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The Low-Volatility Anomaly in the Norwegian Stock Market

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Abstract

In this thesis, we find that the low-volatility anomaly is present on Oslo Stock Exchange in the period 1990 to 2016. The study is performed using a filtered sample of 628 securities, sorted by idiosyncratic volatility on daily returns into quintile portfolios with a holding period of one month. The portfolios are value- and equally weighted, both leading to the same conclusion. The performance evaluation is based on returns, the sign and significance of the alphas, and the Sharpe ratios. We find that the low-volatility portfolio outperforms the high-volatility portfolio, and that performance decreases monotonically with increased risk. Thus we conclude that there exists a low-volatility anomaly on the Norwegian stock market.

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1. Introduction

In this master thesis, we examine the relationship between volatility and returns for portfolios constructed of Oslo Stock Exchange securities in the period from 1990 to 2016. The securities included are sorted into quintile portfolios based on idiosyncratic volatility, in order to reveal whether low-volatility portfolios outperform high-volatility portfolios. This is motivated by the internationally documented low-volatility anomaly, which questions the traditional financial assumption of a positive risk-return relationship. The main goal of this thesis is to examine whether there is empirical evidence of a low-volatility anomaly on the Norwegian stock market.

The evidence of an absent or negative volatility premium has become known as the low-volatility anomaly. Baker, Bradley and Wurgler (2011) even describe this phenomenon as the greatest anomaly in finance, because it challenges the fundamental principle of a risk-return trade-off. According to classic asset pricing models idiosyncratic volatility can be mitigated using diversification and is therefore not rewarded with higher returns. We find a further investigation of the anomaly important, because pricing of idiosyncratic risk challenges the very core of finance.

Most studies conducted on the topic have focused on larger markets, but there is limited research on the anomaly in smaller stock markets such as the Oslo Stock Exchange. We contribute to prior literature by supplementing with an isolated study on the Norwegian stock market, following the methodology of Ang et al. (2006) on idiosyncratic volatility of value weighted portfolios. Additionally we enhance our study by considering equally weighted portfolios, which we believe to increase the power of our analysis. Finally, we seek to find explanatory factors of the anomaly on the Norwegian stock market.

To address our research question, we compare the historical returns of portfolios constructed based on idiosyncratic volatility. The construction is performed using daily returns, and sorted into monthly quintile portfolios. Idiosyncratic volatility is computed from the residuals of the Fama and French (1993) three-factor model. We investigate the performance of the portfolios by looking at returns, alphas and

Sharpe ratios. The difference in performance between the low-volatility portfolio and the high-volatility portfolio allows us to conclude whether a low-volatility anomaly exists. Furthermore, we test if the anomaly is present in different market stages, as well as for other relevant adjustments of the methodology.

The empirical results show that the average monthly excess returns are highest for the low-volatility quintile and decrease monotonically with increased volatility. This suggests that there is evidence of a low-volatility anomaly in the Norwegian stock market. Additionally, the anomaly is confirmed by the alphas of the Fama and French three-factor model, which shows a continuous positive difference between the low-volatility quintile and the high-volatility quintile. However, alpha values are negative for all cases, indicating that the portfolios performed poorly when accounting for the risk involved. The anomaly is further confirmed by the Sharpe ratios which decline monotonically with increased volatility. These findings are robust to variations in choice of model to estimate IVOL, various data filters and tests of different subsamples.

The rest of the paper is organised as follows: Section 2 provides a literature review and background information about the low-volatility anomaly. Section 3 covers relevant background theory and the hypothesis we are testing. Section 4 provides an explanation of the methodological approach we use. Section 5 provides a description of the data, factors and adjustments of data used in the analysis. Section 6 gives the empirical results and section 7 provides a conclusion and suggestions for future research.

2. Background and Literature Review

This section presents the low-volatility anomaly in economic context, empirical research on the anomaly as well as explanations to why it exists.

2.1 The Low-Volatility Anomaly in an Economic Context

In the 1960s there was increasing support for the notion that stock markets were efficient and the capital asset pricing model (CAPM) predicted that more risky stocks would on average earn higher returns (Sharpe, 1964; Lintner, 1965; Mossin, 1966). However, when this relationship was tested, the risk-return relationship was rather flat and sometimes inverted (Jensen, Black and Scholes, 1972; Haugen & Heins, 1972). Fama and French (1993) later found that the market beta when controlling for size does not have significant explanation power for average returns.

In the 2000s academic articles explicitly looking at the low-risk effect appeared (Baker & Haugen, 2012; Ang et al., 2006; Blitz & van Vliet, 2007; Baker et al., 2011). These studies demonstrated that low-risk stocks have high risk-adjusted returns and high-risk stocks have low risk-adjusted returns, which contradicts the concept of a risk premium. Following these findings, low-volatility strategies and indices started to emerge, which led to a rise in low-volatility investing. After the global financial crisis, the focus on volatility blossomed because this was the only factor that offered significant "outperformance" (Blitz & van Vliet, 2015 p.14).

The evidence of an anomaly has increased due to numerous studies by both academics and practitioners which confirm a presence of the anomaly throughout different markets. Following this, the low-volatility market has grown massively over the past 10 years, and volatility has become an accepted new factor (Blitz & van Vliet, 2015 p.15). The highest allocations are found among US private investors and European and Asian institutional investors. Blitz and van Vliet (2015) assume that the total assets amount to USD 200 billion spread over exchange traded funds, passive portfolios and active strategies, amounting to a minor fraction of total equity market value. The small fraction in low-volatility investing may be caused by benchmark constraints and outperformance targets. At the same time academia is constrained by assumptions from classic asset pricing models.

2.2 Empirical Research on the Low-Volatility Anomaly[1]

Between the years of 1991 and 2012, Baker and Haugen published several papers examining the low-volatility anomaly by using total volatility as the risk measure. In 1996, they tested if factor models could predict individual stock's future returns and found that US portfolios with low ex-ante risk achieve equal or higher returns than the market, but with significantly lower risk. The findings showed that the

[1] Empirical research is summarized in Appendix 1

decile portfolio with the highest expected return achieved 35% higher profits than the decile portfolio with the lowest expected return. Furthermore, they observed that low-risk deciles seem to consist of companies with better liquidity and profitability. In 2009 they expanded their analysis from 1996 by using the same method with a larger dataset. These findings supported the original conclusion that the decile with the highest expected and realized return has low risk throughout the sample period. In their latest publication, Baker and Haugen (2012) investigated the relationship between historical volatility and expected return on a global basis including Norway. This study claims that low-risk stocks outperform within all observable markets of the world. One important thing to note is that the size of the anomaly is smaller for the Norwegian stock market compared to most of the other 20 countries included in the analysis.

Clarke, de Silva, and Thorley (2006) performed another study of the low-volatility anomaly based on total volatility by examining the 1000 largest stocks in the U.S. from 1968 to 2005. They further confirmed the findings of Baker and Haugen (1991), that securities with higher idiosyncratic volatility have lower realized returns. Specifically, Clarke et al. (2006) found that volatility declined by 25% and beta declined by 33% compared to a capitalization weighted benchmark. Thus, they found that minimum variance portfolios were capable of delivering similar or higher returns than the market portfolio at a lower risk.

Ang et al. (2006) examined the specific risk component by investigating the relationship between lagged IVOL and average returns. This was the first prominent study to find a negative relation between idiosyncratic risk and return. They found that stocks with high IVOL relative to the Fama and French three-factor model (FF-3 model) have significantly low average returns. These results were proved in U.S. and international markets and the findings were robust to exposures to size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness and dispersion in analysts' forecast. In 2009, Ang et al. expanded their previous study by analysing stock markets in 23 different countries including Norway, to prove that the low-risk anomaly is persistent in a global market. Their conclusion supported that of 2006, even for a skewed exposure to the Fama and French factors.

Based on total volatility and market beta, Blitz and van Vliet (2007) document the presence of a low-risk anomaly in the aggregated global stock market and regional markets, by controlling for value, size and momentum effects to create a more optimal portfolio construction strategy. They used a long-term volatility sample and found a clear volatility effect showing that low risk stocks yield significantly higher returns compared to the market portfolio. In addition, they found that, on a risk adjusted basis, high-risk stocks significantly underperformed. These effects were proved in global, U.S., European and Japanese markets. In 2013, Blitz, Pang and van Vliet found persisting evidence of the low-volatility anomaly in emerging markets. They observed a larger negative alpha for high-volatility portfolios compared to the size of a positive alpha of a portfolio of low-volatility stocks. The Sharpe ratios of this study were significantly higher for low-volatility stocks compared to high-volatility stocks.

2.3 Explanations of the Low-Volatility Anomaly

There has been a vast amount of research trying to explain the low-volatility anomaly, but to date there has been no comprehensive examination of what best explains the puzzle (Hou & Loh, 2016). The following sections provide behavioural explanations based on the works of Baker et al. (2011) and explanations including restrictions on use of benchmark indexes, agency issues and representativeness bias as proposed by Blitz, Falkenstein and van Vliet (2014).

2.3.1 The Irrational Preference for High Volatility

Baker et al. (2011) propose that investors' irrational preference for high-volatility stocks may explain the low-volatility anomaly. Irrational preference comprises biases affecting individual investors, such as the preference for lotteries, the representativeness bias, and the overconfidence bias.

The preference for lotteries regards individuals' preferences of bets involving low probabilities of large gains compared to bets with high probabilities of small losses even if the expected outcomes of the bets are equal (Kahneman & Tversky, 1979). Irrational investors will typically overpay for high-risk stocks and underpay for low risk stocks because of the emphasis on low probabilities in the decision-making processes. This makes investors risk averse in selections involving "safe" gains, but risk-taking in decisions involving "safe" losses (Barberis & Huang, 2008). As investors prefer this form of lottery, the demand of high-risk stocks will increase and thus be overpriced relative to low-risk stocks.

The representativeness bias arises when investors make erroneous assumptions that a small sample is representative of the total sample. Baker et al. (2011) relates this to the volatility anomaly by saying that laymen are likely to consider the value growth of certain IPO stocks, without considering the high failure rates of other IPOs. Following this, inexperienced investors are likely to overvalue high-risk stocks; while more experienced investors will do a more thorough analysis and consider the high-risk stocks less attractive. By ignoring the high probability of impairment costs related to such speculative investments, irrational investors tend to hold too many risky stocks that are overpriced.

Baker et al. (2011) claim that investors with excessive faith in the accuracy of their own estimates will use their own valuation of a stock if they disagree with the market valuation. This is called the overconfidence bias, and especially affects stocks with high-volatility, because investors' personal opinions of a stock's future returns will have greater variation. According to Cornell (2009), overconfident investors will typically invest in high-volatility stocks because these give the highest reward for security selection talent. This bias creates excess demand for high-volatility securities, which may lead to an anomaly in the market.

2.3.2 Benchmarking as a Limit to Arbitrage

Baker et al. (2011) propose benchmarking as a limit on arbitrage to be a possible explanation of the low-volatility anomaly. This is linked to the fact that investors are limited by tracking errors and leverage constraints, which forces them to choose stocks with high-volatility to avoid deviating from the benchmark. Thus, investors will seek volatile stocks to maximize expected excess return, given an aggregated level of risk within the investment mandate. An obvious strategy based on the low-volatility anomaly is shorting or longing stocks with desired qualities. The top volatility quintile tends to be small stocks, which are costly to trade in large quantities, meaning that stocks with high IVOL tend to be overpriced over a longer period than stocks with low IVOL.

2.3.3 Agent's Maximization of Option Value

Blitz et al. (2014) point out that analysts and fund managers are willing to pay for volatility as their incentive structure is designed as a purchase option. When that is the case, the utilizing players will maximize the expected value of the purchase option, by focusing on highly-rated shares with high growth potential and high volatility, which means constructing more volatile portfolios. Baker and Haugen (2012) support this by pointing out that fund managers will typically receive a fixed salary component and a bonus payment if their performance is successful. Such an incentive structure can be considered a purchase option on the portfolio return, meaning that the value of the option will increase with a more volatile portfolio. In other words, fund managers will increase their expected compensation by constructing a more volatile portfolio. This gives the fund managers incentives to focus on high-risk stocks (Blitz et al., 2013).

3. Theory

In this section we present and explain the main theories to examine our research question, as well as the hypothesis to be tested.

3.1 The Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) is a fundamental financial model that describes the relationship between systematic risk and expected returns for assets (Sharpe, 1964; Lintner, 1965). The model is developed from the Markowitz (1959) portfolio theory, and offers predictions on how to measure risk, and the trade-off between risk and return based on certain assumptions[2]. According to the CAPM, investors are compensated for systematic risk by receiving higher expected returns, and the model trusts the assumption that market betas efficiently describe the cross-sectional differences in distribution of expected returns. In later years, this

[2] Assumptions underlying the CAPM: (i) Investors can invest their capital in a risk-free asset. (ii) Investors only care about mean and variance, and wish to maximize their utility of end of period wealth. (iii) Investors have homogenous expectations about asset returns (iv) quantities of assets are fixed. (v) all assets are marketable and perfectly divisible (vi) all investors have access to the same information. (vii) there are perfect capital markets and no transaction costs.

assumption has been modified by including additional factors which are explained in sections 3.2-3.4.

3.2 Size and Book-to-Market Value

The most influential supplement to the CAPM are the size and value factors used in the Fama and French (1993) three-factor model; small minus big (SMB) and high minus low (HML). According to this model companies with the highest book values relative to market values have systematically higher risk-adjusted returns than those with the lowest book value relative to market value. The model also includes a size factor based on the results of Banz (1982), who found that firms with low market value on average have higher risk-adjusted returns.

3.3 Momentum

The Carhart (1997) four-factor model is an extension of the Fama and French three factor model which includes a momentum factor. The factor is a product of Jegadeesh and Titman's (1993) study on the U.S stock market, where they discovered that a momentum strategy of buying stocks that have performed well, and selling stocks that had underperformed in the same period led to excess returns. The momentum factor captures this effect. In the Carhart four-factor model, a momentum factor PR1YR is constructed as a monthly calculation of stock returns over the previous eleven months. The returns are ranked and split into groups containing the top 30%, the median 40% and the bottom 30%. PR1YR is calculated as the difference between the average return of the top and the bottom portfolios. Fama and French proposed a modified version, UMD, which is similar to the PR1YR factors. The difference is a slight modification to remove potentially dominant size effects.

3.4 Liquidity

A fourth characteristic often related to CAPM anomalies is liquidity (Næs, Skjeltorp & Ødegaard, 2009). Liquidity (LIQ) is defined as the standardized turnover-adjusted number of zero daily trading volumes over the prior *x* months. The market is considered to be liquid if traders can buy and sell large amounts of shares quickly with low transaction costs and low price impact.

3.5 Hypothesis

Based on the empirical research and theory presented in the previous sections we investigate the performance of volatility-ranked portfolios in the Norwegian stock market and distinguish whether there is evidence of a low-volatility anomaly. This is done using the following hypothesis:

H0: The Norwegian stock market is efficient, and there is no volatility-anomaly present in the market.

H1: Low volatility portfolios outperform high-volatility portfolios in the Norwegian stock market, and there exists a low-volatility anomaly.

4. Methodology

In this section we describe the methodology used throughout the thesis by explaining the model specifications, estimation of risk, portfolio structuring, performance evaluation and robustness tests.

4.1 Model Specification and Regression Framework

4.1.1 The Capital Asset Pricing Model

The simplest factor model we employ is the CAPM:

$$r_i - r_f = \alpha_i + \beta_i(r_m - r_f) + \varepsilon_i \tag{1}$$

In this model, r_i is the return on security *i*, r_f is the risk-free rate for asset *i*, α_i is the performance measure, $(r_m - r_f)$ is the market risk-premium and β_i is the systematic risk of asset *i*.

4.1.2 The Fama and French Three-Factor Model

The main analysis is performed using the Fama and French (1993) three-factor model:

$$r_i - r_f = \alpha_i + \beta_{i,MKT}(r_m - r_f) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + \varepsilon_i \tag{2}$$

This model specifies the excess return over the risk-free rate ($r_i - r_f$) as a linear relationship of the Fama and French adjusted alpha (α_i), and the three following factors, multiplied with their estimated factor exposures ($\beta_{i,MKT}$, $\beta_{i,SMB}$, and $\beta_{i,HML}$):

MKT: (r_m-r_f) represents the value-weighted market excess return of the specific market portfolio over the risk-free rate. *SMB:* "Small minus big" represents the size premium, and accounts for the spread in between small and large sized firms, which is based on the company's market capitalization. *HML*: "High minus low" represents the value premium, and accounts for the spread in returns between value and growth stocks.

4.1.3 The Carhart Four-Factor Model

To control for exposure to additional factors we employ the Carhart four-factor model:

$$r_i - r_f = \alpha_i + \beta_{i,MKT}(r_m - r_f) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + \beta_{i,UMD}UMD + \varepsilon_i \tag{3}$$

Where $r_i - r_f, \alpha_i, \beta_{i,MKT}, (r_m - r_f), \beta_{i,SMB}SMB, \beta_{i,HML}HML$ and factors *SMB and HML* equal the factors used in regression model 2. UMD "Up minus down", is the momentum factor and represents the premium on winners minus losers. $\beta_{i,UMD}$ is the systematic risk factor of the momentum factor.

4.1.4 The Liquidity Factor

In the last regression we include the liquidity factor (LIQ) of Næs et al. (2009):

$$r_i - r_f = \alpha_i + \beta_{i,MKT}(r_m - r_f) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + \beta_{i,UMD}UMD + \beta_{i,LIQ}LIQ + \varepsilon_i \tag{4}$$

Where $r_i - r_f, \alpha_i, \beta_{i,MKT}, (r_m - r_f), \beta_{i,SMB}SMB, \beta_{i,HML}HML, \beta_{i,UMD}UMD$ equal the factors explained in regression model 3. LIQ is the liquidity factor which represents a zero investment which is long in the least liquid companies and short in the most liquid companies. $\beta_{i,LIQ}$ is the systematic risk factor of liquidity.

4.2 Estimation of Risk

In the estimation of risk, we use idiosyncratic volatility which is the standard deviation of the regression residuals in the asset pricing models from section 4.1:

$$IVOL = \sqrt{Var(\varepsilon_i)} \tag{5}$$

Previous research (Appendix 2) has measured either idiosyncratic or systematic volatility in order to evaluate the risk of an asset. Stocks are then ranked based on the measure of risk, and sorted by ranges of deciles, quintiles or quartiles. The level of risk determines the allocation of assets into portfolios, where the risk and return of the portfolios can be measured to determine whether there is an association between the two. Our research is based on the methodology of Ang et al. (2006), where idiosyncratic volatility represents the risk, and the assets are sorted into quintiles. Consistent with Ang et al. (2006), we examine IVOL with respect to the Fama and French three-factor model rather than the CAPM, due to the wider application of the three-factor model in empirical finance.

4.3 Portfolio Structure

To examine idiosyncratic volatility based on the Fama and French three-factor model we use historical data to form portfolios, following the same portfolio formation strategy as Ang et al. (2006):

We form portfolios based on an estimation period of *L* months, a waiting period of *M* months, and a holding period of *N* months. Consistent with Ang et al. (2006), we have no waiting period in our strategy. During the formation period of *L* months, we compute IVOL from regression (2) on daily historical data. To assess the effect of IVOL in stock returns, we classify the available stocks for each month into portfolios, ranked by IVOL registered in the formation period. The stocks are divided into quintile portfolios, and we calculate each stock's total daily return for a holding period of *N* months. After the ranking and construction of portfolios we measure each quintile portfolio's value and equally-weighted total return for the holding period. The portfolios are rebalanced each month based on new values of IVOL. The ranking and evaluation is repeated until the end of the sample. We then obtain time series of monthly returns for our IVOL portfolios, and measure average returns and the volatility of stock returns in a monthly rolling window.

When deciding the time periods for the strategy we follow Ang et al.'s (2006) strategy of *L/M/N* = 1/0/1. This implies that both the estimation and holding period are one month, hence the first estimation period is April 1990, and the first holding period is May 1990. The 1/0/1 strategy gives us a total of 313 monthly portfolio return observations in the period of 1990-2016.

While Ang et al. (2006) only use value weighted portfolios, we have chosen to also perform the analysis on equally weighted portfolios. This is to assess the higher exposure to the market, size and risk factors in the equally weighted portfolios (Plyakha, Uppal & Vilkov, 2012). The same companies are included in both portfolio weightings, and the value weighted portfolios consist of companies weighted from market capitalization.

4.4 Performance Evaluation

We calculate the mean excess returns of each quintile portfolio, and with the resulting time series we find the portfolio performances by calculating alphas, standard deviations and Sharpe ratios. Ordinary least squares regressions are run relative to the CAPM and Fama and French three-factor model, as well as the Carhart four-factor model and a five-factor model.

4.4.1 Alpha Estimations

We use the alphas as a measure of interest when examining if higher IVOL reflects higher returns. When drawing a conclusion if an anomaly exists we focus on the sign and significance of spread between the portfolios. If α is significantly different from zero, the returns from quintile portfolios are not adequately explained by the size and value factor exposure. If α is not significantly different from zero, the size and value factor exposure explain all the excess returns. However, our main focus is on the alpha of the low-volatility portfolio compared to the high-volatility portfolio, and not the alpha values isolated. We examine whether the phenomenon persists if we control for other anomalies such as value, size, momentum and liquidity effects. We therefore follow the methodology of all mentioned asset pricing models in section 4.1. We use coefficients' standard errors based on Newey and West (1987) heteroskedasticity and autocorrelation consistent variance-covariance matrix to obtain p-values for our coefficients.

4.4.2 Standard Deviation

The standard deviation is defined as the square root of the variance. We compute standard deviations of excess holding period returns for the different portfolios.

4.4.3 Sharpe Ratio

Sharpe ratio was developed by Sharpe in 1966, and is one of the most common measurements of risk-adjusted performance. We measure the Sharpe ratio to test whether the portfolios with low-volatility stocks have a higher return than the portfolios of high-volatility stocks. The ratio is calculated as excess return of the portfolio *(R_p-R_f)* divided by the standard deviation of the excess return:

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p} \tag{6}$$

4.5 Robustness Tests

4.5.1 Computing IVOL With Other Models

In this thesis, the analysis is primarily based on the Fama and French three-factor model to estimate IVOL in accordance with Ang et al. (2006). This volatility calculation is critical in order to sort the firms to correct portfolios. To examine if the results are sensitive to including more factors in the regression we estimate IVOL using the regression residuals from CAPM, the Carhart four-factor model and a five-factor model as described in section 4.1.

4.5.2 Decile Portfolios

In accordance with Ang et al. (2006), we use quintile portfolios in our examination, while most other studies on the anomaly use decile portfolios (Appendix 2). The Norwegian stock market is relatively small compared to other markets that have been examined in similar studies, hence we find it most suiting to use quintile portfolios. However, to approach the methodology used in the majority of literature, we check if our findings are sensitive to being divided into decile portfolios. The results of this test is only considered as supplementary information, as we initially regard a sample sorted in deciles as being too limited due to the lack of possibilities for diversification.

4.5.3 Winsorization, Liquidity Constraints and Penny Stocks

As advised by Ødegaard (2012), we winsorize the sample used in the base case to normalize the most extreme values. As a robustness test we change the sample winsorization from the basic limit of .1% and 99.1% level to 5% and 95% level, to include more extreme return observations. Furthermore, we run a robustness test with penny stocks included in the analysis. Penny stocks fall into the category of "lottery tickets", as mentioned in chapter 2.3.1, and we are therefore interested in whether the behavioural factor on lotterylike investments affects our results by including this type of stocks.

4.5.4 Different Subsamples

Ang et al. (2006) describe a possible explanation for the volatility effect to be asymmetry of return distributions across business cycles. Our sample consists of several different periods regarding market conditions. We therefore test three different subperiods (1990-1998, 1999-2007 and 2008-2016) to evaluate whether the market is sufficiently irrational over time, so that investors can capitalize on the anomaly. Furthermore, we test the global financial crisis as an additional subsample. Blitz and van Vliet (2015, p.14) express volatility to be the only factor offering significant "outperformance" during the crisis, which was one of the main reasons for increased attention on low-volatility investing.

4.5.5 Transaction Costs

Sullivan and Garcia-Feijóo (2014) discover that practical trading purposes make it difficult to take advantage of an anomaly. Constantly rebalancing a portfolio as the volatility of firms changes will result in large transaction costs. We want to consider if the costs of actively trading stocks are noteworthy for the performance results of the different portfolios. Thus, we perform a robustness test by investigating the impact of transaction costs on the low-volatility portfolio versus the high-volatility portfolio.

5. Data

5. 1 Return Data

To analyse if the low-volatility anomaly exists in the Norwegian stock market we use daily return data from Norwegian stock market securities from 1990 to 2016. We start with data from 1990, because this is the first data to be obtained in daily returns. We obtain OSEAX daily prices adjusted for dividends from Bloomberg. The full range of companies listed on the Oslo Stock Exchange for this period includes 953 unique observations. In cases where a company lacks price value in one or more individual months, we have chosen to retain the raw data rather than interpolating courses. This is mainly because interpolation by moving averages or other smoothing methods could lead to artificially low volatility. For a company to be included in the analysis it must meet certain criteria related to size, stock price and liquidity (see detailed explanation in section 5.1.1). After data filtering our sample consists of between 48 and 232 securities for a given year. On average, there are 163 securities that yearly satisfy the criteria for inclusion. The full filtered sample consists of 628 unique securities. When value weighting the securities to their respective portfolios we use market cap values from Bloomberg.

5.1.1 Data Filtering

According to Ødegaard (2012), not all stocks traded at the Oslo Stock Exchange should be used in calculating representative returns for empirical asset pricing investigations. In performing this study we therefore exclude several listed companies based on Ødegaard’s (2012) sample inclusion criteria. Stocks that are rarely traded may have a volatility inaccurately reflecting the stock’s fundamental risk. Thus, we introduce a liquidity criteria to our sample, where stocks must have a minimum number of 10 trading days to be included in the sample. Additionally, we include a size criteria to our sample, where stocks with a total market value outstanding of less than NOK 1 million are excluded. This is in line with Blitz and van Vliet (2007), who state that certain return irregularities tend to disappear or become less pronounced when limiting the amount of small-cap stock in a sample. This filtering is also in line with Ang et al. (2006) who eliminate 5% of firms with the lowest market capitalization. Low value stocks (“penny stocks”) are also problematic because they have exaggerated results. We therefore eliminate stocks that have a value less than NOK 10. These filters are applied to each portfolio formation period, meaning that a company may be included in some periods and excluded in others where the restrictions are met.

5.1.2 Outlier Adjustments and Winsorization

After applying the data filters proposed by Ødegaard (2012), our sample still contains outliers which can possibly affect our results. Studying our filtered daily returns we find several extreme values. For example, we observe 94 returns above 100% in a single day. We therefore winsorize our sample to remove extreme values. By winsorizing, the observations are not entirely removed from the sample, but they are set to a certain percentile of the values in the time series. Thus, the extreme values affect the results in the correct direction without obscuring the analysis. We winsorize on a yearly basis at the of 0.1% and 99.9% level, meaning that all observations above 99.9th percentile and below 0.1th percentile are set to these levels. The dataset now contains daily returns in the interval 75% to -50% (Appendix 3).

5.2 Risk Free Rate

As a proxy for risk-free returns, we use the interest rates obtained from Ødegaard's database on Norwegian asset pricing data (Ødegaard, 2016). These are based on the Norwegian interbank rate, NIBOR, with a maturity of one month.

5.3 Pricing Factors

We obtain daily values for the factors from Ødegaard's (2017) database. The Fama and French (1993) factors *SMB* and *HML*, as well as the *UMD* factor are calculated using Norwegian data. *UMD* is similar to the *PR1YR* factor, but is slightly modified to remove potentially dominant size-effects. To test the significance of different liquidity levels among stocks, we use the *LIQ* factor from Næs et al. (2009), which is calculated based on relative spread for Norwegian companies. For the market factor we use a value weighted index from Ødegaard's (2017) database. The index is constructed with a value weighted average of all the stocks at Oslo Stock Exchange.

5.4 Summary Statistics

Table 1 shows descriptive statistics for the five explanatory factors included in this thesis, as well as the risk-free rate and the two portfolios constructed of all securities meeting the filtering criteria described in section 5.1. The factors are calculated for the Norwegian stock market, and all variables consist of values from April 1990 through December 2016, leaving 313 observations. The market premium displays the highest average monthly return of 1.321%. The *SMB* and *UMD* portfolios are also quite large, with a monthly average of 0.787% and 0.602% respectively. Thereby follow the value portfolio (*HML*) and the liquidity portfolio (*LIQ*) with monthly average returns of 0.120% and 0.020%. The security portfolios exhibit the lowest returns, with a monthly average of -0.352% for the value weighted portfolios and -0.260% for the equally weighted portfolios.

Table 2 shows the correlation matrix of the variables. The highest correlation for both portfolio weightings are for the excess return and the market risk premium (0.784 for the value weighted (VW) portfolio and 0.802 for the equally weighted (EW) portfolio). Other high correlations in absolute value are those of the market risk premium and the *LIQ* factor (-0.647) and the market risk premium and the *SMB* factor (-0.429). This is due to the fact that the liquidity and size factors both consist of short positions in respectively liquid and large companies. Another high correlation value is the one for *SMB* and *LIQ* (0.581), which is natural as liquidity tends to increase with increased firm size.

Table 1: Descriptive Statistics

	Risk Free Rate	Excess Return EW Portfolio	Excess Return VW Portfolio	Market Risk Premium	SMB	HML	UMD	LIQ
Mean	0.396 %	-0.260 %	-0.352 %	1.321 %	0.787 %	0.120 %	0.602 %	0.020 %
Std.Dev.	0.271 %	7.276 %	8.508 %	5.785 %	4.203 %	4.897 %	0.318 %	4.537 %
Minimum	0.068 %	-33.654 %	-68.275 %	-22.182 %	-17.081 %	-16.649 %	-17.061 %	-17.658 %
Maximum	2.074 %	28.917 %	34.495 %	16.228 %	22.140 %	14.661 %	25.484 %	16.420 %
Kurtosis	6.434	5.709	8.760	1.681	3.259	1.128	1.536	0.796
Skewness	1.740	-0.794	-1.211	-0.704	0.229	-0.240	-0.065	0.178

This table shows selected descriptive statistics for the Norwegian 1-month risk free rate, a value weighted and an equally weighted portfolio of the returns in our dataset, as well as the different factors considered and used throughout our analysis. All non-standard measurements are reported as percentages monthly. A thorough description of the risk-free rate, the market portfolio and the factors is provided under “Model Specification and Regression Framework” in section 4.1.

Table 2: Correlation Matrix of Explanatory Variables

	Risk Free Rate	Excess Return Portfolio	Market Risk-Premium	SMB	HML	UMD	LIQ
Panel A: Cross-correlations VW Portfolio							
Risk-Free Rate	1.000						
Excess Return Portfolio	-0.163	1.000					
Market Risk Premium	-0.147	0.784	1.000				
SMB	0.020	-0.282	-0.429	1.000			
HML	0.062	0.028	0.002	-0.120	1.000		
UMD	-0.182	-0.137	-0.098	0.059	-0.121	1.000	
LIQ	0.144	-0.483	-0.647	0.581	0.115	-0.059	1.000
Panel B: Cross-Correlations EW Portfolio							
Risk-Free Rate	1.000						
Excess Return Portfolio	-0.149	1.000					
Market Risk-Premium	-0.147	0.802	1.000				
SMB	0.020	-0.156	-0.429	1.000			
HML	0.062	0.010	0.002	-0.120	1.000		
UMD	-0.182	-0.133	-0.098	0.059	-0.121	1.000	
LIQ	0.144	-0.461	-0.647	0.581	0.115	-0.059	1.000

This table shows cross-correlations for monthly values of the risk-free rate, portfolio excess return, market risk premium as well as the pricing factors described in section 4.1. Panel A shows cross correlations for value weighted (VW) portfolios and Panel B shows cross correlations for equally weighted (EW) portfolios.

6. Results and Analysis

To get an overview of the portfolio performances, we report the regression results of four different model specifications, with the excess return of both value weighted (VW) and equally weighted (EW) portfolios as the dependent variable. The results are based on the main sample period from April 1990 to December 2016, which consists of 313 portfolio month observations. For all results, Quintile 1 (Q1) is the portfolio containing the stocks with lowest risk (lowest IVOL), while Quintile 5 (Q5) contains the stocks with highest risk (highest IVOL). We evaluate whether there exists an anomaly based on the following conditions:

1. The low-volatility portfolio has a higher Alpha than the high-volatility portfolio
2. The low-volatility portfolio has a higher excess return than the high-volatility portfolio
3. The low-volatility portfolio has a higher Sharpe ratio than high-volatility portfolio

6.1 Value Weighted Portfolio Regression Results

6.1.1 Performance Evaluation for Value Weighted Portfolios

First, we present the performance evaluations of the portfolios, providing results for evaluation of whether the anomaly conditions are fulfilled.

In Table 2, we observe that the average monthly excess returns are highest for Q1 (0.43%) and decrease with higher volatility, with the lowest excess return for Q5 (-1.31%). The decrease in returns is consistent, indicating a strictly negative volatility premium. The consistently higher returns for portfolios with low risk, than portfolios with high risk conforms criteria two in the low-volatility anomaly valuation. This result represents the first suggestion that a low-volatility anomaly exists in the Norwegian stock market.

Further we observe that historical volatility for the quintiles provides a good indication for future volatility. This is proposed by the ex post standard deviations, which follow a fairly uniform increase from Q1-Q5. Thus, the realized standard deviation of the portfolios reflects the basis of the portfolio construction, which is ranked by volatility. This is true also for the ex post IVOL observations.

Table 3: Value Weighted Quintile Portfolios Sorted by Volatility

Quintile	Mean return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.428 %	-0.008***	-0.007***	5.717 %	0.075	0.043
Q2	0.382 %	-0.009***	-0.001***	6.653 %	0.057	0.044
Q3	-0.465 %	-0.021***	-0.021***	8.110 %	-0.057	0.048
Q4	-0.791 %	-0.027***	-0.025***	9.300 %	-0.085	0.059
Q5	-1.312 %	-0.038***	-0.032***	11.461 %	-0.115	0.077
Q1-Q5	**1.740 %**	**0.030*****	**0.025*****	**-5.744 %**	**0.189**	**-0.034**

This table shows value weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the mean excess return divided by the ex-post standard deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

The Sharpe ratio, which adjusts the level of return to the risk of each portfolio, is considerably larger for Q1 compared to Q5, indicating that investing in riskier stocks is not compensated with higher returns. The Sharpe ratio decreases for quintiles with higher risk, suggesting that the stock returns fail to compensate for increase in volatility. The greater Sharpe ratio for low-volatility portfolios is consistent with previous research mentioned in chapter 2. Furthermore, the Sharpe ratio results satisfy criteria three in the low-volatility anomaly valuation.

Based on the anomaly criteria, the observations done for value weighted portfolios conclude that there exists a low-volatility anomaly on the Norwegian stock market. The low-volatility portfolios yield higher excess returns compared to the high-volatility portfolios. The anomaly is further confirmed by both a larger alpha and a larger Sharpe ratio for the low-volatility portfolios. These findings are consistent across the various model specifications in computing the alphas.

6.1.2 Model Specifications and Explanatory Variables

Second, we discuss the results on model specifications and the explanatory variables for excess returns in the low-risk and high-risk portfolios. Prior explanations to the low-volatility anomaly have focused largely on the value (Baker & Haugen, 2012) and liquidity-premiums (Haugen & Baker, 1996). We expect to observe that low-volatility portfolio excess returns are larger because the portfolios contain large firms, value stocks and liquid companies. Additionally, we expect to observe that high-volatility portfolios are exposed to illiquidity, growth stocks and small firms.

For the basic CAPM model, the monthly alpha is statistically significant, and is largest for Quintile 1 (-0.75%) and smallest for Quintile 5 (-3.23%). When controlling for additional factors in models (2)-(4), the VW portfolios still have statistically significant alphas, with a continuous positive difference between Q1 and Q5. These results satisfy criteria one in the anomaly evaluation. For the purpose of this thesis we are mostly concerned about the difference between Q1 and Q5, however we see it important to comment on the negative alpha values, which indicate that the portfolios have performed poorly on a risk adjusted basis, and all fail to beat the market.

Table 4: Regression Results of Various Model Specifications for Value Weighted Portfolios

	Model Specification	Quintile	Explanatory Variable Coefficients and t-stats						
			α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	β_{LIQ}	Adj.R^2
(1)	CAPM	Q1	-0.008***	0.891***					80.00 %
			(-5.24)	(-35.39)					
		Q5	-0.032***	1.444***					52.10 %
			(-7.02)	(-18.46)					
(2)	Fama-French 3 factor model	Q1	-0.008***	0.900***	0.027	0.114***			80.90 %
			(-5.25)	(-32.92)	(-0.73)	(-3.93)			
		Q5	-0.038***	1.587***	0.461***	0.11			54.20 %
			(-8.03)	(-18.71)	(-3.91)	(-1.22)			
(3)	Carhart 4 factor model	Q1	-0.008***	0.902***	0.027	0.117***	0.019		80.90 %
			(-5.30)	(-32.83)	(-0.72)	(-3.99)	(-0.74)		
		Q5	-0.037***	1.576***	0.461***	0.094	-0.118		54.40 %
			(-7.80)	(-18.54)	(-3.91)	(-1.03)	(-1.49)		
(4)	5 factor model	Q1	-0.008***	0.936***	-0.011	0.104***	0.027	0.091	81.00 %
			(-5.45)	(-28.46)	(-0.26)	(-3.49)	(-1.05)	(-1.89)	
		Q5	-0.037***	1.549***	0.491***	0.103	-0.124	-0.069	61.60 %
			(-7.73)	(-15.13)	(-3.67)	(-1.11)	(-1.55)	(-0.47)	

This table shows factor loadings for different models. The dependent variable is excess returns of value weighted portfolios. The portfolios are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. Quintile 1 (Quintile 5) contains the stocks with the lowest (highest) volatility. Explanatory variables are market risk premium (MKT), a size factor (SMB), a value factor (HML), a momentum factor (UMD) and a liquidity factor (LIQ). The regression results are the intercept and coefficient estimates with corresponding t-statistics and adjusted R2. P-values are based on robust Newey and West (1986) t-statistics (shown in parentheses). *,**,*** indicates significance at the 10%, 5% and 1% level (respectively).

For all models the coefficient on market risk premium is significantly lower than 1 for Q1, and significantly larger than 1 for Q5. This means that the high-risk portfolios are more sensitive to market increases/decreases, and hereby not only have higher IVOL, but also higher systematic risk than the low-risk portfolios. For example, the Fama and French three-factor model shows a β_M for Q1 equal to 0.90, meaning that portfolio excess return will increase with 0.90% when the market increases with 1%. For Q5 however, a market increase of 1% will increase excess returns with 1.58%. The same reasoning goes for market declines. The fact that Q5 has a beta of 1.58 indicates that the portfolio in theory is 58% more volatile than the market. The market risk premium has distinctly the highest coefficient among all the factors, meaning that changes in the stock market represents the largest explanation of portfolio return variations.

The adjusted R-squared is moderately high for Q1, with a value above 80% in all models. It is lower for Q5, with a value above 50% in all models. Including more factors into the basic model only changes the explanatory power slightly. For Q1 the only increase in explanatory power is present when going from the basic CAPM to adding *SMB* and *HML* to the Fama and French three-factor model. For Q5 the Carhart four-factor model reflects the highest explanatory power. Neither of the coefficients added (β_{UMD} and β_{LIQ}) when going to the four- and five-factor models are statistically significant for any of the portfolios. However, the explanatory power of Q5 excess returns slightly increases when adding the UMD factor.

For Q5, the size factor coefficient β_{SMB} is positive and statistically significant in all models where it is included, but it is not significant in any of the models for Q1. The positive and significant β_{SMB} for Q5 implies that the portfolio has higher excess returns if small-cap stocks outperform large-cap stocks, suggesting that the portfolio is predominantly small-cap stocks. This is consistent with earlier research, suggesting that high-volatility portfolios are often dominated by small stocks, displaying a small cap effect as mentioned by Baker et al. (2011). The coefficient on the value portfolio β_{HML} is positive and statistically significant for Q1, but not for Q5. The positive and significant β_{HML} for Q1 implies that the portfolio has higher excess returns if high value (i.e. high book-to-market) stocks outperform growth (i.e. low book-to-market) stocks, and suggests that the portfolio is predominant value stocks. This is not surprising, as we wouldn't expect growth stocks, e.g. technology stocks, to have low volatilities. Furthermore, the dominance of value stocks in low-volatility portfolios is consistent with the literature described in chapter 2. The liquidity factor β_{LIQ} has a positive coefficient for low-volatility portfolios, and negative for high-volatility portfolios. This contradicts theory that low-volatility portfolios contain liquid stocks, while high-volatility portfolios contain illiquid stocks. However, none of the coefficients are statistically significant, and we cannot conclude significant exposure to this factor.

The regression results in Table 4 reveal the economical differences between high-volatility and low-volatility portfolios. We observe that the high-volatility portfolios include small firms, while low-volatility portfolios seem to consist of larger firms, as well as value stocks. This can explain the differences in risk dimensions between the two portfolios. However, the liquidity factor, which is largely associated with the low-volatility anomaly, is surprisingly a non-significant variable for any of the portfolio's excess returns. We find no evidence of low-volatility portfolios containing liquidity stocks, and high-volatility portfolios containing illiquid stocks. This means that the observed low-volatility anomaly in this case is explained by the market risk premium, the size premium and the value premium.

Based on the criteria presented in the beginning of this chapter, the observations done for value weighted portfolios lets us conclude that there exists a low-volatility anomaly on the Norwegian stock market. The low-volatility portfolios yield higher excess returns compared to the high-volatility portfolios. The anomaly is further confirmed by both a larger alpha, and a larger Sharpe ratio for the low-volatility portfolios compared to the high-volatility portfolios. The findings are consistent across the various model specifications in computing the alphas. These findings are in line with those of Ang et al. (2006), showing that stocks with high idiosyncratic volatility have low average returns. While Ang et al. (2006), shows that the low-volatility quintile outperforms the high-volatility quintile, we find that the performance also increases monotonically with every quintile, which is an even stronger indication of an anomaly. This leaves little evidence of the risk-return relationship stated by CAPM and the additional factor models, i.e. we reject a null hypothesis stating that the Norwegian stock market is efficient, as there is evidence of an anomaly present in the market. Thus, we can acknowledge that low-volatility portfolios outperform high-volatility portfolios in the Norwegian stock market.

6.2 Equally Weighted Portfolio Regression Results

6.2.1 Performance Evaluation for Equally Weighted Portfolios

In Table 5, we observe that the average monthly excess returns are highest for Q1 (0.58%) and decrease with higher volatility, with the lowest excess return for Q5 (-1.92%). The decrease in returns is consistent, indicating a strictly negative volatility premium. The consistently higher returns for portfolios with low risk, than portfolios with high risk conforms with criteria two in the low-volatility anomaly valuation. Additionally, we observe that the returns are higher for equally weighted portfolios than value weighted portfolios, as they tend to do (Plyakha et al. 2012).

Further we observe that historical volatility for the equally weighted quintiles also provide good indications for future volatility. This is proposed by the ex post standard deviations, which follow a fairly uniform increase from Q1-Q5. Thus, the realized standard deviation of the portfolios reflects the basis of the portfolio construction, which is ranking by volatility. This is true also for the ex-post IVOL observations. The equally weighted portfolios also hold lower ex-post risk than the value weighted, with lower values for both standard deviation and IVOL. The Sharpe ratio is considerably larger for Q1 compared to Q5, which indicates that investing in riskier stocks is not compensated with higher returns. The Sharpe ratio decreases when moving to quintiles with higher risk, suggesting that the stock returns fail to compensate growth in volatility. Furthermore, the Sharpe ratio results satisfy criteria three in the low-volatility anomaly valuation.

6.2.2 Model Specifications and Explanatory Variables

Regarding the results of the explanatory variables for the equally weighted portfolios, we expect low-volatility portfolio excess returns to be higher because the portfolios contain large firms, value stocks and liquid companies. Further, we expect high-volatility portfolios to be exposed to illiquidity, growth stocks and small firms and that the equally weighted portfolios have a higher exposure to the market, size and value factors.

Table 5: Performance of Equally Weighted Quintile Portfolios Sorted by Volatility

Quintile	Mean return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.580 %	-0.006***	-0.004**	4.997 %	0.116	0.042
Q2	0.553 %	-0.009***	-0.007***	6.051 %	0.091	0.037
Q3	-0.068 %	-0.011***	-0.014***	7.021 %	-0.010	0.035
Q4	-0.447 %	-0.027***	-0.020***	8.083 %	-0.055	0.041
Q5	-1.921 %	-0.044***	-0.035***	9.226 %	-0.208	0.056
Q1-Q5	**2.501 %**	**-0.038*****	**0.031*****	**-4.229 %**	**0.324**	**-0.015**

This table shows equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns mean excess return and ex-post standard deviation are measured monthly and the Sharpe ratio is the mean excess return divided by the ex-post standard deviation. ex-post IVOL is the portfolio's realized IVOL. FF3 Alpha and CAPM Alpha report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Table 6: Regression Results of Various Model Specifications for Equally Weighted Portfolios

	Model Specification	Quintile	Explanatory Variable Coefficients and t-stats						
			α	β_M	β_{SMB}	β_{HML}	β_{UMD}	β_{LIQ}	Adj.R^2
(1)	CAPM	Q1	-0.004**	0.754***					75.00 %
			(-2.90)	(-30.62)					
		Q5	-0.035***	1.171***					68.80 %
			(-9.46)	(-18.76)					
(2)	Fama-French 3 factor model	Q1	-0.006**	0.797***	0.139***	0.112***			76.90 %
			(-4.10)	(-30.36)	(-3.83)	(-4.00)			
		Q5	-0.044***	1.408***	0.761***	0.023			75.40 %
			(-12.74)	(-22.72)	(-8.85)	(-0.35)			
(3)	Carhart 4 factor model	Q1	-0.006***	0.797***	0.139***	0.112***	-0.001		76.80 %
			(-4.06)	(-30.19)	(-3.82)	(-3.96)	(-0.04)		
		Q5	-0.044***	1.403***	0.761***	0.016	-0.053		76.00 %
			(-12.52)	(-22.55)	(-8.85)	(-0.24)	(-0.92)		
(4)	5 factor model	Q1	-0.006***	0.786***	0.153***	0.116***	-0.004	-0.031	76.80 %
			(-3.99)	(-24.7)	(-3.67)	(-4.01)	(-0.15)	(-0.66)	
		Q5	-0.044***	1.417***	0.746***	0.011	-0.050	0.036	76.40 %
			(-12.50)	(-18.9)	(-7.62)	(-0.16)	(-0.85)	(-0.33)	

This table shows factor loadings for different models. The dependent variable is excess returns of equally weighted portfolios. The portfolios are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. Quintile 1 (Quintile 5) contains the stocks with the lowest (highest) volatility. Explanatory variables are market risk premium (MKT), a size factor (SMB), a value factor (HML), a momentum factor (UMD) and a liquidity factor (LIQ). The regression results are the intercept and coefficient estimates with corresponding t-statistics and adjusted R2. P-values are based on robust Newey and West (1986) t-statistics (shown in parentheses). *,**,*** indicates significance at the 10%, 5% and 1% level (respectively).

For the basic CAPM model, the monthly alpha is statistically significant, and in line with the VW portfolios, while CAPM alphas are largest for Quintile 1 (-0.43%) and smallest for Quintile 5 (-3.47%). The difference between Q1 and Q5 alphas is still positive for all models, and the alphas for both portfolios have increased compared to the VW portfolio. When controlling for additional factors in models (2)-(4), the EW Q1 portfolios have statistically significant alphas in all models. Additionally, we note that the alpha values are negative also for the equally weighted portfolios.

The market risk premium coefficient is significantly lower than 1 for Q1, and significantly larger than 1 for Q5 for all models. This factor has distinctly the highest coefficient among all the factors, meaning that changes in the stock market represents the largest explanation of portfolio return variations. The coefficients on the market risk premium have declined slightly for Q1 compared to the VW portfolio. For Q5 compared to the VW portfolio, the decline applies to all models except the basic CAPM, where there is a slight increase to the market exposure.

As in the value weighted analysis, the adjusted R-squares are moderately high for Q1, with values just below 80% in all models. Including additional factors in the basic model increases the explanatory power slightly, between model (1) and (2) for Q1, but there is no increase in adding more factors. For Q5 the adding of factors increase the explanatory power for all models. R-squared values lies around 70% consistently, where models (2) and (3) have the highest explanatory power. Overall, the models seem to have higher explanatory power in Q5 for the EW portfolios than the VW portfolios. Similar to the VW portfolios, additional factors (*UMD* and *LIQ*) have not significant effect for any of the portfolios.

The coefficient of the size factor β_{SMB} is statistically significant and positive for Q1 in all models, which is different from the VW portfolio. This implies that the size premium is more significant on low-risk portfolios when one does not base the weighting of companies in the portfolio on the market cap size. However, the positive sign means that the portfolios contain small stocks, which contradicts with earlier research. For Q5 the size coefficient is also positive and statistically significant, and quite larger than in Q1. This implies that the high-risk portfolio has a higher dominance of small stocks, and expects a higher excess return when small-cap stocks outperform large-cap stocks. Comparing the coefficient on the value portfolio β_{HML}, we see quite similar results between the EW and VW portfolios. The coefficient is positive and statistically significant for Q1, and not for Q5. This indicates that also the EW low-risk portfolios predominantly consist of value stocks.

An interesting difference between the VW and EW portfolios are the changes in sign of the *LIQ* factor. The coefficients show opposite signs, indicating high liquidity stocks in Q1 and low liquidity stocks in Q5, which is consistent with previous literature. However, none of the coefficients are statistically significant, and we cannot conclude that the portfolios are significantly exposed to this factor. Furthermore, we suspect that this is caused by the high correlation between *SMB* and *LIQ* (Section 5.4), and we attempt to exclude *SMB* when running regressions using the five factor model. Hereby we observe an increase of significance for the high-risk portfolio, where the positive coefficient on β_{LIQ} indicates the portfolio being predominant illiquid stocks (Table 7).

Table 7: Regression Results of Carhart five factor EW Portfolios, removing factor *SMB*

Model	Quintile	Explanatory Variable Coefficients				
		α	β_M	β_{HML}	β_{UMD}	β_{LIQ}
(4) 5 factor model	Q1	-0.005**	0.780***	0.093**	0.004	0.050
	Q5	-0.038***	1.388***	-0.101	-0.013	0.431***

This table shows explanatory variable coefficients and t-statistics for Q1 and Q5 of the 5 factor model. t-statistics report the statistical significance of the alphas. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively).

The regression results in Table 6 reveal the economical differences between equally weighted high-volatility and low-volatility portfolios. We observe that both volatility portfolios are dominated by small firms, but the small-cap effect is largest for the high-risk portfolios. Low-volatility portfolios are significantly affected by the value premium, indicating that they contain value stocks. This can explain the differences in risk dimensions between the two portfolios. However, the liquidity factor, which is largely associated with the low-volatility anomaly, is surprisingly a non-significant variable for any of the portfolio's excess returns. We find no significant evidence for low-volatility portfolios containing liquidity stocks, and high-volatility portfolios containing illiquid stocks. This can be caused by the high correlation between the *SMB* and *LIQ*, and when testing to remove *SMB* from the five-factor model, we find that liquidity has a significant impact on the high-risk portfolio, indicating a predominance of illiquid stocks.

The largest difference between the coefficient results comparing value weighted and equally weighted portfolios is the change in size premium. An increase in the coefficient β_{SMB} for low-volatility portfolios, which for the EW is not significant, indicates that there is a larger exposure for the size factor, which was earlier adjusted by the value weighting in portfolios. However, contrary to the study by Plyakha et al. (2012), there is no sign of a larger exposure to the market and value factor for equally weighted portfolios. This means that the observed low-volatility anomaly in this case is, equal to the value weighted portfolios, explained by the market risk premium, the size premium and the value premium.

Based on the criteria presented in the beginning of this chapter, the observations done for equally weighted portfolios also conclude that there exists a low-volatility anomaly on the Norwegian stock market. All three criteria are fulfilled for the equally weighted portfolios. As for the value weighted portfolios, we find a monotonically decrease in performance when increasing volatility in the portfolios. This leaves little evidence of the risk-return relationship stated by CAPM and the additional factor models, i.e. we reject a null hypothesis stating that the Norwegian stock market is efficient, as there is evidence of an anomaly present in the market. Thus we can acknowledge that low-volatility portfolios outperform high-volatility portfolios in the Norwegian stock market, independent of portfolio weighing.

6.3 Robustness Tests

6.3.1 Computing IVOL With Other Models

Appendices 4-6 investigate the robustness of our findings when using CAPM, the Carhart four-factor model and a five-factor model to estimate IVOL. The different IVOL-calculations give no noteworthy changes in allocation of firms to the different quintile portfolios. Thus, the results obtained when using other factor models are similar to the main results.

6.3.2 Decile Portfolios

In Appendix 7 the descriptive statistics for decile portfolios are shown. When the sample is split into deciles the findings show the same indications as in the base case, with slightly less significant alpha values for Q1. The alpha values do not decline monotonically for each decile, but the first alpha values (Q1) are higher than the last (Q10), which indicates that there is evidence of an anomaly, if not as distinct as the evidence found when using quintiles. As mentioned in the section 4.5, the decile portfolios are unlikely to contain enough stocks to be sufficiently diversified. Thus, we are careful to draw any conclusions based on this test.

6.3.3 Changing Winsorization

The results presented in Appendix 8 show that the mean return increases for all quintiles when including more extreme observations in the analysis compared to the main results. The return of the low-risk portfolio remains insubstantially changed, while the change of the most volatile portfolio is the most distinct. The Sharpe ratio of all quintiles increases with volatility when the winsorization is changed. The fact that the risk-adjusted performance targets are improved indicates that the return on earnings are greater than the resulting increase in volatility. These findings indicate that the anomaly persists when adjusting the winsorization level, but that it becomes less clear. This implies that positive return observations primarily increase the performance of the high-volatility quintile, while they have less effect on the low-volatility quintile.

6.3.4 Including Penny Stocks

As a robustness test we include penny stocks and find that the anomaly presence is robust to including low value observations (Appendix 9). Furthermore, we observe that the difference in returns between Q1 and Q5 has increased due to a large decrease in returns for Q5. Compared to the main results, the results for Q5 have decreased with 70% in the value weighted portfolio, and 35% for the equally weighted portfolio. This is quite interesting, as it implies that the anomaly can partially be explained by irrational preference for high-volatility (Baker et al., 2011). When including stocks under the category of “lottery tickets” the anomaly is strengthened due to a large decline in returns for the high-volatility portfolios.

6.3.5 Testing Different Subsamples

As a robustness test we split the sample into three subperiods of equal size: 1990-1998, 1999-2007 and 2008-2016. The subsamples leave 97, 108 and 108 monthly observations, respectively. As Appendix 10 shows, there are no large changes from the main results. This reflects that the anomaly persists over different subperiods, indicating a consistent anomaly throughout the whole period of our analysis. However, there is a slight decrease in significance levels of alphas for the first and second subperiod for both portfolios. This is not surprising, as the data sample for these periods is quite limited due to few observations. We are therefore careful to draw any conclusions based on this test.

The results in Appendix 11 show evidence of an anomaly during the global financial crisis, with considerable higher differences in alpha and mean return values compared to the main results. This suggests that the anomaly is more prominent in periods with higher volatility.

6.3.6 Transaction costs

For our main results, containing 313 rebalances, we find a monthly average of 9.58 changes of securities for Q1, and 9.63 changes for Q5. The small amount of difference in transactions between the two quintiles indicates that transaction costs have a negligible effect on the difference in performance between the two portfolios. We therefore conclude that the results of overperformance for Q1 compared to Q5 are consistent even when taking transaction costs into account. The low-volatility anomaly is persistent in the Norwegian market regardless of transaction costs. However, the transaction costs might have an impact when considering the performance of a low-volatility portfolio versus the market or other trading strategies with less transactions. Still, this is beyond the subject of this thesis and is not be considered further.

7. Conclusion

Traditional economic theory indicates that investors expect higher returns as a compensation for bearing higher risk. However, several studies have challenged this prediction, and revealed that low-risk assets have higher absolute and risk-adjusted returns than high-risk assets. This deviation from theory has been shown to exist in different equity markets around the world and has become known as the “low-volatility anomaly”.

In our analysis we find evidence revealing that the average monthly excess returns are highest for the low-volatility portfolios and decrease monotonically with increased volatility, consistent with previous studies on the low-volatility anomaly. This negative volatility premium contradicts classic financial theory and indicates that there exists an anomaly in the Norwegian stock market. In particular, the low-volatility portfolio quintile yields average excess returns of 1.74% more than the high-volatility quintile for value weighted portfolios and 2.5% more for equally weighted portfolios. Additionally, the anomaly is confirmed by the alphas of the Fama and French three-factor model, which show a continuous positive difference between the low-volatility quintile and the high-volatility quintile. The anomaly is further confirmed by the Sharpe ratios which decline monotonically with increased volatility. These findings are robust to variations in choice of model to estimate IVOL, various data filters and tests of different subsamples, and are consistent for both value and equally weighted portfolios.

We find that the low-volatility portfolios have negative alpha values for all cases, indicating that the portfolios performed poorly on a risk adjusted basis compared to the market index. Even though the low-volatility portfolio outperforms the high-volatility portfolio, it fails to outperform the market. This suggests that from an investor’s perspective, there exists a volatility effect, but the profitability of volatility based trading may be debatable.

This study contributes to the literature on volatility trading for relatively small markets such as Norway. We hope the results provide valuable information on the Norwegian stock market, and is of interest for readers concerned with market anomalies. In the Norwegian market we find that the difference in performance is driven by the market risk-, size- and value-premiums. The low-volatility portfolios have a lower market beta, and consist of larger firms and value stocks, while high-volatility portfolios have a higher market beta, and seem to consist of small firms. When excluding the *SMB* factor we also find evidence of illiquid stocks in the high-volatility portfolios. We can therefore conclude that the exposure to different factors partially explain the low-volatility anomaly we observe. When running robustness tests, we also found evidence of behavioural explanations for the anomaly. Investor's irrational preference for high-volatility is present in the Norwegian stock market.

For future research we suggest examining the performance of different industries to detect if there are any systematic differences related to industry exposure. Additionally, we suggest an assessment of the interaction between changes in credit ratings and IVOL to investigate if upgrades and downgrades in credit ratings have significant effects on returns of IVOL portfolios. Furthermore, we advise a more thorough look at how other factors such as value and momentum interact with volatility to see if an active low-volatility strategy should avoid negative value and momentum exposure, or aim for positive exposure to these factors.

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Appendices

Appendix 1: Literature Matrix

Author	Publication Date	Market	Period of Analysis	Findings
Baker & Haugen	1991	USA	1972-1989	Low-volatility portfolios outperform the market index
Baker & Haugen	1996	USA	1979-1993	Stocks with low risk often yield high returns and low-risk stocks usually have a higher degree of liquidity compared to high-risk stocks.
Baker & Haugen	2009	USA	1963-2007	Negative risk premium for all measures of risk. The low-risk decile contains big cap companies. Low-volatility stocks are liquid with low transaction costs.
Baker & Haugen	2012	Global	1990-2012	Low-volatility stocks outperform high-volatility within all tested markets.
Clarke de Silva & Thorley	2006	USA	1968-2005	The minimum variance portfolio yields equivalent or higher returns than the market portfolio. These findings are robust for adjustments of HML, SMB and momentum.
Bitz & van Vliet	2007	Global	1986-2006	Finds evidence of a low-volatility anomaly based on total and idiosyncratic volatility in global and regional markets. They show that the volatility effect is a separate effect that is not explained by other factors.
Baker, Bradley & Wurgler	2011	USA	1968-2008	Document a low-volatility anomaly based on both beta and total volatility.
Ang, Hodrick, Xing & Zhang	2006	USA	1963-2000	Low-risk stock yield higher excess returns than high-risk stocks. These results are robust for exposures to the Fama and French factors and liquidity.
Ang, Hodrick, Xing & Zhang	2009	Global	1963-2003	Finds evidence of the low-volatility anomaly in 23 countries including Norway. These results are robust for exposures to the Fama and French factors and liquidity.
Frazzini & Pedersen	2013	USA	1926-2012	Stocks with high market beta have systematically low realized alpha values.

Appendix 2: Methodologies in Previous Research

Methodologies in Previous Research							
Article	**Sample Period**	**Market(s)**	**Sample Selection**	**Risk Measure**	**Risk Measurement Period**	**Return Frequency**	**Portfolio Construction**
Ang, Hodrick, Xing & Zhang (2006)	1983-2000	US stocks	No limits	Volatility	1 month	Daily	Quintiles
Blitz & van Vliet (2007)	1986-2006	US, European and Japanese stocks	Large caps	Beta and Volatility	3 years	Weekly	Deciles
Baker, Bradley & Wurgler (2010)	1968-2008	US stocks	All/top 1000 based on market cap	Beta and Volatility	5 years/ at least 2 years	Monthly	Quintiles
Baker & Haugen (2012)	1990-2011	21 developed and 12 emerging markets	99,5% of the capitalization in each country	Volatility	2 years	Monthly	Deciles
Baker, Bradley & Taliaferro (2013)	1968-2012	US stocks	No limits	Beta	5 years/ at least 2 years	Monthly	Quintiles
Frazzini & Pedersen (2014)	1926-2012	US stocks	No limits	Beta	2 years	Daily	Deciles

Appendix 3: Sample Winsorization

	Number of Observations	Lower Limit (0,1 percentile)	Upper Limit (99,9 percentile)	Number of Winsorizations
Year				
1990	5 179	-31.14 %	26.92 %	13
1991	12 739	-44.44 %	39.99 %	26
1992	16 612	-50.00 %	75.00 %	27
1993	23 236	-30.00 %	57.14 %	48
1994	25 539	-20.21 %	27.27 %	52
1995	27 552	-20.00 %	28.00 %	58
1996	32 920	-20.19 %	26.09 %	64
1997	38 835	-20.00 %	25.00 %	80
1998	39 727	-32.50 %	36.00 %	43
1999	38 012	-27.28 %	44.02 %	78
2000	39 562	-25.71 %	39.45 %	79
2001	37 586	-36.70 %	47.37 %	74
2002	33 639	-44.23 %	61.36 %	68
2003	32 780	-38.24 %	57.89 %	65
2004	37 806	-25.00 %	33.33 %	92
2005	44 761	-15.03 %	25.92 %	89
2006	48 700	-17.58 %	26.51 %	95
2007	53 849	-16.67 %	22.89 %	104
2008	50 943	-29.17 %	37.50 %	100
2009	45 807	-33.33 %	48.89 %	92
2010	55 606	-25.00 %	37.63 %	101
2011	59 350	-29.63 %	46.67 %	118
2012	57 024	-28.57 %	38.00 %	116
2013	53 568	-28.00 %	40.54 %	108
2014	46 269	-22.94 %	34.23 %	94
2015	46 416	-24.42 %	34.55 %	92
2016	46 553	-25.00 %	36.37 %	92
Min/Max		-50.00 %	75.00 %	

Appendix 4: IVOL Computed with CAPM Residuals

Panel A: Value Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.473 %	-0.007***	-0.008***	5.874 %	0.081	0.041
Q2	0.224 %	-0.011***	-0.011***	6.608 %	0.034	0.045
Q3	-0.475 %	-0.022***	-0.021***	8.257 %	-0.058	0.048
Q4	-0.793 %	-0.027***	-0.025***	9.271 %	-0.086	0.059
Q5	-1.370 %	-0.038***	-0.033***	11.389 %	-0.120	0.077
Q1-Q5	**1.843 %**	**0.031*** **	**0.025*** **	**-5.516 %**	**0.201**	**-0.036**
Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.604 %	-0.006***	-0.004**	5.085 %	0.119	0.044
Q2	0.490 %	-0.010***	-0.007***	5.990 %	0.082	0.037
Q3	0.006 %	-0.019***	-0.014***	7.097 %	0.001	0.032
Q4	-0.471 %	-0.027***	-0.020***	8.055 %	-0.058	0.034
Q5	-1.926 %	-0.044***	-0.035***	9.155 %	-0.210	0.050
Q1-Q5	**2.530 %**	**0.038*** **	**0.031*** **	**-4.070 %**	**0.329**	**-0.006**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the CAPM model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 5: IVOL Computed With Carhart Four Factor Model Residuals

Panel A: Value Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.446 %	-0.008***	-0.007***	5.739 %	0.078	0.044
Q2	0.321 %	-0.010***	-0.010***	6.662 %	0.048	0.045
Q3	-0.482 %	-0.022***	-0.021***	8.109 %	-0.059	0.048
Q4	-0.774 %	-0.027***	-0.025***	9.303 %	-0.083	0.059
Q5	-1.275 %	-0.038***	-0.032***	11.438 %	-0.111	0.077
Q1-Q5	**1.721 %**	**0.030*** **	**0.025*** **	**-5.700 %**	**0.189**	**-0.033**
Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.607 %	-0.006***	-0.004**	5.012 %	0.121	0.042
Q2	0.533 %	-0.010***	-0.007***	6.071 %	0.088	0.038
Q3	-0.092 %	-0.020***	-0.015***	7.015 %	-0.013	0.035
Q4	-0.429 %	-0.027***	-0.020***	8.086 %	-0.053	0.041
Q5	-1.926 %	-0.044***	-0.035***	9.236 %	-0.208	0.056
Q1-Q5	**2.533 %**	**0.038*** **	**0.031*** **	**-4.224 %**	**0.330**	**-0.014**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Carhart four factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 6: IVOL Computed With Five Factor Model Residuals

Panel A: Value Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.453 %	-0.008***	-0.007***	5.739 %	0.078	0.044
Q2	0.389 %	-0.010***	-0.010***	6.662 %	0.048	0.045
Q3	-0.466 %	-0.022***	-0.021***	8.109 %	-0.059	0.048
Q4	-0.842 %	-0.027***	-0.025***	9.303 %	-0.083	0.059
Q5	-1.267 %	-0.038***	-0.032***	11.438 %	-0.111	0.077
Q1-Q5	**1.720 %**	**0.030*****	**0.025*****	**-5.700 %**	**0.189**	**-0.033**
Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.583 %	-0.006***	-0.004**	4.976 %	0.121	0.042
Q2	0.541 %	-0.010***	-0.007***	6.080 %	0.088	0.038
Q3	-0.061 %	-0.020***	-0.015***	7.022 %	-0.013	0.035
Q4	-0.450 %	-0.027***	-0.020***	8.082 %	-0.053	0.041
Q5	-1.916 %	-0.044***	-0.035***	9.222 %	-0.208	0.056
Q1-Q5	**2.498 %**	**0.038*****	**0.031*****	**-4.246 %**	**0.330**	**-0.014**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the five factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 7: Test Using Deciles

Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Panel A: Value Weighted Quintile Portfolios						
Q1	0.886 %	-0.002	-0.002	6.241 %	0.142	0.059
Q2	0.208 %	-0.011***	-0.010***	6.540 %	0.032	0.052
Q3	0.485 %	-0.008***	-0.008***	6.986 %	0.069	0.053
Q4	0.312 %	-0.010***	-0.011***	7.521 %	0.041	0.056
Q5	-0.458 %	-0.021***	-0.020***	8.662 %	-0.053	0.059
Q6	-0.475 %	-0.022***	-0.020***	8.476 %	-0.056	0.058
Q7	-0.706 %	-0.025***	-0.022***	10.776 %	-0.079	0.064
Q8	-0.761 %	-0.030***	-0.025***	10.776 %	-0.071	0.075
Q9	-1.326 %	-0.036***	-0.031***	11.311 %	-0.117	0.084
Q10	-1.609 %	-0.042***	-0.035***	12.676 %	-0.127	0.094
Q1-Q10	**2.50 %**	**0.040**	**0.033**	**-6.44 %**	**0.269**	**-0.035**
Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.770 %	-0.003	-0.002	5.100 %	0.151	0.052
Q2	0.370 %	-0.009***	-0.007***	5.565 %	0.066	0.049
Q3	0.676 %	-0.008***	-0.005**	6.137 %	0.110	0.047
Q4	0.431 %	-0.011***	-0.008***	6.547 %	0.066	0.046
Q5	-0.011 %	-0.018***	-0.014***	7.103 %	-0.002	0.046
Q6	-0.126 %	-0.025***	-0.016***	7.749 %	-0.016	0.045
Q7	-0.443 %	-0.026***	-0.020***	8.389 %	-0.053	0.051
Q8	-0.427 %	-0.027***	-0.020***	8.967 %	-0.048	0.054
Q9	-1.336 %	-0.036***	-0.028***	9.086 %	-0.147	0.061
Q10	-2.540 %	-0.052***	-0.042***	11.062 %	-0.230	0.078
Q1-Q10	**3.31 %**	**0.049**	**0.040**	**-5.96 %**	**0.381**	**-0.026**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into deciles, and Decile 1 (10) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 8: Changing Winsorization Level

Panel A: Value Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.485 %	-0.007***	-0.006***	5.404 %	0.090	0.028
Q2	0.420 %	-0.008***	-0.008***	6.030 %	0.070	0.031
Q3	-0.181 %	-0.017***	-0.016***	7.031 %	-0.026	0.038
Q4	-0.457 %	-0.021***	-0.019***	7.484 %	-0.061	0.044
Q5	-0.839 %	-0.026***	-0.022***	8.147 %	-0.103	0.053
Q1-Q5	**1.325 %**	**0.019*** **	**0.016*** **	**-2.743 %**	**0.193**	**-0.025**
Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.602 %	-0.005***	-0.003*	4.648 %	0.129	0.027
Q2	0.509 %	-0.008***	-0.006***	5.409 %	0.094	0.027
Q3	-0.088 %	-0.017***	-0.013***	5.984 %	-0.015	0.028
Q4	-0.664 %	-0.024***	-0.019***	6.467 %	-0.103	0.033
Q5	-1.866 %	-0.036***	-0.030***	6.319 %	-0.295	0.038
Q1-Q5	**2.468 %**	**0.031*** **	**0.027*** **	**-1.671 %**	**0.425**	**-0.011**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama French three factor model. The sample has been winsorized with a level of 95th and 5th percentile. The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio’s realized IVOL. The alphas report the portfolio’s intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 9: Robustness Test Including Penny Stocks

Panel A: Value Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.428 %	-0.008***	-0.008***	5.836 %	0.073	0.038
Q2	0.039 %	-0.013***	-0.014***	7.122 %	0.005	0.046
Q3	-0.330 %	-0.021***	-0.019***	7.990 %	-0.041	0.044
Q4	-1.269 %	-0.035***	-0.030***	9.920 %	-0.128	0.064
Q5	-2.230 %	-0.047***	-0.040***	12.574 %	-0.177	0.096
Q1-Q5	**2.658 %**	**-0.033*** **	**-0.040*** **	**-6.737 %**	**0.251**	**-0.058**

Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.610 %	-0.006***	-0.004**	5.175 %	0.118	0.045
Q2	0.260 %	-0.014***	-0.014***	6.313 %	0.041	0.039
Q3	-0.025 %	-0.021***	-0.015***	7.441 %	-0.003	0.039
Q4	-0.817 %	-0.034***	-0.025***	8.940 %	-0.091	0.046
Q5	-2.598 %	-0.053***	-0.042***	9.856 %	-0.264	0.062
Q1-Q5	**3.208 %**	**0.047*** **	**0.038** **	**-4.681 %**	**0.382**	**-0.017**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model.The sample period is April 1990 to December 2016, with a sample of 313 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 10: Testing Different Subsamples

Panel A: Value Weighted Portfolios 1990-1998						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.533 %	-0.006	-0.004	6.517 %	0.082	0.053
Q2	0.085 %	-0.008*	-0.009**	6.799 %	0.013	0.054
Q3	-0.506 %	-0.020***	-0.016***	7.859 %	-0.064	0.050
Q4	-0.689 %	-0.021**	-0.019*	9.860 %	-0.070	0.075
Q5	-1.291 %	-0.037***	-0.029**	13.310 %	-0.097	0.087
Q1-Q5	**1.823 %**	**0.031**	**0.025**	**-6.793 %**	**0.179**	**-0.034**

Panel B: Value Weighted Portfolios 1999-2007						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	1.231 %	-0.004**	-0.004*	4.861 %	0.253	0.061
Q2	0.436 %	-0.015**	-0.017***	7.044 %	0.062	0.060
Q3	-0.264 %	-0.031***	-0.029***	8.055 %	-0.033	0.051
Q4	-0.512 %	-0.043***	-0.032***	10.177 %	-0.050	0.072
Q5	-1.104 %	-0.051***	-0.046***	13.088 %	-0.084	0.090
Q1-Q5	**2.335 %**	**0.047****	**0.042***	**-8.228 %**	**0.338**	**-0.029**

Panel C Value Weighted Portfolios 2008-2016						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	-0.436 %	-0.012***	-0.013***	5.75 %	-0.076	0.032
Q2	0.168 %	-0.009**	-0.008*	6.88 %	0.024	0.032
Q3	-0.453 %	-0.017***	-0.016***	8.38 %	-0.054	0.039
Q4	-1.776 %	-0.032***	-0.030***	9.22 %	-0.193	0.055
Q5	-1.799 %	-0.030***	-0.029***	9.74 %	-0.185	0.064
Q1-Q5	**1.363 %**	**0.018*****	**0.016*****	**-3.98 %**	**0.109**	**-0.032**

Panel D: Equally Weighted Portfolios 1990-1998

Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.586 %	-0.005	-0.003	5.991 %	0.098	0.043
Q2	0.450 %	-0.007*	-0.006	6.986 %	0.064	0.043
Q3	-0.042 %	-0.015***	-0.011**	7.198 %	-0.006	0.039
Q4	-0.107 %	-0.018***	-0.012**	8.033 %	-0.013	0.040
Q5	-0.650 %	-0.029***	-0.018**	9.168 %	-0.071	0.045
Q1-Q5	**1.235 %**	**0.024**	**0.015**	**-3.177 %**	**0.169**	**-0.002**

Panel E: Equally Weighted Portfolios 1999-2007

Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	1.149 %	-0.003	-0.002	4.284 %	0.268	0.059
Q2	1.262 %	-0.009**	-0.007**	5.874 %	0.215	0.046
Q3	0.467 %	-0.025***	-0.021***	7.640 %	0.061	0.039
Q4	0.481 %	-0.031***	-0.023***	8.987 %	0.054	0.047
Q5	-1.566 %	-0.055***	-0.045***	10.844 %	-0.144	0.068
Q1-Q5	**2.715 %**	**0.052**	**0.043**	**-6.560 %**	**0.413**	**-0.009**

Panel F: Equally Weighted Portfolios 2008-2016

Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	0.008 %	-0.008***	-0.006**	4.646 %	0.002	0.024
Q2	-0.065 %	-0.011***	-0.008**	5.247 %	-0.012	0.025
Q3	-0.628 %	-0.019***	-0.015***	6.186 %	-0.101	0.024
Q4	-1.681 %	-0.031***	-0.026***	7.026 %	-0.239	0.034
Q5	-3.416 %	-0.046***	-0.042***	7.165 %	-0.477	0.046
Q1-Q5	**3.424 %**	**0.038*** **	**0.036*** **	**-2.519 %**	**0.478**	**-0.022**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. Panels show the following: A: VW subsample 1990-1998, B: VW subsample 1999-2007, C: VW subsample 2008-20016, D: EW subsample 1990-1998, E: EW subsample 1999-2007, F: EW subsample 2008 to 20016. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.

Appendix 11: Testing During the Financial Crisis

Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Panel A: Value Weighted Quintile Portfolios						
Q1	-0.801 %	-0.009***	-0.009**	6.678 %	-0.120	0.041
Q2	-1.257 %	-0.015***	-0.014**	7.892 %	-0.159	0.046
Q3	-2.514 %	-0.026***	-0.026***	11.171 %	-0.225	0.061
Q4	-4.061 %	-0.045***	-0.042***	11.845 %	-0.343	0.063
Q5	-5.810 %	-0.063***	-0.059***	12.151 %	-0.478	0.080
Q1-Q5	**5.008 %**	**0.054*** **	**0.050*** **	**-5.473 %**	**0.358**	**-0.039**
Panel B: Equally Weighted Quintile Portfolios						
Quintile	Mean Return	FF3 Alpha	CAPM Alpha	Ex Post Standard Deviation	Sharpe Ratio	Ex Post IVOL
Q1	-0.870 %	-0.011***	-0.009**	5.677 %	-0.153	0.050
Q2	-1.206 %	-0.016***	-0.013***	6.745 %	-0.179	0.043
Q3	-2.030 %	-0.026***	-0.021***	9.026 %	-0.225	0.036
Q4	-3.613 %	-0.044***	-0.037***	10.401 %	-0.347	0.049
Q5	-6.675 %	-0.075***	-0.068***	10.644 %	-0.627	0.066
Q1-Q5	**5.806 %**	**0.064*** **	**0.058*** **	**-4.967 %**	**0.474**	**-0.016**

This table shows a robustness test for value and equally weighted portfolios that are formed and sorted monthly based on the idiosyncratic volatility calculated from daily return data for the past month relative to the Fama and French three-factor model. The sample period is during the global financial crisis 2007-2009, with a sample of 72 monthly portfolio observations. The portfolios are sorted into quintiles, and Quintile 1 (5) contains the stocks with the lowest (highest) volatility. The columns Mean Excess Return and Ex-post Standard Deviation are measured monthly and the Sharpe Ratio is the Mean Excess Return divided by the Ex-Post Standard Deviation. Ex-Post IVOL is the portfolio's realized IVOL. The alphas report the portfolio's intercept with respect to the basic CAPM model and the Fama and French three-factor model. *,**,*** indicates significance at the 10%, 5% and 1% level (respectively), represented by p-values based on Newey and West (1986) t-statistics.