Optimal Monetary Policy with Nominal Rigidities and Lumpy Investment[*]

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New Keynesian theory generally abstracts from the lumpy nature of plant-level investment. Given the prominent role of investment spending for shaping optimal monetary policy, this simplification could be problematic. Our analysis suggests, however, that this is not the case in the context of a New Keynesian model featuring lumpy investment à la Sveen and Weinke (2007).

JEL Codes: E22, E31, E32.

1. Introduction

New Keynesian (NK) theory often abstracts from capital accumulation (see, e.g., Galí 2015), and if capital accumulation is taken into account, then it is common practice to postulate convex adjustment costs of one type or another (see, e.g., Christiano, Eichenbaum, and Evans 2005 and Woodford 2005), which makes the model inconsistent with the observed lumpiness in plant-level investment (see, e.g., Thomas 2002, Khan and Thomas 2008). Given the prominent role of investment spending for shaping optimal monetary policy,[1]

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[1]In the words of Schmitt-Grohé and Uribe (2007): "All the way from the work of Keynes (1936) and Hicks (1939) to that of Kydland and Prescott (1982) macroeconomic theories have emphasized investment dynamics as an important channel for the transmission of aggregate disturbances. It is therefore natural to expect that investment spending should play a role in shaping optimal monetary policy."

this simplification could be problematic. The reason is that lumpy investment is a potential source of significant heterogeneity across firms.[2] Hence, it might play an important role in shaping the optimal monetary policy response to shocks, the same way staggered prices generate inefficient price dispersion and thus provide a motive for inflation stabilization.

Our analysis shows, however, that the implications for the optimal design of monetary policy that obtain in the context of a conventional NK model depend very little on whether investment is lumpy, as in Sveen and Weinke (2007), or smooth, as in Woodford (2005). In Sveen and Weinke (2007) we assumed that both price-setting and investment decisions are conducted in a time-dependent fashion.[3] Up to a first-order approximation, the resulting framework has been shown to be observationally equivalent to the model in Woodford (2005) where he had postulated a convex capital adjustment cost at the firm level. In other words, the two alternative ways of modeling endogenous capital give rise to identical implications for aggregate dynamics. Note that in the field of investment theory, lumpiness is routinely generated by assuming stochastic non-convex capital adjustment costs (see, e.g., Thomas 2002, Khan and Thomas 2008). The recent contribution by Reiter, Sveen, and Weinke (2013) shows, however, that once that modeling choice is embedded into an otherwise conventional NK model, the latter implies large but very short-lived impacts on output and inflation, in a way that goes against the empirical evidence. In other words, it is well established, by now, that standard investment theory gives rise to counterfactual implications in the context of NK theory.[4] We therefore conduct our welfare analysis in the context of the model proposed in Sveen and Weinke (2007). The latter is the only lumpy investment model known so far that gives rise to an empirically relevant monetary transmission mechanism. Apart from the restrictions on capital accumulation, the models under consideration feature sticky prices and wages à la Calvo (1983). The co-existence of the two types of nominal

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[2]In the context of the models presented in this paper, there is no distinction between a firm and a plant. We therefore use those terms interchangeably.

[3]The assumption of time-dependent investment was originally proposed by Kiyotaki and Moore (1997).

[4]The simple reason is the very large interest rate sensitivity of investment implied by that investment theory.

rigidities gives rise to a monetary policy tradeoff and consequently to an interesting monetary policy design problem, as discussed in Erceg, Henderson, and Levin (2000).

Based on Rotemberg and Woodford (1997) and Sveen and Weinke (2009), the present paper presents a second-order accurate expression for the level of welfare[5] in the context of a baseline model featuring time-dependent lumpy investment. The welfare criterion is the unconditional expectation of average household utility. For a benchmark model where firm-level capital is assumed to be smooth as in Woodford (2005), that welfare criterion is of the form calculated in Sveen and Weinke (2009). Our first main result shows that baseline and benchmark imply optimized interest rate rules which are similar in the following sense: if the optimized rule from one model is used in the other one, then the resulting additional welfare loss is negligible. This is an important extension of our earlier work in Sveen and Weinke (2009). In that paper we had shown that optimized interest rate rules, as implied by Woodford's (2005) modeling of endogenous capital, are very similar (in the same sense as above) to the ones that obtain under the assumption of a rental market for capital after an appropriate adjustment of the price stickiness in the latter model. The reason for the adjustment is that Woodford's (2005) modeling of investment implies some endogenous price stickiness, i.e., a price setter internalizes the consequences of a price-setting decision for the marginal costs over the expected lifetime of the chosen price. This effect is, however, absent in the rental market model, where the marginal cost is constant across all firms in the economy. This difference in price stickiness turns out to be the only first-order difference between Woodford's (2005) model and a rental market specification, as analyzed in Sveen and Weinke (2005). After making the two models equivalent up to the first order (by adjusting the price stickiness), the corresponding welfare implications become very similar. Much of the related literature on the optimal design of interest rate rules has adopted the assumption of a rental market for capital.[6] It is therefore interesting to observe that our first result in

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[5]A meaningful welfare analysis can only be conducted based on a second-order approximation to the welfare criterion. See, e.g., Schmitt-Grohé and Uribe (2004).

[6]See, e.g., Schmitt-Grohé and Uribe (2007).

the present paper (when combined with the above-mentioned result in Sveen and Weinke 2009) lends some support to the view that those findings in the related literature appear to hinge very little on the rental market assumption. Second, we compare the welfare implications of each simple interest rate rule to the corresponding outcome under Ramsey optimal policy. This way we demonstrate that optimal policy is well approximated by optimized simple interest rate rules. This increases the practical relevance of our first result.

The remainder of the paper is organized as follows. Section 2 outlines the model structure. In section 3 the results are presented and section 4 concludes.

2.   Model

We use a New Keynesian model with complete markets. Shocks to total factor productivity are assumed to be the only source of aggregate uncertainty. There is a continuum of households and a continuum of firms. Each household (firm) is the monopolistically competitive supplier of a differentiated type of labor (type of good) and we assume sticky wages (sticky prices) à la Calvo (1983), i.e., each household (firm) gets to reoptimize its wage (price) with a constant and exogenous probability. Capital accumulation is assumed to take place at the firm level, and the additional capital resulting from an investment decision becomes productive with a one-period delay. In our baseline model (*LI* for short) lumpiness in investment is modeled as in Sveen and Weinke (2007), i.e., there is a Bernoulli draw conditional on which a firm is allowed to invest, or not. In the benchmark model (*FS* for short) firm-level investment is (unrealistically) assumed to be smooth, as in Woodford (2005).[7] Since the details of the models have been discussed elsewhere (see Erceg, Henderson, and Levin 2000, Woodford 2005, and Sveen and Weinke 2007, 2009), we turn directly to the implied linearized equilibrium conditions. The only two equations that are different in *LI* and *FS* are, respectively, the inflation equation and the law of motion of

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[7]*FS* stands for "firm specific." It should be noted, however, that capital in the baseline model is also firm specific. We refer to the baseline specification as being the lumpy investment model (*LI*) and call the benchmark a model of firm-specific capital since this was Woodford's (2005) original label for this type of model.

capital, as we are going to see. Relying on the insights in Rotemberg and Woodford (1997) and Sveen and Weinke (2009), we will compute our welfare criterion (in each model under consideration) up to the second order.

2.1   Some Linearized Equilibrium Conditions

In what follows we present our baseline model *LI* and make clear in which places it differs from the benchmark *FS*. We consider a linear approximation around a zero-inflation steady state. In what follows, variables are expressed in terms of log-deviations from their steady-state values except for the nominal interest rate, i_t, wage inflation, ω_t, and price inflation, π_t, which denote the levels of the respective variables. The consumption Euler equation reads

$$c_t = E_t c_{t+1} - (i_t - E_t \pi_{t+1} - \rho), \qquad (1)$$

where c_t denotes aggregate consumption and E_t is the expectation operator conditional on information available through time t. Moreover, parameter ρ is the discount rate and the last equation also reflects our assumption of log consumption utility. Up to the first order, aggregate production is pinned down by

$$y_t = x_t + \alpha \ k_t + (1 - \alpha) \ n_t, \qquad (2)$$

where y_t denotes aggregate output, parameter α indicates the capital share, aggregate capital is k_t, aggregate hours are n_t, and x_t represents an exogenous index of technology. The latter is assumed to follow an $AR(1)$ process $x_t = \rho_x \ x_{t-1} + \varepsilon_t$, with $\rho_x \in (0, 1)$ and ε_t denoting an iid shock with variance σ^2. The wage-inflation equation results from averaging and aggregating optimal wage-setting decisions on the part of households, as discussed in Erceg, Henderson, and Levin (2000). It takes the following simple form:

$$\omega_t = \beta \ E_t \omega_{t+1} + \lambda_\omega \ (mrs_t - rw_t), \qquad (3)$$

where parameter $\beta \equiv 1/(1+\rho)$ is the subjective discount factor, rw_t denotes the real wage, and $mrs_t \equiv c_t + \eta n_t$ measures the average marginal rate of substitution of consumption for leisure. In the latter definition parameter η indicates the inverse of the (aggregate)

Frisch labor supply elasticity. Finally, we have used the definition $\lambda_\omega \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\frac{1}{1+\eta\varepsilon_N}$, where parameter θ_w denotes the probability that a household is not allowed to reoptimize its nominal wage in any given period, while parameter ε_N measures the elasticity of substitution between different types of labor. The price inflation equation takes the form

$$\pi_t = \beta\ E_t\pi_{t+1} + \lambda\ mc_t, \tag{4}$$

where $mc_t \equiv rw_t - (y_t - n_t)$ denotes the average real marginal cost. We have also used the definition $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\frac{1}{\xi}$, where parameter θ gives the probability that a firm does not get to reoptimize its price in any given period, while parameter ε denotes the elasticity of substitution between the differentiated goods. Finally, coefficient ξ is a function of the model's structural parameters that is computed numerically as in Sveen and Weinke (2007), using the method developed in Woodford (2005). The details of the respective computations of coefficient ξ depend on whether firm-level investment is lumpy, as in *LI*, or smooth as in *FS*. For future reference, it should be pointed out that Woodford's (2005) method relies on observing that a firm's price-setting rule can be approximated up to the first order by the following expression:

$$\widehat{p}_t^*(i) = \hat{p}_t^* - \tau_1\ \hat{k}_t(i),$$

with $\widehat{p}_t^*(i) \equiv p_t^*(i) - p_t$, $\hat{k}_t(i) \equiv k_t(i) - k_t$, and $\widehat{p}_t^* \equiv p_t^* - p_t$, where $p_t^*(i)$ is the price chosen by firm i in period t and p_t denotes the aggregate price level associated with the usual Dixit-Stiglitz aggregator. Firm i's relative to average capital stock is given by $\hat{k}_t(i)$. Moreover, $\hat{p}_t^*$ is the average relative price in the group of time-t price setters. Finally, coefficient τ_1 can be calculated by using Woodford's (2005) method. Here again the details of the respective computations of coefficient τ_1 depend on whether firm-level investment is lumpy, as in *LI*, or smooth as in *FS*. The law of motion of capital is obtained from averaging and aggregating optimal investment decisions on the part of firms. This implies

$$\Delta k_{t+1} = \beta\ E_t\Delta k_{t+2} + \frac{1}{\epsilon_\psi}\left[(1-\beta(1-\delta))\ E_t ms_{t+1}\right.$$
$$\left. - (i_t - E_t\pi_{t+1} - \rho)\right], \tag{5}$$

where $ms_t \equiv rw_t - (k_t - n_t)$ represents the average real marginal return to capital. The latter is measured in terms of marginal savings in labor costs since firms are demand constrained in our model. Moreover, parameter δ is the rate of depreciation. Finally, coefficient ϵ_ψ measures the smoothness in aggregate capital accumulation. It is a function of the model's structural parameters that is evaluated as in Sveen and Weinke (2007), where we observe that a firm's investment rule can be approximated up to the first order by the following expression:

$$\widehat{k}^*_{t+1}(i) = \hat{k}^*_{t+1} - \tau_2 \, \hat{p}_t(i),$$

with $\widehat{k}^*_{t+1}(i) \equiv k^*_{t+1}(i) - k_{t+1}$, $\hat{k}^*_{t+1} \equiv k^*_{t+1} - k_{t+1}$, and $\hat{p}_t(i) \equiv p_t(i) - p_t$, where $k^*_{t+1}(i)$ is the capital stock chosen by firm i in period t. Moreover, k^*_{t+1} is the average capital stock in the group of time-t investors and $\hat{p}_t(i)$ is firm i's relative price in period t. Coefficient τ_2 can also be calculated by using Woodford's (2005) method. The value of parameter ϵ_ψ in (5) depends crucially on the probability θ_k with which a firm is not allowed to invest in any given period. Note that ϵ_ψ is simply the parameter measuring the convex capital adjustment cost in FS. Finally, we observe that the value of λ in (4) coincides with its counterpart in FS if parameter θ_k is chosen in such a way that the laws of motion of capital coincide in LI and FS. This is the reason why the two models are observationally equivalent, up to the first order.[8] The goods market clearing condition reads

$$y_t = \zeta \, c_t + \frac{1 - \zeta}{\delta} \left[k_{t+1} - (1 - \delta) \, k_t \right], \tag{6}$$

where $\zeta \equiv 1 - \frac{\delta \alpha}{\rho + \delta}$ denotes the steady-state consumption-to-output ratio. Notice that the frictionless desired markup, $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$, does not enter the latter definition. The reason is our assumption of a wage subsidy that makes the steady state of our model efficient. This allows us to approximate our welfare criterion up to the second order relying on the first-order approximation that we have just considered. Next we discuss the welfare criterion that will be used

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[8]For details, see Sveen and Weinke (2007).

to assess the desirability of alternative arrangements for the conduct of monetary policy.

2.2 Welfare

As in Erceg, Henderson, and Levin (2000), the policymaker's period welfare function is assumed to be the unweighted average of households' period utility

$$\mathcal{W}_t \equiv U\left(C_t\right) + \int_0^1 V\left(N_t\left(h\right)\right) dh = U\left(C_t\right) + E_h\left\{V\left(N_t\left(h\right)\right)\right\}. \quad (7)$$

The assumption of complete asset markets combined with a utility function that is separable in its two arguments, consumption and hours worked, implies that heterogeneity in hours worked (a consequence of wage dispersion) does not translate into consumption heterogeneity. This is reflected in (7), where $U\left(C_t\right)$ indicates consumption utility and $V\left(N_t\left(h\right)\right)$ denotes the disutility associated with household h's decision to supply $N_t\left(h\right)$ hours. The notation E_h is meant to indicate a cross-sectional expectation integrating over households. We also follow Erceg, Henderson, and Levin (2000) in expressing period welfare as a fraction of steady-state Pareto-optimal consumption, i.e., we consider $\frac{\mathcal{W}_t - \overline{\mathcal{W}}}{\overline{U}_C \overline{C}}$, where a bar indicates the steady-state value of the original variable and U_C is the marginal utility of consumption. The second-order approximation to period welfare is calculated based on the method by Rotemberg and Woodford (1997).[9] Our welfare criterion is the unconditional expectation of period welfare, which we write in the way proposed by Svensson (2000):

$$\widetilde{\mathcal{W}} \equiv \lim_{\beta \to 1} E_t\left\{(1-\beta)\sum_{k=0}^{\infty} \beta^k \left(\frac{\mathcal{W}_{t+k} - \overline{\mathcal{W}}}{\overline{U}_C \overline{C}}\right)\right\}.$$

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[9]The proof that the method of Rotemberg and Woodford (1997) can be applied to the problem at hand carries over from Edge's (2003) work to ours because the relevant steady-states properties of the two models are identical.

We derive the following expression for that welfare criterion in *LI*:

$$\widetilde{\mathcal{W}} \simeq E \left\{ \Omega_1 y_t^2 + \Omega_2 c_t^2 + \Omega_3 k_t^2 + \Omega_4 n_t^2 + \Omega_5 \omega_t^2 \right.$$
$$\left. + \, \Omega_6 \pi_t^2 + \Omega_7 (\Delta k_t)^2 + \Omega_8 z_t^2 \right\}, \tag{8}$$

where the symbol $\simeq$ is meant to indicate that an approximation is accurate up to the second order. We have also used the definition $z_t \equiv \Delta k_t + \pi_t$, and coefficients Ω_1 to Ω_8 are functions of the structural parameters, as defined in the appendix. For *FS* the welfare criterion is calculated along the lines of Sveen and Weinke (2009). In order to derive (8), one needs to calculate the cross-sectional variances of prices and capital holdings at each point in time as well as their covariance. This is shown in the appendix. In a nutshell, we note that the above-mentioned linearized rules for price setting and for investment can be used to compute the relevant second moments with the accuracy that is needed for our second-order approximation to the welfare criterion.

3.   Results

We consider a conventional family of monetary policy rules and analyze constrained optimal rules, i.e., we restrict attention to a particular subset of possible parameter values that parameterize the rule. It is useful to note that rational expectations equilibrium is locally unique (i.e., determinate) under the constrained optimal policies. The simple interest rate rules under consideration below are therefore also implementable. We first compare the welfare implications of constrained optimal simple and implementable interest rate rules in *LI* and *FS*. In a second step, we ask how well those optimal rules approximate the Ramsey optimal policy.

3.1   Baseline Parameter Values

In our quantitative analysis, the following values are assigned to the model parameters.[10] We consider a quarterly model. The capital share, α, is set to 0.36. The elasticity of substitution between goods,

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[10]To solve the dynamic stochastic system of equations, we use Dynare (www.dynare.org). MATLAB code for our implementation of Woodford's (2005)

ε, takes the value 11. The rate of capital depreciation, δ, is assumed to be equal to 0.025. We set the elasticity of substitution between different types of labor, ε_N, equal to 6. Our choice for the Calvo price stickiness parameter, θ, is 0.75 and the value for the wage stickiness parameter, θ_w, is 0.75. This implies an average expected duration of price and wage contracts of one year. The coefficient of autocorrelation in the process of technology, ρ_x, is assumed to take the value 0.95. Those parameter values are justified in Erceg, Henderson, and Levin (2000), Sveen and Weinke (2007, 2009), and the references therein. Finally, we set the lumpiness parameter $\theta_k = 0.955$. The latter choice makes our model consistent with the evidence in Khan and Thomas (2008), according to which 18 percent of plants undertake lumpy investments in any given year. Conditional on our baseline choices for the remaining parameters, this results in $\epsilon_\psi = 6.359$, a value implying a plausible degree of smoothness in aggregate capital accumulation.[11]

3.2   The Welfare Consequences of Simple Implementable Rules

We consider a family of simple interest rate rules of the form

$$r_t = \rho + \tau_r \ (r_{t-1} - \rho) + \tau_s \left[\tau_\omega \omega_t + (1 - \tau_\omega) \pi_t\right] + \tau_y \ \tilde{y}_t, \quad (9)$$

where parameter τ_s measures the overall responsiveness of the nominal interest rate to changes in inflation, whereas τ_ω is the relative weight put on wage inflation. The weight on price inflation is therefore given by $(1 - \tau_\omega)$. Parameter τ_r denotes the interest rate smoothing coefficient and parameter τ_y indicates the quantitative importance attached to a measure of real economic activity that is taken to be the output gap, $\tilde{y}_t$. To be precise, $\tilde{y}_t \equiv y_t - y_t^{nat}$, where y_t^{nat} is natural output that is computed à la Neiss and Nelson (2003), i.e., the equilibrium output that would obtain in the absence

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method is available upon request. We also thank Junior Maih for code that has been used in the computation of the optimized coefficients of the interest rate rules under consideration.

[11]As we have mentioned already, ϵ_ψ is simply the parameter measuring the convex capital adjustment cost in Woodford (2005).

of nominal frictions. Only positive parameter values are considered, and parameter τ_ω is required to be less than or equal to one.

We follow Erceg, Henderson, and Levin (2000) and consider the business-cycle cost of nominal rigidities. For each model under consideration, we report the optimized coefficients entering the interest rate rule as well as the corresponding welfare measure. Specifically, we compute our welfare measure in each model under the assumption of flexible prices and wages (i.e., $\widetilde{\mathcal{W}}^{nat}$) and subtract the resulting expression from the value in the corresponding model with the nominal rigidities being present (i.e., $\widetilde{\mathcal{W}}$). As they do, we divide this difference by the productivity innovation variance, σ^2, and refer to the resulting number as being a welfare loss. Let us give a concrete example for the interpretation of the welfare loss numbers in our tables. Suppose the productivity innovation variance is 0.00315^2. Under a standard Taylor (1993) rule for the conduct of monetary policy (i.e., $\tau_r = 0.7$, $\tau_s = 1.5$, $\tau_\omega = 0$, and $\tau_y = 0.125$), this choice implies an output variance of 0.016^2, which is well in line with the variance of the cyclical component of output in the U.S. economy, at least for the period starting in the early eighties until the onset of the 2007 financial crisis. Then, a welfare loss of -10 means that the representative household would be willing to give up $10 \times 0.00315^2 \times 100 = 0.01$ percentage points of steady-state (Pareto-optimal) consumption in order to avoid the business-cycle cost associated with the presence of the nominal rigidities.

3.2.1   Wage–Price Rule

We now compare optimized interest rate rules under LI and FS. Our first set of results regards interest rate rules according to which the nominal interest rate is only adjusted in response to changes in price and wage inflation, i.e., parameter τ_y is set to zero. We also state the welfare loss implied by LI if the optimized rule from FS is used in that model. The results are shown in table 1.

The welfare losses under the respective optimized rules are very similar in LI and FS, and the differences between the corresponding optimized parameter values, as implied by the two models, are also small. Moreover, the additional welfare loss is tiny if the optimized rule from FS is used in LI (compared with the outcome that obtains under the optimized rule in LI). In other words, a central banker

Table 1. Wage–Price Rule

Parameter	FS	LI
τ_r	1.0352	1.0353
τ_s	0.8399	0.5837
τ_ω	0.3416	0.3295
Welfare Loss $\left(\frac{\widetilde{\mathcal{W}} - \widetilde{\mathcal{W}}^{nat}}{\sigma^2} \right)$	$-9.1104 \ (-9.1342^a)$	-9.1264

[a] Welfare loss in the lumpy investment model with FS rule.

would only make a tiny mistake by abstracting from lumpiness in investment when optimizing a wage–price interest rate rule. What is the intuition? There is only a little interaction between price setting and investment in the two models under consideration. In fact, an investment decision is almost unaffected by the current price of a firm. The reason is that an investor takes rationally into account the future price changes that are expected to take place over the lifetime of the chosen capital stock. This in turn is so as long as investment decisions are forward-looking enough (regardless of the particular type of restriction on capital accumulation). Put differently, the additional heterogeneity among firms that is implied by the lumpiness in investment does not matter much for the business-cycle cost of the nominal rigidities.

To some extent, the results in table 1 are also surprising. To see this, note a subtle first-order difference between the versions of LI and FS that obtain under the assumption of flexible prices and wages (and keep in mind that those versions of the two models enter our calculations when isolating the business-cycle cost of the nominal rigidities). The reason is that ϵ_ψ is simply a parameter in the law of motion of capital, as implied by FS. On the other hand, ϵ_ψ is a coefficient in the context of LI whose value also depends (among other parameter values) on the degree of price stickiness, as analyzed in Sveen and Weinke (2007). This difference turns out, however, to be inconsequential for the results shown in table 1 according to which the two models have very similar implications for the optimal design of wage–price rules. It is shown next that the similarity between the welfare results in LI and FS is not specific to the type of interest rate rule that we had considered so far.

3.2.2   Taylor-Type Rule

Once again, we compare welfare implications in *LI* and *FS*. The structure of the comparison is the same as in the preceding subsection, but we now consider interest rate rules according to which the nominal interest rate is only adjusted in response to changes in price inflation and the output gap, i.e., parameter τ_ω is set to zero. Also in this case, our analysis brings out a clear message. This is shown in table 2.

There are only very small differences between the welfare losses under the optimized rules in *LI* and *FS*, and the same is true for the respective coefficients parameterizing those rules. Moreover, the additional welfare loss is once again tiny if the optimized rule from *FS* is used in *LI* (compared with the outcome that obtains under the optimized rule in *LI*). We have already mentioned that the law of motion of natural capital is different in the two models because price stickiness affects the smoothness of aggregate capital accumulation in *LI*, but not in *FS*. This observation is particularly relevant for the analysis of Taylor-type interest rate rules because natural output is used in the construction of the output gap. However, as our results clearly show, the quantitative importance of this effect is also very limited in this context. Next we analyze how well the simple and implementable interest rate rules considered so far approximate Ramsey-optimal policy.

3.3   Ramsey-Optimal Policy

We now compute Ramsey-optimal welfare losses in *LI* and *FS*.[12] This is shown in table 3.

In each model the welfare loss under Ramsey-optimal policy is well approximated by the corresponding outcomes that are associated with the optimized interest rate rules considered in the preceding subsection. Taken together, the present paper therefore conveys a positive message. We develop a New Keynesian model featuring lumpy investment using the formalism of a time-dependent rule à la Sveen and Weinke (2007). (As we discussed in the introduction, no alternative lumpy investment model has been proposed so far in the

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[12]To compute Ramsey optimal policy, we use Dynare (www.dynare.org).

**Table 2. Taylor-Type Rule with Neiss
and Nelson Output Gap**

Parameter	FS	LI
τ_r	1.0423	0.9911
τ_s	0.0365	0.0093
τ_y	0.0283	0.0074
Welfare Loss $\left(\frac{\widetilde{\mathcal{W}} - \widetilde{\mathcal{W}}^{nat}}{\sigma^2}\right)$	-9.1914 (-9.4744^a)	-9.1639

[a]Welfare loss in the lumpy investment model with *FS* rule.

Table 3. Ramsey-Optimal Policy

	FS	LI
Welfare Loss $\left(\frac{\widetilde{\mathcal{W}} - \widetilde{\mathcal{W}}^{nat}}{\sigma^2}\right)$	-8.9311	-8.9157

literature that would give rise to an empirically relevant monetary transmission mechanism.) In the context of the resulting framework, it turns out that the conclusions for the desirability of alternative arrangements for the conduct of monetary policy are strikingly similar regardless of whether or not lumpiness in investment is taken into account.

3.4 Sensitivity Analysis

Let us challenge the results obtained so far. Our first sensitivity analysis regards the notion of the output gap. In the context of a model featuring endogenous capital accumulation, Woodford (2003, chapter 5) proposes to refine that notion in the following way. He uses the equilibrium output that would obtain if the nominal rigidities were absent and expected to be absent in the future but taking as given the capital stock resulting from optimizing investment behavior in the past in an environment *with* the nominal rigidities present. Woodford argues that this measure of natural output is more closely related to equilibrium determination than the alternative measure
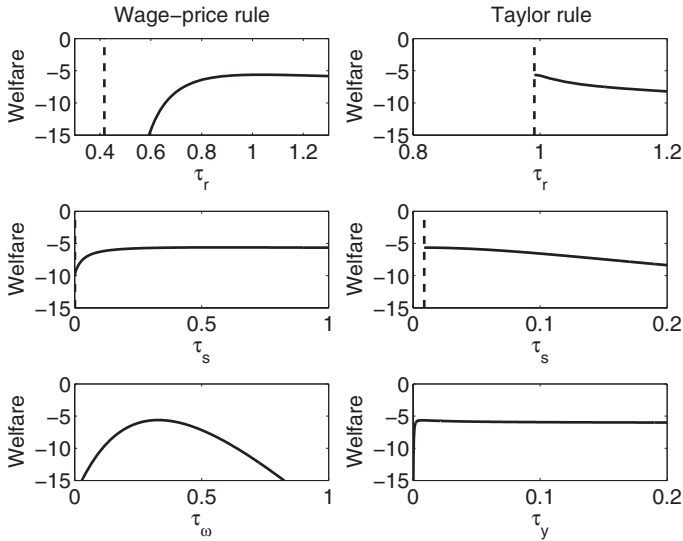
Table 4. Taylor-Type Rule with Woodford Output Gap

Parameter	FS	LI
τ_r	1.0305	1.0080
τ_π	0.0545	0
τ_y	0.0227	0.0343
Welfare Loss $\left(\frac{\widetilde{\mathcal{W}} - \widetilde{\mathcal{W}}^{nat}}{\sigma^2} \right)$	−9.1935 (−9.3054[a])	−9.0641

[a]Welfare loss in the lumpy investment model with *FS* rule.

that has been proposed by Neiss and Nelson (2003) and that we have used so far in our analysis. We have stated already that under their definition natural output is the equilibrium output that would obtain if nominal rigidities were absent. But this means that they are not only currently absent and expected to be absent in the future but that they had also been absent *in the past*. This distinction turns out, however, to be of negligible importance for the results in the present paper. This is shown in table 4.

Our second sensitivity analysis is concerned with the following question: What are the additional welfare losses implied by deviations from the optimized interest rate rules that have been analyzed so far in the context of *LI*? Let us therefore reconsider the general interest rate rule stated in equation (9). The left-hand side of figure 1 shows by how much the welfare loss increases if the policy parameters deviate from their optimal values in the case of the wage–price rule considered above (i.e., the specification where parameter τ_y is set to zero). The right-hand side of figure 1 shows the corresponding analysis for the case of the Taylor-type rule (i.e., the specification where parameter τ_ω is set to zero). Each panel of the figure illustrates the welfare loss associated with a change in one policy parameter at a time while holding the remaining parameters constant at their optimal values. In each case, the figure also shows the corresponding indeterminacy region in the parameter space. More concretely, dashed lines indicate the critical parameter values in the sense that rational expectations equilibrium is locally unique (or determinate) for all parameter values that are larger than those critical values. Finally, let us mention that each panel in the figure shows the same

**Figure 1. Robustness Analysis of Welfare Losses
Associated with Simple Implementable Interest
Rate Rules in the Context of _LI_**

range of welfare losses. That range is chosen arbitrarily, but we believe that our particular choice allows us to highlight the most relevant economic effects. In some cases (more concretely, for the wage–price rule and its parameters τ_r and τ_ω) parameter values that give rise to a determinate equilibrium imply welfare losses outside that range. Those losses are therefore not shown in the corresponding panels of figure 1.

From an economic point of view, our sensitivity analysis with respect to the output gap response in the Taylor-type rule deserves special attention. It is shown that small deviations from the optimal rule result in negligible welfare losses. Only to the extent that the central bank attaches very little importance to the output gap in setting the nominal interest rate does the resulting rule imply a large welfare loss with respect to the optimal policy.[13]

---

[13]Let us also mention here that the conventional wisdom according to which responding to the level of output in an interest rate rule is costly from a welfare point of view (see, e.g., Schmitt-Grohé and Uribe 2007) remains valid in the context of the models analyzed in the present paper.

4.   Conclusion

We augment the model in Erceg, Henderson, and Levin (2000) with a standard textbook treatment of endogenous capital accumulation in the context of New Keynesian theory, namely the firm-specific investment decision considered in Woodford (2005). The latter model abstracts, however, from lumpiness in investment, which is an uncontroversial empirical regularity. The present paper shows that the implications of Woodford's (2005) modeling of capital accumulation turn out to be similar to the ones that emerge in the presence of a lumpy investment decision of the form considered in Sveen and Weinke (2007). In fact, under both alternative assumptions on capital accumulation, we find that simple implementable interest rate rules generate welfare results that are similar. Moreover, the additional welfare loss associated with using the optimized rule from one model in the other one is tiny. Finally, optimized simple interest rate rules are shown to approximate very well the corresponding outcomes under Ramsey-optimal policy.

We had already mentioned that there is no consensus on the way in which endogenous capital accumulation should be modeled in the context of monetary models. Clearly, this issue is high on our research agenda, and, needless to say, alternative ways of modeling lumpiness in investment would change not only the implied monetary transmission mechanism but also the model's welfare implications. Another caveat regards the assumed stochastic driving force of the models under consideration. In each case we restrict attention to stationary exogenous shocks to total factor productivity. This is in line with the assumption in Erceg, Henderson, and Levin (2000) on which we build our analysis. Given the uncontroversial empirical support for the importance of demand factors for aggregate fluctuations, it would however also be interesting to assess the potential role of alternative economic shocks for the results obtained in the present paper. Those caveats notwithstanding, our results demonstrate that time-dependent lumpy investment in itself does not lead to any important changes in the welfare relevant results with respect to standard textbook treatments of endogenous capital in the New Keynesian theory.

Appendix. Welfare with Lumpy Investment

Throughout the appendix we use the notation and the definitions that are introduced in the text. We approximate our utility-based welfare criterion up to the second order. In what follows, we make frequent use of two rules,

$$\frac{A_t - \overline{A}}{\overline{A}} \simeq a_t + \frac{1}{2}a_t^2, \tag{10}$$

where $\simeq$ is meant to indicate that an approximation is accurate up to the second order. We have also used the definition $a_t \equiv \ln\left(\frac{A_t}{\overline{A}}\right)$.

Moreover, if $A_t = \left(\int\limits_0^1 A_t(i)^\gamma \, di\right)^{\frac{1}{\gamma}}$, then

$$a_t \simeq E_i a_t(i) + \frac{1}{2}\gamma Var_i a_t(i), \tag{11}$$

with Var_i indicating a cross-sectional variance. As we have already mentioned in the text, the policymaker's period welfare function reads

$$\mathcal{W}_t \equiv U(C_t) - \int_0^1 V(N_t(h)) \, dh = U(C_t) - E_h\{V(N_t(h))\}. \tag{12}$$

It follows that

$$\mathcal{W}_t \simeq \overline{\mathcal{W}} + \overline{U}_C\overline{C}\left(c_t + \frac{1}{2}c_t^2\right) - \overline{V}_N\overline{N}E_h\left\{n_t(h) + \frac{1}{2}n_t(h)^2\right\}$$
$$+ \frac{1}{2}\overline{U}_{CC}\overline{C}^2 c_t^2 - \frac{1}{2}\overline{V}_{NN}\overline{N}^2 E_h\left\{n_t(h)^2\right\}.$$

The Pareto optimality of the steady state implies $\overline{V}_N\overline{N} = U_C\overline{C}\frac{1-\alpha}{\zeta}$. We therefore have

$$\frac{\mathcal{W}_t - \overline{\mathcal{W}}}{\overline{U}_C\overline{C}} \simeq \left(c_t + \frac{1}{2}c_t^2\right) - \frac{1}{2}c_t^2 - \frac{1-\alpha}{\zeta}E_h\{n_t(h)\}$$
$$- \frac{1}{2}\frac{1-\alpha}{\zeta}(1+\phi)E_h\left\{n_t(h)^2\right\}. \tag{13}$$

Next we show how the linear terms in consumption and employment in the last expression can be approximated up to the second order. We start by analyzing the consumption portion of welfare. To this end we invoke the resource constraint.

The Consumption Portion of Welfare

The resource constraint reads

$$Y_t = C_t + K_{t+1} - (1 - \delta) K_t,$$

where $K_t \equiv \int_0^1 K_t(i)\, di$ and $K_t(i)$ is the amount of composite goods used in firm i's production. It follows that

$$
c_t + \frac{1}{2}c_t^2 \simeq \frac{1}{\zeta}\left(y_t + \frac{1}{2}y_t^2\right) - \frac{1-\zeta}{\zeta}\frac{1}{\delta}\left(k_{t+1} + \frac{1}{2}k_{t+1}^2\right)
$$
$$
+ \frac{1-\zeta}{\zeta}\frac{1-\delta}{\delta}\left(k_t + \frac{1}{2}k_t^2\right). \tag{14}
$$

The Labor Portion of Welfare

Aggregate labor supply is given by $N_t \equiv \left(\int_0^1 N_t(i)^{\frac{\epsilon_N-1}{\epsilon_N}}\, di\right)^{\frac{\epsilon_N}{\epsilon_N-1}}$. Using the second rule, we can write

$$n_t \simeq E_h n_t(h) + \frac{1}{2}\frac{\epsilon_N-1}{\epsilon_N} Var_h n_t(h).$$

Aggregate labor demand reads

$$L_t \equiv \int_0^1 L_t(i)\, di = \int_0^1 \left(\frac{Y_t(i)}{X_t K_t(i)^\alpha}\right)^{\frac{1}{1-\alpha}}\, di = B_t^{\frac{1}{1-\alpha}},$$

where $B_t \equiv \left(\int_0^1 B_t(i)^{\frac{1}{1-\alpha}}\, di\right)^{1-\alpha}$ and $B_t(i) \equiv \frac{Y_t(i)}{X_t K_t(i)^\alpha}$. Clearing of the labor market implies that $N_t = L_t$. We therefore have

$$n_t = \frac{1}{1-\alpha}b_t.$$

Invoking the second rule, we obtain

$$b_t \simeq E_i b_t(i) + \frac{1}{2}\frac{1}{1-\alpha} Var_i b_t(i),$$

and we also note that

$$b_t(i) = y_t(i) - x_t - \alpha\, k_t(i).$$

The last result implies

$$E_i b_t(i) = E_i y_t(i) - x_t - \alpha E_i k_t(i),$$

$$Var_i b_t(i) = Var_i y_t(i) + \alpha^2 \kappa_t - 2\alpha Cov_i(y_t(i), k_t(i)),$$

with Var_i denoting the cross-sectional variance and Cov_i the covariance operator. We can therefore write

$$b_t \simeq E_i y_t(i) - x_t - \alpha E_i k_t(i)$$
$$+ \frac{1}{2}\frac{1}{1-\alpha}\left[Var_i y_t(i) + \alpha^2 \kappa_t - 2\alpha Cov_i(y_t(i), k_t(i)) \right].$$

Let us also observe that

$$E_i k_t(i) \simeq k_t - \frac{1}{2} Var_i k_t(i),$$

$$E_i y_t(i) \simeq y_t - \frac{1}{2}\frac{\varepsilon - 1}{\varepsilon} Var_i y_t(i).$$

Combining the last three results, we arrive at

$$b_t \simeq y_t - x_t - \alpha\, k_t + \frac{1}{2}\frac{\alpha}{1-\alpha} Var_i k_t(i) + \frac{1}{2}\frac{1}{\varepsilon}\left(\frac{1-\alpha+\alpha\varepsilon}{1-\alpha} \right) Var_i y_t(i)$$
$$- \frac{\alpha}{1-\alpha} Cov_i(y_t(i), k_t(i)).$$

It follows that

$$E_h n_t(h) \simeq \frac{1}{1-\alpha}(y_t - x_t - \alpha k_t) + \frac{1}{2}\frac{\alpha}{(1-\alpha)^2} Var_i k_t(i)$$
$$+ \frac{1}{2}\frac{1}{\varepsilon}\frac{1-\alpha+\alpha\varepsilon}{(1-\alpha)^2} Var_i y_t(i)$$
$$- \frac{\alpha}{(1-\alpha)^2} Cov_i(y_t(i), k_t(i)) - \frac{1}{2}\frac{\varepsilon_N - 1}{\varepsilon_N} Var_h n_t(h).$$

Using the demand functions for goods and labor services, we obtain

$$Var_i y_t(i) \simeq \varepsilon^2 \Delta_t,$$
$$Cov_i(y_t(i), k_t(i)) \simeq -\varepsilon \psi_t,$$
$$Var_h n_t(h) \simeq \varepsilon_N^2 \lambda_t,$$

where $\Delta_t \equiv Var_i \widehat{p}_t(i)$, $\psi_t \equiv Cov_i(\widehat{p}_t(i), k_t(i))$, and $\lambda_t \equiv Var_h \widehat{w}_t(h)$, with $\widehat{w}_t(h) \equiv w_t(h) - w_t$. In the latter definition, $w_t(h)$ is household h's nominal wage and w_t denotes the aggregate wage level associated with the corresponding Dixit-Stiglitz aggregator. We therefore have

$$E_h n_t(h) \simeq \frac{1}{1-\alpha}(y_t - x_t - \alpha k_t) + \frac{1}{2}\frac{1-\alpha+\alpha\varepsilon}{(1-\alpha)^2}\varepsilon\,\Delta_t$$
$$+ \frac{1}{2}\frac{\alpha}{(1-\alpha)^2}\,\kappa_t + \frac{\alpha\varepsilon}{(1-\alpha)^2}\,\psi_t - \frac{1}{2}(\varepsilon_N - 1)\varepsilon_N\,\lambda_t,$$

$$(15)$$

with $\kappa_t \equiv Var_i \widehat{k}_t(i)$. Finally, we note that $E_h\left\{n_t(h)^2\right\}$ can be written as

$$E_h\left\{n_t(h)^2\right\} \simeq \varepsilon_N^2\,\lambda_t + n_t^2.$$

$$(16)$$

The Welfare Function

Combining (13), (14), (15), and (16), we arrive at the following approximation to period welfare:

$$\frac{\mathcal{W}_t - \overline{\mathcal{W}}}{\overline{U}_C\overline{C}} \simeq \frac{1}{\zeta}\,x_t - \frac{1}{\zeta}\frac{\alpha}{\rho+\delta}\left[k_{t+1} + (1+\rho)k_t\right] + \frac{1}{2}\frac{1}{\zeta}\,y_t^2 - \frac{1}{2}\,c_t^2$$
$$- \frac{1}{2}\frac{1-\zeta}{\zeta}\frac{1}{\delta}\,k_{t+1}^2 + \frac{1}{2}\frac{1-\zeta}{\zeta}\frac{1-\delta}{\delta}\,k_t^2$$
$$- \frac{1}{2}\frac{1-\alpha}{\zeta}(1+\phi)\,n_t^2 - \frac{1}{2}\frac{\varepsilon}{\zeta}\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}\,\Delta_t$$
$$- \frac{1}{2}\frac{1}{\zeta}\frac{\alpha}{1-\alpha}\,\kappa_t - \frac{1}{2}\frac{(1-\alpha)\varepsilon_N}{\zeta}(1+\phi\varepsilon_N)\,\lambda_t - \frac{1}{\zeta}\frac{\alpha\varepsilon}{1-\alpha}\,\psi_t.$$

$$(17)$$

The first linear term in the last expression is proportional to the level of aggregate (log) technology, x_t, which is exogenous. The remaining linear terms are proportional to current and next period's aggregate capital, k_t and k_{t+1}. Next we compute $\frac{1-\beta}{\overline{U}_C \overline{C}} E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left(\mathcal{W}_{t+k} - \overline{\mathcal{W}} \right) \right\}$, which allows us to invoke a result by Edge (2003). As in her model, the terms in aggregate capital cancel except for the initial one. Following the lead of Svensson (2000), we consider the limit for $\beta \to 1$. This allows us to abstract from initial conditions.[14]

$$\widetilde{\mathcal{W}}^{LI} \simeq \Omega_1 \, E \left\{ y_t^2 \right\} + \Omega_2 \, E \left\{ c_t^2 \right\} + \Omega_3 \, E \left\{ k_t^2 \right\} + \Omega_4 \, E \left\{ n_t^2 \right\}$$
$$+ \, \Omega_\lambda \, E \left\{ \lambda_t \right\} + \Omega_\Delta \, E \left\{ \Delta_t \right\} + \Omega_\kappa \, E \left\{ \kappa_t \right\} + \Omega_\psi \, E \left\{ \psi_t \right\}, \tag{18}$$

with

$$\Omega_1 \equiv \frac{1}{2} \frac{1}{\zeta}, \;\; \Omega_2 \equiv -\frac{1}{2}, \;\; \Omega_3 \equiv -\frac{1}{2} \frac{1 - \zeta}{\zeta},$$

$$\Omega_4 \equiv -\frac{1}{2} \frac{1 - \alpha}{\zeta} (1 + \phi), \;\; \Omega_\lambda \equiv -\frac{1}{2} \frac{(1 - \alpha) \, \varepsilon_N}{\zeta} (1 + \phi \varepsilon_N),$$

$$\Omega_\Delta \equiv -\frac{1}{2} \frac{\varepsilon}{\zeta} \frac{1 - \alpha + \alpha \varepsilon}{1 - \alpha}, \;\; \Omega_\kappa \equiv -\frac{1}{2} \frac{1}{\zeta} \frac{\alpha}{1 - \alpha}, \;\; \Omega_\psi \equiv -\frac{1}{\zeta} \frac{\alpha \varepsilon}{1 - \alpha}.$$

Next we derive recursive formulations for the cross-sectional variances of wages, prices, and capital holdings as well as the welfare relevant covariance of prices and capital holdings. As in Erceg, Henderson, and Levin (2000), the cross-sectional variance of wages, λ_t, can be written in the following way:

$$\lambda_t \simeq \theta_w \, \lambda_{t-1} + \frac{\theta_w}{1 - \theta_w} \, (\omega_t)^2,$$

and the unconditional expected value is

$$E \left\{ \lambda_t \right\} \simeq \frac{\theta_w}{(1 - \theta_w)^2} E \left\{ \omega_t^2 \right\}.$$

---

[14]For a formal proof, see Dennis (2007).

Next we derive recursive formulations for Δ_t, κ_t, and ψ_t. To this end we start by invoking the pricing rule and the capital accumulation rule,

$$\widehat{p}_t^*(i) \overset{FO}{\simeq} \widehat{p}_t^* - \tau_1 \, \widehat{k}_t(i),$$

$$\ln P_t^*(i) \overset{FO}{\simeq} \ln P_t^* - \tau_1 \left(k_t(i) - k_t\right),$$

$$k_{t+1}^*(i) \overset{FO}{\simeq} k_{t+1}^* - \tau_2 \left(\ln P_t(i) - \ln P_t\right),$$

where $\overset{FO}{\simeq}$ is used to indicate that the approximation holds up to the first order. Moreover, we define

$$\widetilde{p}_t \equiv E_i \ln P_t(i),$$

and, in an analogous manner, $\widetilde{p}_t^*$. We should note that $\widetilde{p}_t^* \overset{FO}{\simeq} \ln P_t^*$. For future reference, we first derive the following two expressions:

$$\widetilde{p}_t - \widetilde{p}_{t-1} = E_i\left[\ln P_t(i) - \widetilde{p}_{t-1}\right] = \theta_p E_i\left[\ln P_{t-1}(i) - \widetilde{p}_{t-1}\right]$$
$$+ \left(1 - \theta_p\right) E_i\left[\ln P_t^*(i) - \widetilde{p}_{t-1}\right],$$
$$\overset{FO}{\simeq} \left(1 - \theta_p\right)\left(\widetilde{p}_t^* - \widetilde{p}_{t-1}\right),$$
$$k_{t+1} - k_t = E_i\left[k_{t+1}(i) - k_t\right] = \theta_k E_i\left[k_t(i) - k_t\right]$$
$$+ \left(1 - \theta_k\right) E_i\left[k_{t+1}^*(i) - k_t\right],$$
$$\overset{FO}{\simeq} \left(1 - \theta_k\right)\left(k_{t+1}^* - k_t\right).$$

We rewrite the price dispersion term, Δ_t, in the way proposed by Woodford (2003, chapter 6),

$$\Delta_t = E_i\left[\left(\ln P_t(i) - \widetilde{p}_{t-1}\right)^2\right] - \left(\widetilde{p}_t - \widetilde{p}_{t-1}\right)^2,$$
$$= \theta_p E_i\left[\left(\ln P_{t-1}(i) - \widetilde{p}_{t-1}\right)^2\right]$$
$$+ \left(1 - \theta_p\right) E_i\left[\left(\ln P_t^*(i) - \widetilde{p}_{t-1}\right)^2\right] - \left(\widetilde{p}_t - \widetilde{p}_{t-1}\right)^2.$$

After invoking the price-setting rule, we obtain the following recursive formulation for the measure of price dispersion Δ_t:

$$\Delta_t \simeq \theta_p\, \Delta_{t-1} + (1 - \theta_p)\, E_i \left[(\widetilde{p}_t^* - \tau_1 (k_t(i) - k_t) - \widetilde{p}_{t-1})^2 \right]$$
$$- (\widetilde{p}_t - \widetilde{p}_{t-1})^2$$
$$\simeq \theta_p\, \Delta_{t-1}$$
$$+ (1 - \theta_p)\, E_i \left[\left(\frac{1}{1 - \theta_p} (\widetilde{p}_t - \widetilde{p}_{t-1}) - \tau_1 (k_t(i) - k_t) \right)^2 \right]$$
$$- (\widetilde{p}_t - \widetilde{p}_{t-1})^2$$
$$\simeq \theta_p\, \Delta_{t-1} + (1 - \theta_p)\, \tau_1^2 Var_i k_t(i) + \left(\frac{1}{1 - \theta_p} - 1 \right) (\widetilde{p}_t - \widetilde{p}_{t-1})^2$$
$$\simeq \theta_p\, \Delta_{t-1} + (1 - \theta_p)\, \tau_1^2 \kappa_t + \frac{\theta_p}{1 - \theta_p} \pi_t^2.$$

For the cross-sectional variance of capital holdings, κ_{t+1}, we obtain

$$\kappa_{t+1} = E_i \left[(k_{t+1}(i) - k_t)^2 \right] - (E_i k_{t+1}(i) - k_t)^2$$
$$= \theta_k E_i \left[(k_t(i) - k_t)^2 \right] + (1 - \theta_k)\, E_i \left[(k_{t+1}^*(i) - k_t)^2 \right]$$
$$- (k_{t+1} - k_t)^2$$
$$\simeq \theta_k\, \kappa_t + (1 - \theta_k)\, E_i \left[(k_{t+1}^* - k_t - \tau_2 (\ln P_t(i) - \ln P_t))^2 \right]$$
$$- (k_{t+1} - k_t)^2$$
$$- (1 - \theta_k) \left[(k_{t+1}^* - k_t - \tau_2 (E_i \ln P_t(i) - \ln P_t))^2 \right]$$
$$+ \frac{1}{1 - \theta_k} (k_{t+1} - k_t)^2$$
$$\simeq \theta_k\, \kappa_t + (1 - \theta_k)\, \tau_2^2 \Delta_t + \frac{\theta_k}{1 - \theta_k} (k_{t+1} - k_t)^2.$$

Finally, the covariance between prices and capital holdings, ψ_t, takes the following form:

$$
\begin{aligned}
\psi_t &= E_i \left[(k_t (i) - k_{t-1}) (\ln P_t (i) - \ln P_{t-1}) \right] \\
&\quad - \left[(E_i k_t (i) - k_{t-1}) (E_i \ln P_t (i) - \ln P_{t-1}) \right]. \\
&= \theta_p \theta_k E_i \left[(k_{t-1} (i) - k_{t-1}) (\ln P_{t-1} (i) - \ln P_{t-1}) \right] \\
&\quad + \theta_p (1 - \theta_k) E_i \left[(k_t^* (i) - k_{t-1}) (\ln P_{t-1} (i) - \ln P_{t-1}) \right] \\
&\quad + (1 - \theta_p) \theta_k E_i \left[(k_{t-1} (i) - k_{t-1}) (\ln P_t^* (i) - \ln P_{t-1}) \right] \\
&\quad + (1 - \theta_p) (1 - \theta_k) E_i \left[(k_t^* (i) - k_{t-1}) (\ln P_t^* (i) - \ln P_{t-1}) \right] \\
&\quad - (k_t - k_{t-1}) \pi_t \\
&\simeq \theta_p \theta_k \psi_{t-1} + \theta_p (1 - \theta_k) E_i \left[(k_t^* - k_{t-1} \right. \\
&\quad \left. - \tau_2 (\ln P_{t-1} (i) - \ln P_{t-1})) (\ln P_{t-1} (i) - \ln P_{t-1}) \right] \\
&\quad + (1 - \theta_p) \theta_k E_i \left[(k_{t-1} (i) - k_{t-1}) \right. \\
&\quad \times \left. (\ln P_t^* - \tau_1 (k_{t-1} (i) - k_{t-1} + k_{t-1} - k_t) - \ln P_{t-1}) \right] \\
&\quad + (1 - \theta_p) (1 - \theta_k) E_i \left[(k_t^* - k_{t-1} - \tau_2 (\ln P_{t-1} (i) - \ln P_{t-1})) \right. \\
&\quad \times \left. (\ln P_t^* - \ln P_{t-1} - \tau_1 (k_t^* - k_t) + \tau_1 \tau_2 (\ln P_{t-1} (i) - \ln P_{t-1})) \right] \\
&\quad - (k_t - k_{t-1}) \pi_t \\
&\simeq \theta_p \theta_k \, \psi_{t-1} - \tau_2 (1 - \theta_k) \left[\theta_p + \tau_1 \tau_2 (1 - \theta_p) \right] \Delta_{t-1} \\
&\quad - \tau_1 (1 - \theta_p) \theta_k \, \kappa_{t-1} - (k_t - k_{t-1}) \pi_t.
\end{aligned}
$$

We therefore have

$$
E \{\lambda_t\} \simeq \frac{\theta_w}{(1 - \theta_w)^2} E \{\omega_t^2\},
$$

$$
E \{\Delta_t\} \simeq \theta_p \, E \{\Delta_t\} + (1 - \theta_p) \tau_1^2 E \{\kappa_t\} + \frac{\theta_p}{1 - \theta_p} E \{\pi_t^2\},
$$

$$
E \{\kappa_t\} \simeq \theta_k \, E \{\kappa_t\} + (1 - \theta_k) \tau_2^2 E \{\Delta_t\} + \frac{\theta_k}{1 - \theta_k} E \left\{ (\Delta k_t)^2 \right\},
$$

$$
\begin{aligned}
E \{\psi_t\} &\simeq \theta_p \theta_k E \{\psi_t\} - \tau_2 (1 - \theta_k) \left[\theta_p + \tau_1 \tau_2 (1 - \theta_p) \right] E \{\Delta_t\} \\
&\quad - \tau_1 (1 - \theta_p) \theta_k E \{\kappa_t\} - E \{ (\Delta k_t) \pi_t \},
\end{aligned}
$$

or, equivalently,

$$
B \begin{bmatrix} E\{\Delta_t\} \\ E\{\kappa_t\} \\ E\{\psi_t\} \end{bmatrix} \simeq C \begin{bmatrix} E\{\pi_t^2\} \\ E\{(\Delta k_t)^2\} \\ E\{z_t^2\} \end{bmatrix},
$$

with B

$$
\equiv \begin{bmatrix} (1-\theta_p) & -(1-\theta_p)\tau_1^2 & 0 \\ -(1-\theta_k)\tau_2^2 & (1-\theta_k) & 0 \\ \tau_2(1-\theta_k)[\theta_p + \tau_1\tau_2(1-\theta_p)] & \tau_1(1-\theta_p)\theta_k & (1-\theta_p\theta_k) \end{bmatrix}
$$

and $C \equiv \begin{bmatrix} \frac{\theta_p}{1-\theta_p} & 0 & 0 \\ 0 & \frac{\theta_k}{1-\theta_k} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix},$

or, equivalently,

$$
\begin{bmatrix} E\{\Delta_t\} \\ E\{\kappa_t\} \\ E\{\psi_t\} \end{bmatrix} \simeq A \begin{bmatrix} E\{\pi_t^2\} \\ E\{(\Delta k_t)^2\} \\ E\{z_t^2\} \end{bmatrix},
$$

with $A \equiv B^{-1}C$ and $z_t = \Delta k_t + \pi_t$. We can therefore rewrite our welfare criterion as

$$
\widetilde{\mathcal{W}}^{LI} \simeq \Omega_1\, E\{y_t^2\} + \Omega_2\, E\{c_t^2\} + \Omega_3\, E\{k_t^2\} + \Omega_4\, E\{n_t^2\}
$$
$$
+ \Omega_5\, E\{\omega_t^2\} + \Omega_6\, E\{\pi_t^2\} + \Omega_7\, E\{(\Delta k_t)^2\} + \Omega_8\, E\{z_t^2\},
$$
$$
\tag{19}
$$

with

$$
\Omega_5 \equiv \Omega_\lambda \frac{\theta_w}{(1-\theta_w)^2}, \ \Omega_6 \equiv [\Omega_\Delta A_{11} + \Omega_\kappa A_{21} + \Omega_\psi A_{31}]
$$
$$
\Omega_7 \equiv [\Omega_\Delta A_{12} + \Omega_\kappa A_{22} + \Omega_\psi A_{32}], \ \Omega_8 \equiv \Omega_\psi A_{33}.
$$

The last equation is (8) in the text.

References

Calvo, G. 1983. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–98.

Christiano, L. J., M. Eichenbaum, and C. L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45.

Dennis, R. 2007. "Optimal Policy in Rational Expectations Models: New Solution Algorithms." *Macroeconomic Dynamics* 11 (1): 31–55.

Edge, R. M. 2003. "A Utility-Based Welfare Criterion in a Model with Endogenous Capital Accumulation." Finance and Economics Discussion Series No. 2003-66, Board of Governors of the Federal Reserve System.

Erceg, C. J., D. W. Henderson, and A. T. Levin. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics* 46 (2): 281–313.

Galí, J. 2015. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework.* 2nd ed. Princeton, NJ: Princeton University Press.

Hicks, J. R., Sir. 1939. *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory.* Oxford: Clarendon Press.

Keynes, J. M. 1936. *The General Theory of Employment, Interest, and Money.* New York: Macmillan.

Khan, A., and J. K. Thomas. 2008. "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics." *Econometrica* 76 (2): 395–436.

Kiyotaki, N., and J. Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105 (2): 211–48.

Kydland, F. E., and E. C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (6): 1345–70.

Neiss, K., and E. Nelson. 2003. "The Real-Interest-Rate Gap as an Inflation Indicator." *Macroeconomic Dynamics* 7 (2): 239–62.

Reiter, M., T. Sveen, and L. Weinke. 2013. "Lumpy Investment and the Monetary Transmission Mechanism." *Journal of Monetary Economics* 60 (7): 821–34.

Rotemberg, J., and M. Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy."

In *NBER Macroeconomics Annual 1997*, ed. B. S. Bernanke and J. J. Rotemberg, 297–361. Cambridge, MA: MIT Press.

Schmitt-Grohé, S., and M. Uribe. 2004. "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." *Journal of Economic Dynamics and Control* 28 (4): 755–75.

———. 2007. "Optimal Simple and Implementable Monetary and Fiscal Rules." *Journal of Monetary Economics* 54 (6): 1702–25.

Sveen, T., and L. Weinke. 2005. "New Perspectives on Capital, Sticky Prices, and the Taylor Principle." *Journal of Economic Theory* 123 (1): 21–39.

———. 2007. "Lumpy Investment, Sticky Prices, and the Monetary Transmission Mechanism." *Journal of Monetary Economics* 54 (Supplement): 23–36.

———. 2009. "Firm-Specific Capital and Welfare." *International Journal of Central Banking* 5 (2): 147–79.

Svensson, L. E. O. 2000. "Open-Economy Inflation Targeting." *Journal of International Economics* 50 (1): 155–83.

Taylor, J. B. 1993. "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39 (1): 195–214.

Thomas, J. K. 2002. "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy* 110 (3): 508–34.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ: Princeton University Press.

———. 2005. "Firm-Specific Capital and the New Keynesian Phillips Curve." *International Journal of Central Banking* 1 (2): 1–46.