Ehling, P., & Heyerdahl-Larsen, C. (2017). Correlations. *Management Science, 63*(6), 1919-1937 DOI: https://doi.org/10.1287/mnsc.2015.2413

Correlations *

Paul Ehling[†] Christian Heyerdahl-Larsen[‡]

2015

*We would like to thank Jérôme Detemple (the editor), an associate editor, two referees, Suleyman Basak, João Cocco, Ilan Cooper, Nam Huong Dau, Stephan Dieckmann, Giulia Di Nunno, Bernard Dumas, Mike Gallmeyer, Francisco Gomes, João Gomes, Trond Stølen Gustavsen, Burton Hollifield, Philipp Illeditsch, Tom Lindstrøm, Thomas Maurer (CICF discussant), Iñaki Rodríguez Longarela (EFA discussant), Anna Pavlova, Lasse H. Pedersen, Richard Priestley, Skander Van den Heuvel (FIRS discussant), Costas Xiouros, Amir Yaron (AFA discussant), Knut Kristian Aase, Bernt Øksendal, and participants at a BI Brown Bag, the Workshop on Risk Measures and Stochastic Games with Applications to Finance and Economics at the Department of Mathematics of the University of Oslo, the Arne Ryde Workshop in Financial Economics at Lund University, the NHH-UiO Macro Workshop, Nordic Finance Network (NFN) Workshop, SIFR - Institute for Financial Research, Texas A&M, Fourth Annual Empirical Asset Pricing Retreat at the University of Amsterdam, Symposium on Stochastic Dynamic Models in Finance and Economics at the University of Southern Denmark, EFA 2007 meetings, European Meeting of the Econometric Society 2008, FIRS Conference 2009 Prague, a EIEF Brown Bag, AFA 2011 meetings, 4th Financial Risk International Forum in Paris 2011, a Banco de España internal seminar, a LBS Brown Bag, Bank of Canada, Johannes Gutenberg-Universität Mainz, and the 2013 China International Conference in Finance (CICF) in Shanghai for helpful comments and suggestions. We are grateful to Nam Huong Dau and Jing Yu for excellent research assistance. Paul Ehling thanks the Centre for Asset Pricing Research (CAPR) at BI for funding support. Part of this research was conducted while the first author was a Research Fellow at Banco de España. The views expressed are those of the authors and should not be attributed to the Banco de España.

†Department of Finance, BI Norwegian Business School, Nydalsveien 37, 0484 Oslo, Norway, paul.ehling@bi.no

‡London Business School, Regent's Park, London, NW1 4SA, UK, cheyerdahllarsen@london.edu

Abstract

Correlations of equity returns have varied substantially over time and remain a source of continuing policy debate. This paper studies stock market correlations in an equilibrium model with heterogeneous risk aversion. In the model, preference heterogeneity causes variations in the volatility of aggregate risk aversion from good to bad states. At times of high volatility in aggregate risk aversion, which is a common factor in returns, we see high correlations. The model matches average industry return correlations and changes in correlations from business cycle peaks to troughs and replicates the dynamics of expected excess returns and standard deviations. Model implied aggregate risk aversion explains average industry correlations, expected excess returns, standard deviations, and turnover volatility in the data. We find supportive evidence for the model's prediction that industries with low dividend-consumption correlation have low average return correlation but experience disproportional increases in return correlations in recessions.

Keywords: Dynamics of Equity Return Correlations; Heterogeneous Risk Aversion; Volatility of Aggregate Risk Aversion; Volatility of Turnover

JEL Classification: G10; G11

1 Introduction

During recessions or financial crisis correlations rise even between seemingly unrelated assets. Economists, regulators or the financial press frequently interpret such events as driven by "contagion" and claim that there is "no place to hide." We study correlations in a heterogeneous investor exchange economy to provide a rational consumption-based explanation. In the language of the model, correlations rise in an economic downturn and decline in a boom. Expected excess returns, standard deviations, and many other equilibrium quantities show joint dynamics with correlations. These findings raise the question whether policy makers can cure markets from excess correlations, volatility, and trade without shrinking financial markets' ability to facilitate consumption risk sharing across investors over the business cycle.

Since correlations determine the extent of diversification benefits it might be particularly bad for investors if, as the data suggest, high correlations coincide with economic downturns. We argue that such a view is too simplistic as stock return correlations are not the only drivers of performance. Consistent with our view, Figure 1 shows that the average correlation of US industry portfolios comoves with average stock return volatility and average expected excess return. Further, variations in correlations, volatilities, and expected returns during NBER contractions appear large and can, therefore, cause significant portfolio rebalancing. Indeed, the dynamics of turnover volatility in Figure 1 seem consistent with brisk portfolio rebalancing during contractions. The contributions of this paper, that are most relevant to investors, are to document and justify the dynamics of correlations and to tie correlations theoretically and empirically to expected stock returns and volatilities, turnover volatility, and the state of the economy.

In response to the empirical facts stated above and the questions that emerge from them, we build an economy with many Lucas trees that is populated by consumers with heterogeneous risk aversion and external relative habit formation and show that it accounts for the key empirical features in Figure 1. In the model, when consumption falls, consumers with low risk aversion find it optimal to sell stocks to more risk averse consumers. Consequently, the marginal consumer becomes more risk averse. This, in turn, leads to higher compensation for risk and higher expected excess returns. In equilibrium, the volatility of risk aversion rises when consumption falls. The volatility of aggregate risk aversion is driven by optimal consumption risk sharing between heterogeneous consumers with constant relative risk aversion. When consumption falls significantly, small shocks to aggregate consumption lead to large fluctuations in the distribution of consumption across consumers. When aggregate risk aversion becomes more volatile, then the discount rate volatility rises; hence, we see an increase in stock return volatilities. Intuitively, as the volatility of aggregate risk aversion drives the discount rate of every stock, we see higher return correlations.

The consumption sharing rule is implemented by trade in the stock market and in a risk-free security. Since trade is observable, it provides indirect information about consumer heterogeneity, something that is difficult to measure directly. To understand the heterogeneous risk aversion based origins of turnover volatility, we solve for the volatility, or quadratic variation, of model implied portfolio policies as a measure of trading intensity. What we learn from this exercise is that in the model turnover volatility is high during bad times and that it correlates positively with stock return correlations.

To assess the models ability to quantitatively match the dynamics of stock return correlations, we calibrate our model to ten industry portfolios. Further, to evaluate the heterogeneous risk aversion channel it is import to also aim at matching other key asset pricing moments and their dynamics. Voilà, through the calibration, we learn that the model accounts for the unconditional level of correlations together with the change in correlation over the business cycle. Moreover, the model generates high comovement between stock return correlations, volatilities, and expected excess returns as in the data. We also show that a measure of habit, backed out from our calibration, explains average industry return correlations, volatilities, and expected excess returns both inside the model and in the data.

In the data, industry portfolios exhibit a significant cross-section of dividend-consumption correlations. When we calibrate the model economy to also reflect the cross-section of dividend-consumption correlations, we find interesting asymmetries in return correlations. In good states, the dividend stream with the lowest dividend-consumption correlation produces lower average stock return correlations with other industries than the dividend stream with the highest dividend-consumption correlation. In bad states, the difference washes out. We test this cross-sectional prediction and find supportive evidence.

To have a quantitative impact, the heterogeneity in risk aversion across consumers has to be quite large. Recently, Guiso, Sapienza, and Zingales (2013) provide survey based evidence, validated with actual data on portfolio choices, that is consistent with significant heterogeneity in risk aversion, namely ranging from below 1 to above 10. Several other studies also report significant heterogeneity in risk aversion: Barsky, Juster, Kimball, and Shapiro (1997) provide an estimate for risk aversion of 12.1 with a standard deviation of 16.6; Kimball, Sahm, and Shapiro (2008) report 8.2 for the mean and 6.8 for the standard deviation of the distribution of individual consumers' relative risk aversion. Still, our calibration succeeds in reproducing the quantitative dynamics of equity price moments at the expense of a high upper bound on risk aversion, namely 20.[1] One potential resolution to this problem, within the CRRA framework, would be to introduce an additional source of heterogeneity into the model that correlates with preference heterogeneity, thereby allowing to reduce the required heterogeneity in risk aversion. We leave this extension to future research.[2]

Our paper combines four strands of the literature: Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2013) study asset prices with multiple Lucas trees, where dividend growth rates are i.i.d. over time. Menzly, Santos, and Veronesi (2004) directly model dividend shares as stationary processes; hence, they avoid that one tree dominates in the long-run. We use

[1] Cvitanic and Malamud (2011) show that in complete markets, heterogeneous CRRA preferences always aggregate to a decreasing relative risk aversion for the representative consumer. In this setting, the larger the cross-sectional variance in risk aversion, the steeper is the aggregate risk aversion. Campbell and Cochrane (1999), instead, use a steep aggregate risk aversion without any heterogeneity. To obtain a steep aggregate risk aversion with a small cross-sectional variance in risk aversion requires that individual risk aversion is steep, that is, one has to deviate from the CRRA assumption.

[2] In an earlier version of the paper, we allowed for heterogeneity in beliefs and time preferences in addition to risk aversion. If the most risk averse investor is also relatively pessimistic about dividend growth (of all or at least most trees), then the slope of the sharing rule steepens for a given degree of risk aversion heterogeneity as when risk aversion heterogeneity is heightened.

Menzly, Santos, and Veronesi (2004) dividend shares and match average industry and total industry dividends and their relation to aggregate consumption.

The paper relates to works studying the role of heterogeneous risk aversion in frictionless economies. Dumas (1989) studies risk sharing in a production economy with heterogeneous risk aversion, Wang (1996) analyzes the dynamics of the real interest rate yields in a Lucas economy, Bhamra and Uppal (2009) and Weinbaum (2009) examine the volatility of stock returns, Bhamra and Uppal (2014) derive closed form solutions for asset prices in an economy with heterogeneous preferences and beliefs, Cvitanic, Jouini, Malamud, and Napp (2012) examine equilibrium properties of an economy with differences in preferences and beliefs, Longstaff and Wang (2013) look at the role of leverage for asset prices. Chan and Kogan (2002) study an economy with "Catching up with the Joneses" preferences and show that such preferences lead to stationary asset price moments through a stationary wealth distribution, Garleanu and Panageas (2015) solve an overlapping generation's model with heterogeneous recursive preferences that also leads to a stationary wealth distribution.[3] Zapatero and Xiouros (2010) solve for the consumption sharing rule in closed form and compare the performance of the heterogeneous risk aversion model to Campbell and Cochrane (1999). Common for these papers is that they focus on the aggregate stock market, and hence do not model multiple Lucas trees. We extend this literature by studying correlations that require a cross-section of Lucas trees, by emphasizing the role of the volatility of aggregate risk aversion, and by focusing on the joint implications of heterogeneous risk aversion on correlations, expected excess returns, standard deviations, and turnover volatility.

Our research also relates to the literature that theoretically study stock return correlations. Dumas, Harvey, and Ruiz (2003) match the level of international correlations in a representative consumer framework. Thus, there is no excess correlation puzzle. Chue (2005) employs the Campbell and Cochrane (1999) model to also study international equity correlations. Chue (2005) shows that the diversification benefits tend to be higher in

[3] Cvitanic and Malamud (2011) and Yan (2008) study the survival of agents, among other things, with heterogeneous risk aversion when there is no stationary wealth distribution.

times when stock return correlations are high as the representative consumer values diversification more in bad times. Aydemir (2008) extends the model in Chue (2005) to contrast correlations with perfect and imperfect risk sharing. Ribeiro and Veronesi (2002) analyze fundamental country processes that are jointly affected by an unobservable global business cycle factor. Time variation in correlations of asset returns arises from the learning activity of the representative consumer. Buraschi, Trojani, and Vedolin (2014) study the correlation risk premium in a model with heterogeneous beliefs and multiple consumption goods. Pavlova and Rigobon (2008) study stock prices, exchange rates, and the correlations of stock prices with multiple consumption goods. In their model, spill-over effects arise because of binding portfolio constraints. Finally, Kyle and Xiong (2001) study the role of convergence traders on stock return correlations. In addition to proposing an alternative channel for return correlation dynamics, namely heterogeneous risk aversion, our work differs from the above papers as we quantitatively calibrate our model to unconditional and conditional asset pricing moments. Moreover, our paper highlights the tight connection between stock return correlations and volatilities, expected excess returns, and turnover volatility.

Although there is a large body of empirical literature on time variation in return correlations,[4] there is less recognition that time variation in correlations might have implications for other moments of equity returns. Notable exceptions include Lamoureux and Lastrapes (1990) who argue that trading volume has significant explanatory power for equity standard deviations; Tauchen and Pitts (1983) and Gallant, Rossi, and Tauchen (1992) show that trading volume has a positive relation with volatility;[5] Longin and Solnik (1995), Moskowitz (2003), and Goetzmann, Li, and Rouwenhorst (2005) argue that correlations or covariances and standard deviations move together. Our paper sheds new light on the findings in this strand and related strands of the empirical literature by providing one possible theoretical

[4]See Bollerslev, Engle, and Woolridge (1988), Erb, Harvey, and Viskanta (1994), Ang and Chen (2001), Longin and Solnik (2001), Ledoit, Santa-Clara, and Wolf (2003), Moskowitz (2003), Barberis, Shleifer, and Wurgler (2005), Goetzmann, Li, and Rouwenhorst (2005), and Chordia, Goyal, and Tong (2011).

[5]It is well known in the market microstructure literature that volatility is associated with trading volume.

foundation for the joint cyclicality.[6] Further, our empirical results relate correlations, expected excess returns, standard deviations, and the volatility of turnover to each other and to the business cycle in a way that cannot be read off the extant empirical literature.

2 The Economy

This section introduces a continuous-time exchange economy with infinite horizon, in which N risky securities and one locally risk-free security are traded.

2.1 Aggregate Consumption and Dividends

Aggregate consumption follows the process:

$$dC(t) = C(t)\left(\mu_C dt + \sigma_C dZ_C(t)\right), \tag{1}$$

where μ_C is the mean consumption growth, the scalar $\sigma_C > 0$ denotes the consumption volatility, and Z_C is a Brownian motion. Aggregate consumption consists of the sum of the dividends paid out by the risky securities. Stationary dividend share processes evolve, as in Menzly, Santos, and Veronesi (2004), according to

$$\begin{aligned} ds_i(t) &= \kappa\left(\overline{s}_i - s_i(t)\right)dt + s_i(t)\sigma_{s_i}(t)^\top dZ_s(t), \qquad (2)\\ \text{where}\quad \sigma_{s_i}(t) &= v_i - \sum_{k=1}^{N} s_k(t)v_k, \quad \sum_{k=1}^{N}\overline{s}_k v_k = 0, \quad \text{for} \quad i = 1,...,N, \quad k = 1,...,N,\\ \text{where}\quad s_i(t) &= \frac{\delta_i(t)}{C(t)}, \quad \text{and where} \quad C(t) = \sum_{i=1}^{N}\delta_i(t), \quad \text{for} \quad i = 1,...,N. \end{aligned}$$

In Equation 2, κ is the speed of mean reversion which we assume is the same for all dividend shares, $\overline{s}_i \in [0,1)$ denotes security i's average long-run consumption share, σ_{s_i} is

[6] Fama and French (1989) and Ferson and Harvey (1991) show that expected excess returns increase during economic contractions and peak near business cycle troughs. Harrison and Zhang (1999) and Campbell and Diebold (2009) also show that expected excess returns are countercyclical. Schwert (1989) and Hamilton and Lin (1996) argue that stock market volatility is higher in recessions than in booms.

a N-dimensional vector of volatilities, $Z_s = (Z_{s,1}, \dots, Z_{s,N})$ is a N-dimensional vector of Brownian motions, v_i denotes a N-dimensional vector of constants, and δ denotes dividends. We define the $N+1$ dimensional Brownian motion $Z = (Z_C, Z_{s,1}, \dots, Z_{s,N})$, by stacking the Brownian motion that drives aggregate consumption together with the N-dimensional Brownian motion, Z_s, that drives the dividend shares.[7]

2.2 Endogenous Asset Correlations - An Illustration

In this subsection, we discuss a general mechanism that generates endogenous correlation between asset return volatilities and correlations and their expected returns when dividends evolve as in Section 2.1. To make our case, we study two dividend strips with the same maturity and assume symmetry, i.e., $\nu_{i,i} = \nu$, $\nu_{i,k} = \bar{\nu}$ for $k \neq i$, and $\bar{s}_i = \bar{s}_l$.[8]

Consider a stochastic discount factor with dynamics

$$\frac{d\xi(t)}{\xi(t)} = -r\left(\omega(t)\right)dt - \theta\left(\omega(t)\right)dZ_C(t), \quad \text{where} \quad d\omega(t) = \mu\left(\omega(t)\right)dt + \sigma\left(\omega(t)\right)dZ_C(t), \quad (3)$$

where $\xi(0) = 1$, r is the short rate, θ denotes the market price of risk, ω is a univariate state variable, μ and σ are given functions that guarantee a strong solution for the state variable in Equation (3), and $r(\omega)$ and $\theta(\omega)$ are twice continuously differentiable to ensure that the price-dividend ratio of the claim to aggregate consumption at any time τ is a function of the state variable ω only. The state variable ω is driven by the same shock as consumption growth. We assume that $\sigma(\omega) > 0$ for all ω and refer to ω as procyclical since locally it is perfectly correlated with shocks to consumption growth. Throughout this subsection, we interpret ω as a measure for the state of the economy, where ω is low in a bad state.

The stochastic discount factor in Equation (3) prices any claim in the economy. Consider

[7] Z is defined on a filtered probability space $(\Omega, \mathcal{F}, P, \{\mathcal{F}_t\})$, which is defined over $[0, \infty)$, where Ω is the state space, $\mathcal{F}$ denotes the σ-algebra, P represents the probability measure, and the information structure $\mathcal{F}_{(.)}$ is generated by the natural filtration.

[8] This assumption can be replaced by the assumption that the conditional volatility of the dividend shares are the same at time t, i.e., $\sigma_{s_i}^\top \sigma_{s_i} = \sigma_{s_l}^\top \sigma_{s_l}$ in Proposition 1.

the price of a dividend strip from a Lucas tree i, δ_i, at time $t < \tau$ is

$$P_i^\tau(t) = E_t\left[\frac{\xi(\tau)}{\xi(t)}\delta_i(\tau)\right] = P_C^\tau(t)\left(\bar{s}_i\left(1-e^{-\kappa(\tau-t)}\right)+e^{-\kappa(\tau-t)}s_i(t)\right), \tag{4}$$

where P_C^τ is the price of the claim on (stripped) aggregate consumption at time τ. The dynamics of the claim to (stripped) aggregate consumption is

$$\frac{dP_C^\tau(t)}{P_C^\tau(t)} = dr_C^\tau(t) = \mu_{P_C^\tau}(t)dt + \sigma_{P_C^\tau}(t)dZ_C(t), \quad \text{where} \quad \sigma_{P_C^\tau}(t) = \sigma_C + \frac{\partial log\left(p_C^\tau(t)\right)}{\partial\omega}\sigma\left(\omega(t)\right). \tag{5}$$

The return volatility, $\sigma_{P_C^\tau}$, in Equation (5) is affected by the exogenous consumption volatility and the endogenous term $\frac{\partial log\left(p_C^\tau\right)}{\partial\omega}\sigma(\omega)$. As in Mele (2007), the endogenous term is the stochastic discount factor induced component of return volatility.

Applying Ito's lemma to Equation (4) and using Equation (5) and (2), we obtain

$$\begin{aligned}\frac{dP_i^\tau(t)}{P_i^\tau(t)} &= dr_i^\tau(t) = \mu_{P_i^\tau}(t)dt + \sigma_{P_i^\tau}(t)^\top dZ(t), \qquad (6)\\ \text{where} \quad \sigma_{P_i^\tau}(t) &= \left(\sigma_{P_C^\tau}(t), g_i^\tau\left(s_i(t),t\right)\sigma_{s_i}(t)\right), \quad g_i^\tau\left(s_i(t),t\right) = \frac{e^{-\kappa(\tau-t)}s_i(t)}{\bar{s}_i\left(1-e^{-\kappa(\tau-t)}\right)+e^{-\kappa(\tau-t)}s_i(t)}.\end{aligned}$$

Consider the correlation between the return on dividend strip i and l with maturity τ

$$\rho_{i,l}^\tau(t) = \frac{\sigma_{P_i^\tau}(t)^\top\sigma_{P_l^\tau}(t)}{\left\|\sigma_{P_i^\tau}(t)\right\|\left\|\sigma_{P_l^\tau}(t)\right\|}, \quad \text{where} \quad \left\|\sigma_{P_i^\tau}(t)\right\| = \sqrt{\sigma_{P_i^\tau}(t)^\top\sigma_{P_i^\tau}(t)}. \tag{7}$$

The proposition below relates the variance of the market to the correlation between the dividend strips.

Proposition 1. *Let $\tau < \infty$ and $s_i(t) = s_l(t)$, then we have the following*

$$sign\left(\frac{\partial\sigma_{P_C^\tau}(\omega)^2}{\partial\omega}\right) = sign\left(\frac{\partial\rho_{i,l}^\tau(\omega,s)}{\partial\omega}\right). \tag{8}$$

Proposition 1 shows that the endogenous variance of the claim to the aggregate consumption stream and the correlation between dividend strip i and l move in the same direction when the state variable ω changes. Therefore, the volatility and the correlation show a positive relation. This result is not driven by the correlation between fundamentals as the correlation between the dividend strips does not depend on the state variable ω. Hence, the relation between return volatility and return correlation is purely endogenous.

If $\frac{\partial \sigma_{P_C^T}(\omega)^2}{\partial \omega} < 0$, then volatility is high in bad states of the economy.[9] If, in addition, $\frac{\partial \theta(\omega)}{\partial \omega} < 0$, i.e., the market prices of risk are high in bad states, then expected excess returns, volatilities, and correlations are jointly countercyclical. However, a countercyclical market price of risk is not sufficient to generate countercyclical correlations. For instance, assume that ω follows an Ornstein-Uhlenbeck process and the market price of risk is linear in ω, i.e., we have essentially affine market prices of risk as in Duffee (2002). Then, the volatility of the aggregate consumption claim is constant and correlations do not depend on ω.

Therefore, what kind of general equilibrium model gives rise to a discount factor that replicates the dynamics in Figure 1? In the remainder of the section, we present such an equilibrium model with consumers that exhibit heterogeneous risk aversion; the model generates high correlations, standard deviations and expected excess returns in the bad state. Further, in the next section we show that the model quantitatively matches the data.

2.3 Consumers

Consumers derive utility over consumption through external habit preferences[10]

$$\begin{aligned} U_j(C, X) &= E_0\left[\int_0^\infty e^{-\rho t} u_j(C_j(t), X(t))dt\right], \qquad (9) \\ \text{where} \quad u_j(C_j(t), X(t)) &= \frac{1}{1-\gamma_j} C_j(t)^{1-\gamma_j} X(t)^{\gamma_j - \eta}, \quad \rho > 0, \quad \eta \le min(\gamma_j) = \gamma_L, \end{aligned}$$

[9] Mele (2007) derives conditions for when volatility (of the claim to aggregate consumption) is countercyclical. A popular model that satiesfies those conditions is Campbell and Cochrane (1999). If volatility is countercyclical, then Proposition 1 also implies countercyclical correlations between dividend strips.

[10] See Abel (1990), Chan and Kogan (2002), Zapatero and Xiouros (2010), and Bhamra and Uppal (2014).

and where u represents the instantaneous utility function, C_j stands for individual consumption rates, X denotes the external economy-wide living standard, and γ measures the local curvature of u, i.e., the relative risk aversion parameter. Consumers either have low, $j = L$, or high risk aversion, $j = H$. The parameter η, which is common to all consumers, is set to ensure that the habit level is perceived as a negative externality by consumers.[11]

The economy-wide living standard evolves, as in Chan and Kogan (2002), according to

$$x(t) = x(0)e^{-\lambda t} + \lambda \int_0^t e^{-\lambda(t-u)} \log(C(u))du, \quad \text{where} \quad x(t) = \log(X(t)). \tag{10}$$

In Equation 10, λ governs the dependency of x on past aggregate consumption. With these assumptions, external relative habit, $\omega = c - x$, measures the state of the economy. By Ito's lemma,

$$d\omega(t) = \lambda\left(\overline{\omega} - \omega(t)\right)dt + \sigma_C dZ_C(t), \quad \text{where} \quad \overline{\omega} = \frac{\mu_C - \frac{1}{2}\sigma_C^2}{\lambda}. \tag{11}$$

Variations of ω around $\overline{\omega}$ allow to define good (bad) times.

2.4 Equilibrium

Conditional on endowments and preferences, equilibrium is a collection of allocations and prices such that individuals' consumption are optimal and markets clear. Complete markets allow to solve for the central planner problem in state by state and time by time form[12]

$$u(C(t), X(t), t) = \max_{C_L(t), C_H(t)} \left\{ \begin{array}{c} a e^{-\rho t} \frac{1}{1-\gamma_L} C_L(t)^{1-\gamma_L} X(t)^{\gamma_L - \eta} \\ + (1-a)\, e^{-\rho t} \frac{1}{1-\gamma_H} C_H(t)^{1-\gamma_H} X(t)^{\gamma_H - \eta} \end{array} \right\} \tag{12}$$

[11] External habit preferences are "neutral" to growth, i.e., consumers feel equally happy or unhappy when their consumption growth rate is high or low as consumption and habit level are cointegrated. This neutrality might be seen as a counterintuitive property of preferences. However, such a feature of preferences is consistent with trends of well-being over time, Blanchflower and Oswald (2004), and with the observation that measures of happiness such as "Happy" from the General Social Survey (GSS), www3.norc.org/GSS+Website/, appear stationary and do not trend up in lockstep with consumption.

[12] A sufficient condition for the market to be complete is that the stock price diffusion matrix is invertible for almost all states and times. For general results on completeness in continuous time economies see Anderson and Raimondo (2008) and Hugonnier, Malamud, and Trubowitz (2010).

$$\text{s.t.} \quad C_L(t) + C_H(t) = C(t),$$

where a denotes the weight on consumer type L in the objective of the aggregate consumer.

Heterogeneous consumers optimally share consumption risk. It is well know in the literature that the shape of the sharing rule depends on the degree of preference heterogeneity and the weight on consumers in the objective function of the aggregate consumer.

Proposition 2. *Pareto optimal consumption allocations are given by*

$$\begin{aligned} C_L(t) &= f(t)C(t) \quad \textit{and} \quad C_H(t) = (1 - f(t))\, C(t), && (13) \\ \textit{where} \quad f(t) &= f_L(\omega(t)) = \left(\frac{a}{1-a}\right)^{\frac{1}{\gamma_L}} e^{\left(\frac{\gamma_H}{\gamma_L}-1\right)\omega(t)} (1 - f(t))^{\frac{\gamma_H}{\gamma_L}}, \quad f_H(\omega(t)) = 1 - f(t). \end{aligned}$$

Proposition 2 shows that the consumption share, f, only depends on ω. Moreover, f converges to zero when ω approaches minus infinity and to one when it approaches infinity. Therefore, the least risk averse consumer, γ_L, dominates in very good states of nature, while the most risk averse consumer, γ_H, dominates in bad states. The solution of the sharing rule requires solving a nonlinear algebraic equation which reduces to a polynomial if the ratio of coefficients of relative risk aversions is a natural number. The next proposition, borrowed from Proposition 1 in Bhamra and Uppal (2014), shows an explicit expression for the consumption share as an infinite series using the Lagrange inversion theorem.

Proposition 3. *The consumption share of the consumer with high risk aversion, f_H, is*

$$f_H(t) = \begin{cases} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \binom{n\frac{\gamma_L}{\gamma_H}}{n-1} A(t)^{-\frac{n}{\gamma_H}} & \textit{for} \quad A(t) > Q \\ 1 - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \binom{n\frac{\gamma_H}{\gamma_L}}{n-1} A(t)^{\frac{n}{\gamma_L}} & \textit{for} \quad A(t) < Q \end{cases} \tag{14}$$

where

$$Q = \frac{\gamma_L^{\gamma_H}}{\gamma_H^{\gamma_H}} \left(\frac{\gamma_H}{\gamma_L} - 1\right)^{\gamma_H - \gamma_L}, \quad A(t) = \left(\frac{a}{1-a}\right) e^{\omega(t)(\gamma_H - \gamma_L)} \tag{15}$$

for $z \in \mathbb{C}$, $k \in \mathbb{N}$, and where $\binom{z}{k}$ is the generalized binomial coefficient.

It is well know in the literature that aggregate risk aversion is defined as the consumption share weighted harmonic average of individual consumers' risk aversion.

Proposition 4. *The coefficient of relative risk aversion and the relative prudence of the aggregate consumer are given by*

$$\begin{aligned}\mathcal{R}(t) &= \frac{1}{f(t)\frac{1}{\gamma_L} + (1-f(t))\frac{1}{\gamma_H}} \quad \textit{and} \\ \mathcal{P}(t) &= (1+\gamma_L)\left(\frac{\mathcal{R}(t)}{\gamma_L}\right)^2 f(t) + (1+\gamma_H)\left(\frac{\mathcal{R}(t)}{\gamma_H}\right)^2 (1-f(t)).\end{aligned} \tag{16}$$

The above proposition shows that $\mathcal{R}$ is bounded in between γ_L and γ_H and that it is high in bad states and low in good states. Aggregate relative prudence, however, is not bounded in between the prudence of the two consumers inhabiting the economy.[13]

The next proposition characterizes the diffusion coefficient of aggregate risk aversion.

Proposition 5. *The diffusion coefficient of aggregate risk aversion is given by*

$$\sigma_{\mathcal{R}}(t) = \mathcal{R}(t)\left(1 + \mathcal{R}(t) - \mathcal{P}(t)\right)\sigma_C. \tag{17}$$

We stress that $\mathcal{P}$ considerably drives the volatility of aggregate risk aversion. The volatility of risk aversion remains constant as the economy evolves when the aggregate relative prudence equals $1+\mathcal{R}$, that is, it equals the relative prudence obtained with standard CRRA. However, this can happen only when one consumer type populates the economy.

Equilibrium quantities depend directly, or indirectly via aggregate risk aversion, on consumptions shares and external relative habit as in the model of Chan and Kogan (2002).

Proposition 6. *In equilibrium, the risk-free rate and the market price of risk are*

$$r_f(t) = \rho + \eta\lambda\omega(t) + \mathcal{R}(t)\left(\mu_C - \lambda\omega(t)\right) - \frac{1}{2}\mathcal{R}(t)\,\mathcal{P}(t)\sigma_C^2 \quad \textit{and} \quad \theta(t) = \mathcal{R}(t)\,\sigma_C. \tag{18}$$

[13] See Wang (1996) for a discussion of the consequences of this result for the risk-free interest rate.

Although there are $N+1$ sources of risk in the economy, the market price of risk is one dimensional as there is only one source of uncertainty driving aggregate consumption.

The next proposition characterizes the aggregate wealth-consumption ratio, the expected return of aggregate wealth, and its conditional volatility.

Proposition 7. *Let P_C denote the price of the claim to aggregate consumption:*

$$\begin{aligned}\frac{P_C(t)}{C(t)} &= p_C(t) = p_C(\omega(t)) = E_t\left[\int_t^{\infty} e^{-\int_t^u K_1(\omega(s))ds - \int_t^u K_2(\omega(s))dZ_C(s)} du\right], \quad (19)\\ \textit{where} \quad K_1(\omega) &= r_f(\omega) + \frac{1}{2}\left(\mathcal{R}(\omega)+1\right)\sigma_C^2 - \mu_C \quad \textit{and} \quad K_2(\omega) = \left(\mathcal{R}(\omega)-1\right)\sigma_C.\end{aligned}$$

The cumulative return on the consumption claim, r_c, is described by the process

$$dr_C(t) = \frac{dP_C(t) + C(t)dt}{P_C(t)} = \mu_{P_C}(t)dt + \sigma_{P_C}(t)dZ_C(t), \tag{20}$$

where the expected return is

$$\mu_{P_C}(t) = r_f(t) + \theta(t)\sigma_{P_C}(t), \tag{21}$$

and the diffusion coefficients is

$$\sigma_{P_C}(t) = \sigma_C\left(1 + \frac{p_C'(\omega(t))}{p_C(t)}\right), \quad \textit{where} \quad p_C'(\omega(t)) = \frac{\partial p_C(\omega(t))}{\partial \omega(t)}. \tag{22}$$

The return volatility, σ_{P_C}, in Equation (22) is affected by the exogenous consumption volatility and the endogenous term $\frac{p_C'(\omega)}{p_C}$. The endogenous term is the stochastic discount factor induced component of return volatility.

The next proposition characterizes the price of a claim to a dividend stream, the expected return of the dividend claim, and its diffusion coefficient.

Proposition 8. *Let P_i denote the price of the claim to dividend stream $i = 1, \ldots, N$:*

$$\begin{aligned} P_i(t) &= C(t)\left(\bar{s}_i p_C(t) + (s_i(t) - \bar{s}_i)\, \hat{p}_C(t)\right), & (23) \\ \textit{where} \quad \hat{p}_C(t) &= \hat{p}_C(\omega(t)) = E_t\left[\int_t^\infty e^{-\int_t^u (\kappa + K_1(\omega(s)))ds - \int_t^u K_2(\omega(s))dZ_C(s)} du\right]. \end{aligned}$$

Cumulative returns, r_i, are described by the processes

$$dr_i(t) = \frac{dP_i(t) + \delta_i(t)dt}{P_i(t)} = \mu_{P_i}(t)dt + \sigma_{P_i}(t)^\top dZ(t), \tag{24}$$

where expected returns are

$$\mu_{P_i}(t) = r_f(t) + \theta(t)\left(1 + \frac{1}{P_i(t)}\left(\bar{s}_i p'_C(\omega(t)) + (s_i(t) - \bar{s}_i)\, \hat{p}'_C(\omega(t))\right)\right)\sigma_C, \tag{25}$$

and diffusion coefficients are

$$\sigma_{P_i}(t) = \left(\left(1 + \frac{1}{P_i(t)}\left(\bar{s}_i p'_C(\omega(t)) + (s_i(t) - \bar{s}_i)\, \hat{p}'_C(\omega(t))\right)\right)\sigma_C, \frac{C(t)\hat{p}_C(t)}{P_i(t)} s_i(t)\sigma_{s_i}(t)\right). \tag{26}$$

The return volatility of a claim to a dividend stream has a common part (before the comma), which has an exogenous and an endogenous or stochastic discount factor induced component, and an idiosyncratic part (after the comma).

The next proposition characterizes wealth allocations and portfolio policies.

Proposition 9. *Equilibrium wealth allocations, $Y = (Y_L, Y_H)$, are*

$$\begin{aligned} Y_j(t) &= C(t) f_j(t) y_j(t), & (27) \\ \textit{where} \quad y_j(t) &= y_j(\omega(t)) = E_t\left[\int_t^\infty e^{-\int_t^u K_1(\omega(s))ds - \int_t^u K_2(\omega(s))dZ_C(s)} \frac{f_j(u)}{f_j(t)} du\right]. \end{aligned}$$

The dollar amount invested in stock $i = 1, \ldots, N$ by investor j is

$$\pi_{j,i}(t) = W_i(t) \frac{1}{\sigma_{P_C}(t)} \left(\frac{\mathcal{R}(t)\sigma_C}{\gamma_j} + \frac{y_j'(\omega(t))}{y_j(t)} \right) Y_j(t), \quad \textit{where} \quad W_i(t) = \frac{P_i(t)}{P_C(t)}. \tag{28}$$

The equilibrium portfolio of consumer j decomposes into the local mean-variance optimal portfolio $W_i \frac{\mathcal{R}\sigma_C}{\sigma_{P_C}\gamma_j} Y_j$ and the hedging term $W_i \frac{y_j'(\omega)}{\sigma_{P_C} y_j} Y_j$, where the relative fraction invested in each share is given by the market weight W_i. Hence, the portfolios of the two consumers have the same composition of risky assets, but differ in the composition between the risk-free asset and the market portfolio, i.e, two-fund separation holds:

Corollary 1. *In equilibrium, consumers's portfolios exhibit two-fund separation.*

Since trading volume or turnover in continuous-time economies is infinite, we employ the quadratic variation or volatility of portfolio policies, characterized in the next proposition, as a measure of trading intensity.[14]

Proposition 10. *Equilibrium quadratic variations of portfolio policies in stock i are*[15]

$$RQV_i(t) = \frac{\pi_{j,i}(t)}{P_i(t)} \sqrt{\left(\sigma_{\pi_{j,i}}(t) - \sigma_{P_i}(t)\right)^\top \left(\sigma_{\pi_{j,i}}(t) - \sigma_{P_i}(t)\right)}. \tag{29}$$

3 Calibration and Empirical Analysis

In this section, we calibrate the model to 10 industry portfolios. We construct our sample at monthly frequency from the CRSP files for the period January 1927 to December 2009. We employ all firms, surviving and non-surviving, that appear on CRSP and sort firms into portfolios using the industry classifications from Kenneth French. For each industry

[14] Grossman and Zhou (1996) and Longstaff and Wang (2013), among others, also employ the quadratic variation of portfolio policies to measure trading intensity.

[15] Changing the asset structure in the economy changes the level of RQV. However, as long as assets (trees) pay out dividends and the economy stays dynamically complete, the (cyclical) dynamics of RQV due to heterogeneous risk aversion remain unaffected by the asset structure. For our purposes, only the dynamics of RQV matter as we cannot quantitatively compare RQV to turnover volatility. There would be no trade and consequently no RQV, however, if assets pay out the optimal consumption.

portfolio, we calculate total dividends and market weighted returns. Dividends are adjusted for inflation using the consumer price index and for population growth using population estimates from the U.S. Census Bureau. Stock returns are adjusted for realized inflation. Aggregate per capita real consumption data, available only at annual frequency, for the period 1927 to 2009 are from Robert Shiller's website. We compute the level of the real risk-free rate, for the period January 1927 to December 2009, from monthly nominal risk-free rates obtained from Kenneth French's website by adjusting for realized inflation. To be conservative, we compute the volatility of the real risk-free rate from the estimated quarterly real yields in Chernov and Mueller (2012) merged with quarterly TIPS data. The merged time-series ranges from the first quarter of 1971 to the fourth quarter of 2009.[16]

In Panel A of Table 1, we report the preference parameters. To illustrate the role played by preference heterogeneity in explaining the dynamics of stock return correlations and other asset pricing moments, we consider two different calibrations of the model: One calibration with homogeneous risk aversion and one with heterogeneous risk aversion. We choose the persistence of the habit level, λ, to match the persistence of the price-dividend ratio in the data. The risk aversion pair, 0.5 and 20, together with the utility weight, a, are set to match unconditional asset pricing moments and changes of these moments over the business cycle. Specifically, both calibrations of the model target the unconditional average correlation and the conditional average correlations. The subjective discount factor, ρ, is chosen to match the level of the risk-free rate. Panel B in Table 1 reports the consumption and dividend share parameters. To match that total dividend growth is more volatile than, and imperfectly correlated with, per capita real consumption growth, we include another dividend stream in addition to the ten industries.[17] We set the dividend share of the eleventh tree to match the average dividend to consumption ratio over the period January 1927 to December 2009.

[16]The data used in Chernov and Mueller (2012) are available at personal.lse.ac.uk/muellerp/RealYieldAOT5.xls. The TIPS data are constructed in Gürkaynak, Sack, and Wright (2010) and are available at www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.

[17]Aggregate consumption is then the sum of the eleven Lucas trees. Below we report moments of equilibrium quantities such as correlations based on the ten "industry trees."

As we are interested in the average return correlation between the ten industry portfolios, it is convenient to set homogeneous parameters for industry dividends.[18] In Panel C of Table 1, we report the mean and volatility of aggregate consumption growth and industry dividend growth together with the average correlation between industry dividend growth. To be conservative, we set the correlation between consumption growth and total industry dividend growth of the 10 industries at 0.25, which is close to the value of 0.2 in Campbell and Cochrane (1999). Panel C shows that the model replicates the consumption and average industry data.

3.1 Unconditional Asset Pricing

Table 2 shows unconditional asset pricing moments of the calibrated models. We see that the heterogeneous (0.710) and the homogeneous (0.704) consumer economies match the unconditional correlation of stock returns (0.719). The heterogeneous consumer economy produces a standard deviation of 0.171 for the stock return, which is only slightly lower than the empirical counterpart (0.190), and 0.010 for the level of the risk-free rate, which is only slightly higher than the empirical counterpart (0.006). The homogeneous consumer economy produces a standard deviation for the stock return that is slightly lower than the heterogeneous consumer economy and a level of the risk-free rate that is slightly higher than in the heterogeneous consumer economy. The volatility of the two-year risk-free rate is 2.2% in the heterogeneous consumer economy and 2.7% in the homogeneous consumer economy. The volatility of the two-year real yield, based on data ranging from 1971 to 2009, is 1.6%.

Both the heterogeneous and the homogeneous consumer economies produce a too low equity premium. This is not surprising given that the unconditional relative aggregate risk aversion in the heterogeneous economy is 5.14, while in the homogeneous economy it is 5. Yet, with a larger cross-sectional heterogeneity in risk aversion,[19] the model can also match

[18]Formally, we set $\bar{s}_i = \bar{s}_l$ for $i, l = 1, \ldots, 10$. The matrices ν_i are set such that the volatility of the dividend shares and the correlation between industry dividend shares are the same for the 10 industries.

[19]To match the decreasing or countercyclical aggregate risk aversion in Campbell and Cochrane (1999)

the equity premium.

3.2 Inspecting the Mechanism

We inspect the heterogeneous risk aversion mechanism within the calibrated model in four steps. Throughout this subsection, we plot equilibrium quantities as a function of ω over the range of 0 to 0.4, which corresponds to 6.8 standard deviations in ω.

3.2.1 Consumption Allocations and Aggregate Risk Aversion

The top-left plot in Figure 2 shows the consumption share of the consumer with high risk aversion as a function of ω. We see that in bad states, when ω is low, the consumer with high risk aversion consumes a large fraction of total consumption. This is a standard results in economies with heterogeneity in risk aversion, known since Dumas (1989). The reason for this is that the consumer with low risk aversion takes on more consumption risk than the consumer with high risk aversion. Hence, after negative shocks, the consumption of the consumer with low risk aversion falls proportionally more than that of the consumer with high risk aversion. The equilibrium consumption allocations also explain the variation in the aggregate risk aversion. The top-middle-right plot shows the aggregate risk aversion, $\mathcal{R}$, as a function of ω. We see that the aggregate risk aversion is higher in the bad state than in the good state. Further, from the top-right plot we see that for most of the state space the standard deviation of the risk aversion is decreasing in ω, that is, shocks to aggregate consumption have a larger impact on the variation in risk aversion in bad states than in good states.

3.2.2 Quadratic Variation of Portfolio Policies and Trading

Since investors are heterogeneous they trade with each other. First, we discuss how the quadratic variation in portfolios vary as the state of the economy changes. Second, we

requires a larger cross-section in risk aversion than we use. If individual relative risk aversion is decreasing, then a steeply decreasing aggregate risk aversion with a small cross-section in risk aversion is obtainable.

discuss differences in the portfolios across the two consumers.

The bottom-right plot in Figure 2 shows the relative quadratic variation for the portfolio of the consumer with high risk aversion. The relative quadratic variation measures the rate of change in the portfolio positions in the market portfolio. We see that the relative quadratic variation shows a similar shape as the volatility of aggregate risk aversion, that is, the relative quadratic variation is high when the volatility of aggregate risk aversion is high.

The right plot in Figure 3 shows the fraction of wealth invested in the aggregate stock market by consumer with high risk aversion.[20] First, the consumer with high risk aversion invests less than her total wealth in the risky asset. Consequently, it must be that the consumer with low risk aversion borrows from the more risk averse consumer to lever up in the stock market. Second, from the figure, we see the fraction of wealth invested in the risky asset decreases in ω, that is, in bad states of the economy the consumer with high risk aversion invest a larger fraction in the risky asset than in the good state of the economy. This can be understood from a general equilibrium view; since the consumer with low risk aversion has a more volatile consumption profile, the economy is dominated by the consumer with high risk aversion in the bad state. Therefore, most of the wealth is held by the consumer with high risk aversion, which can be seen from the top-middle-left plot in Figure 2, and her portfolio share in the risky asset approaches one. In very good states, the opposite is true; the wealth share of the consumer with low risk aversion approaches one and, hence, his portfolio share in the risky asset converges to one.

3.2.3 Dynamics of Stock Return Correlations

A consequence of the increased aggregate risk aversion in the bad state is that the market price of risk for consumption shocks is higher in bad states relative to good states. This effect partly explains the higher expected excess return on the market portfolio in bad states than

[20] As Corollary 1 shows, two-fund separation holds true in our economy. Hence, the composition of the risky assets are the same for both consumers. Consequently, it is sufficient to study the investment in the aggregate market portfolio.

in good states, as can be seen from the bottom-middle-right plot in Figure 2. A consequence of the increased standard deviation of aggregate risk aversion in the bad state is that in times when the variation in the aggregate risk aversion is high, the market price of risk is volatile, and this translates into higher volatility of discount rates in the economy. Therefore, stock returns are more volatile in bad states, as can be seen from the bottom-middle-left plot in Figure 2. Since the market price of consumption risk is a common factor for all stocks, the increased variation in stocks are driven by the higher volatility of aggregate risk aversion, which is a common factor. Hence, correlations between stock returns increase in the bad state relative to the good state, which is shown in the bottom-left plot.

To sum up, expected excess returns and standard deviations of the market portfolio, average industry correlations, and market wide turnover volatilities all increase in the bad state through an increase in the standard deviation of aggregate risk aversion.

3.2.4 The Role of Habit Formation

With habit formation, the wealth distribution of heterogeneous consumers in the model is stationary as illustrated in Chan and Kogan (2002). This is a desirable feature; in addition, the model with habit formation has similar asset pricing properties as an equivalent model without habit formation. In contrast, a homogeneous consumer economy with habit formation produces asset pricing moments that differ significantly from a homogeneous consumer economy without habit formation. To show these effects in the model, we compare the main calibration to an equivalent economy without habit formation. For comparison, we keep the risk aversion pairs (γ_L,γ_H), the Pareto weight (a), and the consumption and dividend parameters fixed at the same values as in our main calibration. Figure 4 shows the average stock return correlation, the standard deviation of the market, and the expected excess return on the market for our main calibration with and without habit formation for the heterogeneous and the homogeneous consumer economy. In the economy without habit formation, we use $\omega_t = log\,(C_t)$ to describe the state of the economy.

From Figure 4, we see in the homogeneous consumer economy without habit formation that the conditional (average) correlations, the conditional standard deviation of the return on the market portfolio, and the expected excess return on the market portfolio do not vary with ω. In the homogeneous consumer economy with habit formation the risk-free rate is high in bad states through a low ω and low in good states through a high ω. Further, in good states of the economy the volatility is higher. This variation in the volatility is driven entirely by the variation in the risk-free rate. In good times, when interest rates are low, the duration of the claim to aggregate consumption is high and, therefore, the price is more exposed to variations in the risk-free rate.[21] The largest difference between the economies with and without habit formation is that all equilibrium asset pricing moments are significantly elevated in the economy with habit formation.

In the heterogeneous consumer economy with habit formation the equilibrium asset pricing moments are also elevated in the economy with habit formation. The difference, however, seems small. For instance, in the steady-state, the difference between the correlation with and without habit formation is 0.09. More importantly, the variation in the average stock return correlation, the standard deviation of the market, and the expected excess return on the market due to variations in the state of the economy are very similar. Therefore, habit formation does not drive the variation in asset pricing moments of our calibrated economy.

3.3 Conditional Correlations

We now bridge the calibrated model with the data. Using return data for the 10 industry portfolios, we estimate a multivariate GARCH (DVEC(1,1)). This gives us a time-series for average conditional correlations and standard deviations. As a proxy for expected excess returns, we compute 3-year ahead average returns.[22] The time series of average correlations (top-left plot), expected excess returns (top-right plot), and market return standard deviations (bottom-left plot) are shown in Figure 1 together with the NBER business cycle dates

[21] The yield volatility is constant as it has the same structural form as a Vasicek interest rate model.
[22] Results are robust to using 1 or 5 year ahead average returns.

(gray shaded areas). We see that there is a tendency for correlations, standard deviations, and expected excess returns to increase during recessions. To examine the model's ability to capture the dynamic properties of correlations, standard deviations, and expected returns, we simulate 100 paths of 996 months of prices from the model. On each path, we estimate conditional correlations, standard deviations, and expected excess returns as in the data by estimating a multivariate GARCH (DVEC(1,1)) and by computing 3-year ahead average returns. We run regressions of average correlations (Av. CORR), average 3-year ahead excess market returns (Av. EXR), and average standard deviations (Av. STDV) on external relative habit, ω, for returns from the data and the simulation. To calculate external relative habit in the data, we employ consumption data, also from Robert Shiller's website, from 1889 to 2009. We assume that ω is in its steady state in 1889. Then, we back out ω from the data using the Euler discretization of its dynamics, where shocks are calculated as deviations of the log-consumption growth from its unconditional mean. Since the consumption data are at annual frequency, we interpolate ω to get a monthly series.

In Panel A of Table 3, we report the correlations between Av. CORR, Av. STDV, and Av. EXR in the data and the model. We see that the model correlations are close to the corresponding values in the data. Therefore, the model captures the joint correlations between these endogenous variables. Table 3 also shows the results from the data and model based regressions, where we use the state variable ω as explanatory variable. All regressions show the expected negative sign with highly significant coefficient estimates in Panel B. For the model regressions, we report in Panel C the mean, 5%, 25%, 50%, 75% and 95% percentiles of the regression coefficients. The mean estimate for the correlations is higher and the mean estimate for standard deviations and expected excess returns are lower than the corresponding values in the data. For correlations and standard deviations, the parameter estimates from the data fall within the interquartile range of the simulated data. For expected excess returns, the parameter estimate is slightly outside of the interquartile range of the simulated data; however, since we are not matching the equity premium we would expect a lower slope coefficient in the model than in the data.

An alternative way to categorize the state the of the economy is to use a cut-off between good and bad states. In the data, the most natural candidate to define a bad state is to use NBER dated recessions. Hence, we compute the average industry return correlation, standard deviation, risk-free rate, and expected excess return conditional on the NBER business cycle indicator. We compare the model to the data by calculating the unconditional recession probability based on the NBER business cycle dates in the data, i.e., over the sample period January 1927 to December 2009 with a total of 996 months. Based on 211 recession months, the unconditional recession probability is 21%. To find a corresponding probability in the model, we simulate the distribution of ω from the calibrated model to back out a threshold for ω. In our calibration, this threshold value is 0.103. The average recession length in the data is about 14 months while in the model ω stays on average for 12.5 months below the threshold. The first order autocorrelation of the BCI in the data and the threshold variable in the model are also similar with values of 0.9126 and 0.8980, respectively.

Table 4 shows the results from this exercise. We see that the homogeneous consumer economy fails to replicate the changes in correlations, standard deviations, and excess returns over the "business cycle." This follows from the fact that in the homogeneous consumer economy the market price of risk is constant and volatility is procyclical. In contrast, the heterogeneous consumer economy is capable of simultaneously reproducing changes in correlations, standard deviations, and expected excess returns from good to bad states. In addition, in the heterogeneous consumer economy the volatility of the risk-free rate evolves countercyclically and the data support this prediction.[23] Taken together, these results suggest that the heterogeneous consumer economy outperforms the homogeneous consumer economy by a wide margin and that it replicates the dynamics of equity return correlations and standard

[23]The GARCH(1,1) volatility of the risk-free rate shows the following correlations with industry stock market correlations (0.42), expected excess returns (0.18), standard deviations (0.41), quadratic variations of industry turnover (0.68), and calibrated external relative habit (-0.29).

deviations.

3.4 Cross-Section of Dividend-Consumption Correlations

In this subsection, we explore cross-sectional heterogeneity in the dividend-consumption correlation. In the data, there is considerable heterogeneity across the correlations of industry dividend growth rates and consumption growth. Specifically, from Table 5, we see that dividend-consumption correlations range from -0.01 (Telecom) to 0.53 (Manufacturing). To study the implications of the cross-sectional heterogeneity in dividend-consumption correlations, we adjust the baseline calibration by allowing dividend shares to correlate with aggregate consumption shocks. Specifically, we set dividend consumption correlations for industry one to ten in the range -0.1033 to 0.38. The cross-sectional dispersion in dividend-consumption correlations are similar to the data, but on average slightly lower to reflect an average total dividend-consumption correlation of 0.25.

To examine the cross-sectional relation, we run three sets of regressions in the model and the data: i) average correlation of industry i with the nine other industries on ω, ii) standard deviation of industry i on ω, and iii) excess return over the next three years of industry i on ω. From Panel A in Table 5, we see that on average industries with low (high) dividend-consumption correlation exhibit return correlations with higher (lower) sensitivity to ω and standard deviations and expected excess returns that have lower (higher) sensitivity to ω. To compare the data with the model, we simulate 100 paths of 996 months from the model. The results are reported in the lower panel in Table 5. From the table we see that, just as in the data, the industry with the lowest dividend-consumption correlation also shows the strongest (negative) relation with ω, and that as the dividend-consumption correlation increases the absolute magnitude of the slope coefficient declines. For standard deviations and expected excess returns we see, as in the data, the reverse relation. However, for standard deviations the results are small. To test if the difference in dividend-consumption correlation drives the cross-section, we regress the slope coefficients of the ten industries on the dividend-consumption correlations. The results are presented in Panel B of Table 6. From the table, we see that slope coefficients for correlations, standard deviations, and expected excess returns all have the same sign as in the data. In the data, all three regressions are significant. For the model regressions, we see that the slope coefficients are lower than in the data.

The intuition for the asymmetry in correlations is that for the dividend stream with low correlation with aggregate consumption or the discount factor, most of the variation in good times is explained by the dividend volatility. In bad times, the volatility of aggregate risk aversion increases significantly and for all stocks most of the variation is attributable to the common component or the volatility of aggregate risk aversion. The increase in volatility in the bad states is mainly through the volatility of risk aversion and not through covariance between the dividends and the discount factor for the stock with low dividend-consumption correlation. For the stock with high dividend-consumption correlation, the increase in volatility is due to the discount factor and higher covariance between dividends and the discount factor. Consequently, the increase in bad states is higher than for the dividend stream with low dividend-consumption correlation. The expected excess return is mostly driven by the higher volatility of stock returns.

3.5 Volatility of Trading Volume

We now turn to the relation between correlations and trading activity. As illustrated in Figure 2, the intensity of trade is high when correlation is high. Moreover, this also coincides with high volatility and high expected excess returns. As a measure of trading intensity we calculate a GARCH(1,1) of the log changes of turnover in the market portfolio. The volatility of turnover is our empirical counterpart to the quadratic variation of the portfolio policies in Equation (29). The correlation between the volatility of turnover and average industry correlation, market standard deviations, and expected excess returns are 0.52, 0.39, and 0.16, respectively. Regressing turnover volatility on the backed out habit yields -0.6031 for the coefficient estimate with a Newey-West corrected t-statistics of -6.5560 (using 18 lags) and adjusted R-squared of 0.1946. These estimates are in line with our results in Table 3 and are an important additional piece of evidence in support of the model.

4 Conclusions

In the data, stock return correlations are high in bad states of nature such as NBER dated recessions. In addition, stock return standard deviations, expected excess returns, and the volatility of turnover are also high in recessions. In response, we show in a dynamic consumption based economy with heterogeneous risk aversion, stationary wealth distribution, and stationary dividend shares that return correlations and volatilities, expected excess returns, and trade volatilities move jointly as a function of the habit based state variable. In the model, the variations in the equilibrium quantities are due to consumption risk sharing, which consumers implement by dynamically trading in stocks. The model quantitatively matches the unconditional level of correlations and reproduces changes in correlations from good to bad states.

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A Proofs

Proof of Proposition 1

The diffusion coefficient of the price of dividend strip $i = 1, \ldots, N$ is

$$\sigma_{P_i^\tau}(t) = \left(\sigma_{P_C^\tau}(t), g^\tau\left(s_i(t), t\right)\sigma_{s_i}(t)\right). \tag{A.1}$$

Throughout this proof, we suppress dependency on the dividend shares because we are only interested in the dependency to the aggregate state. Define the following quantities

$$V_C(\omega) = \sigma_{P_C^\tau}(\omega)^2, \quad \text{and} \quad V_{i,l} = g_i^\tau(t) g_l^\tau(t) \sigma_{s_i}(t)^\top \sigma_{s_l}(t). \tag{A.2}$$

The correlation between dividend strip i and l is

$$\rho_{i,l}^\tau(\omega) = \frac{V_C(\omega) + V_{i,l}}{\sqrt{V_C(\omega) + V_{i,i}}\sqrt{V_C(\omega) + V_{l,l}}}. \tag{A.3}$$

We are interested in how the correlation between dividend strip i and l changes with respect to the aggregate state, ω, hence we look at the derivative of the correlation in Equation (A.3)

$$\frac{\partial \rho_{i,l}^\tau(\omega)}{\partial \omega} = \left(\frac{1 - \rho_{i,l}^\tau(\omega) \frac{1}{2} \left(\frac{\sqrt{V_C(\omega)+V_{l,l}}}{\sqrt{V_C(\omega)+V_{i,i}}} + \frac{\sqrt{V_C(\omega)+V_{i,i}}}{\sqrt{V_C(\omega)+V_{l,l}}} \right)}{\sqrt{V_C(\omega) + V_{i,i}}\sqrt{V_C(\omega) + V_{l,l}}} \right) \frac{\partial V_C(\omega)}{\partial \omega}. \tag{A.4}$$

We have $sign\left(\frac{\partial V_C(\omega)}{\partial \omega}\right) = sign\left(\frac{\partial \rho_{i,l}^\tau(\omega)}{\partial \omega}\right)$ if $\frac{1 - \rho_{i,l}^\tau(\omega) \frac{1}{2} \left(\frac{\sqrt{V_C(\omega)+V_{l,l}}}{\sqrt{V_C(\omega)+V_{i,i}}} + \frac{\sqrt{V_C(\omega)+V_{i,i}}}{\sqrt{V_C(\omega)+V_{l,l}}} \right)}{\sqrt{V_C(\omega)+V_{i,i}}\sqrt{V_C(\omega)+V_{l,l}}} > 0$, which is true if $V_{i,i} = V_{l,l}$. This is the condition in Proposition 1.

Proof of Proposition 2

The proposition follows form solving the central planner problem in Equation (12). Details can be found in the Internet Appendix.

Proof of Proposition 3

See Bhamra and Uppal (2014) for a proof.

Proof of Proposition 4

A derivation can be found in the Internet Appendix.

Proof of Proposition 5

The coefficient of relative risk aversion of the aggregate consumer is

$$\mathcal{R}(t) = \mathcal{A}(t)C(t), \quad \text{where} \quad \mathcal{A}(t) = \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1}, \quad \mathcal{A}_j(t) = -\frac{u_{j,CC}(C_j(t), X(t), t)}{u_{j,C}(C_j(t), X(t), t)}. \tag{A.5}$$

Applying Ito's lemma to $\mathcal{R}(t) = \mathcal{A}(t)C(t)$ yields

$$\begin{aligned} d\mathcal{R}(t) &= [.]dt + \left(C(t)\frac{\partial \mathcal{A}(t)}{\partial \omega}\sigma_C(t) + \mathcal{A}(t)C(t)\sigma_C(t)\right)^\top dZ_C(t) \\ &= [.]dt + \mathcal{R}(t)\left(1 + \mathcal{R}(t) - \mathcal{P}(t)\right)\sigma_C(t)^\top dZ_C(t) = [.]dt + \sigma_{\mathcal{R}}(t)^\top dZ_C(t). \end{aligned} \tag{A.6}$$

Proof of Proposition 6

A derivation can be found in the Internet Appendix.

Proof of Proposition 7

This follows from $P_C(t) = C(t)p_C(t)$ and an application of Ito's lemma.

Proof of Proposition 8

The stock price i is

$$\begin{aligned} P_i(t) &= E_t\left[\int_t^\infty \frac{\xi(u)}{\xi(t)}\delta_i(u)du\right] = E_t\left[\int_t^\infty \frac{\xi(u)}{\xi(t)}C(u)s_i(u)du\right] \\ &= C(t)\int_t^\infty E_t\left[\frac{\xi(u)}{\xi(t)}\frac{C(u)}{C(t)}\right]E_t\left[s_i(u)\right]du \\ &= C(t)\int_t^\infty E_t\left[\frac{\xi(u)}{\xi(t)}\frac{C(u)}{C(t)}\right]\left(\bar{s}_i + \left(s_i(t) - \bar{s}_i\right)e^{-\kappa u}\right)du \\ &= C(t)\left(\bar{s}_i p_C(t) + \left(s_i(t) - \bar{s}_i\right)\hat{p}_C(t)\right). \end{aligned} \tag{A.7}$$

Note that

$$E_t\left[\frac{\xi(u)}{\xi(t)}\frac{C(u)}{C(t)}\right] = E_t\left[e^{-\int_t^u\left(r_f(v)+\frac{1}{2}\theta(v)^2\right)dv-\int_t^u\theta(v)dZ_c(v)}e^{\int_t^u\left(\mu_C-\frac{1}{2}\sigma_C^2\right)dv+\int_t^u\sigma_C dZ_C(v)}\right]. \tag{A.8}$$

Inserting Equation (A.8) into Equation (A.7) we get Equation (23). A similar approach can be used to find $\hat{p}_C(t)$. To calculate the diffusion coefficient apply Ito's lemma to Equation

(A.7).

Proof of Proposition 9

The wealth, $Y_j(t)$, of agent $j = L, H$ is

$$
\begin{aligned}
Y_j(t) &= E_t\left[\int_t^\infty \frac{\xi_u}{\xi_t} C_j(u) du\right] = E_t\left[\int_t^\infty \frac{\xi_u}{\xi_t} C(u) f_j(u) du\right] \\
&= C(t) f_j(t) E_t\left[\int_t^\infty \frac{\xi_u}{\xi_t} \frac{C(u) f_j(u)}{C(t) f_j(t)} du\right] \\
&= C(t) f_j(t) E_t\left[\int_t^\infty e^{-\int_t^u K_1(\omega(s))ds - \int_t^u K_2(\omega(s))dZ_C(s)} \frac{f_j(u)}{f_j(t)} du\right].
\end{aligned} \tag{A.9}
$$

By Ito's lemma, the dynamics of the wealth of agent j is

$$
dY_j(t) = [.]dt + Y_j(t)\left(\frac{\mathcal{R}(t)}{\gamma_j} + \frac{y_j'(\omega(t))}{y_j(t)}\right)\sigma_C dZ_C(t). \tag{A.10}
$$

Let $\pi_{j,i}(t)$ be the dollar amount invested by agent $j = L, H$ in stock $i = 1, \ldots, N$. The dynamics of the wealth can also be calculated as

$$
dY_j(t) = [.]dt + \sum_{i=1}^{N} \pi_{j,i}(t) \sigma_{P_i}(t)^\top dZ(t). \tag{A.11}
$$

Comparing the diffusion coefficients in Equation (A.10) and (A.11) we must have that the portfolio that finances the optimal consumption stream satisfies

$$
\sum_{i=1}^{N} \pi_{j,i}(t)\left(1 + \frac{1}{P_i(t)}\left(\bar{s}_i p_C'(\omega(t)) + (s_i(t) - \bar{s}_i)\hat{p}_C'(\omega(t))\right)\right)\sigma_C = Y_j(t)\left(\frac{\mathcal{R}(t)}{\gamma_j} + \frac{y_j'(\omega(t))}{y_j(t)}\right)\sigma_C \tag{A.12}
$$

and

$$
\sum_{i=1}^{N} \pi_{j,i} \frac{C(t)\hat{p}_C(t)}{P_i(t)} s_i(t) \sigma_{s_i}(t) = \mathbf{0}, \tag{A.13}
$$

where $\mathbf{0}$ is an N-dimensional vector of zeros. The optimal wealth only loads on the aggregate consumption shock and, therefore, an optimal portfolio must be a portfolio where all the shocks to the dividend shares are diversified away. This is achieved by holding the market portfolio. To see this, let $\pi_{j,i} = A_j \frac{P_i(t)}{P_C(t)}$, where A_j is a proportionality factor, depending on ω, to be determined. Insert this into Equation (A.13)

$$\begin{aligned}
\sum_{i=1}^{N} A_j \frac{P_i(t)}{P_C(t)} \frac{C(t)\hat{p}_C(t)}{P_i(t)} s_i(t)\sigma_{s_i}(t) &= \mathbf{0} \\
\sum_{i=1}^{N} A_j \frac{C(t)\hat{p}_C(t)}{P_C(t)} s_i(t)\sigma_{s_i}(t) &= \mathbf{0} \\
A_j \sum_{i=1}^{N} s_i(t)\sigma_{s_i}(t) &= \mathbf{0}.
\end{aligned} \tag{A.14}$$

Equation (A.14) is satisfied by $\pi_{j,i} = A_j \frac{P_i(t)}{P_C(t)}$ because $\sum_{i=1}^{N} s_i(t)\sigma_{s_i}(t) = \mathbf{0}$ by the construction of the dividend shares. To determine A_j, insert $\pi_{j,i} = A_j \frac{P_i(t)}{P_C(t)}$ into Equation (A.12)

$$A_j \sum_{i=1}^{N} \frac{P_i(t)}{P_C(t)} \left(1 + \frac{1}{P_i(t)} \left(\bar{s}_i p'_C(\omega(t)) + (s_i(t) - \bar{s}_i)\, \hat{p}'_C(\omega(t))\right)\right) \sigma_C = Y_j(t) \left(\frac{\mathcal{R}(t)}{\gamma_j} + \frac{y'_j(\omega(t))}{y_j(t)}\right) \sigma_C.$$

Manipulation leads to:

$$\begin{aligned}
A_j \sigma_{P_C}(t) &= Y_j(t) \left(\frac{\mathcal{R}(t)}{\gamma_j} + \frac{y'_j(\omega(t))}{y_j(t)}\right) \sigma_C \\
A_j &= \frac{\sigma_C}{\sigma_{P_C}(t)} Y_j(t) \left(\frac{\mathcal{R}(t)}{\gamma_j} + \frac{y'_j(\omega(t))}{y_j(t)}\right).
\end{aligned} \tag{A.15}$$

Inserting Equation (A.15) into $\pi_{j,i} = A_j \frac{P_i(t)}{P_C(t)}$ and defining $W_i(t) = \frac{P_i(t)}{P_C(t)}$ completes the proof.

Proof of Proposition 10

Applying Ito's lemma to Equation (28) yields the dynamics of the portfolio policies

$$\frac{d\pi_j(t)}{\pi_j(t)} = \mu_{\pi_j}(t)dt + \sigma_{\pi_j}(t)^\top dZ(t). \tag{A.16}$$

The diffusion of the fraction of stock i held by consumer j is given by $\frac{\pi_{j,i}(t)}{P_i(t)} \sigma_{\frac{\pi_{j,i}}{P_i}}(t)$, where

$$\sigma_{\frac{\pi_{j,i}}{S_i}}(t) = \sigma_{\pi_{j,i}}(t) - \sigma_{P_i}(t). \tag{A.17}$$

Market clearing implies $\frac{\pi_{L,i}(t)}{P_i(t)} + \frac{\pi_{H,i}(t)}{P_i(t)} = 1$ and applying Ito's lemma to both sides leads to

$$\frac{\pi_{L,i}(t)}{P_i(t)}\sigma_{\frac{\pi_{L,i}}{P_i}}(t) + \frac{\pi_{H,i}(t)}{P_i(t)}\sigma_{\frac{\pi_{H,i}}{P_i}}(t) = 0. \tag{A.18}$$

Finally, the relative quadratic variation, RQV, measures trade volatility in stock i

$$RQV_i\,(t) = \sqrt{\left(\frac{\pi_{j,i}(t)}{P_i(t)}\right)^2 \sigma_{\frac{\pi_{j,i}}{P_i}}(t)^\top \sigma_{\frac{\pi_{j,i}}{P_i}}(t)}. \tag{A.19}$$

Table 1: **Calibration — Parameters.** This table summarizes preference parameters for a heterogeneous aggregate consumer and a homogeneous consumers economy (Panel A), consumption and dividend parameters (Panel B), and population adjusted moments of real consumption, real total industry dividends, and real average industry dividends (Panel C) for the period January 1927 to December 2009 and the corresponding moments in the model economies. Setting the utility weight $a = 0.09$ is equivalent to a steady state wealth share of 0.08. Model moments are calculated by simulating 10 000 paths of 996 months of data and taking the average over all paths.

Panel A: Preference Parameters						
	γ_L	γ_H	a	ρ	η	λ
Heterogeneous	0.5	20	0.09	0.015	0.5	0.13
Homogeneous	5	5	1	0.005	1	0.13
Panel B: Consumption and Dividend Parameters						
	μ_C	σ_C				
Aggregate consumption	0.02	0.03				
	κ	$\bar{s}_i$	$\nu_{i,i}$	$\nu_{i,k}$	$\nu_{i,11}$	
Industry dividend shares ($i, k = 1, \ldots, 10$)	0.01	0.0055	0.1209	-0.0522	-0.0995	
	κ	$\bar{s}_{11}$	$\nu_{11,11}$	$\nu_{11,k}$		
11'th tree dividend share	0.01	0.9450	0.0058	-0.0238		
Panel C: Consumption and Dividend Moments						
	Data	Model				
Mean consumption growth	0.020	0.020				
Standard deviation of consumption growth	0.030	0.030				
Mean aggregate dividend growth	0.023	0.020				
Standard deviation of aggregate dividend growth	0.135	0.120				
Average industry dividend correlation	0.260	0.230				

Table 2: **Calibration — Asset Pricing Moments.** This table summarizes real moments of the market portfolio, average industry portfolio, risk-free rate, and price-dividend ratio of the market for the period January 1927 to December 2009 and corresponding moments from a heterogeneous and a homogeneous consumer model calibration. The volatility of the real risk-free rate is for the period $Q1$ 1971 to $Q4$ 2009. Returns are annualized from monthly frequency. ACF(1) denotes the first-order autocorrelation coefficient. Model moments are calculated by averaging across 10 000 paths of 996 months.

	Data	Model	
		Heterogeneous	Homogeneous
Expected excess return of the market portfolio	0.079	0.028	0.021
Standard deviation of the market portfolio	0.190	0.171	0.156
Average industry correlation	0.719	0.710	0.704
Risk-free rate	0.006	0.010	0.013
Standard deviation of risk-free rate	0.016	0.022	0.027
ACF(1) of log price-dividend ratio of the market portfolio	0.874	0.837	0.855

Table 3: **Calibration — Regressions.** Panel A reports the correlation between average of industry return correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Panel B summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied external relative habit as explanatory variable for Av. CORR, Av. EXR, and Av. STDV. Newey-West corrected t-statistics with 18 lags are in parentheses. Correlations and standard deviations are estimated using a DVEC(1,1) model. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions in the data columns use 996 monthly observations with data ranging from January 1927 to December 2009. The regressions in the model are based on the parameters in Table 1. We simulate 100 paths, each with 996 monthly observations. For every path we calculate the average correlations, 3-year ahead expected excess returns and standard deviations. The reported results are averages over the 100 sample paths. Panel C shows the distribution of regression coefficients in the model: the mean, 5%, 25%, 50%, 75% and 95% percentiles.

Panel A: Joint Correlations						
	Av. CORR - Av. STDV		Av. CORR - Av. EXR		Av. STDV - Av. EXR	
	Data	Model	Data	Model	Data	Model
Correlations	0.5836	0.6932	0.3202	0.2454	0.1554	0.1753

Panel B: Regressions						
	Av. CORR		Av. EXR		Av. STDV	
	Data	Model	Data	Model	Data	Model
Intercept	0.7649	0.7515	0.1847	0.0657	0.2939	0.2439
	(28.8627)	(57.4574)	(6.7508)	(2.5818)	(9.0453)	(29.1254)
Habit	-0.4574	-0.7940	-0.6960	-0.3714	-0.6048	-0.4885
	(-2.8642)	(-9.0296)	(-4.3450)	(-2.3491)	(-3.2582)	(-8.9621)
Adjusted R-squared	0.1317	0.3901	0.1221	0.0736	0.3954	0.5046

Panel C: Distribution of Regression Coefficients								
	Data		Model					
Slope	Estimate	S.E.	Mean	5%	25%	50%	75%	95%
β_{CORR}	-0.4574	0.1597	-0.7940	-1.6624	-1.0623	-0.6707	-0.4526	-0.2207
β_{EXR}	-0.6960	0.1602	-0.3714	-0.9445	-0.5393	-0.3489	-0.1610	0.1028
β_{STDV}	-0.6048	0.1856	-0.4885	-0.9240	-0.6687	-0.4924	-0.2859	-0.1346

Table 4: **Calibration — Asset Pricing Moments over the Business Cycle.** This table summarizes unconditional and conditional (booms and recessions) moments of three year ahead excess return and standard deviation of the market portfolio, average industry portfolio correlation, and risk-free rate for the period January 1927 to December 2009 and corresponding moments from a heterogeneous and a homogeneous consumer model calibration. Returns are annualized from monthly frequency. In the data, recessions are defined by NBER recession dates. In the calibrated model, recessions have the same unconditional probability as in the data. Model moments are calculated by averaging across 10 000 paths of 996 months.

	Data	Model	
		Heterogeneous	Homogeneous
Expected Excess Return of Market			
Average	0.079	0.028	0.021
Boom	0.071	0.018	0.021
Recession	0.110	0.063	0.021
Recession minus boom	0.039	0.045	-0.000
Standard Deviation of Market			
Average	0.190	0.171	0.156
Boom	0.156	0.149	0.156
Recession	0.280	0.251	0.155
Recession minus boom	0.124	0.101	-0.001
Average Industry Correlation			
Average	0.719	0.710	0.704
Boom	0.655	0.675	0.706
Recession	0.812	0.846	0.698
Recession minus boom	0.157	0.171	-0.007
Risk-Free Rate			
Average	0.006	0.010	0.014
Boom	-0.001	0.001	0.003
Recession	0.031	0.043	0.052
Recession minus boom	0.032	0.042	0.048

Table 5: **Cross-Section I.** This table summarizes OLS regression results of model implied external relative habit, ω, as explanatory variable for the average correlation of each industry with every other industry, industry standard deviations and expected excess returns measured by the excess return over the next three years. The table reports the slope coefficients (β_{corr}, β_{stdev}, β_{exr}). Newey-West corrected t-statistics with 18 lags are in parentheses. $\rho_{\delta_i,C}$ is the correlation of the dividend growth of industry i with aggregate consumption over the entire sample period. Correlations and standard deviations are estimated using a DVEC(1,1) model. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions in Panel A use 996 monthly observations with data ranging from January 1927 to December 2009. The regressions in Panel B are based on the parameters in Table 1. We simulate 100 paths, each with 996 monthly observations. For every path we calculate the average correlations, 3-year ahead expected excess returns and standard deviations. The reported results are averages over the 100 sample paths.

Panel A: Data										
Industry	Telcm	Utils	Hlth	Enrgy	NoDur	HiTec	Other	Shops	Durbl	Manuf
$\rho_{\delta_i,C}$	-0.014	0.169	0.209	0.318	0.355	0.386	0.409	0.419	0.455	0.530
β_{corr}	-0.611	-0.548	-0.356	-0.678	-0.357	-0.675	-0.261	-0.358	-0.400	-0.330
	(-5.154)	(-3.220)	(-2.508)	(-5.651)	(-3.507)	(-6.472)	(-3.270)	(-3.265)	(-3.657)	(-4.299)
β_{stdev}	-0.239	-0.758	-0.520	-0.464	-0.346	-0.702	-0.771	-0.497	-0.976	-0.776
	(-3.102)	(-7.072)	(-3.414)	(-5.968)	(-3.408)	(-4.003)	(-4.048)	(-3.996)	(-5.297)	(-4.705)
β_{exr}	-0.679	-0.383	-0.513	-0.549	-0.458	-1.239	-0.936	-0.843	-1.850	-1.198
	(-7.361)	(-4.273)	(-2.924)	(-4.280)	(-2.862)	(-6.537)	(-4.484)	(-4.551)	(-4.978)	(-5.233)
Panel B: Model										
Industry	1	2	3	4	5	6	7	8	9	10
$\rho_{\delta_i,C}$	-0.1033	-0.0486	0.0073	0.0677	0.1201	0.1748	0.2318	0.2841	0.3346	0.3800
β_{corr}	-0.8294	-0.7399	-0.7561	-0.7161	-0.7578	-0.7225	-0.7432	-0.6708	-0.6598	-0.6746
	(-5.5700)	(-5.2143)	(-5.4121)	(-5.4307)	(-5.8867)	(-5.6997)	(-6.1913)	(-5.7049)	(-5.7339)	(-5.9012)
β_{stdev}	-0.4716	-0.4950	-0.4664	-0.5048	-0.4587	-0.4747	-0.4543	-0.4869	-0.5201	-0.4868
	(-8.6258)	(-8.7241)	(-8.3077)	(-8.5290)	(-7.9517)	(-7.6300)	(-7.2993)	(-7.6623)	(-7.9239)	(-7.1627)
β_{exr}	-0.5813	-0.6058	-0.5722	-0.5694	-0.5878	-0.5888	-0.6378	-0.6546	-0.6336	-0.6480
	(-3.3505)	(-3.3487)	(-3.0037)	(-3.0100)	(-3.0222)	(-3.0089)	(-3.0961)	(-3.0967)	(-3.1114)	(-2.9432)

Table 6: **Cross-Section II.** This table summarizes OLS regression results of the slope coefficients β_{corr}, β_{stdev} and β_{exr} from Table 5 on the dividend-consumption correlations, $\rho_{\delta,C}$. Newey-West corrected t-statistics with 18 lags are in parentheses. The regressions in the data columns use 996 monthly observations with data ranging from January 1927 to December 2009. The regressions in the model are based on the parameters in Table 1. We simulate 100 paths, each with 996 monthly observations. For every path we calculate the average correlations, 3-year ahead expected excess returns and standard deviations. The reported results are averages over the 100 sample paths.

	Correlations		Stdevs		Exr	
	Data	Model	Data	Model	Data	Model
Intercept	-0.604	-0.7648	-0.331	-0.4775	-0.318	-0.5852
	(-12.708)	(-56.0615)	(-3.152)	(-71.7257)	(-1.294)	(-67.2861)
Slope	0.453	0.2606	-0.846	-0.0306	-1.689	-0.5852
	(3.666)	(4.7018)	(-3.000)	(-0.9194)	(-2.292)	(-4.8202)
R^2	0.220	0.7103	0.356	0.0582	0.352	0.6258

Figure 1: **Mean Industry Correlations, Returns, Standard Deviations, and Quadratic Variation of Turnover.** This figure shows plots of average pairwise industry correlations from a multivariate GARCH model, average annualized 3-year ahead continuously compounded market excess returns, average industry standard deviations, also from the multivariate GARCH model, and average quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover for the sample period 1927 to 2009 using Kenneth French's industry classification. Gray shaded areas denote NBER recessions. Sample based on the CRSP/Compustat files, using PERMNO.

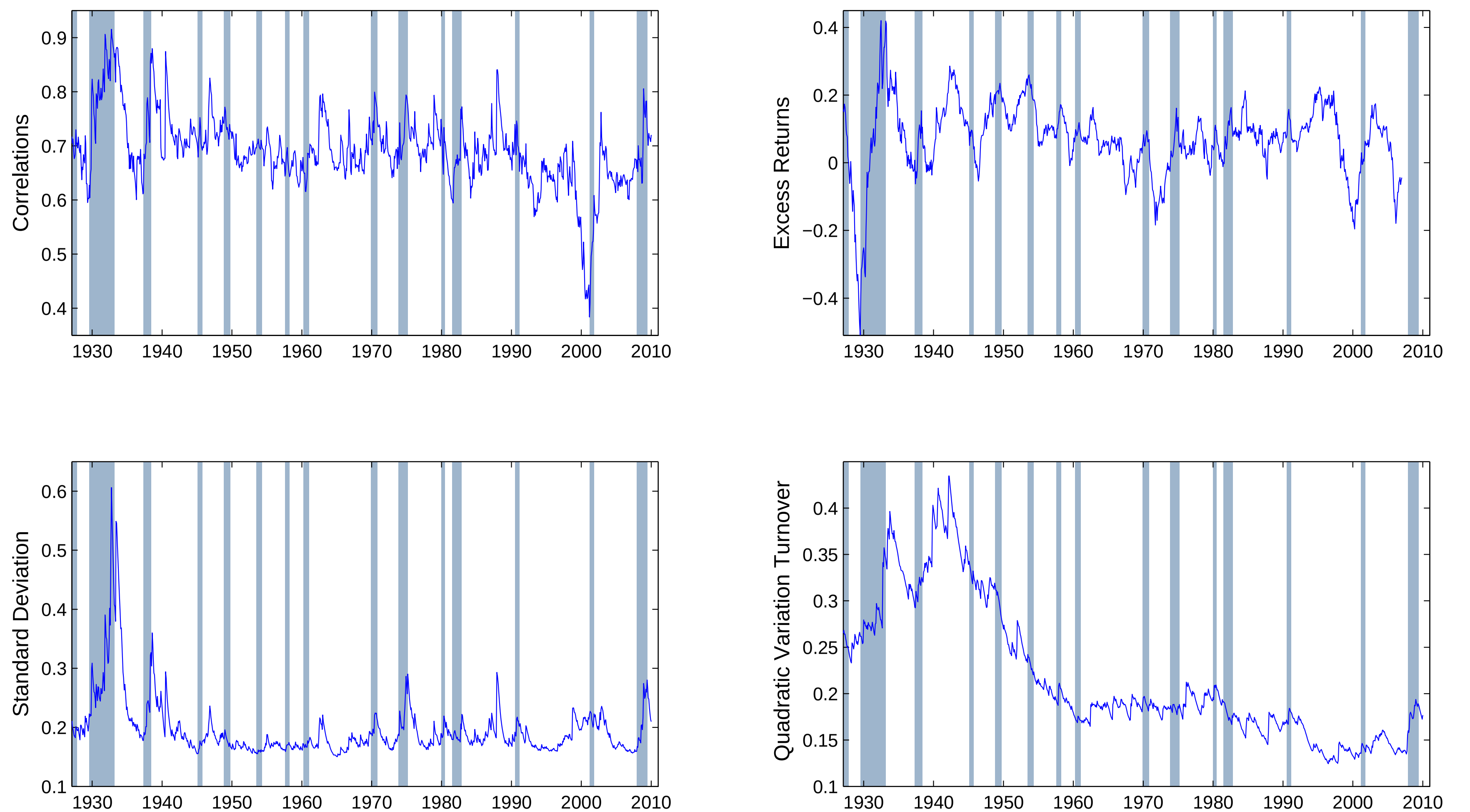


Figure 2: **Risk Aversion Volatility Effect.** This figure shows plots of the consumption share of consumers with high risk aversion, H, the wealth share of consumers with high risk aversion, the risk aversion of the representative agent ($\mathcal{R}$), the standard deviation of risk aversion of the representative agent, the average conditional return correlations, the conditional standard deviation of the market portfolio, the expected excess return on the market portfolio, and the relative quadratic variation of the portfolio of H consumers as functions of ω. Parameters are set as in Table 1.

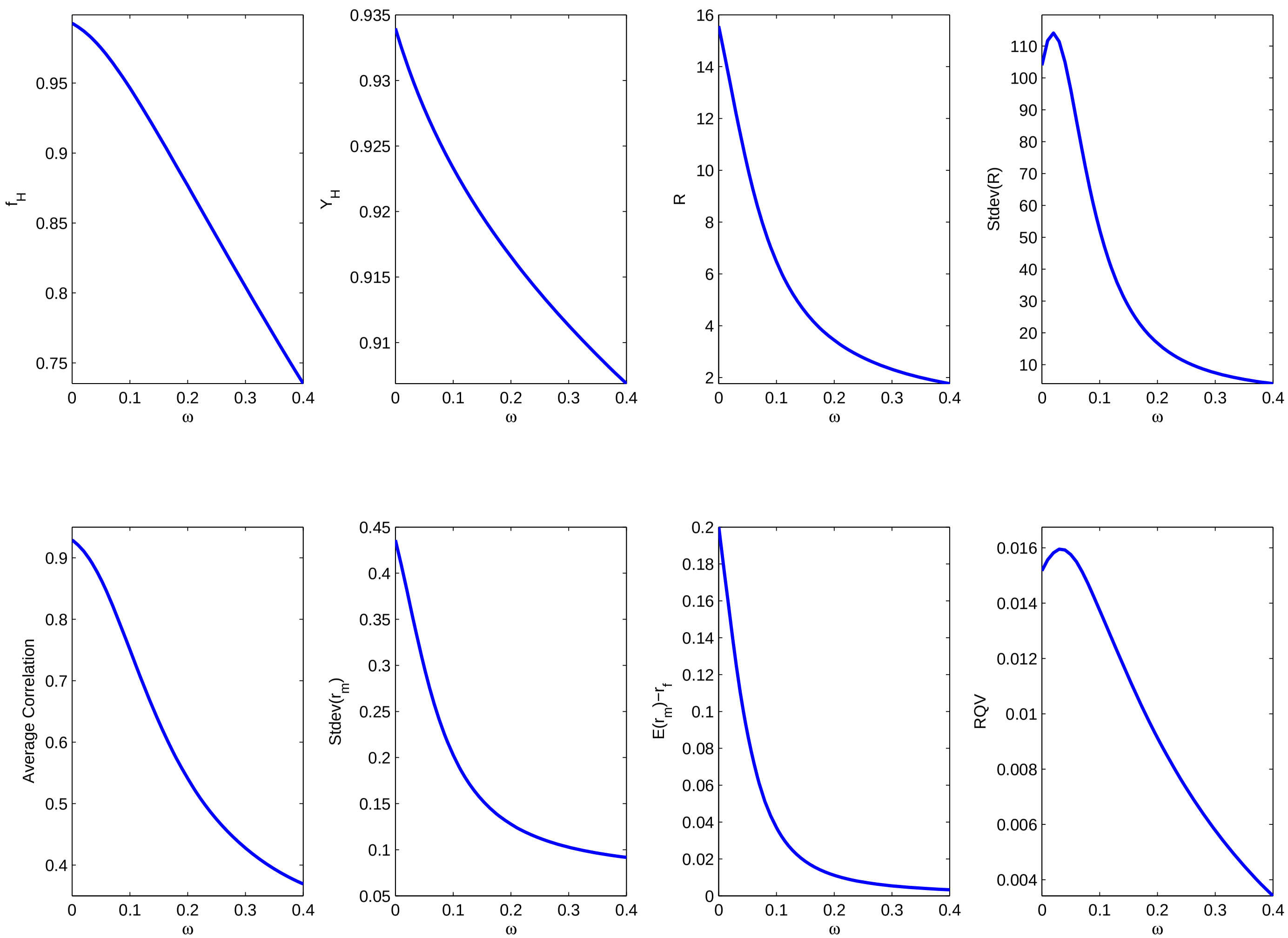


Figure 3: **Leverage and Trading.** This figure shows plots of the levarage and the portfolio share in the market portfolio of consumers with high risk aversion, H, as functions of ω. Parameters are set as in Table 1.

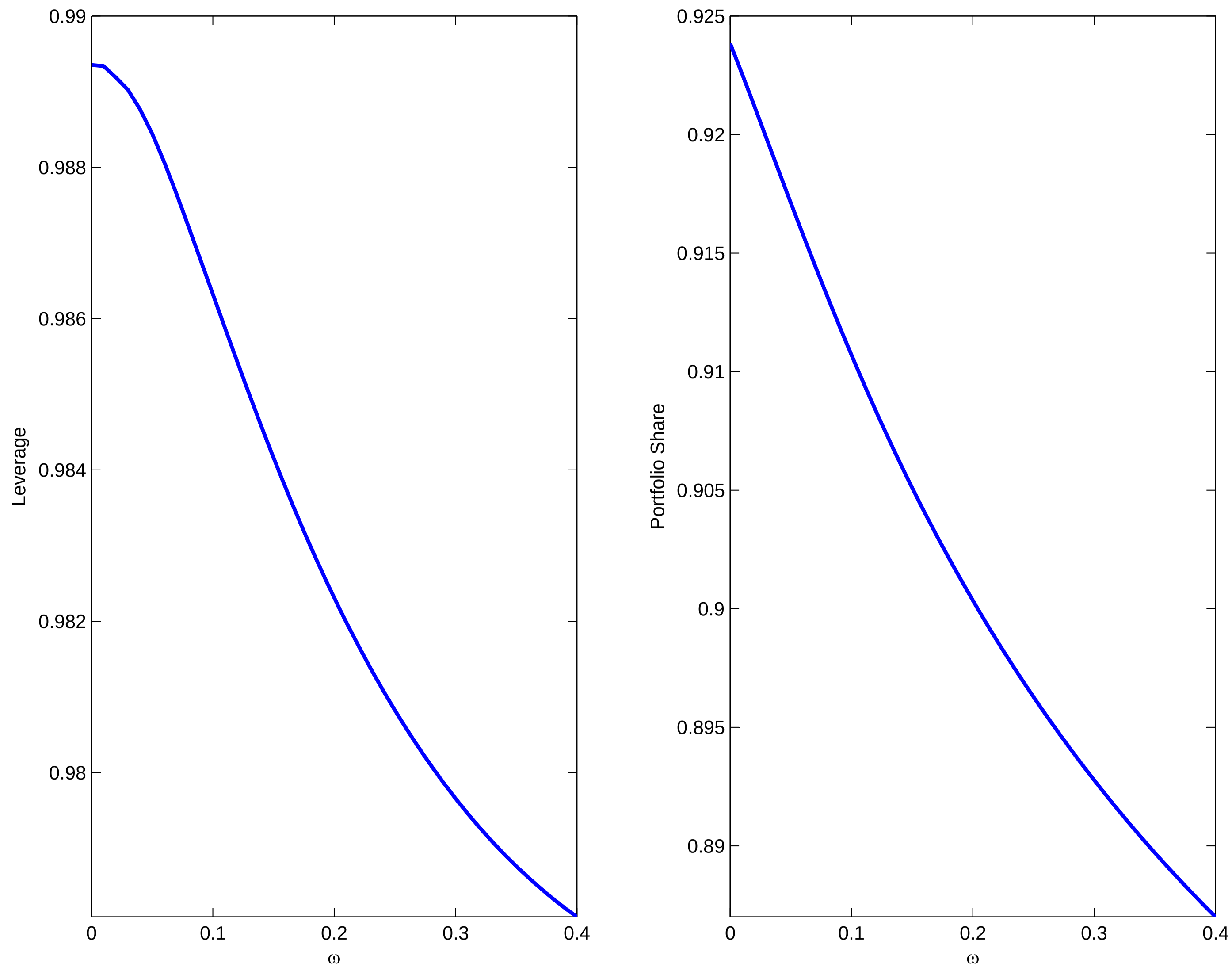
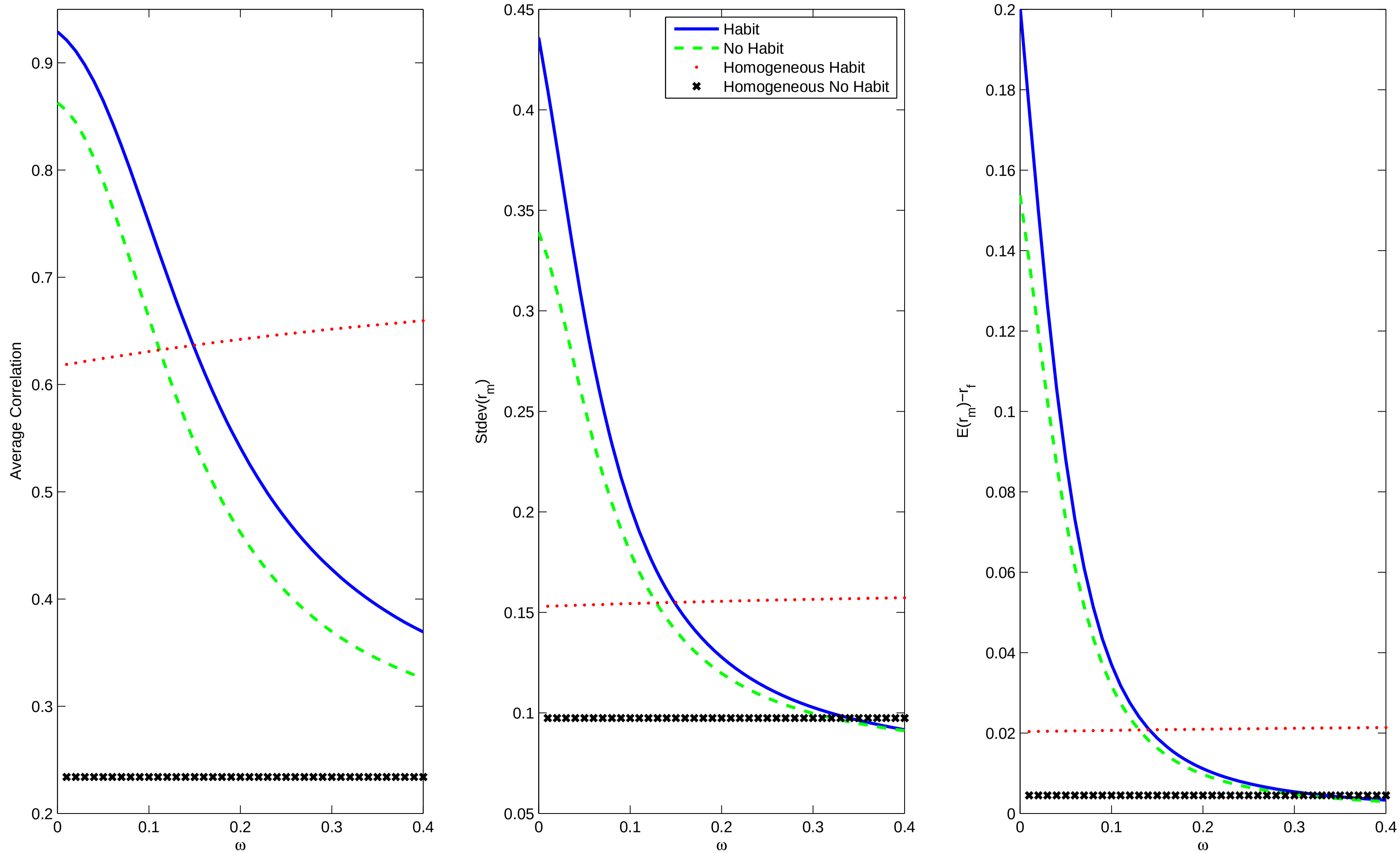


Figure 4: **Homogeneous, Heterogeneous, and Habit.** This figure shows plots of the conditional return correlations, the conditional standard deviation of the market portfolio, and the expected excess return on the market portfolio for a heterogeneous economy with habits, a heterogeneous economy without habits, a homogeneous economy with habits, and a homogeneous economy without habits as functions of ω. Parameters are set as in Table 1.



The material contained herein is supplementary to the article named "Correlations" published in Management Science

Paul Ehling[*] Christian Heyerdahl-Larsen[†]

2015

[*]Department of Finance, BI Norwegian Business School, Nydalsveien 37, 0484 Oslo, Norway, paul.ehling@bi.no

[†]London Business School, Regent's Park, London, NW1 4SA, UK, cheyerdahllarsen@london.edu

This Internet Appendix provides additional results that are left out of the main text of the paper. The appendix is organized as follows: Section 1 presents proofs of the propositions in the main text of the paper. Section 2 performs an extensive principal component analysis and shows that ratio habits explain the first principal component in the time series of correlations and other asset pricing related time series. Section 3 shows the performance of the log price-dividend ratio as explanatory variable instead of ratio habits. Next, Section 4 presents regressions analysis that shows that model implied ratio habit or aggregate risk aversion predicts excess returns in-sample and out-of-sample. Finally, Section 5 presents regression analysis with portfolios sorted on size, book-to-market, and momentum instead of industry sorted portfolios.

1 Proofs of Propositions

Proof of Proposition 2

We solve for equilibrium using the martingale approach (see Cox and Huang (1989) and Karatzas, Lehoczky, and Shreve (1987)). Each investor solves the static optimization problem

$$\max_{C_j} E\left[\int_0^\infty e^{-\rho t}\frac{1}{1-\gamma_j}C_j(t)^{1-\gamma_j}X(t)^{\gamma_j-\eta}dt\right] \tag{1}$$

$$s.t.$$

$$E\left[\int_0^\infty \xi(t)C_j(t)dt\right] \leq f_{Y,j}(0)E\left[\int_0^T \xi(t)C(t)dt\right], \tag{2}$$

where $f_{Y,j}(0)=\frac{Y_j(0)}{Y_L(0)+Y_H(0)}$ *is the initial wealth fraction of investor type* j*. Necessary and sufficient conditions for optimality are*

$$C_j(t)=\left(y_j e^{\rho t}X(t)^{\eta-\gamma_j}\xi(t)\right)^{-\frac{1}{\gamma_j}}, \tag{3}$$

where $y_j>0$ *is such that*

$$E\left[\int_0^\infty \xi(t)\left(y_j e^{\rho t}X(t)^{\eta-\gamma_j}\xi(t)\right)^{-\frac{1}{\gamma_j}}dt\right]=f_{Y,j}(0)E\left[\int_0^\infty \xi(t)C(t)dt\right], \tag{4}$$

i.e., that the budget condition holds with equality. To solve for equilibrium, it is convenient to introduce an aggregate investor

$$u(C(t),X(t),t)=\max_{C_L(t),C_H(t)}\left\{\begin{array}{c} ae^{-\rho t}\frac{1}{1-\gamma_L}C_L(t)^{1-\gamma_L}X(t)^{\gamma_j-\eta} \\ +(1-a)\,e^{-\rho t}\frac{1}{1-\gamma_H}C_H(t)^{1-\gamma_H}X(t)^{\gamma_j-\eta}\end{array}\right\} \tag{5}$$

$$s.t.$$

$$C_L(t) + C_H(t) = C(t). \tag{6}$$

From the first-order conditions (FOC) of the aggregate investor's problem we have

$$ae^{-\rho t}\left(\frac{C_L(t)}{X(t)}\right)^{-\gamma_L} X(t)^{-\eta} = (1-a)\, e^{-\rho t}\left(\frac{C_H(t)}{X(t)}\right)^{-\gamma_H} X(t)^{-\eta}. \tag{7}$$

Defining the consumption share $f(t) = \frac{C_L(t)}{C(t)}$ *of* L *investors and imposing market clearing, Equation 6, we can rewrite Equation 7 as*

$$f(t) = \left(\frac{a}{1-a}\right)^{\frac{1}{\gamma_L}} e^{\left(\frac{\gamma_H}{\gamma_L}-1\right)\omega(t)} \left(1-f(t)\right)^{\frac{\gamma_H}{\gamma_L}}. \tag{8}$$

Proof of Proposition 4

First note that the utility function of the aggregate investor is defined through Equation 5. The coefficient of relative risk aversion is

$$\mathcal{R}(t) = -\frac{u_{CC}(C(t), X(t), t)}{u_C(C(t), X(t), t)} C(t), \tag{9}$$

where u_C *and* u_{CC} *denote the first and second partial derivative with respect to aggregate consumption, respectively. To calculate* $\mathcal{R}$*, we need to compute the partial derivatives of the aggregate investor's utility function. To this end, note that from the FOC of the aggregate investor problem we have that*

$$a\ u_{L,C}(C_L, X(t), t) = (1-a)\ u_{H,C}(C_H, X(t), t). \tag{10}$$

Consequently, we have that

$$\begin{aligned} u_C(C(t), X(t), t) &= a\ u_{L,C}(C_L, X(t), t)\frac{\partial C_L}{\partial C} + (1-a)\ u_{H,C}(C_H, X(t), t)\frac{\partial C_H}{\partial C} \\ &= a\ u_{L,C}(C_L, X(t), t)\left(\frac{\partial C_L}{\partial C} + \frac{\partial C_H}{\partial C}\right) \\ &= a\ u_{L,C}(C_L, X(t), t), \end{aligned} \tag{11}$$

where the second equality follows from Equation 10 and the third equality follows from differentiating both sides of the market clearing condition in Equation 6. Next we calculate the *second derivative of the aggregate investor's utility function*

$$u_{CC}(C(t), X(t), t) = a\ u_{L,CC}(C_L, X(t), t)\frac{\partial C_L}{\partial C}. \tag{12}$$

Define the absolute risk aversion of investor type j as

$$\mathcal{A}_j(t) = -\frac{u_{j,CC}(C_j(t), X(t), t)}{u_{j,C}(C_j(t), X(t), t)}. \tag{13}$$

We have that

$$\begin{aligned} \mathcal{A}(t) &= -\frac{u_{CC}(C(t), X(t), t)}{u_C(C(t), X(t), t)} \\ &= -\frac{a\ u_{L,CC}(C_L(t), X(t), t)}{a\ u_{L,C}(C_L(t), X(t), t)}\frac{\partial C_L}{\partial C} \\ &= \mathcal{A}_L(t)\frac{\partial C_L}{\partial C}. \end{aligned} \tag{14}$$

Thus, we also have that $\frac{\partial C_L}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}$. Similarly, we get that $\frac{\partial C_H}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_H(t)}$. Using the fact that $\frac{\partial C_L}{\partial C} + \frac{\partial C_H}{\partial C} = 1$, we obtain

$$\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)} + \frac{\mathcal{A}(t)}{\mathcal{A}_H(t)} = 1, \tag{15}$$

or

$$\mathcal{A}(t) = \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1}. \tag{16}$$

Using $\mathcal{R}(t) = \mathcal{A}(t)C(t)$ together with Equation 16, we find

$$\begin{aligned} \mathcal{R}(t) &= \mathcal{A}(t)C(t) \\ &= \left(\frac{1}{\mathcal{A}_L(t)} + \frac{1}{\mathcal{A}_H(t)}\right)^{-1} C(t) \\ &= \left(\frac{C_L}{C(t)\gamma_L} + \frac{C_H}{C(t)\gamma_H}\right)^{-1} \\ &= \left(\frac{1}{\gamma_L} f(t) + \frac{1}{\gamma_H}(1 - f(t))\right)^{-1}. \end{aligned} \tag{17}$$

The absolute prudence of the representative investor, $\mathcal{P}^A(t)$, is

$$\mathcal{P}^A(t) = -\frac{u_{CCC}(C(t), X(t), t)}{u_{CC}(C(t), X(t), t)}. \tag{18}$$

Similarly, we define the absolute prudence of investor j as

$$\mathcal{P}_j^A(t) = -\frac{u_{j,CCC}(C_j(t), X(t), t)}{u_{j,CC}(C_j(t), X(t), t)}. \tag{19}$$

To evaluate Equation 18, we need to calculate $u_{CCC}(C(t), X(t), t)$

$$\begin{aligned}
u_{CCC}(C(t), X(t), t) &= \frac{\partial^2 \left(a\ u_{L,C}(C_L(t), X(t), t)\right)}{\partial C^2} \\
&= \frac{\partial \left(a\ u_{L,CC}(C_L(t), X(t), t)\right) \frac{\partial C_L(t)}{\partial C}}{\partial C} \\
&= a\ u_{L,CCC}(C_L(t), X(t), t) \left(\frac{\partial C_L(t)}{\partial C}\right)^2 \\
&\quad + a\ u_{L,CC}(C_L(t), X(t), t) \frac{\partial^2 C_L(t)}{\partial C^2}.
\end{aligned} \tag{20}$$

Similarly, we calculate

$$\begin{aligned}
u_{CCC}(C(t), X(t), t) &= (1-a)\ u_{H,CCC}(C_H(t), X(t), t) \left(\frac{\partial C_H(t)}{\partial C}\right)^2 \\
&\quad + a\ u_{H,CC}(C_H(t), X(t), t) \frac{\partial^2 C_H(t)}{\partial C^2}.
\end{aligned} \tag{21}$$

Using Equation 20 and Equation 21, allow to compute

$$\frac{\partial C_L(t)}{\partial C} \mathcal{P}^A(t) = -\frac{\partial^2 C_L(t)}{\partial C^2} + \mathcal{P}_L^A(t) \left(\frac{\partial C_L(t)}{\partial C}\right)^2 \tag{22}$$

and

$$\frac{\partial C_H(t)}{\partial C} \mathcal{P}^A(t) = -\frac{\partial^2 C_H(t)}{\partial C^2} + \mathcal{P}_H^A(t) \left(\frac{\partial C_H(t)}{\partial C}\right)^2. \tag{23}$$

Adding up Equations 22 and 23 and noting that $\frac{\partial^2 C_L(t)}{\partial C^2} + \frac{\partial^2 C_H(t)}{\partial C^2} = 0$, *we get*

$$\mathcal{P}^A(t) = \mathcal{P}_L^A(t) \left(\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}\right)^2 + \mathcal{P}_H^A(t) \left(\frac{\mathcal{A}(t)}{\mathcal{A}_H(t)}\right)^2. \tag{24}$$

The relative prudence of the representative investor is

$$\begin{aligned}
\mathcal{P}(t) &= \mathcal{P}^A(t) C(t) \\
&= (1+\gamma_L) \left(\frac{\mathcal{R}(t)}{\gamma_L}\right)^2 f(t) + (1+\gamma_H) \left(\frac{\mathcal{R}(t)}{\gamma_H}\right)^2 (1 - f(t)).
\end{aligned} \tag{25}$$

Proposition IA-1

Consumers' consumption dynamics evolve according to

$$\begin{aligned}
dC_j(t) &= C_j(t)\left(\mu_{C_j}(t)dt + \sigma_{C_j}(t)dZ_C(t)\right), \\
\text{where} \quad \mu_{C_j}(t) &= \left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\mu_C(t) + \left(1 - \frac{\mathcal{R}(t)}{\gamma_j}\right)\lambda\omega(t) \\
&\quad + \frac{1}{2}\left[(1+\gamma_j)\left(\frac{\mathcal{R}(t)}{\gamma_j}\right) - \mathcal{P}(t)\right]\left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\sigma_C^2, \\
\sigma_{C_j}(t) &= \left(\frac{\mathcal{R}(t)}{\gamma_j}\right)\sigma_C.
\end{aligned} \tag{26}$$

Proof of Proposition IA-1

First, note that the individual consumption is only a function of aggregate consumption C and the habit level X. By Ito's lemma we have

$$dC_j(t) = \frac{\partial C_j(t)}{\partial C}dC(t) + \frac{\partial C_j(t)}{\partial X}dX(t) + \frac{1}{2}\frac{\partial^2 C_j(t)}{\partial C^2}\left(dC(t)\right)^2. \tag{27}$$

To evaluate Equation 27, we need the partial derivatives $\frac{\partial C_j(t)}{\partial C}$, $\frac{\partial C_j(t)}{\partial X}$ and $\frac{\partial^2 C_j(t)}{\partial C^2}$. From the proof of Proposition 2 we have that

$$\frac{\partial C_j(t)}{\partial C} = \frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}, \tag{28}$$

and

$$\frac{\partial^2 C_j(t)}{\partial C^2} = \mathcal{P}_j^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}\right)^2 - \mathcal{P}^A(t)\left(\frac{\mathcal{A}(t)}{\mathcal{A}_j(t)}\right). \tag{29}$$

Next, we compute

$$\begin{aligned}
\frac{\partial C_L(t)}{\partial X} &= \frac{\partial f(t)C(t)}{\partial X} \\
&= C(t)\frac{\partial f(t)}{\partial X} + f(t)\frac{\partial C(t)}{\partial X} \\
&= C(t)\frac{\partial f(t)}{\partial \omega}\frac{\partial \omega}{\partial X} \\
&= -C_L(t)\left(\frac{\mathcal{R}(t)}{\gamma_L} - 1\right)\frac{1}{X(t)},
\end{aligned} \tag{30}$$

where in the above we have used the fact that $\frac{\partial \omega}{\partial X} = -\frac{1}{X(t)}$ and $\frac{\partial f(t)}{\partial \omega} = f(t)\left(\frac{\mathcal{R}(t)}{\gamma_L} - 1\right)$. Similarly, we get that

$$\frac{\partial C_H(t)}{\partial X} = -C_H(t)\left(\frac{\mathcal{R}(t)}{\gamma_H} - 1\right)\frac{1}{X(t)}. \tag{31}$$

Inserting the partial derivatives together with the dynamics of C and X into Equation (27) yields the proposition.

Proof of Proposition 6

The expression for the state price density follows from the standard result that the state price density is proportional to the marginal utility of the representative investor

$$\xi(t) = \frac{u_C(C(t), X(t), t)}{u_C(C(0), X(0), 0)}. \tag{32}$$

The dynamics of the state price density follow, Duffie (2001),

$$\frac{d\xi(t)}{\xi(t)} - (r(t)dt + \theta(t)dZ_C(t)). \tag{33}$$

Next, applying Ito's lemma to $u_C(C(t), X(t), t)$ we obtain

$$\begin{aligned} du_C(C(t), X(t), t) &= u_{Ct}(C(t), X(t), t)dt + u_{CC}(C(t), X(t), t)dC(t) + u_{CX}(C(t), X(t), t)dX(t) \\ &\quad + \frac{1}{2}u_{CCC}(C(t), X(t), t)\,(dC(t))^2 \\ &= (u_{CC}(C(t), X(t), t)C(t)\mu_C(t) + u_{CX}(C(t), X(t), t)X(t)\lambda\omega(t))\,dt \\ &\quad + \left(\frac{1}{2}u_{CCC}(C(t), X(t), t)C(t)^2\sigma_C^2 + u_{Ct}(C(t), X(t), t)\right)dt \\ &\quad + u_{CC}(C(t), X(t), t)C(t)\sigma_C dZ_C(t). \end{aligned} \tag{34}$$

To evaluate Equation 34, we need in addition to $u_{CC}(C(t), X(t), t)$ also expressions for $u_{Ct}(C(t), X(t), t)$ and $u_{CX}(C(t), X(t), t)$. First note that

$$u_{Ct}(C(t), X(t), t) = -\rho u_C(C(t), X(t), t). \tag{35}$$

Next, we calculate $u_{CX}(C(t), X(t), t)$ as follows

$$\begin{aligned} u_{CX}(C(t), X(t), t) &= \frac{\partial a\ u_{L,C}(C_L(t), X(t), t)}{\partial X} \\ &= (\gamma_L - \eta)\,a\ u_{L,C}(C_L(t), X(t), t)X(t)^{-1} \\ &\quad + \gamma_L \frac{C(t)}{C_L(t)} f'(t) a\ u_{L,C}(C_L(t), X(t), t)X(t)^{-1} \\ &= \left(\gamma_L - \eta + \gamma_L \frac{C(t)}{C_L(t)}\left[\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)} - f(t)\right]\right) a\ u_{L,C}(C_L(t), X(t), t)X(t)^{-1} \\ &= (\mathcal{R}(t) - \eta)\,u_C(C(t), X(t), t)X(t)^{-1}. \end{aligned} \tag{36}$$

Since $f(t)=f(\omega(t))$, its derivative is given by $f'(t)=\frac{df(\omega(t))}{d\omega}$. Next, we use the fact that $\frac{\partial C_L(t)}{\partial C(t)}=\frac{\partial f(t)C(t)}{\partial C(t)}=f(t)+C(t)f'(t)\frac{\partial\omega(t)}{\partial C(t)}=f(t)+f'(t)$ together with $\frac{\partial C_L(t)}{\partial C(t)}=\frac{\mathcal{A}(t)}{\mathcal{A}_L(t)}$. Now, note that we have that

$$u_{CCC}(C(t),X(t),t)=u_C(C(t),X(t),t)\mathcal{R}(t)\mathcal{P}(t)\frac{1}{C(t)^2}. \tag{37}$$

Inserting Equations 12, 35, 36 and 37 together with the corresponding dynamics of $C(t)$ and $X(t)$ into Equation 34 we get

$$\begin{aligned}\frac{du_C(C(t),X(t),t)}{u_C(C(t),X(t),t)} &= -\left(\rho+\eta\lambda\omega(t)+\mathcal{R}\left(t\right)\left(\mu_C-\lambda\omega(t)\right)-\frac{1}{2}\mathcal{R}\left(t\right)\mathcal{P}(t)\sigma_C^2\right)dt \\ &\quad -\mathcal{R}(t)\sigma_C dZ_C(t). \qquad (38)\end{aligned}$$

Finally, matching the drift and diffusion coefficients in Equation 33 with Equation 38 we obtain

$$r(t) = \rho+\eta\lambda\omega(t)+\mathcal{R}\left(t\right)\left(\mu_C-\lambda\omega(t)\right)-\frac{1}{2}\mathcal{R}\left(t\right)\mathcal{P}(t)\sigma_C^2 \tag{39}$$

$$\theta(t) = \mathcal{R}(t)\sigma_C. \tag{40}$$

2 Empirics – Principal Component Analysis

We calculate the first principal component of the forty-five correlation series (PCA CORR), the ten series of 3-year ahead excess returns (PCA EXR), the ten series of standard deviations (PCA STD) and the ten series of quadratic variations of turnover (PCA QV) separately. To calculate the first principal component of all the series, we compute the average of the four sets of series to reduce the impact of the forty-five correlation series and obtain from the averages the first principal component (PCA TOTAL). First, we regress the first principal component of these four series onto model implied external relative habit. Second, we regress the first principal component from all the series, PCA TOTAL, onto model implied external relative habit.[1]

Table 1 shows the results from these principal component regressions. All regression coefficients show negative sign consistent with a heterogeneous consumer version of the model.

[1]The first principal component explains 54%, 86%, 87% and 71% of the variation in the 45 correlation coefficients, 10 standard deviations, 10 quadratic variations of turnover and 10 three years ahead excess returns. The first principal component explains 51% of the variation of the averages of the four series.

Further, all coefficient estimates for external relative habit show highly significant Newey-West corrected t-statistics. The adjusted R-squared range from 14.06% to 40.95%. Our results regarding excess returns are essentially unchanged if we correct the nominal short rate with expected inflation instead of realized inflation.[2] Overall, we conclude that signs of the coefficients as well as the explanatory power of the regressions support our theory.

3 Empirics – Log Price-Dividend Ratio Regressions

In our model the log price-dividend ratio is increasing in ω, i.e., the relation represents a one-to-one mapping. Indeed, the log price-dividend ratio leads to comparable results for the principal component analysis, Table 2, as well as for regressions that explain the averages of the series we study, Table 3.

4 Predicting Excess Returns In-Sample and Out-of-Sample

Model implied relative consumption forecasts excess returns in-sample and out-of-sample. Because relative consumption does not include the level of market prices, it is unlikely to produce spurious results. In the sense that relative consumption forecasts excess returns in the model, it is a natural predictor of stock market returns.[3]

The relation between the excess return on the market portfolio and relative consumption is

$$Corr_t(E_t(dR_M(t) - r(t)), \omega(t)) < 0, \tag{41}$$

which is negative for most of the distribution of ω. Hence, on average, the model implies a negative relation between expected excess returns and relative consumption. A discrete time formulation implies the following slope coefficient in a predictive regression

$$\beta_t = \frac{Cov_t(E_t(R_{M,t+1} - r_t), \omega_t)}{Var_t(\omega_t)}, \tag{42}$$

which is negative whenever Equation 41 is negative. Therefore, the model predicts on average negative relations between relative consumption and expected excess return of the market portfolio as well as other portfolios.

The first three rows of Table 4 show the coefficient estimate, Newey-West corrected t-statistics and the adjusted R-squared for in-sample regressions using relative consumption

[2]Model implied external relative habit and the real short rate show statistically significant negative relation.

[3]Cooper and Priestley (2009) argue that it is important to link predictability to economic fundamentals.

from the heterogeneous investor economy as the predictive variable. The table contains 3 sets of regressions: 1 year excesses returns, 3 year excess returns and 5 year excess returns with each set containing regressions for 10 industries using Kenneth French's industry classification and the market excess return. Regressions include a constant with data ranging from 1927 to 2009. The predictive impact of relative consumption is statistically significant at least at the 10 percent level except for the following sectors: Non-Durables (five year horizon), Energy (one and five year horizons), and Health and Utils for all horizons. Importantly, all coefficients appear with negative sign favoring the model with heterogeneity in risk aversion. The adjusted R-squared statistics indicate that regressions with significant coefficients explain at least 2.8 percent of the variation in excess industry and market returns. Overall, adjusted R-squared statistics first increase when the prediction horizon increases, 3 year versus 1 year, but decrease when the prediction horizon is 5 years.

Next, we ask whether predictive regressions perform also out-of-sample by making nested forecast comparisons. The comparisons are between a model which includes only the constant and a model which includes a constant and relative consumption as a predictor. Theil's U, in the fourth row for each prediction horizon in Table 4, is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number larger than one for Theil's U indicates that the restricted model (with only a constant) has a lower root-mean-squared error than the model with relative consumption as an explanatory variable. The root-mean-squared error of the regressions which include relative consumption are always lower than the regressions with a constant, except for Non-Durables (three and five year horizons) and Health and Utilities for all horizons. Another standard out-of-sample test is the MSE-F statistic which tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The fifth row for each prediction horizon in Table 4 shows the test statistics from our data. The ENC-NEW statistic, in the 6th row for each prediction horizon, tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. At the 10% level, we obtain a picture very similar to the previous results.

5 Empirics – Alternative Portfolio Sorts

In the main body of the paper we calibrate the model to ten industry portfolios. According to our model, stock market correlations, standard deviations and expected returns as well as quadratic variation of portfolio policies have negative relation with ω. This negative relation, however, is independent of portfolio sorts. Therefore, we repeat the regression from the main text of the paper and the predictive regression in Section 4, but use alternative portfolio sorts. In particular, we consider ten portfolios sorted on size, book-to-market and momentum, respectively. The Tables 5 - 10 confirm the prediction of our model and, thus, show that our empirical results in the main text of the paper are not an artifact of industry sorted portfolios.

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Table 1: **Empirics — Principal Component Analysis.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied external relative habit as explanatory variable for the first principal component of industry market correlations (PCA CORR), 3-year ahead expected excess returns (PCA EXR), standard deviations (PCA STDV), quadratic variations of industry turnover (PCA QV), and the first principal component of the four means of the respective series (PCA TOTAL). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover. Model implied external relative habit is linearly interpolated from the heterogeneous consumer model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	PCA CORR	PCA EXR	PCA STDV	PCA QV	PCA TOTAL
Intercept	0.5387	0.3617	0.3232	0.4423	0.1687
	(2.9547)	(3.6869)	(3.0444)	(6.7586)	(4.7019)
Model implied external habit, ω	-3.3680	-2.2556	-2.0205	-2.7656	-1.0524
	(-3.0384)	(-4.0211)	(-3.3327)	(-7.9762)	(-4.8871)
Adjusted R-squared	0.1406	0.1501	0.4095	0.2392	0.2719

Table 2: **Empirics with the Log Price-Dividend Ratio — Principal Component Analysis.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied log Price-Dividend ratio (pd) as explanatory variable of the first principal component of industry market correlations (PCA CORR), 3-year ahead expected excess returns (PCA EXR), standard deviations (PCA STDV), quadratic variations of industry turnover (PCA QV), and the first principal component of the four means of the respective series (PCA TOTAL). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover are estimated by a GARCH(1,1) model based on log changes in turnover. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	PCA CORR	PCA EXR	PCA STDV	PCA QV	PCA TOTAL
Intercept	2.3299	0.8976	0.3122	1.5327	0.5088
	(8.1551)	(5.3126)	(2.3408)	(13.0112)	(8.7718)
pd-ratio	-0.70161	-0.27175	-0.094015	-0.46155	-0.15404
	(-8.1361)	(-5.3423)	(-2.5066)	(-14.1999)	(-8.7895)
Adjusted R-squared	0.41733	0.14183	0.059489	0.45431	0.37974

Table 3: **Empirics with the Log Price-Dividend Ratio — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of model implied log Price-Dividend ratio (pd) as explanatory variable of the average of industry market correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), standard deviations (Av. STDV), and quadratic variations of industry turnover (Av. QV). Newey-West corrected t-statistics are in parentheses. Industry market correlations and standard deviations are calculated using a DVEC(1,1) model. Quadratic variations of industry turnover is estimated by a GARCH(1,1) model based on log changes in turnover. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV	Av. QV
Intercept	1.007	0.3739	0.29082	0.59651
	(25.3129)	(6.6404)	(7.1048)	(18.2762)
pd-ratio	-0.094924	-0.089417	-0.028203	-0.11399
	(-7.9479)	(-5.1959)	(-2.4477)	(-12.6464)
Adjusted R-squared	0.38805	0.14389	0.057661	0.47446

Table 4: **Empirics — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 industry returns, and the market return on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 industry returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other	Mkt
1 year	Coefficient	-0.7645	-2.2214	-1.3620	-0.5428	-1.0977	-0.8883	-1.1093	-0.3020	-0.1509	-0.8508	-0.8969
	t-statistics	-2.7532	-3.7688	-3.2671	-1.4148	-2.2542	-2.0395	-3.7377	-0.9408	-0.2421	-2.3849	-2.8392
	Adj. R-squared	0.0375	0.1101	0.0828	0.0065	0.0348	0.0527	0.0526	-0.0061	-0.0108	0.0280	0.0457
	Theil's U	0.9731	0.8976	0.9225	0.9970	0.9603	0.9711	0.9550	0.9966	1.0073	0.9648	0.9536
	MSE-F	2.2993	9.8919	7.1780	0.2493	3.4613	2.4753	3.9562	0.2815	-0.5932	3.0437	4.0887
	ENC-NEW	1.6596	6.5249	5.2799	0.3866	1.9220	1.5971	2.5484	0.1757	-0.2791	1.8737	2.5655
3 year	Coefficient	-1.2595	-4.3025	-3.0497	-1.3942	-2.9738	-2.1803	-1.9299	-0.8658	-0.6338	-1.8276	-2.0394
	t-statistics	-2.0500	-3.3514	-3.6276	-1.8990	-4.1732	-4.6367	-3.3049	-1.4796	-0.8203	-3.2314	-3.8722
	Adj. R-squared	0.0386	0.2071	0.1817	0.0331	0.1000	0.1154	0.0710	0.0048	-0.0024	0.0561	0.1008
	Theil's U	0.9729	0.8257	0.8085	0.9896	0.9025	0.9491	0.9464	0.9859	0.9845	0.9344	0.8994
	MSE-F	2.2630	18.6636	21.1884	0.8427	9.1083	4.4055	4.6619	1.1552	1.2670	5.8098	9.4460
	ENC-NEW	1.9320	13.6465	17.5287	1.3477	5.3968	3.2575	3.4095	0.6535	0.7912	3.7521	6.4276
5 year	Coefficient	-1.2952	-4.9389	-3.4826	-1.0383	-3.5356	-2.5932	-2.2884	-1.3108	-0.8938	-1.9686	-2.2946
	t-statistics	-1.5359	-3.5891	-3.1063	-0.9750	-3.3304	-4.7007	-3.1951	-1.3886	-1.3221	-2.0412	-2.8981
	Adj. R-squared	0.0258	0.2259	0.1845	0.0053	0.1043	0.1087	0.0845	0.0118	0.0014	0.0443	0.0939
	Theil's U	0.9726	0.7830	0.7734	0.9784	0.9079	0.9453	0.9294	0.9863	0.9772	0.9420	0.8856
	MSE-F	2.2283	24.6203	26.2042	1.7443	8.3154	4.6483	6.1528	1.0913	1.8392	4.9469	10.7276
	ENC-NEW	1.5420	16.1885	18.2481	1.2092	4.8803	3.4253	4.2000	0.64116	1.2122	3.0579	6.7181

Table 5: **Size Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten size sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Size sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.92608	0.17465	0.3775
	(134.7149)	(7.4878)	(9.2808)
Relative consumption, ω	-0.083886	-0.6404	-0.92581
	(-2.057)	(-4.7258)	(-3.957)
Adjusted R-squared	0.038919	0.10923	0.44339

Table 6: **BM Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten book-to-market sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Book-to-market sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.89778	0.28147	0.37096
	(82.2972)	(4.2825)	(9.1625)
Relative consumption, ω	-0.19937	-0.91342	-0.96231
	(-3.1788)	(-2.5318)	(-4.1551)
Adjusted R-squared	0.10979	0.096109	0.49111

Table 7: **Momentum Sorted Portfolios — Averages.** This table summarizes OLS regression results (intercept, coefficient estimate and adjusted R-squared) of relative consumption as explanatory variable of the average of ten momentum sorted portfolio correlations (Av. CORR), 3-year ahead expected excess returns (Av. EXR), and standard deviations (Av. STDV). Newey-West corrected t-statistics are in parentheses. Momentum sorted correlations and standard deviations are calculated using a DVEC(1,1) model. Relative consumption is linearly interpolated from the heterogeneous investor model calibration employing annual consumption data from Robert Shiller's web page. The regressions use 996 monthly observations with data ranging from January 1927 to December 2009.

	Av. CORR	Av. EXR	Av. STDV
Intercept	0.8573	0.21641	0.3113
	(60.3286)	(4.8501)	(8.0865)
Relative consumption, ω	-0.18107	-0.40609	-0.63045
	(-2.135)	(-1.6217)	(-2.9175)
Adjusted R-squared	0.052038	0.029092	0.28267

Table 8: **Size Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 size sorted portfolios on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 size sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-2.5331	-2.3642	-2.0631	-1.766	-1.5606	-1.522	-1.3315	-1.2363	-1.0641	-0.76187
	t-statistics	-3.7912	-3.9303	-3.4419	-3.678	-3.542	-3.5886	-3.4128	-3.3881	-3.0504	-2.4094
	Adj. R-squared	0.10335	0.12343	0.11171	0.093707	0.084875	0.086283	0.069929	0.072247	0.057857	0.034455
	Theil's U	0.90734	0.90559	0.90648	0.91603	0.93167	0.91258	0.94562	0.9418	0.94773	0.96313
	MSE-F	8.587	8.7749	8.6791	7.6691	6.0827	8.0307	4.7331	5.0969	4.5336	3.1212
	ENC-NEW	6.6204	7.1416	7.0036	5.7212	4.5479	5.6783	3.5434	3.7448	3.1748	1.8997
3 year	Coefficient	-5.4825	-5.0902	-4.4132	-3.915	-3.2542	-3.4706	-2.627	-2.4402	-2.3836	-1.9029
	t-statistics	-2.9939	-3.211	-3.0312	-3.7668	-3.4398	-4.2756	-3.0354	-2.9019	-3.4775	-4.1285
	Adj. R-squared	0.13887	0.17569	0.1748	0.17274	0.14305	0.17962	0.1134	0.12062	0.11901	0.089665
	Theil's U	0.88625	0.85769	0.86816	0.8739	0.89822	0.86268	0.91162	0.88741	0.88705	0.91651
	MSE-F	10.6536	14.0158	12.7447	12.0672	9.3394	13.4044	7.9286	10.5242	10.5642	7.4286
	ENC-NEW	9.312	12.8505	12.1593	10.5713	8.3344	11.5743	6.8144	8.5001	8.1435	4.7055
5 year	Coefficient	-6.2674	-5.5616	-4.7686	-4.5781	-3.6692	-4.0739	-2.7641	-2.5305	-2.668	-2.2446
	t-statistics	-2.6992	-2.7431	-2.7775	-3.4134	-3.1552	-3.8067	-2.4722	-2.4207	-2.7209	-2.9326
	Adj. R-squared	0.12387	0.15503	0.157	0.17529	0.14107	0.19233	0.10297	0.10742	0.11654	0.08405
	Theil's U	0.9001	0.86455	0.87416	0.87178	0.89524	0.85526	0.88538	0.86682	0.85659	0.90839
	MSE-F	8.9036	12.8402	11.7284	12.0001	9.4139	13.9507	10.4762	12.5739	13.7892	8.0513
	ENC-NEW	7.0617	10.0728	9.5515	9.7094	7.7083	11.1572	7.4234	8.497	9.217	4.7559

Table 9: **BM Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 book-to-market sorted portfolios on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 book-to-market sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-0.82169	-0.8233	-0.76847	-1.1144	-0.90286	-1.2255	-0.97298	-1.1637	-1.206	-1.5153
	t-statistics	-2.3926	-2.8631	-2.898	-2.8092	-2.32	-2.3933	-2.4825	-2.7033	-2.4605	-2.5837
	Adj. R-squared	0.026426	0.041171	0.034161	0.064196	0.036934	0.07056	0.036174	0.046697	0.044599	0.050486
	Theil's U	0.97336	0.95999	0.97043	0.96016	0.9571	0.93103	0.96008	0.93666	0.92978	0.9342
	MSE-F	2.2191	3.4037	2.4746	3.3882	3.6665	6.1463	3.3957	5.5929	6.2696	5.8331
	ENC-NEW	1.3681	2.1445	1.7087	2.7441	2.4577	4.3464	2.3702	3.7847	4.2616	4.0595
3 year	Coefficient	-1.9894	-1.8852	-1.4569	-2.436	-1.8727	-3.5065	-2.1905	-2.4929	-2.5124	-2.3348
	t-statistics	-3.1101	-3.8753	-2.9266	-3.003	-2.6954	-4.4321	-3.7471	-3.3561	-3.0991	-1.8417
	Adj. R-squared	0.07155	0.10998	0.059748	0.13227	0.066723	0.19868	0.09042	0.098653	0.084035	0.043883
	Theil's U	0.9377	0.90651	0.95494	0.93532	0.90063	0.84483	0.91744	0.85973	0.88695	0.90971
	MSE-F	5.3543	8.4587	3.7672	5.5802	9.081	15.6425	7.3346	13.7645	10.5757	8.126
	ENC-NEW	3.4442	5.7063	2.9289	5.9387	5.9431	14.7487	5.9752	10.2236	8.0908	5.7096
5 year	Coefficient	-2.4954	-2.3527	-1.5835	-2.4631	-1.7428	-4.1236	-2.4591	-2.4624	-2.6212	-1.3322
	t-statistics	-2.911	-3.1794	-2.3926	-2.3457	-1.6343	-3.9262	-2.6903	-2.2095	-2.3383	-0.75529
	Adj. R-squared	0.083094	0.12797	0.054045	0.10096	0.036314	0.19746	0.088374	0.067935	0.073144	0.0020927
	Theil's U	0.92249	0.88364	0.93753	0.91574	0.93415	0.80051	0.89156	0.8689	0.88681	0.96878
	MSE-F	6.6544	10.6666	5.2324	7.3151	5.5462	21.2986	9.8059	12.3314	10.3195	2.4883
	ENC-NEW	3.9994	6.7739	3.3904	5.7852	3.2813	17.3771	6.8504	7.7807	7.1821	1.5034

Table 10: **Momentum Sorted Portfolios — Predictive Regressions.** This table reports in-sample coefficient estimates from OLS regressions of 1 year, 3 year and 5 year excess returns on 10 momentum sorted portfolios on a constant and relative consumption, ω, implied by the model. The constant is not tabulated. Newey-West corrected t-statistics appear in parentheses below the coefficient estimate. The regressions use 83 annual (monthly are available) observations with data ranging from 1927 to 2009. The table also reports one period ahead (for 1 year, 3 year and 5 year) nested forecast comparisons of excess returns on 10 momentum sorted portfolio returns and the market return. The comparisons are between a model which includes a constant and a model which includes a constant and relative consumption. Theil's U is the ratio of the root-mean-squared forecast errors for the unrestricted and restricted models. A number less than one indicates that the restricted model (with only a constant) has lower root-mean-squared error than the model with relative consumption as explanatory variable. The MSE-F statistic tests the null hypothesis that the MSE for the unrestricted model forecasts is less than the MSE for the restricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 3.467, 1.636 and 0.819, respectively. The ENC-NEW statistic tests the null hypothesis that restricted model forecasts encompass the unrestricted model forecasts. The test statistics at critical p-values 1%, 5% and 10% are 2.566, 1.334 and 0.842, respectively. The out-of-sample regressions use 41, 40 and 39 annual observations with 83 total observation ranging from 1927 to 2009.

		Low 10	Dec 2	Dec 3	Dec 4	Dec 5	Dec 6	Dec 7	Dec 8	Dec 9	High 10
1 year	Coefficient	-1.4134	-1.3645	-0.64953	-1.1966	-0.81324	-0.98901	-0.91562	-0.90842	-1.0725	-0.74576
	t-statistics	-3.2985	-3.1765	-1.8873	-3.3075	-2.4874	-3.2436	-2.5844	-2.4358	-2.7167	-1.5307
	Adj. R-squared	0.042124	0.063928	0.010663	0.068581	0.037633	0.053421	0.047874	0.04295	0.053237	0.0091011
	Theil's U	0.95863	0.95785	0.98744	0.94953	0.95497	0.94866	0.93964	0.95353	0.95291	0.98042
	MSE-F	3.527	3.5974	1.0238	4.3657	3.8612	4.4468	5.3037	3.9942	4.0511	1.6139
	ENC-NEW	2.2202	2.6941	0.67696	3.3608	2.3963	2.9122	3.3162	2.6541	2.8495	0.97364
3 year	Coefficient	-2.4182	-3.2404	-1.2261	-2.4368	-1.7952	-2.072	-2.3744	-1.9882	-2.3738	-1.9526
	t-statistics	-3.1389	-4.0428	-2.8487	-3.2629	-4.1023	-3.7515	-5.4936	-2.7736	-2.912	-2.4118
	Adj. R-squared	0.060771	0.16318	0.019003	0.11417	0.090235	0.098443	0.1265	0.081617	0.1037	0.043638
	Theil's U	0.94223	0.91922	0.98599	0.90756	0.89059	0.89092	0.85984	0.91074	0.91304	0.95019
	MSE-F	4.9292	7.1561	1.1161	8.3493	10.1706	10.1344	13.7511	8.0195	7.7828	4.1957
	ENC-NEW	3.3809	7.9184	1.0365	7.205	6.6044	6.8477	9.4183	5.2956	5.9867	2.7626
5 year	Coefficient	-3.0006	-4.2482	-1.6043	-2.776	-2.1352	-2.1092	-3.0671	-2.0398	-2.1098	-1.8455
	t-statistics	-2.5646	-4.3167	-2.664	-2.8204	-2.9244	-2.6365	-4.5601	-1.9102	-1.8279	-1.8869
	Adj. R-squared	0.071098	0.20035	0.026245	0.11867	0.08461	0.071213	0.1501	0.058496	0.058741	0.028411
	Theil's U	0.92636	0.85332	0.96323	0.85252	0.87129	0.89095	0.81768	0.93075	0.93319	0.95987
	MSE-F	6.2819	14.1862	2.9563	14.2846	12.0557	9.8713	18.8358	5.8653	5.636	3.2442
	ENC-NEW	4.1534	13.0081	2.1192	10.0006	7.2018	5.9492	12.3747	3.5552	3.6923	1.9515