BI Norwegian Business School - campus Oslo

GRA 19502

Master Thesis

Component of continuous assessment: Thesis Master of Science
Final master thesis – Counts 80% of total grade

The Low Volatility Anomaly on Oslo Stock Exchange

INavn: Adi Grozdanic, Herman Sandnes Munthe-Kaas

Start: 02.03.2018 09.00

Finish: 03.09.2018 12.00

Abstract

This is our final thesis for the study MSc in Business with major in finance. Our topic is within the field of asset pricing, where we want to explore if the low volatility anomaly is present in the Norwegian stock market in the time period 1995-2017. We find the low volatility anomaly to be present on Oslo Stock Exchange as the low-volatile portfolios outperform the high-volatile portfolios on all performance measures. The anomaly is robust for different pricing-models and proxies for risk. Our conclusion is that the low volatility anomaly is present on Oslo Stock Exchange.

Acknowledgement

We would like to express our gratitude to our supervisor, Associate Professor Costas Xiouros at BI Norwegian Business School, for detailed feedback and relevant input to our thesis.

Further, we want to thank Bernt Arne Ødegaard for allowing us access to asset pricing data at Oslo Stock Exchange and giving us details regarding usage of his data for empirical research.

Finally, we want to thank our parents, friends and fellow students for throughout support during the last five years at BI Norwegian Business School.

Oslo, August 2018

Adi Grozdanic and Herman S. Munthe-Kaas

Table of content

1. Introduction

The basic tenant of finance states that higher risk leads to higher expected returns. This relationship is studied through several empirical papers with different conclusions. If assets with low risk outperform assets with higher risk, it contradicts the very core of financial theory and the popular capital asset pricing model (CAPM). The real relationship between risk and return is addressed using several methods. Realized returns are observable in the market, while risk and expected returns needs to be estimated. The approach to estimate risk is not given. Most studies investigating this particular financial phenomenon use CAPM´s beta, total volatility or idiosyncratic volatility as a measurement for risk.

One of the most interesting studies investigating the anomaly is Ang et al. (2006), providing empirical results showing stocks with low idiosyncratic volatility outperform stocks with high idiosyncratic volatility. Numerous of other studies have exploited the low volatility anomaly, including those using other proxies for risk, such as beta and total volatility. Further, most of the papers have also tried to explain its existence with several rational and irrational explanations.
Based on these academic articles, which in turn are presented later in the thesis, we investigate if the anomaly is present in the Norwegian stock market during the time period 1995 to 2017. We use monthly data from Oslo Stock Exchange to calculate simple returns over the period. Further, we follow Fama and French (1993) three-factor model to calculate both idiosyncratic volatility and use their pricing factors to find the portfolios alphas for performance measurements. When measuring the performance of our portfolios we also use mean returns and Sharpe ratios.

Empirical tests confirm our hypothesis of a low volatility anomaly in the Norwegian stock market. We find low volatile portfolios outperform high volatile for both equal- and value-weighted constructions of the portfolios. When including additional factors in our pricing model, with respect to momentum and liquidity, the anomaly still exists. Using total volatility as a proxy does not change our conclusion.
Examining penny stocks, we show preferences of stocks that perform like lotteries could partly explain the anomaly, especially when looking at the value-weighted

portfolios. We also find that return reversals are present in the market as the high-volatile portfolios have strong evidence of short-term return reversals in the following month.

The remaining of this thesis is organized as follows:
Section 2 gives a theoretical background and presents a literature review of relevant empirical papers (see Appendix 1 and Appendix 2) explaining the different findings, proxies for risk and possible explanations behind the low volatility anomaly. Section 3 describes our data sample, different sources from where we collect our data and covers our filter requirements. Section 4 explains our methodological approach and includes theory behind our research. Section 5 shows our main results, with tables underlining those, including results from our robustness tests. Finally, in section 6 we conclude our thesis.

2. Literature review[1]

2.1 Theoretical background

The traditional capital asset pricing model (CAPM) was introduced and supported through the 1960s stating more risky stocks earn higher returns on average (Sharpe, 1964; Lintner, 1965; Mossin, 1966). The pricing model presumes the presence of an efficient stock market and that investors holds a diversified portfolio. Thus, only systematic risk is priced in the model as the idiosyncratic is diversified away

Studies criticizing CAPM's beta are traced back to the beginning of the seventies. Black, Jensen and Scholes (1972) proved the security market line (SML) is flatter than CAPM's prediction, and further explained CAPM as a better pricing model including restricted borrowing (Black et al., 1972). Haugen and Heins (1975) is the first empirical paper, to our knowledge, to find evidence of a low volatility anomaly in the US stock market while using CAPM´s beta as proxy for risk.

If high idiosyncratic volatility stocks have low risk-adjusted returns, it still proves the presence of an anomaly as idiosyncratic volatility can be diversified away and not be related to returns. When presuming investors do not hold a well-diversified portfolio the relationship between idiosyncratic volatility and returns should be positive (Levy, 1978; Merton, 1987).

Today, empirical papers studying the relationship between risk and returns lead to mixed results.

2.2 Different proxies for risk

The three most common proxies for risk are CAPM's beta, total volatility and idiosyncratic volatility. Baker et al. (2012), who used both beta and total volatility, and Blitz and Vliet (2007) found that beta and total volatility have a high correlation and using one over the other will not give significantly different results. Trainor (2011) also supported this implication that stocks with high (low) total volatility usually have high (low) idiosyncratic volatility. Riley (2014) provided evidence for this correlation and concluded that the two different measures for volatilities are in fact hard to separate. Since the three different

[1] Appendix I shows a literature matrix with the most important information and findings

proxies for risk are highly correlated, all studies with different measurements for risk and their findings are highly relevant for this thesis.

2.3 Literature supporting the low volatility anomaly

There are articles supporting the low volatility anomaly for all three proxies for risk. Some empirical papers, such as Baker, Bradley and Wurgler (2011), find the anomaly present for both beta and total volatility as a proxy for risk.

2.3.1 Beta

Haugen and Heins (1975) used beta to dig deeper into the relationship between risk and returns. They measured the trade-off between risk and return over several time-periods using monthly data to reveal the severity of the bull-bear market problem. The results do not support the theory that risk generates any reward, but indicates that less volatile stock portfolios experience greater average return compared to stock portfolios with higher variance.

In more present years, most studies use either total volatility or idiosyncratic volatility as proxy for risk. An exception is Frazzini and Pedersen (2011), who similar to Haugen and Heins (1975), argued that the low volatility anomaly exists in the stock market. They explored the relation between the beta and returns, using daily data and beta as a measurement of volatility. While Haugen and Heins (1975) used US stocks in their research, Frazzini and Pedersen (2011) found the same results for global markets. In order to exploit the anomaly, they used CAPM with the three-, four- and five-factor extensions to find alphas for the different decile portfolios. Their results find presence of the low volatility anomaly across the observed global markets (including the Norwegian market).

2.3.2 Total volatility

Blitz and van Vliet (2007) studied global large-cap stocks using weekly returns with total volatility as proxy for risk. They formed decile portfolios ranked on total volatility based on the last three years, and found that low-volatility stocks outperform high-volatility stocks, using both Sharpe ratios and different CAPM-alphas as performance measurements. The research found stronger evidence of the anomaly in global markets with low- versus high-volatility alpha spread of 12%. The anomaly is also found to be present after adjusting for value, size and momentum. An extension of the study, Blitz et al. (2013) later found similar results when investigating for the same phenomenon in emerging markets.

Baker and Wurgler (2011) sorted their portfolios using both beta and total volatility and showed that low-risk portfolio outperform high-risk portfolios for both measurements of risk. To demonstrate their findings, an investment in the lowest volatile portfolio would give a remarkable increase of 5955% from 1968 to 2008, while an investment in the highest volatile portfolio would decrease in value by 42%. They used US data to find monthly alphas from CAPM.

Baker and Haugen (2012) did the first study of the low volatility anomaly, to our knowledge, including data and detailed results from the Norwegian financial stock market. The different portfolios are sorted out looking at the Sharpe ratio, total volatility and the realized return between the high and low volatile portfolios. The results show that the low volatility effect exists in 21 developed countries and 12 emerging markets, including the Norwegian. Their data covered the time horizon from 1990 to 2011. The research shows that the low-volatility portfolios earn higher return than the high-volatility portfolios in the Norwegian financial market.

2.3.3 Idiosyncratic volatility

Ang et al. (2006) investigated the US stock market from 1963 to 2000. The findings showed that there is a significant relation between firms with high idiosyncratic volatility and abysmally low average returns. The Fama and French (1993) model is used to define the idiosyncratic volatility and is measured by using daily data over previous months. The cross-sectional relationship between high idiosyncratic volatility and low average return could not be explained by exposures to size, book to market, leverage, liquidity, volume, turnover, bid-ask spreads, skewness, or dispersions in analysts' forecasts characteristics. In a later study done in 2009, they extended their former research from 2006 to not only focus on the US market, but now also including several other global markets. Past idiosyncratic volatility were sorted out from stocks across 23 developed markets, including the Norwegian stock market[2], in order to search for the low volatility anomaly. The results are out-of-sample relative to the earlier findings from Ang. et al. (2006) and implicates that the low volatility anomaly is not just a sample-specific or country-specific effect but is observed worldwide. Further, the findings show that global idiosyncratic volatility effect, only captured by a simple US idiosyncratic volatility factor, are insignificant. The low returns of stocks with

[2] Detailed results from the Norwegian market was not given

high idiosyncratic volatility cannot be explained by standard factors, but is a result from exposure to systematic risk.

2.4 Literature undermining the low volatility anomaly

Using financial time-series with the EGARCH-model, Fu (2009) measured idiosyncratic volatility and found no evidence of a low volatility anomaly. His research finds positive correlation between returns and idiosyncratic volatility. Further, he criticises Ang et al. (2006) results to be driven by short-term return reversal effect and a set of small-cap stocks with extremely high idiosyncratic volatility. He also argues past volatility is a poor measure. Guro, Kassa and Ferguson (2010) pointed out the usage of EGARCH is driven by a look-ahead bias[3]. A look-ahead bias is occurring when information or data in a study, which would not have been known/available during the period being analyzed, possibly affects its results. When correcting for it, Guro et al. (2010) did not find any evidence of a positive correlation between returns and idiosyncratic volatility, and by doing so proving the existence of a low volatility anomaly.

Bali and Cakici (2008) criticized previous methodology; especially usage of daily data, arguing it is less reliable due to micro-noise. To prove this, they replicated Ang et al. (2006) and sampled monthly data. After doing so, they indeed did not find any significant evidence of the low volatile portfolio outperforming the high volatile one. Bali and Cakici (2008) also replaced equal-weighted portfolios with value-weighted portfolios in their research.

2.5 Possible explanatories behind the low volatility anomaly[4]

Several explanations have been suggested as possible factors to the existence of low volatility anomaly. We can differ the explanations into rational, irrational and mathematical.

Lottery preferences are irrational preferences of stocks, which behave like lotteries. Investors often prefer stocks with high probability of small negative returns, but still with the chances of exceptional high returns. Baker and Wurgler (2011) suggested this behaviour could be linked to representativeness. Many irrational investors remember success stories of small tech-companies who had enormous returns (i.e. Microsoft IPO). This bias could in turn increase the

---

[3] Fu acknowledges the bias in his research.

[4] Matrix with overview of possible explanations is given in Appendix II

demand for small and volatile stocks. Blitz and van Vliet (2007) also pointed out lottery preferences to be a possible cause of under-priced low volatile stocks, and over-priced high volatile stocks.

Overconfidence is also an irrational explanation suggested by Baker and Wurgler (2011). This is a human bias, which has been researched and documented beyond finance. This bias suggests the following: "People tend to overestimate the precision of their beliefs or forecasts, and they tend to overestimate their abilities" (Bodie, Kane, and Marcus 2011, 411). This is especially interesting in the topic of investments. Overconfident investors are likely to choose high volatile stocks due to the high reward of a superior talent in picking stocks. Baker and Wurgler (2011) also believed overconfident investors who disagree on a given stock valuation, are more to likely stick to their own valuation. They further explained it with the example of a market with restrictions on short selling; the demand for a particular security would come from those with the most positive assessment of its returns.

Limits to arbitrage and short sale constrains is a rational explanation suggested by Baker and Haugen (2011) as to why the anomaly does not go away. The key here is that small-cap stocks, which again are costly to trade in a large scale, often compose high-volatility portfolios. This correlation prevents the strategy to short the high-volatile portfolio and go long on the low-volatile one.

Leverage constraints are also a possible rational explanation to why the anomaly continues to exist. Black (1972) found that the security market line (SML) is flatter then suggested by CAPM, and further noted the borrowing constrain for this relationship. Frazzini and Pedersen (2011) heavily argued its existence in their paper. They illustrated it by looking at constrained investors (i.e. mutual funds) and unconstrained investors (i.e. hedge funds) preferences on portfolios. Due to borrowing restrictions for some investor groups (e.g. mutual funds, pension funds), the investors are prevented to overweight their investments in low volatility portfolio. In theory this could lever their portfolio to match their preferences of risk. This causes the investors to have a high demand for high-risk assets, which will lead to increasing prices and further lead to lower risk-adjusted returns compared to low risk assets.

The urge to beat benchmark is a rational explanation that could explain the existence of a low risk anomaly. The reasoning is linked to their limitations to tracking errors and leverage constrains. Both Ang et al. (2006/2009) and Baker and Wurgler (2011) blamed this agency problem as a key driver behind the high demand for highly risky stocks. Arguing that if benchmarking contracts exist, the anomaly would continue to exist as well. Investors will seek high volatility stocks to improve their expected excess returns. The highest volatility portfolios are often consisting of small stocks, which are costlier to trade, and therefore stocks with high volatility tend to be overpriced over a longer time-period compared to stocks with low volatility.

Volatility estimation is a potential reason that affects the results of papers exploiting the low volatility anomaly. Since volatility is unobservable, it has to be estimated. Fu (2009) argued the lagged value is used as an estimate of the expected value and disregards the one-month lagged idiosyncratic volatility measured by Ang et al. (2006). Bali and Cakici (2012) further showed using monthly and daily volatility yields different results. More concrete, they showed that using monthly data gives no evidence of a low volatility anomaly. Trainor (2012) also criticized Ang et al. (2006) and Baker and Wurgler (2011) for their compounding. He found that in periods where there is low volatility, high-beta portfolios outperform low-beta portfolios, while high-beta portfolios performed worse in the long run.

Return reversals in the short-term of stocks with high volatility are suggested as a bias in Ang et al. (2006) by Fu (2009). Stock with high volatility tends to have reverse returns in following month. This correlation leads to low returns in the following month and causes a bias in the results. Return reversals are also confirmed as an explanation behind the anomaly by Huang et al. (2010).

3. Data

3.1 Data from Oslo Børs

We collect monthly prices from all stocks at Oslo Børs from 01.01.1988 to 31.12.2017, giving us raw data for a period of 30 years. The list of stocks is given to us directly from Oslo Børs' customer services. Our dataset is without survivorship bias, and includes stocks that are delisted during the period. The list consists of 916 individual securities. Further, we download monthly prices from Compustat[5] using Wharton Research Data Services (WRDS). The data consists of daily stock-prices where a monthly indicator enables us to transform them into a monthly basis. The extracted data is closing prices of the given date. All pricing data (closing prices included) from Compustat are unadjusted ("Understanding the data[6]", chapter 6) in terms of dividends, stock splits and corporate events. Hence, to adjust our stock prices we use the adjustment factor in order to provide us with adjusted prices (Adjusted Price = PRCCD/AJEXDI), which we later use to compute simple returns.

3.2 Filtering data and return computation

Not all stocks traded at the Oslo Stock Exchange should necessarily be used for empirical asset pricing investigation (Ødegaard 2018). With his reasoning, we choose to remove stocks that do not meet our set of filtering requirements.
All shares that are not ordinary shares are retained in our sample. This excludes for instance B shares. We also choose to remove abnormal stock prices above NOK 10.000.

Ødegaard (2018) points out how low valued stocks (penny stocks) could cause problems due to their overemphasized returns. He suggests to only keeping stocks with value above NOK 10 in a sample for empirical research. In our case, we find this criterion to remove almost 1/5 of our sample. We do not find it expedient to remove such a large amount of our sample. Instead, by removing all stocks with value less then NOK 1, our sample only excluded 3% of the ordinary shares.

[5] Following variables are downloaded: **Date, International Security ID (ISIN), PRCCD, AJEXDI, MONTHEND, TPCI, CSHOC.**

[6] Support → Compustat manuals and overviews

In line with Blitz and van Vliet (2007), we add a size criterion. Stocks with a market capital[7] less then NOK 1 mill, as suggested by Ødegaard (2018), are removed. This criterion removes 0.3 % of our sample. Ang et al (2006/2009) removes 5% of the companies with lowest market capital.

When computing simple returns, adjusted prices are used. The simple returns are derived on a monthly basis. For stock *i,* the simple return (R_i) in month *t* is:

$$R_i^t = \frac{P_i^t - P_i^{t-1}}{P_i^{t-1}}$$

We also want to filter our returns in case outliers affect our results. When studying the returns, several abnormal monthly returns are discovered. To get rid of this issue we choose to remove outliers by trimming our data at the 0.01% level. This means all monthly returns above 121% and under –72% are removed from our sample. Monthly returns now have a mean of 0.67%.

After implementing our filtering, we see that number of firms vary from 64 in 1990 to 240 in 2007. Overview with number of firms each year is given in Appendix III.

3.3 Risk-Free Rate

Monthly Norwegian interest rates, which are downloaded directly from Bernt Arne Ødegaards database[8], are used in our study**.** The risk-free rates are based on the Norwegian interbank rate (NIBOR) with maturity of one month.

3.4 Pricing factors

All pricing factors are collected from Ødegaards database in terms of monthly values. The Fama French factors, *SMB* and *HML*, are computed in accordance with Fama and French (1993) using Norwegian data. The Carhart Momentum factor, *UMD,* is computed as by Carhart (1997). The fifth factor, *LIQ*, considers a liquidity effect. It is computed by Næs et al. (2009) and based on Oslo Børs' spread. The market factor (*MKT*) is derived every month consisting of value-weighted excess return across all stocks in our sample. The value-weighted excess returns are also downloaded from Ødegaards database.

[7] Last price (PRCCD) * Shares outstanding (CSHOC)

[8] http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/index.html.

4. Methodology and theoretical framework

This chapter will present our main approach using idiosyncratic volatility, following Ang et al. (2006), and the theoretical framework behind it. Explanations behind the constructed portfolios and the chosen performance measures are also discussed. In section 4.4 on robustness test, some different modifications to the main methodology are added to see if, and how, it influences our results.

4.1 Volatility estimation

We follow Ang et al (2006) when estimating the volatility. The estimation is done through using the Fama and French (1993) three-factor model. The two other factors are later used in the robustness analysis. They developed the model as a supplement to CAPM in order to include size (SMB) and value (HML) factors. They believe companies with highest book values, relative to market values, have systematically higher risk-adjusted returns compared with the lowest book value relative to market value. Further, the model also includes a size factor based on earlier results from Banz (1982). He found that firms with low market value on average have higher risk-adjusted returns. The pricing model is as follows:

$$r_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,MKT}\left(r_{m,t} - r_{f,t}\right) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \varepsilon_i$$

The factors are in standard format, as defined by Fama and French (1993). $r_{i,t}$ is the return on stock *i* in month *t*, and $r_{f,t}$ is the monthly risk-free rate at time *t*. $\left(r_{m,t} - r_{f,t}\right)$ gives us the value-weighted market excess return of the specific market portfolio over the risk-free rate. SMB_t is the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks, while HML_t is the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio with a low book-to-market ratio. $\alpha_{i,t}$ is the pricing error-term, which will be used as one of the performance measures (see detailed in 4.3).

It is the last term, ε_i, called the error-term, which will be used to estimate the idiosyncratic volatility. Idiosyncratic volatility is defined as the standard deviation of the error-term.

$$IVOL_{t+1} = \sqrt{VAR(\varepsilon_{i,t})} = \sqrt{\frac{1}{N-1}\sum_{K=0}^{N-1}(\varepsilon_{t-k} - \bar{\varepsilon})^2}$$

Following Bali and Cakici (2008), who replicated Ang et al. (2006) using monthly data, we estimate volatility based on the previous 24 months (N=24). Further, we only consider stocks with at least 12 return observations in the previous 24 months to be included in one of the quintile portfolios.

4.2 Portfolio constructions

Portfolios are first sorted by idiosyncratic volatility, before they are split into quintiles where P1 (P5) is the portfolio with lowest (highest) volatility. We start constructing the portfolios in 01.01.1995 and rebalance them each month. The reason for this particular date is that we choose to not include the previous years due to few stocks in each portfolio (Appendix 3). While Ang et al. (2006) used equal-weighted portfolios when constructing the portfolios, we want to examine if the anomaly exists with value-weighted portfolios, similar to Bali and Cakici (2008). Value-weighted portfolios are constructed at time *t* by investing with different percentage in every firm, based on its market capital in time *t+1*. Equal-weighted portfolios are made by investing the same percentage amount in each firm in the portfolio.

4.3 Performance measurements

We calculate several performance measurements of each portfolio. All the measurements are on a monthly basis. The different performance measures are mean excess returns, alpha estimations based on FF-3 regression and Sharpe ratios.

Mean excess returns are defined as the weighted returns in a portfolio minus the risk-free rate[9]. We also compute standard deviations of excess holding period returns for all of the portfolios.

4.3.1 Alpha estimation

Following the three factor model from Fama and French (1993) in 4.1, we also evaluate a portfolios performance with respect to its alpha ($\alpha_{i,t}$). When α is significantly different from zero, the returns from the quintile portfolios are not

[9] See 3.2 and 3.3 for further details on returns and risk-free rates.

only explained by exposures from the size and value factors. On the other hand, if α is not significantly different from zero, the exposure from size and value explain the total of the excess returns.

Alphas standard errors and p-values are based on Newey and West (1987) t-statistics. These estimators are chosen to try to overcome autocorrelation and heteroscedasticity in the error terms in our models. These t-statistics are especially designed for time series data.

4.3.2 Sharpe ratio

Sharpe ratios are used for each of the quintile portfolios. This performance measurement is introduced by Sharpe (1966) and is calculated as:

$$SR = \frac{\overline{r_p} - \overline{r_f}}{\sigma_p}$$

where $\overline{r_p} - \overline{r_f}$ is the mean excess return and σ_p is the standard deviation of excess holding period return.

4.4 Robustness tests

When performing different robustness tests we want to see if, and how, it influence our results. Tests in 4.4.3 and 4.4.4 are based on two of the explanations in 2.4 from our literature review, namely *lottery preferences* and *return reversals*.

4.4.1 Different pricing models and IVOL estimations

The pricing model from 4.1 is expanded with introducing the four-factor and five-factor pricing models. The main motivation behind this robustness test is to examine if our results are sensitive to including additional factors in the regression. The four-factor model includes a momentum factor (*UMD)* while the five-factor pricing model also adds a liquidity factor *(LIQ).* The five-factor pricing model is now:

$$r_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,MKT}(r_{m,t} - r_{f,t}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t \\ + \boldsymbol{\beta_{i,UMD}UMD_t} + \boldsymbol{\beta_{i,LIQ}LIQ_t} + \varepsilon_i$$

The factor UMD_t, up minus down, was introduced by Carhart (1997) and is a product of a study on the US stock market in by Jegadeesh and Titman (1993) proving going long on stocks with momentum, and short on stocks who underperformed the same period, led to positive excess returns.

The last factor LIQ_t is defined as the standardized turnover-adjusted number of zero daily trading volumes over the prior x months (Næs, Skjeltorp & Ødegaard, 2009). The factor is calculated on relative spread of liquidity based on Norwegian stocks. The remaining factors are similar to chapter 4.1.

We will now perform two new regressions. The first will only add UMD_t to give us the Carhart four-factor model, while the last also includes LIQ_t and gives us the five-factor model.

The new four- and five-factor pricing models will differentiate our method in two ways. Firstly, it will give us new error-terms (ε_i) when estimating volatility[10]. Secondly, we get different alphas ($\alpha_{i,t}$) when evaluating the performance for our quintile portfolios.

4.4.2 Using total volatility as proxy

In this part, we use an additional technique in estimating volatility. Our main approach is the idiosyncratic volatility, but we also want to investigate if the anomaly exists with total volatility. Total volatility has a similar formula as idiosyncratic volatility given in 4.1. The only exception is that we now look at the standard deviation of the returns and not the error-term. The volatility is now given as:

$$TVOL_{t+1} = \sqrt{VAR(R_{i,t})} = \sqrt{\frac{1}{N-1}\sum_{K=0}^{N-1}(R_{t-k} - \overline{R})^2}$$

where R_i is as defined in 3.2. We continue to follow Bali and Cakici (2008) and Ang et al. (2006) by estimating volatility based on the previous 24 months (N=24).

Total volatility as the proxy for risk is tested with both the three-factor, four-factor and five-factor models.

4.4.3 Adjusted filtering

In chapter 4.2, our data from Compustat are adjusted by adding several filter requirements. In the filtering, stocks with value beneath NOK 1 are removed to exclude the smallest penny stocks. As mentioned in 2.4, preferences of stocks

[10] See 4.1 for volatility estimation, where formula for IVOL is given.

with performance similar to lottery tickets is suggested as an explanation to the findings of a low volatility anomaly. Since penny stocks often fall under this category, the original filter requirements are adjusted in two ways. We want to examine if our results are different when:

1. Include all stocks, also with value lower then NOK 1.

2. Remove stocks with value lower than NOK 10 of our sample[11].

With these mechanisms, we examine if preferences of stocks that behave like lotteries could explain the anomaly by including both more and less low-valued stocks to our original methodology.

4.4.4 Investigating return reversals

Finally, we will investigate if return reversals as documented by Huang et al (2010) are present in our sample. This is done by computing mean excess returns of portfolios on time t (where the volatility is estimated) and comparing it with time $t+1$. If return reversals are present, the results will be negative in t and positive in $t+1$, or vice versa.

[11] Stock with value lower then NOK 1 removes 3% of our sample and 19% when removing stocks with lower value then NOK 10

5. Results

The results are presented with focus on our tables and appendices. Tables 1-7 shows the numerical results with respect to our performance measurements given in 4.3 for different methodologies. In appendix 4 we include detailed regression results to help us explain our findings. In all our tables, Portfolio 1 (P1) is the portfolio with lowest volatility, while Portfolio 5 (P5) contains the stocks with highest volatility. P-values in brackets are based on robust Newey and West (1987) t-statistics. All the results are shown for both equal-weighted and value-weighted portfolios.

5.1 Fama-French 3-factor

Bali and Cakici (2012) showed that using monthly data gives no evidence of a low volatility anomaly. When looking at FF-3 alphas in the equally- and value-weighted portfolios in Table 1 we actually find evidence of low volatility anomaly, which differ from Bali and Cakici (2012) findings, which also used monthly data. On the other hand, the results are consistent with Ang et al. (2006) findings that used daily data. Ang et al. (2006) found average returns drop dramatically in quintile 5, which has the highest idiosyncratic volatility. The observations are quite similar to the Norwegian stock market data. In Table 1.1 average return also has a precipitously drop in portfolio 5, while the largest fall is in P4 for value-weighted portfolios in Table 1.2.

5.1.1 Equal-weighted

We find the monthly excess mean return in P1 to be 0.37% and 0.24% in P5. The returns do not fall monotonically; with both P2 and P4 gaining higher returns then P1. Going long on P1 and short on P5 yields a positive return of 0.13% (P1-P5), but yet insignificant. Bali and Cakci (2008) found similar results to ours, proving the anomaly with respect to mean returns, but however insignificant. These results show us the first suggestion of a possible low volatility anomaly in the Norwegian stock market.

The Sharpe ratios show the same tendency. P1 gains a Sharpe ratio of 0.09 while P5 has 0.03. Since the Sharpe ratios fall from P1 to P5, the stock returns does not compensate for higher volatility. When examining the returns while controlling

for traditional Fama and French (1993) factors, the same pattern is present, and in this case also significant. The (P1 – P5) alpha is 1.5% with a robust t-statistic of 4.36 for the equally weighted portfolio. The alphas decrease from P1 to P5, with expectation of the difference between portfolios P3 and P4, where a slightly increase is present. These findings also provide us with new evidence of a low volatility anomaly. All alphas are significant at the 1 percent level.
Observing the past volatility for our constructed portfolios provides a good indication for future risk. This is proposed by our ex post standard deviations, which follow a linearly increase from P1-P5.

5.1.2 Value-weighted

The monthly excess mean return in P1 is 0.60% and -0.40% in P5. The returns do not fall linearly for value-weighted portfolios either, with a slightly increase from P2 to P3. The difference portfolio (P1-P5) has a higher value of 1% compared to only 0.13% using equal-weighted. The results show a larger anomaly compared to equal-weighted portfolios. Ang et al. (2006) found significant positive return, using daily data.
Sharpe ratios have a similar relationship as equal-weighted portfolios. The stock returns does not compensate for higher volatility using value-weighted portfolios as well. P1 gains a Sharpe ratio of 0.10 while P5 has negative ratio of -0.03. The (P1 – P5) alpha is 2.28% with a robust t-statistic of 3.23 for the value-weighted portfolio, and all alphas are significant at all levels. The alphas decrease from P1 to P5 with ex post standard deviations higher than for all the portfolios for equal-weighted. Interestingly, portfolios across P1 to P5 do not earn higher returns for equally weighted portfolios than for value-weighted portfolios, which is not consistent with Huang et al. (2010)

To help us explain the results we look at Appendix 4.1 and 4.2 where model specifications and explanatory variables are given. From P1 to P5, loadings on our market factor ($\beta1$) increase. This indicates that higher idiosyncratic volatility implies higher market beta. Idiosyncratic volatility follows the systematic risk in the stock market. The loadings on the market factor ($\beta1$) for the low minus high volatility portfolios are negative for both equally- and value-weighted portfolios, and significant at all levels.
Loadings on the small minus big factor ($\beta2$) are significant and increasing with

volatility for both equally- and value-weighted portfolios. This result underlines our theory that size is negatively related with risk. P1-P5 is -0.40 and -0.54, for equally- and value-weighted portfolios respectively, and significant at all levels. The finding could indicate that firm size does not explain the low volatility anomaly, but we should be cautious with this interpretation in search for a possible explanation of a low volatility anomaly. The value-factors, high minus low ($\beta 3$), are mostly close to zero for the value-weighted portfolios and significant, but still low, for the equal-weighted ones. Ang et al. (2006) and Bali and Cakici (2008) found almost none dispersion in B/M across P1 to P5, which is another factor for value. How, and if, value influences our results is further investigated when experimenting with penny stocks in section 5.4.

Based on our measurement criterions in 4.3 we conclude that there exist a low volatility anomaly using idiosyncratic volatility following the Fama and French (1993) three-factor model. Mean returns, Sharpe-ratios and FF-3 alphas do not follow any pattern across portfolios P1 to P5, but P1 outperforms P5 at all performance measurement. We also acknowledge that low-risk portfolios outperform high-risk portfolios using idiosyncratic volatility in the Norwegian stock market, independent of portfolio weighting.

Table 1: Idiosyncratic volatility computed with three-factor model

Table 1.1 - Equally Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha
1	0.37 %	3.92 %	0.09	-0.98*** (-5.86)
2	0.50 %	4.80 %	0.10	-1.13*** (-5.69)
3	0.34 %	6.11 %	0.06	-1.81*** (-6.40)
4	0.63 %	7.05 %	0.09	-1.72*** (-7.19)
5	0.24 %	8.44 %	0.03	-2.47*** (-7.25)
P1 - P5	**0.13 %**	**-4.52%**	**0.06**	**1.50*** (4.36)**

Table 1.2 - Value Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha
1	0.60 %	6.09 %	0.10	-0.80** (-2.21)
2	0.44 %	7.25 %	0.06	-1.33** (-3.36)
3	0.47 %	8.65 %	0.05	-1.35** (-2.12)
4	-0.26 %	9.47 %	-0.03	-2.57*** (-4.42)
5	-0.40 %	12.52 %	-0.03	-3.08*** (-4.44)
P1 - P5	**1.00 %**	**-6.43 %**	**0.13**	**2.28*** (3.23)**

Table 1.1 and 1.2 shows portfolios sorted and calculated using the monthly data from the last 24 months. The calculation is based on idiosyncratic volatility relative to the Fama-French three-factor model. The sample period is valid from January 1995 to December 2017. Table 1.1 present the equally weighted portfolios, while table 1.2 present the value weighted portfolios. Portfolio 1 is the portfolio of stocks with the lowest idiosyncratic volatility and portfolio 5 is the one with the highest. Mean access return (Mean) and standard deviation (Std. Dev) is measured monthly and the sharp ratio is (Mean) divided by the (Std. Dev). The Alpha is estimates are in monthly percentage terms and reported relative to the Fama-French three-factor model. T-statistics are represented by p-values based on Newey and West (1987). Significant values are represented at the 10 percent level (*), 5 percent level (**) and 1 percent level (***).

5.2 Extensions of Fama-French

Further, we form quintile portfolios by extending the Fama and French (2003) model and adding one and two additional factors. In Table 2 the factor (UMD) is added to the three-factor model and in Table 3 the fifth factor (LIQ) is added. The results are compared to the findings in 5.1 to give insight in why the results differ when they do.

5.2.1 Four-factor

The monthly excess mean return for equal-weighted (value-weighted) portfolios in P1 is 0.38% (0.81%) and 0.26% (-0.03%) in P5 respectively. The equal-weighted show only small differences while the value-weighted portfolios. Going long on P1 and short on P5 yields a positive return of 0.12% (0.84%), but is again insignificant, as expected when adding a new variable.
Similar to the three-factor model, we observe that the standard deviation increases from the low to the high volatile portfolios. The high volatile stocks in the equally weighted portfolios earns significant negative four-factor at a 1 percent level. Even though not all the 4-factor alphas are significant with the value-weighted portfolios, the P1-P5 has a positive significant value of 1.76% at a 10 percent level, while it is 1.42%, and significant for all levels, when equal-weighted.

5.2.2 Five-factor

Including the fifth factor in the Fama and French pricing model increases our P1-P5 for equal-weighted and decreases for the value-weighted. However, they are still positive and show sign of a low volatility anomaly, but still insignificant. FF-5 alphas for P1-P5 are also both positive. We see that for equal-weighted portfolios the difference in the alpha is almost none, while it drops from 1.76 (10% significance level) to 0.92 (not significant) when adding the fifth factor.

The momentum factor, UMD ($\beta 4$), shows low loadings in appendix 4.1 and 4.2. It has a positive sign of 0.16 (0.07) for the equal- (value-) weighted control portfolio P1-P5, but only significant for the equal-weighted (5% level). Equal-weighted portfolios are more sensitive to the momentum factor, and P3 and P4 are the portfolios with lowest loadings on $\beta 4$.
An interesting observation is the changes in sign of the LIQ ($\beta 5$) factor. The

coefficients show opposite signs from P1 to P5, indicating high liquidity stocks in Q1 and low liquidity stocks in Q5 (0.55 and 0.50 for P1-P5 for equal- and value-weighted), which is consistent with previous literature. Coefficients are statistically significant and 1% and 10%, and we can say that the portfolios are exposed to this factor.

Testing of the four-factor and five-factor model underlines the same conclusions we got by testing the Fama and French (2003) model, that there is evidence of a low volatility anomaly according to all of our performance measurements. With respect to these results, we will focus on three-factor residuals in the further robustness tests, as there is no evidence that any of the specific models will provide us with different results. The observations are quite similar to the three-factor model, and the additional factors do not seem to have any major impact on the results. However, an important notice is that both the equally- and value-weighted portfolios show an even larger decrease in the average return from P4 to P5 when adding one and two more factors. These results are similar to Ang et al. (2006) who only used the three-factor model when finding the dramatic drop in returns from P4 to P5. We also see that there are larger changes in the value-weighted portfolios when adding more factors, compared to the equal-weighted ones.

Table 2: Idiosyncratic volatility computed with four factor model

Portfolio	Mean	Std Dev	Sharpe Ratio	FF-4 Alpha
Table 2.1 - Equally Weighted Portfolios				
1	0.38 %	3.87 %	0.10	-0.94*** (-5.66)
2	0.42 %	4.93 %	0.09	-1.18*** (-6.01)
3	0.33 %	6.04 %	0.05	-1.68*** (-6.81)
4	0.71 %	6.80 %	0.10	-1.32*** (-6.14)
5	0.26 %	8.55 %	0.03	-2.36*** (-7.30)
P1 - P5	**0.12 %**	**-4.68 %**	**0.07**	**1.42*** (4.38)**
Table 2.2 - Value Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-4 Alpha
1	0.81 %	5.83 %	0.14	-0.52 (-1.60)
2	0.56 %	6.83 %	0.08	-0.92** (-2.08)
3	1.08 %	9.33 %	0.12	-0.51 (-0.89)
4	0.93 %	10.01 %	0.09	-1.06* (-1.70)
5	-0.03 %	12.54 %	-0.0024	-2.28*** (-2.86)
P1 - P5	**0.84 %**	**-6.71 %**	**0.14**	**1.76* (1.96)**

Table 2.1 and 2.2 shows portfolios sorted and calculated using the monthly data from the last 24 months. The calculation is based on idiosyncratic volatility relative to the Fama-French four-factor model. The sample period is valid from January 1995 to December 2017. Table 2.1 present the equally weighted portfolios, while table 2.2 present the value weighted portfolios. Portfolio 1 is the portfolio of stocks with the lowest idiosyncratic volatility and portfolio 5 is the one with the highest. Mean access return (Mean) and standard deviation (Std. Dev) is measured monthly and the Sharpe ratio is (Mean) divided by the (Std. Dev). The Alpha is estimates are in monthly percentage terms and reported relative to the Fama-French four-factor model. T-statistics are represented by p-values based on Newey and West (1987). Significant values are represented at the 10 percent level (*), 5 percent level (**) and 1 percent level (***).

Table 3: Idiosyncratic volatility computed with five factor model

Table 3.1 - Equally Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-5 Alpha
1	0.45 %	3.88 %	0.12	-0.93*** (-5.81)
2	0.37 %	4.95 %	0.07	-1.24*** (-6.17)
3	0.41 %	6.11 %	0.07	-1.56*** (-6.33)
4	0.74 %	6.78 %	0.11	-1.13*** (-4.50)
5	0.13 %	8.45 %	0.02	-2.34*** (-6.73)
P1 - P5	**0.32 %**	**-4.57 %**	**0.10**	**1.40*** (4.65)**
Table 3.2 - Value Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-5 Alpha
1	0.67 %	6.61 %	0.10	-0.74* (-1.86)
2	0.58 %	6.28 %	0.09	-0.85** (-2.29)
3	-0.02 %	8.42 %	-0.0024	-1.78*** (-3.47)
4	1.08 %	9.75 %	0.11	-0.98* (-1.80)
5	0.43 %	10.83 %	0.04	-1.66*** (-2.98)
P1 - P5	**0.24 %**	**-4.22 %**	**0.06**	**0.92 (1.38)**

Table 3.1 and 3.2 shows portfolios sorted and calculated using the monthly data from the last 24 months. The calculation is based on idiosyncratic volatility relative to the Fama-French five-factor model. The sample period is valid from January 1995 to December 2017. Table 3.1 present the equally weighted portfolios, while table 1.2 present the value weighted portfolios. Portfolio 1 is the portfolio of stocks with the lowest idiosyncratic volatility and portfolio 5 is the one with the highest. Mean access return (Mean) and standard deviation (Std. Dev) is measured monthly and the Sharpe ratio is (Mean) divided by the (Std. Dev). The Alpha is estimates are in monthly percentage terms and reported relative to the Fama-French five factor model. T-statistics are represented by p-values based on Newey and West (1987). Significant values are represented at the 10 percent level (*), 5 percent level (**) and 1 percent level (***).

5.3 Total volatility

The main focus in this thesis relies on testing the low volatility anomaly with idiosyncratic volatility. Inspired by Baker and Haugen (2012) study of the low volatility anomaly, including data and detailed results from the Norwegian financial stock market, we also wish to test for total volatility. Contrary to Baker and Haugen (2012) we use data from 1995-2017.

Baker and Haugen (2012) notice that the difference in the total return (lowest risk minus highest risk) and the different in Sharpe ratio between the lowest and the highest portfolio are positive, with some exceptions. They also use a different filter ruling when formatting their portfolios.

The observations in Table 4 resemble similar results. We find that the difference in the total return P1-P5 and the Sharpe ratios are positive for both equally weighted and value weighted portfolios. This also shows that total volatility portfolios exhibit patterns similar to idiosyncratic volatility portfolios.

Blitz and van Vliet (2007) use weekly returns, while we use monthly, with total volatility as proxy and finds strong evidence of the anomaly in global markets with low versus high alpha spread of 12%. Our results are not that extreme, but we still examine that all the low minus high volatility alphas P1-P5 are positive and significant at either a 1 or 5 percentage level.

Like Baker and Haugen (2012), we also find evidence that the low volatility portfolios earn higher return than the high volatility portfolios in the Norwegian financial stock market, using total volatility as a proxy.

Table 4: Total volatility computed with three factor model residuals

Table 4.1 - Equally Weighted Portfolios						
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha	FF-4 Alpha	FF-5 Alpha
1	0.33 %	4.20 %	0.08	-1.04*** (-5.47)	-1.06*** (-5.27)	-1.13*** (-5.73)
2	0.81 %	4.77 %	0.17	-0.81*** (-4.61)	-0.78*** (-4.47)	-0.78*** (-4.26)
3	0.33 %	5.89 %	0.06	-1.73*** (-7.52)	-1.60*** (-7.05)	-1.54*** (-6.30)
4	0.44 %	7.28 %	0.06	-2.05*** (-8.52)	-1.85*** (-7.99)	-1.78*** (-7.19)
5	0.10 %	8.36 %	0.01	-2.54*** (-8.62)	-2.37*** (-8.06)	-2.14*** (-7.45)
P1 - P5	**0.23 %**	**-4.16 %**	**0.07**	**1.50*** (3.83)**	**1.30*** (3.46)**	**1.00*** (2.86)**

Table 4.2 - Value Weighted Portfolios						
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha	FF-4 Alpha	FF-5 Alpha
1	0.62 %	6.61 %	0.09	-0.79** (-2.23)	-0.75** (-2.15)	-0.69* (-1.87)
2	1.09 %	6.79 %	0.16	-0.32 (-0.98)	-0.31 (-0.83)	-0.26 (-0.75)
3	0.88 %	8.82 %	0.09	-1.06*** (-2.65)	-0.95** (-2.43)	-0.78* (-1.86)
4	0.37 %	11.55 %	0.03	-2.36*** (-4.51)	-2.15*** (-3.77)	-2.04*** (-3.58)
5	0.56 %	12.59 %	0.04	-2.46*** (-3.44)	-2.31*** (-3.31)	-2.20*** (-3.07)
P1 - P5	**0.06 %**	**-5.98 %**	**0.05**	**1.67*** (2.58)**	**1.55** (2.38)**	**1.50** (2.21)**

Table 4.1 and 4.2 shows portfolios sorted and calculated using the monthly data from the last 24 months. The calculation is based on total volatility relative to the Fama-French three factor (1993) model. The sample period is valid from January 1995 to December 2017. Table 4.1 present the equally weighted portfolios, while table 4.2 present the value weighted portfolios. Portfolio 1 is the portfolio of stocks with the lowest total volatility and portfolio 5 is the one with the highest. Mean access return (Mean) and standard deviation (Std. Dev) is measured monthly and the sharpe ratio is (Mean) divided by the (Std. Dev). The Alpha is estimates are in monthly percentage terms and reported relative to the Fama-French three factor (1993) model, four-factor model and 5 factor model. T-statistics are represented by p-values based on Newey and West (1987). Significant values are represented at the 10 percent level (*), 5 percent level (**) and 1 percent level (***).

5.4 Penny stocks

A possible explanation to the low volatility anomaly is the preference of stocks as lottery tickets. Investors prefer stocks with low value and high volatility due to a behavioural bias. This robustness test is divided in two parts. Firstly, by taking a stricter filtering by only including stocks with above NOK 10 (as suggested by Ødegaard (2017)), and secondly by including all stocks. If the preference of stocks as lottery tickets is present, the hypothesis is that P1-P5 would increase when including all stocks, and decrease when removing stocks with lower value.

When excluding stocks with lower value then NOK 10, only small changes are present when equal-weighting the portfolios in Table 5.1. The standard deviation drops in all portfolios, as expected, and the mean return drops for both P1 and P5. We observe a difference of 0.06% when looking at P1-P5. The Sharpe ratio difference is almost the same (0.06 to 0.07), and the FF-3 alpha drops from 1.50 to 1.39.
On the other hand, when value-weighting the portfolios, different results appear. Mean excess return on P5 is 1.58%, and a Sharpe ratio of 0.14 with a standard deviation of 11.18%, all numbers the highest so far for all portfolios. The control portfolio P1-P5 yields a negative return of -1.06%. However, the alpha of P5 is not significant, same as for P1-P5. This is an important result that could help us to explain why the anomaly is present in the market. The results indicate that a significant part of the effect is coming from the stocks with values lower then NOK 10.

Table 6 shows the results when including all the stocks (value beneath NOK 1). The anomaly is slightly strengthening for both equal- and value-weighted portfolios. Alphas of P1-P5 are positive and significant at all levels.

These results are interesting, as they suggest the anomaly is partly driven by the preference of stocks with high volatility due to the high correlation between low value stocks and idiosyncratic volatility. Further, a research only including liquid stocks (e.g. bid-ask spread) could help us to examine if low liquidity drives the presence of the anomaly. "Lottery-like" stocks are mentioned by both Baker and Wurgler (2011) and Blitz and van Vliet (2007) as explanations to the low volatility anomaly. The conclusion of the Norwegian stock market is that including low-valued stocks increases the presence of the anomaly.

Table 5: Idiosyncratic volatility with stock value above NOK 10

Table 5.1 - Equally Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha
1	0.45 %	3.71 %	0.12	-0.84*** (-5.60)
2	0.45 %	4.80 %	0.09	-1.11*** (-5.67)
3	0.54 %	5.80 %	0.09	-1.51*** (-5.45)
4	0.53 %	6.88 %	0.07	-1.69*** (-8.15)
5	0.39 %	8.21 %	0.05	-2.23*** (-7.34)
P1 - P5	**0.06 %**	**-2.63%**	**0.07**	**1.39*** (4.29)**

Table 5.2 - Value Weighted Portfolios				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha
1	0.51 %	5.94 %	0.09	-0.94*** (-2.88)
2	0.45 %	6.61 %	0.07	-0.91** (-1.96)
3	0.66 %	8.61 %	0.08	-1.40*** (-2.81)
4	0.06 %	9.67 %	0.01	-1.79*** (-3.30)
5	1.58 %	11.18 %	0.14	-0.80 (-1.40)
P1 - P5	**-1.06 %**	**-5.24%**	**-0.05**	**-0.13 (-0.22)**

Table 5.1 and 5.2 shows portfolios sorted and calculated using the monthly data from the last 24 months. The calculation is based on idiosyncratic volatility relative to the Fama-French three factor (1993) model. The sample period is valid from January 1995 to December 2017. Table 5.1 present the equally weighted portfolios, while table 5.2 present the value weighted portfolios. Portfolio 1 is the portfolio of stocks with the lowest idiosyncratic volatility and portfolio 5 is the one with the highest. Mean access return (Mean) and standard deviation (Std. Dev) is measured monthly and the Sharp ratio is (Mean) divided by the (Std. Dev). The Alpha is estimates are in monthly percentage terms and reported relative to the Fama-French three factor (1993) model. T-statistics are represented by p-values based on Newey and West (1987). Significant values are represented at the 10 percent level (*), 5 percent level (**) and 1 percent level (***).

Table 6: Idiosyncratic volatility including all stocks

Table 6.1 - Equally Weighted Portfolio				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha
1	0.37 %	3.93 %	0.09	-0.98*** (-5.94)
2	0.49 %	4.83 %	0.10	-1.14*** (-5.81)
3	0.29 %	6.12 %	0.05	-1.86*** (-6.54)
4	0.50 %	7.12 %	0.07	-1.88*** (-7.53)
5	0.22 %	8.51 %	0.03	-2.55*** (-7.45)
P1 - P5	**0.15 %**	**-4.58%**	**0.06**	**1.57*** (4.67)**
Table 6.2 - Value Weighted Portfolio				
Portfolio	Mean	Std Dev	Sharpe Ratio	FF-3 Alpha
1	0.54 %	5.79 %	0.09	-0.76** (-2.10)
2	0.66 %	7.59 %	0.09	-1.22*** (-3.03)
3	0.32 %	9.04 %	0.04	-1.70*** (-2.68)
4	0.39 %	10.51 %	0.04	-2.07*** (-3.84)
5	-0.47 %	12.49 %	-0.04	-3.27*** (-4.33)
P1 - P5	**1.01 %**	**-6.70 %**	**0.12**	**2.51*** (3.25)**

Table 6.1 and 6.2 shows portfolios sorted and calculated using the monthly data from the last 24 months. The calculation is based on idiosyncratic volatility relative to the Fama-French three factor (1993) model. The sample period is valid from January 1995 to December 2017. Table 6.1 present the equally weighted portfolios, while table 6.2 present the value weighted portfolios. Portfolio 1 is the portfolio of stocks with the lowest idiosyncratic volatility and portfolio 5 is the one with the highest. Mean access return (Mean) and standard deviation (Std. Dev) is measured monthly and the sharp ratio is (Mean) divided by the (Std. Dev). The Alpha is estimates are in monthly percentage terms and reported relative to the Fama-French three factor (1993) model. T-statistics are represented by p-values based on Newey and West (1987). Significant values are represented at the 10 percent level (*), 5 percent level (**) and 1 percent level (***).

5.5 Return reversals

Huang et al. (2010) argued that “return reversals imply that, all else being equal, expected VW portfolios returns will be less than expected EW portfolios returns, given that the portfolios weights are dependent on the market capitalization of the component stocks in the portfolio formation month”, and means that VW (*t+1*) will have lower expected monthly returns then EW (*t+1*). The portfolios with high idiosyncratic volatility tend to have higher return than the portfolios with low volatility for both the equally weighted (EW) and the value weighted (VW) portfolios. Huang et al (2010) presents in his article that the VW portfolio is based on market capitalization in period *t* and therefore the winner stocks receive a greater weight then the loser stocks. The return reversal in the following month makes VW (*t+1*) receive a lower return. When looking at the VW returns with high idiosyncratic volatility, the expected returns exhibits a remarkable different in the low and high volatility stocks. Looking at the return reversals in Table 7 smaller differences appear in the first two low volatile portfolios, while suddenly in portfolio four and five, a large change from positive to negative values is observed. This could be explained by the high concentration of winner and looser stocks in the high volatile portfolios, which makes a greater return reversal effect. The reversal effect on the EW on the other hand is not that great due to equally weights of loser and winner stocks.

We find strong evidence of short-term return reversal in the following month, and therefore consider that there indeed still could be a positive relation between idiosyncratic risk and expected returns.

Table 7: Return reversals

Portfolio	*1*	*2*	*3*	*4*	*5*	***1-5***
EW (t)	0.30 %	0.81 %	0.95 %	0.35 %	0.50 %	**-0.20 %**
VW (t)	0.60 %	0.47 %	1.09 %	0.61 %	0.75 %	**-0.15 %**
EW (t+1)	0.37 %	0.50 %	0.34 %	0.63 %	0.24 %	**0.13 %**
VW (t+1)	0.60 %	0.44 %	0.47 %	-0.26 %	-0.40 %	**1.00 %**

Table 7 exhibit portfolios sorted and calculated using monthly data from the last 24 months. The calculations are based on idiosyncratic volatility relative to the Fama-French (1993) three-factor model and demonstrate average monthly excess returns. The portfolios are replaced each month and consist of five different portfolios of risk. Portfolio 1 is the portfolio of stocks with the lowest idiosyncratic volatility and portfolio 5 is the one with the highest. (1-5) reports the different in the highest and the lowest volatile portfolio. The notation (t) defines the portfolio formation period and (t+1) represents the returns for the month following the portfolio formation period (t). EW (t) and EW (t+1) present equally weighted average monthly returns, while VW (t) and VW (t+1) present the value weighted average monthly returns. The weights of VW (t+1), are based on market capitalization at the end of month (t). The sample period is from January 1995 to December 2017.

6. Conclusion

Financial theory states investors demand higher expected returns when bearing higher risk. Our findings show that low-risk stocks outperform high-risk stocks on the Oslo Stock Exchange, making a great anomaly that contradicts the very core of traditional financial principles. Several other papers investigating the correlation between risk and returns in global markets confirm the low volatility anomaly. Therefore, we find our results to not be that surprising, even if it contradicts the very core of financial theory.

The anomaly is confirmed using several performance measurements, including Fama and French (1993) alphas, Sharpe ratios and mean excess returns. Looking at the control-portfolio P1-P5, which represents going long on P1 and short P5, we find positive mean returns on all our pricing models and changing to total volatility as a proxy for risk. Positive values are also found when looking at the alphas and Sharpe ratios.

We confirm that low valued stocks could potentially explain the presence of the anomaly. The confirmation of short-term return reversals on high volatile stocks also helps us to understand why our findings contradict the principle of higher risk leading to higher returns.

Reference list

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. 2006. "The Cross-Section of Colatility and Expected Returns". *The Journal of Finance*, 61(1): 259–299.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. 2009. "High idiosyncratic volatility and low returns: International and further U.S. evidence." *Journal of Financial Economics,* 91(1): 1-23.

Baker, M., Brendan B., and Wurgler, J. 2011. "Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly." *Financial Analysts Journal,* 67(1): 40-54.

Baker, N. L. and Haugen, R. A. 2012. "Low Risk Stocks Outperform within all Observable Markets of the World". *Working paper*.

Bali, T. G. and Cakici, N. 2008. "Idiosyncratic Volatility and the Cross Section of Expected Returns". *Journal of Financial and Quantitative Analysis*, 43(01): 29–58.

Bali, T. G., Cakici, N. and Chabi-Yo, F. 2012. "Does Aggregate Riskiness Predict Future Economic Downturns?," *Working Paper Series 2012-09*, Ohio State University, Charles A. Dice Center for Research in Financial Economics.

Baker N.L., Haugen R.A. 2010. Case Closed. In: Guerard J.B. (eds) *Handbook of Portfolio Construction.* Springer, Boston, MA

Banz, R. 1982: "The Small Firm Effect Revisited", *Un-published manuscript (Northwestern University)*

Black, F., Jensen, M. C., and Scholes, M. S. 1972. "The Capital Asset Pricing Model: Some Empirical Tests". *In Studies in the Theory of Capital Markets*, edited by Michael C. Jensen, 79-121. New York: Praeger Publishers Inc.

Blitz, D., and van Vliet, P. 2007. "The Volatility Effect". *Journal of Portfolio Management*, 34(1): 102-113.

Blitz, D., Pang, J., & van Vliet, P. 2013. "The Volatility Effect in Emerging Markets". *Emerging Markets Review*, 16, 31-45.

Carhart, M. M. 1997. "On Persistence in Mutual Fund Performance". *The Journal of Finance*, 52(1): 57-82.

Fama, E. F., and French, K. R. 1993. "Common Risk Factors in the Returns on Stocks and Bonds". *Journal of financial economics*, 33(1): 3–56.

Frazzini, A., and Pedersen, L. 2011. "Betting Against Beta".

Fu, Fangjian. 2009. "Idiosyncratic Risk and the Cross-Section of Expected Stock Returns." *Journal of Financial Economics,* 91: 24-37.

Haugen, R. A., & Heins, A. J. 1972."On the Evidence Supporting the Existence of Risk Premiums in the Capital Market".

Jegadeesh, N. and Titman S. 1993. “Return of Buying Winners and losers: Implications for Stock Market Efficiency” *Journal of Finance,* 48: 65-91.

Levy, H. and Markowitz, H.M. 1979. “Approximating Expected Utility by a Function of Mean and Variance”. *American Economic Review*, 69, 308-317.

Lintner, J. 1965. “The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets”. *The Review of Economics and Statistics, 47*(1), 13-37. doi:10.2307/1924119

Mossin, J. 1966. “Equilibrium in a Capital Asset Market”. *Econometrica*, 34(4), 768-783.

Merton, R. C. 1987. “A Simple Model of Capital Market Equilibrium with Incomplete Information”. *The Journal of Finance.* 42(3), 483-510.

Newey W. K., and West, K. D. 1987. "Hypothesis Testing with Efficient Method of Moments Estimation". *International Economic Review*. Vol. 28, No. 3 (Oct., 1987), pp. 777-787

Næs, R., Skjeltorp, J., & Ødegaard, B. A. 2009. "What Factors Affect the Oslo Stock Exchange". *Norges Bank (Central Bank of Norway), Working Paper.*

Riley, T. B. 2014. "Two Essays on the Low Volatility Anomaly". *Theses and Dissertations—Finance and Quantitative Methods. Paper 1.*

Sharpe, W. F. 1966. Mutual Fund Performance. *The Journal of Business,* 39(1):119– 138.

Trainor, W. J. 2012. "Volatility and Compounding Effects on Beta and Returns". *International Journal of Business and Finance Research.*

Ødegard, B. A. 2018. "Asset pricing data at OSE". *http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/index.html*

Appendices

Appendix 1

Paper	*Sample horizons*	*Markets*	*Proxy*[12]	*Datatype*	*Method*	*Anomaly?*
Haugen and Heins (1975)	1926-1971	Global	Beta	Monthly	CAPM	YES
Ang et al. (2006)	1963-2000	US	IVOL	Daily	FF	YES
Blitz and van Vliet (2007)	1986-2006	Global	TVOL	Weekly	FF	YES
Bali and Cakici (2008)	1963-2000	US	IVOL	Monthly	FF	NO
Ang et al. (2009)	1963-2003	Global	IVOL	Daily	FF	YES
Fu et al. (2009)	1963-2006	US	IVOL	Monthly	GARCH	NO
Frazzini and Pedersen (2011)	1926-2012	Global	Beta	Daily	CAPM	YES
Baker and Wurgler (2011)	1968-2008	US	TVOL, Beta	Monthly	CAPM	YES
Baker and Haugen (2012)	1990-2011	Global	TVOL	Monthly	CAPM	YES
Riley (2014)	1990-2012	US	TVOL, IVOL	Daily	FF,CAPM	YES

[12] IVOL is idiosyncratic volatility, TVOL is total volatility.

Appendix 2

Paper(s)	***Explanation***
Blitz and van Vliet (2007), Baker and Wurgler (2011).	Lottery preferences
Baker and Wurgler (2011).	Overconfidence
Baker and Wurgler (2011).	Limited arbitrage
Frazzini and Pedersen (2011).	Leverage constraints
Ang et al (2006/2009), Baker and Wurgler (2011).	The urge to beat benchmark
Riley (2014).	Volatility estimation
Fu et al (2009), Huang et al (2010).	Return reversals

Appendix 3

Year	*Number of firms*[13]
1990	64
1991	68
1992	75
1993	67
1994	84
1995	97
1996	107
1997	137
1998	147
1999	154
2000	141
2001	145
2002	169
2003	176
2004	174
2005	184
2006	215
2007	240
2008	237
2009	213
2010	205
2011	195
2012	186
2013	174
2014	163
2015	150
2016	144
2017	139

[13] Since we compute volatility and construct portfolios on a monthly basis, number of firms are the average amount of firms every month in the respective year..

Appendix 4.1

		Equally-weighted					
Fama-French Models:		**Fama-French 3 factor**					
Portfolio (1-5)		*P1*	*P2*	*P3*	*P4*	*P5*	***P1-P5***
Constant	(α)	-0.98***	-1.13***	-1.81***	-1.72***	-2.47***	**1.50*** **
		(-5.86)	(-5.69)	(-6.40)	(-7.19)	(-7.25)	**(4.36)**
MKT	(β1)	0.69***	0.83***	1.05***	1.17***	1.29***	**-0.60*** **
		(16.65)	(17.48)	(17.92)	(17.77)	(16.03)	**(-8.23)**
SMB	(β2)	0.22***	0.25***	0.43***	0.42***	0.62***	**-0.40*** **
		(3.76)	(4.15)	(5.17)	(4.70)	(5.67)	**(-3.34)**
HML	(β3)	0.13***	0.06	-0.05	-0.03	-0.23**	**0.36*** **
		(3.28)	(1.37)	(-0.91)	(-0.34)	(-1.97)	**(3.10)**
		Fama-French 4 factor					
Constant	(α)	-0.94***	-1.18***	-1.68***	-1.32***	-2.36***	**1.42*** **
		(-5.66)	(-6.01)	(-6.81)	(-6.14)	(-7.30)	**(4.38)**
MKT	(β1)	0.67***	0.85***	1.04***	1.12***	1.31***	**-0.64*** **
		(15.84)	(19.51)	(21.05)	(21.46)	(19.25)	**(-10.09)**
SMB	(β2)	0.22***	0.24***	0.43***	0.45***	0.66***	**-0.44*** **
		(3.97)	(4.32)	(7.37)	(5.60)	(7.28)	**(-3.75)**
HML	(β3)	0.13***	0.06	-0.05	-0.07	-0.24**	**0.38*** **
		(3.15)	(1.19)	(-0.98)	(-1.22)	(-2.54)	**(3.61)**
UMD	(β4)	0.0003	-0.06	-0.12**	-0.26***	-0.15**	**0.16** **
		(0.01)	(-1.51)	(-2.39)	(-4.96)	(-2.20)	**(2.45)**
		Fama-French 5 factor					
Constant	(α)	-0.93***	-1.24***	-1.56***	-1.13***	-2.34***	**1.40*** **
		(-5.81)	(-6.17)	(-6.33)	(-4.50)	(-6.73)	**(4.65)**
MKT	(β1)	0.73***	0.83***	1.02***	0.98***	1.13***	**-0.40*** **
		(14.41)	(16.10)	(18.69)	(15.73)	(12.44)	**(-4.92)**
SMB	(β2)	0.20***	0.26***	0.43***	0.50***	0.79***	**-0.58*** **
		(3.50)	(4.19)	(6.66)	(5.79)	(7.72)	**(-5.53)**
HML	(β3)	0.11***	0.07	-0.04	-0.06	-0.21**	**0.32*** **
		(2.66)	(1.46)	(-0.93)	(-1.05)	(-2.20)	**(3.66)**
UMD	(β4)	-0.01	-0.05	-0.13***	-0.25***	-0.17***	**0.17*** **
		(-0.18)	(-1.19)	(-2.75)	(-4.84)	(-2.81)	**(2.91)**
LIQ	(β5)	0.12**	-0.05	-0.05	-0.25***	-0.38***	**0.50*** **
		(2.28)	(-0.78)	(-0.63)	(-2.76)	(-2.77)	**(3.33)**

Appendix 4.2

	Value-weighted					
Fama-French Models:	**Fama-French 3 factor**					
Portfolio (1-5)	*P1*	*P2*	*P3*	*P4*	*P5*	***P1-P5***
Constant (α)	-0.80**	-1.33***	-1.35**	-2.57***	-3.08***	**2.28*** **
	(-2.21)	(-3.36)	(-2.12)	(-4.42)	(-4.44)	**(3.23)**
MKT (β1)	0.71***	0.93***	0.90***	1.11***	1.22***	**-0.51*** **
	(10.01)	(7.24)	(7.57)	(10.11)	(8.53)	**(-3.74)**
SMB (β2)	0.23**	0.19	0.36***	0.51***	0.76***	**-0.54*** **
	(2.06)	(1.18)	(2.92)	(3.40)	(4.74)	**(-3.20)**
HML (β3)	-0.01	0.07	0.02	-0.01	-0.16	**0.15**
	(-0.10)	(0.99)	(0.16)	(-0.08)	(-0.85)	**(0.74)**
	Fama-French 4 factor					
Constant (α)	-0.52	-0.92**	-0.51	-1.06*	-2.28***	**1.76***
	(-1.60)	(-2.08)	(-0.89)	(-1.70)	(-2.86)	**(1.96)**
MKT (β1)	0.65***	0.78***	0.84***	1.00***	1.11***	**-0.46*** **
	(9.88)	(11.69)	(9.55)	(7.42)	(8.13)	**(-3.08)**
SMB (β2)	0.17	0.29**	0.48***	0.60***	0.43*	**-0.25**
	(1.58)	(2.37)	(3.31)	(4.70)	(1.84)	**(-0.89)**
HML (β3)	-0.02	-0.05	-0.04	0.09	-0.05	**0.02**
	(-0.24)	(-0.56)	(-0.41)	(0.84)	(-0.26)	**(0.12)**
UMD (β4)	0.06	-0.10	-0.24**	-0.17	-0.01	**0.07**
	(0.93)	(-1.45)	(-2.44)	(-1.57)	(-0.06)	**(0.46)**
	Fama-French 5 factor					
Constant (α)	-0.74*	-0.85**	-1.78***	-0.98*	-1.66***	**0.92**
	(-1.86)	(-2.29)	(-3.47)	(-1.80)	(-2.98)	**(1.38)**
MKT (β1)	0.83***	0.76***	1.06***	1.06***	0.95***	**-0.12**
	(6.46)	(7.51)	(8.02)	(7.64)	(5.63)	**(-0.55)**
SMB (β2)	0.13	0.24**	0.32*	0.61***	0.69***	**-0.55*** **
	(1.39)	(2.23)	(1.91)	(3.57)	(3.99)	**(-2.86)**
HML (β3)	0.05	-0.04	-0.01	0.06	-0.06	**0.12**
	(0.61)	(-0.53)	(-0.14)	(0.56)	(-0.47)	**(0.75)**
UMD (β4)	-0.08	-0.07	-0.29***	-0.23*	-0.12	**0.04**
	(-0.74)	(-1.20)	(-3.55)	(-1.87)	(-1.22)	**(0.27)**
LIQ (β5)	0.24	0.04	0.19	-0.08	-0.31	**0.55***
	(1.48)	(0.35)	(1.01)	(-0.41)	(-1.29)	**(1.90)**