BI Norwegian Business School - campus Oslo

GRA 19502

Master Thesis

Component of continuous assessment: Thesis Master of Science

Master Thesis

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Start: 02.03.2017 09.00

Finish: 01.09.2017 12.00

Master Thesis

Do hidden liabilities unravel the credit spread puzzle?

Hand-in date:

01.09.2017

Course code and name:

GRA 19502 Master Thesis

Programme:

Master of Science in Financial Economics

Master of Science in Business, Major in Finance

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ABSTRACT

This work is based on the so-called "Credit Spread Puzzle" phenomenon. It suggests a new way to deal with the phenomenon, namely, the consideration of hidden debt in specific firms' balance sheets. This information serves as an input in assessing a firm's credit risk and default probabilities. The work's aim is to test, through known credit risk models, whether the modified input helps to bridge the gap between theoretical and observed credit prices and spreads and to determine the extent to which it does make a change.

We would like to thank our supervisor Assoc. Professor Johann Reindl for professionally guiding us through the whole preparation of this Master Thesis. We also would like to thank our families and friends for their unconditional support.

1. INTRODUCTION

1.1. Some Background

The *Credit Spread Puzzle* is a widely-discussed phenomenon of the recent years as it directly affects investors' ability to correctly determine the quality of their investments and to properly price them. Brokerage firm analysts and investors use credit risk models to detect and quantify a company's ability to repay back its debt, the overall ability to meet its financial obligations and the chances of potential credit default of this company. Based on the results of these models, the interested parties, then, determine interest rates/prices of their investments. Therefore, the level of calibration of these models is crucial in protecting the interested parties from mispricing their investments or at worst case, from failure to receive the prospects of their investments. However, this anomaly is actually happening, as all of the available models, arrive at biased results, most of them very low results compared to the actual spreads. There exists a lot of tested ideas that suggest reasons for this bias and ways on how to improve it.

1.2. Research question

One potential reason for this mismatch is the imprecise level of liabilities taken from the balance sheets of these companies. Sometimes the flaw relies on mistakes made by company's accountants, but most of the times, it's as a consequence of accountants neglecting to include certain types of liabilities in their balance sheets. The reason for this is obvious: companies want to appear stronger or to better comply with their loan covenants. For example, borrowers may forget to accrue liabilities for salary or vacation time. Some might underreport payables by holding checks for weeks (or months). Other borrowers might hide bills in a drawer at year end to avoid recording the payable and the expense. These actions tamper revenues and expenses, understate liabilities and artificially enhance profits. Delayed payments can also hurt the company's credit score and cause suppliers to restrict their credit terms.

We are aimed to analyze only some forms of hidden debt (leases, pension liabilities) and investigate consequences for the companies. Hence, for the chosen companies we try to estimate the value of the potential hidden liabilities and use these data in our testing. Our estimation of company's credit risk is based on Merton model using option pricing Black-Scholes formula. The model is implemented twice, once with the original data and finally with the modified data. After comparing the results, we get some minor improvements to the current spreads. This doesn't completely resolve the credit spread puzzle phenomenon, but creates new opportunities for further investigation both in terms of the complexity of the model and hidden liabilities.

1.3. Structure of the paper

The rest of this paper will be organized as follows:

The second chapter provides a general overview of the most important works related to the topic in question. It starts with the big picture of credit spread models being used and then it narrows down to how some of the empirical authors try to solve the credit spread puzzle.

The third chapter gives the theoretical background behind the credit risk and goes step by step to explain the credit spread puzzle. Moreover, it presents how the credit spread puzzle is associated with hidden liabilities, which also constitutes the hypothesis underlying our study.

The fourth chapter presents the methodology being used to address our topic. There is first, a walk through the implementation of the Merton Model on raw data and the discovery of the credit spread puzzle. Next, we present how we proceed with the necessary adjustments of hidden debt and finally some remarks on how to implement the model again with the adjusted data.

The fifth chapter introduces the companies that we will be working with in all steps of the analysis. It explains the reasons behind chosen companies and the process of obtaining all the data needed to conduct our analysis.

The sixth chapter displays the progression of each phase of the analysis, the difficulties we encounter on the way to the results and how we deal with them. The results are rather straightforward, but special attention is given to interpret and compare the results of the first and the last step of the process. Finally, since our thesis is restricted in terms of the number of companies, the simplicity of the chosen model and data precision, in the conclusion, we also bring about some remarks and recommendations that might be considered if a more thorough analysis of this thesis would be needed to be conducted.

2. BACKGROUND AND LITERATURE

2.1. Why is it important to correctly model the credit risk?

Recently, the interest in the framework of credit risk models and measures is increasing. This is because of two main reasons: Firstly, the Capital Accord of 2006, or Basel II, allows large banks to use their internal models to assess their capital requirements instead of the more constraining standardized models. And secondly, the huge increase of off balance-sheet derivatives and the rising use of the securitization of loans call for more developed credit analysis methods (Laajimi 2012).

2.2 Literature on Credit Risk Models and variations

There are three main approaches to credit modelling and the pricing of credit risk – structural models (Merton, 1974), reduced-form models (Jarrow et. al 1995) and statistical models like SEBRA model used by Norges Bank (The Norwegian central bank) (Bernhardsen & Larsen, 2007) and Altman's Z-score method (Altman E., 1968).

The *structural models* originate from the option pricing theory introduced by Black & Scholes (1973), which found its application in the area of corporate bond pricing. The main idea of structural models is to value corporate debt using a contingent-claims approach. The crucial assumption is that information held by the firm's managers is completely available, i.e. the modeler has a comprehensive knowledge of all the firm`s assets and liabilities. These models are used to estimate the spread of bonds issued by public firms, since stock prices are a major component of the model input parameters. Structural models are valued by practitioners due to intuitive economic interpretation, which allows for consistent discussion regarding a variety of credit risk exposures and understanding of transaction implications (impact on credit quality due to increased borrowing, acquisitions, share buybacks, etc.).

The paper "Structural Credit Risk Models: A Review" by Laajimi presents the major structural models. These models differ from one-another in their respective underlying assumptions. All the structural models share a common theoretical framework, based on the Merton model (1974), an exogenous model.

Merton's model is extended by Kim, Ramaswamy and Sundaresan (1993) to incorporate both default before maturity date risk and interest rate risk. The authors assume that the default is triggered by a cash-flow shortage. Other exogenous work include Longstaff and Schwartz (1995), Nielseen, Saa-Requejo and Santa-Clara (1993) and Briys and de Varenne (1997).

There exist three other major contributions. Black and Cox (1976) deal with the timing issue of the Merton model (the default time is restricted to the maturity of debt, independently of the evolution of the asset's value before maturity). In their model, the firm defaults as soon as the value of its assets reaches a non-random default barrier V_B. In this case, bondholders get V_B and equity holders get nothing. Whereas, the Leland (1994) model contributes in terms of the tax shield of debt and bankruptcy costs. On one hand, the debt issuance reduces the firm's value due to the increasing value of bankruptcy costs, and on the other hand, higher value of interest payments imply more tax shield, increasing in this way the value of the firm. Finally, Leland and Toft (1996) model, differently from Leland (1994) model that assumes a perpetual debt, account for debt that is continuously rolled over. This ensures that the principal, coupons and debt maturity are independent of time.

The *reduced from models* were originated with Jarrow and Turnbull (1992) research and subsequently studied by Jarrow and Turnbull (1995), Duffie and Singleton (1999) among others. In contrast to structural models, reduced form models assume incomplete knowledge of the firm`s condition, implying that a firm's default time is inaccessible or unpredictable, and therefore estimated as a stochastic process. This informational assumption is a key difference between two models – structural model can be transformed into reduced form model as the information set changes. In addition, in structural models, the recovery rate process is prespecified by a knowledge of the liability structure, while in reduced form models this process is exogenously supplied. Due to limited information and default time assumptions, reduced form method is considered to be more theoretically accurate, but lack the clear economic rationale for defining the nature of the default process and require detailed bond price data.

Statistical models use various forms of econometric techniques to identify determinants of default. They are less reliant on economic theory as their model framework but are limited by their poor out-of-sample-power.

2.3. Discover the "Credit Spread Puzzle"

Following the review by Laajimi, there is an immense number of empirical work that test the ability of different models to predict the credit spread on bonds and CDS.

As a major contribution to the field, Eom, Helwege and Huang in their paper "Structural Models of Corporate Bond pricing: An Empirical Analysis" test five of these structural models, namely Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001), for the period 1986-1997, using data from 182 bond prices in firms with simple capital structures. Their findings on the implementation of Merton's model, approve the convention that the predicted spreads are too low compared to the real spreads. Nevertheless, the other models display too high spreads on average. They suggest that the problem relies on the accuracy of the models and that better models should account for higher spreads for safer bonds, avoiding excess dispersion of spreads in riskier bonds.

2.4. Resolving the "Credit Spread Puzzle"

Two other authors, Feldhütter and Schaefer test the Merton Model in both their papers. "The Credit Spread Puzzle – Myth or Reality" (2013) paper indicates that current authors, testing for the credit spread puzzle, fail to distinguish some strong biases and their approaches suffer from low statistical power. The problems come from the fact that authors usually compare the predicted spreads to average observed spreads. First, considering the typical convexity of the spreads in firm variables, the spreads of average firm variables are significantly lower than the average of spreads. Secondly, in these type of studies that use samples for a long period, the classical assumption that historical default probabilities serve as a good proxy for expected default frequencies, does not hold. Hence, while correcting for these issues, they find almost no evidence of credit spread puzzle. Contrariwise, the model captures both the average level and time series variation of 10-year BBB-AAA spreads. The authors also suggest that considering the cross-sectional variation of firms and time series variation of firm leverage is crucial in testing the structural models.

The updated version of their first paper, namely “The Myth of the Credit Spread Puzzle” (2016), calibrates the model by using a much longer time series of data compared to previous studies, specifically 92 years. They test both the Merton and Black-Cox models. The authors explain that such long history of ex-post default rates contribute to more precise ex-post default probabilities as they can abbreviate the effect of high skewness in the distribution of realized default rates. They again, highlight that the problems in previous model tests come from how they implement the models rather than deficiencies of the models.

In addition, Huang and Huang (2012) support the idea that if the structural models are calibrated in terms of stochastic interest rates, endogenous default, stationary leverage ratios and strategic default, they will always match the default rates and the equity premium.

Another important paper “On the relation between the credit spread puzzle and the equity premium puzzle” by Chen, Collin-Dufresne and Goldstein 2008, suggests resolving the credit spread puzzle by accounting for the fact that default rates and Sharpe ratios strongly covary with each other.

2.5. Literature on Unconsidered Liabilities

In our study, we propose that the value of liabilities generally used in the models, is not all-inclusive, i.e. incorrect to a certain level, as a lot of companies try to hide some of their debts. The reason behind these actions of companies is to receive more appealing financial ratios and therefore by obtaining a better picture of the corporate health, they rank better in credit ratings.

Here, hiding doesn’t always mean illegality, but it rather entails different approaches on the “consideration” of liabilities. Companies take advantage of all the flawed and incomplete regulations and unwritten rules about firm accountancy by regulating institutions and they individualize law interpretation for their own interest.

The study that this paper introduces is a new approach in academia to solve the credit spread puzzle. There are no released articles in journals that thoroughly link the credit spread puzzle to hidden debt and analyze the phenomenon in specific companies. However, the topic of "Unconsidered Liabilities" alone, is very large and widely discussed among authors and rating companies. Among other works in this field, we chose to base the theory of our thesis on the book of J. Edward Ketz titled "Hidden Financial Risk". The book provides great insights on Ethics in Financial Accounting and Reporting. After giving backgrounds on recent Financial scandals related to the omission of important liabilities in their public balance sheets, the author proceeds by thoroughly analyzing different ways that account managers use to hide debt, elaborating to details on each of them. He comprises here not only how to distinguish any discrepancies and distortion of financial reports, but also how to make adjustments in a guided and proper way.

3. THEORY AND HYPOTHESIS

3.1. Explaining credit risk

To answer the defined question that this thesis will elaborate on, there is the need to explain more in detail the underlying terminology. To begin with, *Credit risk* measures the prospects that a loan will be repaid on the arranged time under original defined terms. The credit risk of a company is important to Financial institutions or individual investors to assess the financial stability of these companies when the latter are considering to loan money to or buy bonds from the former. It is also important to company's shareholders and loan covenants.

The level of *credit risk score* or *credit rating* shows to investors where is this company positioned in terms of risk and according to this risk, investors will charge the most convenient interest rate. Credit risk of a company is assessed by evaluating the five C's: the company's credit history, its capacity to repay, its capital, loan's conditions and associated collateral. If the company (as issuer or its issuance) is rated high, this signalizes to the market players that this investment is less risky. Therefore, it could be charged a lower interest rate and vice versa. However, an AAA rating is not a guarantee against default, it only implies that it is less probable that this firm will default than another firm with a lower rating.

Modelling of the credit risk provides the framework for investors and financial institutions to estimate the credit spread risk of a certain company. The following section will explain in detail the model being used. Nevertheless, from an immense number of studies, it is observed that the standard structural models forecast credit spreads that are very low compared to the real credit spreads. This phenomenon is called "*the credit spread puzzle".* As we suggest, one of the reasons of this puzzle might be the incorrect value of firm's liabilities, the last section of this chapter will narrow down the most important aspects related to this issue.

3.2. Structural Models. Intuition behind the theory

Structural models are cause-and-effect models. First of all, one needs to identify and impose conditions under which the borrowers are expected to default, and then estimate the probability that these conditions will be satisfied. In this case, borrowers are the companies which issue bonds, and debt-holders are the lenders of funds (individuals, other companies, banks). In case of limited liability entities, when a company is unable to cover all payments to its debt-holders, it defaults.

Recalling the balance sheet equation:

$$\text{Assets} = \text{Liability} + \text{Equity}, \tag{3.1}$$

and the fact that equity holders are the residual claimants, we conclude that when company`s liabilities exceed its assets, the value of equity is negative, implying equity holders are willing to give it away at no cost. Simply saying, they exercise the walk-away option, which can be priced with approaches from the option pricing theory.

The first structural model of default risk valuation based on the option pricing theory was proposed by R. Merton in 1974. The model assesses the equity and debt value of listed companies through the Black and Scholes (B&S) option pricing model. The intuition behind is rather straightforward. Assume the company has outstanding debt in form of zero coupon bond equal to L with maturity T, meaning no matter how much the company will profit in the future ($A > L$), the debtholder will definitely receive just a notional L. On the other hand, the equity holders as residual claimants will benefit in this situation – their upside potential is unlimited, they pocket the positive difference. However, if $A < L$ and company defaults, equity holders will receive nothing and remaining assets are claimed by the creditors. Therefore, the pay-off to equity holders may be described as a European call option:

$$E = \max(0, A - L), \tag{3.2}$$

where underlying is the asset value and strike is the outstanding liability L.

The pay-off to debt holders equals to the portfolio of risk-free zero bonds with notional L and a payoff of a short European put option:

$$D = L + \min(0, A - L) = L - \max(0, L - A) \tag{3.3}$$

According to Merton`s simplified setup, the probability of default at time T implies the probability of $A < L$ – the value of assets is below the value of liabilities, or in other words, when the put option is exercised.

3.3. Merton model

The theory of defaultable bond pricing has its bases on Black and Scholes (1973) and Merton (1974). The key feature of the model relies on the fact that corporate securities are seen as contingent claims on the market value of firm's assets. Therefore, the focus will be on the degree to which firms' asset sales are used to finance coupon payments on debt.

Using option-backed equity and debt set up from the previous section, we now discuss the basic Merton credit risk model. The model is based on the following assumptions (Merton,1974):

1. No transaction costs, taxes or indivisibilities of assets
2. There are a sufficient number of investors with comparable wealth levels so that each investor believes that he can buy and sell as much of an asset as he wants at the market price
3. There exists an exchange market for borrowing and lending at the same rate of interest
4. Short-sales of all assets, with full use of the proceeds, is allowed
5. Trading in assets take place continuously in time
6. The Modigliani-Miller (MM) theorem that the value of the firm is invariant to its capital structure obtains.
7. The term structure is flat and known with certainty; i.e., the price of a riskless discount bond that promises a payment of $1 at time T in the future is $P(t,T)=e-r(T-t)$, where r is the (instantaneous) riskless rate of interest, the same for all time
8. The dynamics for the value of the firm, V (in our case we use A for Assets notation), through time, can be described by a diffusion-type stochastic process.

Merton emphasizes that "perfect market" assumptions 1-4 can be weakened, while assumption 7 is chosen to distinguish risk structure from term structure effects on pricing. In some later versions of structural models the term structure is assumed to be a stochastic process. However, continuous trading and following the stochastic process for assets are crucial assumptions.

As in the B&S option price formula set up, where stock price follows a Brownian motion (or Wiener process), the company`s assets are described with the stochastic process:

$$dA_t = \mu A_t dt + \sigma A_t dZ_t, \tag{3.4}$$

where μ is the instantaneous expected rate of return on the firm per unit time dt, σ^2 is the instantenious variance of the return on the firm per unit of time dt, and dZ_t is a change of a normally distributed variable Z that follows Brownian motion.

Wiener process (Brownian motion) is a particular type of Markov stochastic process with a mean change of zero and a variance rate of 1.0 per year. In turn, Markov process assumes that only the current value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant. For example, stock prices are assumed to follow a Markov process, which supports the weak form of market efficiency. A stochastic process equation where the drift and volatility are depended on the variable is called Ito`s process, and may be rewritten as:

$$\frac{dA_t}{A_t} = \mu dt + \sigma dZ_t \tag{3.5}$$

Here the percentage change in A_t is normally distributed with instantaneous mean μ and variance σ^2. This process is also known as geometric Brownian motion. Because the mean and variance at time t are proportional to A_t, the evolution of A_t generates compounding (the change in A_t is proportional to A_t:$\ln(A_t)$ and thus non-normality. Therefore, a variable that follows geometric Brownian motion is lognormally distributed. While A_t is not normal, $\ln(A_t)$ is normally distributed:

$$\ln(A_t) \sim N(\ln(A_0) + (\mu - 0.5\sigma^2)t, \sigma^2 t) \tag{3.6}$$

And $A_t = A_0 e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}Z}$, where $Z \sim N(0,1)$

Using B&S option pricing formula, the value of equity can be expressed as call option:

$$E_0 = A_0 N(d_1) - Le^{-rT} N(d_2), \tag{3.7}$$

where

$$d_1 = \frac{\ln(A_0/L) + (r + 0.5\sigma_A^2)T}{\sigma_A\sqrt{T}} \tag{3.8}$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

and $N(\cdot)$ represents the cumulative normal distribution, r is continuously compounded risk free rate, σ_A is the asset volatility, T is time to maturity. Here $N(d_2)$ has a fairly simple interpretation – it is a probability that a call option will be exercised in a risk-neutral world, namely, it is a probability that the firm will not default as long as value of assets exceeds the value of liabilities. Due to a symmetric property of normal distribution, $N(d_2) = 1 - N(-d_2)$, where $N(-d_2)$ is a risk-neutral probability of default. Variables d_1 and d_2 are results of the standard z-score calculation in B&S formula, where d_2 is also known as distance to default (DD) and measures the number of standard deviations the expected asset value A is away from the default. And to calculate this we need A_0 and σ_A, which are not directly observable. At the same time, for listed firms via stock prices we can determine E_0. From Ito`s lemma:

$$\sigma_E E_0 = \frac{\partial E}{\partial A} \sigma_A A_0 = N(d_1) \sigma_A A_0 \tag{3.9}$$

This expression gives us other condition which has to be satisfied simultaneously with equation (3.7), and σ_E is estimated from historical data or options.

Because it is common in discussions of bond pricing to talk in terms of yields rather than prices, the difference between the corporate bond yield and risk free rate (continuous) can be expressed as:

$$Spread = y - r = \left(\frac{L}{A-E}\right)^{\frac{1}{T}} - 1 - (e^r - 1) = \left(\frac{L}{A-E}\right)^{\frac{1}{T}} - e^r, \tag{3.10}$$

where $D = A - E = \frac{L}{(1+y)^T}$ is the current value of debt.

This spread is thought to be a bond`s credit risk premium. Therefore, our goal is to use available market data, imply the credit risk spread and afterwards compare it to the actual data, which is a credit default swap (CDS) contracts.

3.4. Credit default swaps

In the essence, credit default swap (CDS) is an insurance contract on a bond. The CDS seller (protection seller) pays the buyer (protection buyer) when the reference entity experiences a credit event – fail to cover promised payments. If such event occurs, a CDS buyer receives:

$$CDS\ buyer\ payoff = Bond\ par\ value - Bond\ market\ value \tag{3.11}$$

The International Swaps and Derivatives Association (ISDA) plays a central role in the CDS market: provides the standard documentation, pronounces an occurrence of a default event, controls the settlement auctions that determine the payout.

An investor who buys a bond and CDS on this bond owns a synthetic default-free bond, implying that a CDS premium is approximately equal to the default premium on the bond:

$$Default\ swap\ premium = y - r, \tag{3.12}$$

where y – defaultable bond yield and r – default-free bond yield.

However, this equation does not hold exactly. Blanco et al. (2005) found that the average difference between the CDS premium and the bond credit spread is a positive 5.5 basis points and that CDS prices reflect news before bond prices. In practice, we need to take into account the time variation in the credit spread and transaction costs (Duffie, 1999). Moreover, many other issues arise while a party to a CDS contract may be able to affect the value of the bonds, which eventually influences the value of the CDS contract.

But since we expect our model to yield strong mispricing results, we assume that equation (3.12) holds precisely and equals to the defaultable bond credit spread.

3.5. Unconsidered liabilities

What we use as input in the Merton Model, are periodic financial data about liabilities, interest expense and dividends paid. These data can be extracted from corporates' financial reports. The accuracy of the data found on these reports depends heavily on the attempts of firms to modify their public information using different accountancy techniques. The purpose is usually to deflate the liabilities, but meanwhile, there can be noticed some slight amendments on other financial voices as well.

Lately, we have heard a lot about accounting scandals, like in the case of Enron, Adelphia, and WorldCom. Even though these attempts were revealed and debunked, there will always be managers out there hiding company's liabilities in order to cover mistakes or to make the company seem less risky in financial terms. Some of them might hide a considerable amount of debt and some of them just minor liabilities. However, these activities cause huge discrepancies and negatively affect the results of financial models who use debt data as inputs. Not to mention, the consequences if these scandals are revealed: increased financial risk reporting premium, higher cost of capital and lower stock and bond prices.

There exist two sets of accounting techniques that accountants use to sweep liabilities. The first group entails the equity method, lease accounting, and pension accounting. All the three methods, are reader- and adjustment-friendly since the disclosures made by the CEO and CFO regarding these methods are comprehensive and allow the readers to perform adjustments. However, the level of understanding and the accuracy of the adjustments will depend on the quality of such disclosures. Nonetheless, the second set of accounting methods, that includes the application of asset securitization, SPE borrowings, and synthetic leases, cannot be clearly unraveled from the footnotes on the financial reports. Therefore, analytical adjustments are hard to be undertaken.

In this chapter, we are going to theoretically break down everything related to the first set of accounting methods used to hide debt. We start with the equity method and continue with lease accounting and pension accounting.

The equity method

The equity method relies on the affiliates a company has and the influence of the company over these affiliates. Since the parent company (investor) invests a particular share on the affiliate, we can refer to the latter as investees. When the parent's shares in another company are greater than 20%, it means that the parent has significant influence over the investee. Here, we distinguish two separate cases. When this ownership is more than 20%, but less than 50%, the parent should apply the equity method to consolidate the accounts. On the other hand, if the significance level becomes even more substantial, by owning more than 50% of the subsidiary, then the consolidation method is required.

Specifically, under the equity method, the investor should include on its balance sheet an investment account that represents the investor's proportionate share on the investee. Meanwhile, under the consolidation method, there is no investment account. Instead, the assets and liabilities of the investee are added on the investor's balance sheet. What remains unchanged in both cases, is the net income.

Managers who engage in hiding debt activities, find the equity method a great way to not account at all for large amounts of either liabilities or interest expense that an important subsidiary entails. Sometimes, these interest expenses can be as large as the investor's interest expense, the exclusion of which totally changes return metrics. Furthermore, there are parent companies, like Elan, that use these affiliates to load them up with undesirable debt and generate a better impression for the parent company.

According to the Statement of Financial Accounting Standards (SFAS), No. 94, the consolidation is required by law, only when the share on the investee is more than 50%. But, shall we satisfy our investigative intentions by only adhering to this law? History shows that, even though the companies recognize only a share of less than 50% over some smaller firms, they do control everything on these firms, including operating, investing and financing decisions. Very large affiliate's interest expenses can be used as a fact that the parent company parks most of its liabilities with the former. Experience shows that firms that control the operations of other smaller firms are usually very negatively affected if these small firms default or are in financial depression. Hence, we can deduce that the level of control the investor has over the investee, needs to be a better indicator of determining the model of merging financial data between the two, rather than some reported percentage. In these cases, the readers of financial statements, must make analytical analysis and compare the consolidated balance sheet and income statement to what is being reported by these firms.

Lease accounting

A lease is a contract that gives the lessee the right to use the lessor's property for a certain amount of payments. We can distinguish between two types of leases: operating leases and capital leases. Operating leases essentially are rentals. An accountant can simply recognize it as a rental expense either paid by cash or put on the balance sheet as a payable. While this method seems legit for short-term rentals, it affects credulity when the rental period extends for a substantial time. Capital leases are treated as long-term assets, recognized on both sides of the balance sheet, as an asset as well as a long-term liability. Moreover, accountants have to systematically recognize interest expense on the liability and depreciate the lease.

Managers sometimes report capital leases as operating leases. The reason why these managers try to “cheat” and prefer operating leases instead, is because they tend to avoid high expenses as a result of capital leases. While capital leases show higher expenses in the early years of the contract life, operating leases show the opposite. Statistically, by using lease accounting, managers can understate their firm’s financial structure by 10 to 15 percentage basis points. Another reason lies on the financial ratios, that are crucial in attracting investors. As there is no asset recognized for operating leases, the ROA would appear inflated and therefore appealing to investors.

Statement No. 13 of FASB states four criteria that should make managers recognize leases as capital leases. Table 3.5.1 shows a summary of these criteria.

Table 3.5.1: Lease criteria

Criteria for a capital lease:
1. Passage of title to the lessee 2. Bargain purchase option 3. Lease term equals or is greater than 75% of the useful life of the asset 4. Present value of the minimum lease payments equals or is greater than 90% of the fair value of the property

Firstly, if a purchase will occur in the future, then a capital lease should be the precursor of it. Secondly, the rational lessor usually exercises the purchase option since it is indicated that the lessee offers to sell the property to the lessor at the end of the lease period for a very low price. Thirdly, 75% represents a great portion of the useful life of an asset. Therefore, according to FASB, obtaining property rights for more than 75% of the asset’s life is in essence like a purchase of the asset. As the last criterion says, if the amount that the lessee pays is closer to the property’s fair value, the lease can be classified as a capital lease.

No matter the criteria, managers find tools to trick their financial papers. There are ways that lawyers and Certified public accountants (CPAs) can design contracts to avoid the proper recognition of capital leases. Three of the most notable strategies the managers use will be discussed below:

1. The discount rate as a tool to avoid falling in the 4th criteria

On the one hand, the lessor knows the fair value of the asset and establishes a rate of return that she would like to receive from the property, referred to as *the implicit rate of return*. Using this rate, she then determines the monthly rentals on the asset necessary to produce this rate.

On the other hand, the lessee might not know this rate and if so, FASB introduces *the borrower's incremental borrowing rate*, which will serve him as a discount rate for the lease payments. However, if the lessee knows the implicit rate of return, then he should choose as a discount rate the lowest between the two. This last restriction makes it more probable that the last criteria is met. That is why the best trick here would be ignorance. If the incremental borrowing rate is beneficial for the firm in discounting terms, then his best strategy would be to tell the lessor that he doesn't want to know the implicit rate of return.

2. Unguaranteed residual values to reduce the value of payments in today's terms

The residual value is the estimated value of the property at the end of the lease term. This value might be guaranteed or unguaranteed. If guaranteed, then according to FASB it should be part of the minimum lease payments which means that if at the end of the period the value of the property is less than the guaranteed value, then the lessee should pay for any shortcomings. On the other hand, there are no strings attached if the residual value is unguaranteed. With this in mind, when computing the present value of the payments, the guaranteed residual value in today's terms should always be part of the total sum. This additional amount will bring this sum closer to the critical point of 90% of the fair value. On the other hand, there are no strings attached if the residual value is unguaranteed. That is why managers will try to make the impossible to keep the residual value unguaranteed and therefore avoid capitalization.

3. Contingencies to lower the minimum lease payments

Contingent rental fees are usually used when the lease deals directly with the main activity of the lessee firm. When its monthly sales are relatively stable, the firm can, instead of paying a full fixed rental amount, pay some part of it as a percentage of its monthly guaranteed sales. In this way, the minimum lease payment claimed on the financial notes is reduced and therefore also the present value of the stream of cash flows is substantially reduced. To this extent, there is no evidence to refrain the firm from accounting the lease as an operating lease.

Pension accounting

There are a lot of companies, especially large corporations, who promise employees pension benefits after working for a couple of years in the company. A *pension plan* is an agreement between an employer and its employees that, under prespecified conditions, the former will regularly provide the latter cash payments when the latter retires. The pension plan is financed by pension plan assets. As all types of investments, these assets carry some risk. Usually, the status of the plan, being either overfunded or underfunded, allows the interested parties to measure the risk it entails.

There exist two major types of pension plans: defined benefit plans and defined contribution plans. The second category has no link to the hidden debt since the contribution to the plan depends on short term and performance premises and therefore the firm disentangles itself from long-term payables to its employees. Hence, in this analysis, we will be dealing only with defined benefit plans. A *defined benefit plan* is a plan in which the employer promises to pay its employees a guaranteed amount when they retire and therefore burdens the employers with long-term liabilities towards its employees.

Before exploring the opportunities that pension plans offer to hide debt, we need to have a brief overview of pension accounting.

There are two important amounts related to pension accounting that firm accountants recognize: one goes to the income statement and the other to the balance sheet. The first one is Pension expenses and is calculated as follows:

$$\begin{aligned}&Pension\ expenses\\&= Service\ cost\ +\ Interest\ on\ the\ projected\ benefit\ obligation\\&- Expected\ return\ on\ plan\ assets\\&+ The\ amortization\ of\ various\ unrecognized\ items (ex.prior\ service\ cost)\end{aligned}$$

As we see, it is expressed as a function of what it is promised to the employees (service cost), an interest rate and the outcomes of plan asset management. The amount that is recognized on the balance sheet is either a prepaid pension asset or an accrued pension cost. Literally, this amount should entail all the below-mentioned items:

$$\begin{aligned}&Prepaid\ pension\ asset\ or\ Accrued\ pension\ cost\\&= Prepaid\ pension\ asset\ or\ Accrued\ pension\ cost_{t-1} - Pension\ expense\\&+ Funding\ status\end{aligned}$$

There are three aspects related to pension accounting that allow managers to distort the balance sheet and income statement in the firm's interest. The first element to consider is related to the amount recognized in the balance sheet. Accounting states that employers should report on their financial books and reports in today's terms what it owes to its employees and on the other side the fair value of the assets held in the pension plan. However, the reality seems to be smoother. Following the rules constituted by FASB in pension accounting terms, entities are allowed to net pension assets against pension liabilities, by including in the balance sheet an amount either on the right or the left side depending on the netting result. This is rather inappropriate since it leaves you only with the illusion that the assets will cover the liabilities, but it doesn't explicitly track the performance of the assets and conceals the true liabilities.

The second aspect deals with the expected return on the assets. This amount is computed using an expected rate of return on the assets, which in itself gives an inaccurate picture of the corporation's performance. As part of the pension cost computations, it distorts the income statement. Instead, an actual return should be included.

The last element participating in hiding debt activities is the prior service cost. The *prior service cost* appears as an element at the moment the company starts a pension plan and decides to allocate the employees some pension benefits for the prior years that they have been working with the company. Now, this sum appears as a whole in one particular moment. Hence, it brings in two questions to be answered one after the other. When is the best time to instill this cost into the income statement? Normally, it should be injected in the year that the commitment is made. However, FASB allows amortizing this cost gradually over the remaining working period of the employee. In this way, firms save that particular year from large changes in the financial ratios. This being applied, another question comes into our mind. Why there is no unamortized prior service cost recognized in any account? It seems like this question is left unanswered by most of the companies that follow this strategy, even though it is substantially erroneous.

Hypothesis

Based on our thorough review of existing literature about credit risk pricing, we did not find any paper which investigated the potential impact of certain types of liabilities inclusion on the credit pricing model's outcome. Therefore, our paper is a new contribution to the asset pricing research area. In this paper, we will test our assumption about hidden liabilities in the balance sheets of the companies, the inclusion of which may help to reduce a pricing gap between model output and real data.

4. METHODOLOGY

4.1. Model selection

In order to test our hypothesis about hidden liabilities impact on credit risk pricing, we decided to proceed with a structural model, namely Merton model, due to its economic intuitiveness, ease of interpretation, and our existing knowledge of the Black & Scholes and Merton framework. As numerous empirical tests of Merton model showed its inability to generate sufficiently high-yield spreads to match ones observed in the market, a set of extensions and improvements was introduced to the original model (allowing for coupons, default before maturity, stochastic interest rates, etc.). However, in our case, in addition to the time limit on the writing of our master thesis, the whole process of hidden liabilities extraction turned out to be quite challenging and time-consuming, since we had to deal with complete financial statements and their footnotes manually. Therefore, we decided to test out assumption based on the original model with a minor extension taken from Löffler and Posch (2011). This extension treats company`s liabilities as a zero-coupon bond which apart from promised payment also includes accrued interest and dividend for the time until maturity, but assumes that bond cannot default before maturity. Thus, accrued interest and dividend payments are shifted their actual payment dates into the future (over $(T-t)$ horizon). Even though they are actually paid before maturity, we treat them as liabilities with a higher priority than promised bond payment. Dividend payments due are obtained as:

$$D = \sum_{\tau=t+1}^{T} D_0 \cdot (1+g)^{\tau-t} \cdot e^{r(T-\tau)}, \tag{4.1}$$

where D_0 – reported paid dividends for the latest period, g – dividend growth rate, r – risk-free rate. Although dividends are risky, they are treated senior to debt and we consider that accruing dividends with risk-free rate is the optimal choice.

Interest payments due are calculated as:

$$I = \sum_{\tau=t+1}^{T} c \cdot L \cdot e^{r(T-\tau)}, \tag{4.2}$$

Where c – annual coupon rate, L – total liabilities, r – risk-free rate. As we expected, our selected companies have a rather complex capital structure. Therefore, as the annual coupon rate proxy, we decided to take the average of $\frac{Interest\ paid}{Total\ Liabilities}$ ratios for the previous years.

As dividend and interest payments have equal priority over promised amount L, there are three possible cases:

1. $A_T < L + I$: dividend claim is not covered, equity holders receive $\frac{D}{(D+I)} A_T$
2. $L + D + I > A_T > I + D$: principal claim is not covered, equity holders receive D
3. $A_T > L + D + I$: all claims are covered, equity holders receive $A_T - L - I$, including dividends

As result, the standard B&S option pricing formula for equity value will change to:

$$E_t = A_t N(d_1) - (L + D + I)e^{-r(T-t)} N(d_2) + \frac{D}{D+I} A_t + \frac{D}{D+I}\left(- A_t N(k_1) + (D + I)e^{-r(T-t)} N(k_2)\right), \quad (4.3)$$

where

$A_t N(d_1) - (L + D + I)e^{-r(T-t)} N(d_2)$ – call on assets with strike $L + D + I$ (case 3),

$\frac{D}{D+I} A_t$ – share of $\frac{D}{D+I}$ in assets (case 1),

$\frac{D}{D+I}\left(- A_t N(k_1) + (D + I)e^{-r(T-t)} N(k_2)\right)$ – share of $\frac{D}{D+I}$ in a short call on assets with strike $D + I$ (case 2)

and

$$d_1 = \frac{\ln\left(\frac{A_t}{L + D + I}\right) + (r + 0.5\sigma^2)(T - t)}{\sigma\sqrt{(T - t)}}$$

$$d_2 = d_1 - \sigma\sqrt{(T - t)} \quad (4.4)$$

$$k_1 = \frac{\ln\left(\frac{A_t}{D + I}\right) + (r + 0.5\sigma^2)(T - t)}{\sigma\sqrt{(T - t)}}$$

$$k_2 = k_1 - \sigma\sqrt{(T - t)}$$

As in original Merton model, $N(-d_2)$ with corrected liabilities claim gives a probability of default. Since in our framework we use over 1-year horizon and the model does not allow for interim defaults, we can infer an annual default probability assuming that default probabilities are constant across time:

$$Default\ probability\ (annual) = 1 - (1 - Cummulative\ probability)^{1/_{horizon}} \quad (4.5)$$

Next, we calculate the implied credit spread, which will be discussed in details in the following section.

4.2. Parameter estimation

The model has observable and unobservable variables. Observable variables include stock prices, risk-free rate, cash dividends paid, interests paid, outstanding shares, liabilities. All data can be found either through the financial data providers or in financial reports. Among unobservable variables, there are asset value and volatility. Additionally, we need to specify default barrier and recovery rate. As all variables are known, our model will return the probability of default and finally, credit spread, which we compare to the market data.

4.2.1. Asset value and volatility

As we discussed in a previous chapter (3.3 Merton model), one way to solve for unknown asset value and volatility is to infer the second equation from Ito`s lemma and solve both equations simultaneously. However, we decided to search for different approaches.

Another way is to apply a Maximum Likelihood Estimator (MLE) method. However, the rearranged B&S formula, which expresses asset value, has a complex structure for obtaining a log-likelihood function, which increases chances to get errors while deriving it manually.

Fortunately, the iterative scheme approach proposed by Vassalou and Xing (2004) seems to produce good results, giving not identical but extremely close to MLE estimates. The advantage of this method is that it is quickly implemented and easy to set up. The procedure is the following:

1. Calculate asset values $A_{t0} \ldots . A_{tN}$ from $E_{t0} \ldots . E_{tN}$ using inverse of B&S formula:

$$A_{t_N} = \frac{E_{t_N} - D_{t_N} e^{-r_{t_N}(T-t_N)} N(k_2) + \left(L_{t_N} + D_{t_N} + I_{t_N}\right) e^{-r_{t_N}(T-t_N)} N(d_2)}{N(d_1) + \frac{D_{t_N}}{D_{t_N} + I_{t_N}} - \frac{D_{t_N}}{D_{t_N} + I_{t_N}} N(k_1)}$$

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$$A_{t_0} = \frac{E_{t_0} - D_{t_0} e^{-r_{t_0}(T-t_0)} N(k_2) + \left(L_{t_0} + D_{t_0} + I_{t_0}\right) e^{-r_{t_0}(T-t_0)} N(d_2)}{N(d_1) + \frac{D_{t_0}}{D_{t_0} + I_{t_0}} - \frac{D_{t_0}}{D_{t_0} + I_{t_0}} N(k_1)} \tag{4.6}$$

2. Estimate σ^{n+1} by thinking of $A_{t0}(\sigma^n) \ldots . A_{tN}(\sigma^n)$ as a geometric Brownian motion:

$$\sigma^{n+1} = \sqrt{\frac{1}{N\Delta t} \sum_{i=1}^{N} \left(\ln\left(\frac{A_{t_i}}{A_{t_{i-1}}}\right) - \xi \right)^2}, \tag{4.7}$$

where

$$\xi = \frac{1}{N\Delta t} \left(\ln\left(A_{t_N}\right) - \ln\left(A_{t_0}\right) \right)$$

3. Use the updated value σ^{n+1} in place of σ^n and repeat until the procedure converges. We check convergence by examining the change in asset values from iteration to iteration. In our case, if the sum of squared differences between consecutive asset values is below 10^{-7}, we stop.

The iterative scheme gives us A_{t_N} and σ^n, where the latter should be annualized using a root-T-rule: for example, if we use monthly data, to get annualized measure we multiply monthly volatility σ^n by $\sqrt{12}$.

The starting value for the asset value is equity market value plus reported liabilities, while the starting value for the asset volatility is equity volatility adjusted for leverage:

$$\sigma^n = \sigma_{equity} \frac{E_t}{A_t} \tag{4.8}$$

However, in practice, the convergence is fast and the starting value seems of little importance.

4.2.2. Default barrier

Default barrier is a certain level that company`s assets have to reach in order for default event to happen. Many researchers argued about the approach which defines this level: whether to use total book liabilities or its fraction. Moreover, the method of default barrier estimation varies from model to model. Thus, the models, which assume maturity under 1 year, should have a default barrier less than total liabilities. The reason is that companies are allowed to renegotiate or postpone debt with longer maturities. What is more, the whole debt is rarely due within the estimation period.

One of the most popular approaches among researchers is mentioned in the KMV model (Crosbie & Bohn, 2003). Here, the default barrier consists of the sum of total current liabilities and a fraction of long-term liabilities, where the fraction coefficient is usually 0.5. Some researches tried to test the impact of the coefficient value on the model accuracy (Afik, Arad, Galil, 2012), which proved to be non-significant.

Eom et al (2004) explains the use of book value of liabilities instead of the face value of the bond. The reason is that in most structural models, equity holders earn the residual value of the firm after all debt (par bond value and other debt) is paid. As a majority of firms has several sources of debt, which is especially true for the companies in our sample, we consider the sum of the book value of total liabilities and accrued interest and dividend is the correct measure of the default boundary.

4.2.3. Risk-free rate

We usually assume that yields of the government treasuries with various maturities are the best proxy for the risk-free rate. However, government bonds are not completely without risk but are considered as the asset class with the lowest risk. This is especially true for the US bonds which are considered as ones of the safest. However, government bonds are unique in certain aspects. Prices of government bonds include a liquidity premium and sometimes reflect special tax attributes (exemption from state taxes, etc.). In fact, the yield on a default-free bond is unlikely to be the government yield curve, and may not be directly observable (Blanco et al. 2005). Houweling and Vorst (2002) estimated a credit swap pricing model and found that, empirically, credit swap premiums are more related to the interest rate swap curve than to the government yield curve. Thus, some researchers choose to use the bank swap rate or swap rate less 20-30 bps as a risk-free rate, but this does not seem to impact the credit model results significantly, therefore we agreed to use the government bond yield curve as a source for our risk-free rate.

4.2.3. Recovery rate

The recovery rate is the amount the debt-holders receive as a fraction of what they are owed. Initially, we planned to estimate it empirically using a set of initially defaulted firms. The model is based on a multivariate OLS regression, which incorporates industry-, issuer-, as well as bond-specific information. This approach was taken from our colleagues, master students in NHH (master thesis, Ytterdal and Knappskog, 2015), however, with data used (Scandinavian market) they failed to find the model with a good explanatory and predictive power. Explanatory variables in this model are certain ratios from financial analysis (equity ratio, receivables, long term debt, intangibles, profitability ratios), distance to default and dummy variable (industry). As in Ytterdal and Knappskog study, our preliminary estimates also did not yield any good results.

Another way is to apply the convention among the researchers in this field, 51.31%, as a recovery rate. However, since we deal with S&P500 constituents, most likely those companies` debts have the investment grade ratings and recovery rate of nearly 50% for them would be too low. Therefore, we decided to try another approach: implied by model expected recovery rate conditional on default (McDonald, 2014):

$$E^*(A_T | A_T < L + D + I) = A_t e^{r(T-t)} \frac{N\left[-\frac{ln\left(\frac{A_t}{L+D+I}\right)+(r+0.5\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}\right]}{N\left[-\frac{ln\left(\frac{A_t}{L+D+I}\right)+(r-0.5\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}\right]}, \tag{4.9}$$

where $L + D + I$ – our default barrier and r – risk free rate (replaced asset drift rate which is used to provide estimate of the empirically observed measure, but not appropriate for pricing).

4.2.4. Credit spread

The credit spread is the difference between the yield to maturity on a defaultable bond and an equivalent default-free bond. Let's denote promised payment $L + D + I$ as $\bar{B}$ and B_T as market value of bond at time T. Market value of bond today is:

$$B_0 = e^{-rT}\{E^*(A_T|Default) \cdot Pr^*(Default) + \bar{B} \cdot (1 - Pr^*(Default))\}, \quad (4.10)$$

where $E^*(A_T|Default) \cdot Pr^*(Default)$ – partial expectation of asset value conditional on bankruptcy and $\bar{B} \cdot (1 - Pr^*(Default))$ – promised bond payment assuming that company is not bankrupt.

As $B_T = A_T$ in default:

$$B_0 = e^{-rT}\{E^*(B_T|Default) \cdot Pr^*(Default) + \bar{B} \cdot (1 - Pr^*(Default))\} \quad (4.11)$$

Assume $Pr^*(Default) = 0$ market value of bond today is $B_0 = e^{-rT}\bar{B}$, which leads to the annual yield to maturity on the bond:

$$y = \frac{1}{T} ln\left(\frac{\bar{B}}{B_0}\right) \quad (4.12)$$

From equation (4.10) one can express expected recovery rate as:

$$E^*(Recovery\ rate) = \frac{E^*(B_T|Default)}{\bar{B}} \quad (4.13)$$

And the loss given default (which is the difference between what bondholders are owed and what they receive, as a fraction of promised payment) as:

$$E^*(Loss\ given\ default) = 1 - E^*(Recovery\ rate) \quad (4.14)$$

After some arrangements, using equations (4.12) - (4.14) from equation (4.11) we obtain an expression for the credit spread:

$$y - r = \frac{1}{T} ln\left[\frac{1}{1 - Pr^*(Default) \cdot E^*(Loss\ given\ default)}\right], \quad (4.15)$$

which can be approximated by a Taylor series expansion of equation (4.15) and results in:

$$y - r \approx \frac{1}{T} Pr^*(Default) \cdot E^*(Loss\ given\ default) \quad (4.16)$$

In our calculations, we use a formula from equation (4.15).

4.3. Hidden liabilities approach

The next step in our analysis would be to search for hidden liabilities. As mentioned earlier, accounting for hidden debt changes not only the data we have for liabilities, but it affects many other elements of the balance sheet and the income statement. Which ones and how we will see as we go step by step in adjusting with each of the methods we implement.

The easiness of implementing one method or the other varies from company to company. For instance, a company that has ownership in a number of other small companies would attract our interest in finding out how much ownership does it have on each of them and how does it account for it. In the same way, a company that operates globally and has a lot of chains might find a way not to account as capital leases for the proprieties it uses to run the business. Likewise, a company that uses and rents large machinery to operate, might always consider them as operating leases even in cases when the renting period or renting fee makes us believe that they should be recognized on the balance sheet.

In order to give voice to all of these issues, let's go through each of the methods that we are going to implement one by one.

4.3.1. Equity Method

In the previous chapter, we distinguished between the equity method and the consolidation method and presented some consequences of using the former instead of the latter. Here, we will go step by step through the method to adjust the financial data of an affiliate, that has been previously registered in the financial books with the equity method.

We first make sure that the control the investor has over the investee is substantial and impactful in the life of the investor. This can be made sure, by researching for both companies. Their financial reports are a way to check for information regarding operational control. However, individual research is necessary to confirm the presumed control.

We then go through the financial reports of both companies to get the data from the balance sheets and income statements. If we don't have fair values of the assets and liabilities of the investee, we assume that the difference between the fair value and the book value has been completely depreciated. To make this assumption more reasonable, we experiment with companies that have been on the market for a considerate amount of time. On the other hand, Goodwill will be the difference between the carrying amount in the investments account and the book values of the net assets. This information can be easily found on the investor's financial reports.

These assumptions being made, we continue on to consolidation. We first exclude from the parent's balance sheet the investment on the affiliate. That amount is a proportionate of the book net assets of the affiliate. On the left-hand side, we add the goodwill. On the right side of the balance sheet, we should add the minority interest, as part of the liabilities. The minority interest represents the equity interest in the affiliate by its other shareholders. The last step would be to add up all other voices of the balance sheet and income statement.

4.3.2. Lease Accounting

In order to adjust for the leases, we need to look carefully for notes that show the true nature of the leases. Entities that have operating leases are legally required (by FASB and SEC) to disclose information about those leases. Our goal is to search for capital leases that might have been tricked to be disclosed as operating leases. After finding out these leases we can make assumptions about the life of the assets, tax rates and cash flow patterns to help us adjust the numbers.

The adjustment process will track the following six steps:

1. *We find the lease future cash commitments*

   Companies that have operating leases, report future cash flows related to the lease payments for the next 5 years and a cumulative sum thereafter.
2. We choose a proper discount rate

   If we know the implicit rate of return on the lease, the process is simple. If not, we need to look at debt notes in the financial report to find an appropriate interest rate. Hence, we seek to find financial risks comparable to the leasing risk, so that we can use the same interest rate associated with that debt.
3. *We compute the asset value and the lease obligation*

   Assuming that the cash flows occur at the end of each year, we discount the cash flows to the chosen rate and obtain a present value. This sum represents both the asset value and the lease obligation.

4. *We choose an appropriate life for the leased assets and estimate their present age. We then calculate the depreciation expense and accumulated depreciation.*
   First, we assume a remaining life of the leased asset, being consistent with the firm's depreciation policy and by following a straight-straight line depreciation, we calculate an annual amount. Information about the cost of the property/equipment already on the books and their depreciation will help to estimate an average age of the assets. This last number will serve in finding the accumulated depreciation of the leased asset.

At this point, we start adjusting by taking out the rental expense and adding in the annual depreciation expense in the income statement.

5. *We estimate the interest expense and add it back to the income statement*
   Since the beginning balance in leases is mostly unknown, we shall assume that all the leases were active at the beginning of the year and will continue to be after this year. We conduct the following simple computation to estimate the balance at the beginning of the year:

$$Lease\ obligation_{Beginning\ of\ year} + Interest\ rate * Lease\ obligation_{Beginning\ of\ year} - Cash\ Payment = Lease\ Obligation_{End\ of\ year}$$

The interest rate multiplied by the Lease obligation at the beginning of the year represents the interest expense. We also add this amount to the income statement.

6. *We compute the change in the income tax expense and deferred income taxes*
   From the previous steps, we calculate that the net impact on earnings before taxes is an increase by the rent expense amount and a decrease in the sum of the annual depreciation expense and the interest expense. We employ to the net earnings the tax rate found on the tax notes and thus receive the change in the income tax expense, which is transferred as a change in the deferred income taxes as well. Moreover, the increase of the old depreciation also produces a change in the deferred income taxes.

4.3.3 Pension Accounting

In the previous chapter, we explained that it is not accurate with respect to readers of the financial reports to net the projected benefit obligation against the pension plan assets and to not include the prior service cost in the pension cost and the projected benefit obligation. Therefore, in order to start the adjustment process, we have to unnet the projected benefit obligation and the pension assets. The amount already injected in the balance sheet equals the difference between these two, we don't have to worry about balancing the balance sheet. We place the assets on the left side of the balance sheet and the projected benefit obligation in the right side of the balance sheet under liabilities.

A second adjustment would entail putting all the unrecognized prior service cost and other unrecognized items into the pension expense or the projected benefit obligation.

It has to be noted that before engaging in these adjustment steps we have to take a better look at the interest rates used to calculate some important elements of the pension plan. The interest rate on the projected benefit obligation used to compute the interest cost and present value of the firm's cash promises needs to accurately constitute a rate that a third party would charge the company to settle the pension debt. Using this logic, we might assume an interest rate that can be different from what is stated. Managers tend to overstate the interest rate so that they can lower the liabilities. To overcome this misrepresentation, we must compute the annual cash flow by the manager's rate and then divide by "our" interest rate to obtain a more appropriate projected benefit obligation. The effect in the liabilities, even for small changes in the interest rate, will be substantial. The same should be performed with the interest cost.

5. DATA

Due to the data accessibility issue, we decided to proceed with listed large capital companies which comprise the US market index S&P500. In our opinion, it is easier to search for the substantial hidden liabilities, which may influence the credit risk pricing, in financial reports of large and stable companies. For our task, we picked out only mature companies which were in S&P500 since 1991 (earliest available date in Datastream) until December 2016. This sorting gave us 163 active companies, which is still quite a big sample. Next, we excluded all financial companies in order to make the leverage ratios (assets over default barrier) comparable across companies. The reason is that financial firms have leverage ratios above 90%, while only the least creditworthy firms use as much debt (Lyden and Saraniti, 2000). Moreover, they have complex capital structure. The same reasoning was behind the exclusion of insurance companies. In addition, we excluded gas, oil, and electric companies, as the return on equity, revenues and the risk of default, are strongly influenced by regulators (Eom et al., 2004). Our final sample consists of 49 companies, which operate in various industries (see Appendix 1).

Since all companies in a sample have 5-year actively traded credit default swaps, to simplify the analysis, for each company in our sample we decided to calculate a 5-year credit spread based on reported and market data. Moreover, as shown by Duffie (1999) and Hull and White (2000), the five-year credit default swap spread is in theory very close to the credit spread of the yield on a five-year par yield bond issued by the reference entity over five-year par yield risk-free rate.

In order to be consistent, we agreed to take share prices and CDS contract quotes from the same source, Datastream. Since the earliest data on CDS for our sample is available starting from June 2008, to price a zero bond with 5 years to maturity we take monthly data of the previous 5 years (use a 5-year rolling window as calculation moves forward), meaning that for modeling we need data from June 2003 until December 2016. Our risk-free rate is a 5-year government bond yield and the data was taken from Macrobond.

The company financial data, namely, total liabilities, shares outstanding, interests and dividends paid, was taken from the quarterly reports through Bloomberg, Orbis, Compustat and manually for the missing companies or dates. When linking the monthly data on market equity value with quarterly liabilities, we took the most recent data, namely, the date of quarterly report release is considered a date when the financial information became available to the market. All adjustments were made based on report released dates which were taken from PI Navigator.

All modeling and data filtering are done using R software, while MS Excel is used in the intermediate stages for data preparation.

After carefully choosing seven companies among the original set of companies, we gathered for each company quarterly financial reports (both 10-Q and 10-K) through PI Navigator. To make the necessary adjustments, we went through the information provided not only in the main notes of the reports but also in the footnotes. This process was concentrated rather in what was disclosed and written on the reports than in the numeric values. The main sectors of the financial reports that we used were the notes about Benefit plans, the notes describing the general business and subsidiaries and the notes about the long-term debt. When it came to switching from equity method to the consolidate method, we again used PI Navigator to collect the financial reports of the affiliates, some of them were SEC form 20-F. Additionally, since the quality of some disclosures was lacking, we performed some individual research on subsidiaries and acquisitions and the type of leases companies of the same industry usually use.

In some of the cases, we had to use the proper exchange rate to get all the data in US dollars. All the adjustments were still performed in MS Excel, using simple computations and formulas. The changes were added or subtracted from the previous values we had for the financial voices at stake.

6. RESULTS AND ANALYSIS

In our analysis, we will follow a three-step process:

1. Prove that the credit spread puzzle exists and is substantial in different industries
2. Try to find hidden liabilities and adjust the data accordingly
3. Implement the model again using the updated inputs and compare results

6.1. Is there a room for the credit spread puzzle?

We ran our augmented version of Merton model with all 49 companies (see Appendix 1 for company details), got credit spread estimates and compared them to the actual ones. As numerous studies before us showed, Merton model in most cases severely underestimates the credit spread. However, there are several explanations of why it may be the case. First of all, Merton model is not the best representation of real world, since the companies do not actually issue only zero coupon bonds. As a matter of fact, there is no a single company from our set, which issued a zero bond during the period from June 2003 until December 2016. Moreover, a number of other factors besides the asset value influence a company`s decision to default on its obligation. Finally, the credit spread we retrieve from the model is based on the difference between yields on zero bonds, while the CDS spread is the spread between the yields on par yield bonds (Hull 2004, 2002).

As expected, the selected companies have high credit ratings, and this is supported by the model implied low annual probabilities of default and high recovery rates (see Appendix 2).

To start the analysis, we regressed the actual spread on the modelled one. The average results across all companies are shown in a table below (t-value in brackets).

Table 6.1.1: CDS on model spread regress

Intercept, basis points	Slope, basis points	R2, %
46.783 (25.582)	107.963 (6.184)	29.966

It was decided to exclude Colgate Palmolive Co. as its regression results would strongly distort the average (see Appendix 3 for details). The results reveal a positive relationship between observed CDS spreads and implied by the model. Almost for all companies the slope and most of the time the intercept are statistically significant.

Table 6.2 presents a brief summary of the modelled and the actual spreads, both measured in basis points. The average modelled spread mean is nearly 60% lower than the actual spread mean. And this is in line with our expectations for this slightly extended Merton model. Our predicted spread has smaller range of spreads, varying from minimum nearly 0 to maximum 127.4. The interesting is that actual mean standard deviation is twice higher than modelled one. Looking closely at firm-by-firm results shows that in many cases the model predicted nearly 0 spread which is quite far from reality (see Appendix 4 for details). This is precisely what we are looking for. Hypothetically, the hidden liabilities may be one of the possible causes of such severely under prediction in credit pricing.

Table 6.1.2.: Model vs. Actual credit spread

Results	Mean	Standard deviation	Max	Min
Model	29.361	32.851	127.352	0.089
Actual	86.273	67.065	430.826	33.511

Next, we analyzed the mispricing between the model and the actual spreads. We rely on both absolute and relative measures to analyze the mispricing, using similar definitions as Sæbø (2014). Thus, the absolute mispricing is the absolute difference between model and actual spread measured in basis points, while the relative mispricing is calculated as:

$$Relative\ mispricing = 1 - \frac{Model\ spread}{Actual\ spread}$$

Mispricing analysis results are in the table below.

Table 6.1.3.: Mispricing results

Results	Mean	Standard deviation	Max	Min
Absolute mispricing, bp	63.690	62.559	379.835	17.193
Relative mispricing, %	68.371	35.727	99.901	-26.416

As can be seen, both measures again point out at the strong credit spread mispricing we got from the model. Looking at the relative mispricing measure, it turned out that on average only 100 – 68.371 = 31.629% of the credit spread calculations is explained by the default risk based on our model. Negative mispricing measure indicates that in some rare cases the Merton model overprices the credit spread. For more details see Appendix 5.

6.2. "The chosen" ones

After careful investigation, where not only our preliminary analysis of the model output but also the difficulty of hidden liability extraction influenced our choice, we decided to proceed with the following companies: Coca-Cola Company, Ford Motor Company, IBM corporation, MC Donald's corporation, Norfolk Southern Corp, CVS health and 3M company.

The main goal of the choice was to have a set of companies representing different industries. Considering that managers in similar industries might follow same patterns and strategies when engaging in the process of hiding liabilities, we assumed that one representative from each industry would be sufficient to reveal the effect of hidden debt on the credit spread. The comparability in the patterns can mostly be observed when using the lease accounting, since companies in the same industry have similar leasing approach. The depicted industries lie in the Beverage sector, Automotive manufacturing, Information technology, Restaurant chains, Transportation and Retail industry. The last company is a conglomerate in the chemicals, rubber, plastics and non-metallic products. Since the industry and the characteristics of each company shape their operations and therefore their accountancy and financial reports, they behave differently from each other. Therefore, we didn't perform all the three techniques on all the chosen companies. The table below attaches each company to all the adjustment methods we performed on it:

Company	From equity method to consolidation	Lease accounting	Pension Accounting
Coca-Cola company	√		√
Ford Motor company		√	√
IBM corporation			√
MC Donald's corporation		√	
Norfolk Southern corporation			√
CVS health		√	
3M company		√	√

Table 6.2.1: Adjusting methods applied on the set of chosen companies

Insights from pension accounting weren't part of the pickup decision since the companies we have, are all big corporations with a lifelong of activity and all do have pension plans. The long-term existence of these companies makes the value of the pension plans very large and substantial in firms' balance sheets. Thus, the adjustments, especially due to unnetting will be large and will have a considerable effect on the credit spread results. Moreover, large prior service cost can be anticipated when thinking about the life of these companies.

Let's take here, for instance, Ford Motor company. Only by unneting and adding the appropriate amount to the left side of the balance sheet, we get values of the liabilities that are around 20% higher than the previous ones. When it comes to IBM the projected benefit obligation added to the balance sheet, is as large as half the original value of the liabilities. Such great amount must be due to the large number of employees these companies have and the time they started the retirement benefit plan. Additionally, in the case of Coca-Cola, we performed the same thing with other benefits and postretirement benefits, which constituted a considerable value on its own.

Before adjusting pension-wise, we read through all the methods these companies used to calculate service cost and interest cost, and we found alterations. Most of them didn't allow us to make adjustments in this aspect. Moreover, we kept track of the laws and regulations and performed accordingly. It has to be noticed that after 2005, there was an amendment in recognizing the prior service cost.

In order to make use of the equity method approach, the choice among companies was also affected by the information we obtained when researching about the subsidiaries, acquisitions, and partnerships of the given companies. What made the choice attractive, was the ownership stake that our companies had on other companies. Critical stakes make a good starting point in establishing whether the equity method is the right approach or one needs to switch to the consolidation method instead. However, during the research phase, we encountered some difficulties: On one hand, a lot of investees, which our companies had considerable control of, were not listed firms, and therefore we couldn't find the necessary data for consolidation. We are talking here for instance, about Conrail Inc. and TTX Company, where Norfolk Southern Corp has both a very large investment in and control over. On the other hand, some of the investments in affiliates were immaterial and made it difficult for us to decide whether the consolidation was sensible or not. This particular scenario was observed in both the affiliates of CVS Health Corporation.

Due to these issues, we ended up applying the consolidation method only on the Coca-Cola company. There are some investments on affiliates that the company has total control of, even though the ownership on these affiliates is less than 50%. Following the rules, Coca-Cola has the right to apply the equity method on these affiliates. However, following a stricter logic, when it comes to Coca-Cola creditors, what is under firm's control has a very large impact on the firm's health. That is why we decided to engage here in an analytical adjustment process and consolidate Coca-Cola FEMSA with the parenting company, even though the disclosed stake in the subsidiary is only 28%. The results show that Coca-Cola company places a considerate amount of the liabilities with the affiliates. In most of the years, the added amount is one-third of the previous long term debt. We can anticipate that such change might reflect a substantial impact on our model, or at least on Coke's financial ratios.

The table below shows how some important financial ratios change when shifting from equity method to consolidation method. It is hard not to notice the huge discrepancies and the negative effect on the ratios. These ratios serve as indicators of the firm's health in the eyes of creditors and investors and anticipate some changes in the credit spreads. We chose to display only the ratios at the end of 2016, but the situation persists with the same magnitude also for the previous years. As we can see, the ratios increase after adjustments, raising a question in the ability to cover the liabilities.

Table 6.2.2: Financial ratios of Coca-Cola company before and after adjustments (year end 2016)

Financial ratios	Equity Method	Consolidated
LTD to Equity	1,62	1,95
LTD to assets	0,43	0,48
Interest expense to LTD	19,54	24,16

In choosing among the set of the companies, the intuition behind the lease accounting had a big impact. McDonald's in the restaurant industry, presented a good case to be studied in lease terms since it has a lot of chains all around US and world. It literally needs at least one place to run the business on each of the cities it is operating and usually, these places are rented rather than bought. Usually, experience and success show that wherever McDonald's opens a new restaurant, it stays and will continue to remain there for a long history of years. We, therefore, assumed that the company might recognize some of the capital leases as operating leases and we performed the adjustment on the operating leases dedicated to restaurants. On the other side, CVS health as a retail firm displayed similar characteristics due to its market expansion and the large number of stores it makes use of in its activity. Again, we observe added amounts of interest expense that are as much as 10% of the original values of interest expense.

Finally, to sum up, appendix 6 shows how much liabilities and interest expense of the chosen companies changed when applying each method and the methods altogether compared to the reported values. The table is build up on the data we received for the year end 2016.

6.3. Impact of the hidden liabilities inclusion

After rerunning the model again with the modified data, we obtain new output for the credit spreads. In the table below we compare these spreads to the ones we got from the first step.

Table 6.3.1: Model vs. Actual spread: before and after

#	Company	Ticker	mean PD, %		mean RR, %		mean spread model, bp		mean spread actual, bp
			Before	After	Before	After	Before	After	
1	CVS HEALTH CORP	CVS	1.426	1.666	82.914	84.655	41.768	41.301	48.906
2	COCA-COLA COMPANY	KO	0.001	0.030	93.496	91.966	0.012	0.286	38.097
3	FORD MOTOR CO	F	4.253	9.636	89.247	80.080	54.068	295.902	748.316
4	INTERNATIONAL BUSINESS MACHINES CORP	IBM	0.012	0.077	93.280	93.399	0.101	0.583	45.116
5	NORFOLK SOUTHERN CORP	NSC	0.561	1.277	88.402	87.277	8.035	19.551	40.675
6	MCDONALD'S CORPORATION	MCD	0.000	0.006	94.607	93.810	0.001	0.046	32.635
7	3M COMPANY	MMM	0.004	0.079	92.006	91.299	0.037	0.809	31.760

As we can see, recalculation of liabilities together with interest payments positively affected our model spread results – for all our selected companies the credit spread increased (CVS Health Corp showed a slight drop in the modelled credit spread). However, compared to the actual CDS quotes, calculated credit spread is still far from reality.

To check whether we still have a relationship between the model output and real quotes, we again ran simple linear regressions. It turned out that for Ford Motor Co the slope coefficient became insignificant, which eventually resulted in near 0 the goodness-of-fit measurement, R^2, indicating that the model explains none of the variability of the response data around its mean. While R^2 provides an estimate of the strength of the relationship between the model and the response variable, it does not provide a formal hypothesis test for this relationship. The F-test of overall significance determines whether this relationship is statistically significant. However, the p-value for F-test is 0.4836, meaning that our model does not provide a better fit than the intercept-only model. However, neither R^2 nor F-test doesn't tell us the entire story. Thus, we decided not to dive in details with Ford, since for all other companies both intercept and slope are statistically significant. Therefore, we agreed that there is a positive relationship between the model output and the real credit data. In the table below are shown constant and slope coefficients and R^2 (mean) of 6 out of 7 companies (Ford Motor Co is excluded). For more details see Appendix 6.

Table 6.3.2: Regression results: before and after

Results	Intercept, basis points	Slope, basis points	R2, %
Before	34.375 (25.623)	273.347 (6.918)	30.323
After	32.706 (23.108)	13.493 (7.642)	33.133

Finally, we also compared our results before and after considering extra liabilities using mispricing measures.

Table 6.3.3: Absolute and Relative mispricing: before and after

#	Company	Ticker	Absolute mispricing, bp		Relative mispricing, %	
			Before	After	Before	After
1	CVS HEALTH CORP	CVS	33.945	7.605	34.232	15.551
2	COCA-COLA COMPANY	KO	38.085	37.811	99.972	99.250
3	FORD MOTOR CO	F	694.247	452.414	86.518	60.458
4	INTERNATIONAL BUSINESS MACHINES CORP	IBM	45.015	44.534	99.839	98.709
5	NORFOLK SOUTHERN CORP	NSC	32.640	21.124	81.921	51.933
6	MCDONALD'S CORPORATION	MCD	32.634	32.589	99.998	99.859
7	3M COMPANY	MMM	31.723	30.952	99.916	97.454

Both absolute and relative mispricing measures decreased for all companies: for some significantly (CVS Health Corp, Ford Motor Co, Norfolk Southern Corp), and for others just barely sensible (McDonalds Corp). Therefore, we can conclude that even though the inclusion of hidden in financial reports liabilities really influences the credit risk pricing, there are also some other factors responsible for the correct corporate debt pricing.

6.4. Conclusion

To test our hypothesis about the impact of the hidden liabilities on the credit risk pricing, we have implemented the original Merton model with a slight extension: we included accrued interest and dividend payments over the next 5 years from the pricing date. First, we calculated the credit spread for 49 companies in our sample and compared it to the quoted CDS data. In line with the theory, the tested version of Merton model indeed underestimates the credit risk. Therefore, we moved to the next step where we searched for the hidden liabilities in the financial reports of 7 selected companies. And finally, we reran our model and found that even though reassessed liabilities and consequently interest payments do really narrow the gap between the real quoted data and the model, they cannot fully explain a pricing difference.

There may be numerous reasons for this: the choice of the model (Merton model has a lot of assumptions which contradicts the reality), convenient model assumptions (bond maturity is set to 5 years for all companies, no default before maturity, neglected issued debt - promised payment is total liabilities and coupon is interest payment, etc.), hidden liabilities processing complexity which resulted in strong assumptions from our side, etc. Therefore, one way to improve the results is to test the hypothesis on a more complex model, which allows for default before maturity (for example, Eom et al, 2004) or uses estimates of the model's parameters from the implied volatilities of options on the company's equity (Hull et al, 2004), or any other known model. Here, when assuming in the model the default before maturity one should take care of recovery rate and how to model it in a proper way. The other way is to improve the selection of the companies in a sample: as Eom suggests, to take only companies with the simple capital structure and having no more than 2 bonds outstanding. Moreover, in terms of hidden liabilities, the research can be extended and become more impactful if more information would have been available. Researchers on the field, who can have access to additional firm documents, can broaden the hidden debt adjustment horizon in the application of asset securitization, SPE borrowings, and synthetic leases. Even on the set of accounting methods that we analyzed there is still a lot more to do. An adjustment of interest rates or discounting rates needs not only more expertise on the field, but also more information in terms of comparable debts. Also, some changes in financial voices of the firms might come from different causes rather than to appear better on the ratings. Therefore, for instance, some attention should go also in the difference among trading and available-for-sale securities and the equity method.

And finally, the difference in credit spread may not be caused solely by the credit risk mispricing. Although Longstaff et al. (2005) examined the components of credit spreads and found that credit risk accounts for the majority of the spread, there might be other sources as well. Hull et al. (2012) concludes that part of the puzzle may be explained by compensation for bearing non-default related risk factors. Apart from the defaultable risk, the most common factors examined by researchers are tax premium, liquidity premium and risk premium. Tax premium comes from the different taxation regimes, which apply to the corporate or the government debt, and are quite diverse especially in the US. Liquidity premium comes from the fact that part of the corporate bond market suffers from low trading volumes, which leads to higher and more volatile bid ask spreads. Risk premium comes from the bond's sensitivity to systematic risk factors. For example, as the quality of the bond deteriorates, it resembles the equity features, meaning that its price will be more affected by the same market factors as the equity. Thus, the investors require a compensation for the equity-like non-diversifiable risks.

To sum up, our model and assumptions did not yield immediately prominent results. Nevertheless, our work opens a potentially vast research area in the credit risk pricing where reassessing of the company's liabilities could help to explain the defaultable part of the credit spread.

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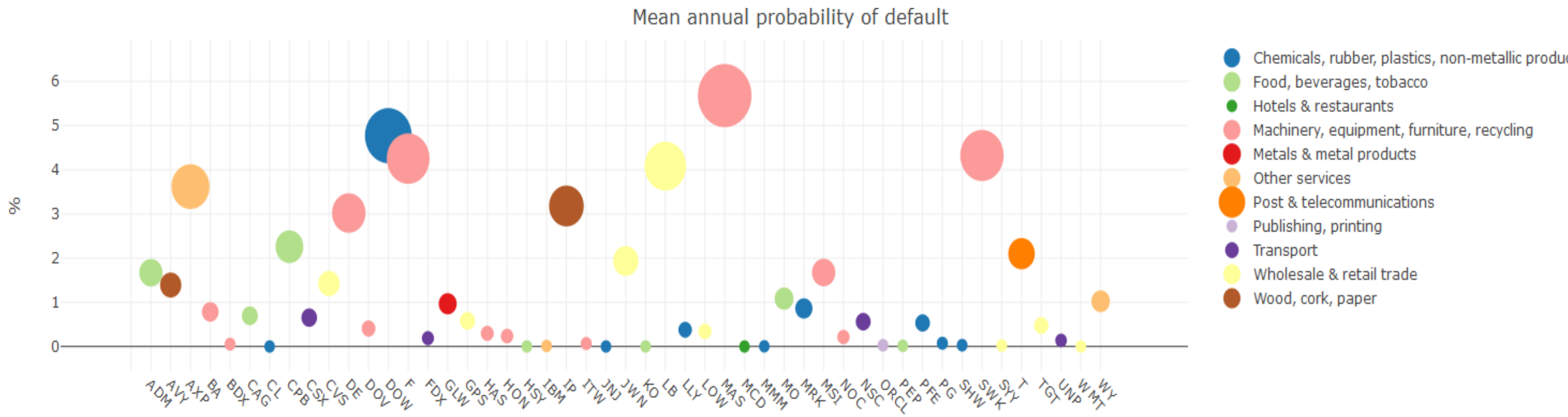
APPENDIX 1. Final sample

#	Full name	Ticker	Major industry sector	Credit rating 2016[1]
1	AT&T INC.	T	Post & telecommunications	BBB+
2	ALTRIA GROUP, INC.	MO	Food, beverages, tobacco	A-
3	AMERICAN EXPRESS COMPANY	AXP	Other services	BBB +
4	ARCHER-DANIELS-MIDLAND COMPANY	ADM	Food, beverages, tobacco	A
5	AVERY DENNISON CORP	AVY	Wood, cork, paper	BBB
6	BECTON, DICKINSON AND COMPANY	BDX	Machinery, equipment, furniture, recycling	BBB+
7	BOEING COMPANY (THE)	BA	Machinery, equipment, furniture, recycling	A
8	CSX CORP	CSX	Transport	BBB+
9	CVS HEALTH CORP	CVS	Wholesale & retail trade	BBB+
10	CAMPBELL SOUP CO	CPB	Food, beverages, tobacco	BBB+
11	COCA-COLA COMPANY (THE)	KO	Food, beverages, tobacco	AA-
12	COLGATE PALMOLIVE CO	CL	Chemicals, rubber, plastics, non-metallic products	AA-
13	CONAGRA BRANDS, INC.	CAG	Food, beverages, tobacco	BBB
14	CORNING INC	GLW	Metals & metal products	BBB+
15	DEERE & CO	DE	Machinery, equipment, furniture, recycling	A
16	DOVER CORP	DOV	Machinery, equipment, furniture, recycling	A-
17	DOW CHEMICAL COMPANY (THE)	DOW	Chemicals, rubber, plastics, non-metallic products	BBB
18	FEDEX CORP	FDX	Transport	BBB
19	FORD MOTOR CO	F	Machinery, equipment, furniture, recycling	BBB
20	GAP INC	GPS	Wholesale & retail trade	BB+
21	HASBRO INC	HAS	Machinery, equipment, furniture, recycling	BBB
22	HERSHEY COMPANY (THE)	HSY	Food, beverages, tobacco	A
23	HONEYWELL INTERNATIONAL INC	HON	Machinery, equipment, furniture, recycling	A
24	ILLINOIS TOOL WORKS INC	ITW	Machinery, equipment, furniture, recycling	A+
25	INTERNATIONAL BUSINESS MACHINES CORP	IBM	Other services	AA-
26	INTERNATIONAL PAPER CO	IP	Wood, cork, paper	BBB
27	JOHNSON & JOHNSON	JNJ	Chemicals, rubber, plastics, non-metallic products	AAA
28	PROCTER & GAMBLE CO	PG	Chemicals, rubber, plastics, non-metallic products	AA-
29	PFIZER INC	PFE	Chemicals, rubber, plastics, non-metallic products	AA
30	PEPSICO INC	PEP	Food, beverages, tobacco	A
31	ORACLE CORP	ORCL	Publishing, printing	AA-
32	NORTHROP GRUMMAN CORPORATION	NOC	Machinery, equipment, furniture, recycling	BBB+
33	NORFOLK SOUTHERN CORP	NSC	Transport	BBB+
34	NORDSTROM INC	JWN	Wholesale & retail trade	BBB+
35	MOTOROLA SOLUTIONS, INC.	MSI	Machinery, equipment, furniture, recycling	BBB-
36	MERCK & CO., INC.	MRK	Chemicals, rubber, plastics, non-metallic products	AA
37	MCDONALD'S CORPORATION	MCD	Hotels & restaurants	BBB+
38	MASCO CORP	MAS	Machinery, equipment, furniture, recycling	BBB
39	LOWE'S COMPANIES, INC.	LOW	Wholesale & retail trade	A-
40	ELI LILLY AND COMPANY	LLY	Chemicals, rubber, plastics, non-metallic products	AA-
41	L BRANDS, INC.	LB	Wholesale & retail trade	BB+
42	WEYERHAEUSER CO	WY	Other services	BBB-
43	WAL-MART STORES, INC.	WMT	Wholesale & retail trade	AA
44	UNION PACIFIC CORP	UNP	Transport	A
45	3M COMPANY	MMM	Chemicals, rubber, plastics, non-metallic products	AA-
46	TARGET CORP	TGT	Wholesale & retail trade	A
47	SYSCO CORP	SYY	Wholesale & retail trade	BBB+
48	STANLEY BLACK & DECKER, INC.	SWK	Machinery, equipment, furniture, recycling	A
49	SHERWIN-WILLIAMS CO	SHW	Chemicals, rubber, plastics, non-metallic products	A

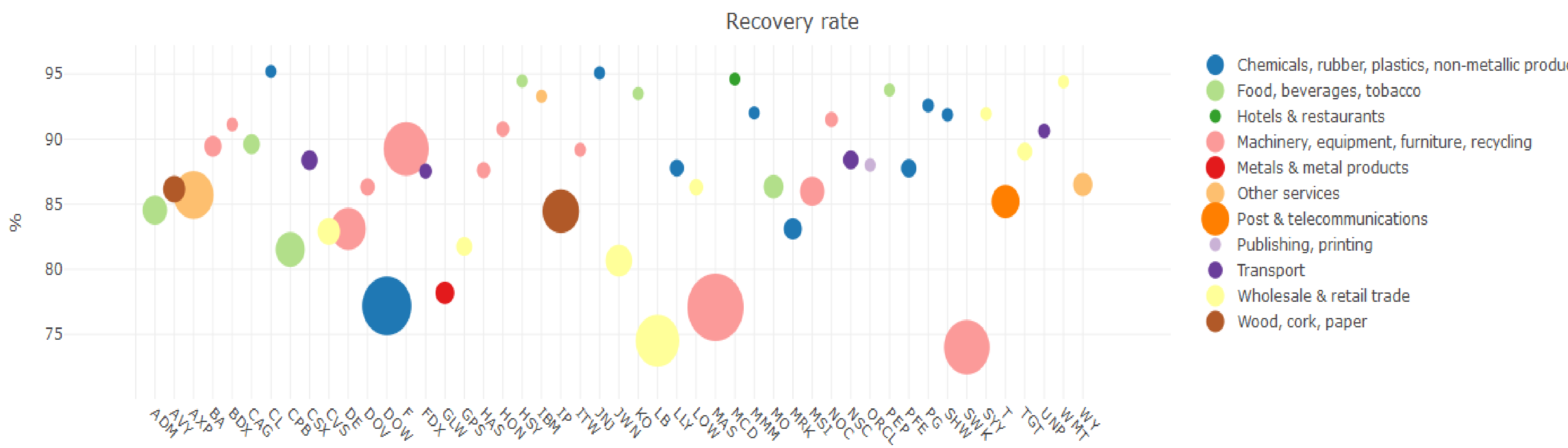
[1] Credit rating is the S&P domestic long term issuer credit rating, taken from Warton research data services (Compustat, Dec. 2016). Bonds are divided into two main credit risk categories, investment grade (low risk) and speculative grade (high risk). Investment grade is defined as bonds with a credit rating of BBB- or higher, while high yield is bonds with credit rating BB+ or lower (S&P).

APPENDIX 2. Probability of default (PD, %) and recovery rate (RR, %)

#	Full name	Ticker	mean PD	std PD	max PD	min PD	mean RR	std RR	max RR	min RR
1	AT&T INC.	T	2.101	2.502	6.907	0.005	85.213	10.457	95.477	70.407
2	ALTRIA GROUP, INC.	MO	1.083	1.252	4.759	0.000	86.347	7.318	96.016	75.165
3	AMERICAN EXPRESS COMPANY	AXP	3.619	4.238	20.076	0.006	85.709	8.615	95.546	66.003
4	ARCHER-DANIELS-MIDLAND COMPANY	ADM	1.669	1.371	4.648	0.040	84.542	4.497	92.580	77.918
5	AVERY DENNISON CORP	AVY	1.391	1.435	6.941	0.000	86.165	5.259	95.070	78.155
6	BECTON, DICKINSON AND COMPANY	BDX	0.053	0.099	0.452	0.000	91.114	2.358	95.822	86.566
7	BOEING COMPANY (THE)	BA	0.783	0.755	3.689	0.004	89.446	2.914	94.307	84.673
8	CSX CORP	CSX	0.653	0.700	3.916	0.002	88.377	3.345	93.820	82.781
9	CVS HEALTH CORP	CVS	1.426	1.651	4.979	0.000	82.914	10.972	95.481	67.918
10	CAMPBELL SOUP CO	CPB	2.258	3.207	8.422	0.000	81.526	16.518	94.623	55.966
11	COCA-COLA COMPANY (THE)	KO	0.001	0.002	0.008	0.000	93.496	1.553	96.151	90.965
12	COLGATE PALMOLIVE CO	CL	0.000	0.000	0.000	0.000	95.195	1.362	97.373	93.016
13	CONAGRA BRANDS, INC.	CAG	0.695	0.955	3.392	0.004	89.607	4.054	94.916	81.229
14	CORNING INC	GLW	0.967	0.874	3.133	0.011	78.195	5.921	88.690	70.222
15	DEERE & CO	DE	3.021	1.999	7.085	0.014	83.106	6.412	94.790	74.266
16	DOVER CORP	DOV	0.406	0.403	1.990	0.010	86.329	2.766	92.110	80.712
17	DOW CHEMICAL COMPANY (THE)	DOW	4.772	4.043	14.389	0.014	77.195	11.194	93.281	59.923
18	FEDEX CORP	FDX	0.188	0.207	0.824	0.000	87.541	2.696	93.120	82.511
19	FORD MOTOR CO	F	4.253	3.963	19.629	0.419	89.247	3.212	93.915	83.063
20	GAP INC	GPS	0.581	0.646	2.646	0.016	81.750	3.257	88.120	74.924
21	HASBRO INC	HAS	0.297	0.345	1.345	0.000	87.611	3.385	93.754	82.186
22	HERSHEY COMPANY (THE)	HSY	0.001	0.002	0.015	0.000	94.457	1.210	96.653	91.950
23	HONEYWELL INTERNATIONAL INC	HON	0.236	0.308	1.776	0.000	90.769	3.436	96.864	85.784
24	ILLINOIS TOOL WORKS INC	ITW	0.067	0.089	0.435	0.000	89.180	2.869	94.974	84.605
25	INTERNATIONAL BUSINESS MACHINES CORP	IBM	0.012	0.027	0.182	0.000	93.280	1.012	94.918	90.225
26	INTERNATIONAL PAPER CO	IP	3.180	3.172	17.835	0.053	84.460	6.167	93.762	73.189
27	JOHNSON & JOHNSON	JNJ	0.000	0.000	0.001	0.000	95.079	1.297	97.483	93.104
28	PROCTER & GAMBLE CO	PG	0.073	0.147	0.783	0.000	92.579	4.592	97.185	83.044
29	PFIZER INC	PFE	0.533	0.620	2.644	0.000	87.759	4.830	95.658	79.295
30	PEPSICO INC	PEP	0.015	0.024	0.100	0.000	93.767	2.745	97.693	89.577
31	ORACLE CORP	ORCL	0.028	0.023	0.105	0.000	88.011	1.367	91.361	85.452
32	NORTHROP GRUMMAN CORPORATION	NOC	0.214	0.261	1.168	0.000	91.498	2.477	97.435	86.519
33	NORFOLK SOUTHERN CORP	NSC	0.561	0.592	3.363	0.008	88.402	2.987	93.251	81.881
34	NORDSTROM INC	JWN	1.935	1.862	9.726	0.009	80.670	6.847	92.552	68.924
35	MOTOROLA SOLUTIONS, INC.	MSI	1.675	2.629	12.419	0.009	85.995	5.415	93.121	73.029
36	MERCK & CO., INC.	MRK	0.861	0.807	2.576	0.000	83.107	7.076	95.608	74.577
37	MCDONALD'S CORPORATION	MCD	0.000	0.000	0.001	0.000	94.607	1.368	96.968	92.315
38	MASCO CORP	MAS	5.679	5.050	17.807	0.001	77.089	11.988	93.966	60.219
39	LOWE'S COMPANIES, INC.	LOW	0.340	0.416	2.079	0.004	86.307	3.201	91.338	79.291
40	ELI LILLY AND COMPANY	LLY	0.378	0.422	1.710	0.000	87.777	6.366	96.492	78.933
41	L BRANDS, INC.	LB	4.083	3.950	11.872	0.004	74.523	13.211	91.425	54.097
42	WEYERHAEUSER CO	WY	1.024	1.178	6.875	0.016	86.516	3.036	93.560	80.442
43	WAL-MART STORES, INC.	WMT	0.002	0.003	0.015	0.000	94.398	0.765	95.669	93.025
44	UNION PACIFIC CORP	UNP	0.137	0.199	1.298	0.000	90.619	2.798	95.681	86.858
45	3M COMPANY	MMM	0.004	0.009	0.076	0.000	92.006	1.584	95.089	89.727
46	TARGET CORP	TGT	0.475	0.590	3.269	0.001	89.039	3.641	94.150	82.080
47	SYSCO CORP	SYY	0.022	0.030	0.158	0.000	91.940	2.170	95.878	88.487
48	STANLEY BLACK & DECKER, INC.	SWK	4.324	3.614	10.386	0.001	74.021	13.895	94.148	58.052
49	SHERWIN-WILLIAMS CO	SHW	0.029	0.043	0.237	0.000	91.862	2.941	96.755	87.372



Y-axis: mean annual default probability, %; **X-axis:** tickers; **bubble size**: value of default probability; **bubble color:** industry company operates in



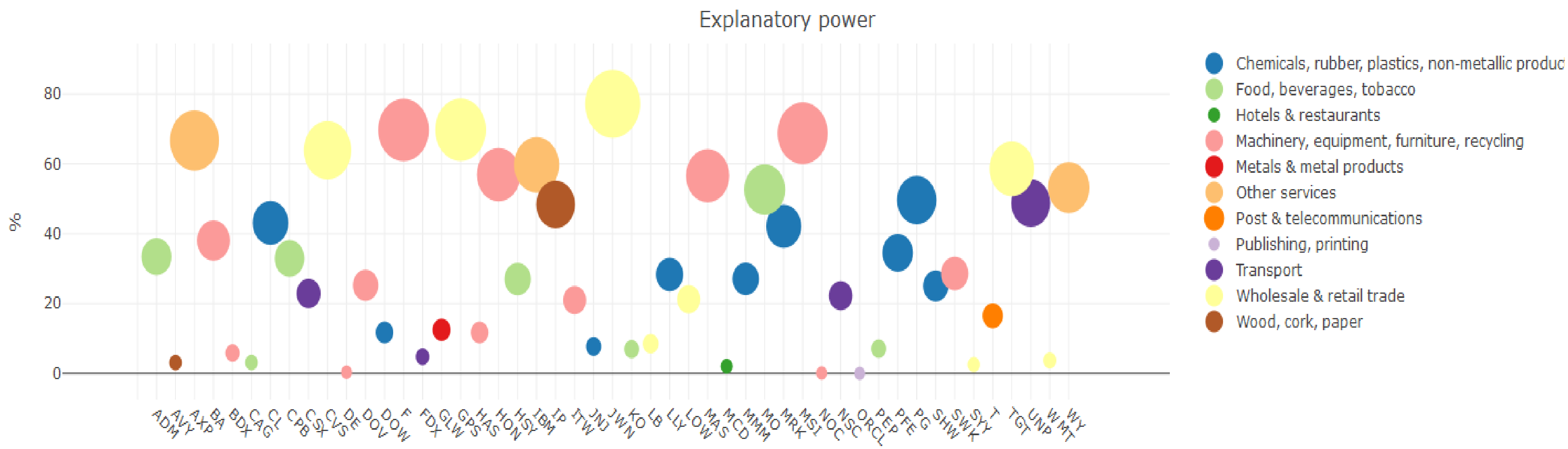
Y-axis: mean recovery rate, %; **X-axis:** tickers; **bubble size**: value of default probability; **bubble color:** industry company operates in

APPENDIX 3. Model spread vs. Actual spread

#	Full name	Ticker	mean spread model	std spread model	max spread model	min spread model	mean spread act	std spread act	max spread act	min spread act
1	AT&T INC.	T	56.585	70.123	204.393	0.025	82.730	24.624	201.662	41.050
2	ALTRIA GROUP, INC.	MO	22.804	28.903	118.197	0.000	72.587	36.927	178.982	27.270
3	AMERICAN EXPRESS COMPANY	AXP	86.003	129.789	682.546	0.028	102.223	100.202	604.800	36.400
4	ARCHER-DANIELS-MIDLAND COMPANY	ADM	31.521	29.080	101.495	0.297	69.481	27.449	212.490	40.080
5	AVERY DENNISON CORP	AVY	22.072	24.648	93.318	0.001	89.446	2.407	97.620	87.930
6	BECTON, DICKINSON AND COMPANY	BDX	0.639	1.270	6.077	0.000	78.169	14.045	93.010	30.000
7	BOEING COMPANY (THE)	BA	10.248	10.628	52.368	0.024	62.537	42.275	226.656	15.220
8	CSX CORP	CSX	9.564	11.354	67.431	0.013	53.643	34.788	182.329	16.750
9	CVS HEALTH CORP	CVS	41.768	49.480	159.744	0.000	48.906	19.307	122.772	28.540
10	CAMPBELL SOUP CO	CPB	93.716	136.053	370.863	0.000	49.654	14.408	84.360	20.130
11	COCA-COLA COMPANY (THE)	KO	0.012	0.021	0.076	0.000	38.097	13.447	89.718	21.990
12	COLGATE PALMOLIVE CO	CL	0.000	0.000	0.003	0.000	35.974	12.536	75.552	21.320
13	CONAGRA BRANDS, INC.	CAG	10.662	16.173	63.677	0.019	68.549	21.295	122.603	25.150
14	CORNING INC	GLW	25.406	25.949	93.307	0.164	96.266	66.210	415.181	40.080
15	DEERE & CO	DE	62.765	51.273	164.815	0.090	57.413	30.236	215.495	26.640
16	DOVER CORP	DOV	6.465	7.277	32.926	0.081	62.812	21.047	148.886	34.370
17	DOW CHEMICAL COMPANY (THE)	DOW	150.746	141.509	451.140	0.095	128.283	100.831	627.246	62.430
18	FEDEX CORP	FDX	2.816	3.468	14.405	0.003	81.994	35.645	253.369	42.600
19	FORD MOTOR CO	F	54.068	58.670	298.832	2.547	748.316	1555.480	10303.040	85.610
20	GAP INC	GPS	12.369	15.374	62.620	0.195	137.494	84.567	399.240	23.890
21	HASBRO INC	HAS	4.680	5.875	23.962	0.000	111.363	50.080	286.420	36.290
22	HERSHEY COMPANY (THE)	HSY	0.005	0.017	0.118	0.000	47.595	14.252	92.748	28.300
23	HONEYWELL INTERNATIONAL INC	HON	2.932	3.976	24.030	0.000	35.610	19.662	125.497	13.950
24	ILLINOIS TOOL WORKS INC	ITW	0.923	1.310	6.084	0.000	48.807	19.437	141.564	30.180
25	INTERNATIONAL BUSINESS MACHINES CORP	IBM	0.101	0.258	1.782	0.001	45.116	13.502	111.303	25.250
26	INTERNATIONAL PAPER CO	IP	65.325	74.970	425.529	0.333	141.793	110.679	737.526	57.050
27	JOHNSON & JOHNSON	JNJ	0.001	0.001	0.007	0.000	29.493	13.806	73.222	11.150
28	PROCTER & GAMBLE CO	PG	1.091	2.358	13.271	0.000	41.771	23.269	141.660	16.950
29	PFIZER INC	PFE	9.810	12.478	54.738	0.000	40.634	17.851	88.880	14.240
30	PEPSICO INC	PEP	0.141	0.244	1.037	0.000	47.345	11.622	94.440	32.706
31	ORACLE CORP	ORCL	0.354	0.312	1.350	0.001	46.279	22.288	165.971	29.610
32	NORTHROP GRUMMAN CORPORATION	NOC	2.354	3.260	15.741	0.000	52.514	6.533	80.270	30.300
33	NORFOLK SOUTHERN CORP	NSC	8.035	9.967	60.943	0.057	40.675	17.979	127.872	15.450
34	NORDSTROM INC	JWN	47.630	55.169	302.248	0.068	120.824	104.291	613.305	39.550
35	MOTOROLA SOLUTIONS, INC.	MSI	35.621	64.724	334.947	0.075	137.931	91.331	582.550	63.490
36	MERCK & CO., INC.	MRK	19.603	20.136	65.489	0.000	33.897	15.927	70.830	10.950
37	MCDONALD'S CORPORATION	MCD	0.001	0.001	0.006	0.000	32.635	10.844	56.832	14.480
38	MASCO CORP	MAS	187.197	181.378	656.858	0.005	224.757	116.532	604.986	89.840
39	LOWE'S COMPANIES, INC.	LOW	5.778	8.064	43.046	0.033	54.835	30.193	155.826	15.790
40	ELI LILLY AND COMPANY	LLY	6.916	8.212	36.031	0.000	41.665	14.605	77.090	18.473
41	L BRANDS, INC.	LB	153.277	167.105	495.070	0.034	216.957	100.536	735.929	124.480
42	WEYERHAEUSER CO	WY	16.060	19.776	122.857	0.143	127.603	58.051	291.257	39.100
43	WAL-MART STORES, INC.	WMT	0.014	0.021	0.103	0.000	40.280	19.060	110.345	13.430
44	UNION PACIFIC CORP	UNP	1.640	2.497	17.063	0.000	39.237	21.056	134.983	14.400
45	3M COMPANY	MMM	0.037	0.086	0.757	0.000	31.760	18.973	118.050	14.250
46	TARGET CORP	TGT	6.976	9.777	58.580	0.005	53.657	35.342	241.522	21.470
47	SYSCO CORP	SYY	0.230	0.329	1.814	0.000	52.956	9.591	87.867	24.590
48	STANLEY BLACK & DECKER, INC.	SWK	161.379	145.817	435.674	0.006	67.198	15.467	152.791	50.290
49	SHERWIN-WILLIAMS CO	SHW	0.329	0.519	2.890	0.000	57.601	25.701	153.941	18.570

APPENDIX 4. CDS on model spread regression results

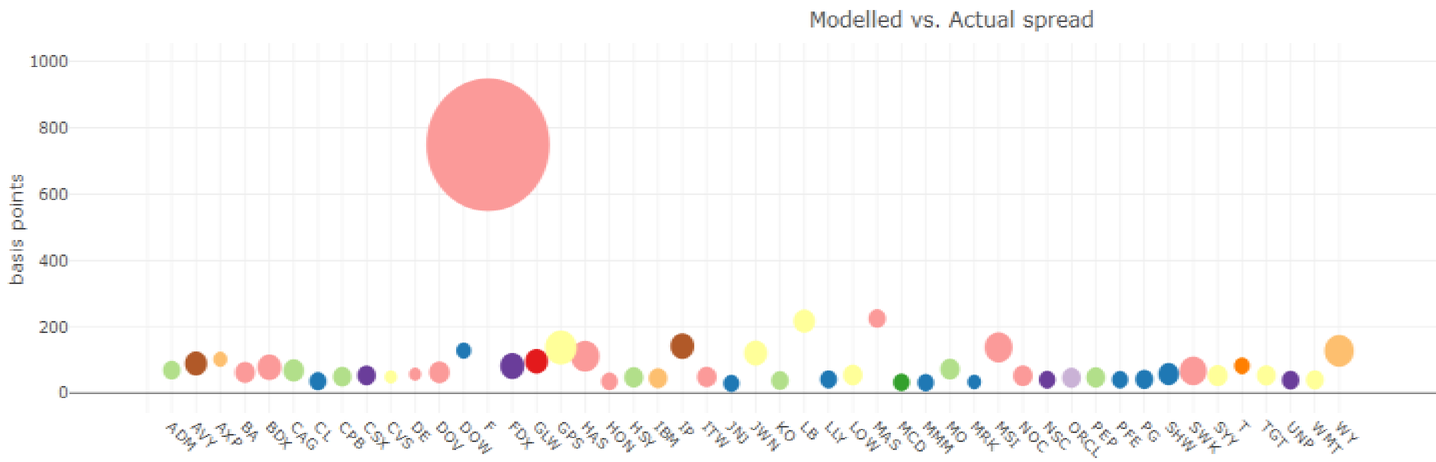
#	Full name	Ticker	Intercept	Intercept t-value	Slope	Slope t-value	R2
1	AT&T INC.	T	74.660	26.154	0.143	4.489	16.495
2	ALTRIA GROUP, INC.	MO	51.450	16.099	0.927	10.647	52.635
3	AMERICAN EXPRESS COMPANY	AXP	48.002	7.011	0.630	14.289	66.686
4	ARCHER-DANIELS-MIDLAND COMPANY	ADM	52.275	16.021	0.546	7.158	33.439
5	AVERY DENNISON CORP	AVY	89.067	260.531	0.017	1.655	3.087
6	BECTON, DICKINSON AND COMPANY	BDX	76.462	50.803	2.671	2.513	5.829
7	BOEING COMPANY (THE)	BA	37.393	8.191	2.453	7.915	38.049
8	CSX CORP	CSX	39.624	10.048	1.466	5.502	22.889
9	CVS HEALTH CORP	CVS	35.877	23.938	0.312	13.438	63.905
10	CAMPBELL SOUP CO	CPB	55.349	39.143	-0.061	-7.077	32.933
11	COCA-COLA COMPANY (THE)	KO	36.056	24.411	167.232	2.757	6.937
12	COLGATE PALMOLIVE CO	CL	32.695	32.573	25825.343	8.789	43.093
13	CONAGRA BRANDS, INC.	CAG	71.026	28.668	-0.232	-1.810	3.113
14	CORNING INC	GLW	73.375	8.569	0.901	3.812	12.469
15	DEERE & CO	DE	55.273	11.722	0.034	0.585	0.334
16	DOVER CORP	DOV	53.419	22.222	1.453	5.868	25.237
17	DOW CHEMICAL COMPANY (THE)	DOW	91.529	6.693	0.244	3.678	11.708
18	FEDEX CORP	FDX	75.662	17.105	2.249	2.264	4.786
19	FORD MOTOR CO	F	-448.578	-3.902	22.137	15.324	69.716
20	GAP INC	GPS	80.662	13.693	4.595	15.347	69.779
21	HASBRO INC	HAS	97.717	16.451	2.916	3.676	11.699
22	HERSHEY COMPANY (THE)	HSY	45.432	36.338	444.192	6.146	27.024
23	HONEYWELL INTERNATIONAL INC	HON	24.671	15.586	3.731	11.606	56.909
24	ILLINOIS TOOL WORKS INC	ITW	42.532	20.386	6.796	5.204	20.981
25	INTERNATIONAL BUSINESS MACHINES CORP	IBM	41.036	45.236	40.452	12.299	59.727
26	INTERNATIONAL PAPER CO	IP	74.720	7.174	1.027	9.776	48.371
27	JOHNSON & JOHNSON	JNJ	27.698	19.180	2805.562	2.920	7.713
28	PROCTER & GAMBLE CO	PG	34.185	19.048	6.952	10.025	49.629
29	PFIZER INC	PFE	32.387	16.404	0.841	6.735	34.529
30	PEPSICO INC	PEP	45.560	35.680	12.664	2.783	7.058
31	ORACLE CORP	ORCL	46.530	13.960	-0.709	-0.100	0.010
32	NORTHROP GRUMMAN CORPORATION	NOC	52.356	65.864	0.067	0.338	0.112
33	NORFOLK SOUTHERN CORP	NSC	33.840	16.832	0.851	5.402	22.242
34	NORDSTROM INC	JWN	41.718	6.420	1.661	18.578	77.189
35	MOTOROLA SOLUTIONS, INC.	MSI	96.247	16.748	1.170	14.988	68.773
36	MERCK & CO., INC.	MRK	23.834	14.270	0.513	8.617	42.127
37	MCDONALD'S CORPORATION	MCD	31.937	27.509	1316.464	1.456	2.037
38	MASCO CORP	MAS	134.276	12.321	0.483	11.533	56.598
39	LOWE'S COMPANIES, INC.	LOW	44.867	13.790	1.725	5.244	21.234
40	ELI LILLY AND COMPANY	LLY	35.119	22.003	0.946	6.349	28.324
41	L BRANDS, INC.	LB	190.071	14.749	0.175	3.078	8.500
42	WEYERHAEUSER CO	WY	93.218	18.456	2.141	10.768	53.201
43	WAL-MART STORES, INC.	WMT	37.942	17.344	171.167	1.984	3.716
44	UNION PACIFIC CORP	UNP	29.575	16.620	5.890	9.854	48.771
45	3M COMPANY	MMM	27.503	15.808	114.768	6.156	27.090
46	TARGET CORP	TGT	34.353	12.459	2.767	12.015	58.597
47	SYSCO CORP	SYY	51.896	45.525	4.619	1.620	2.507
48	STANLEY BLACK & DECKER, INC.	SWK	76.356	39.650	-0.057	-6.395	28.621
49	SHERWIN-WILLIAMS CO	SHW	49.450	19.022	24.755	5.836	25.035



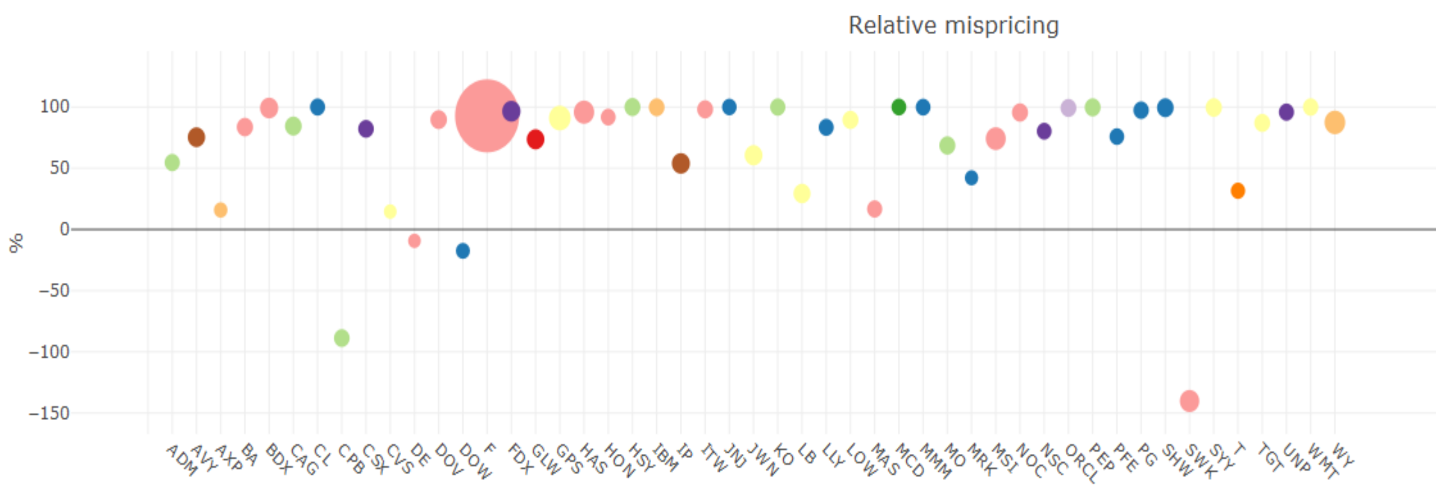
Y-axis: mean R2, %; **X-axis:** tickers; **bubble size**: value of R2; **bubble color:** industry company operates in

APPENDIX 5. Absolute and Relative mispricing

#	Full name	Ticker	Absolute misprice, bp				Relative misprice, %			
			mean	std	max	min	mean	std	max	min
1	AT&T INC.	T	63.7	26.6	114.9	0.7	35.6	80.1	100.0	-173.9
2	ALTRIA GROUP, INC.	MO	51.5	21.9	114.0	3.5	77.0	30.2	100.0	-39.6
3	AMERICAN EXPRESS COMPANY	AXP	61.3	45.9	261.1	3.8	40.0	73.2	99.9	-221.1
4	ARCHER-DANIELS-MIDLAND COMPANY	ADM	39.9	22.9	111.0	3.3	58.2	36.3	99.6	-54.6
5	AVERY DENNISON CORP	AVY	67.5	23.9	88.6	1.6	75.4	27.7	100.0	-6.1
6	BECTON, DICKINSON AND COMPANY	BDX	77.5	13.8	92.4	30.0	99.2	1.5	100.0	92.7
7	BOEING COMPANY (THE)	BA	52.3	36.7	209.2	15.0	86.0	13.5	100.0	46.6
8	CSX CORP	CSX	44.1	31.0	160.7	16.6	83.7	16.3	100.0	41.6
9	CVS HEALTH CORP	CVS	33.9	13.5	83.8	4.4	34.2	77.3	100.0	-119.9
10	CAMPBELL SOUP CO	CPB	115.9	96.7	334.4	6.9	-170.2	414.2	100.0	-1300.6
11	COCA-COLA COMPANY (THE)	KO	38.1	13.4	89.7	22.0	100.0	0.0	100.0	99.8
12	COLGATE PALMOLIVE CO	CL	36.0	12.5	75.6	21.3	100.0	0.0	100.0	100.0
13	CONAGRA BRANDS, INC.	CAG	58.1	28.4	122.3	1.6	82.7	27.2	100.0	-22.9
14	CORNING INC	GLW	70.9	62.0	360.8	5.5	73.0	24.9	99.6	5.6
15	DEERE & CO	DE	48.3	32.2	138.1	1.1	-25.8	113.9	99.8	-296.7
16	DOVER CORP	DOV	56.3	18.5	135.6	28.8	90.3	10.2	99.8	57.1
17	DOW CHEMICAL COMPANY (THE)	DOW	114.5	87.7	423.5	20.8	-17.7	121.4	99.8	-326.0
18	FEDEX CORP	FDX	79.2	35.1	252.8	42.5	96.7	4.2	100.0	81.7
19	FORD MOTOR CO	F	694.2	1506.8	10004.2	58.4	86.5	12.6	98.0	34.7
20	GAP INC	GPS	125.1	72.2	339.6	22.8	93.0	5.6	99.8	75.8
21	HASBRO INC	HAS	106.7	48.4	283.1	28.3	95.8	5.7	100.0	73.5
22	HERSHEY COMPANY (THE)	HSY	47.6	14.2	92.7	28.3	100.0	0.0	100.0	99.8
23	HONEYWELL INTERNATIONAL INC	HON	32.7	16.9	113.5	13.9	93.7	6.9	100.0	71.8
24	ILLINOIS TOOL WORKS INC	ITW	47.9	18.9	140.9	30.1	98.4	2.3	100.0	90.0
25	INTERNATIONAL BUSINESS MACHINES CORP	IBM	45.0	13.3	109.5	25.2	99.8	0.3	100.0	98.3
26	INTERNATIONAL PAPER CO	IP	83.9	71.6	436.6	0.1	58.1	40.6	99.8	-43.5
27	JOHNSON & JOHNSON	JNJ	29.5	13.8	73.2	11.1	100.0	0.0	100.0	100.0
28	PROCTER & GAMBLE CO	PG	40.7	21.7	138.8	16.9	98.3	3.2	100.0	85.9
29	PFIZER INC	PFE	31.1	14.0	73.0	0.3	80.8	25.7	100.0	-21.0
30	PEPSICO INC	PEP	47.2	11.6	94.4	32.7	99.7	0.5	100.0	98.0
31	ORACLE CORP	ORCL	45.9	22.3	165.7	28.8	99.1	0.9	100.0	95.6
32	NORTHROP GRUMMAN CORPORATION	NOC	50.2	7.2	78.1	29.3	95.5	6.2	100.0	70.0
33	NORFOLK SOUTHERN CORP	NSC	32.6	15.9	116.6	7.2	81.9	19.7	99.7	22.2
34	NORDSTROM INC	JWN	73.2	61.7	348.4	15.9	64.2	27.6	99.9	23.8
35	MOTOROLA SOLUTIONS, INC.	MSI	102.3	52.2	352.7	8.5	81.3	25.6	99.9	11.8
36	MERCK & CO., INC.	MRK	17.1	12.4	57.7	0.2	54.0	44.2	100.0	-47.9
37	MCDONALD'S CORPORATION	MCD	32.6	10.8	56.8	14.5	100.0	0.0	100.0	100.0
38	MASCO CORP	MAS	102.7	73.8	352.4	4.5	32.4	60.3	100.0	-74.9
39	LOWE'S COMPANIES, INC.	LOW	49.1	27.4	153.9	15.7	91.4	10.0	99.9	47.6
40	ELI LILLY AND COMPANY	LLY	34.7	12.4	63.4	1.2	85.3	19.7	100.0	4.8
41	L BRANDS, INC.	LB	166.5	65.9	383.3	29.7	28.9	85.1	100.0	-210.4
42	WEYERHAEUSER CO	WY	111.5	45.7	238.5	38.6	90.1	9.4	99.8	49.6
43	WAL-MART STORES, INC.	WMT	40.3	19.1	110.3	13.4	100.0	0.0	100.0	99.8
44	UNION PACIFIC CORP	UNP	37.6	19.4	130.0	14.4	96.9	3.6	100.0	82.2
45	3M COMPANY	MMM	31.7	18.9	118.0	14.2	99.9	0.1	100.0	99.1
46	TARGET CORP	TGT	46.7	28.6	200.5	21.5	89.6	10.9	100.0	58.8
47	SYSCO CORP	SYY	52.7	9.5	86.9	24.4	99.6	0.6	100.0	97.2
48	STANLEY BLACK & DECKER, INC.	SWK	156.4	90.4	377.4	44.0	-171.2	250.3	100.0	-647.8
49	SHERWIN-WILLIAMS CO	SHW	57.3	25.4	153.4	18.6	99.5	0.7	100.0	97.3



Y-axis: actual spread, basis points; **X-axis:** tickers; **bubble size**: absolute mispricing; **bubble color:** industry company operates in



Y-axis: relative mispricing, %; **X-axis:** tickers; **bubble size**: absolute mispricing; **bubble color:** industry company operates in

APPENDIX 6. Before and after adjustment changes

#	Full name	Increase in Liabilities due to Consolidation	Increase in Liabilities due to Lease accounting	Increase in Liabilities due to Pension accounting	**Total Increase in Liabilities**	Increase in Interest expense due to Consolidation	Increase in Interest expense due to Lease accounting	**Total Increase in Interest expense**
1	CVS HEALTH CORP	-	26,49%	10,12%	**36,61%**	-	58,33%	**58,33%**
2	COCA-COLA COMPANY (THE)	11,36%	-	12,57%	**24,23%**	49,39%	-	**49,39%**
3	FORD MOTOR CO	-	0,5%	20,10%	**20,6%**	-	12,35%	**12,35%**
4	IBM CORPORATION	-		50,87%	**50,87%**	-	-	-
5	NORFOLK SOUTHERN CORP	-	9,40%	10,02%	**19,42%**	-	11,66%	**11,66%**
6	MCDONALD'S CORPORATION	-	24,23%	-	**24,23%**	-	41,86%	**41,86%**
7	3M COMPANY	-	35,97%	26,42%	**62,39%**	-	15,67%	**15,67%**

APPENDIX 7. R code

```
shares <- data.frame(read.csv("Shares_final.csv"))
equity_price <- data.frame(read.csv("Equity_price_final.csv"))
liabilities <- data.frame(read.csv("Liabilities_final.csv"))
dividends <- data.frame(read.csv("Dividends_final2.csv"))
interests <- data.frame(read.csv("Interests_final2.csv"))
risk_free <- data.frame(read.csv("Rf_final.csv"))
#drift <- data.frame(read.csv("Drift_final.csv"))
index500 <- data.frame(read.csv("index500_final.csv"))
CDS <- data.frame(read.csv("CDS.csv"))

companies_names <- colnames(shares)
companies_names <- companies_names[-1]

f <- data.frame(interests[,1])
f2 <- data.frame(interests[,1])
f3 <- data.frame(interests[,1])
reg_names <- c("Const","Const t-value","Slope","Slope t-value","R2")
reg_results_by_firm <- data.frame(reg_names)
misprice_names <- c("Absolute misprice mean","Absolute misprice std","Absolute misprice max","Absolute
misprice min","Relative misprice mean","Relative misprice std","Relative misprice max","Relative misprice min")
mispricing_results <- data.frame(misprice_names)
firm_stats_names <- c("mean PD","std PD", "max PD","min PD","mean RR","std RR", "max RR","min RR","mean
spread model", "std spread model", "max spread model","min spread model","mean spread act", "std spread
act","max spread act", "min spread act")
firm_stats_results <- data.frame(firm_stats_names)

# Iteration function

iteration <- function(asset_value, stddev, liabilities, dividend, interest, risk_free, equity_market, equity_price, step,
n,k, horizon){
  d1 <- c()
  d2 <- c()
  k1 <- c()
  k2 <- c()
  asset_iter_k <- c()
  asset_logreturn_k <- c()

    for (i in step:k){
    d1[i-step+1] <- (log(asset_value[i-step+1]/(liabilities[i,n] + dividend + interest)) +  (risk_free[i,2] +
          0.5*stddev^2)*horizon)/(stddev*(horizon^0.5))
    d2[i-step+1] <- d1[i-step+1] - stddev*(horizon^0.5)
    if (dividend == 0  &&  interest == 0)
    {asset_iter_k[i-step+1] <- (equity_market[i-step+1]+(liabilities[i,n])*exp(-risk_free[i,2])*pnorm(d2[i-step +
          1]))/(pnorm(d1[i-step+1]))
     }
    else
    {k1[i-step+1] <- (log(asset_value[i-step + 1]/(dividend  + interest)) + (risk_free[i,2] +
           0.5*stddev^2)*horizon)/(stddev*(horizon^0.5))
     k2[i-step+1] <- k1[i-step+1] - stddev*(horizon^0.5)
     asset_iter_k[i-step+1] <- (equity_market[i-step+1]-dividend*exp(-risk_free[i,2])*pnorm(k2[i-step  + 1]) +
               (liabilities[i,n]+dividend+interest)*exp(-risk_free[i,2])*pnorm(d2[i-step+1]))/(pnorm(d1[i-
               step+1])+dividend/(dividend+interest)-dividend/(dividend+interest)*pnorm(k1[i-step+1]))
     }
```

```
# calculate log returns
    if (i > step)
    {asset_logreturn_k[i-step+1] <- log(asset_iter_k[i-step+1]/asset_iter_k[i-step+1-1])
    }
  }

  asset_logreturn_k <- asset_logreturn_k[-1]
  stddev_k1 <- sd(asset_logreturn_k)*(12)^0.5
  error <- sum((asset_value - asset_iter_k)^2)
  result <- cbind.data.frame(error, stddev_k1, asset_iter_k)
  return(result)
}

for (n in 2:50)
{
EA <- c()
PD_annual <- c()
RR <- c()
spread <- c()
spread2 <- c()
spread_simple <- c()
abs_misprice <- c()
rel_misprice <- c()
reg <- c()
firm_stats <- c()
step <- 0
  for (k in 60:163){
      step <- step + 1
      asset_initial_value <- c()
      asset_logreturn_initial <- c()
      equity_market <- c()
      equity_logreturns <- c()
      end_window <- k
      for (i in step:end_window){
          equity_market[i-step+1] <- shares[i,n]*equity_price[i,n]
          asset_initial_value[i-step+1] <- equity_market[i-step+1] + liabilities[i,n]

         if (i > step)
            {asset_logreturn_initial[i-step+1] <- log(asset_initial_value[i-step+1]/asset_initial_value[i-step+1-1])
            equity_logreturns[i-step+1] <- log(equity_price[i,n]/equity_price[i-1,n])
            }

          }
      asset_logreturn_initial <- asset_logreturn_initial[-1]
      equity_logreturns <- equity_logreturns[-1]
      stddev_initial_equity <- sd(equity_logreturns)*(12)^0.5


      stddev_initial <-
equity_market[length(equity_market)]*stddev_initial_equity/asset_initial_value[length(asset_initial_value)]

      # while accruing annual data was used
      dividend <- dividends[step,n]/4
      interest <- interests[step,n]/4
      horizon <- 5
```

```
    iterations <- 0
    asset_value_new <- c()
    results <- iteration(asset_initial_value, stddev_initial,liabilities, dividend, interest, risk_free, equity_market,
                         equity_price, step, n,k, horizon)
    iter_error <- results[1,1]
    stddev_new <- results[1,2]
    asset_value_new <- results[,3]

    while (iter_error > 0.000001){
      results <- iteration(asset_value_new, stddev_new,liabilities, dividend, interest, risk_free, equity_market,
                         equity_price, step, n,k, horizon)

     iter_error <- results[1,1]
     stddev_new <- results[1,2]
     asset_value_new <- results[,3]
     iterations <- iterations + 1
     }

    # calculate drift rate

    asset_logreturn_drift <- c()
    for (i in 2:length(asset_value_new))
    {
     asset_logreturn_drift[i-1] <- asset_value_new[i]/asset_value_new[i-1]
    }
    x <- c()
    y <- c()
    for (i in step:end_window){
     x[i-step+1] <- index500[i, 2]
     y[i-step+1] <- asset_logreturn_drift[i-step+1] - (1 + (exp(risk_free[step, 2])-1)/12)
    }
    beta <- lm(y ~ x)$coefficients[2]

    #drift_rate <- log((exp(risk_free[end_window, 2])-1) + 0.06*beta + 1)
    drift_rate <- risk_free[59+step, 2]

    PD <- pnorm(-(log(asset_value_new[length(asset_value_new)]/(liabilities[59+step, n] + dividend + interest)) +
                  (drift_rate - stddev_new^2*0.5)*horizon)/(stddev_new*(horizon^0.5)))
    PD_annual[step] <- 1-(1-PD)^(1/horizon)
    bankruptsy_level <- liabilities[59+step,n] + dividend + interest
    N1 <- (pnorm(-(log(asset_value_new[length(asset_value_new)]/(liabilities[59+step, n] + interest + dividend)) +
                  (drift_rate + stddev_new^2*0.5)*horizon)/(stddev_new*(horizon^0.5))))
    N2 <- (pnorm(-(log(asset_value_new[length(asset_value_new)]/(liabilities[59+step, n] + interest + dividend)) +
                  (drift_rate - stddev_new^2*0.5)*horizon)/(stddev_new*(horizon^0.5))))
    # expected asstet value conditional on default
    EA[step] <- asset_value_new[length(asset_value_new)]*exp(drift_rate*horizon)*N1/N2
    RR[step] <- EA[step]/bankruptsy_level
    LGD <- 1 - RR[step]
    spread[step] <- PD_annual[step]*(1-RR[step])*10000
    }

# CDS on Model regression results: 1 - const,2 - const t-value, 3 - slope, 4 - slope t-value, 5 - R2
y <- CDS[,n]
```

```
reg[1] <- summary(lm( y ~ spread))$coefficients[1,1]
reg[2] <- summary(lm( y ~ spread))$coefficients[1,3]
reg[3] <- summary(lm( y ~ spread))$coefficients[2,1]
reg[4] <- summary(lm( y ~ spread))$coefficients[2,3]
reg[5] <- summary(lm( y ~ spread))$r.squared
reg_results_by_firm <- cbind.data.frame(reg_results_by_firm, reg)
colnames(reg_results_by_firm)[n] <- c(companies_names[n-1])

#absolute (bp) and relative (%) mispricing: 1 - mean, 2 - std, 3 - max, 4 - min

abs_misprice[1] <- mean(abs(spread - CDS[,n]))
abs_misprice[2] <- sd(abs(spread - CDS[,n]))
abs_misprice[3] <- max(abs(spread - CDS[,n]))
abs_misprice[4] <- min(abs(spread - CDS[,n]))

rel_misprice[1] <- mean((1 - (spread/CDS[,n]))*100)
rel_misprice[2] <- sd((1 - (spread/CDS[,n]))*100)
rel_misprice[3] <- max((1 - (spread/CDS[,n]))*100)
rel_misprice[4] <- min((1 - (spread/CDS[,n]))*100)
comb <- append(abs_misprice, rel_misprice)
mispricing_results <- cbind.data.frame(mispricing_results, comb)
colnames(mispricing_results)[n] <- c(companies_names[n-1])

# firm stats: 1 - mean PD, 2 - std PD, 3 - max PD, 4 - min PD,
# 5 - mean RR, 6 - std RR, 7 - max RR, 8 - min RR,
# 9 - mean spread model, 10 - std spread model, 11 - max spread model, 12 - min spread model,
# 13 - mean spread act, 14 - std spread act, 15 - max spread act, 16 - min spread act
firm_stats[1] <- mean(PD_annual)
firm_stats[2] <- sd(PD_annual)
firm_stats[3] <- max(PD_annual)
firm_stats[4] <- min(PD_annual)
firm_stats[5] <- mean(RR)
firm_stats[6] <- sd(RR)
firm_stats[7] <- max(RR)
firm_stats[8] <- min(RR)
firm_stats[9] <- mean(spread)
firm_stats[10] <- sd(spread)
firm_stats[11] <- max(spread)
firm_stats[12] <- min(spread)
firm_stats[13] <- mean(CDS[,n])
firm_stats[14] <- sd(CDS[,n])
firm_stats[15] <- max(CDS[,n])
firm_stats[16] <- min(CDS[,n])
firm_stats_results <- cbind.data.frame(firm_stats_results, firm_stats)
colnames(firm_stats_results)[n] <- c(companies_names[n-1])

print(paste(n,iterations, iter_error))
}
write.csv(reg_results_by_firm, "regression_results.csv")
write.csv(mispricing_results, "mispricing_results.csv")
write.csv(firm_stats_results, "firm_stats_results.csv")
```

BI Norwegian Business School - campus Oslo

GRA 19502

Master Thesis

Component of continuous assessment: Forprosjekt, Thesis MSc

What determines the credit default swap/ credit risk premium of a company?

ID number:	0986661, 0987038
Start:	01.12.2016 09.00
Finish:	01.03.2017 12.00

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Preliminary Thesis Report

What determines the credit default swap/ credit risk premium of a company?

Hand-in date:
01.03.2017

Course code and name:

Programme:
Master of Science in Financial Economics
Master of Science in Business, Major in Finance

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CONTENTS

ABSTRACT

This work is based on the so-called “Credit Spread Puzzle” phenomenon. It suggests a new way to deal with the phenomenon, namely, the consideration of Hidden debt in specific firms’ balance sheets. This information serves as an input in assessing firm’s credit risk and default probabilities. Our work’s aim is to test, through known credit risk models, whether the inclusion of this modified input helps to bridge the gap between theoretical and observed credit spreads and credit prices.

1. INTRODUCTION

1.1. Some Background

The *Credit Spread Puzzle* is a widely-discussed phenomenon of the recent years as it directly affects investors' ability to correctly determine the quality of their investments and to correctly price them. Brokerage firm analysts and investors use credit risk models to detect and quantify a company's ability to repay back its debt, the overall ability to meet its financial obligations and the chances of potential credit default of this company. Based on the results of these models, the interested parties, then determine the interest rates/prices of their investments. Therefore, the level of calibration of these models is crucial in protecting the interested parties from mispricing their investments or at worst case, from failure to receive the prospects of their investments.

This anomaly is actually happening, as all of the available models, arrive at biased results, most of them very low results compared to the actual spreads. There exists a lot of tested ideas that suggest reasons for this bias and ways on how to improve it.

1.2. Research question

Nevertheless, one potential reason of this mismatch is the imprecise level of liabilities taken from the balance sheets of these companies. Sometimes the flaw relies in mistakes made by company's accountants, but most of the times, it's as a consequence of accountants neglecting to include certain types of liabilities in their balance sheets. The reason for this is obvious: companies want to appear stronger or to better comply with their loan covenants. For example, borrowers may "forget" to accrue liabilities for salary or vacation time. Some might underreport payables by holding checks for weeks (or months). Other borrowers might hide bills in a drawer at year end to avoid recording the payable and the expense. This mismatches revenues and expenses, understates liabilities and artificially enhances profits. Delayed payments can also hurt the company's credit score and cause suppliers to restrict their credit terms.

We are aimed to analyze only some forms of hidden debt (especially leases and pension liabilities) and investigate consequences for the companies. Hence, for the chosen companies, we will try to estimate the value of the potential hidden liabilities and use these data in our testing. Our estimation of company's credit risk will be based on Merton model (and probably on others as well) using option pricing Black-Scholes formula. Afterwards, we will test our data with the market and conduct a matching bond analysis.

When choosing the country/ area of our observation, we consider exchange rate conventions and particular regulations in terms of companies' public documents.

1.3. Structure of the paper

The rest of this paper will be organized as follows:

The second chapter provides a general overview of the most important works related to the topic in question. It starts with the big picture of credit spread models being used and then it narrows down to how some of the empirical authors try to solve the credit spread puzzle.

The third chapter gives the theoretical background behind the credit risk and goes step by step to explain the credit spread puzzle. Moreover, it presents how the credit spread puzzle is associated with hidden liabilities, which also constitutes the hypothesis underlying our study.

Finally, the fourth chapter presents some details on how are we going to approach the research question both methodologically and in terms of the data we are going to use.

2. BACKGROUND AND LITERATURE

2.1. Why modeling correctly the credit risk is important?

Recently, the interest in the framework of credit risk models and measures is increasing. This is because of two main reasons. First, the Capital Accord of 2006, or Basel II, allows large banks to use their internal models to assess their capital requirement instead of the more constraining standardized model. Second, the huge increase of off balance-sheet derivatives and the rising use of the securitization of loans necessitate more developed credit analysis methods (Laajimi 2012).

2.2 Literature on Credit Risk Models and variations

There exist a wide number of models that assess the credit risk. Crouhy, Galai and Mark 2000, in their paper "A comparative analysis of current credit risk models" reviews some of these Credit Value-at-Risk methodologies: These models differentiate in their approach to the risk. JP Morgan proposes the credit mitigation approach, Merton proposes the structural approach, Credit Suisse Financial Products propose the actuarial approach and McKinsey proposes a multi-period macro based approach. What is important in our investigation, is the structural approach. This approach, utilizes information about the capital structure of the firm. It signals default at the moment the assets of the firm decline to a certain critical level.

The paper "Structural Credit Risk Models: A Review" by Laajimi presents the major structural models that differ from one-another in their respective underlying assumptions. All the structural models share a common theoretical framework, based on the Merton model (1974), which is an exogenous model. In this model, the firm defaults when its assets reach an exogenous level. Hence, the starting point of this model is to set the diffusion process of the assets. Moreover, it assumes a simple capital structure and refers to firm liabilities being represented by a single-zero coupon bond maturing at T. Here, the default barrier is equal to the nominal value of the debt.

Merton's model is extended by Kim, Ramaswamy and Sundaresan (1993) to incorporate both default risk and interest rate risk. The authors assume that the default is triggered by a cash-flow shortage. Other exogenous work include Longstaff and

Schwartz (1995), Nielseen, Saa-Requejo and Santa-Clara (1993) and Briys and de Varenne (1997).

In the endogenous group, there are three major contributions. Black and Cox (1976) deal with the timing issue of the Merton model (the default time is restricted to the maturity of debt, independently of the evolution of the asset's value before maturity). In their model, the firm defaults as soon as the value of its assets reaches a non-random default barrier V_B. In this case, bondholders get V_B and equity holders get nothing. Whereas, the Leland (1994) model contributes in terms of the tax shield of debt and bankruptcy costs. On one hand, the debt issuance reduces the firm's value due to the increasing value of bankruptcy costs, and on the other hand, higher value of interest payments implies more tax shield, increasing in this way the value of the firm. Finally, Leland and Toft (1996) model, differently from Leland (1994) model that assumes a perpetual debt, accounts for debt that is continuously rolled over. This ensures that the principal, coupons and debt maturity are independent of time.

2.3. Discover the "Credit Spread Puzzle"

Following the review by Laajimi, there is an immense number of empirical work that test the ability of different models to predict the credit spread on bonds and CDS.

As a major contribution to the field, Eom, Helwege and Huang in their paper "Structural Models of Corporate Bond pricing: an Empirical Analysis" test five of these structural models, namely Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001), for the period 1986-1997, using data from 182 bond prices in firms with simple capital structures. Their findings on the implementation of Merton's model, approve the convention that the predicted spreads are too low compared to the real spreads. Nevertheless, the other models display too high spreads on average. They suggest that the problem relies on the accuracy of the models and that better models should account for higher spreads for safer bonds, avoiding excess dispersion of spreads in riskier bonds.

2.4. Resolving the "Credit Spread Puzzle"

Two other authors, Feldhütter and Schaefer test the Merton Model in both their papers. "The Credit Spread Puzzle – Myth or Reality" (2013) paper indicates that current authors, testing for the credit spread puzzle, fail to distinguish some strong biases and their approaches suffer from low statistical power. The problems come from the fact that authors usually compare the predicted spreads to average observed spreads. First, considering the typical convexity of the spreads in firm variables, the spreads of average firm variables are significantly lower than the average of spreads. Secondly, in these type of studies that use samples for a long period, the classical assumption that historical default probabilities serve as a good proxy for expected default frequencies, does not hold. Hence, while correcting for these issues, they find almost no evidence of credit spread puzzle. Contrariwise, the model captures both the average level and time series variation of 10-year BBB-AAA spreads. The authors also suggest that considering the cross-sectional variation of firms and time series variation of firm leverage is crucial in testing the structural models.

Their next paper "The Myth of the Credit Spread Puzzle" (2016), calibrates the model by using a much longer time series of data compared to previous studies, specifically 92 years. They test both the Merton and Black-Cox models. The authors explain that such long history of ex post default rates contribute to more precise ex post default probabilities as they can abbreviate the effect of high skewness in the distribution of realized default rates. They again, highlight that the problems in previous model tests come from how they implement the models rather than deficiencies of the models.

In addition, Huang and Huang (2012) support the idea that if the structural models are calibrated in terms of stochastic interest rates, endogenous default, stationary leverage ratios and strategic default, they will always match the default rates and the equity premium.

Another important paper "On the relation between the credit spread puzzle and the equity premium puzzle" by Chen, Collin-Dufresne and Goldstein 2008, suggest to resolve the credit spread puzzle by accounting for the fact that default rates and Sharpe ratios strongly covary with each other.

2.5. Literature on Unconsidered Liabilities

In our study, we propose that the value of Liabilities generally used in the models, is not correct, as a lot of companies try to hide some of their debts, to rank better in credit ratings. Given that there is no known paper that has studied this particular issue in any country, our study will be a new approach in the field.

As for the Unconsidered Liabilities, J. Edward Ketz in "Hidden Financial Risk" provides great insights on Ethics in Financial Accounting and Reporting. After giving backgrounds on recent Financial scandals related to the omission of important liabilities in their public balance sheets, he proceeds by thoroughly analyzing different ways that account managers use to hide debt, particularly with the Equity Method, with Lease Accounting, with Pension Accounting and with Special-Purpose Entities. The author provides knowledge on how potential investors might account for these hidden liabilities in order to reduce their financial risk to real terms.

3. THEORY AND HYPOTHESIS

3.1. Explaining credit risk

To answer the defined question that this thesis will elaborate on, there is the need to explain more in detail the underlying terminology. To begin with, *Credit risk* measures the prospects that a loan will be repaid on the arranged time under original defined terms. The credit risk of a company is important to Financial institutions or individual investors to assess the financial stability of these companies when the latter are considering to loan money to or buy bonds from the former. It is also important to company's shareholders and loan covenants. The level of *credit risk score* or *credit rating* shows to investors where is this company positioned in terms of risk and according to this risk, investors will charge the most convenient interest rate. Credit risk of a company is assessed by evaluating the five C's: the company's credit history, its capacity to repay, its capital, loan's conditions and associated collateral. If the company is rating high, it means it is less risky. Therefore, it could be charged a lower interest rate and conversely when the company is rating low in credit risk.

Modelling of the credit risk provides the framework for investors and financial institutions to estimate the credit spread risk of a certain company. The following three sections will explain in detail the general model being used.

From an immense number of studies, it is observed that the standard structural models forecast credit spreads that are very low compared to the real credit spreads. This phenomenon is called "*the credit spread puzzle".* As we suggest that one of the reasons of this puzzle, might be the incorrect value of firm's liabilities, the last section of this chapter will narrow down the most important aspects related to this issue.

3.2. Credit risk pricing models. Overview

There are three main approaches to credit modelling and the pricing of credit risk – structural models (Merton, 1974), reduced-form models (Jarrow et. al 1995) and statistical models like SEBRA model used by Norges Bank (The Norwegian central bank) (Bernhardsen & Larsen, 2007) and Altman's Z-score method (Altman E., 1968).

The *structural models* originate from the option pricing theory introduced by Black & Scholes (1973), which found its application in the area of corporate bond pricing. The main idea of structural models is to value corporate debt using a contingent-claims approach. The crucial assumption is that information held by the firm's managers is completely available, i.e. the modeler has a comprehensive knowledge of all the firm`s assets and liabilities. These models are used to estimate the spread of bonds issued by public firms, since stock prices are a major component of the model input parameters. Structural models are valued by practitioners due to intuitive economic interpretation, which allows for consistent discussion regarding a variety of credit risk exposures and understanding of transaction implications (impact on credit quality due to increased borrowing, acquisitions, share buybacks, etc.).

The *reduced from models* were originated with Jarrow and Turnbull (1992) research, and subsequently studied by Jarrow and Turnbull (1995), Duffie and Singleton (1999) among others. In contrast to structural models, reduced form models assume incomplete knowledge of the firm`s condition, implying that a firms' default time is inaccessible or unpredictable, and therefore estimated as a stochastic process. This informational assumption is a key difference between two models – structural model can be transformed into reduced form model as the information set changes. In addition, in structural models the recovery rate process is prespecified by a knowledge of the liability structure, while in reduced form models this process is exogenously supplied. Due to limited information and default time assumptions, reduced form method is considered to be more theoretically accurate, but lack the clear economic rationale for defining the nature of the default process and require detailed bond price data.

Statistical models use various forms of econometric techniques to identify determinants of default. They are less reliant on economic theory as their model framework, but are limited by their poor out-of-sample-power.

3.3. Structural Models. Intuition behind the theory

Structural models are cause-and-effect models. First of all, one needs to identify and impose conditions under which the borrowers are expected to default, and then estimate the probability that these conditions will be satisfied. In this case borrowers are the companies which issue bonds, and debt-holders are lenders of funds (individuals, other companies, banks). In case of limited liability entities, when the company is unable to cover all payments to its debt-holders, it defaults.

Recalling the balance sheet equation

$$\text{Assets} = \text{Liability} + \text{Equity},$$

and the fact that equity holders are the residual claimants, we conclude that when a company`s liabilities exceed its assets, the value of equity is negative, implying equity holders are willing to give it away at no cost. Simply saying, they exercise the walk-away option, which can be priced with approaches from option pricing theory.

The first structural model of default risk valuation based on the option pricing theory was proposed by R. Merton in 1974. The Model assesses the equity and debt value of listed companies through the B&S option pricing model. The intuition behind is rather straightforward. Assume the company has an outstanding debt in form of zero coupon bond equal to L with maturity T, meaning no matter how much the company will profit in the future ($A>L$), the debtholder will definitely receive just a notional L. On the other hand, the equity holders as residual claimants will benefit in this situation – their upside potential is unlimited, they pocket the positive difference. However, if $A<L$ and company defaults, equity holders will receive nothing and remaining assets are claimed by the creditors. Therefore, the pay-off to equity holders may be described as European call option

$$E = \max(0, A - L),$$

where the underlying is the asset value and the strike is the outstanding liability L.

The pay-off to debt holders equals to the portfolio of risk-free zero bond with notional L and a payoff of a short European put option:

$$D = L + \min(0, A - L) = L - \max(0, L - A)$$

According to Merton`s simplified set up, the probability of default at time T implies the probability of A<L – value of assets is below the value of liabilities, or in other words, when put option is exercised.

3.4. Merton model

The key feature of the model relies on the fact that corporate securities are seen as contingent claims on market value of firm's assets. Therefore, the focus will be on the degree to which firms' asset sales are used to finance coupon payments on debt.

Using option-backed equity and debt set up from previous section, we now discuss the basic Merton credit risk model. The model is based on the following assumptions (Merton,1974):

1. No transaction costs, taxes or indivisibilities of assets
2. There are a sufficient number of investors with comparable wealth levels so that each investor believes that he can buy and sell as much of an asset as he wants at the market price
3. There exists an exchange market for borrowing and lending at the same rate of interest
4. Short-sales of all assets, with full use of the proceeds, is allowed
5. Trading in assets take place continuously in time
6. The Modigliani-Miller (MM) theorem that the value of the firm is invariant to its capital structure obtains.
7. The term structure is flat and known with certainty; i.e., the price of a riskless discount bond that promises a payment of $1 at time T in the future is $P(t,T)=e-r(T-t)$, where r is the (instantaneous) riskless rate of interest, the same for all time
8. The dynamics for the value of the firm, V (in our case we use A for Assets notation), through time can be described by a diffusion-type stochastic process.

Merton emphasizes that "perfect market" assumptions 1-4 can be weakened, while assumption 7 is chosen to distinguish risk structure from term structure effects on pricing. In some later versions of structural models the term structure is assumed to be a stochastic process. However, continuous trading and following the stochastic process for assets are crucial assumptions.

As in the B&S option price formula set up, where stock price follows a Brownian motion (or Wiener process), the company`s assets are described with the stochastic process:

$$dA_t = \mu A_t dt + \sigma A_t dZ_t,$$

where μ is the instantaneous expected rate of return on the firm per unit time dt, σ^2 is the instantenious variance of the return on the firm per unit of time dt, and dZ_t is a change of a normally distributed variable Z that follows Brownian motion.

Wiener process (Brownian motion) is a particular type of Markov stochastic process with a mean change of zero and a variance rate of 1.0 per year. In turn, Markov process assumes that only the current value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant. For example, stock prices are assumed to follow a Markov process, which supports the weak form of market efficiency. A stochastic process equation where the drift and volatility are depended on the variable is called Ito`s process, and may be rewritten as:

$$\frac{dA_t}{A_t} = \mu dt + \sigma dZ_t$$

Here the percentage change in A_t is normally distributed with instantaneous mean μ and variance σ^2. This process is also known as geometric Brownian motion. Because the mean and variance at time t are proportional to A_t, the evolution of A_t generates compounding (the change in A_t is proportional to A_t:$\ln(A_t)$ and thus non-normality. Therefore, a variable that follows geometric Brownian motion is lognormally distributed. While A_t is not normal, $\ln(A_t)$ is normally distributed:

$$\ln(A_t) \sim N(\ln(A_0) + (\mu - 0.5\sigma^2)t, \sigma^2 t)$$

And $A_t = A_0 e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}Z}$, where $Z \sim N(0,1)$

Using B&S option pricing formula, the value of equity can be expressed as call option:

$$E_0 = A_0 N(d_1) - Le^{-rT} N(d_2),$$

where

$$d_1 = \frac{\ln(A_0/L) + (r + 0.5\sigma_A^2)T}{\sigma_A\sqrt{T}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T}$$

and $N(\cdot)$ represents the cumulative normal distribution, r is continuously compounded risk free rate, σ_A is the asset volatility, T is time to maturity. Here $N(d_2)$ has a fairly simple interpretation – it is a probability that a call option will be exercised in a risk-neutral world, namely, it is a probability that the firm will not default as long as value of assets exceeds the value of liabilities. Due to a symmetric property of normal distribution, $N(d_2) = 1 - N(-d_2)$, where $N(-d_2)$ is a risk-neutral probability of default. Variables d_1 and d_2 are results of the standard z-score calculation in B&S formula, where d_2 is also known as distance to default (DD) and measures the number of standard deviations the expected asset value A is away from the default. And to calculate this we need A_0 and σ_A, which are not directly observable. At the same time, for listed firms via stock prices we can determine E_0. From Ito`s lemma:

$$\sigma_E E_0 = \frac{\partial E}{\partial A}\sigma_A A_0 = N(d_1)\sigma_A A_0$$

This expression gives us other condition which has to be satisfied simultaneously with the previous equation of E_0 , and σ_E is estimated from historical data or options.

Because it is common in discussions of bond pricing to talk in terms of yields rather than prices, the difference between the corporate bond yield and risk free rate (continuous) can be expressed as

$$Spread = y - r_{rf} = \left(\frac{L}{A-E}\right)^{\frac{1}{T}} - 1 - (e^{r_{rf}} - 1) = \left(\frac{L}{A-E}\right)^{\frac{1}{T}} - e^{r_{rf}},$$

where $D = A - E = \frac{L}{(1+y)^T}$ is the current value of debt.

Assuming outstanding bond is a zero bond, the spread can be described as a product of implied from each model risk neutral probability of default (PD) and static recovery rate (RR), adjusted for maturity T:

$$Credit\ spread = \frac{PD \cdot LGD}{T} = \frac{PD \cdot (1 - RR)}{T}$$

Where LGD - loss given default - is the difference between what the bond holders are owed and what they receive, as a fraction of promised payment.

3.5. Unconsidered liabilities

Lately, we have heard a lot about accounting scandals, like in the case of Enron, Adelphia and WorldCom. Even though these attempts were revealed and debunked, there will always be managers out there hiding company's liabilities in order to cover mistakes or to make the company seem less risky in financial terms. Some of them might hide a considerable amount of debt and some of them just minor liabilities. However, these activities cause huge discrepancies and negatively affect the results of financial models who use debt data as inputs. Not to mention, the consequences if these scandals are revealed: increased financial risk reporting premium, higher cost of capital and lower stock and bond prices.

Some of the ways these managers use, are: the equity method, operating leases, pension accounting or the creation of special-purpose entities (SPE) to place their debt there. They usually make use of flawed laws and regulations to "justify" their actions.

The equity method

When a company's shares in another company are greater than 20% and less than 50%, it should apply the equity method to consolidate the accounts. Specifically, the investor should include on its balance sheet the proportional net assets Assets less Liabilities) of the investee. The managers make sure that a net amount always goes to the left-hand side of the balance sheet, by underreporting the investee's liabilities. Moreover, in the case when the parent company controls the operations of another company (>50% shares), and it uses the equity method instead of the consolidation, it can usually be inferred that it is somehow hiding liabilities. In these cases, the readers of financial statements, have to make analytical analysis and compare the consolidated balance sheet and income statement to what is being reported by these firms.

Lease accounting

The lease is a contract that gives the lessee the right to use the lessor property for a certain amount of payments. Managers sometime report these leases as operating leases to avoid the recognition of higher liabilities. Statistically, by using lease accounting, managers can understate their firm's financial structure by 10 to 15 percentage basis points. However, leases are long-term contracts, different from operating leases, and as the rental period extends for a substantial time, it should have a considerate impact on the balance sheet of the firm. There should appear an asset in the left side of the balance sheet and the long-term liability in the right side of the balance sheet. Moreover, in the income statements, accountants should recognize not only the rental expense, but also interest expenses on the liability and the depreciation of the asset. The reason why managers try to "cheat" is because they tend to avoid high expenses in the early years of the lease, while capital leases show higher expenses in the early years of the contract life and lower expenses thereafter. Another reason would be the fact that as there is no asset recognized for operating leases, the ROA would appear inflated and attract the investors more.

The job of analysts, creditors and investors in this case, is to look carefully for footnotes in the financial reports, that show the true nature of the leases.

Pension accounting

There are a lot of companies, especially large companies, who promise employees pension benefits after working for a couple of years in the company. Everything related to these liabilities is important for the balance sheet, income statement and the cash flow. Literally, Pension expenses should entail all the below mentioned items:

$$\begin{aligned} &Pension\ expenses \\ &= Service\ cost\ +\ Interest\ on\ the\ projected\ benefit\ obligation \\ &- Expected\ return\ on\ plan\ assets \\ &+ The\ amortization\ of\ various\ unrecognized\ items (ex.prior\ service\ cost) \end{aligned}$$

However, the regulations on recognizing these elements are not very strict and allow for the netting of the projected benefit obligations and pension assets. What we find in the balance sheet is either the prepaid pension asset or the accrued pension cost. Managers most of the time try to exclude the prior service cost and the gains/losses on plan assets. They hide behind the illusion that the netting gives: assets will be able to cover pension obligations at any point, which in fact is not true.

Therefore, in order to get the true value of these liabilities, the interested parties should "unnet" them. A special attention should also be given to the assumed interest rates that were used in calculating the present value of these liabilities.

Special purpose entities

Usually, the purpose of a SPE is to act as an intermediary between a company and a group of creditors (investors). These creditors lend money to SPE and the SPE transfers money to the company, while the company transfers a sufficient amount of quality assets to the SPE. SPE uses the cash generated by these assets to pay off the debt to the creditors.

Illegally, companies make use of SPE's to hide debt, simply since the GAAP does not require firms to reveal the liability. These actions usually come in two forms: Securitization and Synthetic Leases.

Hypothesis

In this paper we will test our assumption about hidden liabilities in the balance sheets of the companies, including which may help to reduce a pricing gap between model output and real data.

4. METHODOLOGY AND DATA

4.1. Methodology

According to our thorough review of existing literature about credit risk pricing, we did not find any paper which investigated the potential impact of certain types of liabilities inclusion on the credit pricing models outcome. Therefore, our paper is a new contribution to the asset pricing research area.

In order to test our hypothesis about hidden liabilities in credit risk pricing, we decided to proceed with structural model, namely Merton model, due to its economic intuitiveness, ease of interpretation, and our existing knowledge of the Black & Scholes and Merton framework. As empirical test of Merton model showed its inability to generate sufficiently high-yield spreads to match ones observed in the market, a set of extensions and improvements were introduced to the original model (allowing for coupons, default before maturity, stochastic interest rates). Therefore, in our paper in addition to the original Merton model, we are also planning to use the extended version taken from Eom et al (2004) research on structural models (or possibly other simpler versions). This extension treats a coupon bond as if it were a portfolio of zero-coupon bonds, each of which can be priced using the zero-coupon version of the model. Moreover, it incorporates payout ratio (including dividends, share repurchases, interest paid, all divided by asset value) and default before maturity.

Both original and extended versions of the model use observable and unobservable variables. Observable variables include stock prices, bond`s coupons, time to maturity, government bond yield as risk free rate, dividends, interests paid, value of liabilities from firms` reports. Among unobservable variables there are asset value and volatility. For extended version we also need to specify default barrier and recovery rate. For default barrier we will assume the sum of current liabilities plus a half of long term liabilities (KMV approach, Crosbie & Bohn, 2003). As for the recovery rate, we are planning to estimate it empirically using a set of initially defaulted firms. The model is based on a multivariate OLS regression, which incorporate industry-, issuer-, as well as bond-specific information. This approach was taken from our colleagues, master students in NHH (master thesis, Ytterdal and Knappskog, 2015), however, with data used (Scandinavian market) they failed to find the model with a good explanatory and predictive power. Explanatory variables in this model are certain ratios from financial analysis (equity ratio, receivables, long term debt, intangibles, profitability ratios), distance to default and dummy variable (industry). Again, depending on the results and our new findings as for the recovery rate modeling, we will introduce some changes into this proposed model.

Alternatively, recovery rates can be taken from Moody`s data (Annual default study, 2017) – as historical average for the industry or specific to bond`s credit rating (if bond is not rated, we will try to assign to it a rating based on the probability of default), or simply we can follow the market convention of 51.31% (Keenan, Shtogrin and Sobehart, 1999). To arrive at the credit spread, we will take a difference of implied yield and risk free rate for each firm individually. Afterwards, we will try to aggregate our results and compare across industries and investment grade/junk classes.

Since time to maturity varies for each bond and risk free rates are rarely observed directly for non-standard maturities, we will use extended by Svendsson Nelson-Siegel model to interpolate and extrapolate the missing dates. In order to implement all calculations and regressions, we are planning to use R software and MS Excel.

4.2. Data

Following closely the Eom, Helwege and Huang (2004) paper, we will use US market and listed firms with simple capital structure. One way to calibrate the data is to take large capital companies which comprise the US market index S&P500, and test our assumption just on a plain-vanilla bond (if it exists), while assuming more complex bonds issued by the firm (convertible, callable, etc.) as a part of book value of debt. In this way we will artificially achieve a simplicity of capital structure.

Depending on the data availability, as an alternative we can take all listed companies in US excluding financial ones, since they may have abnormal leverage. Moreover, chosen firms should have a simple capital structure - with maximum two simple bonds outstanding, and five years of market stock data prior to the bond price observation (taken from Eom et al, 2004).

We will test our assumption on the data in a range from 1986 until today. Additionally, we exclude bonds with maturity of less than one year, since they are unlikely to trade (Warga, 1991). All data will be extracted from Datastream, CRSP, Compustat, Bloomberg.

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