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Executive Summary

This paper compares for the the first time to our best knowledge the forecasting performances of linear univariate and multivariate models for hourly day-ahead electricity prices. Our set of models includes AR and VAR models with only dummy variables for seasonality which are used as baseline for the corresponding formulations enlarged by including also forecasted demand and renewable energy generation, analysed from both the frequentist and the Bayesian perspective.

The accuracy of point and density forecasts are inspected in four main European markets characterized by different levels of renewable energy power generation. The analysis of these performances covers all 24 hours from 2015 to 2016.

The first important finding is that both AR and VAR specifications with demand and renewable energy dominate models without RES.

Secondly, the Bayesian approach leads to improvements in the univariate but also (and especially) in the multivariate models.

Thirdly, and for the first time since the increasing RES penetration, we show that the models with included forecasted demand and renewable energy power yield statistically more accurate forecasts in all studied countries. Therefore, we provide a strong empirical evidence of the influence of the renewable power generation during the day, and consistently with the country intra-daily profiles.

Finally and more importantly from an energy forecasting perspective, we conclude that these linear multivariate autoregressive models with renewable energy seem to have interesting and important advantages over the widely used univariate ones.

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Abstract

This paper compares alternative univariate versus multivariate models, probabilistic versus Bayesian autoregressive and vector autoregressive specifications for hourly day-ahead electricity prices, with and without renewable energy sources. The accuracy of point and density forecasts are inspected in four main European markets (Germany, Denmark, Italy and Spain) characterized by different levels of renewable energy power generation. Our results show that the Bayesian VAR specifications with exogenous variables dominate other multivariate and univariate specifications, in terms of both point and density forecasting.

Keywords: Density Forecasting; Electricity Market; Forecasting; Hourly Prices; Renewable Energies.
JEL codes: C11, C53, C55, Q42, Q47

1 Introduction

Despite the recent availability of high frequency data for forecasted demand and renewable generation, the literature on forecasting electricity prices with these exogenous variables is still relatively scarce. Therefore, we aim at filling this gap looking at linear models, in both univariate and multivariate frameworks, while comparing the probabilistic with the Bayesian approach and evaluating both point and density forecasts.

This paper shows that hourly prices can be efficiently predicted by taking advantage of intra-daily information available to market participants. We have indeed explored linear autoregressive (AR) and vector autoregressive (VAR) models with and without exogenous variables, which are fundamental drivers as forecasted demand, forecasted wind and solar power generation. These exogenous variables play an important role in formulating day-ahead conditional expectations, and their effects have motivated extensive research. Furthermore, in the last ten years, electricity generated from renewable energy resources (RES–E) has grown significantly thank to the political and financial support for these sources which may play an essential role in reducing country energy dependence (on imported fossil fuels), but, more importantly, in mitigating global warming (by reducing greenhouse gas emissions). The RES share of the total power capacity increased from 24% to 44% from 2000 to 2015 in Europe, reaching a total of more than 2,000 GW in 2016. Wind increased from 2.4% to 15.6% with a total generation around 300TWh covering more than 10% of EU demand. Denmark and Germany were among the leading countries for total wind

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power capacity per inhabitant. Notwithstanding ongoing economic crises, austerity measures and retroactive policy changes, Spain ranked second in Europe for total wind operating capacity (with more than 20GW in 2015); whereas, Italy contracted significantly its wind annual installations in 2016. However, high RES penetration levels were observed: 38% in Denmark, 19% in Spain, and 13% in Germany (with 11% from onshore and 2% from offshore). Instead lower shares were observed for solar PV which accounted for more than 7% in Italy, and 6.4% in Germany. The global solar PV capacity totalled an estimated 106 GW in Europe at the end of 2016, which is more than 32 times its capacity observed in 2006. Here, we find Germany, Italy and Spain belonging to the group of top 10 world countries for capacity and additions; with Germany and Italy being also the leaders for solar PV capacity per inhabitant (see REN21, 2017).

However, on the operational side, RES have added complexity to the management of the electricity system, and consequently to the electricity price modelling and forecasting because of their high variability and partial predictability. As a consequence, a growing literature has investigated the effects of RES on electricity price dynamics, in several markets all around the world (Europe, United States, Canada, and Australia).

Considering univariate models, Koopman et al. (2007) include water levels and power consumption when studying NordPool over the years 2000-2005, but they do not perform forecasting and do not consider RES given that their market penetration increased substantially only in subsequent years. Karakatsani and Bunn (2008a) include a comprehensive set of fundamental factors with nonlinear and time-varying formulations, accounting for distinct intra-day specifications in the British market. In addition, Karakatsani and Bunn (2008b) test out-of-sample predictions of linear and non-linear price models over the period 2001-2002. Chen and Bunn (2010) and Chen and Bunn (2014) consider the effect of fuel prices (natural gas, coal and carbon emission), drivers (as forecasted demand, reserve margin and imbalance spread), and market shares of major generators to compare the forecasting performances of the logistic smooth transition regression, linear regression and structural finite mixture regression models. Bunn et al. (2016) suggest a quantile factor model with volatility and several exogenous variables (such as forecasted demand, forecasted reserve margins, coal, gas and CO_2 prices) for the UK half-hourly data from 2005 to 2010. They provide better calibrated and more accurate forecasts using traditional coverage tests. Using the same dataset, Maciejowska and Weron (2016) show that forecasting accuracy of several univariate AR and multivariate factor models increases when natural gas, coal and demand are included. Gianfreda and Grossi (2012) study the Italian daily electricity zonal prices over 2006-2008 by means of Reg-ARFIMA-GARCH models with student-t innovations, accounting for technologies, market concentration, congestions and traded volumes and show that these factors increase the (point) forecasting performance of their enlarged models.

But all these studies did not include any forecasted renewable power generation. More recently, instead, several papers have analyzed the impact of RES on wholesale electricity price dynamics see Jónsson et al. (2010), Gelabert et al. (2011); Woo et al. (2011); Mauritzen (2013), Ketterer (2014), Paraschiv et al. (2014), Martinez-Anido et al. (2016), Pircalabu et al. (2017) and Rintamäki et al. (2017); among many others. And, it is worth to emphasize that most of the authors have modelled each hourly time series individually (that is 24 hourly time series separately), as in Misiorek et al. (2006) and in García-Martos et al. (2007); hence, ignoring the relationships among different hours of the day.

To our best knowledge, only few papers have compared the forecasting performances of different models when RES are included. Cruz et al. (2011) consider the forecasted wind power generation to empirically compare the predictive accuracy of univariate, multivariate, linear and nonlinear methods to forecast day-ahead electricity prices in Spain, while implementing only point and not considering density forecasting. They find higher accuracy when including explanatory variables (as forecasted load and wind generation). However, it is important to observe that forecasted solar power was not considered, since there was a low RES penetration in those studied years (2007-2008). Based on Weron and Misiorek (2008) who use temperatures to predict NordPool log-prices from 1998 to 1999, Kristiansen (2012) presents an autoregressive model with dummy variables for the days of the week, 'historical' demand and wind power as exogenous variables to compute price predictions for all 24 hours of a given day, using a more recent sample from 2004-2007 to 2011. Maciejowska and Nowotarski (2016) consider 24 separate models for each hourly (log)prices, using an autoregressive process with exogenous terms including historical prices (at lags 1 and 7), a constant plus a weekend dummy, current and lagged system (log)loads (again at lags 1 and 7) together with the ratio of the smallest to the largest load for a given day. Furthermore, the last prices (at hours 23 and 24) from the previous day are included in the models for the early morning hours (up to 4 a.m.).

Therefore, following Conejo et al. (2005), Misiorek et al. (2006), and Maciejowska and Weron (2015), we select AR models as benchmarks because of their widespread use in the literature and their relatively good performance in predicting electricity prices. But, we also consider VAR representations to detect improvements in the forecasting performances. Indeed, we expect better forecasts from multivariate than univariate models given the larger information contained in a panel of data, as suggested by Stock and Watson (2002). We aim at pushing these models forward by including among regressors the forecasted demand and RES generation in both our univariate AR and multivariate VAR models; hence managing a total of 158 parameters for each hour.

Moreover, as emphasized by Weron (2014) and Nowotarski and Weron (2018), there is an increasing interest in electricity price forecasting, but only few studies have considered the density forecasting properly. Among their references, we find Panagiotelis and Smith (2008), Serinaldi (2011), Huurman et al. (2012), and Jónsson et al. (2014). Serinaldi (2011) forecasts only the prediction intervals, obtained as quantiles of the density forecasts for Californian and Italian hourly electricity prices ranging between years 1999-2000 (for the US market) and between 2004-2006 (for the EU market). Recently, a similar approach has been used in Gianfreda and Bunn (2017) to investigate the shape-shifters of German hourly electricity prices, when considering also forecasted renewable energy from 2010 to 2016. However, they use the forecasting performance of their 'multi-factor skew t' models at selected quantiles only to test their models against overfitting. Another recent paper by Pape et al. (2017) study the German market, exploring the dynamics of hourly electricity prices during 2013-2014 trying to characterize and forecast the price distributions in 2015, while accounting for the forecasted residual load (as difference between 'actual' load and forecasted solar in-feed), the forecasted wind power generation, and two variables representing costs for coal and gas (computed considering plant efficiency, emission intensity, fuels and CO_2 emission prices). Again, the hourly electricity prices are analyzed separately by means of 24 multiple regressions with residuals inspected for common factors and for conditional heteroskedasticity. Whilst, in our paper, we focus on Bayesian models and on multivariate models. Moreover, we exploit more in details the different forecasting measures of accuracy, such as point and density forecast.

On the other hand, Huurman et al. (2012) also perform properly density forecasting of Scandinavian day-ahead electricity prices by means of GARCH-type time-varying volatility models. Using the probability integral transform scores, the Berkowitz's likelihood ratio test and the Kullback-Leibler information criterion, they provide interesting evidence that models augmented with weather forecasts statistically outperform specifications which ignore this information. Jónsson et al. (2014), instead, use a different framework of a parametric GARCH and a time-adaptive quantile regression model (for the 5%–95% quantiles) with an exponential distribution for the tails. They generate only prediction densities of day-ahead electricity prices in Western Denmark from 2008 to 2011 using hourly forecasts for system load and wind power production; and, assess their forecasts by computing the average Continuous Ranked Probability Score (CRPS) and the related Continuous Ranked Probability Skill Score (CRPSS).

More close to our approach, Panagiotelis and Smith (2008) propose a VARX model for hourly electricity prices in Australia from 2003 to 2006. They use a first order vector autoregressive model with exogenous effects accounting for a parsimonious dependency structure in which the current price depends on the previous two, and also on the price observed at the same time the day before. However, given the low RES penetration in those studied years, they only consider variables to capture seasons, day types and trend; finally, the forecasting performance is evaluated using the continuous ranked probability score.

More recently, but without accounting for fundamental drivers and looking only at point forecasts, Raviv et al. (2015) compare the performances of models for the full panel of 24 hourly prices studying NordPool from 1992 to 2010. Based on univariate AR and multivariate VAR models, they compute forecast combinations and empirically demonstrate that the useful predictive information contained in disaggregated hourly prices improves the forecasts of multivariate models. They show that shrinking VAR models leads to further better forecasts, with the Bayesian VAR outperforming the unrestricted VAR. However, no density forecasting is performed and no RES are included in their models, as in Ziel and Weron (2018). They propose 58 multi-parameter regression univariate and multivariate models accounting for different forms of seasonality. They straggle to show the uniform superiority of multivariate specifications across all 12 studied markets, seasons or hours. More specifically and close to our analysis, they conclude that in Spain more often the multivariate specification outperforms the univariate in the morning hours, whereas in Germany and in the two Danish zones the univariate more often outperforms the multivariate in the late evening/night hours. These results are however depending on the specifications of their models and may produce different results if forecasted demand and RES generation are included. Therefore, this further supports our investigation and we aim at providing even more clear evidence on linear univariate and multivariate forecasting performances comparing probabilistic and Bayesian models, when more complexity is induced by uncertain and intermittent renewable generation.

Our results show that demand and renewable energies improve the point and density accuracy of the predictive models, especially during peak hours. Moreover, we find evidence of better forecasting of Bayesian univariate and multivariate models with respect to probabilistic ones.

The paper proceeds as follows. Section 2 contains the details on the data used. Section 3 presents our models and estimation methodology, whereas details on priors are provided in the Appendix A. Section 4 presents the metrics used to assess our results and explains in details the major findings, and finally Section 5 concludes.

2 Data Description

We use hourly day-ahead prices (in levels) to estimate models for electricity traded/sold in Denmark, Germany, Italy and Spain. These markets are particularly interesting given their high levels of RES penetration.

We obtained national electricity prices directly from the corresponding power exchanges. We collected the German hourly auction prices of the power spot market from the *European Energy Exchange* EEX. We averaged the two hourly zonal prices from *Nordpool* for Denmark. Whereas, the hourly *single national prices* PUN were collected from the Italian ISO, *Gestore dei Mercati Energetici*, (GME); and, the *precios horario del mercado spot diario* for Spain from the *Operador del Mercado Ibérico, Polo Español*, (OMIE).

As main drivers, we considered both supply and demand sides. As far as the supply side is concerned, we consider here only the forecasted renewable generation and disregarded fossil fuels since they hold a constant structure over the 24-hour daily horizon (and this adds operational problems as explained later for solar). Forecasted values for supply and demand were downloaded directly from the corresponding TSOs, apart for the German and Italian forecasts which were instead provided by Thomson Reuters at hourly frequency. In these two latter cases, the results from two weather providers (the *European Centre for Medium-Range Weather Forecast* - EC or ECMWF - and the *Global Forecast System* - GFS - of the American weather service of the National Centers for Environmental Prediction) have been inspected. Both use two types of weather models: the *operational* one, which is deterministic with no involved randomness and high resolution; and, the *ensamble* one, which is a probabilistic model with lower resolution and variations around the initial set of weather conditions, hence providing different weather scenarios and consequently an idea of the weather instability.

Both providers use one single run for the operational model and different runs for the ensamble at specific hours. We decided to use only the EC operational model running at 00, because this updates from 05.40 a.m. to 06.55 a.m., hence representing the latest information available to market operators to formulate their day-ahead bidding strategy.

While demand forecast models make use of weather forecasts accounting for temperature, precipitation, pressure, wind speeds, and cloud cover or radiation; forecasted wind values are obtained using the information on wind speeds and installed capacity. Finally, forecast solar power production only considers PV installations, solar radiation and installed capacity, given the predominance of photovoltaic plants over solar thermal ones. It is worth to recall that the time series for solar power exhibits a block structure of null values in hours early in the mornings and late in the evenings, hence we pre-processed these series by linear transformation.[1]

To summarize, we use hourly data for prices, forecasted demand, wind and solar PV generation from 01 January 2011 to 31 December 2016 for Germany and Denmark, whereas from 13 June 2014 to 13 June 2017 for Italy and Spain. We use the first four years as estimation sample for Germany and Denmark, and the first two for Italy and Spain, whereas we use the last two/one years as forecast evaluation period. The historical dynamics of these series observed in Germany are reported in Figure 1 (see Figure 3, 4 and 5 for the other countries). Prices clearly show the new stylized fact of "downside" spikes together with mean-reversion, whereas forecasted demand and solar generation exhibit more clear seasonal patterns, with increasing trend for solar power generation according to the new capacity additions through years. On the other hand, forecasted wind shows more variability due to weather conditions albeit with an increasing trend corresponding to investments in new capacity.

Following the literature and considering the monthly profiles for electricity prices, forecasted demand, wind and solar generation, we confirm clear monthly seasonal pattern (these graphs are omitted for lack of space). Furthermore, the intra-daily profiles for demand and RES-E generation clearly exhibit scenarios of high/low demand and/or RES-E which affect price dynamics and influence forecasts. To this aim, these dynamics are presented in Figure 2 across drivers and markets. We can observe that the ramp-up hours (during which the demand for electricity is expected to grow substantially) as well as the ramp-down hours (when demand is expected to sharply decrease) change across markets according to day- and night-time and geographical locations. However, they confirm higher demand levels in peak period (roughly between 8 a.m. and 8 p.m. for all markets). The intra-daily profiles for wind show instead different dynamics: we can again identify scenarios for high wind generation during peak hours in Denmark and Italy, while the opposite occurs in Germany and Spain. Obviously, the intra-daily profiles for solar PV generation is instead common for all markets, where available. Therefore, we can expect a stronger combined effect of high demand-wind in Denmark, and high demand-wind-solar in Italy; but contrasting scenarios for demand-wind-solar during the day in Germany and Spain: a low-high-low one (low demand-solar versus high wind) in the early and late hours versus high-low-high one (high demand-solar versus low wind) for peak hours.

3 Forecasting Models

In this section, we introduce our different models to forecast hourly day-ahead electricity prices. As anticipated, we consider univariate and multivariate models for hourly prices with the introduction

[1]For solving this problem, we draw from an Uniform, $\mathcal{U}(0+\delta, 0+\varepsilon)$, where $(\delta, \varepsilon) > 0$ are small number in the neighbourhood of zero. Instead of having columns of zeros, we have columns of different small values closed to zero. Similar problems could be experienced with fuel prices since they hold a constant value across all 24 hours.

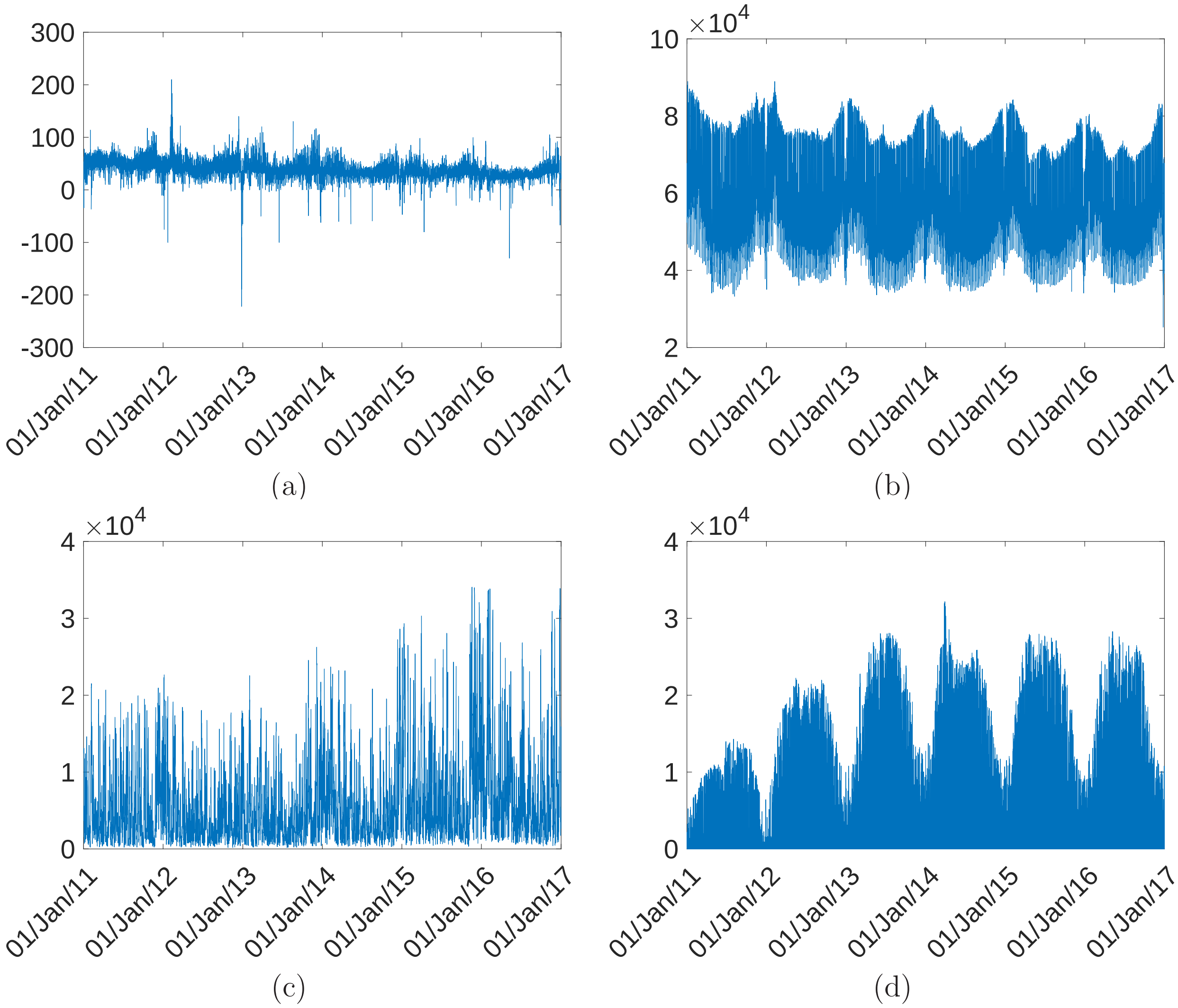

Figure 1: Hourly Series for Electricity Day-ahead Prices (panel a), Forecasted Demand (panel b), Forecasted Wind Generation (panel c) and Forecasted Solar PV Generation (panel d) observed in Germany from 01/01/2011 to 31/12/2016.

of exogenous variables relative to the forecasted demand and forecasted electricity generated by renewable energy sources.

We write our univariate and multivariate models with a general lag order p, which is commonly set equal to 7 given the strong persistence and the weekly seasonality observed in electricity prices.However, following Knittel and Roberts (2005), Weron and Misiorek (2008) and Raviv et al. (2015), we include only the first, second and the seventh lag of the hourly prices (that is we adopt a reduced 7-lag structure) and, with an abuse of notation, in the remainder of the paper, $p = 3$ is used to denote the number of included lags instead of the maximum lags in all our univariate and multivariate models.

3.1 Multivariate Models

We consider and compare the performances of two different multivariate model specifications with and without exogenous variables, used as benchmarks for the corresponding multivariate models. These are the vector autoregressive model (VAR), the vector autoregressive model with exogenous variables (VARX), and their Bayesian formulations (BVAR and BVARX, respectively) with two different priors.

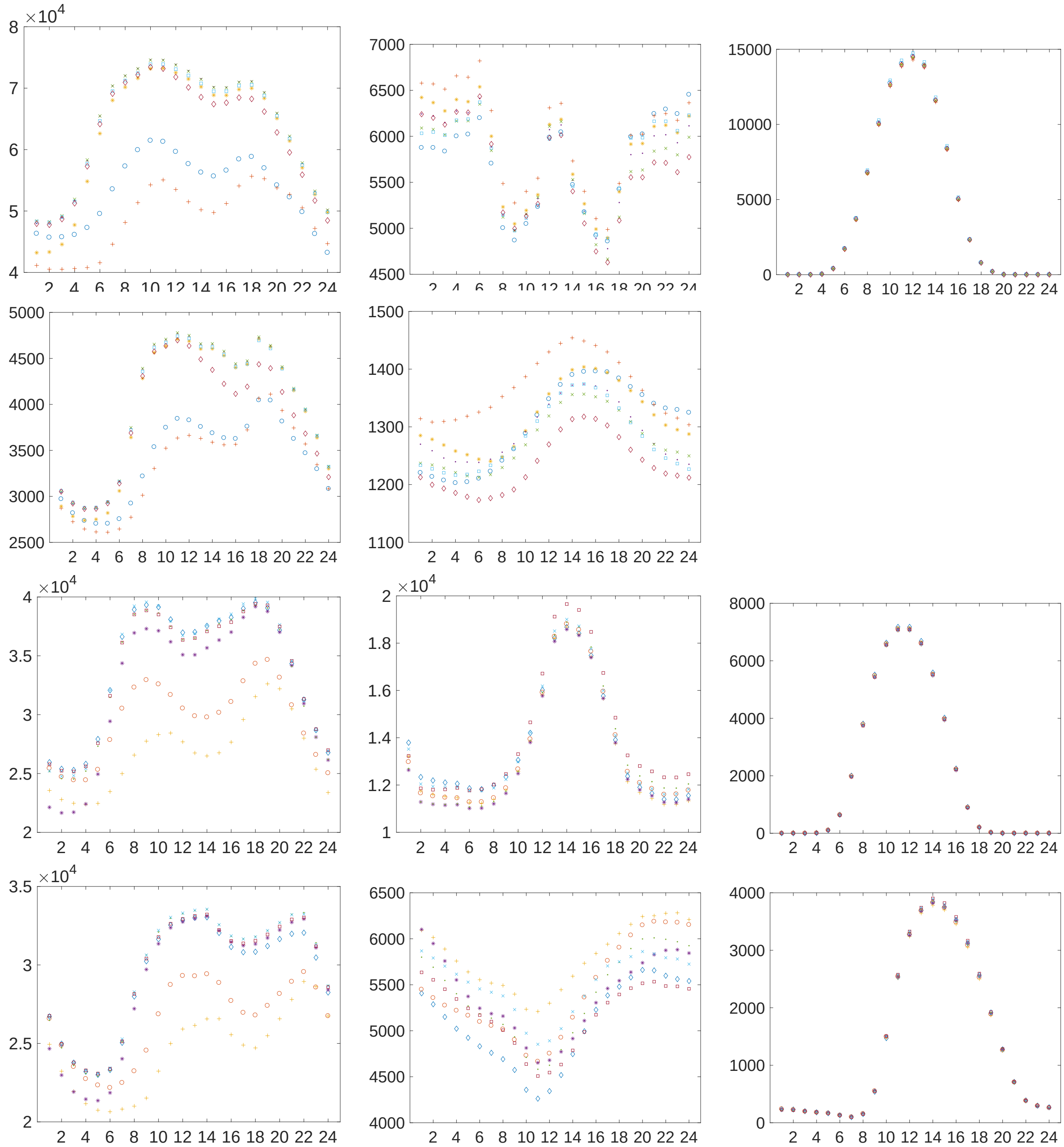

Figure 2: Intra-daily profiles of different days of the week for Forecasted Demand (on the left), Forecasted Wind Generation (in the middle) and Forecasted Solar PV Generation (on the right) in Germany (first row), Denmark (second row), Italy (third row) and Spain (last row). [Saturday ($\circ$), Sunday ($+$), Monday ($\star$), Tuesday ($\bullet$), Wednesday ($\times$), Thursday ($\square$), Friday ($\diamond$)].

3.1.1 Vector Autoregressive Model – VAR

Let $\mathbf{y}_t = (y_{1t}, \ldots, y_{Ht})'$ denote the $(H \times 1)$ vector of hourly electricity prices, with $H = 24$. Moreover, we denote with $\mathbf{d}_t = (d_{1t}, \ldots, d_{Kt})'$ the $(K \times 1)$ dummy vector with $(d_{1t}, \ldots, d_{12t})$ representing the twelve months of the year and (d_{13t}, d_{14t}) representing Saturdays and Sundays, hence $K = 14$. The VAR model of order p is formulated as follows:

$$\mathbf{y}_t = \Phi' X_t + \mathbf{e}_t, \quad t = 1, \ldots, T, \tag{1}$$

where Φ is the $((Hp+K) \times H)$ matrix containing the autoregressive coefficients as well as the coefficients for all dummy variables, and $X_t = (\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}, \mathbf{d}_t)$ is the matrix $((Hp+K) \times H)$ made by the lagged electricity prices and the dummy variables. The vector of errors $\mathbf{e}_t$ is assumed to be serially uncorrelated and normally distributed with zero mean and covariance matrix Σ.

3.1.2 Vector Autoregressive Model with Exogenous Variables – VARX

The vector autoregressive model with exogenous variables (VARX) includes the forecasted demand, as well as forecasted wind and solar power generation, when availables. These exogenous variables are represented by the following vectors of dimensions $(H \times 1)$, $\mathbf{x}_t = (x_{1t}, \ldots, x_{Ht})'$, $\mathbf{z}_t = (z_{1t}, \ldots, z_{Ht})'$ and $\mathbf{w}_t = (w_{1t}, \ldots, w_{Ht})'$, respectively. From the formula defined in equation 1, we re-define the matrix X_t and consequently the matrix of coefficients Φ of size $((Hp+K+3H) \times H)$ as follows:

$$X_t = (\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}, \mathbf{d}_t, \mathbf{x}_t, \mathbf{z}_t, \mathbf{w}_t),$$

The matrix X_t is now composed by the vector of lagged hourly electricity prices, and by the vectors of the dummy and the exogenous variables.

From equation 1, since the observations are varying with time $t = 1, \ldots, T$, the VAR and VARX models of order p can be rewritten in a compact way

$$\mathbf{Y} = \mathbf{X}\Phi + \mathbf{E}, \tag{2}$$

where $\mathbf{Y} = (\mathbf{y}'_1, \ldots, \mathbf{y}'_T)$ is an $(T \times H)$ matrix, $\mathbf{X} = (X_1, \ldots, X_T)'$ is the $(T \times (Hp + K + 3H))$ matrix of explanatory variables containing all the exogenous variables[2]). The $(T \times H)$ error matrix $\mathbf{E} = (\mathbf{e}'_1, \ldots, \mathbf{e}'_T)$ is normally distributed and serially uncorrelated with covariance matrix Σ.

3.1.3 Bayesian Vector Autoregressive Models – BVARs

Our multivariate models with or without exogenous variables have been additionally estimated with the Bayesian methodology. From equation 2, a Bayesian Vector Autoregressive model (BVAR or BVARX) has the following stacked form:

$$\mathbf{y} = (I_H \otimes \mathbf{X})\boldsymbol{\alpha} + \boldsymbol{\varepsilon}, \tag{3}$$

where $\boldsymbol{\alpha} = \mathrm{vec}(\Phi)$, $\mathbf{y} = \mathrm{vec}(\mathbf{Y})$ are vectorized matrices, $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \Sigma \otimes I_T)$ with I_T being a T-dimensional identity matrix.

In particular, we define prior information on the matrix of coefficients and on the covariance matrix using two different priors: the Minnesota and the conjugate Normal-Wishart prior; with models indicated respectively with BVAR-Min or BVARX-Min, and with BVAR-NW or BVARX-NW. Details on both priors are reported in the Appendix A.

3.2 Univariate Models

For all previous models, we formulate the univariate AR specification as in the corresponding VAR specifications with the same assumptions on the lag order whereas the errors are assumed to be normally distributed with zero mean and σ_h^2, $h = 1, \cdots, 24$, variance. The autoregressive model with only dummy variables is used as benchmark in the forecasting comparisons. Even in the univariate case, we use both the frequentist and the Bayesian estimation procedures.

[2]We have also performed the forecasting exercises including the lags (1,2, and 7) for exogenous variables, but the results were unchanged although time-demanding. For these reasons and having proper forecasts, we prefer to adopt the former formulations without lagged exogenous variables.

4 Results

Our results are based on one-step-ahead forecasting process with a rolling window approach of 4 years for Germany and Denmark and of 2 years for Italy and Spain. Hence, we have two different forecast evaluation periods: the last two years from 01 January 2015 to 31 December 2016 (for Germany and Denmark, hence 731 observations), whereas only the last year from 14 June 2016 to 13 June 2017 for Italy and Spain (hence, only 365 observations given the unavailability of older historical data). Furthermore, for the multivariate models, we also run a model 'restricted' to only the peak hours in which we have both forecasted wind and solar power generation.

Before evaluating the out-of-sample results, our in-sample evidence provides statistically significant coefficients for the RES variables in all markets; hence confirming the empirical findings in previous literature on univariate models augmented with RES variables, and extending similar conclusions also to multivariate models. In particular, coefficients of wind and solar are negative in Germany, Italy and Spain. Also in Denmark, wind has a negative coefficient. These results confirm that renewable energy sources are significantly connected to and reduce electricity prices. Therefore, we continue our analysis by investigating whether these relationships can result in forecast gains.

The following results show the performance of our different univariate and multivariate models from the simplest ones with only dummy variables (the benchmarks) to more complex ones with forecasts for demand, wind and solar for Germany, Italy and Spain; whereas, no solar PV power is available in Denmark.

We assess the goodness of our forecasts using different point and density metrics. Considering the accuracy of point forecasts, we use the root mean square errors (RMSEs) for each of the hourly prices as well as the RMSEs on the daily average and on a restricted daily average, as specified below. The root mean square error for $h = 1, \ldots, 24$ hourly price is computed as

$$\mathrm{RMSE}_h = \sqrt{\frac{1}{T-R} \sum_{t=R}^{T-1} \left(\hat{y}_{h,t+1|t} - y_{h,t+1}\right)^2}, \tag{4}$$

where T is the number of observations, R is the length of the rolling window and $\hat{y}_{h,t+1|t}$ are the individual hourly price forecasts. In addition, we analyse the average root mean squared errors on all the 24 hours ($\mathrm{RMSE}_{\mathrm{Avg}}$) and on the hours from 8 a.m. to 8 p.m. (peak hours, $\mathrm{RMSE}^P_{\mathrm{Avg}}$), computed as follows:

$$\mathrm{RMSE}_{\mathrm{Avg}} = \frac{1}{24} \sum_{h=1}^{24} \mathrm{RMSE}_h, \tag{5}$$

$$\mathrm{RMSE}^P_{\mathrm{Avg}} = \frac{1}{13} \sum_{h=8}^{20} \mathrm{RMSE}_h. \tag{6}$$

To evaluate density forecasts, we use both the average log predictive score and the average continuous ranked probability score (CRPS). The log predictive score is commonly viewed as the broadest measure of density accuracy, see Geweke and Amisano (2010). We compute it for each hour h as follows

$$s_{h,t}(y_{h,t+1}) = \ln\left(f(y_{h,t+1}|I_{h,t})\right), \tag{7}$$

where $f(y_{h,t+1}|I_{h,t})$ is the predictive density for $y_{h,t+1}$ constructed using information up to time t and hour h. We construct similarly the average log predictive score of the 24 hours and of the 'restricted' hours on day $t+1$.

As indicated in Gneiting and Raftery (2007) and Gneiting and Ranjan (2011), some researchers view the continuous ranked probability score as having advantages over the log score. In particular, the CRPS does a better job of rewarding values from the predictive density that are close to - but not equal to - the outcome, and it is less sensitive to outlier outcomes. The CRPS, defined such that a lower number is a better score, is given by

$$\mathrm{CRPS}_{h,t}(y_{h,t+1}) = \int_{-\infty}^{\infty} \left(F(z) - \mathbb{I}\{y_{h,t+1} \leq z\}\right)^2 dz = E_f|Y_{h,t+1} - y_{h,t+1}| - 0.5E_f|Y_{h,t+1} - Y'_{h,t+1}|,$$

where F denotes the cumulative distribution function associated with the predictive density f, $\mathbb{I}\{y_{h,t+1} \leq z\}$ denotes an indicator function taking value 1 if $y_{h,t+1} \leq z$ and 0 otherwise, and $Y_{h,t+1}$ and $Y'_{h,t+1}$ are independent random draws from the posterior predictive density. In the same way we can construct the average CRPS of the price over the 24 hours and of the restricted price for the peak hours on day $t+1$.

More specifically, in our tables, we report the RMSEs, average log scores and average CRPS for both the baseline AR and VAR models. For the other AR (VAR) models, we report: the ratios of each model's RMSE to the baseline AR (VAR) model, such that entries less than 1 indicate that the given model yields forecasts more accurate than those from the baseline; differences in score relative to the AR (VAR) baseline, such that a positive number indicates a model beats the baseline; and ratios of each model's average CRPS relative to the baseline AR (VAR) model, such that entries less than 1 indicate that the given model performs better.

To provide a rough gauge of whether the differences in forecast accuracy are significant, we apply Diebold and Mariano (1995) t-tests for equality of the average loss (with loss defined as squared error, log score, or CRPS)[3] The differences in accuracy that are statistically different from zero are denoted with one, two or three asterisks, corresponding to significance levels of 10%, 5% and 1%, respectively. The underlying p-values are based on t-statistics computed with a serial correlation-robust variance, using the pre-whitened quadratic spectral estimator of Andrews and Monahan (1992). Our use of the Diebold-Mariano test with forecasts from models that are, in many cases, nested is a deliberate choice as in Clark and Ravazzolo (2015). We report p-values based on one-sided tests, taking the AR (VAR) as the null and the other current models as the alternative.

4.1 Point Forecasts: RMSEs

Looking at the evaluation of our point forecasts, the results in terms of RMSEs are presented in Tables from 1 to 4 for Germany, Denmark, Italy and Spain, respectively.

Considering the average errors (RMSEs_{Avg}) and consistently across Germany, Denmark and Spain, we can firstly observe that they decrease when moving from the univariate to the multivariate case, as suggested by Stock and Watson (2002) and anticipated by Raviv et al. (2015). Further important reductions are found for the alternative enlarged and Bayesian univariate and multivariate specifications. We detect statistically substantial low errors in Spain when the enlarged Bayesian multivariate models are used. In addition, we observe further improvements in Germany, where the average RMSEs move from 8.259 €/MWh in the univariate case to 6.839 €/MWh in the multivariate case; with further reductions especially for Bayesian multivariate specifications. On the contrary, we observe lighter forecast improvements in Italy.

Secondly, looking at the values of RMSEs across the 24 hours, we confirm that those by simple multivariate models are much lower than those produced by univariate ones across all hours and markets. However, in Italy we notice that multivariate errors are higher than univariate ones at hours 23 and 24. Furthermore, they are higher during peak hours in Germany and Italy, whereas in Denmark we observed the highest error values at hours 8-12 and 18-19 and no special hourly pattern in Spain.

Thirdly, we observe slight improvements across all markets when the Bayesian approach is used in both univariate and multivariate cases (that is in BARs versus AR, and BVARs versus VAR models).

[3] In our application for testing density forecasts, we use equal weights and not adopt a weighting scheme, as in Amisano and Giacomini (2007).

Fourthly, when forecasted demand and RES are included and the Bayesian approach is used, we are able to find substantial improvements during hours 8-24 in Germany, Denmark, Italy; whereas in Spain there are relevant for all hours. However, the advantage of VARX and BVARX over the VAR model tends to be larger and more significant during hours 8-24. In fact, early in the morning, when there is a low generation of wind (and solar) as observed in Denmark (and Italy), the gain in adding these variables is null, hence making the baseline models preferred to the corresponding alternatives. Instead, during peak hours, RMSEs strongly decrease if the demand and renewable energies are introduced. We also note that the errors are higher in Denmark and Italy, and we believe that these results are due to the lack of or limited solar generation in these countries (which then reduces the information available). Whilst in Germany and Spain, solar PV decreases the RMSE for AR and consequently the ratios.

Fifthly, we empirically show that generally the Minnesota prior outperforms the Normal-Wishart; especially in Spain and Italy, where, for instance, the ratio BARX-Min/AR and BVARX-Min/VAR are 0.972 and 0.878 at 13 (with respect to RMSEs of 6.389 and 6.144 respectively).

Finally, when we restrict our models only to peak hours (hence focussing on hours during which we have forecasted solar), the performances of our models are in line with previous results, supporting our solar data preprocessing.

In these point forecasts, the gains in the forecast accuracy provided by adding (forecasted demand and) renewable energies can be explained by considering the high RES penetration in all these countries; and it is well known that the intra-daily solar generation (as well as wind) acts as negative demand, hence it can be considered as new and relevant information supporting the forecasting.

4.2 Density Forecasts: Log Predictive Scores

The results in Tables 5-8 for log predictive scores firstly indicate that they increase when moving from the univariate to the multivariate case. Furthermore, we observe that the log scores generally decrease from early to peak hours, to increase again towards the end of the day.

Secondly and as found for the RMSEs, we observe accuracy gains across all markets when the Bayesian approach, in both univariate but especially in multivariate cases.

Thirdly, when forecasted demand and RES are included and the Bayesian approach is used, the accuracy of density forecasts relative to models without RES substantially improve across all hours in Germany and Spain, and from 8 in the morning in Italy and Denmark. In particular, the relative gains in average predictive scores are typically larger than the differences in RMSEs. As an example, in Germany at hour 15 the BARX-NW model improves on the RMSE of the AR baseline by 23%, while the BARX-NW model has a predictive score that is 46% higher than that of the AR specification. On the other hand, for Denmark, there is an increment in the log score for forecasts of model with RES, but lower with respect to Germany; given our previous considerations on intra-daily demand and RES profiles.

Overall, these results confirm that including renewable energies in univariate autoregressive models typically yields sizable gains in density accuracy as measured by log scores. On the other hand, their inclusion in multivariate models lead to gains in density accuracy during the peak hours as measured with the point forecast measures, in particular for Denmark.

4.3 Density Forecasts: CRPS

Looking at this metrics in Tables 9-12, we observe again that the accuracy of density forecasts improve in the multivariate models, where we observe substantial low CRPS across all hours and markets. Indeed the average values, computed over the 24 hours, decrease by 82% (from 4.427 to 3.643) in Germany, by 87% (from 4.901 to 4.273) in Denmark, by 95% (from 3.658 to 3.469) in Italy, and by 80% (from 3.517 to 2.831) in Spain.

As before, we observe higher CPRS values in peak hours in Germany and Italy, whereas in Denmark only across hours 8-12 (when the demand is at its highest values); and no specific hourly trend is detected in Spain.

The Bayesian approach leads to improvements in the univariate but especially in the multivariate models. Indeed, in Germany, we observe the average ratio of CRPS of BARs over the AR baseline model (and similarly for the ratio between BVARs over the VAR baseline) equal to 0.999 (with respect to 4.427 for AR, and with respect to 3.643 for VAR); similar results are found in the other markets. However, when demand and RES are included, the ratios are further and substantially reduced in Germany, Denmark and Spain. In Germany, the new average values are equal to 0.80 for the BARXs, and to 0.87 for the BVARXs; in Denmark, we find 0.91 and 0.97, respectively; and in Spain, with 0.83 for both BARXs and BVARXs. Instead, in Italy, they are less substantial on average, but the improvements at hourly levels are remarkable, especially at hours 13-15.

In further hourly details, in Germany, the ratio of the CRPS of the Bayesian and frequentist ARX model relative to the AR model is 0.854 (at 4) or 0.741 (at 13). In the same way, in Denmark, the CRPS for the benchmark AR model is higher with respect to Germany, but as above, the ratio of the CRPS of ARX model relative to the AR model is significant and equal to 0.968 (at 4) or 0.919 (at 13). As a result, as the forecasted hours increases (in particular during the peak hours), the models with RES do not improve in relative terms under the CRPS measure as they did under the log score measure. This pattern reflects the fact that the CRPS is less sensitive to outliers.

With VAR models, the ratio of the VARX CRPS, both Bayesian and frequentist, to VAR CRPS is 1.035 (at 4), while it is 0.799 at 12 in Germany. On the other hand, in Denmark, the models with RES are dominated during early morning by VAR baseline with CRPSs that exceed the baseline by 11%. This percentage decreases during the peak hours, where the models with renewable energies improve the CRPS of the VAR baseline of 10% or more.

The reduced models confirm that demand and renewable energies improve the density accuracy. Once again, adding these exogenous variables to the univariate and multivariate models typically improves forecast accuracy, especially during peak hours[4]

5 Conclusions

This paper compares for the the first time to our best knowledge the forecasting performances of linear univariate and multivariate models with enlarged specifications. Our set of models includes AR and VAR models with only dummy variables for seasonality which are used as baseline for the corresponding formulations enlarged by including also demand and renewable energy sources, analysed from both the frequentist and the Bayesian perspective.

Our analysis of point and density forecasting performances cover all 24 hours from 2015 to 2016. Our results indicate that the AR and VAR specifications with demand and renewable energy dominate models without RES, in terms of both point and density forecasting.

The first important finding is that the Bayesian approach leads to improvements in the univariate but also (and especially) in the multivariate models; with the Minnesota prior outperforming the conjugate Normal-Wishart.

Secondly and for the first time since the increasing RES penetration, we show that the models with included forecasted demand and renewable energy power yield statistically more accurate point and density forecasts in all studied countries. Specifically, univariate models which further include these exogenous drivers (that is forecasted demand and RES-E in addition to seasonal dummies) provide forecasts statistically more accurate than those produced by the equivalent models with only monthly and weekday dummy variables during all day, consistently across the

[4]These results are confirmed by graphical evidence of the interval forecasts, represented by 5th, 25th, 50th 75th and 95th percentile of the predictive densities.

frequentist and Bayesian approach. In other words, during all hours of the day, and in particular during the peak hours, the frequentist and Bayesian ARX forecasts are more accurate than those of the corresponding frequentist and Bayesian AR models; and similar results hold for the multivariate models.

Therefore, we provide a strong empirical evidence of the influence of the renewable power generation during the day, and consistently with the country intra-daily profiles. In fact, during the first hours of the day, the models without forecasted RES generation are more accurate than those with them, and again with errors from multivariate models lower than those from univariate ones. Whilst, the increasing RES generation during the day leads to more accurate forecasts from augmented models. Furthermore, our results are consistent across all adopted scoring rules, such as the RMSRs, the average log predictive scores and the CRPS.

Finally, we conclude that from an energy forecasting perspective these linear multivariate autoregressive models with renewable energies seem to have interesting and important advantages over the widely used univariate ones.

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A Prior Information

We estimate all the Bayesian models described in Section 3 using the Bayesian Markov chain Monte Carlo (MCMC) methods. We have used the Gibbs sampling algorithm for both prior distributions and all our results are based on samples of 6.000 posterior draws, with a burn-in period of 1.000 iterations.

Hereafter, we will describe the two different priors used in the Bayesian models.

A.1 The Minnesota Prior

In this case, we make some restrictions on the coefficients, but these restrictions are less strict and are in the form of shrinkage or regularization. In particular, we shrink the coefficient estimates by combining their unconstrained least squares estimates with certain prior distribution. As said above, we choose the Minnesota prior distribution (Doan et al., 1984; Litterman, 1986) on the vector of coefficients $\boldsymbol{\alpha}$. The usual representation of the Minnesota prior is a normal distribution, $\mathcal{N}(\underline{\boldsymbol{\alpha}}, \underline{\mathbf{V}})$, with prior mean $\underline{\boldsymbol{\alpha}}$ and prior covariance matrix, $\underline{\mathbf{V}}$. In particular, we specify the prior mean and variance such that the estimates are shrunk towards a random walk specification for the individual hourly prices.

For the prior mean, $\underline{\boldsymbol{\alpha}}$, the Minnesota prior involves setting most or all of its elements to zero. In our case, we set all the elements to zero, except for the element corresponding to the first own lag of the dependent variable in each equation, which are set to 0.9 for simplicity (as in Koop and Korobilis (2010)).

The Minnesota prior assumes the prior covariance matrix $\underline{\mathbf{V}}$ to be diagonal. If we let $\underline{\mathbf{V}}_h$ be the block associated with the coefficients in equation h and $\underline{\mathbf{V}}_{h,jj}$ be its diagonal elements $(i = 1, \ldots, M)$, then a common implementatin of the Minnesota prior would set:

$$\underline{\mathbf{V}}_{h,jj} = \begin{cases} \frac{\lambda_1}{r^2} & \text{for coefficients on own lags for lag } r = 1, \ldots, p, \\ \frac{\lambda_2}{r^2} \cdot \frac{\sigma_i}{\sigma_h} & \text{for coefficients on cross-lags of } y_{it} \text{ for lag } r = 1, \ldots, p, \\ \lambda_3 \cdot \sigma_h & \text{for coefficients on dummy and exogenous variables.} \end{cases} \tag{8}$$

We estimate σ_h recursively at every time point using the standard error of the residuals from an univariate regression model for each of the 24 hourly prices. The ratio $\frac{\sigma_i}{\sigma_h}$ adjusts for differences in the units that the variables are measured in. Moreover, this form captures the sensible properties that, as lag length increases, coefficients are increasingly shrunk towards zero and that own lags are more likely to be important predictors than lags of other variables. Following the choiche of Koop and Korobilis (2010), we set $\lambda_1 = \lambda_2 = 0.5$ and $\lambda_3 = 100$, which means that the coefficients of the exogenous variables are not shrunk and their estimated coefficients are equal to the least squares estimates.

An advantage of the Minnesota prior is that it leads to simple and well known posterior distribution for the coefficients. In particular the posterior distribution for $\boldsymbol{\alpha}$ is given by:

$$\boldsymbol{\alpha}|\mathbf{y} \sim \mathcal{N}(\overline{\boldsymbol{\alpha}}, \overline{\mathbf{V}}), \tag{9}$$

where

$$\overline{\mathbf{V}} = \left[(\underline{V})^{-1} + \hat{\Sigma}^{-1} \otimes \left(\mathbf{X}'\mathbf{X} \right) \right]^{-1},$$

$$\overline{\boldsymbol{\alpha}} = \overline{\mathbf{V}} \left[(\underline{\mathbf{V}})^{-1} \underline{\boldsymbol{\alpha}} + \left(\hat{\Sigma}^{-1} \otimes \mathbf{X} \right)' \mathbf{y} \right].$$

A.2 Natural conjugate priors - Normal-Wishart prior

The natural conjugate priors allow us to have the same family of distributions for the prior, likelihood and posterior. In the Bayesian Vector autoregressive models, the natural conjugate prior has the form:

$$\begin{aligned}
&\boldsymbol{\alpha}|\Sigma \sim \mathcal{N}\left(\underline{\boldsymbol{\alpha}}, \Sigma \otimes \underline{V}\right), \\
&\Sigma^{-1} \sim \mathcal{W}\left(\underline{S}^{-1}, \underline{\nu}\right),
\end{aligned}$$

where $\underline{\boldsymbol{\alpha}}, \underline{V}, \underline{S}, \underline{\nu}$ are the prior hyperparameters.

From the prior distribution, we have the following posterior distribution for the matrix of coefficients $\boldsymbol{\alpha}$

$$\boldsymbol{\alpha}|\Sigma, \mathbf{y} \sim \mathcal{N}(\overline{\boldsymbol{\alpha}}, \Sigma \otimes \overline{V}),$$

where

$$\begin{aligned}
&\overline{V} = \left[\underline{V}^{-1} + \mathbf{X}'\mathbf{X}\right]^{-1}, \\
&\overline{A} = \overline{V}\left[\underline{V}^{-1}\underline{A} + \mathbf{X}'X\hat{A}\right] \\
&\overline{\boldsymbol{\alpha}} = \operatorname{vec}\left(\overline{A}\right),
\end{aligned}$$

where $\underline{A}$ is the matrix made by the unstacking of the vector $\underline{\boldsymbol{\alpha}}$ and $\hat{A}$ is its least square estimator.

On the other hand, the covariance matrix Σ has posterior distribution as follows

$$\Sigma^{-1}|\mathbf{y} \sim \mathcal{W}\left(\overline{S}^{-1}, \overline{\nu}\right),$$

where

$$\begin{aligned}
&\overline{S} = S + \underline{S} + \hat{A}'\mathbf{X}'\mathbf{X}\hat{A} + \underline{A}'\underline{V}^{-1}\underline{A} - \overline{A}'\left(\underline{V}^{-1} + \mathbf{X}'\mathbf{X}\right)\overline{X}, \\
&\overline{\nu} = T + \underline{\nu},
\end{aligned}$$

with $S = \left(\mathbf{Y} - \mathbf{X}\hat{A}\right)'\left(\mathbf{Y} - \mathbf{X}\hat{A}\right)$ the least square estimate of the covariance matrix.

B The Root Mean Square Errors - RMSEs

Table 1: RMSE values for AR and VAR benchmark models, and RMSE ratios for all other models in Germany

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	7.240	7.333	8.067	7.387	6.869	6.229	8.027	9.514	9.646	8.905	8.832	8.834	9.214
ARX	0.783***	0.798**	0.793**	0.822**	0.828**	0.841***	0.842***	0.809***	0.784***	0.778***	0.754***	0.740***	0.743***
BAR-Min	1.001	0.999	0.999	1.000	1.000	1.000	1.001	1.000	0.999	0.999**	1.000	1.000	0.999
BAR-NW	0.998**	1.000*	1.000	1.000	1.000	0.998**	0.998**	0.997***	0.998***	0.998***	0.998***	0.998***	1.000
BARX-Min	0.781***	0.799**	0.792**	0.821**	0.828**	0.840***	0.842***	0.808***	0.783***	0.778***	0.754***	0.740***	0.743***
BARX-NW	0.779***	0.794***	0.788**	0.815**	0.825**	0.839***	0.838***	0.804***	0.780***	0.774***	0.750***	0.739***	0.743***
Multivariate													
VAR	4.278	4.373	5.230	4.944	4.817	4.791	6.271	7.455	7.513	6.905	7.113	7.443	7.934
VARX	1.052	1.080	1.043	1.094	1.096	1.078	1.015	0.944	0.917	0.885***	0.846***	0.809***	0.799***
BVAR-Min	0.999	0.999**	0.997***	0.992***	0.995**	0.990***	0.985***	0.994***	0.991***	0.995*	0.993***	0.991***	0.996*
BVAR-NW	1.001	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.001	1.000	1.000	1.000
BVARX-Min	1.049	1.073	1.034	1.080	1.087	1.067	1.004	0.939	0.907*	0.882***	0.841***	0.807***	0.802***
BVARX-NW	1.051	1.081	1.041	1.089	1.094	1.086	1.021	0.940	0.911*	0.880***	0.841***	0.807***	0.799***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	1.062	1.058	1.070	1.070	1.068	1.071
VARX	–	–	–	–	–	–	–	0.984	0.937	0.929	0.878	0.840	0.823
BVAR-Min	–	–	–	–	–	–	–	1.054	1.051	1.069	1.069	1.061	1.070
BVAR-NW	–	–	–	–	–	–	–	1.060	1.057	1.069	1.069	1.067	1.070
BVARX-Min	–	–	–	–	–	–	–	0.977	0.931	0.929	0.876	0.839	0.824
BVARX-NW	–	–	–	–	–	–	–	0.972	0.928	0.924	0.876	0.841	0.825

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	10.372	11.176	9.669	9.124	8.844	8.692	8.512	7.094	6.277	6.118	6.236	8.259	9.333
ARX	0.757***	0.776***	0.763***	0.775***	0.786***	0.821***	0.828***	0.816***	0.820***	0.813***	0.789***	0.791	0.777
BAR-Min	1.000	1.000	1.000*	1.000	0.998**	1.000	0.998***	1.000	1.000	1.000	0.999	0.999	0.999
BAR-NW	0.999***	0.999***	0.999***	0.999***	1.000*	0.997***	0.997***	0.998***	0.997***	0.998***	0.999***	0.998	0.998
BARX-Min	0.758***	0.776***	0.763***	0.775***	0.784***	0.820***	0.828***	0.815***	0.818***	0.812***	0.789***	0.791	0.777
BARX-NW	0.756***	0.774***	0.761***	0.772***	0.780***	0.814***	0.825***	0.814***	0.818***	0.812***	0.788***	0.788	0.774
Multivariate													
VAR	8.923	9.773	8.350	7.956	8.111	8.290	8.150	6.859	6.164	6.127	6.369	6.839	7.994
VARX	0.829***	0.855***	0.838***	0.815***	0.794***	0.798***	0.774***	0.761***	0.771***	0.754***	0.739***	0.873	0.837
BVAR-Min	0.999	1.003	0.994***	0.999	0.993***	0.990***	0.990***	0.993***	0.994***	0.994***	0.993**	0.994	0.995
BVAR-NW	1.000	0.999	0.999	0.999	0.999	1.000	0.999	0.999	0.999	0.999**	1.001	1.000	0.999
BVARX-Min	0.831***	0.859***	0.834***	0.814***	0.787***	0.786***	0.764***	0.755***	0.768***	0.751***	0.736***	0.868	0.834
BVARX-NW	0.829***	0.853***	0.834***	0.810***	0.790***	0.791***	0.768***	0.757***	0.770***	0.752***	0.735***	0.870	0.834
Reduced Multivariate													
VAR	1.061	1.051	1.053	1.051	1.028	1.013	1.016	–	–	–	–	–	1.051
VARX	0.843	0.857	0.840	0.833**	0.817***	0.809**	0.802***	–	–	–	–	–	0.859
BVAR-Min	1.060	1.054	1.046	1.050	1.022	1.005	1.007	–	–	–	–	–	1.047
BVAR-NW	1.061	1.051	1.052	1.049	1.026	1.010	1.013	–	–	–	–	–	1.050
BVARX-Min	0.843	0.860	0.837	0.833**	0.813***	0.804***	0.797***	–	–	–	–	–	0.856
BVARX-NW	0.843	0.855	0.837	0.830**	0.813***	0.806***	0.800***	–	–	–	–	–	0.855

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the RMSEs computed according to equation 5 and 6.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the RMSEs (first row of each panel); for all other AR models, the table reports the ratio between the RMSE of the current model and the RMSE of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the RMSE ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal RMSEs.

Table 2: RMSE values for AR and VAR benchmark models, and RMSE ratios for all other models in Denmark

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	5.850	6.282	6.626	6.566	6.097	5.605	6.857	12.435	13.490	13.162	12.637	12.306	8.226
ARX	0.959	0.965	0.963	0.974	0.987	0.984	0.957	0.864***	0.832***	0.841***	0.848***	0.859***	0.883**
BAR-Min	1.000	1.000	0.998**	1.000	0.998	0.998	0.998*	0.996***	0.994***	0.996**	0.996***	0.995***	0.998**
BAR-NW	0.998**	0.999***	1.000*	0.998***	0.998***	0.997***	0.998***	0.996***	0.998**	0.995***	0.995***	0.998**	0.998***
BARX-Min	0.958	0.964	0.962	0.973	0.988	0.983	0.955	0.863***	0.830***	0.836***	0.845***	0.855***	0.883**
BARX-NW	0.959	0.965	0.961	0.973	0.986	0.983	0.955	0.864***	0.832***	0.842***	0.851***	0.860***	0.884**
Multivariate													
VAR	3.413	3.721	3.998	4.131	4.195	3.990	5.159	10.021	11.322	10.913	10.212	9.681	7.607
VARX	1.283	1.359	1.333	1.367	1.332	1.302	1.090	0.981	0.940**	0.963	0.967	0.968	0.888**
BVAR-Min	0.978***	0.976***	0.975***	0.976***	0.978***	0.959***	0.977***	0.940***	0.949***	0.937***	0.934***	0.922***	0.987**
BVAR-NW	1.000	1.001	1.001	1.001	1.001	0.999	1.001	1.000	1.001	1.000	0.999	0.999	0.999***
BVARX-Min	1.270	1.343	1.323	1.355	1.328	1.283	1.079	0.920***	0.887***	0.900***	0.893***	0.890**	0.883**
BVARX-NW	1.283	1.360	1.334	1.367	1.332	1.303	1.090	0.980	0.940**	0.963	0.967	0.969	0.889**
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	1.022	1.008	1.004	1.005	1.010	1.027
VARX	–	–	–	–	–	–	–	0.984	0.938	0.952	0.952	0.958	0.914
BVAR-Min	–	–	–	–	–	–	–	0.973	0.971	0.951	0.956	0.950	1.026
BVAR-NW	–	–	–	–	–	–	–	1.020	1.007	1.002	1.003	1.009	1.026
BVARX-Min	–	–	–	–	–	–	–	0.937	0.903	0.903	0.898	0.898	0.916
BVARX-NW	–	–	–	–	–	–	–	0.983	0.938	0.951	0.949	0.956	0.915

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	8.089	7.834	8.029	9.412	11.362	10.300	8.762	7.462	6.012	5.021	4.804	8.469	10.465
ARX	0.883***	0.871***	0.854***	0.855***	0.838***	0.841***	0.861***	0.891*	0.931	0.963	0.960	0.888	0.854
BAR-Min	0.999	0.999*	1.000	1.000	1.000	1.000	1.000*	1.001	0.999**	0.999**	1.000	0.998	0.998
BAR-NW	0.998***	0.999**	0.999***	1.000***	1.000**	0.998***	0.999***	0.998***	0.998***	0.997***	0.998***	0.998	0.998
BARX-Min	0.882***	0.870***	0.854***	0.854***	0.838***	0.841***	0.862***	0.891*	0.931	0.962	0.959	0.887	0.853
BARX-NW	0.884***	0.872***	0.856***	0.856***	0.839***	0.841***	0.861***	0.892*	0.930	0.963	0.959	0.888	0.855
Multivariate													
VAR	7.465	7.205	7.466	8.986	11.264	10.441	8.678	7.213	5.897	5.008	4.741	7.197	9.328
VARX	0.877***	0.863***	0.853***	0.865***	0.854***	0.842***	0.853***	0.866***	0.891***	0.924	0.922	0.967	0.901
BVAR-Min	0.987**	0.991*	0.993*	0.989**	0.987**	0.985***	0.983***	0.987***	0.991***	0.985***	0.983***	0.970	0.966
BVAR-NW	0.999***	0.999***	0.998***	0.999***	0.999	0.999**	0.999**	0.999**	0.999**	0.998**	0.998**	0.999	0.999
BVARX-Min	0.870***	0.860***	0.849***	0.856***	0.846***	0.831***	0.841***	0.858***	0.887***	0.918	0.918	0.941	0.872
BVARX-NW	0.878***	0.864***	0.855***	0.866***	0.856***	0.845***	0.855***	0.867***	0.890***	0.923	0.921	0.967	0.906
Reduced Multivariate													
VAR	1.037	1.051***	1.045***	1.026***	1.015	1.015	1.032	–	–	–	–	–	1.021
VARX	0.907	0.907***	0.892***	0.880***	0.851***	0.845**	0.881	–	–	–	–	–	0.913
BVAR-Min	1.034	1.051***	1.047***	1.021***	1.007	1.006	1.023	–	–	–	–	–	0.997
BVAR-NW	1.036	1.050***	1.044***	1.025***	1.015	1.015	1.032	–	–	–	–	–	1.019
BVARX-Min	0.907	0.911***	0.893***	0.876***	0.846***	0.837**	0.872	–	–	–	–	–	0.891
BVARX-NW	0.909	0.909***	0.895***	0.881***	0.851***	0.846**	0.882	–	–	–	–	–	0.914

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the RMSEs computed according to equation 5 and 6.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the RMSEs (first row of each panel); for all other AR models, the table reports the ratio between the RMSE of the current model and the RMSE of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the RMSE ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal RMSEs.

Table 3: RMSE values for AR and VAR benchmark models, and RMSE ratios for all other models in Italy

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	4.560	4.491	4.354	4.405	4.404	4.365	5.162	7.791	9.968	9.107	8.126	7.462	6.389
ARX	1.016	1.011	1.021	1.032	1.024	1.033	1.009	0.990*	0.990*	0.986*	0.976**	0.975**	0.973*
BAR-Min	0.998	1.001	1.001	1.000	1.001	0.999	1.000	0.998*	0.999	1.000	1.000	0.998***	1.001
BAR-NW	0.998	0.998	0.999	0.999	0.998*	0.998	0.999	0.998*	0.998**	0.999**	1.000	0.999	1.000
BARX-Min	1.009	1.009	1.019	1.032	1.023	1.030	1.009	0.989*	0.988*	0.986*	0.976**	0.975*	0.972*
BARX-NW	1.012	1.009	1.020	1.032	1.023	1.033	1.009	0.990*	0.991*	0.986*	0.976**	0.975*	0.973*
Multivariate													
VAR	3.884	3.987	3.967	4.105	4.145	4.224	4.753	7.186	9.160	8.569	7.692	7.008	6.144
VARX	1.054	1.037	1.027	1.017	1.032	1.043	1.026	0.973	0.952	0.949	0.930**	0.922**	0.907***
BVAR-Min	0.966***	0.963***	0.960***	0.962***	0.960***	0.963***	0.969***	0.976***	0.981***	0.986***	0.974***	0.977**	0.968***
BVAR-NW	0.999	0.999**	0.999	0.999	0.999**	0.999	1.000	1.000	0.999	1.000	1.000	1.000	1.001
BVARX-Min	1.015	0.992	0.984	0.982	0.986	1.003	0.988	0.939	0.925	0.923**	0.897***	0.896***	0.878***
BVARX-NW	1.043	1.026	1.015	1.010	1.023	1.037	1.020	0.965	0.945	0.942	0.921**	0.914***	0.897***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	0.985	0.998	1.002	1.003	1.020	1.022
VARX	–	–	–	–	–	–	–	0.944	0.962	0.959	0.940	0.951	0.944
BVAR-Min	–	–	–	–	–	–	–	0.980	0.987	0.997	0.989	1.007	1.000
BVAR-NW	–	–	–	–	–	–	–	0.984	0.997	1.001	1.002	1.019	1.021
BVARX-Min	–	–	–	–	–	–	–	0.942	0.954	0.959	0.932	0.946	0.931
BVARX-NW	–	–	–	–	–	–	–	0.944	0.965	0.961	0.940	0.951	0.942

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	6.808	8.118	8.122	8.753	9.185	9.835	8.641	8.233	7.104	4.608	4.118	6.838	8.331
ARX	0.981	0.982	0.979	0.983	0.985	0.996	0.996	0.996	0.995	1.010	1.019	0.994	0.985
BAR-Min	1.000	1.001	1.001	0.999	1.0101	1.001	1.001	1.002	1.002	1.000	0.999	1.000	1.000
BAR-NW	0.998	1.000	1.000*	1.000**	1.000*	1.000	1.001	1.000**	1.001	0.999**	0.999	0.999	0.999
BARX-Min	0.980	0.983	0.980	0.982	0.986	0.996	0.997	1.000	0.999	1.009	1.016	0.994	0.985
BARX-NW	0.980	0.982	0.980	0.983	0.985	0.996	0.997	0.997	0.997	1.006	1.013	0.994	0.985
Multivariate													
VAR	6.447	7.624	7.650	8.259	8.855	9.582	8.532	8.147	6.893	4.890	4.454	6.507	7.901
VARX	0.902***	0.912**	0.923**	0.947	1.004	1.010	1.007	1.009	1.014	1.018	1.007	0.977	0.953
BVAR-Min	0.963***	0.981**	0.976***	0.989**	0.988***	0.985***	0.980***	0.979**	0.994**	0.957***	0.954***	0.976	0.980
BVAR-NW	1.001	1.001	1.000	1.000**	1.000	0.999**	0.999**	0.999**	1.000*	1.001	1.001	1.000	1.000
BVARX-Min	0.865***	0.896***	0.893***	0.928*	0.977	0.985	0.971	0.981	1.004	0.971	0.954	0.946	0.925
BVARX-NW	0.893***	0.904**	0.913**	0.940	0.994	1.003	0.994	1.001	1.006	1.008	0.995	0.968	0.944
Reduced Multivariate													
VAR	1.023	1.008	1.012**	1.002	1.009	1.006	1.010	–	–	–	–	–	1.001
VARX	0.947	0.960	0.970***	0.983	1.007	1.021	1.000	–	–	–	–	–	0.971
BVAR-Min	0.993	0.998	0.998***	0.996*	1.003	0.997	1.000	–	–	–	–	–	0.996
BVAR-NW	1.021	1.008	1.011**	1.002	1.009	1.005	1.010	–	–	–	–	–	1.006
BVARX-Min	0.929	0.959	0.962***	0.976	1.001	1.012	0.990	–	–	–	–	–	0.964
BVARX-NW	0.944	0.959	0.969***	0.982	1.007	1.020	0.997	–	–	–	–	–	0.970

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the RMSEs computed according to equation 5 and 6.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the RMSEs (first row of each panel); for all other AR models, the table reports the ratio between the RMSE of the current model and the RMSE of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the RMSE ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal RMSEs.

Table 4: RMSE values for AR and VAR benchmark models, and RMSE ratios for all other models in Spain

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	7.036	6.919	6.689	6.741	6.871	6.766	7.066	7.294	7.074	6.421	6.139	6.164	6.140
ARX	0.797***	0.786***	0.783***	0.791***	0.786***	0.798***	0.820***	0.835***	0.829***	0.831***	0.829***	0.829***	0.841***
BAR-Min	0.998**	0.999	0.999	0.998	0.998	0.999	1.000	1.001	0.999	0.999	0.999	0.999**	0.996***
BAR-NW	0.999*	0.997**	0.998	0.998***	0.998***	0.998***	0.997***	0.996***	0.994***	0.994***	0.997***	0.997***	0.997***
BARX-Min	0.795***	0.785***	0.783***	0.790***	0.785***	0.797***	0.819***	0.836***	0.829***	0.830***	0.828***	0.827***	0.838***
BARX-NW	0.796***	0.786***	0.783***	0.791***	0.786***	0.797***	0.819***	0.835***	0.830***	0.833***	0.827***	0.826***	0.837***
Multivariate													
VAR	3.943	4.360	4.331	4.638	4.973	5.016	5.227	5.376	5.284	4.761	4.624	4.826	5.018
VARX	1.006	0.972	0.961	0.935*	0.920**	0.892***	0.872***	0.846***	0.849***	0.860***	0.858***	0.850***	0.854***
BVAR-Min	0.978***	0.984**	0.984***	0.971***	0.964	0.972***	0.961***	0.960***	0.967***	0.973***	0.970***	0.962***	0.958***
BVAR-NW	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	0.998*	0.998	0.999*	0.999	0.999
BVARX-Min	0.982	0.953	0.940*	0.907**	0.886***	0.872***	0.842***	0.819***	0.823***	0.848***	0.846***	0.830***	0.831***
BVARX-NW	1.006	0.976	0.965	0.939	0.923**	0.894***	0.870***	0.841***	0.846***	0.857***	0.855***	0.846***	0.847***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	1.089	1.067	1.092	1.099	1.088	1.081
VARX	–	–	–	–	–	–	–	0.891	0.890	0.912	0.920	0.902**	0.903*
BVAR-Min	–	–	–	–	–	–	–	1.076	1.059	1.086	1.091	1.073	1.066
BVAR-NW	–	–	–	–	–	–	–	1.086	1.061	1.086	1.095	1.085	1.077
BVARX-Min	–	–	–	–	–	–	–	0.885	0.883	0.910	0.918	0.894**	0.893**
BVARX-NW	–	–	–	–	–	–	–	0.888	0.889	0.910	0.918	0.901**	0.901**

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	7.036	6.919	6.689	6.741	6.871	6.766	7.066	7.294	7.074	6.421	6.139	6.299	6.389
ARX	0.797***	0.786***	0.783***	0.791***	0.786***	0.798**	0.820**	0.835**	0.829	0.831*	0.829***	0.838	0.849
BAR-Min	0.998	0.999	0.999	0.998	0.998**	0.999	1.000**	1.001***	0.999***	0.999	0.999***	0.999	0.999
BAR-NW	0.999***	0.997***	0.998***	0.998***	0.998***	0.998***	0.997***	0.996***	0.994***	0.994**	0.997***	0.997	0.996
BARX-Min	0.795***	0.785***	0.783***	0.790***	0.785***	0.797***	0.819**	0.836**	0.829**	0.830*	0.828***	0.837	0.848
BARX-NW	0.796***	0.786***	0.783***	0.791***	0.786***	0.797**	0.819**	0.835**	0.830	0.833*	0.827***	0.837	0.848
Multivariate													
VAR	5.360	5.728	5.908	5.906	5.758	5.363	5.206	5.079	4.823	5.130	5.995	5.110	5.317
VARX	0.820***	0.798***	0.769***	0.761***	0.783***	0.808***	0.856***	0.864**	0.844***	0.791***	0.757***	0.849	0.822
BVAR-Min	0.961***	0.957***	0.954***	0.969***	0.956***	0.960***	0.955***	0.962***	0.964***	0.965***	0.962***	0.965	0.961
BVAR-NW	0.999	0.998**	0.997**	0.998	0.996**	0.993***	0.994***	0.994***	0.994***	0.996**	0.997**	0.998	0.997
BVARX-Min	0.805***	0.784***	0.753***	0.748***	0.768***	0.789***	0.823***	0.829***	0.826***	0.770***	0.733***	0.828	0.803
BVARX-NW	0.814***	0.795***	0.766***	0.757***	0.780***	0.806***	0.854***	0.862**	0.842***	0.789***	0.756***	0.847	0.818
Reduced Multivariate													
VAR	1.070	1.068	1.058	1.043	1.026	1.028	1.022	–	–	–	–	–	1.063
VARX	0.876**	0.870	0.845	0.822	0.845	0.870	0.889	–	–	–	–	–	0.878
BVAR-Min	1.052	1.047	1.033	1.035	1.011	1.008	0.993	–	–	–	–	–	1.047
BVAR-NW	1.067	1.063	1.052	1.037	1.019	1.020	1.014	–	–	–	–	–	1.057
BVARX-Min	0.868**	0.860	0.839	0.822	0.840	0.865	0.872	–	–	–	–	–	0.871
BVARX-NW	0.874**	0.868	0.843	0.820	0.842	0.867	0.888	–	–	–	–	–	0.875

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the RMSEs computed according to equation 5 and 6.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the RMSEs (first row of each panel); for all other AR models, the table reports the ratio between the RMSE of the current model and the RMSE of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the RMSE ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal RMSEs.

C The Average Log Predictive Scores

Table 5: Average log predictive score (scores for AR and VAR benchmarks, score differences in all others) for Germany.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	-3.519	-3.562	-3.777	-3.714	-3.675	-3.514	-3.714	-3.796	-3.785	-3.619	-3.606	-3.614	-3.745
ARX	0.305***	0.244***	0.241***	0.206***	0.169***	0.126***	0.078	0.155***	0.129	0.219***	0.253***	0.300***	0.310***
BAR-Min	-0.028	0.024	0.027	0.024	-0.009	-0.011	-0.007	-0.007	0.001	0.004	-0.003	-0.002	-0.001
BAR-NW	0.002	0.021	0.011	0.035*	-0.014	0.009	0.001	0.001	0.003	-0.001	-0.001	0.002	-0.006
BARX-Min	0.308***	0.237***	0.252***	0.208***	0.187***	0.113***	0.070	0.148***	0.173***	0.226***	0.283***	0.298***	0.308***
BARX-NW	0.262***	0.249***	0.262***	0.215***	0.212***	0.128***	0.081	0.166***	0.201***	0.236***	0.259***	0.301***	0.306***
Multivariate													
VAR	-3.144	-3.107	-3.453	-3.278	-3.302	-3.275	-3.464	-3.543	-3.588	-3.407	-3.402	-3.450	-3.583
VARX	0.139*	0.070*	0.093	0.063*	0.062*	0.037*	0.052***	0.031	0.101**	0.161***	0.156***	0.228***	0.249***
BVAR-Min	0.100*	0.028	0.143	0.007	0.037	-0.003	-0.017	0.006	0.048	0.032	0.002	-0.002	-0.010
BVAR-NW	0.050	0.037*	0.125	0.022**	0.009**	0.029*	0.014***	0.027**	0.063*	0.039*	0.014**	0.011***	0.010***
BVARX-Min	0.188*	0.044	0.240*	0.046	0.039	0.010	0.029**	0.093***	0.165***	0.187***	0.205***	0.229***	0.244***
BVARX-NW	0.198**	0.080**	0.248**	0.116*	0.164*	0.051**	0.065***	0.099***	0.167***	0.176***	0.204***	0.250***	0.254***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	-0.082	-0.012	-0.029	-0.075	-0.069	-0.076
VARX	–	–	–	–	–	–	–	-0.035*	0.100**	0.112***	0.132***	0.188***	0.200***
BVAR-Min	–	–	–	–	–	–	–	-0.093	-0.012	-0.036	-0.070	-0.072	-0.092
BVAR-NW	–	–	–	–	–	–	–	-0.084	-0.004	-0.027	-0.063	-0.063	-0.077
BVARX-Min	–	–	–	–	–	–	–	0.013**	0.121***	0.128***	0.143***	0.185***	0.200***
BVARX-NW	–	–	–	–	–	–	–	0.029**	0.125***	0.140***	0.152***	0.194***	0.206***

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	-4.047	-4.136	-3.929	-3.851	-3.664	-3.609	-3.570	-3.385	-3.266	-3.236	-3.344	-3.653	-3.767
ARX	0.422***	0.465***	0.328***	0.326***	0.152*	0.180***	0.197***	0.208***	0.195***	0.207***	0.176***	0.233	0.264
BAR-Min	0.057*	0.031	0.015	0.012	0.018*	-0.001	-0.005	-0.013	-0.053	-0.011	0.037	0.004	0.009
BAR-NW	0.056*	0.086	-0.003	0.005	0.031*	-0.002	-0.002	-0.035	-0.062	-0.029	0.005	0.005	0.013
BARX-Min	0.414***	0.464***	0.325***	0.317***	0.118	0.190***	0.187***	0.201***	0.197***	0.210***	0.196***	0.235	0.265
BARX-NW	0.417***	0.467***	0.329***	0.265***	0.111	0.205***	0.189***	0.205***	0.202***	0.211***	0.240***	0.238	0.266
Multivariate													
VAR	-3.849	-3.975	-3.785	-3.712	-3.613	-3.557	-3.526	-3.370	-3.240	-3.256	-3.479	-3.473	-3.615
VARX	0.240***	0.329**	0.209***	0.070	0.048	0.245***	0.262***	0.271***	0.174**	0.279***	0.408***	0.166	0.179
BVAR-Min	-0.036	0.005	-0.035	-0.014	0.057**	-0.007	-0.000	-0.000	-0.008	0.015*	0.120**	0.019	0.003
BVAR-NW	-0.035	0.012***	0.027**	-0.007	0.037	0.007**	0.002	-0.002	-0.033	0.017*	0.112**	0.025	0.016
BVARX-Min	0.293***	0.351***	0.262***	0.232***	0.119*	0.228***	0.253***	0.280***	0.243***	0.318***	0.491***	0.199	0.220
BVARX-NW	0.303***	0.386***	0.285***	0.158**	0.113	0.260***	0.266***	0.291***	0.257***	0.317***	0.519***	0.218	0.225
Reduced Multivariate													
VAR	-0.047	-0.054	-0.083	-0.065	-0.037	-0.019	-0.028	–	–	–	–	–	-0.052
VARX	0.256***	0.318**	0.238***	0.125*	0.115*	0.212***	0.209***	–	–	–	–	–	0.167
BVAR-Min	-0.051	-0.023	-0.025	-0.033	-0.011	-0.020	-0.020	–	–	–	–	–	-0.043
BVAR-NW	-0.042	0.002	-0.030	-0.025	-0.005	-0.020	-0.028	–	–	–	–	–	-0.036
BVARX-Min	0.246***	0.326**	0.229***	0.142**	0.089	0.201***	0.212***	–	–	–	–	–	0.172
BVARX-NW	0.260***	0.334**	0.242***	0.133**	0.057	0.218***	0.218***	–	–	–	–	–	0.178

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the Average log predictive score for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average values of log predictive density scores (first row of each panel); for all other AR models, the table reports the average score of each model less the average score of the AR baseline; similarly for multivariate models. Entries greater than 0 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

[4] ***, ** and * indicate that the average scores differences are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average scores.

Table 6: Average log predictive score (scores for AR and VAR benchmarks, score differences in all others) for Denmark.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	-3.354	-3.456	-3.526	-3.644	-3.606	-3.581	-3.511	-4.341	-4.364	-4.380	-4.362	-4.325	-3.601
ARX	0.130***	0.052	-0.056	0.120***	0.106***	0.087***	0.066***	0.059***	0.073***	0.064***	0.054***	0.055***	0.071
BAR-Min	0.011	-0.024	-0.017	-0.033	0.005	-0.007	0.034	-0.009	-0.007	-0.004	-0.009	-0.007	0.021
BAR-NW	0.003	0.009	-0.042	-0.042	-0.052	0.003*	-0.005	0.000	0.000	0.002**	-0.001	0.001	-0.019
BARX-Min	0.111***	0.060	-0.065	0.128***	0.066**	0.055***	0.076***	0.050***	0.063***	0.054***	0.043***	0.043***	0.014
BARX-NW	0.122***	-0.010	-0.034	0.124***	0.082***	0.069***	0.075***	0.061***	0.074***	0.065***	0.054***	0.056***	0.036
Multivariate													
VAR	-2.916	-3.062	-3.115	-3.186	-3.214	-3.239	-3.278	-4.253	-4.279	-4.300	-4.287	-4.242	-3.705
VARX	-0.172	-0.164	-0.175	-0.171	-0.110	-0.060	-0.021	0.005*	0.016***	0.008**	0.005*	0.006**	0.123***
BVAR-Min	-0.013	-0.013	-0.028	-0.021	0.015	-0.026	-0.027	-0.034	-0.033	-0.032	-0.032	-0.032	0.038**
BVAR-NW	0.026*	0.005*	0.005	0.027*	0.043	0.012***	0.027**	0.009***	0.009***	0.009***	0.009***	0.009***	0.033*
BVARX-Min	-0.075	-0.115	-0.079	-0.054	-0.015	0.022	0.030	-0.070	-0.059	-0.069	-0.070	-0.071	0.168***
BVARX-NW	-0.060	-0.065	-0.052	-0.013	0.017	0.013	0.074*	0.006*	0.016***	0.009**	0.007**	0.007**	0.148***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	-0.005	-0.005	0.000	0.001	0.001	0.114
VARX	–	–	–	–	–	–	–	0.011***	0.022***	0.018***	0.017***	0.016***	0.111*
BVAR-Min	–	–	–	–	–	–	–	-0.025	-0.022	-0.018	-0.020	-0.021	0.162
BVAR-NW	–	–	–	–	–	–	–	-0.002	-0.001	0.003	0.002	0.001	0.183*
BVARX-Min	–	–	–	–	–	–	–	-0.031	-0.018	-0.022	-0.025	-0.027	0.158**
BVARX-NW	–	–	–	–	–	–	–	0.013***	0.024***	0.020***	0.017***	0.016***	0.246**

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	-3.650	-3.647	-3.764	-4.093	-4.350	-3.937	-3.579	-3.417	-3.225	-3.044	-3.069	-3.743	-4.030
ARX	0.004	0.153***	0.182***	0.080	0.314***	0.170***	0.153***	0.110***	0.044	0.010	-0.056	0.085	0.110
BAR-Min	0.042	0.007	0.015*	0.002	0.141**	-0.019	-0.000	-0.000	0.001	-0.006	-0.028	0.005	0.013
BAR-NW	0.025	0.026**	0.006	0.043	0.148**	-0.000	0.006	-0.038	0.015*	-0.004	0.002	0.004	0.018
BARX-Min	0.007	0.110**	0.142***	0.098	0.265***	0.157***	0.138***	0.122***	0.013	0.035	-0.033	0.073	0.091
BARX-NW	-0.022	0.156***	0.159***	0.083	0.378***	0.148***	0.129***	0.086**	0.085***	0.005	-0.018	0.082	0.106
Multivariate													
VAR	-3.677	-3.665	-3.733	-4.122	-4.119	-3.890	-3.619	-3.417	-3.239	-3.064	-3.162	-3.616	-3.992
VARX	0.075	0.153**	0.141**	0.023	-0.035	0.103**	0.201***	0.041	-0.108	-0.037	0.039	-0.005	0.063
BVAR-Min	0.029*	0.045*	0.058	0.058	-0.002	0.045**	0.058	0.023*	0.046**	0.033	0.043	0.008	0.013
BVAR-NW	0.028**	0.042**	0.027	0.017	-0.085	0.060*	0.071*	-0.006	0.051*	0.019	0.027	0.019	0.018
BVARX-Min	0.067	0.276***	0.235***	0.383***	0.091	0.192***	0.229***	0.162***	0.134**	0.102***	0.144***	0.065	0.100
BVARX-NW	0.107**	0.204***	0.237***	0.091	0.045	0.161***	0.200***	0.147***	0.014	0.076**	0.146***	0.064	0.095
Reduced Multivariate													
VAR	0.056	0.080	-0.034	0.141	-0.091	0.029	-0.003	–	–	–	–	–	0.022
VARX	-0.017	0.110	0.093	0.138	0.048	0.069	0.185***	–	–	–	–	–	0.063
BVAR-Min	0.069	0.080	0.008	0.207*	-0.079	0.046	0.026	–	–	–	–	–	0.032
BVAR-NW	-0.012	0.033	0.033	0.196	-0.127	0.036	0.021	–	–	–	–	–	0.028
BVARX-Min	0.062	0.221**	0.164**	0.350**	0.095	0.162***	0.192***	–	–	–	–	–	0.098
BVARX-NW	0.030	0.204**	0.183***	0.251**	0.080	0.175***	0.155**	–	–	–	–	–	0.109

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the Average log predictive score for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average values of log predictive density scores (first row of each panel); for all other AR models, the table reports the average score of each model less the average score of the AR baseline; similarly for multivariate models. Entries greater than 0 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

[4] ***, ** and * indicate that the average scores differences are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average scores.

Table 7: Average log predictive score (scores for AR and VAR benchmarks, score differences in all others) for Italy.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	-2.954	-2.928	-2.913	-2.918	-2.932	-2.909	-3.087	-3.579	-4.053	-3.805	-3.638	-3.444	-3.270
ARX	-0.006	-0.008	-0.013	-0.015	-0.009	-0.025	-0.010	-0.275	-0.004	0.103*	0.107**	0.010	-0.025
BAR-Min	-0.004	-0.019	-0.001	-0.012	-0.012	-0.011	0.019	0.015	0.078	0.127*	0.027	-0.010	-0.012
BAR-NW	0.005**	-0.009	0.003	-0.002	-0.008	-0.005	0.016	-0.043	0.172	0.071	0.070	0.005	-0.013
BARX-Min	-0.006	-0.014	-0.011	-0.029	-0.016	-0.026	-0.002	-0.126	0.092**	0.009	0.127*	0.036*	0.023
BARX-NW	-0.006	-0.009	-0.007	-0.022	-0.012	-0.028	0.001	-0.265	0.051	0.044	0.088**	0.006	-0.005
Multivariate													
VAR	-2.792	-2.906	-2.832	-2.899	-2.945	-2.903	-3.046	-4.140	-4.021	-3.719	-3.545	-3.379	-3.260
VARX	-0.043	0.015	-0.023	0.028	0.026	-0.112	-0.057	-0.036	-0.026	-0.258	0.008	-0.091	0.040
BVAR-Min	0.033	0.137	0.050	0.066	0.111	0.065*	0.088*	0.573**	0.243**	0.124**	0.127**	0.044**	0.070**
BVAR-NW	0.029	0.048*	0.007	0.052	0.082	0.055*	0.076**	0.153**	0.068**	-0.033	0.081**	0.020	0.025*
BVARX-Min	-0.028	0.088	0.010	0.045	0.069	0.022	0.067	0.690**	0.381**	0.183***	0.192***	0.115***	0.152***
BVARX-NW	-0.012	0.069	0.021	0.031	0.034	-0.019	0.023	0.201*	0.051	-0.060	0.126**	0.110***	0.069
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	0.316*	-0.117	-0.140	0.027	-0.016	-0.007
VARX	–	–	–	–	–	–	–	0.262*	0.070	-0.204	0.076*	0.035	0.040
BVAR-Min	–	–	–	–	–	–	–	0.469**	0.185**	0.087**	0.078*	0.009	0.026
BVAR-NW	–	–	–	–	–	–	–	0.522*	-0.035	0.025	0.098*	-0.017	-0.005
BVARX-Min	–	–	–	–	–	–	–	0.547**	0.366*	0.129**	0.169***	0.077**	0.109***
BVARX-NW	–	–	–	–	–	–	–	0.285*	0.118	0.038	0.065	0.046	0.060*

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	-3.368	-3.539	-3.535	-3.882	-4.016	-4.181	-3.746	-3.841	-3.818	-2.967	-2.839	-3.424	-3.697
ARX	-0.060	-0.032	-0.085	-0.090	-0.024	-0.161	-0.019	-0.043	0.017	-0.036	-0.030	-0.030	-0.043
BAR-Min	0.022**	-0.010	0.010	-0.016	0.100	-0.002	0.077	0.005	-0.132	-0.028	-0.007	0.008	0.031
BAR-NW	0.000	0.016	0.020	-0.017	0.053*	-0.049	0.092	-0.025	0.012	-0.009	-0.006	0.014	0.029
BARX-Min	0.014	-0.040	0.004	0.007	-0.060	-0.049	0.070	-0.006	-0.117	-0.022	-0.012	-0.006	0.008
BARX-NW	-0.004	0.031*	0.020	0.030*	0.009	-0.051	-0.012	-0.025	0.023	-0.053	-0.022	-0.009	-0.004
Multivariate													
VAR	-3.305	-3.537	-3.500	-3.850	-3.925	-4.440	-3.925	-3.761	-3.842	-3.141	-3.018	-3.443	-3.734
VARX	-0.172	-0.130	-0.141	-0.157	-0.326	0.233*	0.161	-0.257	-0.166	-0.060	-0.105	-0.069	-0.069
BVAR-Min	0.061**	0.117**	0.070**	0.093*	0.250**	0.589***	0.352*	0.236**	0.162*	0.158	0.149**	0.165	0.209
BVAR-NW	0.024**	0.021	0.047***	0.026	0.129**	0.277**	0.102**	0.067**	0.066	0.062	0.079**	0.065	0.072
BVARX-Min	0.158***	0.187***	0.108*	0.146**	0.244 *	0.591**	0.390*	0.160*	0.277**	0.139	0.134*	0.188	0.272
BVARX-NW	0.018	-0.036	-0.094	0.014	-0.161	0.326**	0.287*	-0.081	-0.120	0.099	0.100	0.042	0.066
Reduced Multivariate													
VAR	-0.011	0.006	-0.061	-0.034	-0.007	0.377**	0.166	–	–	–	–	–	0.038
VARX	-0.011	-0.094	-0.081	-0.049	-0.211	0.091	0.147*	–	–	–	–	–	0.006
BVAR-Min	0.036	0.072*	0.038	0.049	0.134	0.528**	0.198*	–	–	–	–	–	0.147
BVAR-NW	0.002	0.036	-0.000	0.050	0.030	0.500**	0.180*	–	–	–	–	–	0.107
BVARX-Min	0.094**	0.110**	0.091**	0.046	0.111	0.549**	0.334*	–	–	–	–	–	0.210
BVARX-NW	0.035	0.042	0.035	-0.082	-0.275	0.271	0.220*	–	–	–	–	–	0.066

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the Average log predictive score for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average values of log predictive density scores (first row of each panel); for all other AR models, the table reports the average score of each model less the average score of the AR baseline; similarly for multivariate models. Entries greater than 0 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

[4] ***, ** and * indicate that the average scores differences are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average scores.

Table 8: Average log predictive score (scores for AR and VAR benchmarks, score differences in all others) for Spain.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	-3.405	-3.393	-3.363	-3.378	-3.395	-3.365	-3.395	-3.437	-3.393	-3.303	-3.306	-3.307	-3.316
ARX	0.220***	0.264***	0.266***	0.261***	0.264***	0.215***	0.212***	0.165***	0.142**	0.198***	0.228***	0.214***	0.193***
BAR-Min	-0.007	-0.020	-0.010	0.003	0.003	-0.000	-0.002	-0.004	-0.003	-0.005	0.043	-0.006	-0.011
BAR-NW	0.003	0.000	0.002	0.017	0.003	0.001	0.001	0.017**	0.000	-0.001	0.027	-0.050	-0.079
BARX-Min	0.224***	0.269***	0.265***	0.275***	0.274***	0.221***	0.210***	0.173***	0.183***	0.199***	0.256***	0.228***	0.221***
BARX-NW	0.253***	0.270***	0.270***	0.248***	0.266***	0.233***	0.199***	0.207***	0.197***	0.201***	0.255***	0.165***	0.235***
Multivariate													
VAR	-2.994	-2.966	-2.903	-2.996	-3.088	-3.098	-3.092	-3.160	-3.169	-3.037	-2.986	-3.111	-3.111
VARX	0.093	0.012	-0.010	-0.127	-0.254	-0.255	-0.025	0.083	0.154**	0.177***	-0.006	0.197	0.134
BVAR-Min	0.142	0.009	-0.014	0.048*	0.059*	0.073*	0.029*	0.047	0.075	0.026	-0.018	0.070	0.013
BVAR-NW	0.041*	-0.110	-0.033	-0.027	-0.028	0.017**	-0.039	0.019***	0.057*	0.048*	-0.005	0.010***	0.014
BVARX-Min	0.173	0.084	0.010	0.085**	0.150**	0.110	0.163***	0.238***	0.261***	0.183***	0.162***	0.265**	0.241***
BVARX-NW	0.137	0.094	-0.013	-0.081	-0.142	-0.079	0.051	0.151***	0.192***	0.211***	0.150**	0.236*	0.240***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	-0.064	-0.036	-0.078	-0.093	-0.044	-0.046
VARX	–	–	–	–	–	–	–	0.143***	0.186**	0.140***	0.105**	0.215**	0.170**
BVAR-Min	–	–	–	–	–	–	–	-0.067	-0.020	-0.076	-0.139	-0.059	-0.082
BVAR-NW	–	–	–	–	–	–	–	-0.069	-0.040	-0.070	-0.094	-0.031	-0.068
BVARX-Min	–	–	–	–	–	–	–	0.160**	0.189**	0.121**	0.088**	0.206**	0.172***
BVARX-NW	–	–	–	–	–	–	–	0.159***	0.191***	0.150***	0.122***	0.220**	0.185***

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	-3.420	-3.378	-3.414	-3.383	-3.326	-3.196	-3.142	-3.111	-3.059	-3.058	-3.192	-3.309	-3.332
ARX	0.213***	0.143**	0.209***	0.167***	0.185***	0.168***	0.149***	0.144***	0.148***	0.156***	0.177***	0.196	0.183
BAR-Min	0.006	-0.019	0.007	0.006	0.007	-0.015	0.000	-0.010	-0.009	-0.008	-0.003	-0.002	0.001
BAR-NW	0.009*	-0.007	0.006	0.011*	0.009***	-0.015	0.005	-0.000	0.003	-0.001	0.005*	-0.001	-0.005
BARX-Min	0.239***	0.181***	0.191***	0.164***	0.189***	0.161***	0.140***	0.136***	0.138***	0.154***	0.178***	0.203	0.194
BARX-NW	0.256***	0.138**	0.208***	0.167***	0.183***	0.167***	0.145***	0.146***	0.142***	0.160***	0.182***	0.204	0.194
Multivariate													
VAR	-3.220	-3.289	-3.256	-3.252	-3.208	-3.149	-3.120	-3.103	-3.066	-3.089	-3.234	-3.112	-3.159
VARX	0.184*	0.235**	0.299***	0.317***	0.231***	0.253***	0.187***	0.202***	0.240***	0.235***	0.240***	0.117	0.188
BVAR-Min	0.019	0.058	0.032	0.015	0.008	0.005	-0.011	-0.022	-0.028	-0.007	0.006	0.026	0.026
BVAR-NW	-0.008	0.018*	0.015**	0.024***	0.022***	0.021***	0.013***	0.004	0.009***	0.012***	0.010***	0.004	0.019
BVARX-Min	0.320***	0.349***	0.322***	0.311***	0.271***	0.231***	0.187***	0.183***	0.203***	0.263***	0.305***	0.211	0.257
BVARX-NW	0.287***	0.322***	0.328***	0.351***	0.280***	0.259***	0.177***	0.216***	0.249***	0.255***	0.288***	0.173	0.245
Reduced Multivariate													
VAR	-0.048	-0.033	-0.041	-0.025	-0.025	-0.028	-0.027	–	–	–	–	–	-0.045
VARX	0.247***	0.245***	0.204***	0.231***	0.194***	0.178***	0.156***	–	–	–	–	–	0.186
BVAR-Min	-0.050	-0.030	-0.048	-0.043	-0.028	-0.024	-0.032	–	–	–	–	–	-0.054
BVAR-NW	-0.070	-0.052	-0.064	-0.036	-0.012	-0.016	-0.016	–	–	–	–	–	-0.049
BVARX-Min	0.242***	0.256***	0.206***	0.227***	0.196***	0.170***	0.152***	–	–	–	–	–	0.183
BVARX-NW	0.237***	0.233***	0.185***	0.257***	0.211***	0.193***	0.166***	–	–	–	–	–	0.193

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the Average log predictive score for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average values of log predictive density scores (first row of each panel); for all other AR models, the table reports the average score of each model less the average score of the AR baseline; similarly for multivariate models. Entries greater than 0 indicate that forecasts from the indicated model are more accurate than forecasts from the associated baseline model.

[4] ***, ** and * indicate that the average scores differences are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average scores.

D The Average Continuous Ranked Probability Scores - CRPS

Table 9: Average CRPS (CRPS for AR and VAR benchmarks, CRPS ratios in all others) for Germany.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	3.770	3.934	4.229	4.062	3.793	3.505	4.467	5.090	5.129	4.942	4.939	4.987	4.962
ARX	0.830***	0.840***	0.846***	0.860***	0.861***	0.868***	0.863***	0.825***	0.795***	0.772***	0.752***	0.737***	0.741***
BAR-Min	1.003	1.002	1.003	1.004	1.006	1.003	1.005	1.003	1.000	0.999	1.001	1.001	1.000
BAR-NW	1.000	0.999	1.000	1.000	1.002	0.998**	0.998*	0.997***	0.998**	0.997***	0.999	0.999	0.999*
BARX-Min	0.829***	0.845***	0.850***	0.862***	0.865***	0.870***	0.863***	0.826***	0.795***	0.772***	0.752***	0.738***	0.741***
BARX-NW	0.826***	0.838***	0.842***	0.854***	0.859***	0.865***	0.858***	0.820***	0.792***	0.769***	0.748***	0.736***	0.741***
Multivariate													
VAR	2.261	2.495	2.846	2.901	2.794	2.697	3.443	3.896	3.903	3.772	3.946	4.158	4.185
VARX	1.065	1.053	1.037	1.045	1.042	1.046	0.997	0.962	0.939**	0.889***	0.842***	0.804***	0.791***
BVAR-Min	1.011	1.014	1.013	1.011	1.010	1.007	1.002	0.998	0.997	1.000	0.997	0.997	1.002
BVAR-NW	0.997**	0.995***	0.995***	0.995***	0.994***	0.995***	0.995***	0.996***	0.997***	0.998**	0.998**	0.998*	0.998**
BVARX-Min	1.074	1.067	1.050	1.057	1.055	1.055	1.000	0.952*	0.925***	0.884***	0.836***	0.805***	0.796***
BVARX-NW	1.058	1.046	1.029	1.035	1.034	1.045	0.996	0.951**	0.926***	0.879***	0.834***	0.799***	0.789***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	1.080	1.072	1.085	1.080	1.076	1.083
VARX	–	–	–	–	–	–	–	1.009	0.968	0.939**	0.884***	0.844***	0.832***
BVAR-Min	–	–	–	–	–	–	–	1.079	1.070	1.085	1.081	1.073	1.085
BVAR-NW	–	–	–	–	–	–	–	1.077	1.071	1.084	1.078	1.074	1.081
BVARX-Min	–	–	–	–	–	–	–	1.004	0.962	0.940**	0.881***	0.844***	0.833***
BVARX-NW	–	–	–	–	–	–	–	0.993	0.956	0.934**	0.880***	0.843***	0.831***

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	5.265	5.386	4.970	4.683	4.730	4.792	4.652	3.885	3.423	3.341	3.322	4.427	4.964
ARX	0.746***	0.762***	0.761***	0.778***	0.785***	0.827***	0.836***	0.825***	0.837***	0.830***	0.817***	0.804	0.778
BAR-Min	1.001	1.001	1.001	1.001	0.999	1.000	1.001	1.002	1.001	1.001	1.001	1.001	1.000
BAR-NW	0.998**	0.999*	0.998**	0.999	0.998**	0.995***	0.998*	0.998**	0.996***	0.998**	0.998**	0.998	0.998
BARX-Min	0.747***	0.762***	0.762***	0.777***	0.785***	0.828***	0.837***	0.826***	0.836***	0.830***	0.818***	0.805	0.779
BARX-NW	0.745***	0.761***	0.759***	0.774***	0.780***	0.821***	0.834***	0.825***	0.835***	0.829***	0.816***	0.801	0.775
Multivariate													
VAR	4.388	4.544	4.208	3.997	4.277	4.525	4.442	3.728	3.347	3.317	3.347	3.643	4.172
VARX	0.823***	0.856***	0.833***	0.812***	0.799***	0.805***	0.789***	0.774***	0.791***	0.778***	0.766***	0.874	0.840
BVAR-Min	1.004	1.009	1.001	1.001	0.997	0.999	0.996	0.995*	0.996	0.997	0.999	1.002	1.000
BVAR-NW	1.000	0.999	0.996***	0.997***	0.996***	0.996***	0.997**	0.997**	0.996***	0.997***	0.997***	0.997	0.997
BVARX-Min	0.826***	0.862***	0.832***	0.809***	0.794***	0.803***	0.784***	0.770***	0.788***	0.773***	0.764***	0.874	0.838
BVARX-NW	0.820***	0.850***	0.827***	0.805***	0.792***	0.798***	0.781***	0.768***	0.788***	0.772***	0.759***	0.868	0.833
Reduced Multivariate													
VAR	1.082	1.070	1.062	1.060	1.030	1.018	1.019	–	–	–	–	–	1.062
VARX	0.860***	0.880***	0.860***	0.852***	0.826***	0.828***	0.818***	–	–	–	–	–	0.875
BVAR-Min	1.083	1.075	1.060	1.062	1.029	1.016	1.014	–	–	–	–	–	1.062
BVAR-NW	1.079	1.067	1.060	1.058	1.028	1.014	1.014	–	–	–	–	–	1.060
BVARX-Min	0.860***	0.885***	0.860***	0.853***	0.824***	0.829***	0.817***	–	–	–	–	–	0.874
BVARX-NW	0.858***	0.876***	0.856***	0.850***	0.821***	0.824***	0.816***	–	–	–	–	–	0.870

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the average cumulative ranked probability score (CRPS) for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average CRPS (first row of each panel); for all other AR models, the table reports the ratio between the average CRPS of the current model and the average CRPS of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the average score ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average CRPS.

Table 10: Average CRPS (CRPS for AR and VAR benchmarks, CRPS ratios in all others) for Denmark.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	3.236	3.508	3.651	3.690	3.444	3.213	3.896	8.460	8.808	8.844	8.678	8.385	4.400
ARX	0.973	0.971	0.971	0.969*	0.979	0.975	0.959**	0.919***	0.897***	0.909***	0.915***	0.919***	0.894***
BAR-Min	1.002	1.003	1.003	1.002	1.002	1.002	1.003	1.004	1.001	1.002	1.004	1.004	0.998*
BAR-NW	0.998**	0.998*	0.999	0.998**	0.997**	0.996***	0.999	0.998**	0.997**	0.997***	0.999	0.999	0.997***
BARX-Min	0.974	0.971	0.971	0.971	0.981	0.976	0.958**	0.924***	0.902***	0.911***	0.921***	0.924***	0.895***
BARX-NW	0.973	0.968*	0.969*	0.968*	0.976	0.973	0.956**	0.918***	0.896***	0.908***	0.915***	0.919***	0.896***
Multivariate													
VAR	2.019	2.334	2.509	2.644	2.613	2.489	3.035	7.427	7.783	7.829	7.657	7.301	3.925
VARX	1.144	1.158	1.150	1.166	1.166	1.144	1.045	0.992	0.973***	0.987**	0.989*	0.991	0.908***
BVAR-Min	1.010	1.011	1.009	1.013	1.011	1.002	1.003	1.009	1.008	1.009	1.009	1.007	0.991**
BVAR-NW	0.994***	0.994***	0.993***	0.995***	0.994***	0.992***	0.995***	0.992***	0.994***	0.995***	0.993***	0.994***	0.994***
BVARX-Min	1.161	1.172	1.166	1.180	1.182	1.154	1.056	1.036	1.014	1.030	1.033	1.033	0.908***
BVARX-NW	1.135	1.146	1.140	1.156	1.156	1.134	1.039	0.990	0.973***	0.986**	0.988*	0.990	0.907***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	1.008	1.007	1.000	1.001	1.004	1.046
VARX	–	–	–	–	–	–	–	0.990	0.970***	0.977***	0.977***	0.982**	0.948**
BVAR-Min	–	–	–	–	–	–	–	1.009	1.007	1.001	1.003	1.005	1.046
BVAR-NW	–	–	–	–	–	–	–	1.006	1.003	0.998	0.998	1.002	1.042
BVARX-Min	–	–	–	–	–	–	–	1.008	0.989	0.995	0.996	1.000	0.952**
BVARX-NW	–	–	–	–	–	–	–	0.988	0.969***	0.976***	0.976***	0.980**	0.947**

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	4.291	4.131	4.156	4.627	5.693	5.379	4.734	3.963	3.188	2.703	2.542	4.901	6.199
ARX	0.893***	0.883***	0.875***	0.858***	0.836***	0.850***	0.875***	0.915***	0.947***	0.974	0.977	0.914	0.891
BAR-Min	1.001	1.000	1.001	0.999	1.000	1.000	0.998*	1.001	1.001	1.002	0.999	1.002	1.002
BAR-NW	0.998**	0.999	0.998**	0.998***	0.999	0.998**	0.997***	0.998***	0.998**	0.997***	0.996***	0.998	0.998
BARX-Min	0.891***	0.884***	0.874***	0.858***	0.835***	0.849***	0.876***	0.913***	0.946***	0.974	0.978	0.916	0.895
BARX-NW	0.892***	0.884***	0.875***	0.859***	0.835***	0.848***	0.874***	0.914***	0.944***	0.974	0.977	0.914	0.891
Multivariate													
VAR	3.823	3.685	3.761	4.278	5.471	5.329	4.614	3.785	3.100	2.658	2.476	4.273	5.605
VARX	0.893***	0.877***	0.870***	0.866***	0.862***	0.850***	0.867***	0.887***	0.894***	0.926***	0.939**	0.964	0.932
BVAR-Min	0.993**	0.993*	0.995*	0.994**	0.996	0.989***	0.988***	0.992**	0.994**	0.994*	0.994	1.001	1.001
BVAR-NW	0.995***	0.994***	0.992***	0.993***	0.996***	0.996***	0.996***	0.996***	0.996***	0.994***	0.993***	0.994	0.994
BVARX-Min	0.892***	0.877***	0.868***	0.862***	0.861***	0.844***	0.860***	0.881***	0.894***	0.928***	0.939**	0.981	0.953
BVARX-NW	0.891***	0.874***	0.867***	0.862***	0.860***	0.849***	0.867***	0.885***	0.891***	0.922***	0.933***	0.961	0.931
Reduced Multivariate													
VAR	1.055	1.066	1.061	1.046	1.024	1.018	1.035	–	–	–	–	–	1.022
VARX	0.941**	0.941**	0.931***	0.904***	0.864***	0.856***	0.902***	–	–	–	–	–	0.944
BVAR-Min	1.056	1.067	1.065	1.045	1.023	1.012	1.029	–	–	–	–	–	1.022
BVAR-NW	1.053	1.063	1.058	1.042	1.020	1.015	1.032	–	–	–	–	–	1.019
BVARX-Min	0.942**	0.943**	0.931***	0.902***	0.864***	0.853***	0.895***	–	–	–	–	–	0.954
BVARX-NW	0.940**	0.939**	0.930***	0.902***	0.862***	0.856***	0.901***	–	–	–	–	–	0.943

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the average cumulative ranked probability score (CRPS) for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average CRPS (first row of each panel); for all other AR models, the table reports the ratio between the average CRPS of the current model and the average CRPS of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the average score ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average CRPS.

Table 11: Average CRPS (CRPS for AR and VAR benchmarks, CRPS ratios in all others) for Italy.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
AR	2.547	2.499	2.431	2.460	2.467	2.441	2.851	4.090	5.231	4.772	4.336	4.026	3.494
ARX	1.009	1.010	1.021	1.029	1.025	1.032	0.996	0.985	0.983	0.983	0.974**	0.971**	0.968**
BAR-Min	1.001	1.004	1.003	1.002	1.003	1.003	1.003	0.997*	1.001	1.002	1.000	0.999	1.001
BAR-NW	0.997**	1.001	0.999	0.998	0.997**	0.999	1.001	0.997**	0.998*	0.999	1.000	0.999	0.999
BARX-Min	1.004	1.009	1.018	1.034	1.026	1.029	0.999	0.983	0.981*	0.984	0.976*	0.971**	0.968**
BARX-NW	1.005	1.007	1.017	1.032	1.025	1.030	0.997	0.984	0.983	0.983	0.974**	0.969**	0.969**
Multivariate													
VAR	2.147	2.206	2.184	2.265	2.283	2.345	2.588	3.671	4.764	4.509	4.134	3.791	3.342
VARX	1.063	1.051	1.039	1.029	1.036	1.058	1.047	1.007	0.985	0.968	0.939**	0.925**	0.908***
BVAR-Min	0.976***	0.968***	0.971***	0.976***	0.976***	0.971***	0.977***	0.972***	0.978***	0.980**	0.973***	0.978***	0.971***
BVAR-NW	0.999	0.997**	0.998	0.999	0.999	0.996***	0.997*	0.995***	0.994***	0.994***	0.995***	0.996***	0.996**
BVARX-Min	1.037	1.007	1.008	1.004	1.009	1.015	1.008	0.971	0.948**	0.937**	0.911***	0.907***	0.884***
BVARX-NW	1.049	1.037	1.023	1.020	1.026	1.049	1.036	0.996	0.975	0.955*	0.925***	0.910***	0.894***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	0.993	1.006	1.003	1.005	1.021	1.014
VARX	–	–	–	–	–	–	–	0.967	0.989	0.967	0.950**	0.957*	0.941**
BVAR-Min	–	–	–	–	–	–	–	0.986	0.991	0.994	0.989	1.009	0.996
BVAR-NW	–	–	–	–	–	–	–	0.988	1.001	0.998	1.001	1.015	1.011
BVARX-Min	–	–	–	–	–	–	–	0.965	0.975	0.964*	0.940**	0.953*	0.929**
BVARX-NW	–	–	–	–	–	–	–	0.963	0.988	0.965*	0.944**	0.952*	0.935**

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	3.720	4.376	4.390	4.546	4.741	5.037	4.607	4.353	3.616	2.494	2.253	3.658	4.414
ARX	0.972*	0.974	0.977	0.983	0.989	0.993	1.001	0.999	0.997	1.015	1.017	0.992	0.982
BAR-Min	1.000	1.002	1.003	1.001	1.002	1.001	1.003	1.003	1.000	1.000	1.001	1.001	1.001
BAR-NW	0.998	0.997*	0.999	0.999	0.999	0.999	1.001	0.998	0.998**	0.996***	0.999	0.999	0.999
BARX-Min	0.971*	0.975*	0.980	0.983	0.991	0.995	1.000	1.001	0.999	1.014	1.017	0.993	0.982
BARX-NW	0.969*	0.974	0.979	0.983	0.988	0.991	0.999	0.997	0.997	1.007	1.012	0.991	0.981
Multivariate													
VAR	3.480	4.062	4.095	4.292	4.648	4.954	4.562	4.342	3.569	2.613	2.401	3.469	4.177
VARX	0.896***	0.910***	0.923**	0.955	1.022	1.045	1.028	1.027	1.044	1.052	1.029	0.993	0.967
BVAR-Min	0.971***	0.982*	0.981**	0.987**	0.980***	0.981***	0.981***	0.980***	0.992	0.968***	0.964***	0.977	0.979
BVAR-NW	0.997	0.996**	0.995***	0.994***	0.991***	0.994***	0.995***	0.995***	0.995***	0.997**	0.995***	0.995	0.995
BVARX-Min	0.870***	0.897***	0.902***	0.937**	0.985	1.010	0.994	0.996	1.021	1.011	0.982	0.963	0.938
BVARX-NW	0.882***	0.897***	0.906***	0.941**	1.007	1.035	1.013	1.016	1.032	1.036	1.009	0.979	0.953
Reduced Multivariate													
VAR	1.022	1.009	1.011	0.997	0.989	1.003	1.009	–	–	–	–	–	1.006
VARX	0.950*	0.963	0.971	0.993	1.005	1.040	1.006	–	–	–	–	–	0.979
BVAR-Min	0.999	0.999	1.002	0.993	0.981	0.993	0.998	–	–	–	–	–	0.995
BVAR-NW	1.019	1.004	1.007	0.992	0.982	0.996	1.004	–	–	–	–	–	1.001
BVARX-Min	0.934**	0.960	0.968	0.986	0.993	1.026	0.998	–	–	–	–	–	0.971
BVARX-NW	0.945**	0.959	0.968	0.986	0.999	1.033	0.998	–	–	–	–	–	0.975

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the average cumulative ranked probability score (CRPS) for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

[3] For the AR and VAR baseline models, the table reports the average CRPS (first row of each panel); for all other AR models, the table reports the ratio between the average CRPS of the current model and the average CRPS of the AR benchmark; similarly for multivariate models. Entries less than 1 indicate that forecasts from the current model are more accurate than forecasts from the corresponding baseline model.

[4] ***, ** and * indicate that the average score ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average CRPS.

Table 12: Average CRPS (CRPS for AR and VAR benchmarks, CRPS ratios in all others) for Spain.

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
Univariate													
Hour	1	2	3	4	5	6	7	8	9	10	11	12	13
AR	3.914	3.849	3.740	3.757	3.809	3.774	3.948	4.022	3.907	3.555	3.401	3.412	3.402
ARX	0.784***	0.786***	0.779***	0.779***	0.775***	0.788***	0.817***	0.827***	0.826***	0.834***	0.837***	0.831***	0.844***
BAR-Min	1.001	0.999	1.000	0.999	1.002	1.001	1.004	1.003	1.001	1.003	1.002	1.000	0.998
BAR-NW	0.999	0.996***	0.998*	0.997**	0.997***	0.998*	1.001	0.993***	0.994***	0.996**	0.995***	0.993***	0.996**
BARX-Min	0.784***	0.785***	0.781***	0.777***	0.774***	0.788***	0.816***	0.828***	0.828***	0.834***	0.833***	0.830***	0.842***
BARX-NW	0.784***	0.784***	0.780***	0.777***	0.774***	0.786***	0.816***	0.825***	0.827***	0.835***	0.832***	0.827***	0.839***
Multivariate													
VAR	2.128	2.407	2.393	2.536	2.709	2.763	2.897	2.938	2.910	2.644	2.555	2.651	2.743
VARX	1.016	0.973	0.953	0.938**	0.918**	0.883***	0.872***	0.848***	0.845***	0.857***	0.858***	0.845***	0.851***
BVAR-Min	0.992	0.988*	0.994	0.980***	0.970***	0.978***	0.969***	0.972***	0.978***	0.987**	0.985**	0.976***	0.972***
BVAR-NW	0.997**	0.997**	0.997**	0.996***	0.996***	0.996***	0.996***	0.994***	0.993***	0.995***	0.996**	0.994***	0.993***
BVARX-Min	1.016	0.964	0.950*	0.916***	0.890***	0.865***	0.846***	0.829***	0.828***	0.857***	0.859***	0.841***	0.841***
BVARX-NW	1.013	0.974	0.956	0.940*	0.921**	0.884***	0.868***	0.839***	0.840***	0.850***	0.850***	0.838***	0.839***
Reduced Multivariate													
VAR	–	–	–	–	–	–	–	1.110	1.082	1.099	1.109	1.102	1.100
VARX	–	–	–	–	–	–	–	0.908**	0.901**	0.913**	0.923**	0.911**	0.916**
BVAR-Min	–	–	–	–	–	–	–	1.100	1.078	1.099	1.109	1.093	1.087
BVAR-NW	–	–	–	–	–	–	–	1.103	1.072	1.088	1.101	1.095	1.092
BVARX-Min	–	–	–	–	–	–	–	0.905**	0.897***	0.918**	0.929**	0.910**	0.911**
BVARX-NW	–	–	–	–	–	–	–	0.903**	0.899**	0.907**	0.917**	0.907**	0.911**

Hour	14	15	16	17	18	19	20	21	22	23	24	Avg	Avg_{8-20}
Univariate													
AR	3.506	3.688	3.844	3.824	3.592	3.139	2.935	2.839	2.669	2.723	3.170	3.517	3.557
ARX	0.850***	0.846***	0.846***	0.844***	0.862***	0.876***	0.900***	0.897***	0.908***	0.892***	0.854***	0.833	0.846
BAR-Min	1.003	1.000	1.001	1.004	1.000	1.003	1.002	1.002	1.001	1.002	1.001	1.001	1.001
BAR-NW	0.997*	0.995***	0.997**	0.996***	0.995***	0.993***	0.995***	0.996**	0.996**	0.997*	0.996***	0.996	0.995
BARX-Min	0.849***	0.846***	0.847***	0.845***	0.863***	0.879***	0.902***	0.897***	0.908***	0.892***	0.855***	0.833	0.847
BARX-NW	0.847***	0.846***	0.845***	0.843***	0.861***	0.876***	0.902***	0.897***	0.911***	0.892***	0.856***	0.832	0.845
Multivariate													
VAR	2.930	3.142	3.267	3.303	3.233	3.019	2.948	2.871	2.745	2.857	3.354	2.831	2.945
VARX	0.818***	0.801***	0.760***	0.750***	0.768***	0.798***	0.836***	0.856***	0.827***	0.787***	0.751***	0.844	0.815
BVAR-Min	0.974***	0.976**	0.974**	0.983**	0.971***	0.975***	0.978**	0.986*	0.990	0.982**	0.975***	0.979	0.977
BVAR-NW	0.993***	0.994***	0.993***	0.994***	0.992***	0.990***	0.991***	0.990***	0.991***	0.991***	0.992***	0.993	0.993
BVARX-Min	0.812***	0.794***	0.758***	0.750***	0.765***	0.793***	0.821***	0.836***	0.827***	0.777***	0.734***	0.834	0.809
BVARX-NW	0.807***	0.793***	0.753***	0.743***	0.763***	0.793***	0.835***	0.850***	0.821***	0.785***	0.748***	0.839	0.808
Reduced Multivariate													
VAR	1.084	1.082	1.067	1.046	1.028	1.029	1.019	–	–	–	–	–	1.072
VARX	0.887***	0.876***	0.841***	0.815***	0.828***	0.861***	0.876***	–	–	–	–	–	0.879
BVAR-Min	1.072	1.068	1.053	1.046	1.020	1.017	1.005	–	–	–	–	–	1.064
BVAR-NW	1.079	1.075	1.059	1.040	1.019	1.019	1.009	–	–	–	–	–	1.064
BVARX-Min	0.881***	0.868***	0.841***	0.819***	0.829***	0.861***	0.866***	–	–	–	–	–	0.877
BVARX-NW	0.883***	0.873***	0.839***	0.812***	0.825***	0.857***	0.873***	–	–	–	–	–	0.875

Notes:

[1] Forecast errors are calculated using a rolling window estimation. 'Avg' and 'Avg_{8-20}' stand for the average cumulative ranked probability score (CRPS) for average values.

[2] Please refer to Section 3 for details on model formulations. The capital letter 'X' indicates models with exogenous variables. The letter 'B' stands for Bayesian, whereas '-Min' and '-NW' refer to the usage of the Minnesota and the conjugate Normal-Wishart priors. All forecasts are produced with recursive estimation of the models.

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[4] ***, ** and * indicate that the average score ratios are significantly different from 1 at the significance levels of 1%, 5% and 10%, according to the Diebold-Mariano t-statistic test for equal average CRPS.

E Graphical Appendix

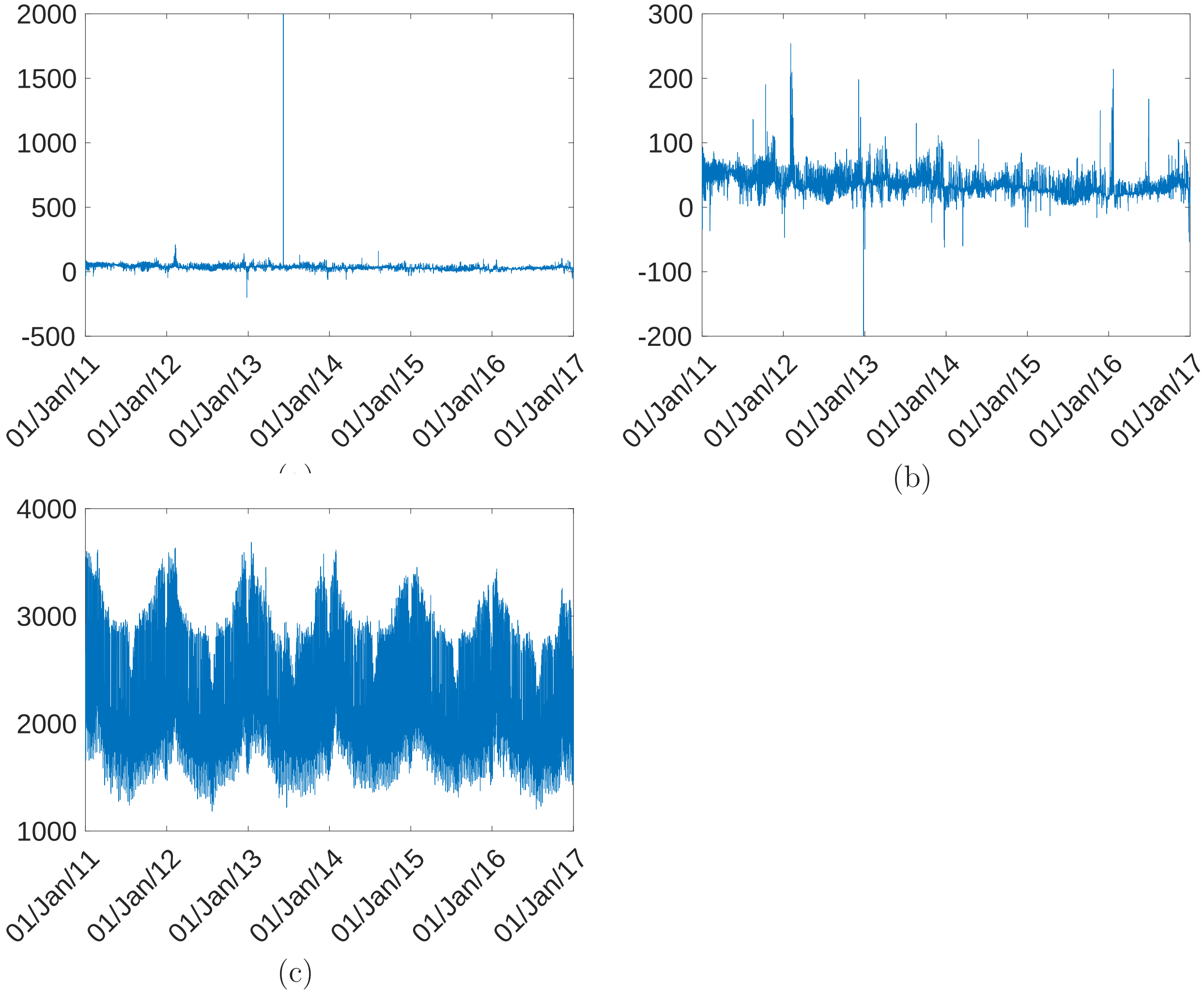

Figure 3: Hourly Series for Electricity Day-ahead Prices (panel a), Forecasted Consumption (panel b) and Forecasted Wind Generation (panel c) observed in Denmark from 01/01/2011 to 31/12/2016.

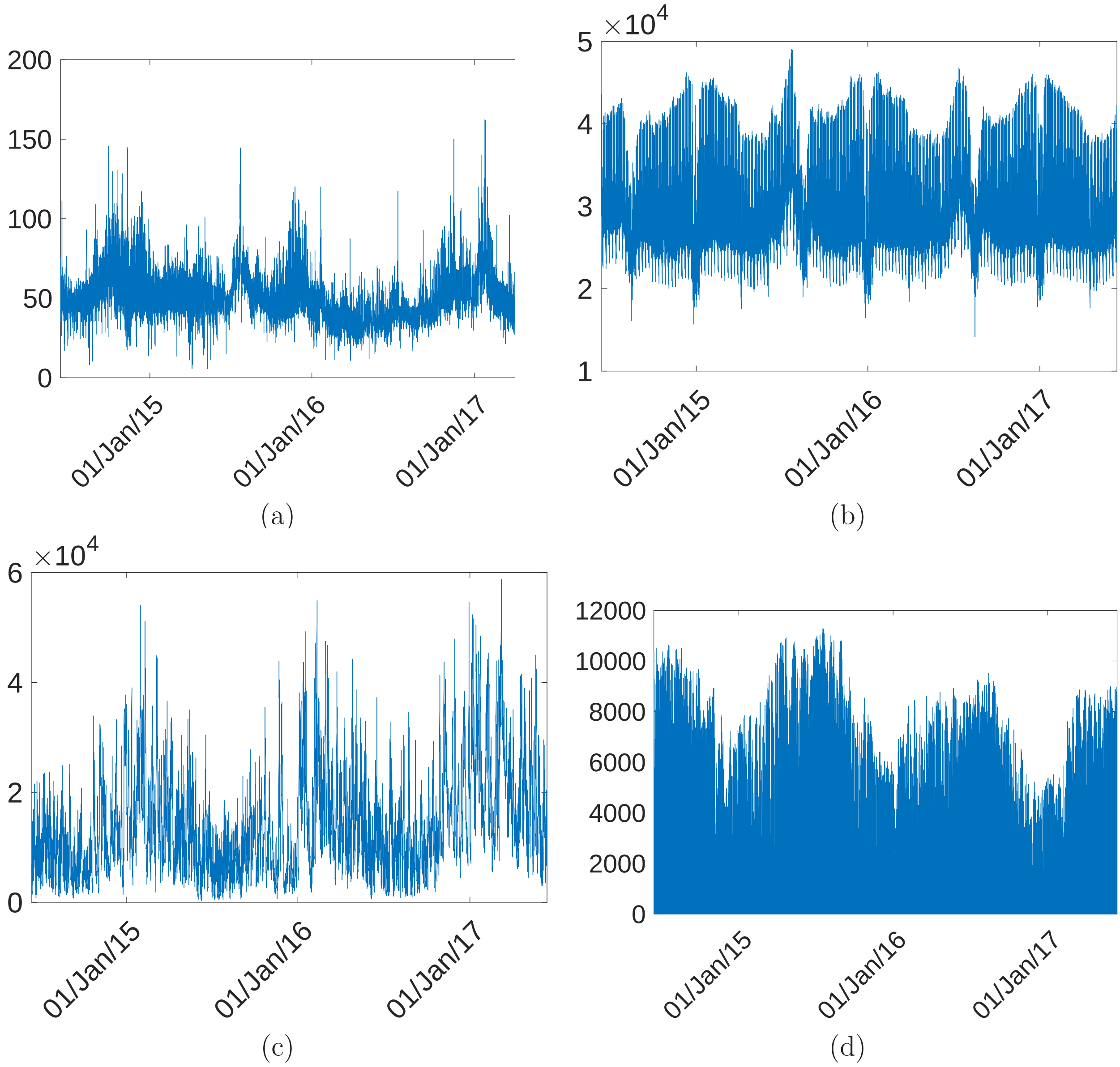

Figure 4: Hourly Series for Electricity Day-ahead Prices (panel a), Forecasted Demand (panel b), Forecasted Wind Generation (panel c) and Forecasted Solar PV Generation (panel d) observed in Italy from 13/06/2014 to 13/06/2017.

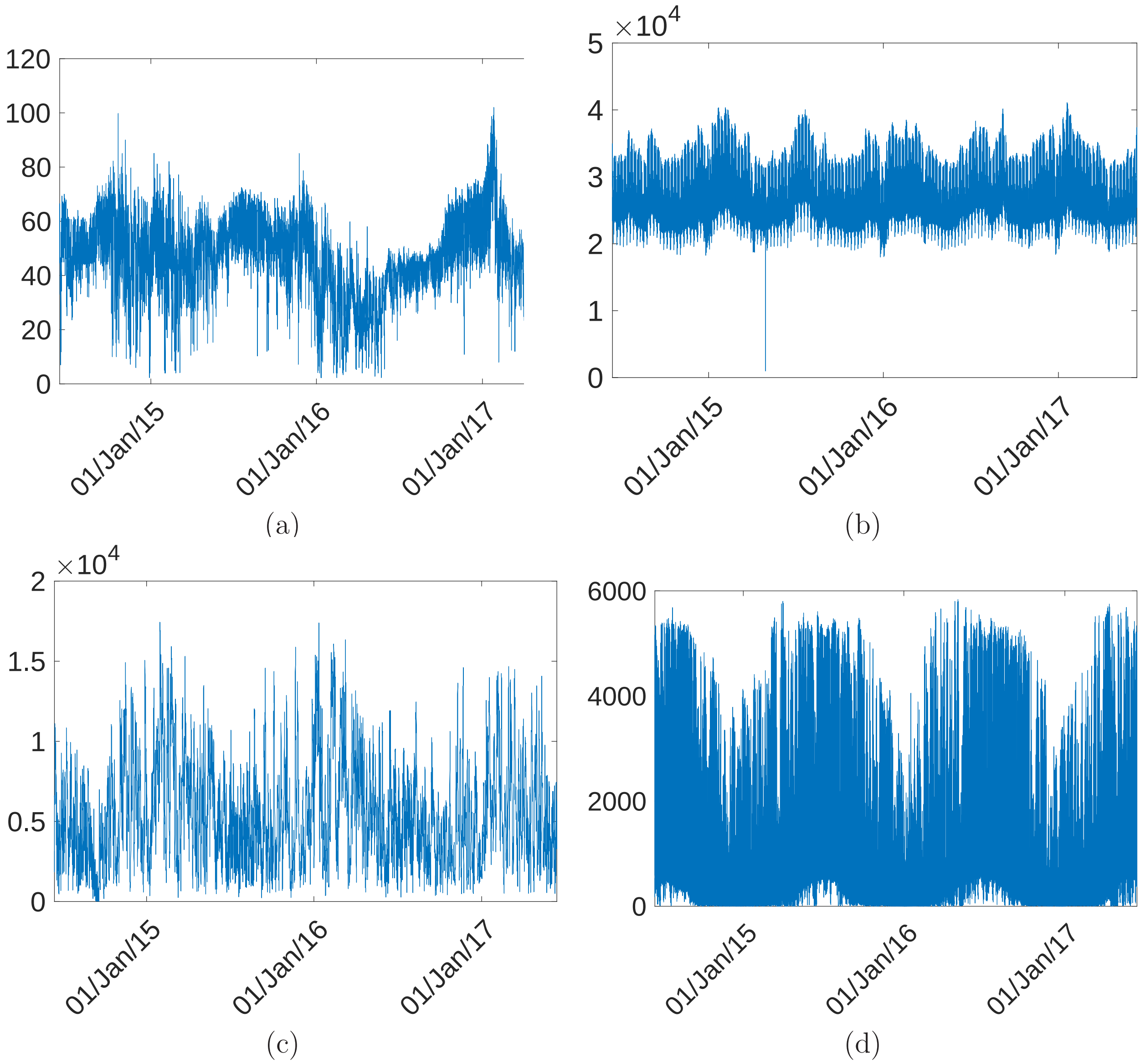

Figure 5: Hourly Series for Electricity Day-ahead Prices (panel a), Forecasted Demand (panel b), Forecasted Wind Generation (panel c) and Forecasted Solar PV Generation (panel d) observed in Spain from 13/06/2014 to 13/06/2017.

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