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Mutual Fund Management Costs and Lifetime Wealth Accumulation – A Consumption Utility Function

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Abstract

I am studying how mutual fund management costs affect lifetime wealth accumulation and thus consumption utility. I take an individual who is 25 years old and starts saving for retirement, retires at age 65 and stays retired until age 90. Such an investor is faced with the decision whether to invest in actively managed mutual funds or passively managed mutual funds. A simulation of 1,000 scenarios is performed, where returns are randomly drawn from the sample of net-of-fees real fund returns from the period of 1977 to 2016. I find that in 92.8% of cases an investor experiences greater consumption utility if she is investing in passively managed mutual funds. I conclude that higher fees of actively managed mutual funds outweigh supposedly higher returns and do not result in higher consumption utility.

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I Introduction

With combined value of net total assets of $16.34 trillion, mutual funds in the US represent one of the most popular securities (Statista, 2017). Investors are looking for the best performers and are predominantly focused on high returns, but often neglect management fees. Yet, these on-a-first-glance miniscule costs may significantly affect wealth accumulation over a longer period of time. Differences between management fees of different mutual funds vary from 1.5 to 2 percentage points and may seem irrelevant in absolute terms, however, as Ellis (2012) pointed out, if we calculate the management fees as percentage of returns instead of percentage of invested assets, fees no longer look low (Ellis, 2012, p. 4). Traditionally, lower fees are associated with passively managed mutual funds or index funds, which mimic a selected benchmark. On the other hand, actively managed mutual funds try to outperform a benchmark, thus experience larger asset turnover and are incurring higher costs that translate in higher management fees. For the purpose of this thesis, passively managed mutual costs are sometimes referred to as a low-cost fund or a low-cost investment, and actively managed mutual costs are hence referred as a high-cost fund or a high-cost investment.

What differences in management costs convert to in the long period of time and which type of investment is better may be interesting especially for small retail investors who are saving for retirement. Several studies offer an answer to the puzzle. The most famous being Sharpe's (2013) and Bogle's (2014). William F. Sharpe (2013) assumes expense ratios of 0.06% and 1.12% for a low-cost fund and a high-cost fund, respectively. A real return on investment is randomly drawn from normal distribution with average of 6.9% and standard deviation of 17.7% – it is the same for both investment alternatives, as he believes there are no abnormal returns in the long run. A 30-year simulation with 1,000,000 scenarios shows that a low-cost fund investor ends up with more than 20% greater wealth compared to a high-cost fund. Another study made by John C. Bogle (2014) argues that difference is in fact even larger if all costs are considered. Therefore, in addition to management fees he includes also transaction costs, cash drag, fees to brokers, and sales load. With differences in management fees of 2.21 percentage points and 7%

annual return for both alternatives, he calculates that in 40 years of compounding the advantage of a low-cost fund grows to staggering 65%.

My work contributes to the literature by expanding existing research in three aspects: (1) use of real fund data, (2) application of a lifetime approach, and (3) focus on consumption utility rather than only on total wealth. I introduce the model which observes an investor of age 25 who retires at age 65 and expects to live for another 25 years. First 40 years represent a period of funds accumulation as an investor saves 10% of her salary for retirement in mutual funds. When she retires, she sells a fraction of her funds each year and these proceeds represent her retirement income. That is a period of funds decumulation. If after death the fund is not fully exhausted, the remaining part is called a bequest wealth. Returns on investments are simultaneously drawn randomly with replacement from the sample of net-of-fees real returns of actively managed and passively managed mutual funds, respectively, from the period of 1977 to 2016. Both types of funds are domiciled in the US and have a mandate to invest solely in large capitalization US stocks. After simulating 1,000 possible scenarios, aggregate consumption utilities of both investment alternatives are being compared.

I find that in 92.8% of the cases an investor who chooses a low-cost alternative enjoys greater consumption utility than the one choosing a high-cost alternative. The lifetime utility of a low-cost investment is on average 4.4% greater than the one of a high-cost alternative. Although the size of an average benefit may not seem too striking, translated in monetary terms it means on average 19.1% higher retirement income and 117% higher bequest wealth – a considerable advantage of the passive over the active investment style. A low-cost alternative is also more reliable in providing retirement income larger than the last salary. The latter situation happens in 60.1% of years for a low-cost investment and decreases to 52.1% of years for a high-cost alternative.

Even extreme scenarios are exposing a larger potential of a low-cost alternative. Maximum advantage in utility that the latter can provide over a high-cost counterpart is 19.2%, compared with other extreme – a maximum advantage of a high-cost alternative of only 7.9%.

The rest of the thesis is organized as follows. Section 2 presents a review of the literature on mutual fund management costs. Section 3 outlines the methodology used to compare the aggregate consumption utilities. Section 4 describes the data. Section 5 presents my findings. Section 6 concludes the thesis.

II Background and literature

What differences in management costs translate into in a couple of decades is the starting point of the study conducted by William F. Sharpe (2013). In his paper, he introduces the *terminal wealth ratio* (TWR) term. It is an easy-to-calculate measure of what effect differences in management fees make over a certain period of time. In its final form it is defined as shown in Equation (2.1):

$$TWR = \left[\frac{(1 - x_1)}{(1 - x_2)}\right]^n \frac{G_{n1}}{G_{n2}} \tag{2.1}$$

where compounded gross return for each investment is denoted as G_n, with *1* being a low-cost investment and *2* being a high-cost investment, expense ratios are x_1 and x_2, and n is the number of years an investment is held for.

Sharpe even performs a Monte Carlo simulation where future gross returns differ between funds and which generates one million possible 30-years scenarios, computes the TWR and the range of the ratios across those scenarios. As a management fee of a low-cost fund, he takes Vanguard Total Stock Market Index Fund Admiral Shares' expense ratio of 0.06% a year, whereas the average expense ratio of a high-cost fund is assumed to be 1.12%. A return on a low-cost fund is drawn randomly from normal distribution. The return distribution is based on the historical performance of an index of a global real stock return; average annual real return is 6.9% with a standard deviation of 17.7% a year. The return on a high-cost fund is the sum of the return on a low-cost fund and the tracking error as shown in the equation below (2013, p. 37, 38).

$$\tilde{r}_n = \tilde{r}_l + \tilde{\varepsilon} \tag{2.2}$$

In addition, Sharpe calculated the probabilities that TWR exceeds 1.0 given the expense ratios and tracking error standard deviation. In the scenario of lump-sum investment with uncertain returns, with greatest active risk (0.050) for more than 90% of the cases the TWR will exceed 1.0, meaning that the investor in a high-cost fund will be poorer than low-cost fund investor (2013, p. 39). The analysis of recurring investment with uncertain returns yielded that after 30 years there is only

10% chance a low-cost fund will provide an investor with less wealth than a high-cost fund with the similar investment style (2013, p. 40).

Although I pay great respect to his work, Sharpe makes calculations with uncertain returns obtained from normal distribution. This is an aspect where my thesis introduces real funds returns. Using his approach, Sharpe concluded that differences in management fees translate in more than 20% higher standard of living if a person chooses low-cost investments (2013, p. 34).

This number was later challenged by John C. Bogle (2014), who takes different path and shows that differences are in fact even greater and can reach as high as 65% (2014, p. 17). Sharpe's approach is quantitatively superior to Bogle's. Bogle takes, as he calls it, the "all-in" approach, including not only expense ratios, but also fund transaction costs, sales loads, and cash drag. He argues that whereas index funds are fully invested, portfolios of actively managed funds always carry a cash position of about 5%, causing the funds to lose a portion of the long-term equity premium (2014, p. 14). Furthermore, index funds are also more tax efficient as they operate with minimal turnover portfolio and do not realize capital gains. Although tax represents an additional drag, it is not of immediate concern for investors in tax-deferred retirement plans.

The first of the hidden costs he includes in the analysis are transaction costs. There are different estimates of these costs in academic circles, ranging from 0.30% to 1.44%, so not risking the overstatement and for rounding purposes, he takes the number 30 bp. Assuming 5% cash position, as mentioned above, and annual long-term equity premium of 6%, he calculates an additional 30 bp cash drag for actively managed funds. Front-end sales loads averaged about 8% of the dollar amount of sales purchased until late 1970s, and have dropped to 5% since then. As he explains, in the new environment, fees paid by investors to brokers and investment advisers typically run to about 1% per year. For annual broker and advisor costs and sales loads, he takes the number 50 bp (2014, p. 15-16).
Putting it all together, his "all-in" investment expenses calculation results in 2.27% expenses for actively managed funds and 0.06% for index funds. As he puts this into perspective, after 40 years of savings, index advantage is 65%, assuming 7% nominal annual return on both actively managed and index funds.

Bogle then includes also tax inefficiency and inflation rate of 2% and shows that real return would then fall from original 7% to 1.98% for actively managed funds and 4.64% for index funds. The initial investment of $10,000 compounded for 40 years would grow to $22,000 and staggering $61,000 in real terms in the case of an actively managed fund and an index fund, respectively.

Bogle many times emphasises the imprecision of his data, which together with the lack of qualitative support is a major drawback of his paper. In his analysis, he uses constant 7% annual return with no standard deviation for both investment alternatives. Nevertheless, he offers a good intuition for understanding the differences between index and actively managed funds.

III Methodology

The design of my model closely follows the real world to provide as meaningful results as possible. The US Bureau of Labor Statistics (2017) reports that in the second quarter of 2017 the median annual salary for 20 to 24-year-olds was $27,300 and the median annual salary for 25 to 34-year-olds was $40,352. Using interpolation, it can be estimated that the median annual salary for 25-year-old person is $36,532. Median annual salary of 55 to 64-year-olds in the same quarter was $50,232 (US Bureau of Labor Statistics, 2017). This suggests that current 25-year-olds could experience 37.5% pay raise in real terms before retirement. To get an estimation of a constant year-on-year salary increase, assuming the 40-year working period, we calculate 40^{th} root of 1.375, which amounts to 0.00079. Current life expectancy in the US for a 25-year-old, the average of both genders, is 88.75 years (US Social Security Administration, 2017).

By developing and applying a proprietary code in R, my model considers an investor who is 25 years old, employed, and with no prior retirement savings. She has an annual salary of $36,000, which is expected to increase in real terms by 0.8% every year. She saves for retirement 10% of her annual income and once a year buys a share of a mutual fund. At age 65 she retires and is expected to be drawing funds from her portfolio until age 90. When retired, she wishes to receive a retirement payout of the same amount as her last salary. If an actual retirement payout exceeds her last salary, she considers the year as favorable, and if an actual retirement plan does not reach even a half of her last salary, she considers the year as a very poor. From such a payout, she receives disutility. She is faced with the decision whether to invest in actively managed mutual funds or passively managed mutual funds.

To observe which investment style provides greater lifetime consumption utility and to quantify the benefit, the *Utility ratio* (UR) is formed in Equation (3.1). Since the previous research suggest the supremacy of a low-cost investment, it is put in the numerator for an easier interpretation of the results.

$$UR = \frac{U_{0,p}}{U_{0,a}} \tag{3.1}$$

The calculation of the UR requires an aggregate utility of each investment style; aggregate utility when investing in passively managed mutual funds, $U_{0,p}$, and aggregate utility when investing in actively managed mutual funds, $U_{0,a}$. It is computed as follows:

$$U_{0,x}(C_1, C_2, \dots, C_{T,x}, B_{T,x}) = \sum_{s=1}^{T} \rho^s U_{s-1}(C_{s,x}) + \rho^T V_{T-1}(B_{T,x}) \qquad (3.2)$$

where x can be either p or a, denoting *passive* or *active* investment style, respectively. Time of death less the current age is the time T, which in this model equals to 65. It is assigned to the last consumption when alive and to a bequest wealth, denoted as $B_{T,x}$. The discount factor, ρ, is set to value 0.98 for the purposes of this thesis. It implies that the early consumption is valued more than the later one. People still care about the bequest wealth, although not in the same magnitude as for consumption when alive. To reflect this characteristic, I assigned a factor ϕ of value 0.25 in the model. This demonstrates that a bequest wealth of $100,000 gives a person the same satisfaction as $25,000 of consumption when alive. The shape of the function is a square-root function, a widely-used function for calculating the utility due to its concave down increasing shape. These changes are implemented in Equation (3.3).

$$U_{0,x}(C_1, C_2, \dots, C_{T,x}, B_{T,x}) = \sum_{s=1}^{T} 0.98^s \sqrt{C_{s,x}} + 0.98^T \sqrt{0.25 \cdot B_{T,x}} \qquad (3.3)$$

When working, an investor consumes an annual income, I_s, less the savings for retirement in this year, S_s. That holds until the time n, which is the difference between the retirement age and current age, in our case $n = 40$.

$$C_s = I_s - S_s\,; \quad \forall\, s = 1, \dots, n \qquad (3.4)$$

Savings for retirement, S_s, are a function of an annual income, I_s. In this model, the savings rate remains constant and is denoted as sr.

$$S_s = sr \cdot I_s \; ; \quad \forall\, s = 1, \dots, n \tag{3.5}$$

An annual income, I_s, is increasing from its base value when 25 years old, I_0, with a constant fraction i.

$$I_s = I_0(1+i)^s \; ; \quad \forall\, s = 1, \dots, n \tag{3.6}$$

Now that we have established what defines consumption when working, let us focus on the consumption when retired. It is the same as the payout from the pension plan for a given year in retirement. Note that now it differs depending on an investment alternative.

$$C_{s,x} \;=\; payout_{s,x} \;\; ; \quad \forall\, s = n+1, \dots, T \tag{3.7}$$

I formed a simple payout rule that considers your life expectancy and value of the funds in the retirement plan. Such a characteristic insures lower payouts in years with negative returns and gives the plan a chance to accumulate funds in the future and not get completely exhausted too early.

$$payout_{s,x} = \frac{fund\ balance_{s,x}}{(T-S)} \; ; \quad \forall\, s = n+1, \dots, T \tag{3.8}$$

The fund balance, $fund\ balance_{s,x}$, consists of a period of funds accumulation and a period of funds decumulation. In the first period, retirement savings are being added to the fund and compounded by a coefficient of return, $r_{n,x}$, drawn randomly with replacement from the sample of returns from 1977 to 2016. This is done simultaneously for both investment alternatives: $r_{n,a}$ and $r_{n,p}$, actively managed mutual funds and passively managed mutual funds, respectively. The coefficients of return in the sample are in real terms and cleared of management costs. More about this can be found in section 4. At no point in time the pension plan adds other investments (e.g. bonds) to the portfolio. The purpose of this thesis is to show how mutual fund management costs affect lifetime consumption utility, therefore I stick with the same investment for the whole lifetime.

$$fund\ balance_{s,x} = S_1 \prod_{m=1}^{s} r_{m,x} + \cdots + S_n \prod_{m=n}^{s} r_{m,x} - payout_{n+1} \prod_{m=n+1}^{s} r_{m,x} - \cdots - payout_{s-1} \cdot r_{s,x} \quad (3.9)$$

A maximum payout is capped at 3.5-times the last salary, I_n, as it is not realistic that a person would consume more.

$$payout_{max} = 3.5 \cdot I_n \quad (3.10)$$

A minimal payout, which an investor wishes to receive and still gets a positive utility from, is a half of the last salary:

$$payout_{min} = 0.5 \cdot I_n \quad (3.11)$$

However, payouts can drop even below that value. Should such a situation occur, a disutility is assigned to the aggregate consumption function. To ensure that the aggregate utility does not become negative, a special disutility form was developed as shown by Equation (3.12). For example, if a payout reaches only 80% of the minimum payout, a disutility of 20% of the minimum payout is assigned. Recall Equation (3.2), due to the discount factor, ρ, an earlier low payout is penalized more than a later one. If an annual payout drops to \$6,000 or less, it is counted as a bankruptcy. That does not necessarily mean that the plan will fail to provide future payouts – in case of favorable future returns it may grow back to the level where it can provide higher payouts.

$$payout_s < payout_{min} \Longrightarrow$$

$$U(C_s) = -\sqrt{\left(1 - \frac{payout_s}{payout_{min}}\right) \cdot payout_{min}}\ ; \quad \forall\ s = n+1, \dots, T \quad (3.12)$$

In the last year of retirement, the plan pays out the greater of the two: either the amount of the last salary, I_n, or all the remaining funds in the plan, $fund\ balance_{T,x}$.

$$payout_{T,x} = \max\left(I_n, fund\ balance_{T,x}\right) \tag{3.13}$$

After the last payout, $payout_{T,x}$, if there is still some wealth left in the fund, it is called a bequest wealth, $B_{T,x}$.

$$B_{T,x} = fund\ balance_{T,x} - payout_{T,x} \tag{3.15}$$

Similarly to Sharpe (2013), the model calculates the TWR when an investor retires. However, Sharpe includes a 30-year period, whereas in my model a 40-year period is used. That does not make the statistics directly comparable, as due to the longer compounding period this TWR should surpass the value of Sharpe's TWR.

$$TWR = \frac{fund\ balance_{n,p}}{fund\ balance_{n,a}} \tag{3.16}$$

IV Data

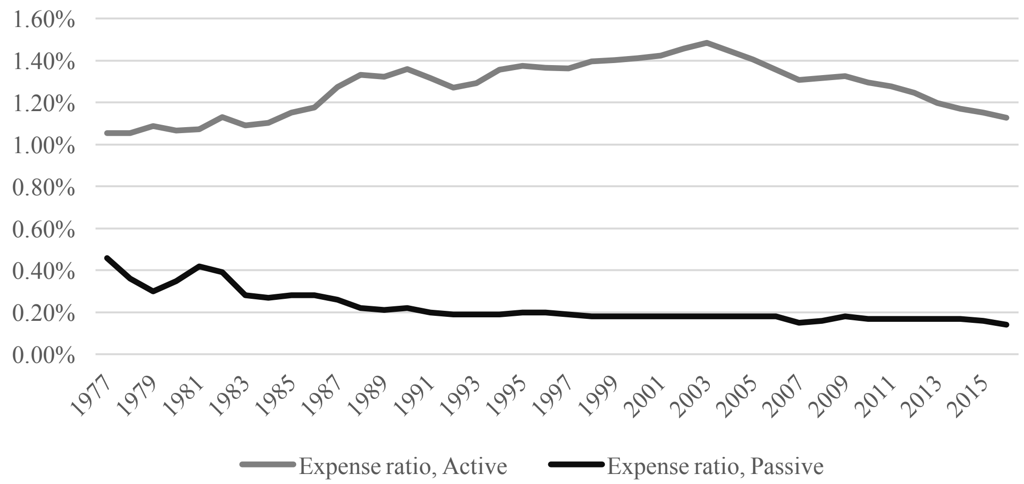
In this study, I use yearly returns, annual net expense ratios, and yearly net assets on 10,829 mutual funds during the period from 1977 to 2016. Dataset includes all US domiciled mutual funds with a mandate to invest in US large capitalization stocks. The data is obtained from Morningstar's platform. Morningstar is an investment research and investment management firm with a comprehensive database on mutual funds.

The mutual funds are divided with respect to their management style. Survivorship bias is avoided as the dataset includes even funds that ceased to exist. All except for one are actively managed mutual funds. The return on an actively managed investment is in fact the weighted average of returns of 10,828 actively managed funds weighted by funds' net assets. The larger the net assets, the larger the weight in return calculation. This characteristic ensures that funds with larger net assets are represented to a greater extent. Apart from the high returns which are inflating net assets, the other explanation is that a greater number of investors chose to trust their money to a mutual fund, which leads to higher net assets. However, there was no option to filter the funds on the minimal investment, so the funds that are not within the reach of a retail investor would be excluded from the population. That may be an advantage over the passive investment.

In the group of passively managed mutual funds I included just Vanguard 500 Index Investor, the very first index fund, with first observable return in 1977. For that reason, year 1977 is the starting year in the dataset. Moreover, the minimal investment in Vanguard 500 Index Investor is $3,000 and is as such appropriate to use it in my model (Vanguard, 2017). Vanguard 500 Index Admiral offers even lower expense ratio, yet the minimal investment is set at $10,000. Nevertheless, it represents an important constraint of the data.

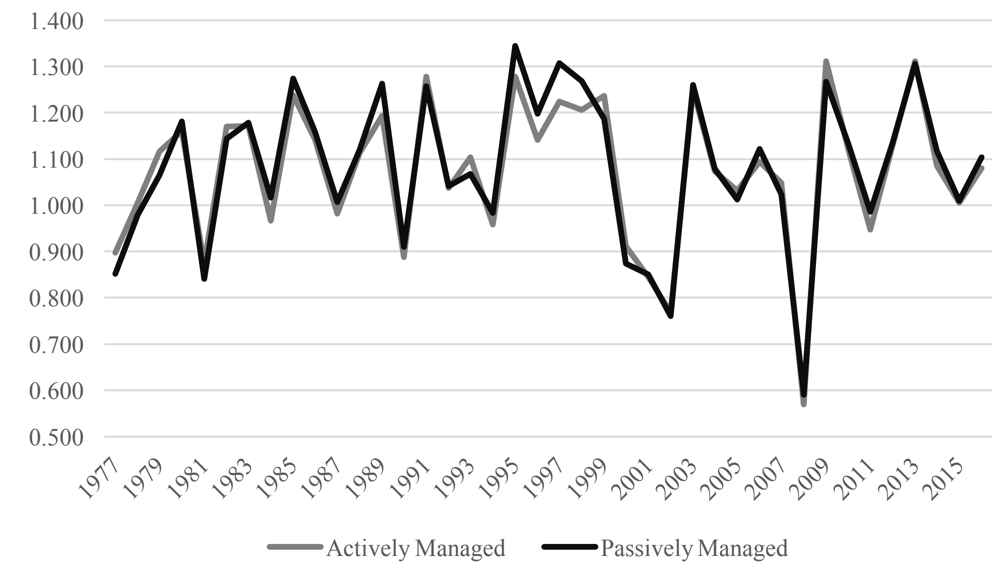
As the other costs from Bogle's approach (2014) are difficult to estimate, this study is concentrated only on expense ratios published in annual reports. These are collected by Morningstar. Figure 1 shows the evolution of expense ratios over the last 40 years. For actively managed mutual funds an average is calculated. The constant decline in expense ratio of a passive investment is not matched with the decline in the same ratio of an active investment; the latter trend being present as late as from year 2003 onwards.

Figure 1: Comparison of an average expense ratio through years



Having cleared the nominal returns of expense ratios and inflation, obtained from Federal Reserve of St. Louis database, coefficients of net-of-fees real returns were calculated and are presented in Figure 2 below.

Figure 2: Coefficients of net of fees real returns



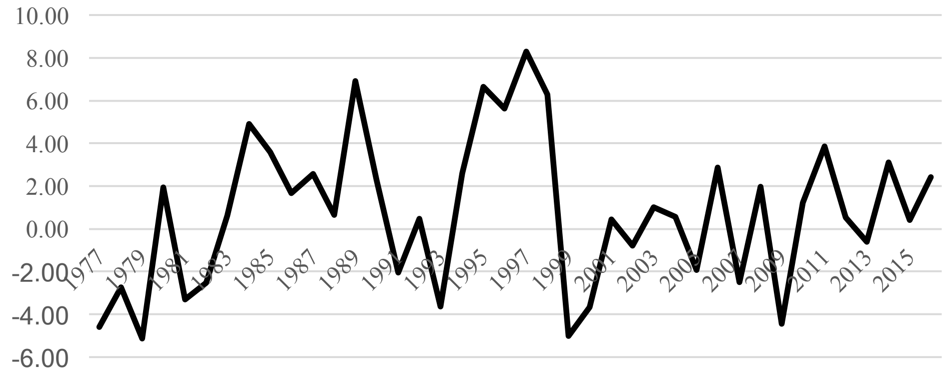
As it can be seen in Table 1, a simple comparison of descriptive statistics between the two investment alternatives shows the dominance of the passively managed investment. It provides 0.8 percentage points greater mean return, 1.2 percentage points greater median return, 2 percentage points lower minimal return, and 3.3 percentage points higher maximal return. Additionally, annual net of fees real return of the passive investment is in 65% of the years greater than the one of the active investment.

Table 1: Descriptive statistics

	Actively Managed	*Passively Managed*
Mean	1.074	1.082
Standard Error	0.025	0.026
Median	1.099	1.111
Standard Deviation	0.158	0.166
Minimum	0.570	0.590
Maximum	1.311	1.344
N	40	40

Perhaps even more revealing visual comparison is the difference in net-of-fees real returns between the two investments. It can be again noticed that differences fall in negative part of the scale just in 14 out of 40 years. See that deviations in a positive direction are also larger.

Figure 3: Difference in net-of-fees real returns in percentage points; (passive vs. active)



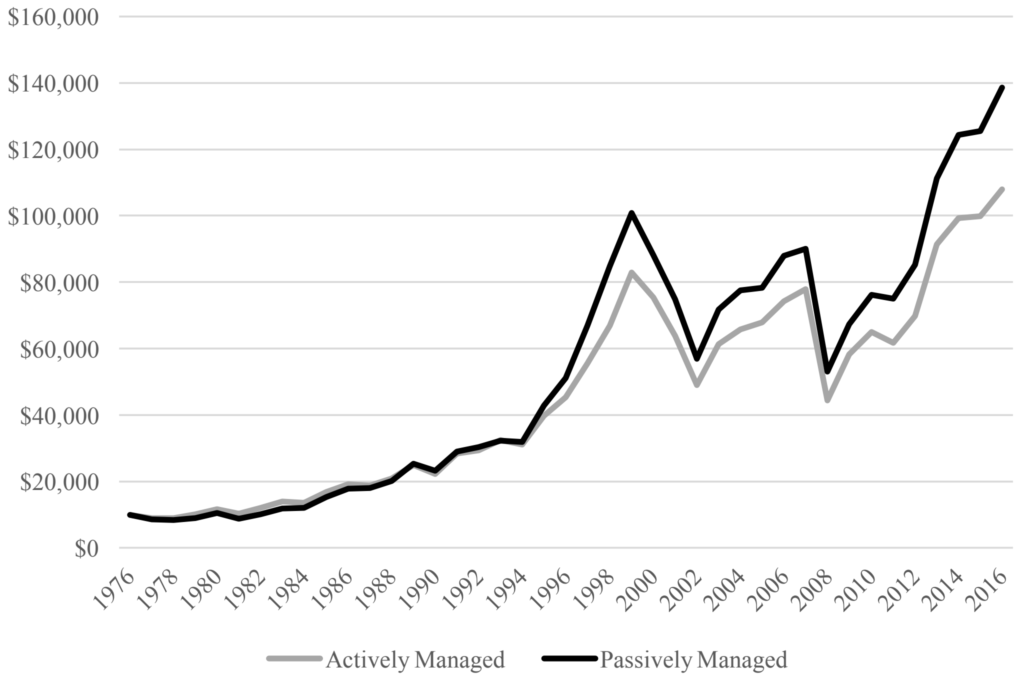
Being aware of the limitations of the dataset with only 40 histories, I still argue that the dataset provides decent variability of returns as it covers periods of the bear as well as bull markets.

V Results

As Sharpe (2013) and Bogle (2014) do this simulation, firstly I observe what happens to a lump-sum of $10,000 invested in both alternatives in 1976. Returns are net of fees and net of inflation and are applied chronologically as they actually occurred. Sharpe (2013), who takes an average annual real return of 6.9% and a standard deviation of 17.7% a year, reports “more than 20% difference” in favor of the low-cost investment after the investment period of 30 years. From Figure 4 it can be observed that in 2006 the investment in actively managed funds grew to $74,237 as opposed to $88,006 in passively managed funds, which represents 18.5% increase in favor of low cost investment.

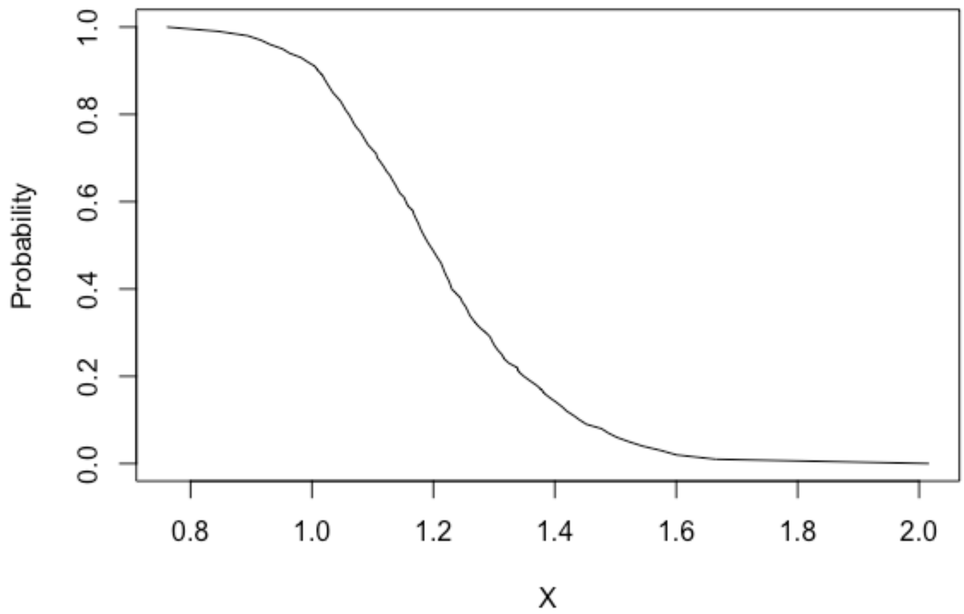
Bogle (2014) uses a different, “all-in” approach with constant 7% nominal annual return, making the results even less directly comparable. Nevertheless, in 2016, after 40 years of compounding, the investment in actively managed funds was worth $107,897 compared to $138,684 in passively managed funds – an increase of 28.5%, far from his stated index advantage of 65%.

Figure 4: Lump-sum of $10,000 invested for 40 years with real fund net-of-fees return



Sharpe also introduces the *terminal wealth ratio* (TWR) term and finds that in more than 90% of the cases TWR will exceed 1.0, meaning that the investor in a high-cost fund will be poorer than the low-cost fund investor (2013, p. 39). Figure 5 shows that when using real fund data, the probability for such a scenario is 91.3%. He further argues that there is a 50% chance that the TWR will be roughly 1.38 or greater (2013, p. 39). In my simulation, there is a 50% chance that the TWR will be equal or greater than 1.194.

Figure 5: Probability that the Terminal Wealth Ratio exceeds X

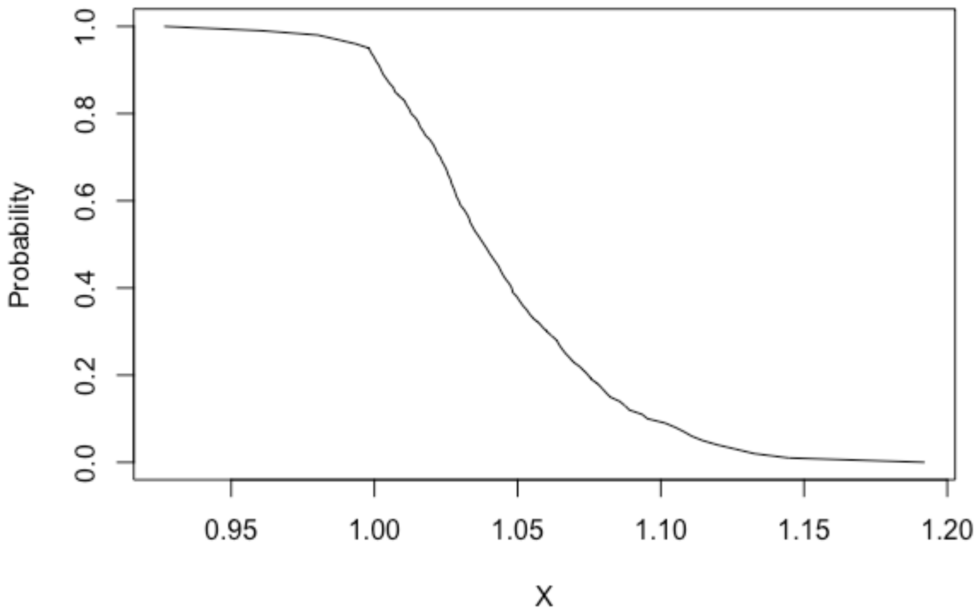


Finally, applying the methodology on the data, the results suggest that an investor who is saving in a low-cost alternative experiences, on average, 4.4% greater aggregate consumption utility than an investor who is saving in a high-cost alternative. Summary statistic of the Utility ratio is presented in Table 2. Recall the Equation (3.12) of aggregate utility function; most of the utility value is gained in early years, due to the discount factor ρ^s. Such a feature implies that a relatively large difference in consumption in retirement would result in a modest increase of the aggregate utility. However, in 92.8% of scenarios low-cost funds do provide greater aggregate utility, as the value of the $UR > 1$. They also perform better on the extremes as it can be observed from Figure 6. In best case scenario for a high-cost investment, it can outperform the low-cost counterpart by 7.9%, whereas the maximum advantage of a low-cost investment could be up to 19.2%. Again, note that the shape of the utility function is a power function, with applied discounting factor.

Table 2: Summary of the Utility ratio

	Utility ratio
Minimum	0.9266
1st quartile	1.0179
Median	1.0383
Mean	1.0442
3rd quartile	1.0667
Maximum	1.1921

Figure 6: Probability that the Utility ratio exceeds X

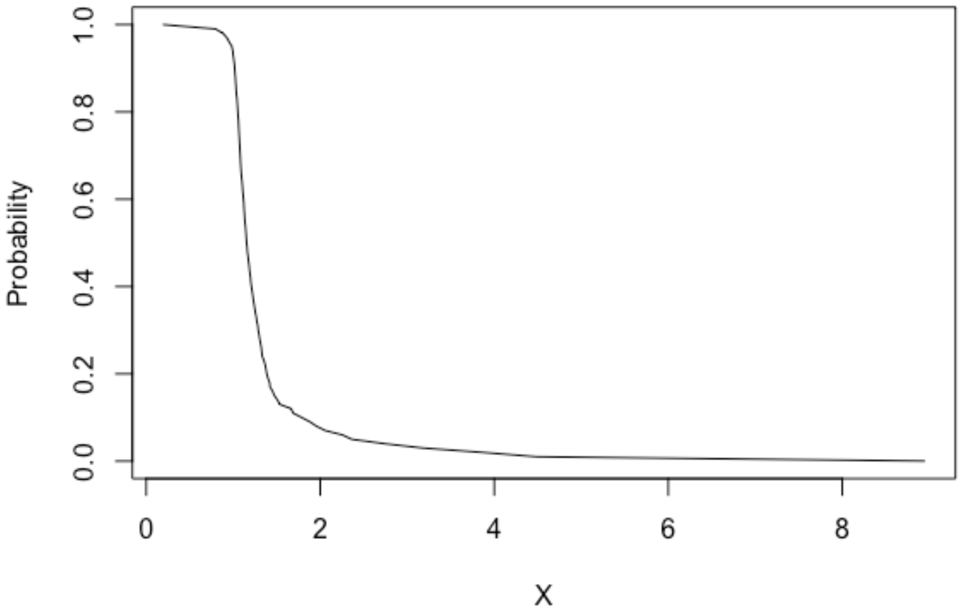


Now, for a moment assume that we are interested only in the utility in a retirement period, as the consumption when working is the same regardless of the investment style. The procedure is outlined in Equations A.1, A.2 and A.3 in Appendix. The results are substantially different. Summary statistic of retirement utility ratio is presented in Table 3. Mean benefit of the low-cost investment now increases to 34.9%. Again, it is heavily influenced by some outliers, and therefore a median is more suitable parameter to observe. In 50% of scenarios, the retirement utility of the low-cost investment is greater by at least 15.4%. Figure 7 clearly exposes that the range of minimum and maximum utilities is wider now. In the best case scenario for a high-cost investment it can outperform the low-cost counterpart by 422%, whereas the maximum advantage of the low-cost investment could be up to 795%. Yet, these are extremely unlikely scenarios.

Table 3: Summary of the Utility ratio at retirement

	Retirement utility ratio
Minimum	0.1913
1st quartile	1.0671
Median	1.1537
Mean	1.3488
3rd quartile	1.3282
Maximum	8.9504

Figure 7: Probability that the Utility ratio at retirement exceeds X



People usually understand numbers expressed in monetary or percentage terms better than utilities, therefore such an explanation is offered below. Table 4 summarizes the findings.

Table 4: Summary of retirement benefits

	Actively Managed	*Passively Managed*
The last salary	$49,513	$49,513
Mean retirement benefit	$71,299	$84,964
Mean bequest wealth	$1,151,762	$2,495,382
Mean retirement benefit, %**	144%	171.6%
Retirement benefit > last salary*	52.1%	60.1%
Retirement benefit < 1/2 last salary*	24.3%	18.3%
Probability of at least one bankruptcy	65.8%	53.1%
Mean years of bankruptcy*	3.7%	2.9%

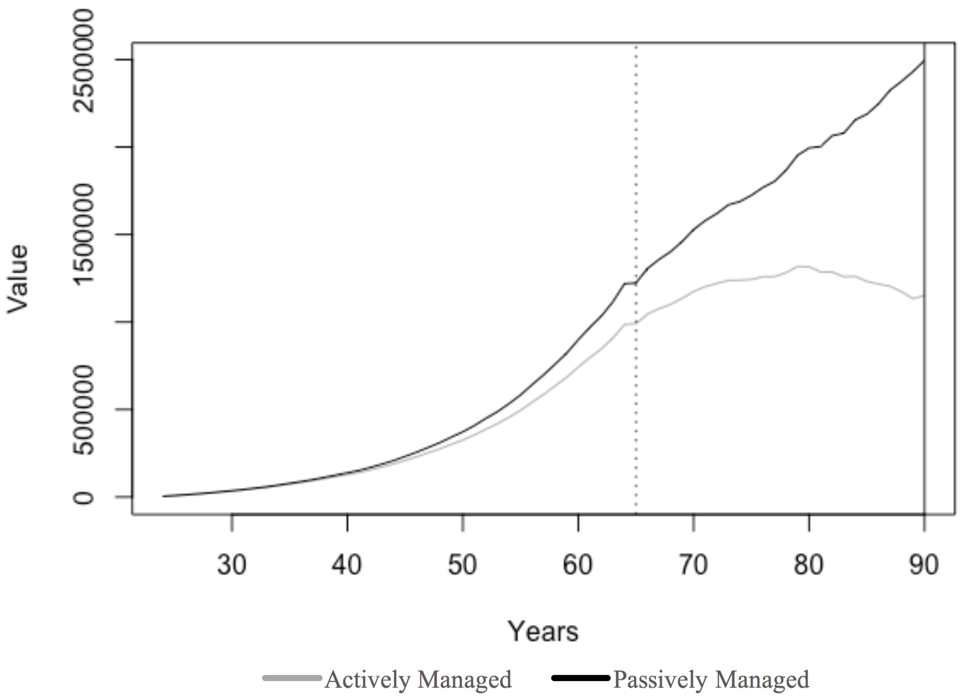
Note: * - in percentage of years in retirement, ** - as a percentage of the last salary

After assigning increases in annual income, the last salary rises to $49,513. The goal was to receive at least three quarters of the last salary when retired. Simulation shows that the mean retirement benefit of the high-cost investment is 144% of the last salary, whereas the mean retirement benefit of the low-cost investment is at even higher 171.6% of the last salary. Not only low-cost investment provides greater payouts, it is also superior in other aspects. For 60.1% of years in retirement it is providing a payout higher than the last salary, compared to 52.1% of years in retirement for high-cost investment. The latter investment provides payouts lower than a half of the last salary in, on average, 24.3% of years in retirement, whereas a low-cost investment does it in 18.3% of years in retirement.

Recall that if a payout falls below $6,000 for a given year, it is counted as a bankruptcy. Probabilities that this occurs at least once in a retirement are surprisingly high: 53.1% for a low-cost investment and 65.8% for a high-cost investment. It should be pointed out, however, that this happens, on average, in 3.7% retirement years for a high-cost investment and 2.9% for a low-cost investment. Therefore, the mean is a biased indicator as it is influenced by a small number of extremely bad scenarios and as such overstates its importance. More expressive is the statistic that in 75% of the scenarios the number of such years is smaller than 2. Realistically speaking, it is extremely unlikely that we would have witnessed a whole decade or more of negative returns only.

Last but not least, there is also an important difference in bequest wealth. This is the part that contributes only a little to aggregate consumption utility, but it is by no means insignificant. On average, a bequest wealth of a low-cost investment is 2.17 times larger than the one of a high-cost investment. Figure 8 shows what is the mean value of invested funds in the portfolio. Vertical dashed line represents an age of retirement and separates the periods of accumulation and decumulation, while vertical solid line represents the age of death.

Figure 8: Mean value of retirement portfolio; 1,000 replications



VI Conclusion

Unlike the methodology of several previous studies, my model includes real fund data, rather than assumed numbers, and implements a lifetime approach with periods of funds accumulation and decumulation. This feature ensures more realistic outcomes with simple payout rules. Moreover, it is not concentrating only on averages, but also observes other parameters to conclude about persistency of the benefits.

The biggest puzzle in such studies, with this model being no exception, are indeed the returns. There is no guarantee that returns in the future will be the same as those in the last 40 years. The economy will surely experience the periods of prosperity as well as recessions. As Mark Twain said: “History does not repeat itself, but it often rhymes.” For this reason, I believe it is not meaningful to focus too much on the extremes of the simulation of 1,000 repetitions, as it is unlikely to have, for example, a period of 40 years of either positive or negative returns. On the other hand, in the future we can expect a pressure on lowering the management costs as robo-advisors are gaining on popularity.

The study just confirmed previous findings that recommend saving in passively managed mutual funds, as supposedly higher return of actively managed mutual funds not necessarily outweigh higher management costs. Investors should care about their net-of-fees real returns. However, the magnitude of the benefits of a low-cost investment does not match that reported by Sharpe (2013) and Bogle (2014). Nevertheless, it is no question about persistency of the passively managed mutual funds dominance, as they are proven to provide greater lifetime consumption utility in 92.8% of the cases. The magnitude of those benefits yields in more stable and, on average, 19.1% higher pension payout during the retirement and significantly larger bequest wealth. The aforementioned results may provide added value to individuals that are deciding between the two retirement investment strategies.

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Appendix

Equation (A.1): Calculation Utility ratio at retirement (URR)

$$URR = \frac{U_{R,p}}{U_{R,a}} \qquad (A.1)$$

Equation (A.2): Utility of retirement consumption at the retirement, low-cost investment

$$U_{R,p}\left(C_{n+1,p}, C_{n+2,p}, \dots, C_{T,p}, B_{T,p}\right) = \sum_{s=1}^{T-n} \rho^{s} \sqrt{C_{n+s,p}} + \rho^{(T-n)} \sqrt{\phi \cdot B_{T,p}} \qquad (A.2)$$

Equation (A.3): Utility of retirement consumption at the retirement, high-cost investment

$$U_{R,a}\left(C_{n+1,a}, C_{n+2,a}, \dots, C_{T,a}, B_{T,a}\right) = \sum_{s=1}^{T-n} \rho^{s} \sqrt{C_{n+s,a}} + \rho^{(T-n)} \sqrt{\phi \cdot B_{T,a}} \qquad (A.3)$$