BI Norwegian Business School - campus Oslo

GRA 19502

Master Thesis

Component of continuous assessment: Thesis Master of Science

Optimization of Icenet's call centre performance using queuing theory approach

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Start:	02.03.2017 09.00
Finish:	01.09.2017 12.00

Written by: Prakash Santhanam

Master thesis - BI Norwegian Business School

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Examination code and name:

GRA 19502 - Master thesis

Hand-in date:

01.09.2016

Campus:

BI Oslo

Supervisor:

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Programme:

Master of Science in Business and Economics

Logistics, Operations and Supply Chain Management

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Chapter 1 - Introduction

1.0 Introduction

In this thesis, the call center of a big growing mobile broadband company will be analyzed, and this company is icenet. Their call center is receiving a lot of complaints from frustrated customers, that the waiting time before their calls are answered is too long. Icenet think that their customer satisfaction is being reduced because of this. Icenet's call centre will be unit of analysis in this thesis. They want to know whether their call centre is understaffed. In chapter 1, the company icenet will be introduced to show who they are and then their call centre will be introduced in 1.2. After that, the problems which Icenet's call center is facing will be explained in detail. Then in 1.4 the topic of queuing theory approach will be touched upon, but explained more in detail in chapter 2.

1.2 The case company - ice.net

ice.net is a Norwegian mobile broadband provider and mobile phone operator company, which was established in 2003 and they had a vision to offer mobile broadband services to entire Scandinavia (Øksnes, 2017). They took the initiative when other mobile broadband operators in the market were unable to provide mobile broadband for large part of the population in Scandinavia, and ice.net have since then grown to become the third largest provider of mobile broadband services in Norway (Øksnes, 2017). The company operates in Sweden and Denmark as well. This thesis will focus on icenet in the Norwegian market.

Ice.net's main competitors in the Norwegian market for mobile broadband are Telenor and Telia. These two companies are the leading providers of mobile broadband in Norway. In terms of number of subscriptions in the mobile broadband market, Telenor is the market leader with a market share of 45,5% at the end of year 2015. Second comes Telia, with its marketshare at the end of 2015 being 26,9%. Third comes Ice.net which had a market share of 20,7% at the end of 2015 (Nasjonal kommunikasjonsmyndighet, 2015,).

According to ice.net, they are able to provide broadband coverage for 75 percent of Norway's land area. They have also coverage up to 100 km off the coastline of Norway as well (Øksnes, 2017). This is more than what any of the competitors are able to provide. Icenet has a number of 150 employees working for them in

Norway. In 2015 Icenet had operating revenues of MNOK 344,6.

1.3 Icenet's call centre

Icenet has two call centers located at two different places (Øksnes, 2017). They have one call centre in Moss and another one in Fredrikstad. They have outsourced both of their call centers to outside companies that specialize in telephone customer contact (Øksnes, 2017). Icenet's call centers are run by the companies Teleperformance and Transcom, respectively. The call centre in Fredrikstad is the one that experiences the most traffic out of the two. Transcom handles all incoming call traffic, both B2C and B2B, and also all written enquiries ((Øksnes, 2017)). The call centre in Fredrikstad will be the subject of analysis in this thesis, where focus will be on their inbound call centre services. Icenet has around 130 employees working for them in the call centre in Fredrikstad (Øksnes, 2017). Icenet are billed for the work that the outsourcees are doing for them (Øksnes, 2017). They have to pay for the services provided to them by the outsourcees. Transcom also provides outbound call centre services for icenet regarding sales.

1.4 The research problem statement

The current situation nowdays in icenet is that they are in a expansion phase, and the fundamentals to this is because Icenet has experienced a steady growth in its customer base and profitability over the past several years (Øksnes, 2017). The foundation to this expansion phase is built on this growth. There are a lot of changes going on in icenet due to this expansion.

Prior to 2015 icenet was only known as a mobile broadband provider, but in 2015 they launched as a full-fledged mobile network operator as well. Icenet experienced huge growth in the number of customers to their company after this launch. The current situation in icenet is that their mobile carrier customers are using Telia's network because the network that icenet currently has can only operate at a certain frequency(Øksnes, 2017). The frequency that it is able to run on is only capable of supporting mobile broadband and not internet on today's smartphones for icenets mobile carrier subscribers. Mobile phone carrier customers can also not make phone calls using icenets network, they currently have to use Telia's network to enable these services. Icenet is now currently in the process of upgrading their network which will fix these limitations, and new upgrades on frequency will support internet and phone calls for icenet's mobile phone customers. So because of this, icenet has now recently started a big process where they are moving their customers which are using Telia's network, to their own newly upgraded network. This migration process of moving their customers to their own network happens as a gradual process, and this is because their own network is not fully tested yet and not 100% up and running yet. It will take time before it is fully ready. That's why this moving of customers is currently happening as a gradual process. And also Telia was in 2016 awarded the best provider of mobile phone internet in Norway in terms of coverage. So icenet is not loosing anything on having their cooperation with Telia. It is a valuable cooperation for icenet, for the time being as this migration process is taking place. According to icenet, a central aspect of their strategy is that quality of the services provided to their customers should always be on top. That's why they are moving their customers gradually to have the least reduction in quality of services provided to customers(Øksnes, 2017).

As a result of this migration process which happens because of the expansion phase icenet currently is in, there are a lot of changes taking place in icenets systems. Icenet's mobile phone customers are instructed to change sim cards so that they can now be connected to icenet's network and use all of the same services, but now through icenet's network. This switch between networks, also requires that customers using smartphones need to change certain settings on their phones in order to be connected to icenet's network. These changes in settings must be done manually by customers, which a number of customers find it difficult to do on their own and need support from customer service. A lot of the technical equipment related to the internal computer systems of icenet's network is being changed due to this migration process going on. This causes their network to be unstable at certain times, and this affects customers in the way that they are experiencing uneven and sometimes very weak cellular coverage. This whole ongoing expansion process leaves a lot of customers frustrated, and having a lot of questions for customer service.

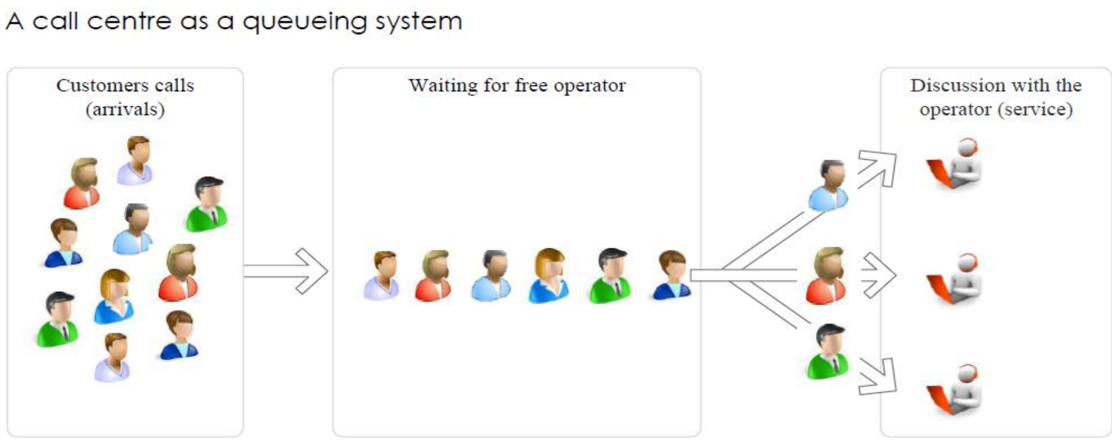
The consequence for icenet's call center from expansion phase

So due to this expansion phase they are in, and the challenges that icenet is currently facing, their customer service section is receiving a much higher volume of customer enquiries than before. Icenet's call centre is experiencing a much higher volume of incoming calls than before. Customer service representatives are at times flooded with work. Call centre management is finding it difficult to handle all the calls while ensuring at the same time that all the callers have a satisfactory waiting time in the waiting queue before their calls are answered. The calling customers are complaining about long waiting times before they reach a CSR. Icenet's call centre is struggling on how to determine how many CSR's to employ in each of the periods during each workday, to get satisfying waiting times, but at the same time achieve optimal costs when doing this. In this thesis, an analysis will be done to determine an optimal solution for their problem, and this will be done based on the queuing theory approach described in the beginning of the introduction. The unit of analysis in this thesis will be icenet's inbound call centre service.

1.4 Queuing theory approach applied on call center optimization

A call center or a contact centre is a physical place where telephone calls from or to customers are handled by an organization (Stolletz, 2012). The operators, agents or customer service representatives (CSR) working in a call center are the ones who answer enquiries from customers by telephone for a business. They can also make calls to customers for telemarketing or market research. Depending on which direction the contact between the customer and the agent is initiated, we can differentiate between inbound and outbound call centers (Stolletz, 2012). The research in this thesis will only focus on inbound call center service where operators receive incoming calls from customers, for example regarding product support or information enquiries from consumers. So therefore this form of call center is driven by random customer call arrivals (Stolletz, 2012). And because of this there will be (on average) both customers waiting for an agent and agents waiting for a customer (Stolletz, 2012).

In this thesis, the call centre will be studied viewed as a queuing system. A call centre as a queuing system is illustrated in the figure below:



(source: Brezavscek and Baggia, 2014, page 11)**Figur 1:** *Basic description of a queuing system in a call centre*

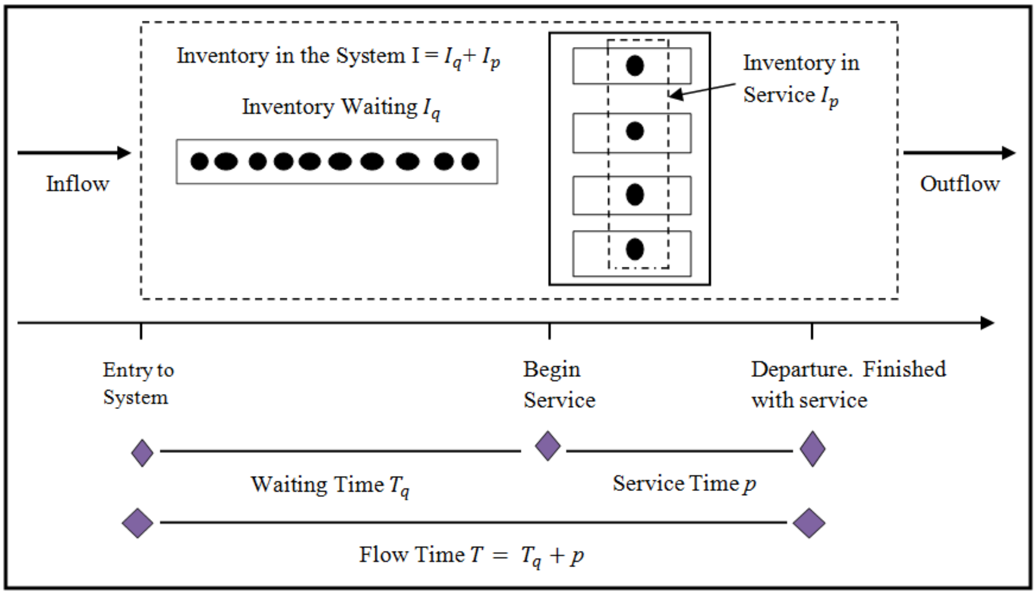


Figure 2: Detailed queuing system showing performance measures. (source: Cachon and Terwiesch, 2009)

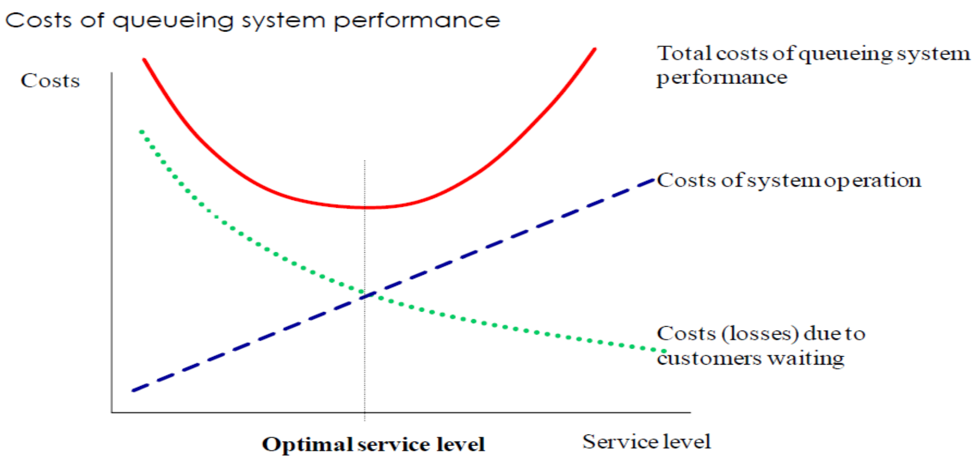
A queuing system in a call centre consists of one or more service units (i.e. servers which are operators ready to service customers), arrivals of customers demanding the service, and the service process. Whenever it is not possible to serve all customers at once, queues are formed. In this case the calling customer will be put in a waiting area or buffer (this is when you listen to music in a call centre) (Cachon and Terwiesch, 2009). Here customers can wait in a queue for an operator to become free regardless the number of customers in the queue (Brezavscek and Baggia, 2014). After waiting, they will be sent to a "service room" where a free operator will serve them.

To achieve customer satisfaction, short expected *waiting times* are important (Stolletz, 2012). Performance of the call center can be measured by looking at waiting times, availability of service or customer abandonment (Stolletz, 2012). Improvement of these technical performance measures are possible if the call center management hires more agents (Stolletz, 2012). However, in a call center the operating expenses are mainly driven by costs associated with these agents. Around 60-70% of the operating expenses consists of personnel-related costs (Stolletz, 2012).

A term that will be constantly used throughout this thesis is service level and it is important to have a clear understanding of what it is in the context of this thesis. Service level is an ambiguous term, especially in the context of call centers (Koole, 2013). Service level defined in the context of this thesis is related to the quality of service that the call center provides. The quality of call centre service is related to how satisfied the customers are with the service, and this often depends on the waiting time for customers before their calls are answered (Brezavscek and Baggia, 2014). And this is how it will be considered in this thesis. So a high service level means low waiting times.

When queues occur, this will lead to waiting costs (losses) for the call centre. For example costs related to customers who hang up while waiting for service because they are tired of waiting (Cachon and Terwiesch, 2009). This is lost throughput (abandoned calls). There will also be increased line charges because of the special phone number (800- numbers) that a lot of call centers use (Cachon and Terwiesch, 2009). Line charges are incurred for the actual talk time as well as for the time the customer is on hold. This is holding costs (line charges) for the call centre (Cachon and Terwiesch, 2009). Also there will be lost goodwill (Cachon and Terwiesch, 2009). These costs will increase with the number of customers in the queue.

So we have to balance these two costs: costs of system operation and waiting costs. As mentioned earlier, if we want to decrease waiting costs and increase service level ensuring better system performance (by improving certain performance measures), new investments must be made, but this will lead to higher costs of the queuing system operation. So we have to find the right balance between costs and service level. The figure below illustrates that it is possible to find the optimal service level which gives the minimum total costs of the queuing system performance by balancing both costs of system operation and costs due to customers waiting (Brezavscek and Baggia, 2014).



(source: Brezavscek and Baggia, 2014) **Figure 3:** *Balance between different costs of queuing system performance.*

To determine the optimal service level various quantitative characteristics (performance measures) of the queuing system is used (Brezavscek and Baggia, 2014). These variables can be calculated using queuing models (queuing models= a mathematical representation of the characteristics and constraints of the queue (Koole and Mandelbaum, 2002)). These variables must be balanced in a certain way so that we achieve optimal service level where total costs of queuing system performance is lowest (Stolletz, 2012).

The imbalances of acceptable technical performance measures and the economic performance of the call center has to be adjusted by the call center management (Stolletz, 2012).

1.2.1 Main objectives and research question

The goal for this thesis, is to study one workday in a busy-week in 2017 that experienced heavy traffic in icenet's call centre. The call centre performance on that day in a selected busy-week will be studied through a performance analysis done on it. A performance analysis on the state the queuing system in icenet's call centre was is in, during various selected small time intervals inside different periods throughout that day will be done (see section 4.3 for more details).

An assumption made prior to the analysis will be that icenet's call centre is understaffed. The background for this assumption is that they are constantly receiving complaints from frustrated customers on long waiting times. This is very likely to occur as icenet is currently going through an expansion phase which was discussed earlier. So, through the performance analysis, suggestions on how to reduce waiting time will be presented, but changes in waiting time will be done in a way so that it also gives optimal or near optimal total costs of the queuing system performance. According to Brezavscek and Baggia, (2014) an important factor in optimization of call centre performance is the determination of the right number of CSR's considering the selected performance measures.

So this particular workday will be studied and an evaluation will be done on whether it was understaffed or overstaffed during different time intervals in various periods of the day. And changes will be suggested to be done accordingly.

In this thesis, the performance of the call centre will be studied using a suitable queuing model which describes the characteristics of the queuing system (details on various queuing models is given in section (nr)), and the values of the performance measures of the queuing system will be calculated accordingly to the selected queuing model.

Based on what is known about icenet's call centers current situation, the initial aim for the time being is to reduce the value of the performance measures that are related to waiting time, lower than what they are already now with the current situation in icenet's call centre. Because they are afraid they are spending too much money on waiting costs and that this consists a great part of their total costs of queuing system performance compared to operating costs. Icenet's call centre management wants this to be investigated. These two costs are the measures of economic performance of the queuing system. They want to know if this possible imbalance between these two costs can be evened out more.

So improving the performance of their queuing system will be a priority, through reducing waiting time related performance measures. But at the same time make sure that these changes reach a point where also the total costs of queuing system performance is optimized or close to optimized. Details regarding which specific waiting time related performance measures that will be focused on will be explained in chapter 2.

It will be illustrated in this thesis, how icenet's call center's historical data on its operations can be exploited through using queuing theory approach to do a performance analysis on their call centre operations on that chosen workday, and based on this, suggestions will be given on how they could have improved icenet's call centre performance on that day, both with regard to waiting related performance measures and economic performance measures. It might give icenet's call centre management some clearer insight into the current situation they are in, regarding facing more traffic due to the expansion phase they are in.

This leads to the research questions for this thesis:

How can the queuing system performance of Icenet's inbound call centre service be improved through using queuing theory approach with the intention to optimize it by analyzing its performance measures?

What changes can be done to the waiting time related performance measures (calculated using a suitable queuing model) and the economic performance measures in order to achieve optimal or near optimal service level where at the same time total costs of the queuing system performance are minimal?

1.4 Relevance of the topic/significance of the study

In this section, a discussion will be done to provide justification for the study in this thesis. A discussion on why this study is important in general for call centers will be given. Then the purpose and practical relevance this study has for icenet's call centre will be explained.

1.4.1 Why study on call centre performance improvement is important

Inbound call centers make up a large and growing part of the global economy (Aksin, Armony and Mehrotra, 2007). Call centers are frequently used by organizations for technical support for end users (Brezavscek and Baggia, 2014). They are a preferred and prevalent way for many companies to communicate with their customers (Koole and Mandelbaum, 2002). A call centre usually represents the first contact of a customer with a given company. Call centers serve as the "public face" for many firms (Kim and Park, 2007). Therefore, the quality of the call center's service is of high importance (Brezavscek and Baggia, 2014). So giving a good first impression to customers is important. In general, the quality of call centre service is related to customer satisfaction with the service (Brezavscek and Baggia, 2014). Providing poor service quality to customers might tarnish the reputation of the company. The image customers have of the company will be bad. That's why it is important for call centres to make sure they provide high quality of service.

Companies that focus on customer loyalty are increasingly using their centers to differentiate their product or service offering and drive customer satisfaction (Miciak and Desmarais, 2001). They use it to differentiate them from competition. Gain competitive advantage. Cutting cost and enhancing effectiveness of call centre are directly connected with strengthening the competitiveness of the companies (Kim and Park, 2007). This shows that the research topic for this thesis, call centre performance optimization, is important because it affects a company's competitiveness. A call centre that is running optimally might give a competitive advantage.

Purpose of the thesis and practical relevance

The purpose of this thesis is to check whether icenet's call center was understaffed or overstaffed during various selected time intervals throughout the day. The day selected will come from busyweek in 2017. This day is from the past. Because the purpose of this thesis is to show icenet how the queuing system of their call centre were running on some selected time intervals on a day from the past. It might give icenet's call centre management some clearer insight into the current situation they are in, regarding facing more traffic due to the expansion phase they are in. It will give a display of whether they were accurate with their forecasting or not. Suggestions will be given for the particular time-intervals on how they could have done it better with a cost optimal and service optimal alternative.

In this thesis, queuing theory approach is used to show icenet the deviation or error they have made between comparing the actual staffing level they had in that interval on the selected day and the staffing level they should have had which would have given them cost optimal and service optimal solution.

Being accurate regarding this is important for icenet when their amount of subscribers and customer base is icreasing. They may have lost a lot of money on overstaffing which they didn't have to, which was an unnecessary loss, and this is displayed through this thesis. This is how it is shown to them how their queuing system was running on a day in a busy-week.

Chapter 2 – literature review and theoretical foundation

2.0 Literature review and theoretical foundation to support the research

2.1 Operations research

It is clear that it focuses on quantitative methods as an aid for decision making. Analyzing complex real life problems is what operations research is mostly used for doing, with the intention and objective of improving or optimizing system performance (Mishra and Sandilya, 2011). A managers main activity is decision making. Certain decisions can be taken by using common sense, experience and sound judgment without employing advanced mathematical models, and in some other cases this may not be possible and other more advanced techniques is necessary to use, if you want to achieve the desired results from the decision making (Mishra and Sandilya, 2011). In complex situations, it is also possible to base the decision you take on common sense and experience, but if the decision you take is supported by mathematical calculations then this will reduce the risk factor and increase the probability of success (Sikkim Manipal University, 2017). In this sense, operations research is highly applicable for the research question in this thesis, regarding icenet's call centre having shortcomings in their call centre performance. It can aid icenet's call centre in achieving optimal outcome of their decision making process. To maximize benefits and to minimize time and effort is the ultimate goal of all decisions (Sikkim Manipal University, 2017). Decision makers are given the power through operations research, to make effective decisions and improve day-to-day operations (Sikkim Manipal University, 2017).

Some of the benefits that operations research provides are:
- Decrease cost or investment
- Increase revenue or return on investment
- Increase market share
- Manage and reduce risk
- Improve quality
- Increase throughput while decreasing delays
- Achieve improved utilization from limited resources
- Demonstrate feasibility and workability

(Heger, 2008).

According to Rajgopal (2001), modeling, deserves a lot of attention because it is a defining characteristic of all operations research projects. Formally defined, a model may be a selective abstraction of reality (Rajgopal, 2001). What this definition means is that modeling is the process of capturing selected characteristics of a system or a process and then you combine these into an abstract representation of the original (Rajgopal, 2001). The main idea behind doing this is that it is usually much easier to do analysis on a simplified model than it is to analyze the original system, and if the model is a fairly accurate representation, then the conclusions that are drawn from such an analysis may be in a valid way extrapolated back to the original system (Rajgopal, 2001). One key point that must be kept in mind when creating a model is that there must be a correct balance between the accuracy of a model and its tractability (Rajgopal, 2001). What is meant by this, is that on one hand you can create a model which is a very detailed and exact representation of the system at hand (Rajgopal, 2001). This model is a highly realistic representation of the original system (Rajgopal, 2001). Going through the very process of creating such a comprehensive model will help you a lot in understanding the system better. According to Rajgopal (2001), from an analytical perspective, such a detailed model may well be useless because constructing it may be extremely time consuming and its complexity prevents any meaningful analysis. That’s the negative side of a detailed model.

On the other hand one can construct a less detailed model that has a lot of simplifying assumptions so that it is easy to do analysis on it (Rajgopal, 2001). But the negative side of this could be that the model may lack so much accuracy that when you extrapolate the results drawn from the analysis back to the original system then this may give serious errors (Rajgopal, 2001). So, it is important to construct a sufficiently accurate model of the original system, but also at the same time it must be tractable (Rajgopal, 2001). So, in the formal definition given above on what a model is, the word “selective” mentioned, is a keyword in this matter. It is very important that you have a clear problem definition because this will make it possible for you to better determine the crucial aspects of a system that must be selected so that it/and is represented by the model (Rajgopal, 2001). The ultimate goal is to create a model that includes all the key elements of the system, but at the same time it remains simple enough to analyze (Rajgopal, 2001).

This discussion is important to consider regarding this thesis when constructing the queuing model later on here which will be a representation of the queuing system in icenet's call centre. Regarding the research question of this thesis, the model's main objective is to provide a way to analyze the queuing system's behavior with the intention to improve its performance.

2.2 Queuing theory

There exists plenty of relevant previous research on how the research question of this thesis can be answered. The goal of this thesis is to make a performance analysis on icenet's current call centre performance and make changes to performance measures to optimize performance and come up with an optimized model. And suggest them this optimized model where optimal service level is achieved and costs are optimal. The aim is to improve an existing model. Results of previous research on call centre optimization shows that it can be done using the queuing theory approach (Brezavscek and Baggia, 2014). Queuing theory can be used to balance the cost of increased capacity against the increased productivity and service (Osahenvemwen and Odiase, 2016).

Kleinrock defines queuing theory

According to professor Leonard Kleinrock (1975) who is a computer scientist that further developed queuing theory in the 1960's after Danish mathematician Agner Krarup Erlang invented the field in the early start of the 1900s, he explains that queuing theory is a mathematical discipline that studies the phenomena of standing, waiting and serving.

Any system in which an arriving flow of customers or units requesting service, and servers waiting to provide service to them, where arrivals place demands upon a finite-capacity resource may be termed a queuing system (Kleinrock, 1975). In particular, if the arrival times of these demands are unpredictable, or if the size of these demands is unpredictable, then conflicts for the use of the resource will arise and queues of waiting customers or units will form (Kleinrock, 1975). In this way it is called a queuing system. Any system where a queue is made and served can be modeled and studied by the use of queuing theory (Kleinrock, 1975). Queuing theory make use of mathematical models and performance measures to assess the flow of customers through a queuing system and hopefully improves it. Though queues are often made up of physical lines of people or things, it is also possible for the queues to be invisible as with telephone calls waiting on hold in a call centre (Green, 2011). In the context of this thesis with icenet's call centre, customers wait in a virtual queue.

Queuing theory – areas of application

Queuing theory has a variety of applications. Since Agner Krarup Erlang started his research on queuing theory, its applications has been apparent in the fields of traffic engineering, telecommunications, computing and when factories, shops, offices and hospitals has been designed (Chuka, Ezeliora, Okoye, Obiafudo, 2014). It has been used largely by the service industries (Nosek and Wilson, 2001). Queuing theory has in the past been used to assess such things as staff schedules, working environment, productivity, customer waiting time, and customer waiting environment (Nosek and Wilson, 2001). So this theory will in this thesis be used to assess staff scheduling and customer waiting environment in icenet's call centre.

Queuing theory – requirements to build queuing models

There are basically three things that queuing theory is based on - a queuing model which is a mathematical representation of the characteristics and constraints of the queue, a real world system such as a telephone network or a call center that you wish to model, and a mapping between the two (Koole and Mandelbaum, 2002). This is exactly what will be done in this thesis regarding icenet's call centre case, it will be presented through a mathematical model so that one can analyze its queuing system performance. There exists a great number of contributions in the literature that proves that queuing models are a useful and applicable tool that can be used to analyze call centre efficiency (Brown, Gans, Mandelbaum, Sakov, Shen, Zeltyn and Zhao 2005; Dombacher 2010 and Koole and Mandelbaum 2002). Relevance and usefulness of the results that are obtained through such an analysis depends on proper selection of a mathematical model, which is based on three elements of the queuing system: the arrival process, the service times and the queuing discipline (Brezavscek and Baggia, 2014). Parameters such as the arrival process, service times and the number of agents, is what queuing models are built from. To apply a queuing model, the call center must first estimate these parameters based on historical data (Koole and Mandelbaum, 2002). In specifying a queuing model, assumptions about the probabilistic nature of the arrival and service processes must be made (Green, 2011). Appropriate selection of a mathematical model, is based on knowledge of the probability density functions of inter-arrival times (which is times between two successive incoming calls) and service times (call duration), that are both random variables (Brezavscek and Baggia, 2014). Cachon and Terwiesch 2009 explains how and why an arrival process is analyzed.

Important to consider before selecting queuing model

A big risk related to any mathematical model is that these tools always provide us with a number (or a set of numbers), independent of the accuracy with which the inputs we enter into the equation reflect the real world (Cachon and Terwiesch, 2009). There are two questions that are important to answer before selecting queuing model and starting to calculate performance measures, answering these questions will improve the predictions of the model substantially (Cachon and Terwiesch, 2009). The selected model will give us more accurate predictions. The questions that should be answered are:

- Is the arrival process stationary; that is, is the expected number of customers arriving in a certain time interval constant over the period we are interested in (Cachon and Terwiesch, 2009)?

- Are the inter-arrival times exponentially distributed, and therefore form a so-called Poisson arrival process (Cachon and Terwiesch, 2009)? And also whether the service times are exponentially distributed or not influences the choice of queuing model.

Collecting historical data about the arrival process and service process and determining its probabilistic nature is important because the mapping between the mathematical model and real world system will be more precise. The mathematical model's predictions about the performance measures will be more accurate and realistic.

In the context of this thesis with icenet’s call center case where various 30 minute intervals throughout the day will be selected for analysis, these intervals are implicitly assumed to be stationary. A 30 minute interval will be studied over a 30 minute period. Each 30 minute interval will be studied separately in the analysis. There is only one arrival rate considered within every separate 30 minute interval to be studied.

To check if interarrival times is exponentially distributed, this can be done by comparing the mean and standard deviation in the distribution of interarrival times. The same approach goes for service times. The mean and standard deviation of the distribution of service times must be compared.

Measure of variability, coefficient of variation CV

Waiting time in inbound call centers occur because of variability. Random customer call arrivals is what drives an inbound call centre (Stolletz, 2011). Because of this, on average we have both customers waiting for an agent and agents waiting for a customer (Stolletz, 2011). The service process being analyzed in this thesis is assumed to exhibit random variability in demand. Queue arise when the demand for service exceeds the capacity, which are often caused by random variation in service times and the times between customer arrivals (Osahenvemwen and Odiase, 2016).

It is possible to measure variability. Variability is measured as the coefficient of variation (CV) of a random variable as:

Coefficient of variation = CV= Standard deviation / Mean. (Cachon and Terwiesch, 2009).

As discussed, there exists variability in both the time between arrivals and the duration of each call. So the variability of an arrival (demand) process can be measured like this: CV_a = Standard deviation of interarrival time/Average interrarival time. Variability in service time or activity time can be measured as: CV_p= Standard deviation of activity time / Average activity time.

If an arrival process has exponentially distributed interarrival times or not, affects the choice of queuing model. It is the same case for the service process. Whether the service process has exponentially distributed service times or not also affects choice of queuing model. If interarrival times are exponential, then the mean is equal to the standard deviation and this would give a CV_a=1. The same would happen to service times if they were exponentially distributed, it would have a CV_p=1. If the interarrival times and service times were not exponentially distributed, then their CV's would have to be calculated using the CV formulas given above.

2.5.1 Characteristics of a queuing system

There are three parts of a queuing system and these are: (1) the arrivals to the system (sometimes called the calling population or arrival population), the waiting line or the queue itself, and (3) the service facility (Render, Stair and Hanna, 2011). Before a queuing model can be built, certain characteristics that each of these three parts have must be examined (Render, Stair and Hanna, 2011).

With regards to the arrivals or inputs to the system there are some characteristics of it which are important to consider. Like the size of the calling population and the behavior of the arrivals (Render, Stair and Hanna, 2011). The size of the calling population can either be infinite or finite (Render, Stair and Hanna, 2011). The calling population is considered unlimited when the amount of arrivals on hand at any given moment is just a small portion of potential arrivals (Render, Stair and Hanna, 2011). When it comes to behavior of the arrivals, it is assumed by most queuing models that the arriving customer is a patient customer (Render, Stair and Hanna, 2011). Customers that wait in the line until they are served and who don’t switch between lines are patient customers (Render, Stair and Hanna, 2011). In icenet’s call centre there are both patient and not patient customers who calls. But in the context of this thesis, the queuing models used here assume that all arriving customers are patient.

The second component of the queuing system is the waiting line itself and the length of it can either be limited or unlimited (Render, Stair and Hanna, 2011). Queue discipline is another waiting line characteristic (Render, Stair and Hanna, 2011). It is about in which order the customers standing in line should be served. First-in, first-out (FIFO) also called first in first served (FIFS) rule can be followed (Render, Stair and Hanna, 2011). LIFS stands for last-in, first served.

The third part of the queuing system is the service facility (Render, Stair and Hanna, 2011). How the service system is configured must be examined. A service system consists of a number of servers who will serve incoming customers. It also consists of a number of phases or a number of service stops that the customer has to go through (Render, Stair and Hanna, 2011). A single-channel system is a service system where you only find one server serving incoming customers all standing in one common line (Render, Stair and Hanna, 2011). This is where several servers are on duty, and all customers are waiting in one common line for the first available server (Render, Stair and Hanna, 2011). In a single-phase system, customers receive service at only one station and then exits the system (Render, Stair and Hanna, 2011). Throughout the whole process from entering the queuing system to exiting it, customers only go through one station and then out (Render, Stair and Hanna, 2011). The one that best fits to represent the queuing system in icenet's call centre would most likely be a multichannel, single-phase system.

2.5 Queuing models

Sanjay Bose (2002) explains in his book "Introduction to Queuing System" that queuing models can be represented using Kendall's notation which comprise of three factors, A/B/C. The basic features of a queuing system is described through using this notation. This is a queuing model's notation and details of its notation is:

A: Interarrival time distribution

B: The service time distribution

C, s or m: Number of parallel servers or service channels. In the context of this thesis with icenet's call centre case, this is the number of CSR's.

Balakrishnan (2010) adds that this notation can be extended to: A/B/C/D/E/F.

Typical notations for A and B (Sanjay B, 2002):
These notations are used for describing the probability density function of inter-arrivals and service times.
M – Poisson process. Exponential distribution. (M stands for memoryless).
Ek – Erlang distribution with k phases.
D – Deterministic times (means that times between two successive events are constant (Brezavscek and Baggia, 2014).
G – General distribution, with mean and variance known (Render, Stair and Hanna, 2011).

According to Balakrishnan (2010) the meaning of the notations D, E and F are:
D – the length of the queue. Can be limited or unlimited. ∞ symbolizes infinity.
E- stands for the size of the calling population. ∞ symbolizes infinity
F – is the queuing discipline.

So based on what is known from the characteristics of a queuing system and knowledge of kendalls notation a queuing model can be built for example like this: M/M/s/infinite/infinite/FIFO. According to Brezavscek and Baggia, (2014), an alternative way of writing this is: M/M/s {∞/∞/FIFO}. Both are the same.

According to the literature, D, E and F extends the queuing model (Balakrishnan, 2017). By default, if these three notations are omitted, then it is assumed that these values are infinite and FIFO (Balakrishnan, 2017). Then the model can simply be written as M/M/s. In the literature, the queuing model is most often written with only these 3 symbols (Balakrishnan, 2017). And if nothing is said about the D, E and F then it is assumed that they are infinite and FIFO.

Kendalls notation is important to know because most of the existing literature on Queuing theory uses these notations to describe the characteristics of a queuing system. In chapter 5 where the analyses of each selected time interval takes place, queuing models will be used to represent the characteristics of the queuing system behavior during the chosen time interval. Since each time interval is analyzed separately, then each of them have their own separate interarrival time distribution and service time distribution, and this must be taken into account when using queuing models to represent the time interval.

M/M/s – A widely used queuing model

The M/M/s model or Erlang C model is the most commonly used queuing model (Green, 2011). The assumptions for this model is that there is a single queue with unlimited waiting room that are going to be serviced by s identical servers (Green, 2011). Customers arrive according to a Poisson process with a constant rate, and the service times has an exponential distribution (Green, 2011). The arrivals are serviced on a FCFS (First-come-first-served) basis (Tiwari, Gupta, Joshi, 2016). For this queuing model there are a set of formulas available that enables the calculation of various performance measures of the queuing system(Chowdhury, Rahman, Kabir, 2013). These formulas calculate performance measures such as: The average number of customers in the system, that is, the number in line plus the number being served (Chowdhury, Rahman, Kabir, 2013). The average time a customer spends in the system, that is, the time spent in line plus the time spent being served (Chowdhury, Rahman, Kabir, 2013). The average number of customers in the queue alone (Chowdhury, Rahman, Kabir, 2013). Probability that wait time is greater than t time.

From M/M/s to G/G/s

The advantages of the M/M/s model is that it requires only 3 parameters, so it can be used to obtain performance estimates with very little data (Green, 2011). There are disadvantages as well with this model. According to Koole and Mandelbaum (2002) M/M/s could turn out highly inaccurate because reality often go against its underlying assumptions. In the M/M/s, the coefficient of variations for interarrival times and service are both equal to 1. If we have non-exponential service times, then the M/G/s model would be more suitable to model the queuing system (Koole and Mandelbaum, 2002). Here one must resort to approximations, out of which it turns out that service time affects performance (efficiency of queuing system, meaning expected waiting time) through its variability, coefficient of variation (Koole and Mandelbaum, 2002). If the CV of service times is substantially different than one, the use of the M/M/s model may significantly underestimate or overestimate actual waiting time (Green, 2011). This is why it is better to use the M/G/s model, its more accurate, it takes into account variability in service times. When using the M/G/s model, the performance weakens when stochastic variability in service times increases (Koole and Mandelbaum, 2002). And improves when variability decreases. The M/G/s 's formula for calculating expected waiting time shows the impact of variability on waiting time (Green, 2011). The G/G/s model takes into account nonexponential interarrival times as well, and here CV_a is different from 1. Variability in interarrival times have the same effect on expected waiting time as variability in service times does (Green, 2011).

In the context of icenet's call centre case, the queuing model that is most preferred to model their queuing system within most of the time intervals to be analyzed, would be a G/G/s model, because their queuing system is assumed to exhibit variability in interarrival times and service time. Which model that fits best will be found out when data are analyzed in chapter 5.

Relationship between waiting time and customer satisfaction – quality of icenet's inbound call centre service

In the context of this thesis with icenet's call center case, it is important to understand the relationship between waiting time for customers in the queue and customer satisfaction. The quality of call centre service is related to customer satisfaction with the service, and this is often dependent on how long calling customers have to wait before their calls are answered (Brezavscek and Baggia, 2014). Short expected waiting times are important to achieve customer satisfaction (Raik Stolletz, 2012).

In this thesis with icenet's call center case, waiting time does affect customer satisfaction (Øksnes, 2017). Ice.net's call centre has received many complaints from their customers about long waiting times in queue and that they are understaffed (Øksnes, 2017). Dissatisfied customers may cancel their subscription with ice.net and be tempted to go for their competitors instead (Øksnes, 2017). This is a loss for ice.net. A highly satisfied customer is more likely to spread the positive experience by word of mouth advertising (Nosek and Wilson, 2001). A dissatisfied customer will most likely be more than willing to share his or her bad experience with whoever will listen, and this will have an obvious negative impact on revenues (Nosek and Wilson, 2001).

Waiting related performance measures

Based on the discussion above, this supports why waiting related performance measures should be used to measure the performance of the queuing system in icenet's call centre. There is a link between waiting time and customer satisfaction. Customer satisfaction is important for icenet (øksnes, 2017).

The two waiting related performance measures that will be used to measure the performance of the queuing system operations in icenet's call centre are:

- Expected waiting time in the queue
- Expected number of customers waiting in the queue.

Cachon and Terwiesch (2009) provide formulas for how to calculate these measures. They explain what data must be gathered first before one can use these formulas. The formulas are shown below:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}}\ -1}{1 - utilization}\right) \times \left(\frac{(CV_a)^2 \times (CV_p)^2}{2}\right)$$

$$I_q = \frac{T_q}{a}$$

The first one is the waiting time formula (T_q) and the second formula is used to calculate the average number of customers waiting in the queue (I_q) (Cachon and

Terwiesch, 2009). Certain data needs to be collected before the formulas can be used. These are shown below:

- Number of servers, m
- average activity time or average service times, p
- average interarrival time, a

(Cachon and Terwiesch, 2009)

These data were explained earlier in this chapter in section (nr). The waiting time formula can be modified depending on what queuing model represents the queuing system. For ex. if an M/M/m model is used then variability can be left out from the model. The third factor of the formula will be gone. Another piece of data that must be found before using the waiting time formula, is the utilization, u (Cachon and Terwiesch, 2009). According to Cachon and Terwiesch (2009) u can be calculated in this way:

utilization, u = $\frac{p}{a \times m}$ or utilization u = p/(a×m) (form used in chapter 5)

Utilization shows the proportion of the time that the servers are in use (Render, Stair and Hanna, 2011).

To use the waiting time formula, it is required that the queuing system is in a steady state (Render, Stair and Hanna, 2011). A steady state condition exists when a queuing system is in its normal stabilized operating condition (Render, Stair and Hanna, 2011). This is when u < 1 or u < 100%, then the steady state condition is satisfied (Cachon and Terwiesch, 2009). If u >1 or equal to 1, then the queue continues to grow and grow (Cachon and Terwiesch, 2009). This is not caused by variability, it is caused because the requested capacity is not there (Cachon and Terwiesch, 2009).

Economic Performance Measures

To measure the economic performance of the queuing system in icenet's call centre, there are two cost factors that must be studied, in relation to the context of this thesis. As mentioned in chapter 1, icenet's call center is receiving a lot of complaints from frustrated customer about long waiting time in the queue before receiving service from a CSR. A solution to this problem is just to hire many more CSR's to reduce the waiting time for customers standing in line, however, this can become expensive (Render, Stair and Hanna, 2011). When it comes to waiting line problems, there are two basic types of costs associated with it, and these are waiting costs and operating costs. (Tiwari, Gupta, Joshi, 2016).

In call centers, operating costs are mainly driven by the costs of the agents (Raik Stolletz, 2012). Around 70 % of operating costs are personnel-related (Raik Stolletz, 2012). So therefore the cost per CSR will be used as a good measure of operating costs. The more CSR's on duty, the higher the operating costs will be. These costs are considered fairly tangible costs compared to waiting costs. Waiting costs are the relatively "intangible" costs (Tiwari, Gupta, Joshi, 2016). Waiting line costs are caused because customers have to wait and these costs could come from lost goodwill, loss of sales, dissatisfied customers and losing customers to competitors (Chowdury, Rahman, Kabir, 2013). This fairly intangible cost is not easy to quantify (Chowdury, Rahman, Kabir, 2013).

Waiting costs and operating costs are the two measures of economic performance on the queuing system operations in icenet's call centre that will be focused on in this thesis. Cost of labor per call will be used to measure operating costs. Cachon and Terwiesch (2009) provides a formula for how to calculate cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

where

w = Estimated average cost of a CSR per unit of time.

u = utilization for chosen staffing level.

p = average activity time per call

w and p must be expressed using the same time unit (for eks. w= 5 kr/min and p=3 min). w can be obtained from icenet's call center management. This is the formula that will be used in this thesis to calculate cost of labor per call.

Waiting cost per call will be used as a measure for waiting costs. Cachon and Terwiesch (2009) provides also a formula for how to calculate waiting cost per call:

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Where expected waiting time per caller is calculated from the expected waiting time formula mentioned in section (nr). Cost of waiting per unit of time has to be found by icenet's call center management and this will be found as a rough approximated estimate, since it is a fairly intangible cost and not easy to measure.

Through economic analysis of waiting costs and operating costs management will get help to make a trade-off between increased operating costs and the decreased waiting costs from customers which are provided that service (Jhala and Bhathawala, 2016). To be able to evaluate and determine the optimum amount of servers in the service system, these two opposing costs must be balanced appropriately. Finding the right balance can be done through the formula for total cost per call which is provided by Cachon and Terwiesch (2009). Which is:

Total cost per call = Cost of labor per call + Waiting costs per call.

For various staffing levels we get different total cost per call. The optimum staffing level is found where total cost per call is the lowest. For various selected staffing levels, various values on utilization, T_q and I_q will also be shown. So the staffing level which is the most appropriate one which also gives the desired values on waiting related performance measures with the right total cost per call can be selected.

The formulas discussed here will be used to calculate the performance measures which will measure the queuing system performance in icenet's call centre during various selected time intervals throughout the day. An optimal or near optimal solution with regard to the choice of the waiting related performance measures and economic performance measures will be decided upon.

Step by step procedure for conducting the performance analysis

So, based on the literature and theories studied in this chapter, a step by step procedure for how to conduct the performance analysis in chapter 5 can be developed and finalized. The performance analysis consists of several steps. These steps must be done in chronological order (as can be seen why, in the performance analysis itself). The 4 steps that explain how to go through the performance analysis in chapter 5, put together after studying the literature and theory, is presented down below:

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

2) *Show the calculations of waiting related performance measures:*

3) *For the chosen staffing level, what are the economic performance measures?*

4) *Show the impact of various staffing levels on waiting time related performance measures and economic performance measures.*

This is the preferred procedure to follow to complete the performance analysis that will be done on icenet's inbound call centre service. Which time intervals that will be chosen for investigation will be discussed in detail in chapter 4. These steps will be understood better when actually doing the performance analysis in chapter 5. The performance analysis itself in chapter 5 explains in detail how these steps are gone through.

Chapter 3 – Research Methodology

3.0 Research methodology

3.1 Research strategy

In this thesis, a mathematical description of the current status of the queuing system of the inbound call centre service in icenet will be provided. Various quantitative characteristics (performance measures) that describes the performance of the queuing system must be measured. Numerical data must be collected to calculate these performance measures. To do the performance analysis, numerical data is required. The fact that numerical data is needed shows what characteristics this thesis has.

The purpose of this thesis is to optimize the service level at minimum total costs for icenet’s inbound call centre service by changing variables that affect queuing system performance. Research in this field has been done many times before and there exists vast amount of literature/theory that describes how it can be achieved (Brezavscek and Baggia, 2014). Results of previous research show that this can be done using queuing theory approach (Brezavscek and Baggia, 2014). Queueing theory approach will guide the research in this thesis, it decides what data to be collected and this will be used to build the queuing models which will be used to conduct a performance analysis on the queuing system of icenet’s call centre. And this analysis will be used to answer the research question. Therefore it is safe to say that the research in this thesis will be of a deductive approach.

The research in this thesis will be mostly quantitative, but also slightly qualitative. Since a queuing model which will be used in this research is a mathematical representation of the queuing system. The queuing model requires numerical data in order to calculate the values of the performance measures which will be balanced to find the optimal service level. But to build a queuing model, it also requires some few qualitative data as well, like what queuing discipline does the queuing system in icenet’s call centre follow, it could for example be the FIFO rule or the LIFO rule. Measuring the economic performance of the queuing system in icenet requires quantitative data like cost data. So therefore the research in this thesis will be of both quantitative and qualitative character. Research that has a combination of both quantitative and qualitative character is in the literature referred to as a mixed method research (Bryman and Bell, 2011). This is the approach of research strategy that will be used in this thesis.

3.2 Research design

The research design of the research in this thesis will be a case study, where the study will be based on how icenet's inbound call centre service can improve the performance of its queuing system and achieve optimal service level through a performance analysis. Only the *inbound* call centre service of icenet's call centre will be studied in this thesis, as they also have an outbound call centre service which is sales focused. Also the study will be undertaken during the current *expansion phase* icenet is undergoing, and an objective in this study is to observe how is the performance of the queuing system in icenet's inbound call centre during this ongoing expansion phase they are in. Queuing theory will be applied in a real world case, for icenet's call centre, and for the current situation they are in, to improve the performance of their queuing system at minimum costs. So therefore it is safe to say that the research design of this thesis is a case study design, because a single company is looked upon and this in a specific case.

3.3 Data collection

This section will explain what type of data that will be collected for this thesis. Data can be collected from different types of sourses. There are two different sources where data can be collected from, and these are primary sources and secondary sources. Primary data is the type of data that is collected by the researcher himself and it is used for the specific purpose or analysis under consideration (Boslaugh, 2007). The data is called secondary data if it was collected by someone else for some other purpose (Boslaugh, 2007). And the researcher uses this for his study.

In order to conduct the research in this thesis, mainly secondary data is required.

3.3.1Secondary data

In this thesis, collecting secondary data are crucial to measuring the efficiency of icenet's inbound call center operations. Data on call arrivals and service times are important data that must be obtained in order to do the performance analysis on icenet's call center. Interarrival times are measured through studying call arrival data. Modern day technology allows automatic logging of all the events regarding operations in a call center, and this is the case for icenet. So data on call arrivals and service times are stored in icenet's IT systems and according to Alf Martin Øksnes it is possible to retrieve this data. These data are automatically logged by the computer systems in icenet as calls arrive. So this can be termed as secondary data. Secondary data on the staff schedules used on each day in the selected busy-week is needed. Other secondary data that are needed are cost data, so that economic performance measures can be calculated, and these cost data are gathered by the finance department in icenet.

There are also certain other data about the characteristics of the queuing system in icenet's call center that are required to do the performance analysis, but all these data are collected and provided by Øksnes. Therefore, these data are secondary data. One of the advantages of secondary data is that it is less time consuming and less costly to obtain (Boslaugh, 2007).

3.3.2 Data analysis

After the data collection process is completed, then analysis of these data can take place. Data on call arrivals will be used to find interarrival times, and then the software excel will be used to compute the mean and standard deviation of interarrival times to determine probability distribution of interarrival times. This was discussed thoroughly in chapter 2. The mean and standard deviation of service times will be calculated through using excel, and also here the probability distribution of service times will be calculated. This analysis is required to build the queuing models, and the selected queuing model determines how to calculate the performance measures, which was discussed in detail in chapter 2. The means and standard deviations found will be used to measure variability in the arrival process (CV_a) and the service process (CV_p). Variability affects the calculations of performance measures. The staff schedule for the particular workday will be studied to find out how many workers were on duty at any moment in time throughout the day.

The figure below is a summary of how the collected data will be analyzed and what they are used for.

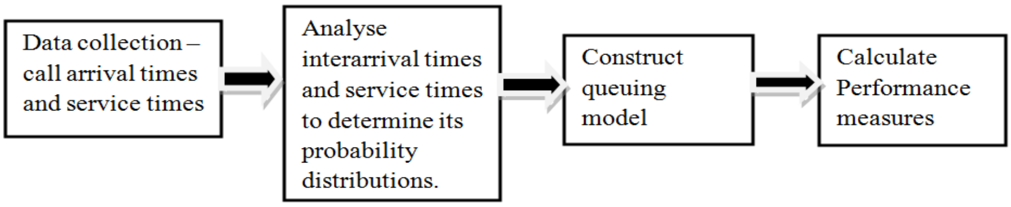


Figure : *Summarizing the process for analysis of data, and its purpose.*

3.4 Quality of the research

To make sure the quality of business research is guaranteed, certain criteria are used to evaluate this. According to Bryman and Bell (2011), reliability, replication and validity are three leading criteria for assessing the quality of business research.

3.4.1 Reliability:

Bryman and Bell (2011) explains that reliability is the question of whether the results of a research study is repeatable. In the context of quantitative research, the question that needs to be asked is whether a measure is stable or not (Bryman and Bell, 2011). In the context of icenet's call center case, the question of whether the results of this thesis are reliable can be cleared up.

Because the purpose of this thesis is to only show the performance of icenet's queuing system only during one day and this day being in the current expansion phase they are in, the scope of this research is small when it comes to amount of data analyzed. Since only some timeintervals from various shifts throughout one particular day will be analyzed, then another researcher who wish to get the same results must only use the exact same data collected on these timeintervals from that particular day. Because all the past call data from icenets call centre operations is stored in their database (Øksnes, 2017), and therefore if one wishes to do the research once again and retrieves the exact same data and uses the same queuing theory methods, then the research will give exactly consistent results. If data from a different day is used in the study, then the results will change. So it all depends on the input data used, they must be same as they were the first time.

3.4.2 Replicability

According to Bryman and Bell (2011), replication or replicability is about how possible it is for other researchers to reproduce the results or findings of a research study. For a replication to take place, the study must be capable of being replicated (Bryman and Bell, 2011).

In the context of this thesis with icenet's call centre case, replication is fully possible. It is because all data about icenet's call centre operations are stored in a database, and these are available for other researchers to get (Øksnes, 2017). Contemporary technology makes it possible to automatically log all the events in the call centre. These data are crucial for building the queuing model and for the mathematical analysis which will both be done in this thesis. Other researchers can access these data through icenet.

Through this thesis a detailed description is shown of the steps and procedures used to obtain the results of this thesis. Which other researchers can see and use to replicate the results that will be obtained in this thesis. All this available information makes it very possible for replicability. The fact that a research is replicable ensures reliability (Bryman and Bell, 2011).

Chapter 4 – Empirical findings

4.0 Introduction

In this chapter the empirical findings will be presented. All the data required to do the research in this thesis to study icenet's call centre performance will be presented here. A graphical representation of the queuing system of icenets inbounds call centre service will be shown. Data required to build the queuing models will be presented. Data collected about call arrivals and service times, current staffing plan, the queuing discipline, the characteristics of the waiting line and service facility will all be presented here. All this data are provided by logistics and forecasting manager in icenet, Alf Martin Øksnes (Øksnes, 2017).

4.1 Description of the queuing system in icenet's call centre

The queuing system in icenet's call centre is quite similar to figure (1) and (2) shown in chapter 1 and the explanation of how a queuing system functions given in chapter 2. The figure below, according to Øksnes (2017) is a very close depiction of the queuing system in their call centre.

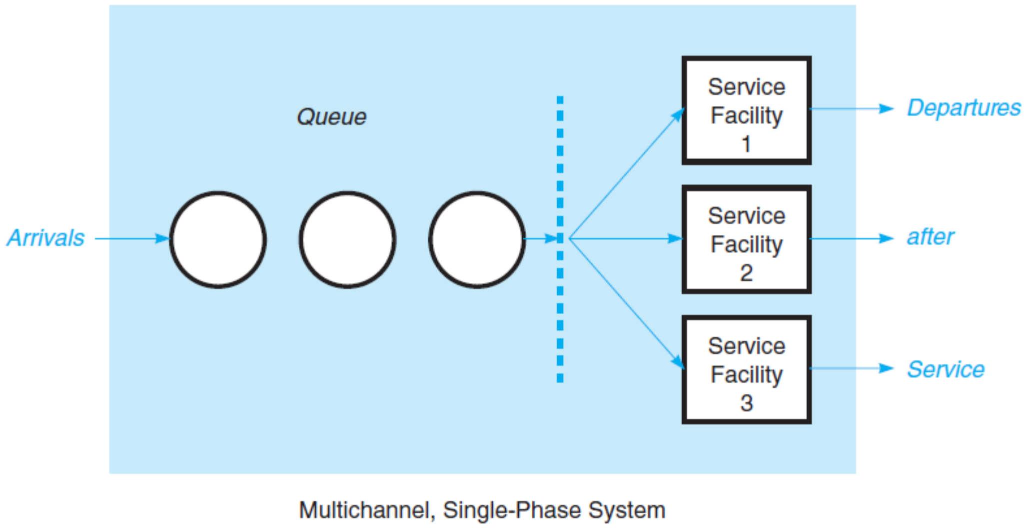


Figure 4. *Illustration of a queuing system with multichannel, single-phase system (Render, Stair and Hanna, 2011)*

Their customers call the call centre and if all CSR's are busy then they are placed in a queue waiting for a CSR to become available. When a CSR becomes available for service, then the customer leaves the queue and will be connected to the free CSR to receive service, and then leave the system when done. The queuing system in icenet's call centre is of a multichannel system (Øksnes, 2017). This is where several CSR's are on duty, and all customers are waiting in one common line for the first available CSR (Render, Stair and Hanna, 2011). Øksnes

(2017) also says that their system is a single-phase system. Here customers receive service at only one station and then exits the system (Render, Stair and Hanna, 2011). Throughout the whole process from entering the queuing system to exiting it, customers only go through one station and then out (Render, Stair and Hanna, 2011). According to Øksnes (2017) there is no limit on how long the length of the queue in icenet's call center can be. The queue length can go up to unlimited (Øksnes, 2017). Icenet's call center follows a first-in, first-out (FIFO) queuing discipline (Øksnes, 2017). The CSR's in the call center are all homogeneous which means that they have the same skill-level (Øksnes, 2017).

The size of the calling population is considered unlimited as anyone can make a phone call to icenet's call centre, for ex. whether they are already customer in icenet or not. It could be any existing person.

4.2 Data collected on call arrivals and service times from icenet's call center operations

Data on call arrivals and service times has been collected from icenet's call center operations during a busy-week in 2017. This selected busy-week is week 2 in the beginning of this year 2017, in the month January. In the busy-week, week 2, data were collected from the date 09.01.2017 Monday to 13.01.2017 Friday. These data were not collected as primary data, but as secondary data since icenet's call center stores all information regarding call arrivals and service times in a database.

A small portion of the data set will be presented here in a table, this is to illustrate the layout of the dataset. Only a small piece extracted from the actual data set will be shown here for the purpose of illustration. The actual full dataset is in a excel file which will be placed as an attachment to this thesis. The full dataset contains approximately 7500 call arrivals which were registered by icenet in the busy-week from Monday 09.01.2017 to Friday 13.01.2017. For the full dataset in the excel file, a clear explanation will be given in the appendices on how to read and interpret the numerical information in the dataset, because these are completely raw data extracted from icenet's computer systems which were stored in a database. Explanation for the numerical data in the full dataset is written in a computer language which is difficult for the common reader to understand. This will be cleared up in the appendices, in appendix 2.

But for now only a small piece taken out from the full data set, which is made understandable for the common reader, will be shown here in a table down below for the purpose of illustration.

Caller	Arrival time into queuing system	Exit time from queuing system	Service time per call (in seconds)
1	08:00:16	08:05:40	96
2	08:00:24	08:09:31	426
3	08:00:41	08:14:40	53
4	08:00:42	08:11:57	475
5	08:01:05	08:10:35	380

Table 1. *Call arrivals to icenet's call center during a minute.*

The table above shows call arrivals to icenet's call center during one minute. These calls arrived in the early opening minute on 09.01.2017 Monday.
Such a data set makes it possible to measure the interarrival times and service times, which are important measures to conduct the performance analysis in this thesis. In an inbound call center, call arrivals and call-handling times are often random (Raik Stolletz, 2012). Øksnes (2017) agrees that in icenet's call center, call arrivals and call handling times are random and independent of one another.

4.3 Icenet's call centre operations – staff schedule and call volumes during the busy-week

The operating hours of icenet's call centre is from 08.00 a.m. – 20.00 p.m. Monday to Friday and 08.00 a.m. – 16.00 p.m. on Saturdays (Øksnes, 2017). On Sundays their inbound call centre service is closed. In this thesis, focus will be on call volumes handled from Monday to Friday, because the data received for study covers only this timeframe, and not a Saturday.

Øksnes (2017) explains that each workday from Monday to Friday is divided into 3 shifts. This is shown in the table below.

Duration of each shift on a typical weekday.	Shift nr.
08.00 – 13.00	1
10.30 – 18.00	2
15.00 – 20.00	3

Table 2. *Work shifts throughout a typical workday and its durations* (Øksnes, 2017).

As mentioned under the main objectives and research question section in the introduction chapter, one typical workday in a busy-week will be chosen for analysis and this particular day is Friday 13.01.2017. On this day, a number of CSR's were scheduled to work on each of these 3 shifts. The staff schedule for

Shifts during a Friday	Shift nr.	Number of CSR's per shift
08.00 – 13.00	1	9
10.30 – 18.00	2	12
15.00 – 20.00	3	11

this day is shown below in a table.

Table 3. *Staff schedule for Friday 13.01.17.*

It can be noticed in this table that the shifts are overlapping each other. By looking at this table with only these three shifts it is difficult to determine exactly how many CSR's are present at any moment in time in a shift because they are overlapping each other.

For the purpose of the analysis in this thesis it is important to have a clear mapping of exactly how many CSR's are present at any moment in time during the workday. A solution to finding out this is to divide the workday into "periods" where we can read the number of CSR's that are left when CSR's from one shift arrives or CSR's from another shift leaves. So for example at 08.00 a.m. 9 CSR's arrive for work, then at 10.30 they will be joined by another 12 CSR's, so from 08.00-10.30 9 CSR's are present and then from 10.30 – 13.00 21 CSR's will be present. At 13.00 the first group of 9 CSR's are finished for the day and the call center is left with 12 CSR's from 13.00. At 15.00 they are joined by 11 CSR's who come for work. From 15.00 to 18.00 there are 23 CSR's present and at 18.00 the CSR's from shift 2 leave (12 CSR's leave). From 18.00 – 20.00 the call center is left with 11 CSR's. See the table below which illustrates the periods that shows how many CSR's are present in the call centre at any moment in time during the workday.

Workday on Friday divided into "periods" with duration in each period	Periods	Number of CSR's present at work within different time periods
08.00 – 10.30	1	9
10.30 – 13.00	2	21
13.00 – 15.00	3	12
15.00 – 18.00	4	23
18.00 – 20.00	5	11

Table 4. *Workday on Friday 13.01.17 divided into periods with CSR's in them.*

This table will be helpful for the upcoming analysis as it clearly shows the exact number of CSR's present in the call centre at any time during the workday.

A performance analysis on the state the queuing system is in during various small time intervals inside periods throughout the day will be done, to find out whether they actually had the right number of CSR's within these timeintervalls in the periods. Based on this, suggestions will be made on alternative staffing levels, which can give more satisfying service and costs.

So this will show whether icenet's call centre had the right amount of people at work or not on those timeintervalls studied on that particular day, Friday 13.01.17.

How the day and the periods are divided into smaller time intervals will be shown in section 4.3.2 and also which of the time intervals from each period that will be studied in the analysis will be mentioned in this section.

4.3.1 Distribution of call arrivals from Monday to Friday in week 2

Here a figure which shows the call volumes received by icenet's call centre each day in week 2 will be presented. These call volumes are distributed over the different periods, which is shown in the figure below.

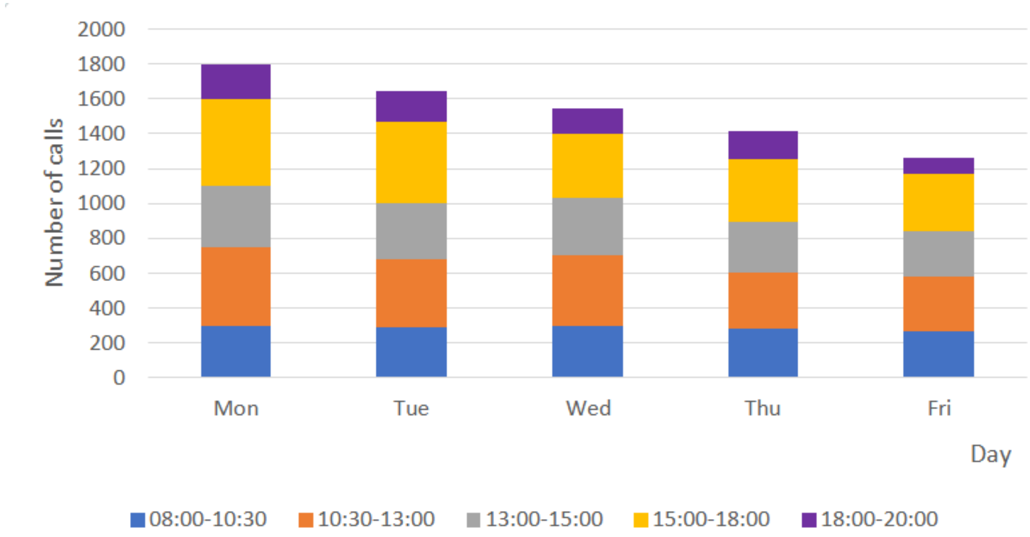


Figure 5. *Distribution of call arrivals per period in each day in week 2.*

In the figure above it can be seen that there is a pattern in the amount of call arrivals in each period in each day throughout the week. In the last period 18.00 – 20.00 the call volumes are much lower than in the other periods. In the 4th period, call volumes have been much higher compared to other periods in the days throughout the week. The periods on Friday will be analyzed in this thesis. Already now by comparing call arrivals on Friday from figure (5) and the staff schedule for Friday from table (4), it is possible to see that icenet's call center management have given some thought when scheduling staff on this day, for ex. there are less CSR's present in the first period and the last period compared to other periods. But it remains to see after the performance analysis coming up in the next chapter, whether icenet have scheduled their staff appropriately to the incoming call volumes with regards to achieving optimal or near optimal service level and costs, or if it could have been made better.

In appendix 3, a table which contains the input data used to create figure (5) can be found. It shows the numbers on how many calls that arrived in different periods in the days throughout the week.

4.3.2 Call arrivals on Friday 13.01.17 distributed over 30 minute intervals throughout the day

The various time intervals within the periods throughout the day will have a length of 30 minutes each. In each of these intervals throughout the day, a number of calls were received by icenet's call centre. The number of call arrivals in each of these intervals has been measured and extracted through studying the dataset used in this thesis. For each 30 minute interval analyzed, the number of call arrivals in that particular interval will be considered when calculating the performance measures which will give a picture of the queuing system performance within the selected interval that is studied. The table below shows the number of call arrivals on Friday 13.01.17 distributed over 30 minute intervals throughout the day.

Period 1		**Period 2**		**Period 3**		**Period 4**		**Period 5**	
Time interval	Call arrivals	Time interval	Call arrivals	Time interval	Call arrivals	Time interval	Call arrivals	Time interval	Call arrivals
08.00 - 08.30 (1)	42	10.30-11.00 (6)	54	13.00-13.30 (11)	59	15.00-15.30 (15)	72	18.00-18.30 (21)	27
08.30-09.00 (2)	29	11.00-11.30 (7)	71	13.30-14.00 (12)	57	15.30-16.00 (16)	57	18.30-19.00 (22)	18
09.00-09.30 (3)	61	11.30-12.00 (8)	60	14.00-14.30 (13)	68	16.00-16.30 (17)	46	19.00-19.30 (23)	22
09.30-10.00 (4)	57	12.00-12.30 (9)	64	14.30-15.00 (14)	76	16.30-17.00 (18)	45	19.30-20.00 (24)	21
10.00-10.30 (5)	76	12.30-13.00 (10)	62			17.00-17.30 (19)	57		
						17.30-18.00 (20)	48		

Table 5. *Call arrivals on Friday 13.01.17 distributed over 30 minute intervals throughout the day*

It can be seen in the table that there are twenty-four 30-minute intervals in total. Each interval from the first one to the last one is given a number from (1) to (24).

In the analysis in the next chapter, this number represents which interval is analyzed. The time intervals selected for the analysis are (1), (5), (6), (11), (15) and (21). An interval from each period is studied.

4.4 Cost data for calculating economic performance measures

In this thesis, the waiting costs and the operating costs of the queuing system operations will be compared to each other. Icenet's call center management thinks that they have too high waiting costs compared to operating costs, and waiting costs are the type of costs they want the least of (Øksnes, 2017). This potential imbalance between these two costs will be investigated through the performance analysis in the next chapter. If there is an imbalance, they want this to be evened out with regard to achieving optimal or near optimal service level and total costs.

As mentioned earlier, waiting costs and operating costs are the two measures of economic performance on the queuing system operations in icenet's call centre that will be focused on in this thesis. Certain cost data's are required to calculate these economic performance measures. Cost of labor per call will be used to measure operating costs. The formula for calculating cost of labor per call was defined in chapter 2 and the formula requires that cost of a CSR per unit of time is known for computation. In addition to this utilization and average activity time per call must be known. The information on cost of a CSR per unit of time is provided by icenet's call center management.

According to Øksnes the estimated average cost of a CSR per unit of time is 400 kr/hour (Øksnes, 2017). This cost represents how much every hired CSR costs per hour for icenet. This cost includes wages, use of equipment and training, altogether put in one parameter (Øksnes, 2017).

Estimated average cost of a CSR per unit of time	**400 kr/hour**

Waiting line costs are "relatively" intangible costs (Chowdury, Rahman, Kabir, 2013). Waiting line costs are caused because customers have to wait and these costs could come from lost goodwill, dissatisfied customers and losing customers to competitors (Chowdury, Rahman, Kabir, 2013). This fairly intangible cost is not easy to quantify (Chowdury, Rahman, Kabir, 2013).

Øksnes has come up with a fairly approximate estimate of the waiting costs which he thinks is 130 kr/hour, and this number reflects lost goodwill, dissatisfied customers and loss of customers (Øksnes, 2017). This parameter will be used to calculate waiting cost per call.

Cost of waiting per unit of time	**130 kr/hour**

Chapter 5 – Performance analysis

5.0 Introduction

In this chapter the performance analysis on the various time intervals in the periods throughout the day will be done. The selected time intervals from section 4.3.2 will be analyzed in this chapter. In chapter 2, the step by step procedure presented in section (nr), drawn from queuing theory approach, for calculation of waiting time related performance measures and economic performance measures will be followed to analyze the queuing system performance of the various time intervals. Based on this, a discussion will be made on which staffing level will be most favorable with regards to achieving optimal service and costs.

The analysis on each time interval in the periods will be done in chronological order from the first time interval of the day to the last one. Only the chosen time intervals will be analyzed, and this in chronological order. For the calculations of waiting related performance measures and economic performance measures in this analysis chapeter, explanation on how to calculate these step by step will be shown only for the staffing level which was the one that was actually used by icenet in the time interval. The calculations on how various staffing levels will impact the waiting related performance measures and the economic performance measures will be shown thoroughly step by step in appendix 1. But here in this chapter, only a table containing only the numerical values on all the performance measures calculated for each staffing level is shown, with no explanation on how they were calculated. This table will be shown in the end of each time interval analysis.

The calculations of the averages and the standard deviations on interarrival times and service times for each selected time interval are explained in excel file called “Calculations for interarrival times and service times (2)”, which is put as an attachment to this thesis.

The values of the performance measures calculated through the analysis will be discussed in the next chapter, chapter 6. And an evaluation of whether they had the right staffing level will be taken, and what favorable changes can be done, and what impact this has on the costs of icenet’s call centre and the service they provide.

5.1 Performance analysis of selected time intervals from Friday 13.01.17

5.1.1 <u>Period 1 – time interval 1 – 08.00-08.30:</u>

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

Average interarrival time: 41,595 seconds.
Standard deviation of interarrival time: 48, 4924 seconds.
Average interarrival time ≠ Standard deviation of interarrival time. So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 301,41 seconds.
Standard deviation of activity time: 209,52 seconds.
Average activity time ≠ standard deviation of activity time. So we have non-exponential service time. There is variability in the service times. So service times have a general distribution.

p = 301,4

a = 41,59

m = 9

The queuing model that represents this time interval is:
G/G/9 {∞/∞/FIFO}

2) *Calculations of waiting related performance measures:*

utilization u = p/(a×m) => u= 301,4/(41,59×9) = 0,805 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?

m= 8

u= 301,4/(41,59×8) = 0,906 < 1

Measuring variability of arrival process:

CV_a= Standard deviation of interarrival time/ average interarrival time

= 48,49/41,59 = 1,1659

Measuring variability in service times:

CV_p = standard deviation of activity time/ average activity time

= 209,52/301,41 = 0,6951

Calculation of expected waiting time using formula for expected waiting time:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}\ -1}}{1-utilization}\right) \times \left(\frac{(CV_a)^2 \times (CV_p)^2}{2}\right)$$

All values to insert into formula are known.

p=activity time = average activity time.

For the chosen input values, T_q= 74,469 seconds = 1,241 minutes.

Average number of customers waiting in the queue:

$I_q = \frac{T_q}{a}$ = 1,79

3) *For this staffing level, what are the economic performance measures?*

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 301,4 seconds = 5,023 min.

Cost of labor per call = $\frac{5{,}023\text{min/call} \times 6{,}667\text{kr/min}}{0{,}805}$ = 41,6 kr/ call

Waiting cost per call and total cost per call:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 74,469 seconds = 1,241 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 1,241min × 2,1667kr/min = 2,689kr
Total cost per call = 41,6 + 2,689 = 44,289 kr/ call.

4) *The table below shows the impact of various staffing levels on waiting time related performance measures and economic performance measures.*

Time interval 1: 08.00 – 08.30						
m	u	T_q	I_q	Cost of labor per call (kr)	Waiting cost per call (kr)	Total cost per call (kr)
8	0,906	268,08	6,446	36,96	9,68	46,64
9	0,805	74,47	1,79	41,6	2,689	44,289
10	0,725	30,81	0,741	46,19	1,11	47,3
11	0,659	14,56	0,35	50,82	0,526	51,35

Table 6. *Calculations of performance measures, time interval 1: 08.00-08.30.*

5.1.2 <u>Period 1 – time interval 5 – 10.00-10.30:</u>

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

Average interarrival time: 22, 207 seconds.
Standard deviation of interarrival time : 22,848 seconds.
Average interarrival time ≠ Standard deviation of interarrival time. So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 265,72 seconds
Standard deviation of activity time: 221,72 seconds
Average activity time ≠ standard deviation of activity time. So we have non-

exponential service times. There is variability in the service times. So service times have a general distribution.

p = 265,72
a = 22,207
m = 9
The queuing model that represents this time interval is:
G/G/9 {∞/∞/FIFO}

2) *Calculations of waiting related performance measures:*

utilization u = p/(a×m) => u= 265,72/(22,207×9) = 1,3295 u>1, It doesn't satisfy the steady state condition. What this means for the queuing system behavior, will be discussed in the next chapter.

5.1.3 Period 2 – time interval 6 – 10.30-11.00:

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

Average interarrival time: 33,73 seconds.
Standard deviation of interarrival time : 29,98 seconds.
Average interarrival time ≠ Standard deviation of interarrival time. So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 277,796 seconds
Standard deviation of activity time: 165,691 seconds
Average activity time ≠ standard deviation of activity time. So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

p = 277,796
a = 33,73

m = 21

The queuing model that represents this time interval is:
G/G/21 {∞/∞/FIFO}

2) *Calculations of waiting related performance measures:*

utilization u = p/(a×m) => u= 277,796/(33,73×21) = 0,39 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?
m=9
u= 277,796/(33,73×9) = 0,915 < 1

Measuring variability of arrival process:
CV_a= 29,98/33,7358 = 0,889

Measuring variability in service times:
CV_p = 165,6918/277,7962 = 0,596

Calculation of expected waiting time using formula for expected waiting time:
T_q

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 0,0617 seconds = 0,0010283 minutes.
Average number of customers waiting in the queue:
$I_q = \frac{T_q}{a}$ = 0,001829

3) *For this staffing level, what are the economic performance measures?*

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 277,796 seconds = 4,62993 min.

Cost of labor per call = $\frac{4{,}62993\text{min/call} \times 6{,}667\text{kr/min}}{0{,}39}$ = 79,14 kr/ call

Waiting cost per call and total cost per call:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 0,0617 seconds = 0,0010283 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,0010283 min × 2,1667kr/min = 0,002228 kr

Total cost per call = 79,14 + 0,002228 = 79,142 kr/ call.

4) *The table below shows the impact of various staffing levels on waiting time related performance measures and economic performance measures.*

Time interval 6: 10.30 – 11.00						
m	u	T_q	I_q	Cost of labor per call (kr)	Waiting cost per call (kr)	Total cost per call (kr)
9	0,915	152,77	4,5292	33,73	5,52	39,25
10	0,823	43,8	1,298	37,51	1,58	39,09
11	0,748	18,5	0,548	41,27	0,668	41,938
12	0,686	9	0,267	44,996	0,325	45,3
21	0,39	0,0617	0,001829	79,14	0,002228	79,142

Table 7. *Calculations of performance measures, time interval 6: 10.30-11.00.*

5.1.4 <u>Period 3 – time interval 11 – 13.00-13.30:</u>

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

Average interarrival time: 30,016 seconds.

Standard deviation of interarrival time : 31,1940 seconds.

Average interarrival time ≠ Standard deviation of interarrival time. So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 340,1967 seconds.
Standard deviation of activity time: 284,17 seconds.
Average activity time ≠ standard deviation of activity time. So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

p = 340,1967
a = 30,016
m = 12

The queuing model that represents this time interval is:
G/G/12 {∞/∞/FIFO}

2) *Calculations of waiting related performance measures:*

utilization u = p/(a×m) => u= 340,1967/(30,016×12) = 0,944 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?
m=12
u= 340,1967/(30,016×12) = 0,944 < 1

Measuring variability of arrival process:
CV_a= 31,1940/30,016 = 1,03924

Measuring variability in service times:
CV_p = 284,1745/340,1967 = 0,8353

Calculation of expected waiting time using formula for expected waiting time:
T_q

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 354,95 seconds = 5,9158 minutes.

Average number of customers waiting in the queue:
$I_q = \frac{T_q}{a}$ = 11,8

3) *For this staffing level, what are the economic performance measures?*

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Cost of labor per call:
Cost of labor per call = $\frac{p \times w}{u}$
p = 340,1967 seconds = 5,6699 min.
Cost of labor per call = $\frac{5{,}6699\text{min/call} \times 6{,}667\text{kr/min}}{0{,}944}$ = 40 kr/ call

Waiting cost per call and total cost per call:
Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 354,95 seconds = 5,9158 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 5,9158 min × 2,1667kr/min = 12,8178 kr
Total cost per call = 40 + 12,8178 = 52,8178 kr/ call.

4) *The table below shows the impact of various staffing levels on waiting time related performance measures and economic performance measures.*

Time interval 11: 13.00 – 13.30						
m	u	T_q	I_q	Cost of labor per	Waiting cost per	Total cost per call

				call (kr)	call (kr)	(kr)
12	0,944	354,95	11,8	40	12,8178	52,8178
13	0,8718	100,6	3,35	43,36	3,63	46,99
14	0,809	43,74	1,4572	46,726	1,5795	48,3
15	0,755	22,21	0,74	50,1	0,802	50,902

Table 8. *Calculations of performance measures, time interval 11: 13.00-13.30.*

5.1.5 <u>Period 4 – time interval 15 – 15.00-15.30:</u>

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

Average interarrival time: 24,69 seconds.
Standard deviation of interarrival time : 24.26 seconds.
Average interarrival time ≠ Standard deviation of interarrival time.
So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 283,917 seconds.
Standard deviation of activity time: 202,2589 seconds.
Average activity time ≠ standard deviation of activity time. So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

p = 283,917
a = 24,69
m = 23

The queuing model that represents this time interval is:
G/G/23 {∞/∞/FIFO} or using an M/G/23 {∞/∞/FIFO} is also a possibility.

The difference between mean and standard deviation for interarrival times are very small. According to Green (2011) if the actual CV_a is just a little bit lower or higher than 1, the calculations through using a M/G/s model will also give quite good estimates of waiting time. If CV_a is substantially larger or smaller than 1, then the M/G/s model could underestimate or overestimate actual waiting times (Green, 2011).

2) *Calculations of waiting related performance measures:*

utilization u = p/(a×m) => u= 283,917/(24,69×23) = 0,50 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?
m=12
u= 283,9178/(24,694×12) = 0,9581 < 1

Measuring variability of arrival process:
CV_a= 24,26/24,694 = 0,9824

Measuring variability in service times:
CV_p = 202,2588/283,9178 = 0,7123

Calculation of expected waiting time using formula for expected waiting time:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}}\ -1}{1-utilization}\right) \times \left(\frac{(CV_a)^2 \times (CV_p)^2}{2}\right)$$

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 0,2983 seconds = 0,0049716 minutes.

Average number of customers waiting in the queue:

$$I_q = \frac{T_q}{a} \quad = 0{,}012$$

3) *For this staffing level, what are the economic performance measures?*

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4{,}731963\text{min/call} \times 6{,}667\text{kr/min}}{0{,}50}$ = 63,095 kr/ call

Waiting cost per call and total cost per call:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 0,2983 seconds = 0,0049716 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,0049716 min × 2,1667kr/min = 0,01 kr

Total cost per call = 63,095 + 0,01 = 63,1 kr/ call.

4) *The table below shows the impact of various staffing levels on waiting time related performance measures and economic performance measures.*

Time interval 15: 15.00 – 15.30						
m	u	T_q	I_q	Cost of labor per call (kr)	Waiting cost per call (kr)	Total cost per call (kr)
12	0,9581	348,81	14,12	32,927	12,5961	45,523
13	0,8844	82,09	3,32	35,67	2,96	38,63
14	0,821	34,49	1,396	38,42	1,245	39,665
15	0,7664	17,28	0,7	41,16	0,624	41,784
16	0,718	9,35	0,378	43,94	0,337	44,28
23	0,5	0,2983	0,012	63,095	0,01	63,1

Table 9. *Calculations of performance measures, time interval 15: 15.00-15.30.*

5.1.6 <u>Period 5 – time interval 21 – 18.00-18.30:</u>

1) *What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?*

Average interarrival time: 62,11 seconds.
Standard deviation of interarrival time : 58,56 seconds.
Average interarrival time ≠ Standard deviation of interarrival time. So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 317,5 seconds.
Standard deviation of activity time: 261,08 seconds.
Average activity time ≠ standard deviation of activity time. So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

p = 317,5
a = 62,11
m = 11

The queuing model that represents this time interval is:
G/G/11 {∞/∞/FIFO}

2) *Calculations of waiting related performance measures:*

utilization u = p/(a×m) => u= 317,5/(62,11×11) = 0,46 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?
m=6
u= 317,5/(62,11×6) = 0,852 < 1

Measuring variability of arrival process:
CV_a= 58,56/62,11 = 0,9428

Measuring variability in service times:

CV_p = 261,08/317,5 = 0,822

Calculation of expected waiting time using formula for expected waiting time:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}\ -1}}{1-utilization}\right) \times \left(\frac{(CV_a)^2 \times (CV_p)^2}{2}\right)$$

All values to insert into formula are known.

p=activity time = average activity time.

For the chosen input values, T_q= 2,02 seconds = 0,03367 minutes.

Average number of customers waiting in the queue:

$I_q = \frac{T_q}{a}$ = 0,0325

3) *For this staffing level, what are the economic performance measures?*

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 317,5 seconds = 5,29167 min.

Cost of labor per call = $\frac{5{,}29167\text{min/call} \times 6{,}667\text{kr/min}}{0{,}46}$ = 76,69 kr/ call

Waiting cost per call and total cost per call:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 2,02 seconds = 0,03367 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,03367 min × 2,1667kr/min = 0,0729 kr

Total cost per call = 76,69 + 0,0729 = 76,76 kr/ call.

4) *The table below shows the impact of various staffing levels on waiting time related performance measures and economic performance measures.*

Time interval 21: 18.00 – 18.30						
m	u	T_q	I_q	Cost of labor per call (kr)	Waiting cost per call (kr)	Total cost per call (kr)
6	0,852	180,27	2,9	41,4	6,51	47,91
7	0,7302	51,2	0,824	48,31	1,848	50,16
8	0,6389	20,11	0,323	55,22	0,726	55,946
9	0,568	8,96	0,144	62,11	0,3235	62,4335
11	0,46	2,02	0,0325	76,69	0,0729	76,76

Table 10. *Calculations of performance measures, time interval 21: 18.00-18.30.*

Chapter 6 – Discussion of analysis results

6.0 Introduction

In this chapter, the outcome of the performance analysis will be discussed. The state of the queuing system performance during each time interval will be discussed. The calculated performance measures from each time interval will be explained. The practical relevance the results of the analysis has for icenet's call center will be explained.

6.1 Discussion of results from performance analysis on each selected time interval

6.1.1 Results obtained for Period 1 – time interval 1 – 08.00-08.30

In this early morning 30 minute time interval, 42 calls arrived in the call center. This amount of calls which arrived in this time interval is smaller than what arrived in many of the other later intervals throughout the day. This is obvious because it is early in the morning, and customers may think that it takes some time before the call centre is up and running. When they call later they expect that the call center is already up and running. In period 1 where the time interval is from, there were 9 CSR's on duty. The utilization of the process was 80,5 %, which is the same as that each of the 9 CSR's had a utilization of 80,5 %. This means each CSR were busy 0,805×30 min. = 24,15 min. of the time in the 30 min. interval. The expected wait time for each customer who is waiting in line is 74,47 seconds = 1,24 min. On table (6), for various staffing levels, different values for cost of labor per call and waiting cost per call is shown. When staffing level increases, the cost of labor per call increases and waiting cost per call decreases. According to the table, 9 CSR's which was their actual staffing level in this first interval, will give the balance between these two costs that will give optimal total cost per call. But if they want lower waiting time, then they can alternatively go for 10 CSR's, but this will not give optimal total cost per call. It will give them slightly higher cost than the optimal total cost level. So icenet's choice of having the staffing level of 9 CSR's gave optimal total cost per call during this time interval, and they hit the target good, with regard to achieving optimal total cost. Alternatively, 10 CSR's would have been a more waiting time pleasant option.

6.1.2 Results obtained for Period 1 – time interval 5 – 10.00-10.30

In this 30 minute time interval there were 76 call arrivals. In period 1, 9 CSR's were on duty as can be seen from table (4). In this time interval, the utilization is higher than 1 and this means that it does not satisfy the steady state condition. In a practical sense, this means that the waiting line grows a lot during this time interval until it reaches a steady state in another time interval. It is not possible to find any more information about the queuing system performance. Queuing analytical formulas for queues with unlimited size are applicable only for steady-state processes (Kolker, 2010). Only when $u<1$, a steady state condition is possible, otherwise the queuing formulas presented in chapter 2 are not applicable and the queue grows indefinitely (Kolker, 2010).

6.1.3 Results obtained for Period 2 – time interval 6 – 10.30-11.00

In this 30 minute time interval there were 54 call arrivals. In period 2, 21 CSR's were on duty as can be seen from table (4). The utilization of the process was 39 %, which is the same as that each of the 21 CSR's had a utilization of 39%. This means each CSR were busy 0,39×30 min. = 11,7 min. of the time in the 30 min. interval. So with regard to this time interval, icenet has been slightly inaccurate when deciding staffing level here, because each of their CSR's have an idle time of 18,3 min, which is not good. The expected wait time for each customer who is waiting in line is 0,0617 seconds. This can be interpreted as, on average almost no waiting time at all for customers. But this comes at a very high cost. The labor cost per call in this time interval is 79,14kr and waiting cost per call is 0,002228 kr, which gives total cost per call 79,14. Here, there is a big imbalance between labor cost per call and waiting cost per call which can be evened out to benefit total cost per call, because raising the expected waiting time from 0 seconds to a tolerable waiting time is not going to cause too much bad customer service. They can afford to raise the waiting costs in this situation. As can be seen from the table, a staffing level of 21 gives a very high total cost per call compared to the other staffing options. A staffing level of 9 CSR's reduces the total cost per call by almost 50 % to 39, 25 kr. This gives a expected waiting time of 152,77 seconds = 2,55 min. There is another option which gives even better numbers on waiting time and total cost per call, and this is a staffing level of 10 CSR's, as can be seen in the table. This is the option that gives optimal total cost per call which is 39,09 kr and also a more pleasant expected waiting time of 43,8 seconds. When the aim is to reduce waiting time but also achieve optimal total cost, the staffing level of 10 CSR's would have been the best option. So icenet missed their target badly, if they were aiming for reducing waiting time and at the same time have optimal total cost per call, by having 21 CSR's on duty during this time interval. They were very overstaffed during this time interval.

6.1.4 Results obtained for Period 3 – time interval 11 – 13.00-13.30

In this 30 minute time interval there were 59 call arrivals. In period 3, 12 CSR's were on duty as can be seen from table (4). The utilization of the process was 94,4 %, which is the same as that each of the 12 CSR's had a utilization of 94,4%. This means each CSR were busy 0,944×30 min. = 28,32 min. of the time in the 30 min. interval. This is good news for icenet's call center management that all their CSR's are busy and have very less idle time. They are more productive. The negative side of this is that customer's expected wait time in the queue will be longer. The expected wait time is 354,95 s = 5,91 min, and a reduction on this should be attempted. By observing on table (8), values for cost of labor per call and waiting cost per call is shown for various staffing levels, and here it can be seen that a staffing level of 12 gives a total cost per call of 52,82 kr. But there is a better option which is a staffing level of 13 and this gives 46,99 kr in total cost per call. It can be seen in the table that this option gives the optimal total cost per call. This option also reduces the expected waiting to 100, 6 s = 1,67 min., and this will increase customer satisfaction. So a staffing level of 13 would have been the best option in this time interval studied.

6.1.5Results obtained for period 4 – time interval 15 – 15.00-15.30

In this 30 minute time interval there were 72 call arrivals. In period 4, 23 CSR's were on duty as can be seen from table (4). The utilization of the process was 50 %, which is the same as that each of the 23 CSR's had a utilization of 50%. This means each CSR were busy 0,5×30 min. = 15 min. of the time in the 30 min. interval. With regard to this time interval, it clearly shows that icenet had to many CSR's on duty because all these 23 CSR's are idle in half of the time in this 30 min. time interval. Expected waiting time is 0,2983 seconds which is close to almost no waiting time. Labor cost per call is 63 kr and waiting cost per call is 0,01 kr and total cost per call is 63,1 kr. As can be seen in table (9), this imbalance can be evened out to benefit total cost per call. And it doesn't hurt to increase the waiting time from 0 to a number that is not too high. So here the option that gives optimal total cost per call is a staffing level of 13. Here, expected wait time is 82 s. = 1,36 min. If a lower waiting time than this is wanted, then staffing levels of 14 and 15 are also very good, as can be seen in table (9). Since these staffing levels give values on total cost per call that are quite close to the optimal total cost per call. So icenet should have had 13 CSR's on duty during time interval 15.

6.1.6 Results obtained for period 5 – time interval 21 – 18.00-18.30

In this 30 minute time interval there were 27 call arrivals. In period 5, 11 CSR's were on duty as can be seen from table (4). The utilization of the process was 46 %, which is the same as that each of the 11 CSR's had a utilization of 46%. This means each CSR were busy 0,46×30 min. =13,8 min. of the time in the 30 min. interval. This means that each of the 11 CSR's were idle 16,2 min. of the time in this 30 min. interval. In this time interval, the cost of labor per call was 76,69 kr and waiting cost per call was 0,0729 kr which is close to zero. This gives a total cost per call of 76,76 kr. The expected waiting time is 2,02 seconds when 11 CSR's are on duty. As can be seen in table (10), there are many other options that decreases total cost per call, but at the same doesn't increase waiting time too much. The optimal choice for total cost per call found in the calculations in this time interval is the total cost per call value that is the lowest. Here, the option that gives the lowest total cost per call is a staffing level of 6, which gives 47,91 kr as total cost per call, but this gives waiting time 180 seconds = 3 min and this is high compared to the other options. As can be seen in the table, a staffing level of 7 would give considerably lower waiting time, with only a 2,25 kr increase in total cost per call for a waiting time of 51,2 seconds. With regard to lower waiting time and total cost per call, a staffing level of 7, would have been the best for icenet during this late time interval.

7 Conclusion and limitations

8.1 Conclusions

The research in this thesis has been conducted to study the queuing system performance of icenets call center during various selected intervals throughout the day in a busy-week. This is to give them insight into how their system was running during the chosen intervals. In some time intervals, icenet's call center was heavily overstaffed and in some time intervals they were understaffed, where the queue grew larger and larger, with no steady state condition satisfied. In some time intervals icenet's call center management were very accurate with the number of CSR's scheduled. In chapter 6 where the discussion of the results of the performance analysis takes place, the results were discussed accordingly to the research question of this thesis. It is important to find a cost optimal staffing level but this must also give optimal or near optimal service level. This was taken under consideration when discussing the results, as there many different alternative staffing levels to chose from. In many of the intervals, icenet's call center was overstaffed, rather than understaffed which was the assumption in the introduction.

8.2 Limitations

One thing that is important to mention in the beginning is that the scope of this thesis is small. Only 6 intervals were analyzed throughout the day on Friday 13.01.17. This doesn't give a big picture of how the queuing system was running on that particular day. It could be that, on the other time intervals during the day, maybe the call center was understaffed. And this caused customer dissatisfaction. The day was divided into twenty four 30 minute intervals, and it could be that many of the other intervals were understaffed, but it was not captured here because only 6 intervals were studied in this thesis.

Also there is a big possibility that some of the cost data where inaccurate, like with the waiting cost. Since this is an fairly intangible cost, which is difficult to measure. It is very likely that the estimate on waiting cost data received could be a far off approximate.

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Appendix 1: detailed step by step calculations of performance measures for all time intervals analyzed in the performance analysis

<u>Period 1 – time interval 1 – 08.00-08.30:</u>

1) What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?

Average interarrival time: 41,595 seconds
Standard deviation of interarrival time : 48, 4924 seconds

Average interarrival time ≠ Standard deviation of interarrival time.

So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 301,41 seconds
Standard deviation of activity time: 209,52 seconds

Average activity time ≠ standard deviation of activity time.

So we have non-exponential service time. There is variability in the service times. So service times have a general distribution.

The queuing model that represents this time interval is:
G/G/9 {∞/∞/FIFO}

p = 301,4
a = 41,59
m = 9

2) Calculations of waiting related performance measures:

utilization u = p/(a×m) => u= 301,4/(41,59×9) = 0,805 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?

u= 301,4/(41,59×8) = 0,906 < 1

Measuring variability of arrival process:
CV_a= Standard deviation of interarrival time/ average interarrival time
= 48,49/41,59 = 1,1659

Measuring variability in service time:
CV_p = standard deviation of activity time/ average activity time
= 209,52/301,41 = 0,6951

Calculation of expected waiting time using formula for expected waiting time:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}\ -1}}{1-utilization}\right) \times \left(\frac{(CV_a)^2 \times (CV_p)^2}{2}\right)$$

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 74,469 seconds = 1,241 minutes.

Average number of customers waiting in the queue:
$I_q = \frac{T_q}{a}$ = 1,79

3) For this staffing level, what are the economic performance measures?

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 301,4 seconds = 5,023 min.

Cost of labor per call = $\frac{5{,}023\text{min/call} \times 6{,}667\text{kr/min}}{0{,}805}$ = 41,6 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 74,469 seconds = 1,241 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 1,241min × 2,1667kr/min = 2,689kr

Total cost per call = 41,6 + 2,689 = 44,289 kr/ call.

4) What is the impact of various staffing levels on waiting time related performance measures and economic performance measures?

Starting with the least number of CSR's that satisfies the steady state condition.

m = 8

u= 301,4/(41,59×8) = 0,906

p and a are not changed.

T_q= 268,08 seconds = 4,468 min

In this calculation, the only values changed, are the u and m. The variability measures on the arrival process and service times remains the same.

I_q = 6,446

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 301,4 seconds = 5,023 min.

Cost of labor per call = $\frac{5{,}023\text{min/call} \times 6{,}667\text{kr/min}}{0{,}906}$ = 36,96 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 268,08 seconds = 4,468 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 4,468 min × 2,1667kr/min = 9,68 kr

Total cost per call = 36,96 + 9,68 = 46,64 kr/ call.

<u>**m = 10**</u>

u= 301,4/(41,59×10) = 0,725

p and a are not changed.

T_q= 30,81 seconds = 0,5135 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,741

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 301,4 seconds = 5,023 min.

Cost of labor per call = $\frac{5{,}023\text{min/call} \times 6{,}667\text{kr/min}}{0{,}725}$ = 46,19 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 30,81 seconds = 0,5135 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 0,5135 min × 2,1667kr/min = 1,11 kr

Total cost per call = 46,19 + 1,11 = 47,3 kr/ call.

m = 11

u= 301,4/(41,59×11) = 0,659
p and a are not changed.
T_q= 14,56 seconds = 0,2426 min
In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.
I_q = 0,35
(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$
p = 301,4 seconds = 5,023 min.
Cost of labor per call = $\frac{5{,}023\text{min/call} \times 6{,}667\text{kr/min}}{0{,}659}$ = 50,82 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 14,56 seconds = 0,2426 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 0,2426 min × 2,1667kr/min = 0,526 kr

Total cost per call = 50,82 + 0,526 = 51,35 kr/ call.

Period 1 – time interval 5 – 10.00-10.30:

1) What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?

Average interarrival time: 22, 207 seconds.
Standard deviation of interarrival time : 22,848 seconds.
Average interarrival time ≠ Standard deviation of interarrival time. So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 265,72 seconds
Standard deviation of activity time: 221,72 seconds
Average activity time ≠ standard deviation of activity time. So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

p = 265,72
a = 22,207
m = 9
The queuing model that represents this time interval is:
G/G/9 {∞/∞/FIFO}

2) Calculations of waiting related performance measures:

utilization u = p/(a×m) => u= 265,72/(22,207×9) = 1,3295 u>1, It doesn't satisfy the steady state condition. What this means for the queuing system behavior, is discussed in chapter 6.

Period 2 – time interval 6 – 10.30-11.00:

1) What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?

Average interarrival time: 33,73 seconds

Standard deviation of interarrival time : 29,98 seconds

Average interarrival time ≠ Standard deviation of interarrival time.

So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 277,796 seconds

Standard deviation of activity time: 165,691 seconds

Average activity time ≠ standard deviation of activity time.

So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

The queuing model that represents this time interval is:

G/G/21 {∞/∞/FIFO}

p = 277,796

a = 33,73

m = 21

2) Calculations of waiting related performance measures:

utilization u = p/(a×m) => u= 277,796/(33,73×21) = 0,39 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?

u= 277,796/(33,73×9) = 0,915 < 1

CV_a= 29,98/33,7358 = 0,889

CV_p = 165,6918/277,7962 = 0,596

Calculation of expected waiting time using formula for expected waiting time:
T_q

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 0,0617 seconds = 0,0010283 minutes.

Average number of customers waiting in the queue:
$I_q = \frac{T_q}{a}$ = 0,001829

3) For this staffing level, what are the economic performance measures?

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Formula for cost of labor per call:
Cost of labor per call = $\frac{p \times w}{u}$
p = 277,796 seconds = 4,62993 min.
Cost of labor per call = $\frac{4{,}62993\text{min/call} \times 6{,}667\text{kr/min}}{0{,}39}$ = 79,14 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 0,0617 seconds = 0,0010283 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 0,0010283 min × 2,1667kr/min = 0,002228 kr

Total cost per call = 79,14 + 0,002228 = 79,142 kr/ call.

4) What is the impact of various staffing levels on waiting time related performance measures and economic performance measures?

Starting with the least number of CSR's that satisfies the steady state condition.

<u>m = 9</u>

u= 277,796 /(33,73×9) = 0,915

p and a are not changed.

T_q= 152,77 seconds = 2,5462 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 4,5292

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 277,796 seconds = 4,62993 min.

Cost of labor per call = $\frac{4,62993\text{min/call} \times 6,667\text{kr/min}}{0,915}$ = 33,73 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 152,77 seconds = 2,5462 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 2,5462 min × 2,1667kr/min = 5,52 kr

Total cost per call = 33,73 + 5,52 = 39,25 kr/ call.

<u>m = 10</u>

u= 277,796 /(33,73×10) = 0,823

p and a are not changed.

T_q= 43,8 seconds = 0,73 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 1,298

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 277,796 seconds = 4,62993 min.

Cost of labor per call = $\frac{4{,}62993\text{min/call} \times 6{,}667\text{kr/min}}{0{,}823}$ = 37,51 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 43,8 seconds = 0,73 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,73 min × 2,1667kr/min = 1,58 kr

Total cost per call = 37,51 + 1,58 = 39,09 kr/ call.

m = 11

u= 277,796 /(33,73×11) = 0,748

p and a are not changed.

T_q= 18,5 seconds = 0,3083 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,548

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 277,796 seconds = 4,62993 min.

Cost of labor per call = $\frac{4{,}62993\text{min/call} \times 6{,}667\text{kr/min}}{0{,}748}$ = 41,27 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 18,5 seconds = 0,3083 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,3083 min × 2,1667kr/min = 0,668 kr

Total cost per call = 41,27 + 0,668 = 41,938 kr/ call.

m = 12

u= 277,796 /(33,73×12) = 0,686

p and a are not changed.

T_q= 9 seconds = 0,15 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,267

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 277,796 seconds = 4,62993 min.

Cost of labor per call = $\frac{4,62993\text{min/call} \times 6,667\text{kr/min}}{0,686}$ = 44,996 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 9 seconds = 0,15 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 0,15 min × 2,1667kr/min = 0,325 kr

Total cost per call = 44,996 + 0,325 = 45,3 kr/ call.

<u>Period 3 – time interval 11 – 13.00-13.30:</u>

1) What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?

Average interarrival time: 30,016 seconds
Standard deviation of interarrival time : 31,1940 seconds

Average interarrival time ≠ Standard deviation of interarrival time.

So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 340,1967 seconds
Standard deviation of activity time: 284,17 seconds

Average activity time ≠ standard deviation of activity time.

So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

The queuing model that represents this time interval is:
G/G/12 {∞/∞/FIFO}

p = 340,1967
a = 30,016
m = 12

2) Calculations of waiting related performance measures:

utilization u = p/(a×m) => u= 340,1967/(30,016×12) = 0,944 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?

u= 340,1967/(30,016×12) = 0,944 < 1

CV_a= 31,1940/30,016 = 1,03924

CV_p = 284,1745/340,1967 = 0,8353

Calculation of expected waiting time using formula for expected waiting time: T_q

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 354,95 seconds = 5,9158 minutes.

Average number of customers waiting in the queue:
$I_q = \frac{T_q}{a}$ = 11,8

3) For this staffing level, what are the economic performance measures?

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 340,1967 seconds = 5,6699 min.

Cost of labor per call = $\frac{5{,}6699\text{min/call} \times 6{,}667\text{kr/min}}{0{,}944}$ = 40 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 354,95 seconds = 5,9158 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 5,9158 min × 2,1667kr/min = 12,8178 kr

Total cost per call = 40 + 12,8178 = 52,8178 kr/ call.

4) What is the impact of various staffing levels on waiting time related performance measures and economic performance measures?

Starting with the least number of CSR's that satisfies the steady state condition.

<u>**m = 13**</u>

u= 340,1967 /(30,016×13) = 0,8718

p and a are not changed.

T_q= 100,6 seconds = 1,6766 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 3,35

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 340,1967 seconds = 5,6699 min.

Cost of labor per call = $\frac{5{,}6699\text{min/call} \times 6{,}667\text{kr/min}}{0{,}8718}$ = 43,36 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 100,6 seconds = 1,676 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 1,6766 min × 2,1667kr/min = 3,63 kr

Total cost per call = 43,36 + 3,63 = 46,99 kr/ call.

m = 14

u= 340,1967 /(30,016×14) = 0,809

p and a are not changed.

T_q= 43,74 seconds = 0,729 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 1,4572

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 340,1967 seconds = 5,6699 min.

Cost of labor per call = $\frac{5{,}6699\text{min/call} \times 6{,}667\text{kr/min}}{0{,}809}$ = 46,726 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 43,74 seconds = 0,729 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,729 min × 2,1667kr/min = 1,5795 kr

Total cost per call = 46,726 + 1,5795 = 48,3 kr/ call.

<u>m = 15</u>

u= 340,1967 /(30,016×15) = 0,755

p and a are not changed.

T_q= 22,21 seconds = 0,37 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,74

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 340,1967 seconds = 5,6699 min.

Cost of labor per call = $\frac{5{,}6699\text{min/call} \times 6{,}667\text{kr/min}}{0{,}755}$ = 50,1 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 22,21 seconds = 0,37 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,37 min × 2,1667kr/min = 0,802 kr

Total cost per call = 50,1 + 0,802 = 50,902 kr/ call.

Period 4 – time interval 15 – 15.00-15.30:

1) What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?

Average interarrival time: 24,69 seconds
Standard deviation of interarrival time : 24.26 seconds

Average interarrival time ≠ Standard deviation of interarrival time.

So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 283,917 seconds
Standard deviation of activity time: 202,2589 seconds

Average activity time ≠ standard deviation of activity time.

So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

The queuing model that represents this time interval is:
G/G/23 {∞/∞/FIFO}
Can also use an M/G/23 {∞/∞/FIFO}

p = 283,917
a = 24,69
m = 23

2) Calculations of waiting related performance measures:

utilization u = p/(a×m) => u= 283,917/(24,69×23) = 0,50 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?

u= 283,9178/(24,694×12) = 0,9581 < 1

CV_a= 24,26/24,694 = 0,9824

CV_p = 202,2588/283,9178 = 0,7123

Calculation of expected waiting time using formula for expected waiting time:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}}-1}{1-utilization}\right) \times \left(\frac{(CV_a)^2\times(CV_p)^2}{2}\right)$$

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 0,2983 seconds = 0,0049716 minutes.

Average number of customers waiting in the queue:

$I_q = \frac{T_q}{a}$ = 0,012

3) For this staffing level, what are the economic performance measures?

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4{,}731963\text{min/call} \times 6{,}667\text{kr/min}}{0{,}50}$ = 63,095 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 0,2983 seconds = 0,0049716 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,0049716 min × 2,1667kr/min = 0,01 kr

Total cost per call = 63,095 + 0,01 = 63,1 kr/ call.

4) What is the impact of various staffing levels on waiting time related performance measures and economic performance measures?

Starting with the least number of CSR's that satisfies the steady state condition.

m = 12

u= 283,9178 /(24,694×12) = 0,9581

p and a are not changed.

T_q= 348,81 seconds = 5,8135 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 14,12

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4{,}731963\text{min/call} \times 6{,}667\text{kr/min}}{0{,}9581}$ = 32,927 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 348,81 seconds = 5,8135 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 5,8135 min × 2,1667kr/min = 12,5961 kr

Total cost per call = 32,927 + 12,5961 = 45,523 kr/ call.

<u>m = 13</u>

u= 283,9178 /(24,694×13) = 0,8844

p and a are not changed.

T_q= 82,09 seconds = 1,368167 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 3,32

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4{,}731963\text{min/call} \times 6{,}667\text{kr/min}}{0{,}8844}$ = 35,67 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 82,09 seconds = 1,368167 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 1,368167 min × 2,1667kr/min = 2,96 kr

Total cost per call = 35,67 + 2,96 = 38,63 kr/ call.

<u>m = 14</u>

u= 283,9178 /(24,694×14) = 0,821

p and a are not changed.

T_q= 34,49 seconds = 0,57483 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 1,396

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4{,}731963\text{min/call} \times 6{,}667\text{kr/min}}{0{,}821}$ = 38,42 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 34,49 seconds = 0,57483 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,57483 min × 2,1667kr/min = 1,245 kr

Total cost per call = 38,42 + 1,245 = 39,665 kr/ call.

<u>m = 15</u>

u= 283,9178 /(24,694×15) = 0,7667

p and a are not changed.

T_q= 17,28 seconds = 0,288 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,70

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4,731963\text{min/call} \times 6,667\text{kr/min}}{0,7667}$ = 41,15 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 17,28 seconds = 0,288 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,288 min × 2,1667kr/min = 0,624 kr

Total cost per call = 41,15 + 0,624 = 41,77 kr/ call.

<u>m = 16</u>

u= 283,9178 /(24,694×16) = 0,718

p and a are not changed.

T_q= 9,35 seconds = 0,15583 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,378

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 283,9178 seconds = 4,731963 min.

Cost of labor per call = $\frac{4{,}731963\text{min/call} \times 6{,}667\text{kr/min}}{0{,}718}$ = 43,94 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 9,35 seconds = 0,15583 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,15583 min × 2,1667kr/min = 0,337 kr

Total cost per call = 43,94 + 0,337 = 44,28 kr/ call.

<u>**Period 5 – time interval 21 – 18.00-18.30:**</u>

1) What kind of distribution does service times and interarrival times have? And which type of queuing model represents this interval?

Average interarrival time: 62,11 seconds

Standard deviation of interarrival time : 58,56 seconds

Average interarrival time ≠ Standard deviation of interarrival time.

So we have non-exponential interarrival times. There is variability in the arrival (demand) process. So interarrival times has a general distribution.

Average activity time in this time interval: 317,5 seconds
Standard deviation of activity time: 261,08 seconds

Average activity time ≠ standard deviation of activity time.

So we have non-exponential service times. There is variability in the service times. So service times have a general distribution.

The queuing model that represents this time interval is:
G/G/11 {∞/∞/FIFO}

p = 317,5
a = 62,11
m = 11

2) Calculations of waiting related performance measures:

utilization u = p/(a×m) => u= 317,5/(62,11×11) = 0,46 u<1, It satisfies the steady state condition.

So what is the minimum number of CSR's required to satisfy the steady state condition?

u= 317,5/(62,11×6) = 0,852 < 1

CV_a= 58,56/62,11 = 0,9428

CV_p = 261,08/317,5 = 0,822

Calculation of expected waiting time using formula for expected waiting time:

$$T_q = \left(\frac{Activity\ time}{m}\right) \times \left(\frac{utilization^{\sqrt{2\times(m+1)}\ -1}}{1-utilization}\right) \times \left(\frac{(CV_a)^2 \times (CV_p)^2}{2}\right)$$

All values to insert into formula are known.
p=activity time = average activity time.

For the chosen input values, T_q= 2,02 seconds = 0,03367 minutes.

Average number of customers waiting in the queue:

$I_q = \frac{T_q}{a}$ = 0,0325

3) For this staffing level, what are the economic performance measures?

Estimated average cost of a CSR per unit of time – w = 400kr/hour = 6,667 kr/min

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 317,5 seconds = 5,29167 min.

Cost of labor per call = $\frac{5{,}29167\text{min/call} \times 6{,}667\text{kr/min}}{0{,}46}$ = 76,69 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 2,02 seconds = 0,03367 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,03367 min × 2,1667kr/min = 0,0729 kr

Total cost per call = 76,69 + 0,0729 = 76,76 kr/ call.

4) What is the impact of various staffing levels on waiting time related performance measures and economic performance measures?

Starting with the least number of CSR's that satisfies the steady state condition.

<u>m = 6</u>

u= 317,5 /(62,11×6) = 0,852

p and a are not changed.

T_q= 180,27 seconds = 3,0045 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 2,90

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 317,5 seconds = 5,29167 min.

Cost of labor per call = $\frac{5{,}29167\text{min/call} \times 6{,}667\text{kr/min}}{0{,}852}$ = 41,4 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 180,27 seconds = 3,0045 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 3,0045 min × 2,1667kr/min = 6,51 kr

Total cost per call = 41,4 + 6,51 = 47,91 kr/ call.

<u>**m = 7**</u>

u= 317,5 /(62,11×7) = 0,7302

p and a are not changed.

T_q= 51,2 seconds = 0,8533 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,824

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 317,5 seconds = 5,29167 min.

Cost of labor per call = $\frac{5{,}29167\text{min/call} \times 6{,}667\text{kr/min}}{0{,}7302}$ = 48,31 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min

Expected waiting time per caller = 51,2 seconds = 0,8533 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,8533 min × 2,1667kr/min = 1,848 kr

Total cost per call = 48,31 + 1,848 = 50,16 kr/ call.

<u>m = 8</u>

u= 317,5 /(62,11×8) = 0,6389

p and a are not changed.

T_q= 20,11 seconds = 0,335167 min

In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.

I_q = 0,323

(All time units in seconds)

Formula for cost of labor per call:

Cost of labor per call = $\frac{p \times w}{u}$

p = 317,5 seconds = 5,29167 min.

Cost of labor per call = $\frac{5{,}29167\text{min/call} \times 6{,}667\text{kr/min}}{0{,}6389}$ = 55,22 kr/ call

Formula for waiting costs:

Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 20,11 seconds = 0,335167 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.
Waiting cost per call = 0,335167 min × 2,1667kr/min = 0,726 kr

Total cost per call = 55,22 + 0,726 = 55,946 kr/ call.

m = 9
u= 317,5 /(62,11×9) = 0,568
p and a are not changed.
T_q= 8,96 seconds = 0,1493 min
In this calculation, the only values changed, are the u and m. The variability measures (coefficient variations) on the arrival process and service times remains the same.
I_q = 0,144
(All time units in seconds)

Formula for cost of labor per call:
Cost of labor per call = $\frac{p \times w}{u}$
p = 317,5 seconds = 5,29167 min.
Cost of labor per call = $\frac{5{,}29167\text{min/call} \times 6{,}667\text{kr/min}}{0{,}568}$ = 62,11 kr/ call

Formula for waiting costs:
Cost of waiting per unit of time = 130kr/h = 2,1667kr/min
Expected waiting time per caller = 8,96 seconds = 0,1493 minutes

Waiting costs per call = expected waiting time per caller × cost of waiting per unit of time.

Waiting cost per call = 0,1493 min × 2,1667kr/min = 0,3235 kr

Total cost per call = 62,11 + 0,3235 = 62,4335 kr/ call.

Appendix 2: Understandable explanation of the numerical information in the dataset with call arrivals and service times data.

An excel file containing a dataset with data on call arrivals and service times for the week 2 the busyweek has been uploaded as an attachment to this thesis. An explanation on how to read and interpret the computer language that represents the various data's in the dataset will be shown here:

callid (column A) = identification number on each caller. Like for ex. 109040048. Every caller into the call centre has a caller id. In the thesis we just give each caller a number like 1 because he is the first caller who calls the call center. And 2 for the second caller etc.

segstart_d (column G) = at what time a call arrived to the queuing system. For ex. in cell G2 09.01.2017 08:00:16. This cell G2 shows when the first call arrived.

segstop_d (column I) = At what time a call exits the queuing system. For ex. in cell I2 the first call exits at 09.01.2017 08:05:40. This cell I2 shows when the first call exited.

talktime (column K) = This shows the service times in *seconds*, activity duration of each call. For ex. K2 shows that the first caller spent 96 seconds talking to a CSR.

Flow time can be calculated like this: 09.01.2017 08:00:16 minus 09.01.2017 08:05:40 which is equal to 324 seconds.
324 seconds minus 96 seconds gives waiting time 228 seconds.

These are the information columns that are relevant for this thesis. The other information columns are irrelevant.

Appendix 3: Table with input data used to create figure (nr)

Periods/days	Mon	Tue	Wed	Thu	Fri
08:00-10:30	300	287	300	283	265
10:30-13:00	450	392	404	318	314
13:00-15:00	350	324	332	297	263
15:00-18:00	500	463	361	354	330
18:00-20:00	200	181	147	164	90
Sum	1800	1647	1544	1416	1262