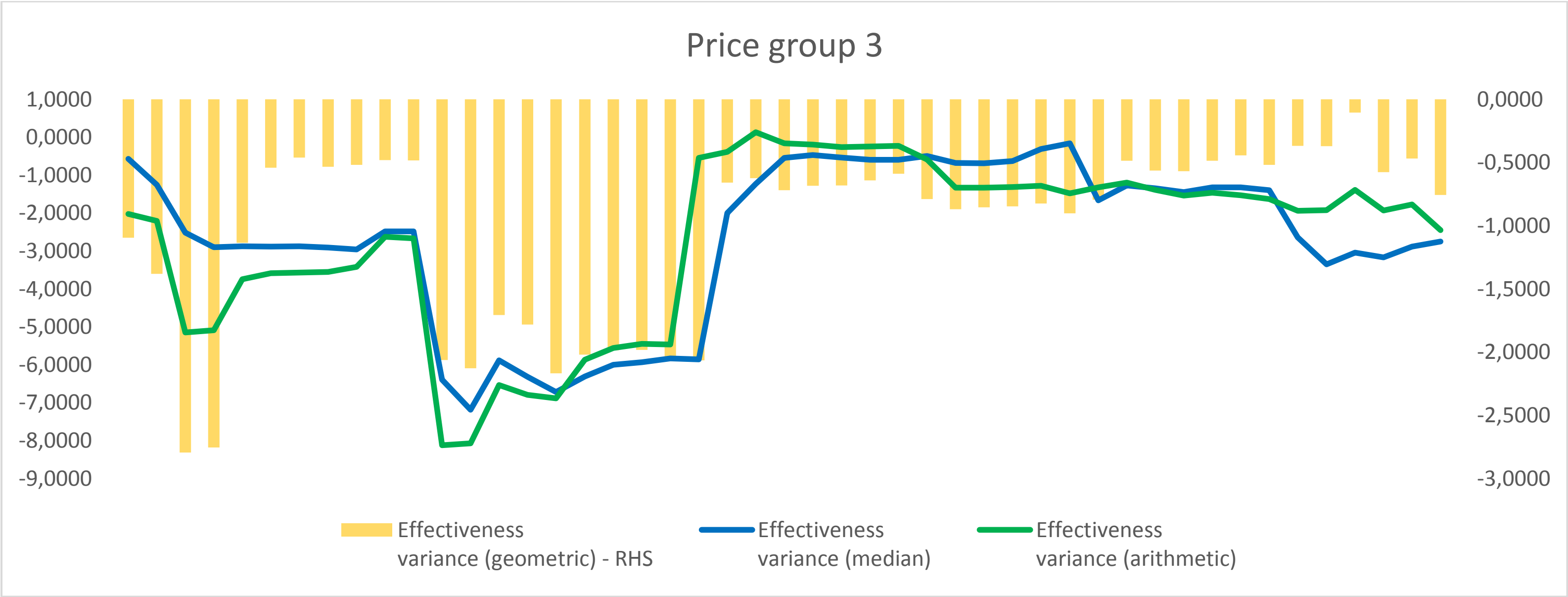
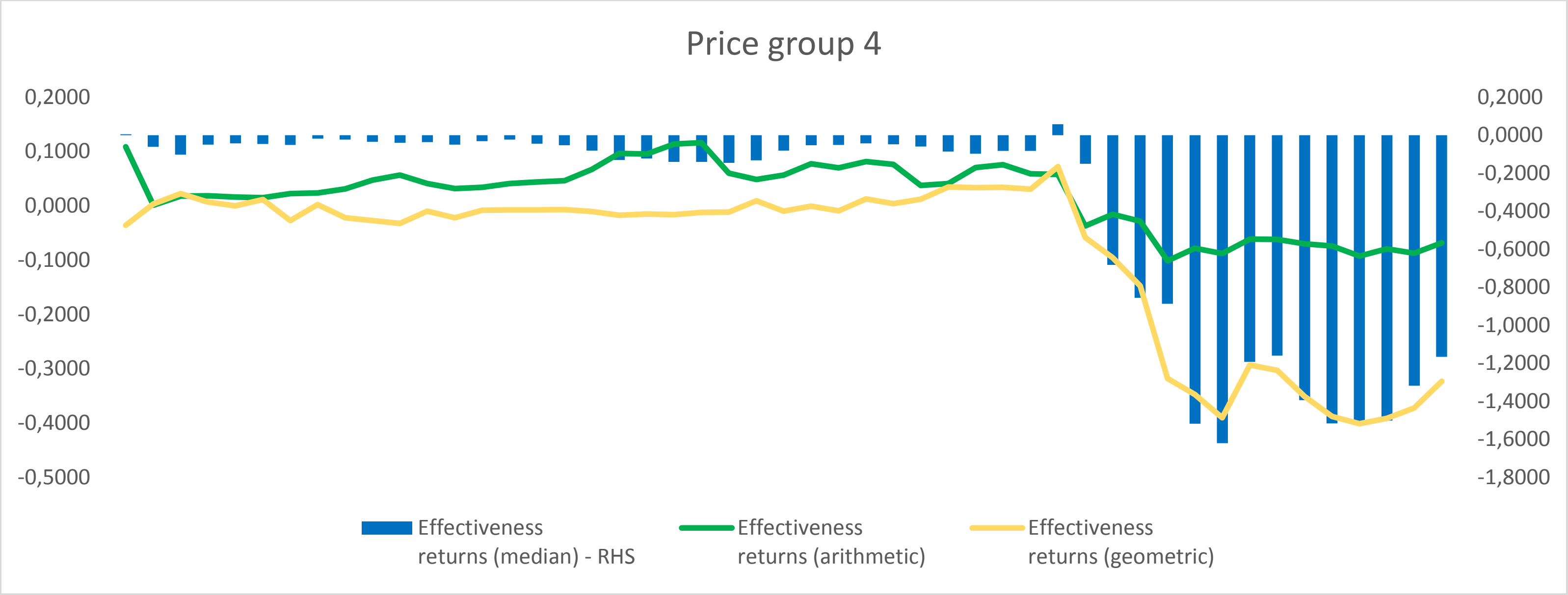
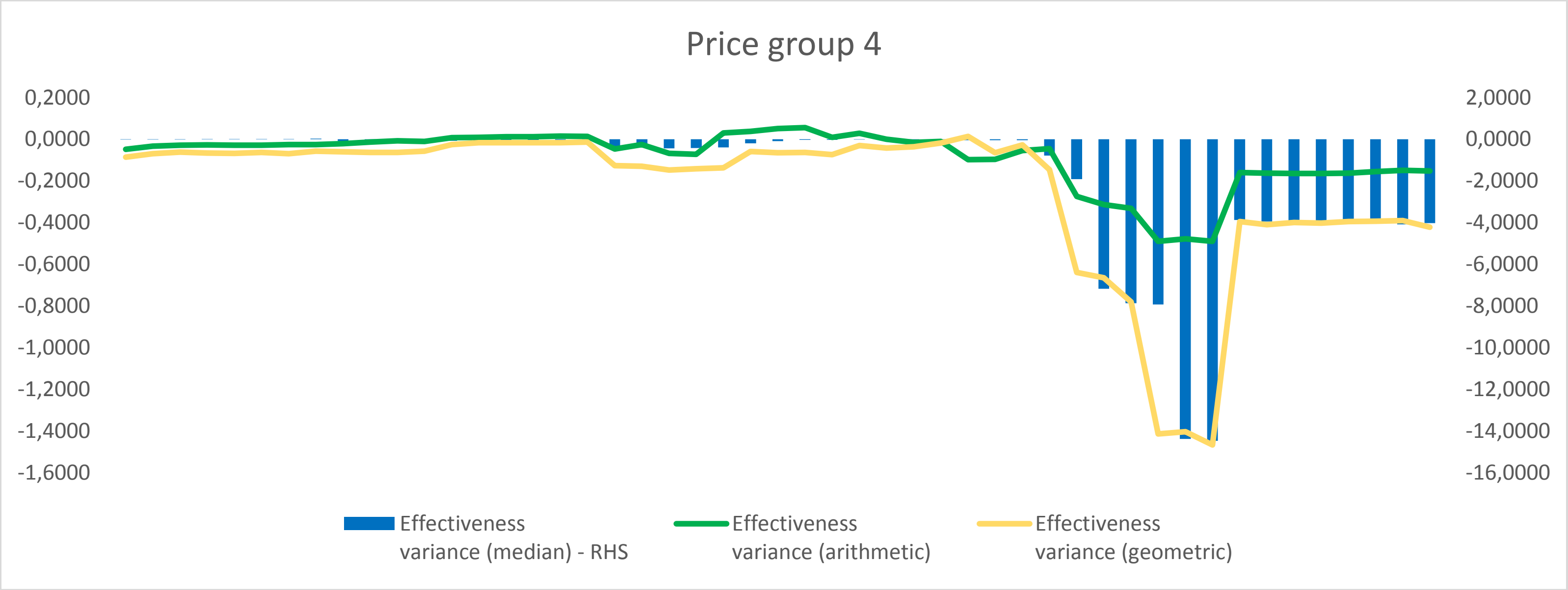


Figure 15 Hedging results of Strategy 3 for Price group 4

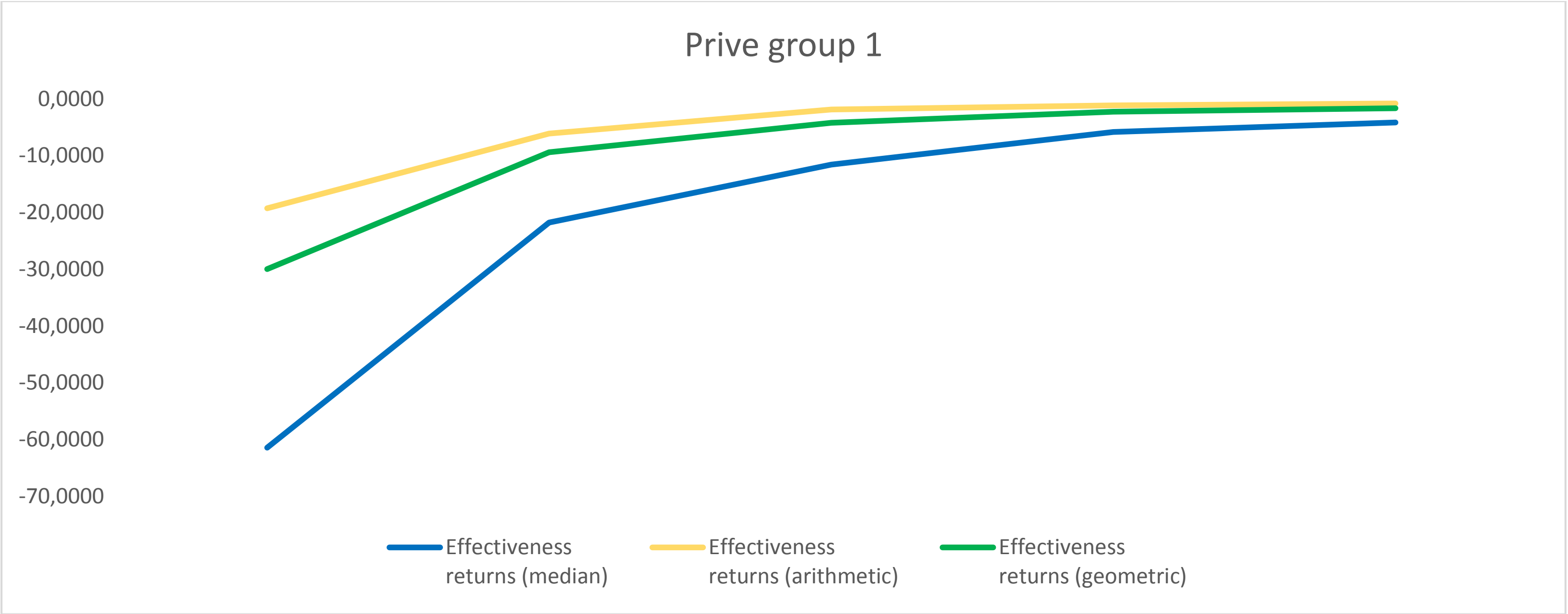




Strategy 4: rollover conditional OLS strategy

Due to less data available for information variables in Strategy 4, we can only estimate the hedging effectiveness for the period from 2011Q2 to 2017Q1 (this makes the total number of 5-year hedge periods equal to 5). Therefore, we will just point at the available pattern while implementing the hedge from strategy 4: hedging performs quite well in increasing the hedged returns (i.e. in Table 5, we notice a couple of times when we have positive hedging effectiveness in 100% of all 5 hedged periods analyzed in this strategy); but it underperforms in decreasing the variability of those returns. Similar to what we have seen in previous strategies, median index tends to show higher magnitudes in hedging effectiveness compared to arithmetic and geometric indices. However, this is mostly observed in median index's extremely bad performance in reducing volatility of hedged portfolio.

Figure 16 Hedging results of Strategy 4 for Price group 1



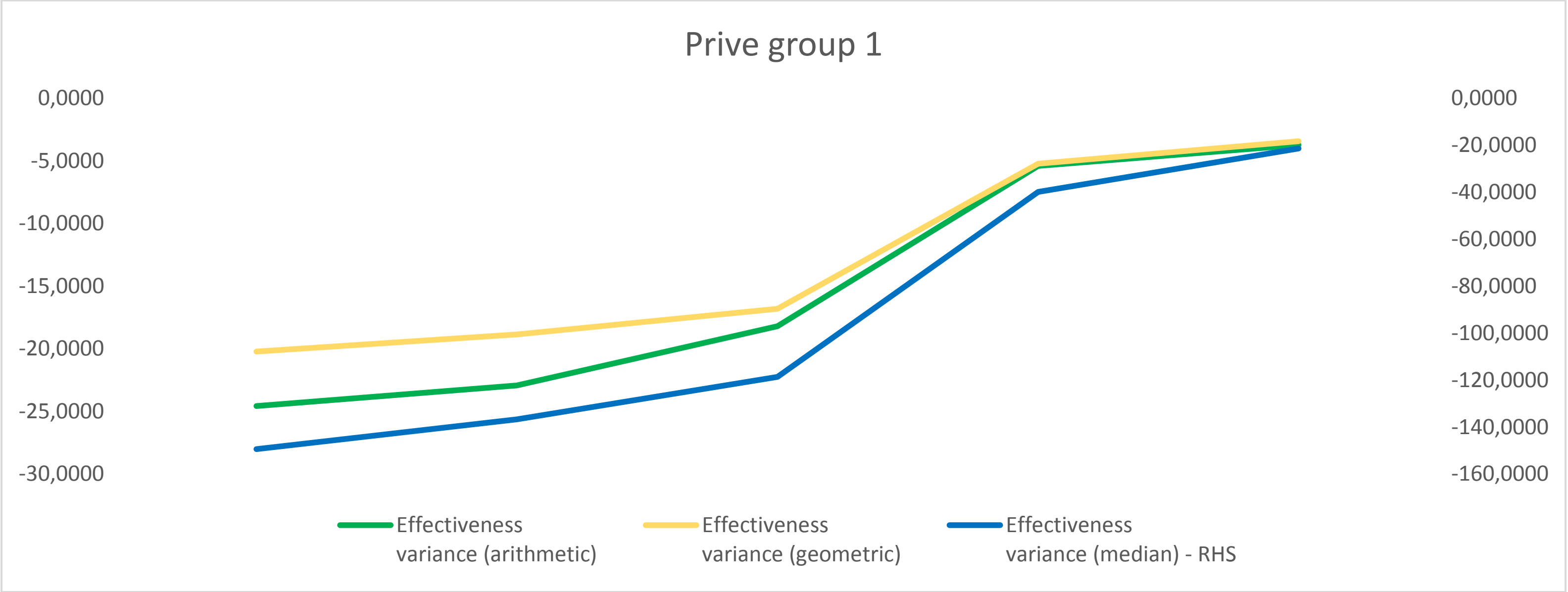
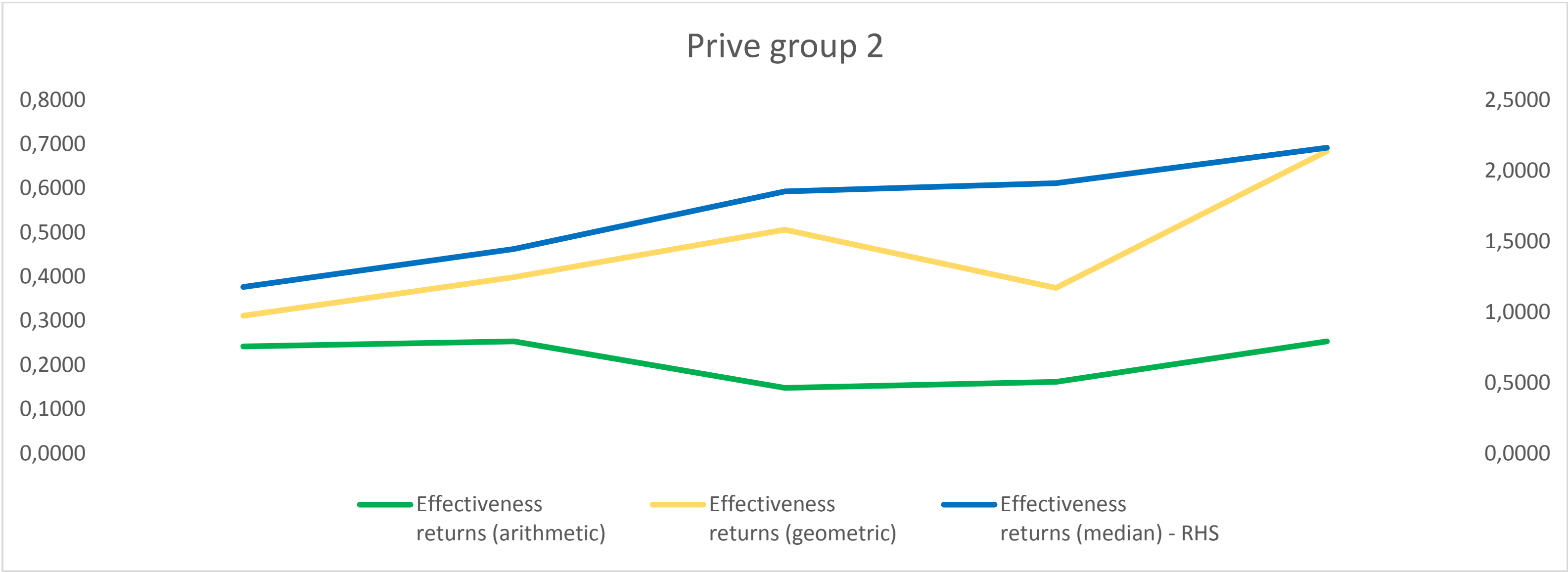


Figure 17 Hedging results of Strategy 4 for Price group 2



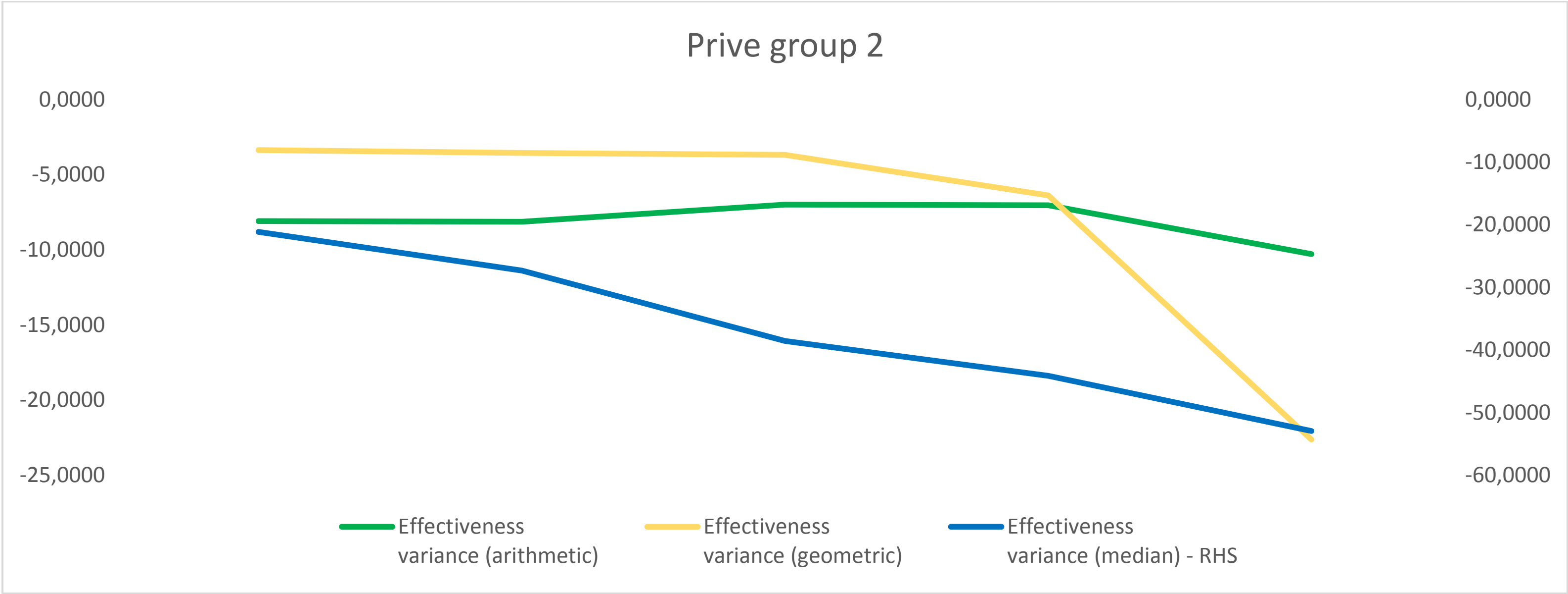
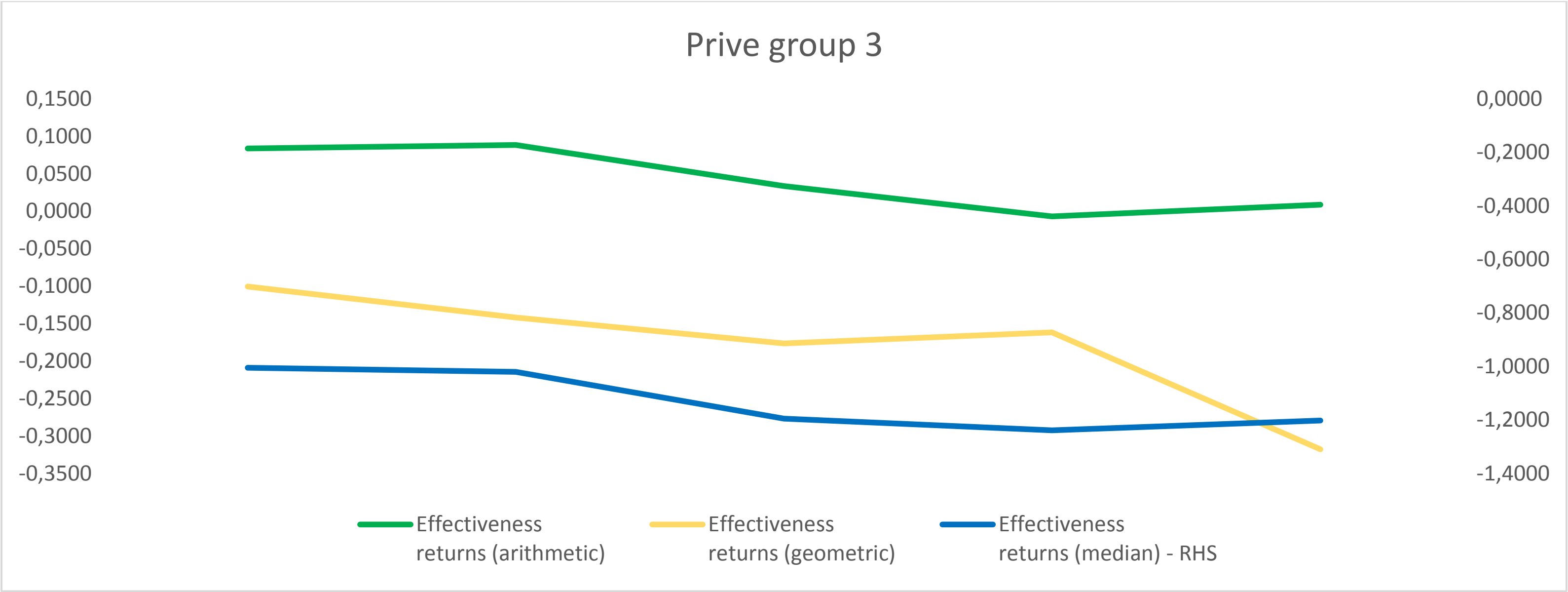


Figure 18 Hedging results of Strategy 4 for Price group 3



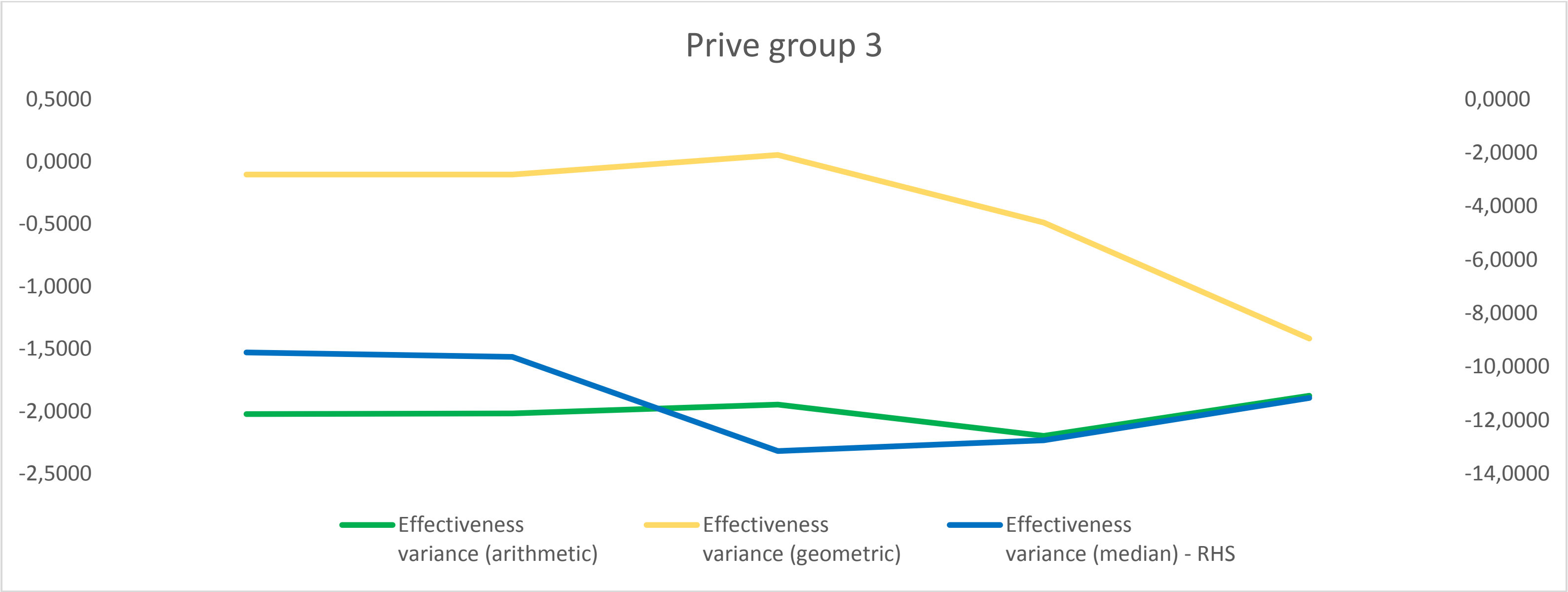
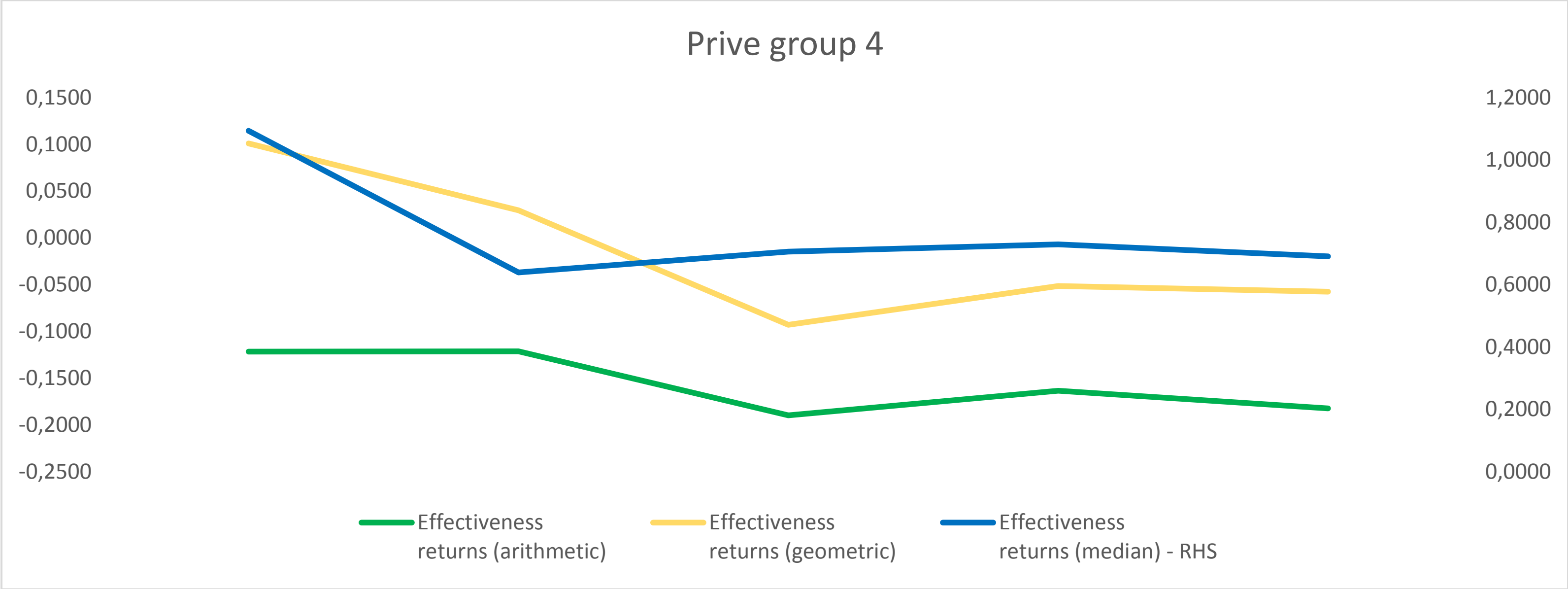
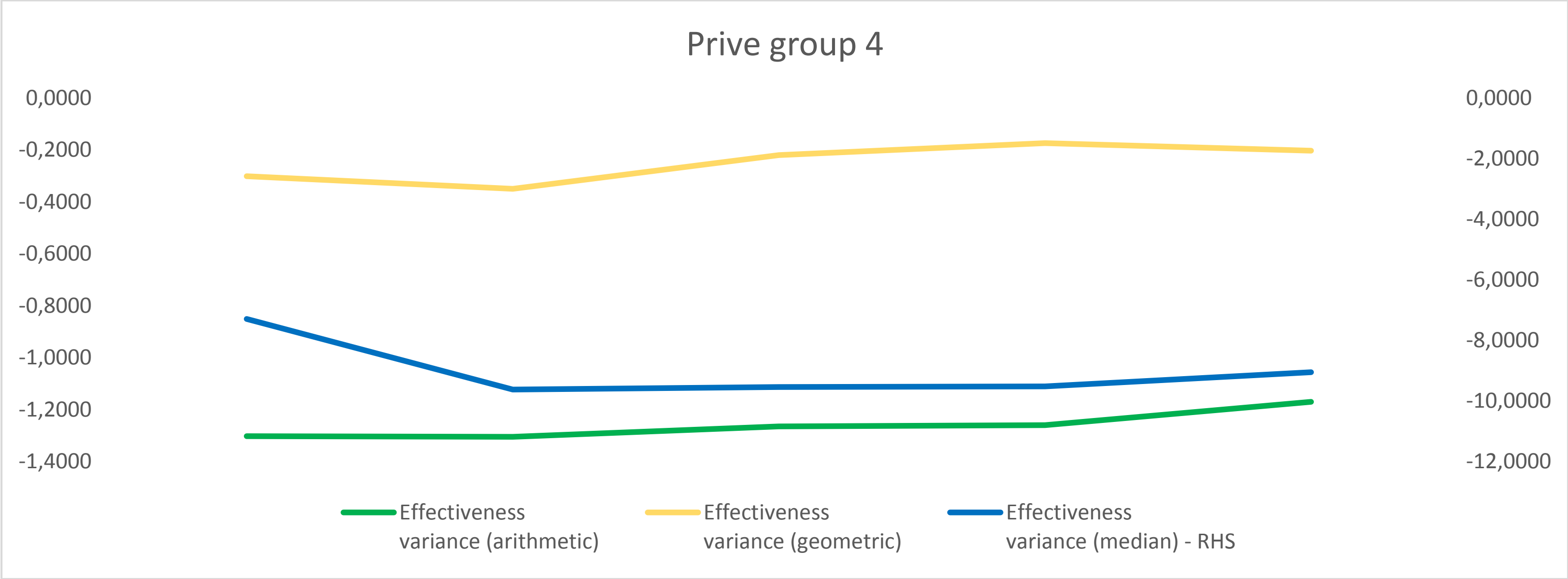


Figure 19 Hedging results of Strategy 4 for Price group 4





Conclusion

The results of hedging presented in Strategy 1 (naïve strategy) and Strategy 4 (rollover conditional OLS strategy) show that hedging cannot protect house owners neither from price decline nor from high variability of their returns. However, strategy 1 is too simple to draw conclusions based on its results, while strategy 4 (due to data limitations) calculates hedging effectiveness only for 5 hedge horizons, which is not enough to make a conclusion. Therefore, we will mostly focus on the hedging results for strategies 2 and 3.

Strategies 2 and 3 show a famous variance-return trade-off: hedging with at least one index increases the returns for house owners in more than 50% of cases for all house price groups, while the decrease in variance is observed at maximum 42.86% of cases. We also found that more sophisticated strategy 3 fails to protect house owners from negative returns during the crisis period (and years preceding and following the crisis), while much simpler strategy 2 gives more stable results.

Based on the hedging effectiveness measured by increased returns, hedging performs the best for cheap and expensive houses (hedging with median index, in this case, is advised for the owners of cheap houses, while hedging with arithmetic index is advised for the owners of expensive homes). In addition, hedging with arithmetic index shows the highest percentage of decrease in variance of returns if compared with other indices.

Therefore, based on the above conclusions, we can say that if the homeowner wants to make his returns more predictable and stable, hedging with housing futures is not the best way to accomplish this. If, on the other hand, the homeowner (or it is better to call him speculator) wants to protect himself against negative returns, there is rather high probability of doing this if he implements the hedge with housing futures (and if he is willing to gamble).

We conclude that if housing futures are to be introduced in Oslo, they would attract speculators who are interested in earning higher returns. However, the actual house owners who are afraid of very volatile returns will not be able to give their risk away. Therefore, housing futures in Norway, if introduced, can face the same problems as in the rest of the world: low demand and low trading volumes, which will only worsen their performance as a hedging instrument.

Further research

For further research relating to the topic of how to hedge house price risk in Norway and what we do not cover in this paper, we would suggest to, firstly, increase the data sample from including only Oslo to adding housing data from other cities in Norway (e.g. Bergen and/or Stavanger). Secondly, it would be interesting to examine the performance of other housing derivatives (e.g. housing options), which can give more flexibility in decreasing risk without affecting housing returns for homeowners. In addition, in this paper, we assume that a typical individual owns the portfolio consisting of a house and housing futures, while it can also be assumed that such a portfolio can additionally include shares, bonds and/or other securities. Finally, since we conclude that housing futures cannot be used by homeowners to give away the risk of house price fluctuations, and therefore, they should not be introduced in Oslo, it can be investigated whether existing derivatives in Norway can help in reducing the risk house owners face.

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Appendix

Construction of arithmetic repeat-sales house price index

First, we define P-matrix with the initial data:

- P-matrix:

House	1993	1994	1995	1996	1997
301/102/238/0/5	700 000	652 000			
301/10/26/0/143	500 000		855 000		
301/10/1086/0/0		1 706 000			2 040 000
301/10/156/0/37			680 000	830 000	
301/10/1186/0/0				3 275 000	1 000 000

Based on the initial data, we construct Z-matrix, X-matrix, and Y-matrix:

- Z-matrix:

House	1994	1995	1996	1997
301/102/238/0/5	1	0	0	0
301/10/26/0/143	0	1	0	0
301/10/1086/0/0	-1	0	0	1
301/10/156/0/37	0	-1	1	0
301/10/1186/0/0	0	0	-1	1

- X-matrix:

House	1994	1995	1996	1997
301/102/238/0/5	652 000	0	0	0
301/10/26/0/143	0	855 000	0	0
301/10/1086/0/0	-1 706 000	0	0	2 040 000
301/10/156/0/37	0	-680 000	830 000	0
301/10/1186/0/0	0	0	-3 275 000	1 000 000

- Y-matrix:

House	1993
301/102/238/0/5	700 000
301/10/26/0/143	500 000
301/10/1086/0/0	0
301/10/156/0/37	0
301/10/1186/0/0	0

After that, we calculate gamma using the formula:

$$\gamma = (Z'X)^{-1}Z'Y = \left(\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}^T \begin{pmatrix} 652000 & 0 & 0 & 0 \\ 0 & 855000 & 0 & 0 \\ -1706000 & 0 & 0 & 2040000 \\ 0 & -680000 & 830000 & 0 \\ 0 & 0 & -3275000 & 1000000 \end{pmatrix}\right)^{-1} \times$$

$$\times \left(\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}^T\right) \begin{pmatrix} 700000 \\ 500000 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.18 \\ 0.51 \\ 0.33 \\ 1.02 \end{pmatrix}.$$

Gamma in the reciprocal house price index. Finally, we can find arithmetic repeat-sales index:

$$Index = \frac{1}{\gamma} = \begin{pmatrix} 0.85 \\ 1.98 \\ 3.01 \\ 0.98 \end{pmatrix}.$$

Therefore, if we take 1993 as the base year (i.e. $Index_{1993} = 1$), we see that house prices decreased in 1994 and 1997, and increased in 1995 and 1996 compared to 1993.

Construction of geometric repeat-sales house price index

Again, we have the following P-matrix with the initial data:

- P-matrix:

House	1993	1994	1995	1996	1997
301/102/238/0/5	700 000	652 000			
301/10/26/0/143	500 000		855 000		
301/10/1086/0/0		1 706 000			2 040 000
301/10/156/0/37			680 000	830 000	
301/10/1186/0/0				3 275 000	1 000 000

Next, we can construct Z-matrix and Y-matrix:

- Z-matrix:

House	1994	1995	1996	1997
301/102/238/0/5	1	0	0	0
301/10/26/0/143	0	1	0	0
301/10/1086/0/0	-1	0	0	1
301/10/156/0/37	0	-1	1	0
301/10/1186/0/0	0	0	-1	1

- Y-matrix:

House	difference in ln(price)
301/102/238/0/5	-0.07
301/10/26/0/143	0.54
301/10/1086/0/0	0.18
301/10/156/0/37	0.20
301/10/1186/0/0	-1.19

Now, we will calculate gamma using formula:

$$\gamma = (Z'Z)^{-1}Z'Y$$

$$= \left(\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}^T \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \right)^{-1} \left(\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}^T \right) \begin{pmatrix} -0.07 \\ 0.54 \\ 0.18 \\ 0.20 \\ -1.19 \end{pmatrix}$$

$$= \begin{pmatrix} -0.18 \\ 0.65 \\ 0.96 \\ -0.12 \end{pmatrix}.$$

By taking the exponential of gamma we obtain geometric repeat-sales index:

$$Index = \exp(\gamma) = \exp \begin{pmatrix} -0.18 \\ 0.65 \\ 0.96 \\ -0.12 \end{pmatrix} = \begin{pmatrix} 0.83 \\ 1.91 \\ 2.61 \\ 0.89 \end{pmatrix}.$$