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Executive Summary

Our research tests the efficiency hypothesis on data from the housing market in Oslo and Stavanger, covering the period 2002-2014. We utilize the Case-Shiller time structure test on a repeated sales house price index and examine the excess return time series for housing investments for each city. This paper is mainly a replication of the research carried out by Røed Larsen and Weum in 2008,[1] it does, however, offer some modifications and extensions.

We conclude that both the repeated sales house price index and the excess return to housing does not contain time structure in any of the cities, and hence the housing markets are characterized as efficient. This is in contrast to the previously concluded inefficiency in Oslo from 1991-2002 by Røed Larsen and Weum. It is quite interesting that the housing market in Oslo has evolved from inefficient to efficient when comparing the last two decades. This paper does not provide an in-depth analysis of outside factors that may have contributed to these changes, as we leave that to further research. It gives, however, a solid conclusion of efficiency in the Oslo and Stavanger housing market for the relevant period. We demonstrate that the stock market consistently yields higher appreciation and higher volatility than both the housing markets in the period of 2002-2014, which is a contrast to the previous research. The housing markets appear to yield the highest risk-adjusted return.

[1] *Rød Larsen E., Weum S. 2008. Testing the efficiency of the Norwegian housing market. J. of Urban Econ. 64:510-516.*

1. Introduction

1.1 The Norwegian Housing Market

For the last two decades, the price growth in the Norwegian housing market has been substantial. As an economic consequence of the Norwegian banking crisis at the end of the 1980s, the house prices reached a bottom in 1992, before the development changed. By examining two different cities in Norway; *Oslo and Stavanger*, one clearly sees that the growth rate of Norwegian housing prices has increased dramatically. The recent period of 1993-2013 stands out as the price levels have been beyond any historical level. During 2005-2014, which includes the effect of the financial crisis on the respective cities, the price per square metre increased 80% in Oslo and 115% in Stavanger according to Statistics Norway *(Appendix, A.1.).* As illustrated in an analysis presented by chief economist *Harald Magnus Andreassen* in Swedbank, the house price growth in Norway after the financial crisis in 2007 has been unique. The Norwegian housing market quickly picked up approximately the same growth rate as before the crisis, while other countries had declining trends. However, after 2012, most countries appear to have an increasing growth rate in the housing prices *(Appendix, A.2.).* The development in the Norwegian housing market have received attention from Nobel laureates such as Robert J. Shiller[2] and Paul Krugman,[3] who voice their concern regarding a housing bubble in Norway.

From the statistics of Norwegian households wealth composition, it is evident Norwegians are inclined to invest in housing. Above 70% of the population between the ages of 35-74 have invested in housing, while only 21-31% have invested in regular bonds and stocks. The participation rate in funds is higher, around 40%, but still significantly lower than the share of people invested in housing. However, these numbers are from 2009, being in the wake of the financial crisis, and might have shifted when capital markets normalized *(Appendix, A.3.).* In a survey conducted by the Norwegian real estate agency *Garanti Eiendomsmegling*

[2] *Dagens Næringsliv, January 11, 2012, "Ekspert frykter norsk boligboble."*
[3] *Dagens Næringsliv, January 7, 2014, "Advarer mot norsk boligboble."*

in 2012,[4] 67% of people living in Norway consider housing to be the best long-term investment, 16% reported bank deposits and 5% said bonds and shares. This underlines housing as the favourable long-term investment by Norwegians, which is not surprising given the last two decades of house price appreciation. In Norway, approximately 84.4% of the population live in owned housing, which is above the European average of 70.1%.[5] Owning a house is considered the biggest asset for most individuals, and account for a significant part of their total budget. The majority of homebuyers consider the acquisition partly as an investment additionally to the desire of finding a good place to live. Owning a house is closely related to the "feeling of being rich," thus, the current and future value of the house is of major importance. The housing market consequently has a huge impact on individual's economic well-being and behaviour.

The overall economic situation at the beginning of 2016 is characterized by low interest rates on a worldwide level. In order to stabilize inflation and avert the risk of deflation, several of the major central banks in Europe, including the European Central Bank (ECB), the Danish National Bank (DNB), the Swedish Riskbank and the Swiss National Bank (SNB) have pushed the short-term policy rates into negative levels.[6] The rates are thereby far below the pre-financial crisis levels, and people are indeed searching for alternative investment opportunities as inflation seems to be eating up people's savings. Norway is no exception from this case as the Norwegian Central Bank is currently operating with a record low key policy rate of 0.5%, while the inflation remains close to the target of 2.5% *(Appendix, A.4.).*

The total gearing ratio of Norwegian households has increased significantly, and total debt of households was 210% of disposable income in 2013[7] *(Appendix, A.5.).* Borgersen and Hungnes (2009) state that mortgages constitute 90% of Norwegian household's total debt. The large exposure towards the housing market underlines the influence this market has on the economic development and sustainability. If housing prices suddenly fall, the reduced value of the house will constrain the household's ability to obtain new mortgages. This leads to ripple effects, such as

[4] *Norges Telegrambyrå, January 6, 2012, "Ny undersøkelse: Tror på boliginvesteringer."*
[5] *http://appsso.eurostat.ec.europa.eu/nui/show.do*
[6] *The World Bank, June, 2015, " Global Economic Prospect June 2015- Negative interest rates"*
[7] *Finanstilsynet, December 17, 2013, "Boliglånsundersøkelsen 2013"*

influence on private consumption and activity level in the economy. With households unable to borrow effectively, the impact of the monetary policy makers' most important tool, the key policy rate, may be reduced. As their goal is to sustain economic stability, it is crucial the tool keeps its impact.

1.2 Housing Prices are of Economic Importance

It is argued that development in the housing market influences wealth formation and inequalities. During real estate market booms, certain agents may face limitations in mobility due to financial barriers. Low-income families may struggle to find safe, quality accommodation and limitations in job- or education opportunities. There is an identified relationship between residential mobility and school performance, which concerns educators, policymakers and parents (Fowler-Finn, 2001; Holloway, 2000; Rothstein, 2000). In this case, learning disruption may occur, which affect the young generation's performance in school (Crowley 2003). This causes social challenges, highlighting this as an important topic in behavioural economics.

Housing debt constitutes a significant proportion of credit creation within an economy. Thus, volatility in the house prices may often imply financial instability in the economy as a whole. A clear example of this is the recent financial crisis where the large-scale default of subprime loans was the triggering factor. This led to the most severe recession since the Great Depression. Financial institutions and investors were effectively betting on increasing housing prices as they were lending to individuals with poor credit score, causing a speculative and inefficient market (Gorton 2009). Real estate market breakdowns have a more severe impact on the economy than stock market busts, underlining the important relationship between housing market and the overall economy. Helbling and Terrones (2003) reported that during 1970-2002, the output effects related to housing price breakdowns were twice as large and twice as persistent as those of equity price busts.

Understanding the housing market's behaviour and its degree of efficiency are of interest to other stakeholders such as *lending institutions*, *homeowners,* and *investors*. The household's act partially as investors in their house purchases, as they want to generate a highest possible return at lowest risk. It would greatly benefit them to understand the market's dynamics, expectations, and to know if entry timing can be optimized. Furthermore, the bankers and lending institutions are also stakeholders in the housing market, as they analyse potential risk and adjust after the guidelines from the government. Financial institutions take the dwelling as collateral, and a sudden drop in the value of this collateral will represent a potential risk. Sommervoll, Borgersen and Wennemo (2010) highlight housing and mortgage markets as shock originators that may potentially destabilize other parts of the economy. They argue that the three groups of agents in the housing market - *sellers, buyers, and mortgagees* - through their interaction increase the price volatility. Also, homes as mortgage collaterals increase the market instability, even when there is a consensus among the market forecasts between the agents.

1.3 Efficiency in the Norwegian Housing Market

The discussion of the Norwegian housing market and its exceptional growth has been a hot topic among interested parties for quite some time and is even more interesting given the current economic situation. There are several points of view trying to explain the past and future development of the Norwegian housing market. The discussion of a potential housing price bubble has been highlighted in the media many times. Jacobsen and Naug (2005) conclude in their research *"What drives house prices?"* that there is no evidence of Norwegian house prices being overvalued in relation to the fundamental value determined by interest rates, income, unemployment, and housing construction. Grytten (2009a) studies the historical prices of housing and the sales-to-rental price ratio in the Norwegian real estate market. He concludes that there exists a housing price bubble in Norway. This has been an ongoing debate, and it is commonly argued that the intensive growth rate in this market cannot sustain indefinitely.

So, can this drastic increase in prices over a sufficient period rationally be explained? Is it sustainable? At the essence of these questions lie the issues of forecastability, entry timing, inertia and time persistence. It is clear that the development in the Norwegian housing market is standing out as something unique and individuals have been receiving massive capital gains courtesy their investment in housing. At the core of understanding the development of housing prices is the theory of market efficiency. In an efficient market, the relevant information is reflected in the prices, and rational behaviour mainly explains the movements. A market is efficient if the prices follow a martingale process, a concept that we will return to. If the market is inefficient, it may lead to bubbles that can harm the economic stability.

There are several reasons to suspect that the Norwegian housing market is characterized as less efficient than other capital- or financial markets. Professional individuals find it difficult to take advantage of the profit opportunity due to transaction costs, high entry barriers, carrying costs, indivisibility, limited liquidity and tax considerations. Furthermore, it is difficult to short the housing market, and few financial derivatives exist to mitigate risk. Syz, Vanini and Salvi, (2008) investigate the exposure owner-occupied households have towards price fluctuations in the housing market and emphasize the lack of financial derivatives to reduce the housing risk. They propose a new type of mortgage that is linked to an underlying price index rather than an interest rate, in other words, the mortgage that is not an interest rate but a house price derivative.

If the Norwegian housing market has been inefficient over an extended period, investors may take advantage and outperform the market, thus earning a "free lunch". As the goal of investment is to generate a return on capital, it is highly relevant to analyse the level of market efficiency. As mentioned above, if the housing market is inefficient, it will have significant consequences for the overall economy, but it will also provide investment opportunities. On the other hand, if the housing market is considered efficient, it will to some extent contradict the policy maker's argument that housing auctions in Norway need more regulation and monitoring.

In the next section, we review relevant literature and commonly applied methods. The third section will provide an in-depth description of the data we utilize in our analysis and an explanation of the market features. In the fourth section, we explain the empirical technics applied, and the fifth section offers our results. The sixth section discusses the results and the limitations. Lastly, we provide our conclusion and the implication of our results.

2. Theory and Literature review

2.1 Efficiency Theory

The question of market efficiency is intriguing and has been subject to heavy scrutiny by researchers and professionals for a long time. At the core of understanding market efficiency, one examines the market dynamics, opportunities, and threats of possible inefficiency. Fama (1970) formulates the efficient market hypothesis and suggest that prices in an efficient market will reflect, at any given time, all available information. Meaning that prices are always fair and technical analysis cannot be used to predict and beat the market. Later, Fama (1991) elaborate the efficiency definition by explaining that prices reflect information to the point where marginal benefits of acting on information (excessive profits) do not exceed the marginal cost. Thus, efficient market follows a discrete-time stochastic process known as a martingale process.

The concept of a martingale process has been subject to extensive research by several papers, including Samuelson (1965, 1973), Fama (1970, 1991) and LeRoy (1989). A martingale process ensures a fair game as it implies that the best prediction of future prices is today's price. If price π follows a martingale process, then the best forecast of π_{t+1} at time t is π_t. If the house prices exhibit the properties of a martingale process, then π_{t+1} - π_t, i.e. the first difference, is purely white noise. On the other hand, if this is not the case and the process exhibit time structure, the stochastic process become $\pi_t = \lambda\pi_{t-1} + u_t$, where λ does not display the characteristics of unity. Thus, previous prices may be used to identify time structure that improves the forecast of future prices. In this paper, we investigate if the prices of housing in Oslo and Stavanger follow the martingale process or if they exhibit time structures, and thereby evaluate the degree of market efficiency.

2.2 House Price Indices

To investigate for time structures in the housing prices, we need to establish a house price index. Previously, different approaches have been established for this purpose, all with particular strengths and weaknesses. A basic method is reporting median changes in prices, as National Association of Realtors for instance provides. There are several weaknesses in this approach, which we carefully assess to highlight the difficulties in measuring house price development. For example, a disproportionate number of high-priced homes might be sold in one period, which will skew the median number up significantly, even though no property price appreciation occurred. Also, as real income is rising over time, the quality of new homes is likely to increase as well. Since the new homes become "existing" in the calculations, it will increase the median level, even if individual properties are not appreciating.

A different measurement approach is the Hedonic Pricing Method. When applying the Hedonic approach in real estate, the housing is decomposed into several characteristics. For example size, the number of bedrooms, distance to city centre, access to collective transportation are some of the factors considered. The price of the housing will be affected by these structural-, environmental- and neighbourhood characteristics. This approach goes back to the general economic price indexing of goods where quality changes over time, and was introduced by Court (1939), then further developed by Griliches (1961). This method was incorporated into real estate by Kain and Quigley (1970), refined and developed further by Rosen (1974) and Goodman (1978). Even though we do not use the hedonic approach when calculating our house price indices in this paper, we use a simple setup of the hedonic approach when calculating rental indices for both Oslo and Stavanger.

The last approach in calculating a house price index that we highlight is the Repeated Sales Method. Baily, Muth and Nourse (1963) introduce the approach referred to as BMN, which is a regression method for real estate price index construction. They solve the problem of estimating a price index for real estate, which is often caused by variation in quality among properties, by using repeated sales of the same objects at different points in time. Their method provides a house price index that produces estimates and standard errors by regressing, using ordinary least squares, the change of log price of each house on a set of dummy variables. In their repeated sales method, they argue that if the log price changes of individual houses are different from the citywide log price change because of an independent, identically distributed noise term, then by the Gauss-Markov theorem, their estimated index is the best linear unbiased estimate of the citywide log price.

Throughout our analysis, we apply a specific version of the Repeated Sales Method introduced by Case and Shiller (1989). The method is a modification of the *BNM* method, used for testing the efficiency hypothesis for single-family homes in Atlanta, Chicago, Dallas and San-Francisco/Oakland from 1970-1986. This construction method for house price indices was presented as the Weighted Repeated Sales *(WRS)* method. This method makes different assumptions about the behaviour of the error terms, as Case and Shiller (1989) argue that the errors are likely to be larger for repeated sales where the time intervals between the sales are greater. The weighted regression is down-weighting observations corresponding to large time intervals. Case and Shiller were not able to reject the efficiency hypothesis in any of the four cities. They do however suggest a trading rule that appears profitable, which is inconsistent with the theory of weak-form efficiency of the market. An in-depth explanation of the Case-Shiller method will be reviewed in our section of empirical techniques.

2.3 Previous Literature

Housing Market and the Stock Market

The real estate market in different geographic areas has been perceived as inefficient by several empirical researchers, confirming that investors have been able to earn abnormal returns. Papers like Wendt and Wong (1965), Coyne, Goulet and Piconni (1980) and Kaplan (1985) find that investment in real estate outperform other assets such as stocks and bonds, both on a risk-adjusted and not risk-adjusted basis. Barkham and Geltner´s (1996) use data on the housing market in Great Britain and apply the Case-Shiller repeated sales index. They analyse how value-relevant information affects the stock market and housing prices, and find that the timing of when the stock market reflects new economic information compared to when the information is fully incorporated into the housing prices leads to the conclusion that the UK housing market is inefficient. Kouwenberg and Zwinkels (2010) distinguish investors into two sub-categories, Fundamentalists and Chartists. The Fundamentalists expect the housing prices to revert to their fundamental value based on present value of rents, while the Chartists extrapolate past price trends and expect these trends to continue in the future. Historically, the proportion of each type of investors has been relative equal, but in the last two decades, the share of Chartists has increased substantially, which is a contributing argument of why housing prices may have moved above the fundamental value.

Arbitrage

An outcome of possible market inefficiencies and misalign pricing is arbitrage opportunities. Poterba (1984) introduced a housing market no-arbitrage condition stating that, in equilibrium, the user cost of accommodation should equal rental price level of a similar dwelling. The intuition is straightforward and appealing because it takes the future expectations into account by a single expected housing appreciation term. The condition mentioned above has been popular in recent studies to assess if house prices are misaligned in different countries and different cities e.g. Finicelli (2007), Girouard et al. (2006), Himmelberg et al. (2005) and McCarthy and Peach (2004). However, the practical complication in applying this condition is severe. Oikarinen (2010) studies 10 Finish cities in the period of 1995-2004. He recommends using the implied expected appreciation derived from the no-arbitrage condition. The implied house price growth will be the appreciation rate at which user cost equals rental cost. Factors such as risk premium and expected inflation need to be adjusted through the time interval. The paper claims that the maintenance cost as a fraction of housing prices are expected to be smaller in the major cities and downward trending in city-centres. Also, rental prices are expected to grow faster than maintenance costs and thus the gross price to rent ratio is likely to trend upwards. His analysis concludes no housing bubble, but rather that the high growth in house prices is an adjustment towards the no-arbitrage condition because prices fell to an abnormally low level during the deep recession in the early and mid-1990s.

Bubbles

Hosios and Pesando (1991) show that the housing market does not process information efficiently, suggesting the prices might rise above equilibrium levels, resulting in bubbles. There have been several studies conducted about the occurrences of bubbles resulting from inefficient markets. Jacobsen and Naug (2005) do not find evidence of bubbles or overpricing in the Norwegian housing market. Furthermore, they conclude that the housing prices respond quickly to adjustments in the interest rate. However, the discussion on housing bubbles is debatable among different geographical areas. Meese and Wallace (1994) conclude that they cannot rule out the presence of non-rational expectations and pricing in some counties in California in the period 1970-1988. Himmelberg et al. (2005) and Cameron et al. (2006), however, do not find support for bubbles in USA and UK.

Alternative Approaches

Rosenthal (2006) uses a hedonic approach with fixed-weight, quality-adjusted measures of the price for different vintage buildings, and compare these to new buildings. In the research, he concludes that the market for residential buildings in the UK housing market over the period of 1991-2001 is indeed efficient. Furthermore, he underlines that the inefficiency in the housing market claimed by Case and Shiller (1989) must reside in the market for the land itself. Hjalmarsson and Hjalmarsson (2009) investigate the efficiency in the housing market in Sweden by examining the relationship between the sales price and the present value of future monthly payments or rents. They find evidence of a systematic failure in pricing the dwellings correctly when considering the discounted future stream of rent payment. Holly, Pesaran, and Yamagata (2011) investigate the spatial and temporary diffusion in a dynamic system using the real house prices in the UK. They conclude that shocks in a dominant region, in their case London, spread to other areas with a delay. This indicates information inefficacy, as the market does not respond is lagging.

Norway

Røed Larsen and Weum (2008) also replicate the Case-Shiller methodology on the Oslo housing market in the time-interval of 1991-2002. As they can reject the null hypothesis of martingale along with the null hypothesis of efficiency, they conclude that the housing market in Oslo is inefficient in this time-interval. Furthermore, they show that the housing market consistently yields a higher return at lower risk than the stock market over the same sample period. Kallåk Anundsen and Røed Larsen (2016) test for micro efficiency in the Norwegian housing market using registered housing transactions from 2002 to 2014, and conclude that the market seems to be relatively micro efficient. That is, an excessively high or low sell price in one transaction is not repeated in the next transaction, and hence, if an investor pays more than expected he cannot anticipate a similar premium when reselling the unit. The market seems to be punishing overpay, and rewarding underpay in an efficient way. They also conclude that there is little scope for profitable arbitrage in the excess of the market return when they adjust for home improvements.

Macro efficiency

Several researchers have documented macro persistence in the housing market. Miles (2011) estimates a component GARCH[8] model to examine the persistence of the Office of Federal Housing Enterprise Oversight (OFHEO) and S&P/CS[9] home price indices, finding evidence of long memory in volatility. This indicates that the probability of significant losses is much higher than standard mean-variance analysis. Elder and Villupuram (2012) also find evidence of persistent long-memory in both the return and volatility of real estate indices, which violates the weak form efficiency. Macro predictability in the housing market is highly supported, and Glaeser et al. (2014) list predictability of house price index changes as a stylized fact about the housing market. Their model correctly predicts that price changes mean revert at a 5-year time horizon, which is also an important stylized fact about the housing market. Even though we do not find time persistence in our model, there is an indicator of persistence when considering a more long-term perspective (i.e. 5 years), but the lack of data limits us from providing a solid conclusion of this.

As summarized above, the topic of market efficiency is widely studied before. The extension to the housing market, whether it is efficient or not, have also been subject to scrutiny. Different approaches have been developed to investigate whether the efficiency hypothesis holds for the housing market. The empirical results offer no clear conclusion across geographical locations. The Case-Shiller method has been used in the Norwegian housing market, but the literature lacks an updated and expanded study using this approach, which we will provide in this paper.

3. Research Question

In our research paper we aim to analyse the market behaviour of the Norwegian housing market, and answer the following research question:

Is the housing market in Oslo and Stavanger efficient?

[8] Generalized Autoregressive Conditional Heteroscedasticity model.
[9] The Standard & Poor/Case-Shiller Home Price Indices.

Our approach is a replication of Erling Røed Larsen and Steffen Weums research in *"Testing the efficiency of the Norwegian housing market"* (2008) on a more recent time series. By this, we apply the method developed by Case and Shiller (1989) on a rich data set of house transactions in Oslo and Stavanger, testing the efficiency hypothesis. We utilize the time-persistence test on a repeated sales model by creating a house price index and returns to housing. We carry out the analysis and compare it to the previously concluded market inefficiency by Røed Larsen and Weum (2008). They also conclude: *"The housing market seems to deliver the most attractive combination of high return and low risk."* We will continue their research by using data from 2002 to 2014, and investigate if this perceived trend has continued for the last 13 years as well.

In extension, we analyse the risk and return in the stock market over the same relevant period. This gives us the opportunity to compare how the two different markets behave over the same time interval and is relevant as investors often face the trade-off between investing in the stock market versus investing in property. This research aims to conclude on which type of investment that generates the highest return compared to risk.

4. Data and Market Features

4.1 Housing Transactions - Sales

We obtained the data employed in this research from Eiendomsverdi AS and our supervisor Erling Røed Larsen. Eiendomsverdi is a company that has an overview and monitors the development of prices in the property market in Norway. Eiendomsverdi has built an extensive and unique database covering the Norwegian real estate market using public information, information directly from real estate agents, the different housing cooperatives, and real estate developers. Thus, the database provides information on every property and sales price of every transaction since 1990. Due to this extensive collaboration between market agents, the database is updated in real time[10].

[10] *Eiendomsverdi AS (https://eiendomsverdi.no)*

We argue that our sample of housing sales transactions gives a more accurate picture of the market dynamics than the OBOS sample used by Røed Larsen and Weum (2008)[11]. First, we observe the exact date of bid and acceptance instead of the judicial registration date, which eliminates the noise of systematic lagging and clustering of the reported sales. Another weakness of studying cooperatives, as done in the OBOS sample, is that people have option rights that might skew the selling price down, as it disregards the actual willingness to pay by agents. Our sample is also more heterogeneous, as we have a larger variation in housing stock, and hence it provides a more general illustration for the apartment market as a whole. A detailed description of our data follows in this section.

We study the cities of Oslo and Stavanger independently in our time interval, which runs from first quarter 2002 until the fourth quarter 2014. The dataset we examined contained 89.934 observations of apartment transactions in total, where the Oslo and Stavanger subsample amounted to 81.294 and 8.640 observations each initially. Some duplicates had to be removed[12].

We identify the apartments that have been sold exactly twice in the period, this leaves us with 28.096 observations in Oslo and 2.744 observations in Stavanger. This condition is partially set because we suspect apartments sold more than twice have certain characteristics that make it undesirable for the owner to keep ownership of the apartment. Apartments sold more than twice is also more likely to have changes in characteristics between the first and last sale. Using objects sold exactly twice also makes our results more comparable to Røed Larsen and Weum (2008). Since we are developing a repeated sales index, it is crucial that the apartments have kept similar characteristics. The dataset report renovation as a binary variable, if an object was renovated it is reported on a "yes/no" basis, the objects renovated between the two sales is removed. Certain objects with major changes in living area and the number of bedrooms are reported with "no" renovation in the dataset these observations have also been removed. This process removes 1.778 observation in Oslo and 242 in Stavanger. In addition, we observe certain large outliers where the square metre price has an abnormal development. This is likely due to recording errors or some other changes in characteristics and

[11] *OBOS: A Norwegian sales cooperative, which organizes housing cooperatives*
[12] *4 observations in Stavanger and 38 observations in Oslo were removed due to duplicates.*

accounts for 582 observations in Oslo and 82 in Stavanger, which are removed. Apartments that consist of more than five bedrooms are also disregarded as such apartments have housing collective characteristics. This is only relevant in Oslo and 36 observations are removed. Summarizing all these adjustments, we are left with an Oslo subsample consisting of 25.698 observations and Stavanger with 2.420. A general observation is that apartments in Oslo, particularly the Frogner area, show a higher degree of renovation and quick sales than apartments in Stavanger.

As each sale has to be assigned to a specific quarter to create a quarterly price index, we use the reported "actual sales date" on the observations. This entry is, however, missing in some observations, and we use "registration date" in these situations. However, 10 observations in Stavanger and 192 observations in Oslo are missing both entries and were removed from the samples. We use the reported sales price including common debt, as this is the accurate value of the property acquisition.

Furthermore, apartments built the same year it was sold the first time and the sales price was equal to ask price are removed as we suspect they are sold at a fixed price and not through a regular auction. Newly build apartments that are sold for the second time within a year at a profit we suspected to be bought below market value and then sold to take advantage of arbitrage and we remove these observations as well. Finally, we remove apartments that are sold twice in the same quarter. This gives us the final sample of 24.854 observations (12.427 individual apartments sold twice) in Oslo and 2.382 (1.191 individual apartments sold twice) in Stavanger *(Appendix, A.6.)*. We use the official CPI level reported by Statistics Norway to adjust for inflation[13]. Table 1 and 2 summarize statistics of the house price data used to create the house price indices in each city:

[13] *Statistic Norway, KPI (http://www.ssb.no/kpi)*

Table 1. Oslo House Price Data, Statistics of Sales Price

Oslo house price data		
Statistic	**2002q1**	**2014q4**
No. of observations	436	428
Minimum	661 901	1 454 527
5th percentile	885 000	2 050000
25th percentile	1 168 350	2 672 522
Median	1 556 800	3 550 000
Mean	1 778 220	4 071 015
75th percentile	2 011 655	4 900 000
95th percentile	3 490 000	7 800 000
Maximum	6 800 000	18 000 000
Standard deviation	844 768	1 986 573

Table 2. Oslo House Price Data, Hedonic Attributes Size and Number of Bedrooms

Oslo house price data - Hedonic characteristics		
Hedonic attribute	**Size**	**Number of bedrooms**
No. of observations	12 427	11 962
Minimum	15	1
5th percentile	32	1
25th percentile	50	1
Median	65	2
Mean	70	1.8
75th percentile	83	2
95th percentile	126	4
Maximum	302	5
Standard deviation	30	0.78

* 465 apartments are lacking reported bedrooms

Table 3. Stavanger House Price Data, Statistics of Sales Price

Stavanger house price data		
Statistic	**2002q1**	**2014q4**
No. of observations	13	42
Minimum	800 000	2 195 000
5th percentile	800 000	2 250 000
25th percentile	1 100 000	2 550 000
Median	1 343 000	2 800 000
Mean	1 411 615	3 069 727
75th percentile	1 510 000	3 290 000
95th percentile	2 175 000	5 050 000
Maximum	2 250 000	5 190 000
Standard deviation	429 291	764 903

Table 4. Stavanger House Price Data, Hedonic Attributes Size and Number of Bedrooms

Stavanger house price data - Hedonic characteristics		
Hedonic attribute	Size	Number of bedrooms
No. of observations	1 191	1 160
Minimum	19	1
5th percentile	32	1
25th percentile	50	1
Median	64	2
Mean	68	1.67
75th percentile	81	2
95th percentile	114	3
Maximum	209	5
Standard deviation	25	0.64

* 31 apartments are lacking reported bedrooms

4.2 Housing Transactions - Rent

We obtained data about the rental market in Oslo and Stavanger from Finn.no. The total data set consists of 101.567 observations, where 85.801 is Oslo and 15.766 is Stavanger, spanning from 2006 to 2014. Noting that the data lack enough observations pre 2008, we cut the sample, and the rental index is developed using data from 2008Q3 to 2014Q4. In regards to creating the rental index, we isolate the housing and leisure homes rents from the CPI and perform backward calculations. Thus we establish the rental index as a starting point of 100 in 2002Q1[14]. Furthermore, the observations were assigned to the month the rental contract was agreed, we converted the observation to a quarterly basis. In a similar fashion, the rent was reported on a monthly basis, and we simply convert this to a quarterly basis.

In the dataset, certain observations were lacking reported number of bedrooms, and these observations are taken out of the sample. Apartments with more than seven bedrooms were also removed because these apartments have characteristics of housing collectives or student homes. We also decide to remove the 2% highest and lowest square meter rental price to remove the large outliers. This data cleaning leaves us with 76.140 observations in Oslo and 14.906 observations in Stavanger,

[14] *This is a limitation in our dataset, and the Oslo and Stavanger rental index therefore move similarly from 2002-2009. This is because the reported CPI measure is not separable between Oslo and Stavanger. After 2009 we have the city-specific measures, which makes rational differences in the rental indices.*

which we use to create our rental indices. Table 5 and 6 summarize the rental data used to create the rental index in Oslo. The data is reported in quarterly rental price while the parenthesis represents rent per square meter quarterly. Table 7 and 8 reports correspondingly for Stavanger.

Table 5. Oslo Rental Data, Statistics of Quarterly Rental Price

Oslo rental data				
quarterly rent total *(rent per squaremeter quarterly)*				
Statistic	**2008q3**		**2014q4**	
No. of observations	110		634	
Minimum	17 000	*(112)*	18 000	*(118)*
5th percentile	18 000	*(132)*	26 700	*(154)*
25th percentile	25 500	*(176)*	33 000	*(200)*
Median	30 000	*(208)*	39 000	*(230)*
Mean	32 181	*(206)*	41 325	*(233)*
75th percentile	38 700	*(223)*	45 000	*(264)*
95th percentile	48 000	*(293)*	66 000	*(325)*
Maximum	75 000	*(350)*	138 000	*(371)*
Standard deviation	9800	*(45)*	31 291	*(49)*

Table 6. Oslo Rental Data, Hedonic Attributes Size and Number of Bedrooms

Oslo rental data		
Hedonic attribute	**Size**	**Number of bedrooms**
No. of observations	76 140	76 140
Minimum	10	1
5th percentile	30	1
25th percentile	45	1
Median	55	1
Mean	60	1.6
75th percentile	70	2
95th percentile	104	3
Maximum	405	6
Standard deviation	25	0.8

Table 7. Stavanger Rental Data, Statistics of Quarterly Rental Price

Stavanger rental data				
quarterly rent total *(rent per squaremeter quraterly)*				
Statistic	**2008q3**		**2014q4**	
No. of observations	48		118	
Minimum	13 500	*(92)*	15 000	*(100)*
5th percentile	18 000	*(100)*	25 500	*(133)*
25th percentile	27 000	*(143)*	31 500	*(169)*
Median	30 000	*(170)*	40 500	*(200)*
Mean	33 069	*(178)*	41 339	*(204)*
75th percentile	42 000	*(214)*	45 000	*(240)*
95th percentile	48 000	*(254)*	63 000	*(297)*
Maximum	48 000	*(273)*	90 000	*(331)*
Standard deviation	8937	*(46)*	12 670	*(50)*

Table 8. Stavanger Rental Data, Hedonic Attributes Size and Number of Bedrooms

Stavanger rental data		
Hedonic attribute	Size	Number of bedrooms
No. of observations	14 906	14 906
Minimum	17	1
5th percentile	37	1
25th percentile	50	1
Median	65	2
Mean	67	1.7
75th percentile	80	2
95th percentile	106	3
Maximum	265	6
Standard deviation	22	0.71

In order to evaluate the excess return, we used an estimation of average mortgage interest rate from Statistics Norway, which displayed a lowering in the interest rates to the households over the relevant period that was taken into account on a quarterly basis[15]. The interest payment tax shield was 28% until the end of 2014, and then lowered to 27%.

5. Empirical techniques

5.1 Visualisation of Data

In our research, we limit the sample of dwellings to apartments. The decision of not looking at all dwellings, but only apartments, is taken on the basis that apartments tend to keep more similar characteristics across time, i.e. same size, the same number of rooms and same quality compared to single family homes. In other words, the apartments sample is more homogeneous, and thereby more comparable. A large proportion of dwellings in Oslo and Stavanger are apartments, and this specification will allow us to create a rental price index in each of the cities, which will become advantageous in our analysis of excess return. This represents a distinction from the Case-Shiller research, which studied single-family homes, and Røed Larsen and Weum, who studied dwellings transactions reported by OBOS. Our research will comprise of apartments sold at least and at most twice. We will

[15] *A monthly reported average of mortgage suppliers and banks lending rate in-cooperated quarterly.*

also exclude transactions not made at an arm’s length because these do not necessarily reflect market value[16].

Through our analysis, we work with panel data on our populations, which are apartments in Oslo and Stavanger, running from first quarter 2002 to fourth quarter 2014. Idealizing the population data is useful to illustrate our purpose and aim, and will highlight the strength and weaknesses of our dataset. With the perfect data, we would know the prices of all the dwellings from 1 to D in every period. The price of the i’th dwelling at time t is denoted as $\pi_{i,t}$, and we can illustrate population data in this matter as a “Big Matrix”:

0	1	2	3	4	…	N
$\pi_{1,0}$	$\pi_{1,1}$	$\pi_{1,2}$	$\pi_{1,3}$	$\pi_{1,4}$	$\cdots$	$\pi_{1,N}$
$\pi_{2,0}$	$\pi_{2,1}$	$\pi_{2,2}$	$\pi_{2,3}$	$\pi_{2,4}$	$\cdots$	$\pi_{2,N}$
$\pi_{3,0}$	$\pi_{3,1}$	$\pi_{3,2}$	$\pi_{3,3}$	$\pi_{3,4}$	$\cdots$	$\pi_{3,N}$
$\pi_{4,0}$	$\pi_{4,1}$	$\pi_{4,2}$	$\pi_{4,3}$	$\pi_{4,4}$	$\cdots$	$\pi_{4,N}$
$\pi_{5,0}$	$\pi_{5,1}$	$\pi_{5,2}$	$\pi_{5,3}$	$\pi_{5,4}$	$\cdots$	$\pi_{5,N}$
$\pi_{6,0}$	$\pi_{6,1}$	$\pi_{6,2}$	$\pi_{6,3}$	$\pi_{6,4}$	$\cdots$	$\pi_{6,N}$
$\pi_{7,0}$	$\pi_{7,1}$	$\pi_{7,2}$	$\pi_{7,3}$	$\pi_{7,4}$	$\cdots$	$\pi_{7,N}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\pi_{D,0}$	$\pi_{D,1}$	$\pi_{D,2}$	$\pi_{D,3}$	$\pi_{D,4}$	$\cdots$	$\pi_{D,N}$

However, we are not emphasising the absolute prices, but rather the growth rate in the prices and thereby define the growth rates as:

$$g_{it}, = \pi_{i,t}/\pi_{i,t-1}$$

We adjust our matrix for this purpose:

1	2	3	4	5		N
$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,1}$	$g_{1,1}$	$\cdots$	$g_{1,N}$
$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,1}$	$g_{2,1}$	$\cdots$	$g_{2,N}$
$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,1}$	$g_{3,1}$	$\cdots$	$g_{3,N}$
$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,1}$	$g_{4,1}$	$\cdots$	$g_{4,N}$
$g_{5,1}$	$g_{5,2}$	$g_{5,3}$	$g_{5,1}$	$g_{5,1}$	$\cdots$	$g_{5,N}$
$g_{6,1}$	$g_{6,2}$	$g_{6,3}$	$g_{6,1}$	$g_{6,1}$	$\cdots$	$g_{6,N}$
$g_{7,1}$	$g_{7,2}$	$g_{7,3}$	$g_{7,1}$	$g_{7,1}$	$\cdots$	$g_{7,N}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$g_{D,1}$	$g_{D,2}$	$g_{D,3}$	$g_{D,4}$	$g_{D,5}$	$\cdots$	$g_{D,N}$

[16] *See Appendix A.6.1 for a detailed description of the data cleaning.*

It is common in the literature to treat the growth rate as a population parameter for the entire population, not for the individual dwelling. We believe emphasising this distinction is useful because we recognize that there exists a distribution of growth rates in the population.

Our actual data sample is not equal to the ideal data. New dwellings are built, and older dwellings are demolished. Therefore, the numbers of houses are dependent on t. The sample will be reduced even further as we are not able to observe the price of the dwelling in all the period the dwelling exists. Rather, we observe the price only in the period that the dwelling is actually sold. At this stage, we still adjust our sample, as we are not looking at all dwellings but only apartments, as mention above. We also eliminate all observations where the apartment is sold less than or more than twice in the interval of N=52. Our sample will therefore lack several objects in the respective cities when we conduct our analysis. The final image of our dataset is illustrated as the following:[17]

0	1	2	3	4	…	N
$\pi_{1,0}$			$\pi_{1,3}$			
			$\pi_{5,3}$			$\pi_{5,N}$
	$\pi_{7,1}$			$\pi_{7,4}$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$

With this visualisation of the data we observe obvious challenges; the sales happened at infrequent intervals, and we lack data on a significant part of the dwellings population. However, we argue that cleaning the dataset this way gives us more comparable and consistent characteristics of the objects, which allows us to extract a representable price appreciation or depreciation in each period[18].

[17] *This visualisation illustrates the general structure of our dataset.*
[18] *We adjust for several differences in characteristics, in order to get an accurate picture of price development. See section 4. Data and Market features*

5.2 House Price Index - Methodology

When testing for housing market efficiency, we replicate the approach by Røed Larsen and Weum (2008). Equivalent to their research, we apply a time-structure test on a repeated sales house price index. The house price index construction gives a reflection of the average change in market prices for constant quality apartments over the relevant period. In this section, we will carefully review all the steps in constructing the repeated sales house price index that we use.

5.2.1 Estimating the Weighted Repeated Sales Indices

The first step in testing the efficiency of the housing markets in Oslo and Stavanger is the construction of a house price index in each city. We are using the Weighted Repeated Sales *(WRS)* method, which is a modified version of the Bailey, Muth and Nourse *(BNM)* method when creating the house price indices. The motivation for the *WRS* method is the assumption that the log price π_{it} of the i'th house at time t is:

$$\pi_{it} = C_t + H_{it} + N_{it}$$

In this equation,

- C_t represents the log of the city-wide level of house prices at time t
- H_{it} is a random walk term that represents the drift in house prices over time (ΔH_{it} has zero mean and constant variance σ_h^2, H_{it} is uncorrelated with C_t and N_{it})
- N_{it} represents a sale specific, serially uncorrelated random error term, with zero mean and constant variance σ_N^2 for all i.

The ultimate goal with the *WRS* method is to estimate the movement in C, i.e. the citywide level of house prices.

5.2.2 Three-step Weighted Generalized Least Squares Procedure

Furthermore, we apply a three-stage weighted least squares regression on the repeated sales object, i.e. the apartments. As we replicate the Case-Shiller setup, the following stages are implemented:

1. In the first stage, the *BNM* method is followed exactly to calculate a vector of the regression residuals. In logarithmic form we get the following equation:

$$\pi_{it} - \pi_{is} = \hat{\varsigma}_1 D_{i1} + \hat{\varsigma}_2 D_{i2} + \cdots + \hat{\varsigma}_{52} D_{i52} + \varepsilon_{it}$$

$$Where\ i \in I: t, s, \in \{1, \dots, 52\}, D_{it} \in \{-1,0,1\}$$

- π_{is} is the logarithm first sale price of object i in period s
- π_{it} is the logarithm second sale price of object i in period t, thus t>s
- D is a dummy variable that takes the value -1 in the first period the object was sold and 1 in the second period it was sold (if the object was sold in the first period (s=1) then the dummy variable takes the value 0, thus D_{i1}= 0 always)
- The $\hat{\varsigma}$'s gives us estimated parameters, capturing the rate of house price appreciation
- ε_{it} is the error term

The *BNM* method assumes a constant variance in the error term across apartments i.e. homoscedasticity. The second stage addresses this.

2. In the second stage, Case and Shiller argue that treating the error terms as heteroscedastic is more realistic, as the variance in this term seems to increase with time. The changes in value across time might occur from factors such as random differences in maintenance and changes in neighborhood attractiveness. As justified in the theory section, the errors in the regression are likely to be larger for apartments where the time interval between sales is larger. Thus, the squared residuals in the first step regression are to be regressed on a constant, and the time interval between the first and second sale is represented by the following:

$$\hat{y}_i^2 = \hat{\alpha} + \hat{\beta} X_i + \varphi_i, \quad \rightarrow \hat{\sigma}_i = \sqrt{\hat{y}_i^2}, \quad i \in I$$

- X_i is a counting variable that denotes the time interval between the first and second sale
- $\hat{\alpha}$ and $\hat{\beta}$ are the parameters relates to the squared residuals of the counting variable.
- φ is the standard zero mean and constant variance noise term
- $\hat{\sigma}$ is the inverse weight that is assigned to the observation, a large $\hat{\sigma}$ means large estimated variance

The effect of this weighting will be to reduce the weight of the observations where the time intervals are larger.

3. In the third stage, we use the inverse weights as calculated in the second step for the corresponding observations in the first step. With these combinations we get:

$$\frac{\pi_{it} - \pi_{is}}{\hat{\sigma}_i} = \frac{\hat{\varsigma}_1 D_{i1}}{\hat{\sigma}_i} + \frac{\hat{\varsigma}_2 D_{i2}}{\hat{\sigma}_i} + \cdots + \frac{\hat{\varsigma}_N D_{iN}}{\hat{\sigma}_i} + \frac{\varepsilon_{it}}{\hat{\sigma}_i}$$
$$Where\ i \in I: t, s, \in \{1, \dots, N\}, D_{it} \in \{-1,0,1\}$$

This is the resulting Feasible General Least Square estimation. In this step, we estimate better coefficients, $\hat{\varsigma}_t$, for the price appreciation index. In our data, we use this weighting four times, as we see that the parameters converge towards a finite number.

5.2.3 Noise in the Error Term

These results lead to the *WRS* index in each city, which is the log price index. It is however not valid to only create one *WRS* index as it will be biased. The reason for this is that the same noise from each sale may occur on both sides of the time structure. The reasoning behind this is illustrated as the following example: *Consider an apartment, A, first sold in period 0, then sold the second time in period 1 (s=0, t=1) and an apartment, B, first sold in period 0 and the second time in period 2 (s=0, t=2). Using the WRS index in period 1 we get:*

$$\pi_{A1} - \pi_{A0} = C_1 - C_0 + H_{A1} - H_{A0} + N_{A1} - N_{A0}$$

In period 2 we get:

$$(\pi_{B2} - \pi_{B0}) - (\pi_{A1} - \pi_{A0}) =$$
$$(C_2 - C_0) - (C_1 - C_0) + (H_{B2} - H_{B0}) - (H_{A1} - H_{A0}) + (N_{B2} - N_{B0}) - (N_{A1} - N_{A0})$$

Rearranging gives:

$$(\pi_{B2} - \pi_{B0}) - (\pi_{A1} - \pi_{A0}) =$$
$$C_2 - C_1 + H_{B2} - H_{B0} + N_{B2} - N_{B0} - (H_{A1} - H_{A0} + N_{A1} - N_{A0})$$

We see that the common terms appear with opposite sign. Thus we get a negative correlation between the index change from 0 to 1 and from 1 to 2. Case and Shiller (1989) also highlights the possibility of serial correlation, which we illustrate with a similar scenario: *Assume that apartment X is sold in period 1 and 3, apartment Y is sold in period 0 and 2, and apartment Z is sold in period 0 and 3. Then estimated changes in the WRS would be:*

In period 1:

$$(\pi_{Z3} - \pi_{Z0}) - (\pi_{X3} - \pi_{X1})$$

And in period 3:

$$(\pi_{Z3} - \pi_{Z0}) - (\pi_{Y2} - \pi_{Y0})$$

In this case, we see apartment Z appear with the same sign in both expressions and is consequently positively correlated in the model, while the apartments X and Y will be independent to specific shocks.

5.2.4 Dealing with the Estimation Error

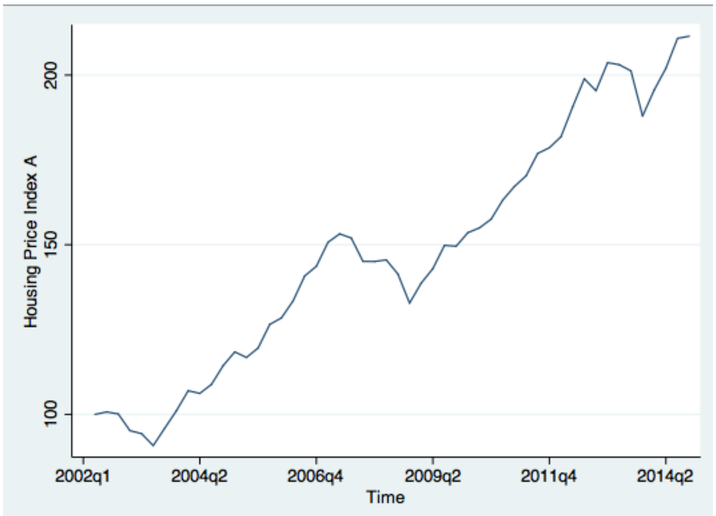
In order to deal with this estimation error, we divide the original sample into two subsamples, estimating two separate *WRS* indices[19]. The apartments are randomly divided into two samples, A and B, each containing half of the original sample[20]. The log price indices, WRS_A and WRS_B, are estimated for each subsample. Adjusting for the growth in general prices, we create the real log house price index, *W*:

$$W_j(t) = WRS_j(t) - \log(CPI\ (t)), \textit{where } j \in \{A, B\},\ t\ \in \{2002q1 - 2014q4\}$$

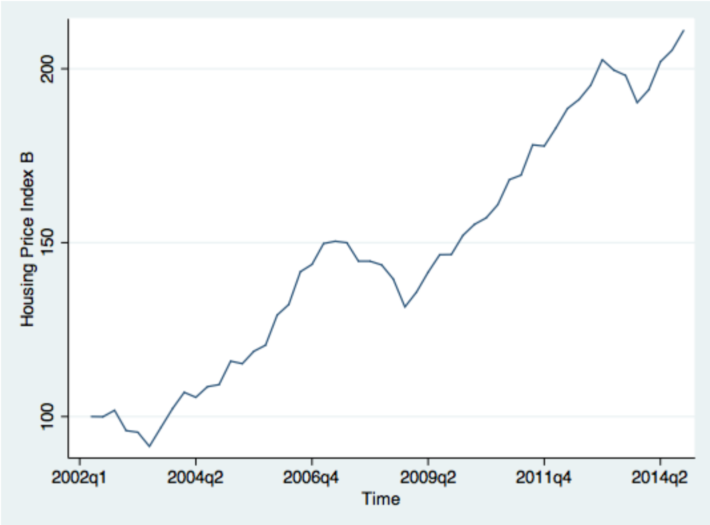
The *CPI(t)* is simply the general consumer price index, originally monthly data that we convert to quarterly. The estimated house price indices are graphed below;

[19] *This procedure is done in each city, hence we create four indices.*
[20] *STATA does this for us. However, checking for balance, we had to do the operation several times in each city before we got approximately randomly divided samples. As Stavanger is a smaller sample, the division of this is less balanced than Oslo, which affects the validity of our findings.*

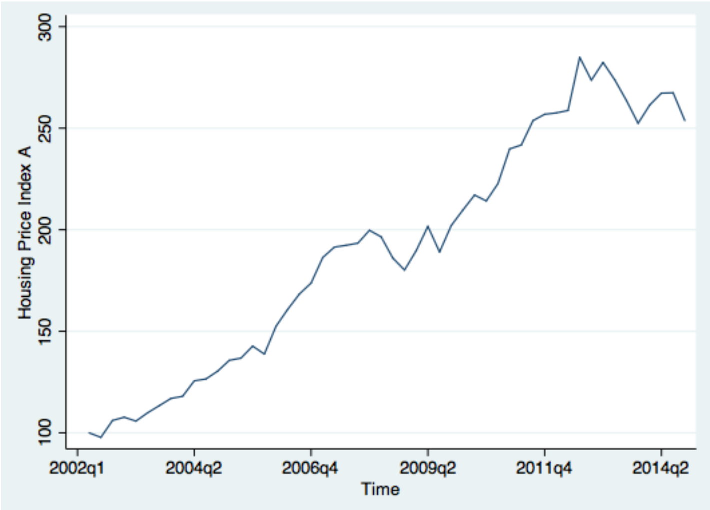
Graph 1. House Price Index A Oslo



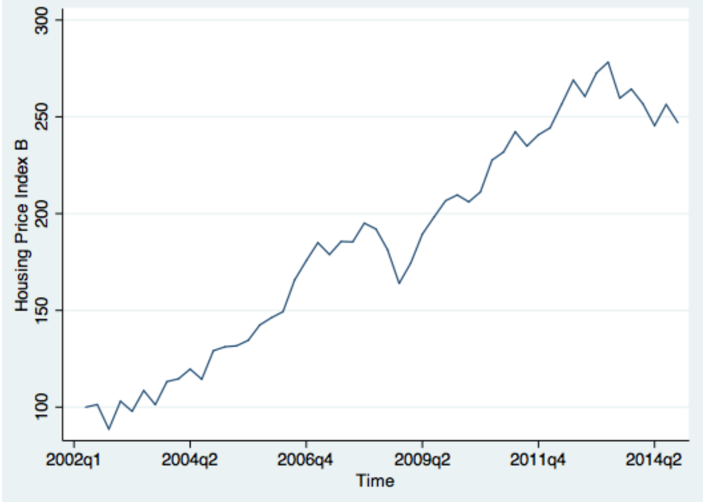
Graph 2. House Price Index B Oslo



Graph 3. House Price Index A Stavanger



Graph 4. House Price Index B Stavanger



5.3 Testing the Efficiency hypothesis

As we now have our house price indices, we continue to construct a test where the difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. As the two indices reflect the same development, we can test for random walks using the following expression[21]:

$$W_j(t) - W_j(t-4) = \hat{\beta}_0 + \hat{\beta}_1(W_k(t-L) - W_k(t-L-4)) + u(t)$$
$$Where\ j,k \in \{A,B\},\ \ j \neq k$$

It is important to empathize that there is still noise on both sides of the equation here, but since the different indices contain different apartments, the noise on each side will not be correlated. Hence, there are no longer systematic trends in the error estimates.

To avoid seasonal effects, we use quarterly data with a four-quarter lag. L is a denotation of a lagged variable and takes the value of zero in a case where both indices are supposed to pick up the same price development in the same period. If indices are measured perfectly, they should have a 0 intercept and a slope of 1. Due to the errors-in-variable problem a slight deviation is expected. By using L=4, we

[21] *Intuition and description of the findings are summarized in section 6.1.*

get a 4-quarter lag and regress the real log price change in one subsample on the lagged real log price change of the previous year in the other subsample.

This is the efficiency test; if there is a time structure, it violates the criteria for information efficiency. If the slope coefficient is statistically significant, we can reject our hypothesis of weak form of efficiency in the housing market.

5.4 Excessive Return

A possible time structure in apartment prices is further investigated by using Case-Shiller´s designed formula for *excessive return*. In this case, we implement factors such as interest rates, tax shield on interest payments and housing rents to calculate an excess return time series on apartment investments.

$$\boldsymbol{ExcessReturn = Growth\ in\ Capital + Avoided\ Rent - Interestpayments\ net\ of\ Tax\ Detuction}$$

$$E * R_j(t) = \left\{\frac{WRS_j(t+4)}{WRS_k(t)} - 1\right\} + C_j * \left(\frac{\left(\frac{\{R_t + \cdots + R_{t+3}\}}{4}\right)}{WRS_j(t)}\right) - \frac{(1-\tau) * r_t}{100}$$

- R_t refers to a rental index in the specific city at time t
- r_t is the mortgage interest rate at time t
- τ is the share of interest payments that is tax deductible

The excess return, $ER_j(t)$, comprise of three elements. The first element is simply the growth in the capital, appreciation or depreciation. The second element is implicit rent, or more specific; the imputed rent. The last element in the equation assesses the interest payments net of tax deduction. The concept behind the second term, imputed rent, is straightforward. If an inhabitant purchases an apartment at price π she would not need to pay rent. The value of the imputed rent is the value the inhabitant would have to pay if she kept renting the apartment, i.e. the opportunity cost. Furthermore, we say that E is the rent at purchasing point i.e. t=1. As rent follows the rental index, the first four quarters time structure are $E * R_1$, $E * R_2$, $E * R_3$ and $E * R_4$. Since R_t is the rental index, the annual imputed rent becomes $E * R_1 + E * R_2 + E * R_3 + E * R_4 \rightarrow E * (R_1 + R_2 + R_3 + R_4)$ and the quarterly average of this is $E * (R_1 + R_2 + R_3 + R_4)/4$. The imputed rent needs to be calculated as a part of the purchasing price, so the above expression becomes:

$$\{E * (R_1 + R_2 + R_3 + R_4)/4\}/\pi$$

In the next period (t=2), an updated rental index and the updated house price development must be included. The house price index, *WRS*, is included by multiplying the apartment value with the house price index. In t=2 the expression become:

$$\{E * (R_2 + R_3 + R_4 + R_5)/4\}/(\pi * WRS_2)$$

Furthermore, we can rewrite the expression on general form:

$$\frac{C(R_t + \cdots + R_{t+3})}{WRS_j}, \qquad Where\ C\ is\ the\ Case - Shiller\ constant, C = \frac{E}{4\pi}$$

Lastly, we standardize the average dividend-price ratio for the number of quarters to 0.03. Thus we get:

$$\frac{\left(\frac{1}{N}\right)\sum_t^N C\,(R_t + \cdots + R_{t+3})}{WRS_t} = 0.03$$

In our research, we tried out several different standardizations. Initially, the Case-Shiller constant is set to 0.05 as *"the best representation of average dividend-price ratio."* However, we found it rational to slightly adjust this down to 0.03, as house prices have increased significantly more than renting in our period compare to Case-Shillers time (1984)[22]. A basic illustration of this is that our rental index in Oslo takes the value of 151 in quarter 52, and the Stavanger rental index ends up in 149 in the same quarter. Further, we observe that our house price indices in Oslo ends up in 213 in the same quarter, and the house price indices in Stavanger end up in approximately 253. Clearly, the housing prices have increased more relatively to

[22] *We tried out 0.05, 0.04 and 0.03, calculated the effect on the constant C, and observed that the implementation of these standardizations in our excess return did not change the findings.*

rental prices, which is natural considering the low long-term interest rates observed in this period. As the tendency of low risk-free rates seems to be a lasting trend, we argue that this adjustment of the Case-Shiller constant is likely to be sustainable in the future as well.

Now that the implicit rent is converted to the standardized form on a dividend-based form, the excess return formula can easily be applied: The implicit rent is added to the capital gain, and we subtract the interest payments net of tax deduction. The illustration on regression form is the following;

$$ER_j(t) = \hat{\beta}_0 + \hat{\beta}_1 ER_k(t-L) + u_t$$

5.4.1 Constructing the Rental Index in Each City (R_t)

When constructing a rental index for Oslo and Stavanger, we use a simple hedonic-pricing setup. Ideally, we would have adjusted for several characteristics in the different rent-objectives such as view, geographic area, access to collective transportation, building year and so on. Due to limitations in the dataset, we only use primary characteristics such as the size of the apartment and number of bedrooms to withdraw an approximate price change over time in each city.

The rental index is created by using the following log-log specification:

$$ln(P_{it}) = \hat{\alpha} + \hat{\beta} ln(S_{it}) + \sum_{\tau=2}^{6} \hat{\phi}_\tau D_\tau + \sum_{t=2}^{24} \hat{\theta}_t T_t + v_{it}$$

In this equation, the logarithm of the reported quarterly rental price, P_{it}, is regressed on the logarithm of the size, S_{it}, i.e. the square meters of the rental object. The transaction date is the time when the apartment actually got rented, which is captured by the dummy variable T, and in the first period T=0 by default. $\hat{\theta}$ is the calculated time coefficients. The period runs from t=2 to t=24, and we observe the movement over six years, using quarterly data. D_τ represent a dummy variable that adjusts for number of bedrooms, and apartments with one bedroom default[23]. $\hat{\phi}$ is the calculated coefficients that measure the price estimate based on the number of bedrooms. The error term v_{it} is assumed to be a nicely behaved mean-zero stochastic variable.

The resulting rental price index R_t is the price increase compared to the price in period t=1 (*Appendix B.2.*), i.e:

$$Rt = \frac{\exp(\ln(P_{is}))}{\exp(\ln(P_{i1}))} = e^{\alpha-\alpha+\ldots+\hat{\theta}_s*1-0} = e^{\hat{\theta}_s}$$

5.4.2 The Capital Asset Pricing Model and Sharpe Ratios

In order to investigate the risk-return relationship in the context of housing asset pricing, we use the original Capital Asset Pricing Model *(CAPM).* This model builds on the work of Harry Makowitz concerning diversification and modern portfolio theory. The model was initially introduced by Treynor (1961, 1962), Sharpe (1964), Lintner (1965a,b) and Mossin (1966) independently. CAPM describe the relationship between risk and expected return for an asset. According to the CAPM, investors should be compensated in two ways; time value of money and the systematic risk. The time value of money is represented by the risk-free interest rate, R_f. The systematic risk is represented by the beta, β_i, and calculates the amount of compensation the investor requires for taking additional risk.

$$E(R_i) = R_f + \hat{\beta}_i(E(R_m) - R_f)$$

- $E(R_i)$ is the expected return on the capital asset
- $\hat{\beta}_i$ is the sensitivity of the expected excess returns to the expected excess market returns, or also $\hat{\beta}_i = \frac{Cov\ (Ri,Rm)}{Var\ (Rm)}$
- $E(\ R_m)$is the expected return of the market and hence, the $E(R_m) - R_f$ is the market premium
- $E(R_i) - R_f$ is the risk premium

[23] *D2=1 for apartment with two bedrooms, D3=1 three bedrooms, D4=1 four bedrooms, D5=1 five bedrooms, D6=1 six bedrooms*

The CAPM states that the expected return of an asset (or a portfolio) should equal the risk-free return plus a systematic risk premium. In other words, if the expected return does not equal or exceed the required return the investment should not be undertaken. The security market line *(SML)* plots the results, where the x-axis represents the systematic risk, and the y-axis represents the expected return. The slope of the SML determines the market risk premium.

Due to the elimination of idiosyncratic risk (unsystematic risk) in diversified portfolios, CAPM suggests that there should be no risk premium for idiosyncratic risk in investment assets. Housing assets differ from other financial assets in respect to their twofold purpose as an investment object and as a place to live i.e. for consumption. The housing market also has considerably higher transaction costs, carrying costs, higher liquidity risk and economic constraints on holding a diversified housing investment portfolio. Therefore, it is important to emphasize that idiosyncratic risk plays a significant role in returns to housing investments. Studies have shown that this idiosyncratic risk is positive and significantly priced in certain markets. Miller and Pandher (2006) find evidence for this in their study of the U.S. metropolitan housing market.

Sharpe (1966) developed the Sharpe ratio as a measurement of reward to variability. The Sharpe ratio is a tool to scrutinise the performance of an investment by incorporating risk. This ratio is a measure of the risk premium per unit of deviation in an investment. In this paper, we utilize ex-post Sharpe ratios because the return and risk we investigate are realized.

$$\hat{S}_i = \frac{R_i - R_f}{\hat{\sigma}_i}$$

- $\hat{S}_i$ is the resulting Sharpe ratio
- R_i is the realized returns of the relevant investment
- R_f is the risk free-rate of return
- $R_i - R_f$ is the excess return of an investment
- $\hat{\sigma}_i$ is the standard deviation of return levels measured at the end of each quarter

6. Empirical Results

6.1 Testing the Efficiency Hypothesis on the House Price Index

When testing the efficiency in each city, we start by regressing the changes of the real log house price index from one half of the sample onto changes in real log house price index from the other half of the sample without lag (i.e. L=0). This is done as a diagnostic test where index A is regressed on index B and vice versa, as both samples should reflect the same development, and thereby have a zero intercept and slope coefficient of one. From Table 9 we note intercept and slope coefficient estimates of (0.0016, 0.9913) and (0.0018, 0.9516) in Oslo. The corresponding adjusted R^2 is 0.9421. This is close to the expected values, and the test concludes that both samples indeed reflect the same price development. We have considerably fewer observations in Stavanger, and hence the test is a bit more ambiguous with values of (0.0234, 0.6979), (0.01499, 0.9849) and a corresponding adjusted R^2 of 0.6805 (See Table 10). It is noted that the validity of the test is lower in Stavanger than in Oslo, as expected due to sample size. We examined several random splits, but where not able to get a more balanced sample.

Secure that the indices in Oslo, and to a moderate extent in Stavanger, report the same price development we proceed by introducing 4-quarter lags (i.e. L=4). This gives the actual hypothesis test, which will expose potential time structure. We observe a statistically significant intercept of 0.0741 and 0.0706 in Oslo. However, the slope coefficient, $\hat{\beta}_1$, is not statistically significant in any of the tests. The corresponding adjusted R^2 is low with values -0.0161 and -0.0211[24], which clearly indicates that we can not predict future price trend deviation by looking at past price trend deviation. Hence, the price development of the previous period does not help us predict the price development of the next period due to statistical insignificant slope coefficient. On average, we observe a slightly above 7% yearly appreciation in the Oslo housing market in the relevant period.

[24] $Adjusted\ R^2 = 1 - \frac{(1-R^2)(N-1)}{N-p-1}$, *where* R^2*=0.0075 and 0.0027, N=44 and p=1* $\Rightarrow$ *Adj.*R^2*=-0.0161 and -0.0211*

The same intuition holds for Stavanger, with statistically significant intercepts of 0.0663 and 0.0713, but not statistically significant slope coefficients. The corresponding adjusted R^2 is -0.0068 and -0.0186. Hence, we observe a yearly appreciation slightly below 7% in the Stavanger housing market over the relevant period. Furthermore, the price development of the previous period does not help us forecast the price of the next period due to the statistically insignificant slope coefficient. These findings differ from the previous conclusion of inefficiency by Røed Larsen and Weum (2008), which we will return to in the next section of comparative statistics. We conclude that there is no time structure in any of the cities, and hence the markets are efficient.

Table 9. Efficiency Test Oslo, Housing Price Indices

$W_j(t) - W_j(t-4) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-4)) + u(t)$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where T ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	0,0016332	(-0,0050329, 0,0082993)	0,0018175	(-0,0047088, 0,0083438)
t-value	*0,49*		*0,56*	
Slope, B1	0,9912965	(0,9191838, 1,063409)	0,9516072	(0,8823818, 1,020833)
t-value	*27,67*		*27,67*	
R²	0.9433		0.9433	
R² Adjusted	0,9421		0,9421	
L=4				
Intercept, B0	0,0740767	(0,0474679, 0,1006855)	0,0706134	(0,043321, 0,0979058)
t-value	*5,62*		*5,22*	
Slope, B1	-0,0771649	(-0,3534518, 0,199122)	-0,0469901	(-0,3301082, 0,231281)
t-value	*-0,56*		*-0,33*	
R²	0.0075		0.0027	
R² Adjusted	-0,0161		-0,0211	

Table 10. Efficiency Test Stavanger, Housing Price Indices

$W_j(t) - W_j(t-4) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-4)) + u(t)$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	0,0233691	(0.0070131, 0,397251)	0.014989	(-0,0196012, 0,022599)
t-value	*2.88*		*0.14*	
Slope, B1	0,697889	(0,5581933, 0,8375846)	0.9848791	(0.787737, 1.182021)
t-value	*10,06*		*10.06*	
R²	0.6873		0.6873	
R² Adjusted	0.6805		0.6805	
L=4				
Intercept, B0	0,066338	(0,03003, 0,102646)	0,0713163	(0,0267673, 0,1158653)
t-value	*3,69*		*3,23*	
Slope, B1	0,1241283	(-0,1733656, 0,4216221)	0,0917226	(-0,3092602, 0,4927053)
t-value	*0,84*		*0,46*	
R²	0.0166		0.005	
R² Adjusted	-0,0068		-0,0186	

6.2 Comparison with Case and Shiller (1989), and Røed Larsen and Weum (2008)

Table 11 summarize our comparing of statistics in the efficiency analysis to Case and Shiller (1989), and Røed Larsen and Weum (2008). We then compare the real house price indices. Notice that our conclusion for Oslo and Stavanger on recent time series is similar to Case and Shiller's analysis of Dallas and Atlanta.

Table 11. Comparing Statistics

Table 6. Comparing statistics

Research	*Market*	*Time period*	*Time structure*	*Regression (t-value)* $\Delta Wt = a + \mathcal{B}\Delta Wt - 1$	*Adjusted* R^2
Case and Shiller (1989)					
	San Francisco	1972q1 - 1986q3	Yes	0.021(1.2) + 0.43(2.5)△ W_{t-1}	0,22
	Dallas	1972q1 - 1986q2	No	0.012(0.9) + 0.312(1.5)△ W_{t-1}	0,046
	Chicago	1972q1 - 1986q2	Yes	-0.0000(-0.0) + 0.502(2.2)△ W_{t-1}	0,23
	Atlanta	1972q1 - 1986q2	No	-0.004(-0.4) + 0.191(1.1)△ W_{t-1}	0,046
Røed Larsen and Weum (2007)					
	Oslo (1)	1991q3 - 2002q4	Yes	0.171(9.3) - 0.332(-3.0)△ W_{t-1}	0,179
	Oslo (2)	1991q3 - 2002q4	Yes	0.165(9.6) - 0.275(-2.79)△ W_{t-1}	0,154
Grønstad and Graarud (2016)					
	Oslo (1)	2002q1 - 2014q4	No	0.074(5.62) - 0.077(-0.56)△ W_{t-1}	-0,016
	Oslo (2)	2002q1 - 2014q4	No	0.071(5.22) - 0.047(-0.33)△ W_{t-1}	-0,021
	Stavanger (1)	2002q1 - 2014q4	No	0.067(3.69) + 0.124(0.84)△ W_{t-1}	-0,007
	Stavanger (2)	2002q1 - 2014q4	No	0.071(3.23) + 0.092(0.46)△ W_{t-1}	-0,019

Furthermore, we see that Røed Larsen and Weum's conclusion of time structure in Oslo has changed. The behaviour of the regression has changed as the intercept and slope estimates are lower. The slope coefficient is no longer statistically significant, which leads us to the conclusion that the housing market in Oslo is efficient on more recent time series. As we find a similar conclusion in Stavanger, we support the argument that the same trend seems likely in other cities in Norway as well. As we notice that our house price indices have several breaks, first being in the wake of the dot-com bubble, then the effect of the financial crisis, we see that such breaks disrupt the conclusion of time structure significantly. As an expansion of the analysis, we changed the period by splitting up the sample to pre- and post-financial crisis, but were still not able to find time structure *(Appendix, B.2.)*. We also did the same analysis without adjusting for seasonal effects, i.e. using L=1, noting that this test is less valid and leads to ambiguous conclusions *(Appendix, B.3.)*.

6.3 Returns to House Investments

Even though we are not able to detect a time structure in the house price development, we continue to analyse possible time structure of excess return to housing, equivalent to Case and Shiller, and Røed Larsen and Weum, as a basis for comparison and further analysis. Table 12 report the results of the excess returns analysis in Oslo, and we see that the L=0 test still holds valid, with intercept and slope estimates of (0.0023, 0.9503) and (0.0023, 0.985). The corresponding adjusted R^2 is 0.9346. Table 13 summarizes the findings for Stavanger, and we have as expected the same problem as previous, intercept and slope estimates of (0.0038, 0.9738), (0.0265, 0.6985) and a corresponding adjusted R^2 of 0.6733. The validity of the test is therefore still lower in Stavanger than in Oslo. Searching for time structure, we analyse the situation with L=4, using 4 quarter lag equivalent to the house price analysis. In Oslo, the findings are consistent with statistically significant intercepts of 0.082 and 0.0837, and again, not statistically significant slope coefficients. In Stavanger we have the same story, with statistically significant intercepts of 0.0833 and 0.0732, but not statistically significant slope coefficients. Thus, we are not able to find time structure in any of the cities in the excess return analysis.

Table 12. Efficiency Test Oslo, Excess Returns (Case-Shiller Constant 0.03)

$ER_j(t) = \beta_0 + \beta_1 ER_k(t-L) + u_t$

Regressing excess returns from one sample on lagged return on the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	ER(a) regressed on ER(b)	95% conf. Interval	ER(b) regressed on ER(a)	95% conf. Interval
L=0				
Intercept, B0	0.0023247	(-0.0052733 , 0.0099228)	0.002334	(-0.0054025 , 0.0100699)
t-value	*0.62*		*0.61*	
Slope, B1	0.9503008	(0.8765521 , 1.02405)	0.984951	(0.9085136 , 1.061389)
t-value	*25.94*		*25.94*	
R^2	0.936		0.936	
R^2 Adjusted	0.9346		0.9346	
L=4				
Intercept, B0	0.0820139	(0.0515295 , 0.1124983)	0.083658	(0.0529594 , 0.1143565)
t-value	*5.43*		*5.5*	
Slope, B1	-0.0443751	(-0.3336952 , 0.2449449)	-0.04187	(-0.3372862 , 0.2535381)
t-value	*-0.31*		*-0.29*	
R^2	0.0023		0.0019	
R^2 Adjusted	-0.0215		-0.0218	

Table 13. Efficiency Test Stavanger, Excess Returns (Case-Shiller Constant 0.03)

$ER_j(t) = \beta_0 + \beta_1 ER_k(t-L) + u_t$

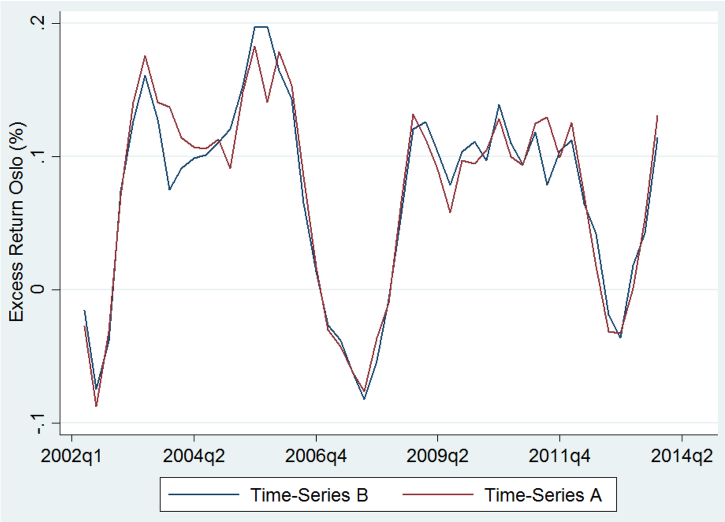
Regressing excess returns from one sample on lagged return on the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	ER(a) regressed on ER(b)	95% conf. Interval	ER(b) regressed on ER(a)	95% conf. Interval
L=0				
Intercept, B0	0.0037795	(-0.0199061 , 1.171982)	0.026472	(0.0079902 , 0.0449541)
t-value	*0.32*		*2.88*	
Slope, B1	0.97381840	(0.7756548 , 1.171982)	0.698505	(0.5563656 , 0.8406452)
t-value	*9.89*		*9.89*	
R^2	0.6802		0.6802	
R^2 Adjusted	0.6733		0.6733	
L=4				
Intercept, B0	0.0833429	(0.0335939 , 0.1330919)	0.073187	(0.0327793 , 0.1135937)
t-value	*3.38*		*3.66*	
Slope, B1	0.0950212	(-0.3050849 , 0.4951272)	0.161705	(-0.139336 , 0.4627474)
t-value	*0.48*		*1.08*	
R^2	0.0054		0.0272	
R^2 Adjusted	-0.0182		0.004	

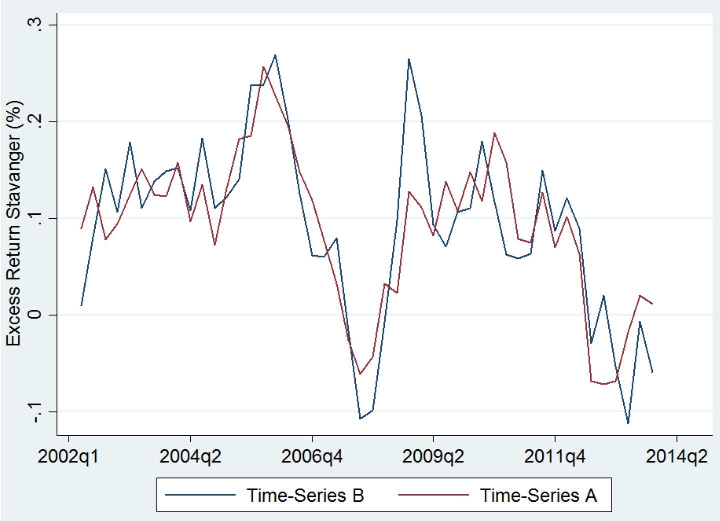
6.3.1 Comparing Excess Return in Oslo and Stavanger

Notice that the excess return is expressed in percentages, and has been mostly above zero in both cities over the relevant period. This indicates that the homeowners can, in general, expect a positive excess return on their investment. Further, the excess return in Stavanger has been more volatile than Oslo. The time series A and B in Oslo are more correlated as they move more smoothly together over time compared to Stavanger, which is mainly due to fewer observations. We highlight that the findings in Oslo are more reliable. It appears that investors can expect a higher excess return by investing in housing in Stavanger, but it involves a higher risk.

Graph 5. Excess Return Oslo



Graph 6. Excess Return Stavanger



6.3.2 Comparing Excess Return Oslo (1991-2002 vs. 2002-2014)

As Table 14 illustrates, the detected time structure from Røed Larsen and Weums research on the excess return is no longer present in our period. In general, we observe that all coefficient estimates are lower for our sample, and the slope coefficients are not statistically significant, as t-values are only (-0.031) and (-0.29) compared to Røed Larsen and Weums t-values of (-2.3) and (-2.2). Also, notice that the adjusted R^2 is low in our sample, which is strong support for the no time-structure argument.

Table 14. Comparing Excess Return Oslo

Comparing excess return Oslo						
Research	***Market***	***Time period***	***Time structure***		***Regression (t-value)*** $ER_j(t) = \beta_0 + \beta_1 ER_k(t-L) + u_t$	***Adjusted*** R^2
Røed Larsen and Weum (2007)						
	Oslo	1991q3 - 2002q4	Yes	*j=B k=A, L=4*	0.219(8.3) - 0.295(-2.3)*ERk(t-L)*	0,107
		1991q3 - 2002q4	Yes	*j=A k=B, L=4*	0.214(8.8) - 0.251(-2.2)*ERk(t-L)*	0,096
Grønstad and Graarud (2016)						
	Oslo	2002q1 - 2014q4	No	*j=B k=A, L=4*	0.082(5.43) - 0.044(-0.31)*ERk(t-L)*	-0,021
		2002q1 - 2014q4	No	*j=A k=B, L=4*	0.084(5.5) - 0.042(-0.29)*ERk(t-L)*	-0.022

These findings are not surprising, given the analysis of the housing indices behaviour in the previous section. It is important to notice that our approach to the excess return analysis is somewhat different from Røed Larsen and Weum, as we use several updated inputs. Firstly, the data on mortgage interest rate used in their analysis is the central bank's target rate plus 1%. In our sample, we use a time series from Statistics Norway on the average lending rate of mortgage suppliers. Secondly, they use a Case-Shiller constant of 0.05, while we deviate to a more updated measure of 0.03. We find this modification of the constant accurate, as the housing prices have increased relative to the rental price compared to Case and Shillers (1989) time. Thirdly, as we created an original, simple hedonic rental index using data from Finn.no, it is not directly comparable to the rental index applied by Røed Larsen and Weum. Lastly, we argue that our sample of housing transactions is more accurate than Røed Larsen and Weum[25]. These differences are highlighted, as they may be explanatory to the difference in results.

[25] *Review arguments in section 4.1 Data and Market features.*

6.4 The Stock Market and House Market

In Table 15, we compare the price appreciation of all the nominal house price indices in both cities, and the Oslo Stock Exchange All Share Index (OSEAX). We observe that the OSEAX have sufficiently higher average quarterly appreciation compared to the house indices over the relevant period. However, it also has a remarkably higher volatility, which is reasonable when considering the trade-off between risk and return. Also, note that Stavanger has a higher average quarterly return and higher volatility than Oslo, which was also the trend in the excess return analysis.

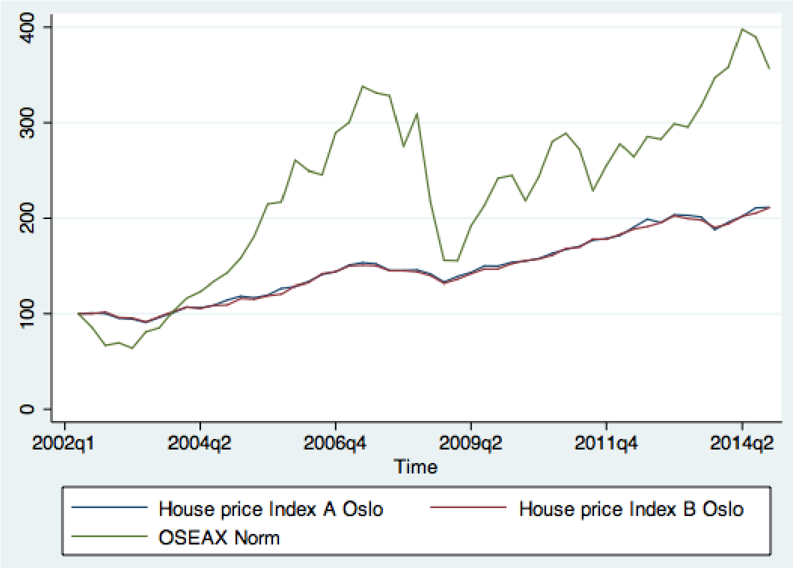
Table 15. Appreciation Rate and Volatility for the Oslo Stock Exchange All Share Index (OSEAX) and the Housing Indices

The Oslo Stock Exchange All Share Index (OSEAX) and the housing indicies	Average quarterly appreciation, in percentage terms; $\mu = \left(\sqrt[51]{(I_{52}/I_1)} - 1\right) * 100$	Volatility measure; $V = \left(\frac{1}{51}\right)\sum_{t=2}^{52}((\left(\frac{I_t}{I_{t-1}} - 1\right) * 100) - \mu)^2$
OSEAX	2.4777	158.3963
House price Index A Oslo	1.4504	35.8546
House price Index B Oslo	1.4463	35.3466
House price Index A Stavanger	1.8071	58.4030
House price Index B Stavanger	1.7542	57.7148

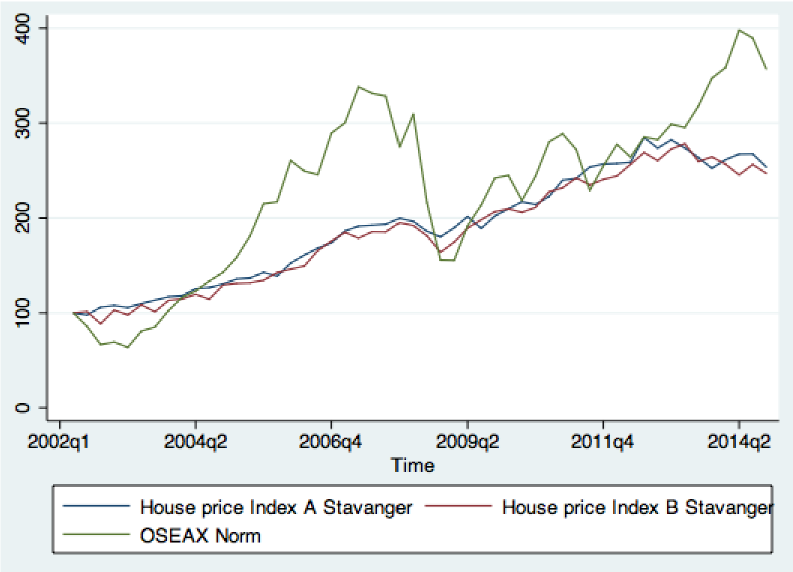
Our findings differ from Røed Larsen and Weums research, as the housing markets for our period yields lower return than the stock market. The OSEAX has a higher average quarterly appreciation and higher volatility in our period. This is reasonable considering that we observe the financial crisis. As illustrated in the following graphs, it is interesting to see that the effect of the financial crisis on the stock market was much greater than the effect on the housing markets, which is consistent with previous research[26].

[26] *Helbling and Terrones (2003)*

Graph 7. House Price Indices Oslo and OSEAX, Normalized to 100 in 2002q1



Graph 8. House Price Indices Stavanger and OSEAX, Normalized to 100 in 2002q1



Furthermore, when analysing the correlation between the house price indices and the OSEAX using the Pearson Correlation coefficients, we observe a high correlation between the four different house price indices. The OSEAX is also highly correlated with the house price indices. The Oslo house price indices have a correlation of 0.86 with the OSEAX, while the Stavanger house price indices have a correlation of 0.82 and 0.83 to the OSEAX *(Appendix, B.7.)*. A high correlation between the housing market and the stock market is expected, as it is known by economic theory that they are influenced by similar factors such as interest rates, unemployment rate, economic growth, etc. Tsai (2015) reports a causal relationship between the real estate market and the stock market. The paper concludes that housing returns over the previous two months affect current stock prices and that the returns on housing are not only affected by self-related factors but also by the stock prices during the previous month. This relationship is stronger in times of financial distress such as the financial crisis in 2008. Kakes and Van Den End (2004) provide evidence that stock prices have causal distributional effects across different segments in the Dutch real estate markets. This implies that the stock market influences the real economy through wealth composition, because it indirectly effects via the housing market.

In order to further examine housing assets as an investment object, we apply the capital asset pricing model and calculate the β for each of the house price indices. We use the average return on 3-month treasury bills issued by Norges Bank in 2014[27] as risk-free interest rate, which yielded an average quarterly return of 0.309%. The expected return from the market we assume to be 2.478%, which is the average quarterly return from the OSEAX from 2002 to 2014[28]. From Table 16 we observe that the house price indices in Oslo have significantly lower β's (0.3345 and 0.34) than the OSEAX (1) and thus a lower expected return. The house price indices in Stavanger have higher β's (0.5266 and 0.5275) than the Oslo indices, however, they are significantly lower than the OSEAX. These results are expected, as the volatility in the housing market is lower than in the stock market. We also observe that the actual quarterly return on the house price index is higher than expected quarterly return on asset for all house price indices. As previously stated, the CAPM suggest that investors should only be compensated for the time value of money and systematic risk, because of the elimination of idiosyncratic risk in a diversified portfolio. However, since this is not the case when acquiring a housing asset, the investors should be compensated for the idiosyncratic risk as well. It is likely that the observed actual quarterly return is higher than the expected quarterly return on this asset because investors are compensated for idiosyncratic risk.

[27] We use the 3-month treasury bill from 2014 because it is the last year in our period (http://www.norges-bank.no/en/Statistics/Interest-rates/Treasury-bills-annual)

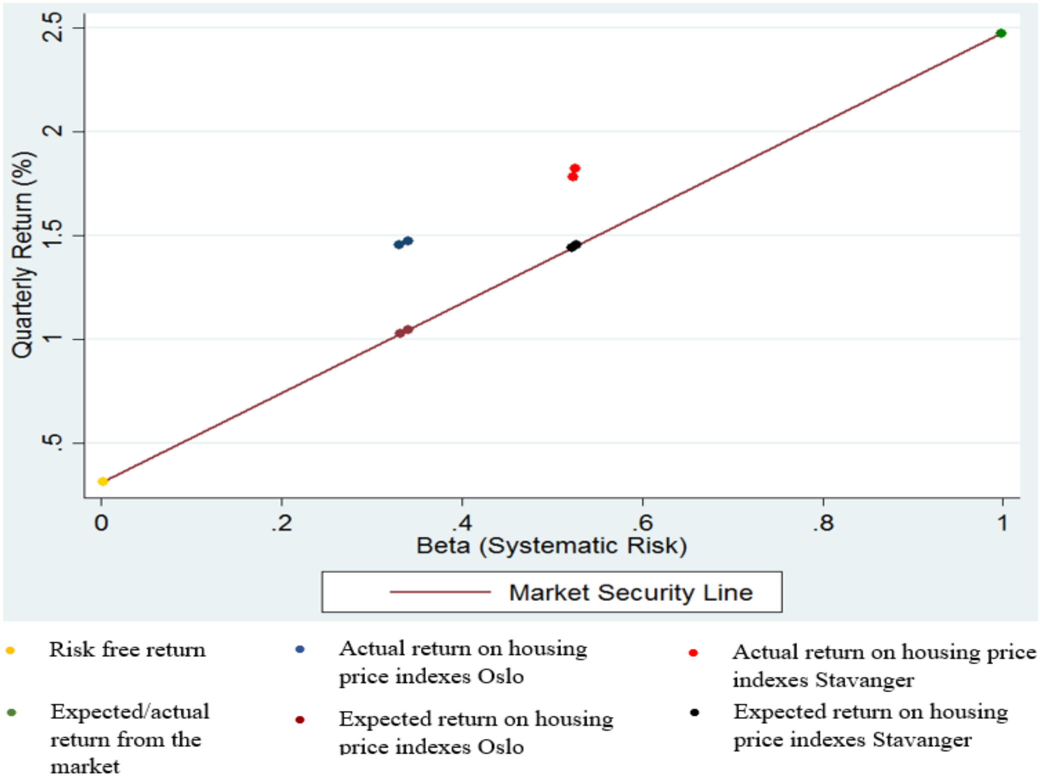
[28] It is noted that the assumed expected return from the market can be measured differently, but we argue that the OSEAX gives a good general approximation of this.

Table 16. Risk and Return Calculations on Different Investment Assets.

Investment Asset	Beta $\beta i = \frac{Cov\,(Ri, Rm)}{Var\,(Rm)}$	Expected Quarterly Return on Asset (%) $E(Ri) = Rf + \beta i(E(Rm) - Rf)$	Actual Quarterly Return (%)
Risk free	0	0.309	0.309
Housing Price Index B Oslo	0.3345	1.034	1.466
Housing Price Index A Oslo	0.34	1.046	1.470
Housing Price Index B Stavanger	0.5266	1.451	1.771
Housing Price Index A Stavanger	0.5275	1.453	1.824
OSEAX	1	2.478	2.478

Furthermore, we plot the security market line. The risk free-rate of return equal 0.309% and by definition have a β=0. The OSEAX yields a quarterly rate of return equal 2.478% and has a β=1. The expected quarterly return of the house price indices in Oslo is 1.034% and 1.046% respectively with corresponding β's of 0.3345 and 0.34 while we observe the actual return of 1.466% and 1.470%. In Stavanger, the expected quarterly return on assets is 1.451% and 1.453% with corresponding β's of 0.5266 and 0.5275 while the observed actual return is 1.771% and 1.824%.

Graph 9. The Security Market Line and Observed Returns



In order to further scrutinize the performance of housing as an investment option, we examine the Sharpe ratios. We observe the highest Sharp ratios for the house price indices in Oslo (0.032 and 0.033), which suggests that housing in Oslo offers the highest level of risk-adjusted return, see Table 17. The Sharpe ratio's for the house price indices in Stavanger (0.026 and 0.025) is lower than in Oslo, however, higher than the OSEAX (0.014). Based on the Sharpe ratios, the OSEAX appears to offer the lowest risk-adjusted return. It is important to emphasize that it is not possible for investors to invest in a house price index. However, the Sharpe ratio indicates housing as a better investment than the OSEAX.

Table 17. Sharpe Ratios

	House Price Index A Oslo	House Price Index B Oslo	House Price Index A Stavanger	House Price Index B Stavanger	OSEAX
Sharpe ratio $Sharpe\ ratio = \frac{\bar{r}_p - r_f}{\sigma_p}$	0.032	0.033	0.026	0.025	0.014

6.5 Investment Portfolio Analysis, Buy and Hold?

We investigate different investment strategies for an artificial portfolio. Assume a portfolio manager have a portfolio of 100 million NOK and want to invest in the housing market because of the historical high returns the market have yielded. The analysis will use the price indices we have developed and the deposit interest rate in the relevant period[29]. We offer two different approaches; a buy-and-sell strategy and a buy-and-hold strategy. In the first approach, the portfolio manager systematically buys in the fourth quarter and sells in the second quarter the next years, i.e. keeping the apartment for two quarters. This approach takes advantage of seasonal effects by buying late in the year where housing prices are historically relatively low, and selling in the late spring/early summer where the prices are relatively high. With this approach the portfolio manager would have to pay extensive transactions cost due to high frequency of sales/buys. The investor would need to pay capital gain taxes. In the second approach, the portfolio manager buys housing assets first quarter in the first year and holds the investment until the fourth quarter in the last year. This approach is similar to how households invest. In the first approach, the portfolio manager invests in bank deposits when the capital is not invested in housing.

[29] *We use the historical deposit rate reported by Statistic Norway*

We assume 2.5% transaction cost concerning buying the housing assets due to the document tax, this tax is included in our calculation because we have consistently worked with freeholder housing. Furthermore, the transaction cost concerning sales of the housing assets is assumed to be 2% due to the real estate agents commission.

First, consider the buy-and-sell strategy, the portfolio manager will invest in housing in the fourth quarter in the first year, i.e. 2002, and sell in the second quarter in the second year, i.e. 2003. He then reinvests in the housing market in the fourth quarter 2003 and so on. At the end of 2014 the value of the portfolio is 104.5 million NOK for the Oslo market and 138.6 million NOK for the Stavanger market. As an extension, we explore this approach with no transaction cost. This is rather unrealistic, however, professional investors might be able to lower the transaction costs from the standard rates. This gives a portfolio value in the Oslo market of 179.6 million NOK and 234.7 million NOK in the Stavanger market (Appendix B.8.).

Secondly, consider the buy-and-hold strategy, the portfolio manager will invest in housing in the first quarter the first year, i.e. 2002, and hold the investment until the fourth quarter in the last year, i.e. 2014. At the end of 2014 the value of the portfolio is 203.5 million NOK for the Oslo market and 257.6 million NOK in the Stavanger market (Appendix B.8.).

We observe that the second approach, buy-and-hold, yields a higher return in this 13-year period in both scenarios with and without transaction costs. This strategy also yields considerably higher return than investing in bank deposit the whole period. This result supports our previously concluded market efficiency. If counterfactual the buy-and-sell approach yielded higher return than the buy-and-hold approach, it would be an indication of informational inefficiency in the housing market, as an optimal trading strategy would be to enter and exit the housing market systematically.

Lastly, we consider investments in the OSEAX. We apply a similar method by investigating a buy-and-sell strategy and a buy-and-hold strategy. In the first approach the investor invest in the OSEAX index in the fourth quarter and sell in the second quarter the year after. We apply the same rule as in the housing investment case in order to calculate comparable results. In the second approach the investor invest in the OSEAX index in the first quarter the first year and sell in the last quarter last year. The transaction costs is assumed to be 1% when selling and buying[30]. The results are similar to the housing investment case. We observe a significant higher return for the buy-and-hold approach than the buy-and-sell approach. From an initial portfolio with value of 100 million NOK the buy-and-sell approach increased it's value to 266.4 million NOK while the buy-and-hold approach reached a value of 382.9 million NOK. This result underlines the significant transaction costs. It is however important to emphasize that the stock market does not have seasonal cycles similar to the housing market in a sense that there is not a price fall late in the year before a price surge in the early summer months.

7. Discussion

We conclude that the housing market in Oslo and Stavanger is characterized as efficient. It is interesting that the market has evolved from inefficient to efficient in the last two decades. The time interval we study include periods with falling housing prices, i.e. the Dot.com bubble and the financial crisis that struck the worldwide market in 2007 and these events had without question implications on the Norwegian housing market and influenced our results. It would be interesting to investigate further why the housing market now appears to be efficient. Intuitively, we see that when people were searching for new homes in the 1990s, it was common to use the newspaper or get contacted by a specific real estate agent who knew something was coming on the market. By the end of the 1990s and the beginning of 2000s, the flow of information in the real estate market changed in a more efficient way. People turned towards the use of websites as information areas, where demanders in the market easily can sort and search up places to buy, and suppliers reach a broad spectre of customers across geographical areas. It is common to use sites such as *finn.no,* which is the dominating platform for housing sales advertisements in Norway. Also in more recent time, the prices apartments sell for are public information, so a person can effectively search up and compare prices of the same apartment, or a geographical area.[31] Hence, the information flow

[30] *1% is chosen as an approximate average trading cost.*
[31] *Dagens Næringsliv, November 13, 2015, «Ny søketjeneste: Se hva boligen er solgt for»*

in the real estate market has changed a lot, and may be a reasonable intuition of our findings. However, we leave the further investigation of why the market has evolved from inefficient too efficient as a suggestion for further research.

Furthermore, we emphasize that the sample used in Stavanger is relatively small, and applying this method to a more comprehensive sample could offer more reliable conclusions. This research concludes that the Stavanger housing market is more volatile than the Oslo housing market, which might rationally be explained as the region is highly related to the offshore oil industry. As an extension of this paper, it could be interesting to compare our Stavanger house price index to the oil price index, to evaluate the causality and thereby scrutinize this statement.

This paper does not address the challenge of incorporating maintenance cost for apartments in the development of house price indices. An accurate measure of maintenance cost could improve the analysis. It might also reduce the return to housing, and therefore affect the comparison of housing asset and stocks. It is clear that all apartments need maintenance investments to keep the characteristics. Concerning our sample, as we look exclusively at apartments, the maintenance cost would exist but be relatively small and certainly lower than for houses. Apartments are part of complexes, thus they achieve economic of scale in maintenance cost, as particular external upkeep will be beneficial. A house has four walls and a roof that need maintenance, while apartments have shared walls and quite few would have an external roof. Harding, Rosenthal, and Sirmans (2007) estimate a yearly housing depreciation of 2.5% using the American Housing Survey. It is, however, questionable if this estimate is correct for the Norwegian housing market and in particular apartments.

The rental index we developed considers the size and number of bedrooms of the rental object. Realistically other factors such as location, access to public transportation, quality, etc. of the object would affect the rental price. Rental contracts also have a time perspective and the lessor would have limited possibilities to adjust the prices during a contract, thus, the actual rental price may not reflect market value in all cases. The observations from Finn.no reports the asking price, this may deviate from the actual rental price as tenants can negotiate the price. The observations had mixed reports whether the rent was per person or for the whole object. We adjusted this so all observations reported rental price for the entire apartment. However, some errors may have occurred in this process. We performed a backwards calculation using the CPI (we isolate the housing and leisure homes rents part of CPI), and believe this is a good approximation as rental contracts commonly have a clause, so the rental price is only adjusted according to the CPI unless a new rental contract is negotiated.

Røed Larsen and Weum conclude that the housing market offers higher return and lower volatility than the OSEAX. Our research offers a different result. We find that the OSEAX offers higher returns and higher volatility than the housing market. The OSEAX increased dramatically before the financial crisis and again after 2009. We consider this development a normalization of the capital markets as this risk-return relationship is more aligned with economic and financial theory. Furthermore as an extension, we apply the CAPM, and acknowledge this model applies to well-diversified portfolios. We use the model to illustrate why the realised return from the housing market is higher than the expected return as investors are likely compensated for idiosyncratic risk.

8. Conclusion and Implications

We show that the house price index and returns to housing in Oslo and Stavanger follow the martingale stochastic process in the period of 2002-2014. There is no evidence of time structure, and hence the housing markets in each city are characterized as efficient. We can conclude an average yearly appreciation of 7% in both Oslo and Stavanger. However, the lack of time structure implies that this yearly prediction is independent of previous development in price, i.e. there are no systematic trends[32]. The finding of efficiency in the housing market leaves less room for arguments supporting market regulation. Policy makers in Norway have commonly voiced the opinion that housing transactions need strict monitoring and regulation. This article presents contradicting evidence, as the trend in housing prices seem to be relatively informative and efficient.

[32] *Example: an increase in house prices one year does not imply an increase next year*

Further, we demonstrate the comparison of appreciation and volatility in the housing markets to the stock market and conclude that the housing markets have less appreciation and lower risk than the OSEAX. We also observe that the actual quarterly return from the housing market is higher than expected return on capital assets according to CAPM. We suspect this excess return occur because housing investors receive compensation for idiosyncratic risk, which is not incorporated in the CAPM. Lastly, we conclude that the housing market yields the highest risk-adjusted return according to Sharpe ratios.

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Appendix

Appendix A: Tables and Graphs

A.1. Price per square meter, Oslo and Stavanger, 2002q1-2014q4 (Statistics Norway).

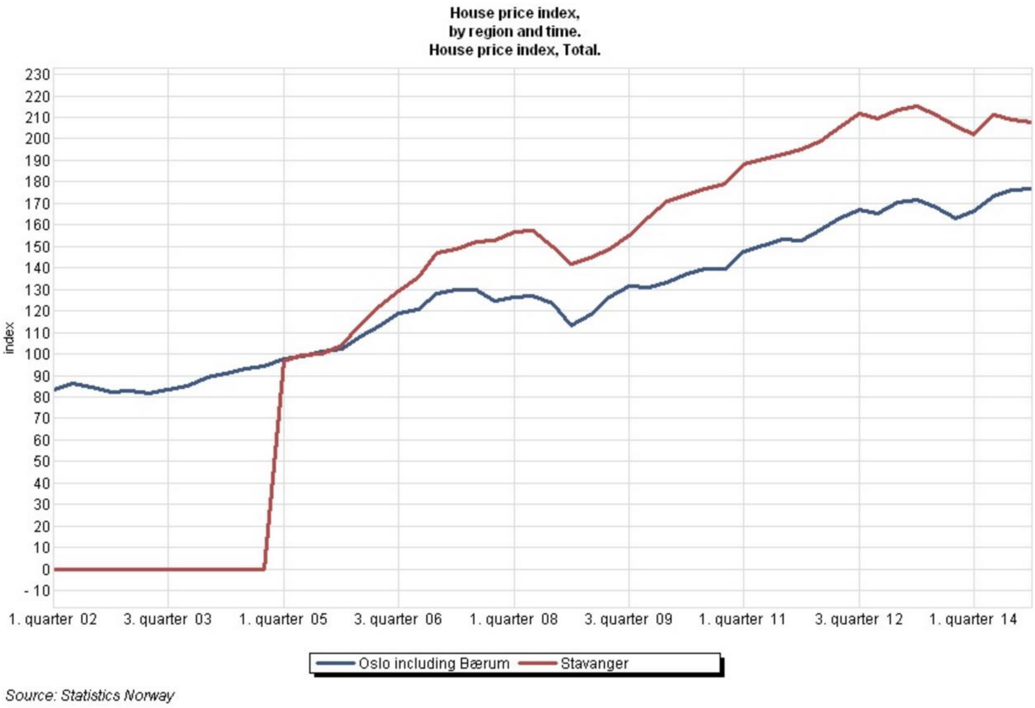
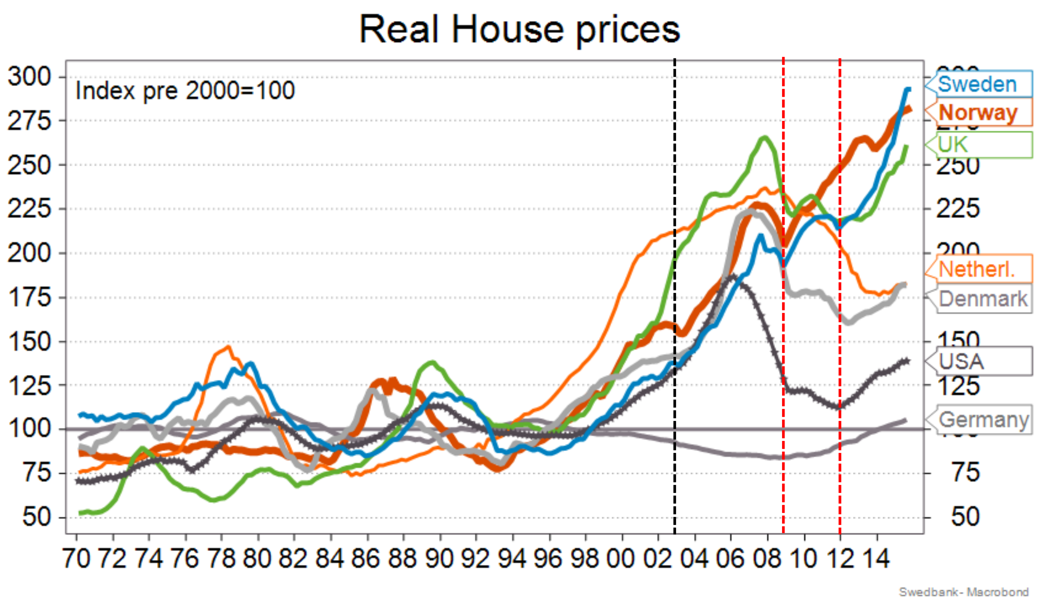


Figure A.1. https://www.ssb.no/statistikkbanken/SelectVarVal/saveselections.asp

A.2. Real House Prices, a comparison with other countries (Swedbank)



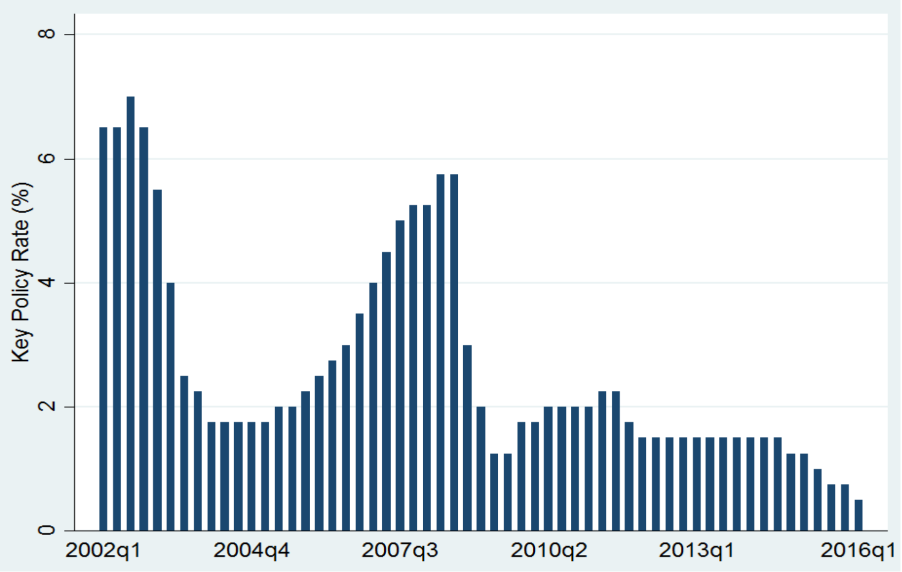
Graph A.2. House Price development. Source: BIS, OECD, Dallas Fed and Swedbank

A.3. Norwegian Households Wealth Composition (Statistics Norway)

Age	Bank Deposits	Stocks and Bonds	Funds	Housing	Debts including student loans
18-24	0.98	0.06	0.22	0.39	0.81
25-34	0.98	0.15	0.31	0.56	0.92
35-44	0.98	0.21	0.40	0.70	0.93
45-54	0.99	0.26	0.42	0.76	0.93
55-64	0.99	0.31	0.44	0.79	0.86
65-74	0.99	0.26	0.38	0.78	0.65
75+	0.98	0.15	0.25	0.62	0.34

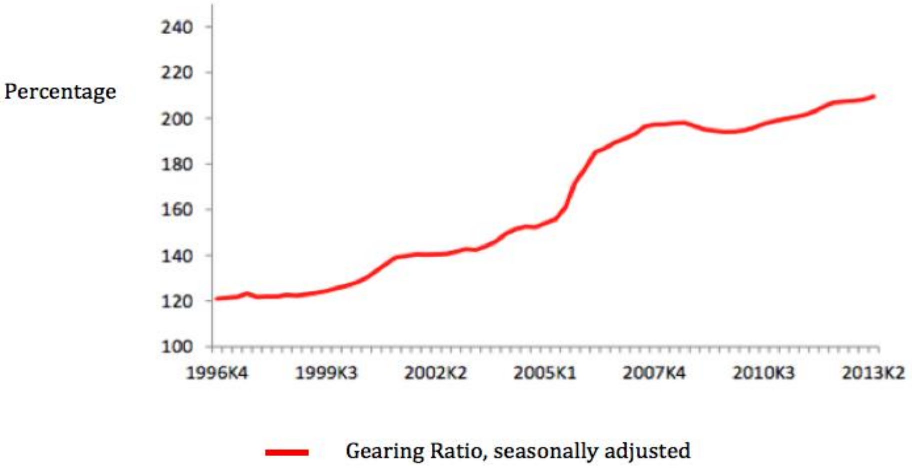
Table A.3. Wealth Composition and ownership by age group (Norway 2009). Source: Statistics Norway. Selvangivelsesstatistikk, Thomassen and Melby (2009)

A.4. Key Policy Rate development, Norway



Graph A.4. Key Policy Rate, Norway, Norges Bank

A.5. Total Gearing Ratio, Norwegian Households



Graph A.5. Total Gearing Ratio, Norwegian Households. Statistic Norway.

A.6. Data Cleaning, a detailed description of each step

A.6.1 Housing Transactions- Sales data

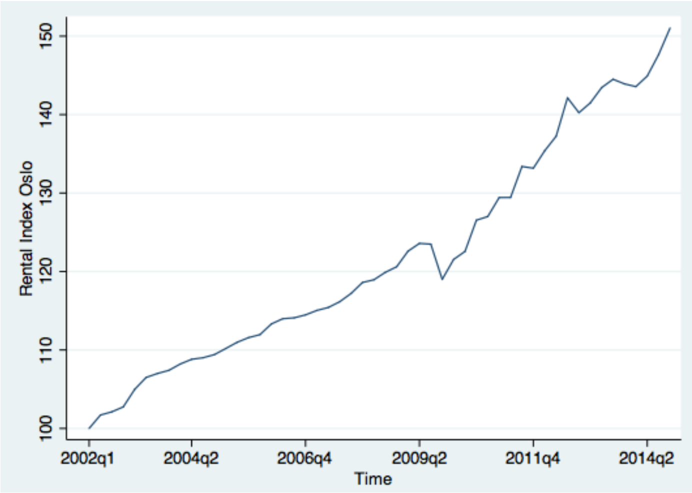
	Oslo, observations (objects)	**Stavanger, observations (objects)**
Recived from Eiendomsverdi	81.294	8.640
Duplicates removed	38	4
Observation removed due to the object is not sold exactly twice	53.160	5.892
Objects renovated between the repeated sales (if one object is removed we have to remove two observations)	1.778 (889)	242 (121)
Objects removed due to large change in square metre living area (if one object is removed we have to remove two observations)	18 (9)	4 (2)
Objects removed due to changes in bedrooms, if the change in bedroom is limited to 1 we keep the observation if the rest of the apartment is the same as before(if one object is removed we have to remove two observations)	408 (204)	20 (10)
Objects removed to change in gross area	158 (79)	26 (13)
Objects removed due to large number of bedrooms	36 (18)	0
Objects removed due to lack of sales date	192 (96)	10 (5)
Objects removed due to lack of auction in sale process	182 (91)	30 (15)
Objects removed due to quick sales (either sold twice in same quarter or newly build apartments sold within a year with a large profit)* here we suspect people take advantage of arbitrage	470 (235)	30 (15)
Total observation used in sample	**24.854 (12427**	**2.382 (1191)**

A.6.2 Housing Transactions- Rental data

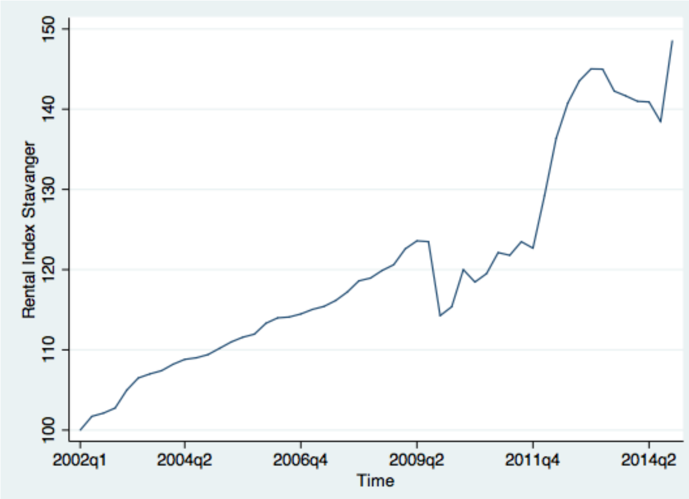
	Oslo, observations	Stavanger, observations
Received from Finn.no	85.801	15.766
Removed all observation pre 2008q2	152	36
Observations reported with 0 or 7 or more bedrooms	6.199	187
Trim the data set, we remove the observation with the 2% highest and lowest square meter price	3.178	622
If (total square meter)/ bedrooms > 10 we remove the observation	132	15
Total observation used in sample	**76.140**	**14.906**

Appendix B: Regression Results

B.1. Rental Indices, Graphics



Graph B.1.1 Rental Index Oslo



Graph B.1.2 Rental Index Stavanger

B.2. Splitting the sample, Testing for Efficiency Pre- and Post Financial Crisis

In general, we see that our conclusion of efficiency holds in each city, even when we split up the sample. Table B.2.1 illustrates that the housing market in Stavanger, covering the time period of 2002q1 to 2007q4 i.e. pre financial crisis, is characterized as efficient. We note that the slope coefficients with t-values of -1.67 and -0.34 are not statistically significant. However, the L=0 test is not to strong as we are dealing with a smaller and not perfectly balanced sample.

Housing Price Indexes Stavanger (2002-2007)

$W_j(t) - W_j(t-4) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-4)) + u(t)$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2007q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	0 .0517595	(0.0174871, 0.0860318)	0.0032825	(-0.0528372, 0.0594022)
t-value	*3.17*		*0.12*	
Slope, B1	0.5737186	(0.3251804, 0.8222568)	0.9873666	(0.5596337, 1.4151)
t-value	*4.85*		*4.85*	
R^2 Adjusted	0.5424		0.5424	
L=4				
Intercept, B0	0.1437226	(0.0815019, 0.2059434)	0.1933606	(0.1200905, 0.2666307)
t-value	*4.95*		*5.66*	
Slope, B1	-0.0728224	(-0.5325785, 0.3869337)	-0.461926	(-1.054671, 0.1308185)
t-value	*-0.34*		*-1.67*	
R^2 Adjusted	-0.0627		0.1068	

Table B.2.1 House Price Indices Stavanger (2002q1-2007q4)

Table B.2.2 illustrates that the housing market in Stavanger, covering the time period of 2009q1 to 2014q4 i.e. post financial crisis, is ambiguous. Even though one of the slope coefficients is statistically significant with t-value of 2.2, this does not hold when we twist the test, as the t-value becomes -0.28. Again note that this is within a smaller sample, Stavanger, and that the L=0 test is not to strong.

Housing Price Indexes Stavanger (2009-2014)

$W_j(t) - W_j(t-4) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-4)) + u(t)$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where T ∈ {2009q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	0.0192281	(-0.0030498, 0.0415061)	-0.0011735	(-0.0306713, 0.0283243)
t-value	*1.79*		*-0.08*	
Slope, B1	0.6337177	(0.4190589, 0.8483764)	0.9760712	(0.6454472, 1.306695)
t-value	*6.11*		*6.11*	
R^2 Adjusted	0.6020		0.6020	
L=4				
Intercept, B0	0.0572539	(-0.0030598, 0.1175675)	-0.0032789	(-0.0622049, 0.055647)
t-value	*2.02*		*-0.12*	
Slope, B1	-0.0655756	(-0.5697816, 0.4386304)	0.5865324	(0.0195551, 1.15351)
t-value	*-0.28*		*2.20*	
R^2 Adjusted	-0.0612		-0,0211	

Table B.2.2 House Price Indices Stavanger (2009q1-2014q4)

Table B.2.3 illustrates that the housing market in Oslo, covering the time period of 2002q1 to 2007q4 i.e. pre financial crisis, is again characterized as efficient. We note that the slope coefficients with t-values of -0.67 and -1.54 are not statistically significant. The L=0 test is strong.

Housing Price Indexes Oslo (2002-2007)

$$W_j(t) - W_j(t-4) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-4)) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where T ∈ {2002q1 - 2007q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	.0032983	(-0.0153202, 0 .0219167)	0.00369	(-0.0146619, 0.0220418)
t-value	*0.38*		*0.43*	
Slope, B1	0.9686597	(0.7986675, 1.138652)	0.9438847	(0.7782404, 1.109529)
t-value	*12.22*		*12,22*	
R^2 Adjusted	0.9082		0.9082	
L=4				
Intercept, B0	0.125839	(0.1075558, 0.1441222)	0.1215757	(0.0912195, 0.151932)
t-value	*15.34*		*8.92*	
Slope, B1	-0.1302356	(-0.3185925, 0.0581213)	-0.0879151	(-0.3804641, 0.2046338)
t-value	*-1.54*		*-0.67*	
R^2 Adjusted	0.1110		-0.0528	

Table B.2.3 House Price Indices Oslo (2002q1-2007q4)

Table B.2.4 illustrates that the housing market in Oslo, covering the time period of 2009q1 to 2014q4 i.e. post financial crisis, is characterized as efficient. We note that the slope coefficients with t-values of -0.72 and -0.9 are not statistically significant. The L=0 test is strong.

Housing Price Indexes Oslo (2009-2014)

$$W_j(t) - W_j(t-4) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-4)) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where T ∈ {2009q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	0.0013695	(-0.007351, 0.0100901)	0.0020223	(-0.006589, 0.0106336)
t-value	*0.32*		*0.49*	
Slope, B1	0.9792408	(0.8728228, 1.085659)	0.9602477	(0.8558937, 1.064602)
t-value	*19.04*		*19.04*	
R^2 Adjusted	0.9377		0.9377	
L=4				
Intercept, B0	0.0930006	(0.0258785, 0.1601227)	0.0844511	(0.0269156, 0.1419867)
t-value	*2.95*		*3.13*	
Slope, B1	-0.3098053	(-1.047054 0.4274432)	-0.216256	(-0.8526819, 0.42017)
t-value	*-0.90*		*-0.72*	
R^2 Adjusted	-0.0125		-0.0306	

Table B.2.4 House Price Indices Oslo (2009q1-2014q4)

B.3. Testing for Efficiency without Seasonal Adjustments (L=1)

Table B.3.1 illustrates the efficiency test without adjusting for seasonal effects in Stavanger. In this case we use one lag instead of four. We note that the L=0 test is bad, as the slope coefficients are 0.36 and 0.174, not even close to 1. The corresponding adjusted R^2 is only 0.04. This highlights the importance of seasonal effects in the Stavanger housing market, and we therefore conclude that testing market efficiency in this form is not strong.

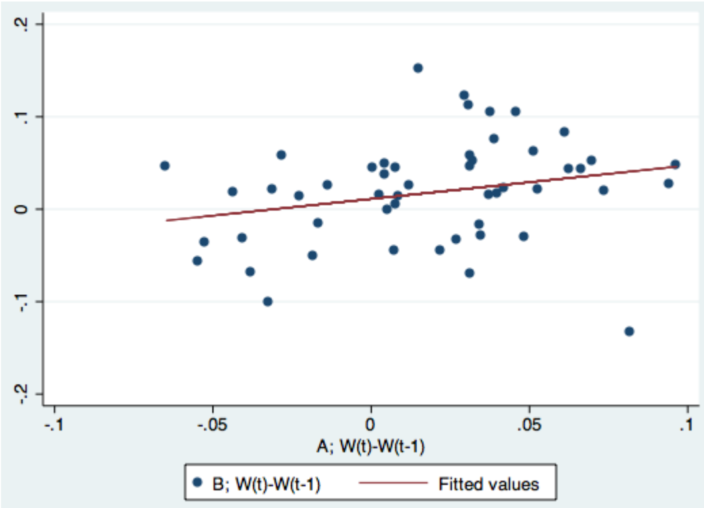
Housing Price Indexes Stavanger, not seasonally adjusted (L=1)

$$W_j(t) - W_j(t-1) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-1)) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B		95% conf. Interval
L=0					
Intercept, B0	0.0151698	(0.0038374, 0.0265023)	0.011123	(-0.0060754, 0.0283215)	(-0.0528372, 0.0594022)
t-value	*2.69*		*1.30*		
Slope, B1	0.174311	(-0.0185497, 0.3671717)	0.3618706	(-0.0385092, 0.7622504)	(0.5596337, 1.4151)
t-value	*1.82*		*1.82*		
R^2	0,0631		0.0631		
R^2 Adjusted	0.0440		0.0440		
L=1					
Intercept, B0	0.0222278	(0.0111198, 0.0333358)	0.0124232		(-0.0057552, 0.0306017)
t-value	*4.02*		*1.37*		
Slope, B1	-0.1792933	(-0.341989, -0.0165976)	0.2742606		(-0.1510213, 0.6995426)
t-value	*-2.22*		*1.30*		
R^2	0.0928		0.0338		
R^2 Adjusted	0.0739		0.0137		

Table B.3.1 House Price Indices Stavanger, Not seasonally adjusted in test



Graph. B.3.1 Stavanger, L=0 Test without seasonal adjustments

Table B.3.2 illustrates the efficiency test without adjusting for seasonal effects in Oslo. Again, we use one lag instead of four. We note that the L=0 test also is worse than when we use L=4 in Oslo, as the slope coefficients are 0.796 and 0.867 (Compared to 0.95 and 0.99 in the initial Oslo test). Again, the seasonal effects are important in Oslo, but not as important as in Stavanger. Even though one of the slope coefficients in the L=1 test is statistically significant with t-value of 2.02, this does not hold when we twist the test, as the t-value becomes 0.17.

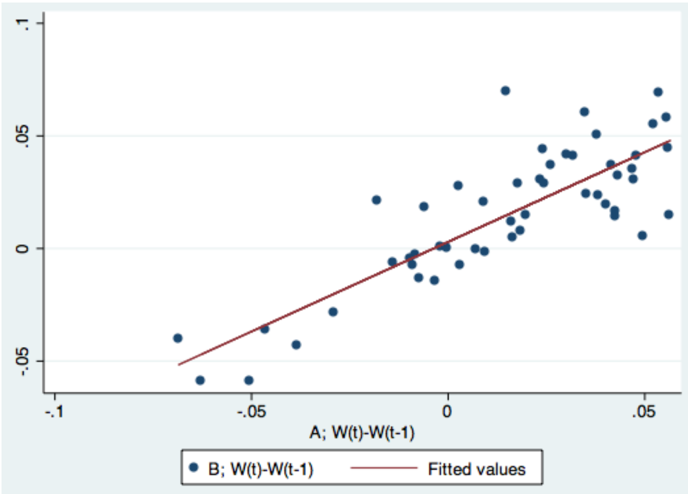
Housing Price Indexes Oslo, not seasonally adjusted (L=1)

$$W_j(t) - W_j(t-1) = \beta_0 + \beta_1(W_k(t-L) - W_k(t-L-1)) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where T ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	95% conf. Interval	Index A regresses on Index B	95% conf. Interval
L=0				
Intercept, B0	0.0019835	(-0.003596, 0.0075631)	0.0029503	(-0.0023562, 0.0082567)
t-value	*0.71*		*1.12*	
Slope, B1	0.8673606	(0.7006982, 1.034023)	0.796226	(0.6432321, 0.9492199)
t-value	*10.46*		*10.46*	
R^2	0,6906		0.6906	
R^2 Adjusted	0.6843		0.6843	
L=1				
Intercept, B0	0.0144736	(0.0043797, 0.0245676)	0.0109442	(0.0016057, 0.0202828)
t-value	*2.88*		*2.36*	
Slope, B1	0.0247073	(-0.26144, 0.3108546)	0.2681345	(0.0015227, 0.5347463)
t-value	*0.17*		*2.02*	
R^2	0.0006		0.0785	
R^2 Adjusted	-0.0202		0.0593	

Table B.3.2 House Price Indices Oslo, Not seasonally adjusted in test



Graph. B.3.2 Oslo, L=0 Test without seasonal adjustments

Graph B.3.1 and B.3.2 are included to show the major difference in Oslo and Stavanger when we computed this test. We conclude that the seasonal effects are important in each city, especially in Stavanger, and that this way of testing for efficiency is not valid. Seasonal adjustments must be taken into account when researching the housing markets this way. This argument also holds for the next part, B.4.

B.4. Testing for Efficiency without Seasonal Adjustments (L=1) with multiple lagged variables

Table B.4.1 illustrate the efficiency test without adjusting for seasonal effects in Stavanger. We observe a low F-test (0.38 and 1.13) and low adjusted R^2 (-0.1116 and 0.0202). We observe no significant coefficients, which support our findings in part 6.

Housing Price Indexes Stavanger, control, not seasonally adjusted

$$W_j(t) - W_j(t-1) = \beta_0 + \beta_1(W_k(t-1) - W_k(t-2)) + \beta_2(W_k(t-2) - W_k(t-3)) + \beta_3(W_k(t-3) - W_k(t-4)) + \beta_4(W_k(t-4) - W_k(t-5)) + \beta_5(W_k(t-5) - W_k(t-6)) + \beta_6(W_k(t-6) - W_k(t-7)) + \beta_7(W_k(t-7) - W_k(t-8)) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	Index A regresses on Index B
Intercept, B0	0.067073	0.139482
t-value	*0.59*	*1.11*
Slope, B1	0.1392015	0.1690352
t-value	*0.95*	*0.88*
Slope, B2	0.1077435	0.2022825
t-value	*0.74*	*1.08*
Slope, B3	0.067205	0.1308909
t-value	*0.45*	*0.7*
Slope, B4	0.06547	0.0271459
t-value	*0.45*	*0.15*
Slope, B5	0.106928	-0.1991036
t-value	*0.78*	*-1.04*
Slope, B6	0.0003711	0.16678
t-value	*0.00*	*0.84*
Slope, B7	0.0206605	-0.2796781
t-value	*0.15*	*-1.37*
F-test	1.38	1.13
R^2	*0.07*	*0.18*
R^2 Adjusted	-0.1116	0.0202

Table B.4.1 Efficiency test without seasonal adjustments with multiple lagged variables (Stavanger)

Table B.4.2 illustrate the efficiency test without adjusting for seasonal effects in Oslo. We observe a higher R^2 (0.1507 and 0.1264) when including multiple lagged variables but still not a significant F-test (1.92 and 1.89), this support our results from part 6.

Housing Price Indexes Oslo, control, not seasonally adjusted

$$W_j(t) - W_j(t-1) = \beta_0 + \beta_1(W_k(t-1) - W_k(t-2)) + \beta_2(W_k(t-2) - W_k(t-3)) + \beta_3(W_k(t-3) - W_k(t-4)) + \beta_4(W_k(t-4) - W_k(t-5)) + \beta_5(W_k(t-5) - W_k(t-6)) + \beta_6(W_k(t-6) - W_k(t-7)) + \beta_7(W_k(t-7) - W_k(t-8)) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	Index A regresses on Index B
Intercept, B0	0.0144899	0.013722
t-value	*2.13*	*2.1*
Slope, B1	0.4382119	0.3485267
t-value	*2.10*	*2.35*
Slope, B2	-0.0681417	-0.0600844
t-value	*-0.38*	*-0.4*
Slope, B3	-0.1386159	-0.0348766
t-value	*-0.80*	*-0.26*
Slope, B4	0.3059387	0.2677462
t-value	*1.82*	*2.02*
Slope, B5	-0.1954807	-0.318154
t-value	*-1.16*	*-2.17*
Slope, B6	-0.0020867	0.1241874
t-value	*-0.01*	*0.8*
Slope, B7	-0.219076	-0.1797546
t-value	*-1.34*	*-1.23*
F-test	1.92	1.89
R^2	*0.29*	*0.2686*
R^2 Adjusted	0.1507	0.1264

Table B.4.2 Efficiency test without seasonal adjustments with multiple lagged variables (Oslo)

B.5. Testing for Efficiency with multiple lagged variables

Table B.5.1 illustrate the efficiency test with multiple lagged variables in Stavanger. We observe low F-tests (0.5458 and 0.4461) and adjusted R^2 (-0.243 and -0.0076). The only statistical significant coefficient is the intercept (t-value equal 2.39) when Index B is regressed on Index A. It does not appear to be a longer price cycle in the Stavanger housing market.

Housing Price Indexes Stavanger, control, seasonally adjusted

$$W_j(t) - W_j(t-4) = \beta_0 + \beta_1\big(W_k(t-4) - W_k(t-8)\big) + \beta_2\big(W_k(t-8) - W_k(t-12)\big) + \beta_3\big(W_k(t-12) - W_k(t-16)\big) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	Index A regresses on Index B
Intercept, B0	0.0964851	0.088921
t-value	*2.39*	*1.8*
Slope, B1	0.0775803	0.145046
t-value	*0.42*	*0.58*
Slope, B2	-0.199502	-0.407959
t-value	*-1.10*	*-1.6*
Slope, B3	-0.1281314	0.079918
t-value	*-0.68*	*0.27*
F-test	0.5458	0.4461
R^2	*0.06*	*0.0788*
R^2 Adjusted	-0.0243	-0.0076

Table B.5.1 Efficiency test with multiple lagged variables (Stavanger)

Table B.5.2 illustrate the efficiency test with multiple lagged variables in Stavanger. In this test we observe a statistical significant F-test (3.35 and 3.14) and a higher adjusted R^2 (0.1678 and 0.0155), this could occur for severall reasons. We observe from the data that it appears to be a cycle in the housing prices in Oslo were prices increase in periodes of 5-6 years before they tend to drop for a shorter period.

Housing Price Indexes Oslo, control, seasonally adjusted

$$W_j(t) - W_j(t-4) = \beta_0 + \beta_1\big(W_k(t-4) - W_k(t-8)\big) + \beta_2\big(W_k(t-8) - W_k(t-12)\big) + \beta_3\big(W_k(t-12) - W_k(t-16)\big) + u(t)$$

The difference in the real log price index from one of the samples is regressed by the lagged real log price index of the other sample. Where t ∈ {2002q1 - 2014q4} and j ≠ k

	Index B regresses on Index A	Index A regresses on Index B
Intercept, B0	0.105557	0.110479
t-value	*4.62*	*4.75*
Slope, B1	-0.0017515	-0.042345
t-value	*-0.01*	*-0.24*
Slope, B2	-0.3991546	-0.326235
t-value	*-2.48*	*-1.94*
Slope, B3	-0.2641665	-0.350241
t-value	*-1.72*	*-2.32*
F-test	3.35	3.14
R^2	*0.24*	*0.2274*
R^2 Adjusted	0.1678	0.155

* In further research we suggest using a longer time series to investigate whether there exist a longer cycle in the housing prices. In our time series it appear to be a 5/6 year cycle where prices increase before they fall

Table B.5.2 Efficiency test with multiple lagged variables (Oslo)

B.6. Testing for Efficiency with index values

Table B.6.1 illustrate the efficiency test in Stavanger when using the price indices as variables in contrast to previous tests where we use the change in price indices between periods. We observe a statistical significant F-test (116.89 and 132.83) and a high adjusted R^2 (0.8991 and 0.9102). We get a different result when running this test, which is expected. It is intuitive that the price indices in previous periodes affect the price index in future periods.

Housing Price Indexes Stavanger, Index test

$$W_j(t) = \beta_0 + \beta_1(W_k(t-4)) + \beta_2(W_k(t-8)) + \beta_3(W_k(t-12)) + u(t)$$

We test the modell by using the indexes and not the differences in indexes across periodes

	Index B regresses on Index A	Index A regresses on Index B
Intercept, B0	1.249318	1.050067
t-value	*5.38*	*4.57*
Slope, B1	0.6719858	0.7877296
t-value	*4.28*	*4.78*
Slope, B2	-0.0824593	-0.576335
t-value	*-0.44*	*-2.44*
Slope, B3	0.1975892	0.609532
t-value	*1.38*	*3.55*
F-test	116.89	132.83
R^2	*0.9069*	*0.9171*
R^2 Adjusted	0.8991	0.9102

*It makes intuitive sense that the price index in the previous periods have influence over the price index in the period. This relationship is not the main topic in this thesis, as we want to investigate if the changes in the price index affect future changes.

Table B.6.1 Testing for Efficiency with index values (Stavanger)

Table B.6.2 illustrate the efficiency test in Oslo when using the price indices as variables in contrast to previous tests where we use the change in price indices between periods. We observe a statistical significant F-test (92.38 and 82.81) and a high adjusted R^2 (0.8755 and 0.8629). This result is expected and intuitive as the price indices in previous periods affect the price index in future periods.

Housing Price Indexes Oslo, index test

$$W_j(t) = \beta_0 + \beta_1(W_k(t-4)) + \beta_2(W_k(t-8)) + \beta_3(W_k(t-12)) + u(t)$$

We test the modell by using the indexes and not the differences in indexes across periodes

	Index B regresses on Index A	**Index A regresses on Index B**
Intercept, B0	0.5989279	0.5392472
t-value	*2.19*	*1.84*
Slope, B1	0.9558856	0.09372862
t-value	*6.01*	*5.43*
Slope, B2	-0.355204	-0.2494937
t-value	*-1.74*	*-1.18*
Slope, B3	0.2970704	0.2177238
t-value	*2.13*	*1.84*
F-test	92.38	82.81
R^2	*0.885*	*0.8734*
R^2 Adjusted	0.8755	0.8629

*It makes intuitive sense that the price index in the previous periods have influence over the price index in the period. This relationship is not the main topic in this thesis, as we want to investigate if the changes in the price index affect future changes.

Table B.6.2 Testing for Efficiency with index values (Oslo)

B.7. Pearson Correlation Coefficients

Pearson Correlation Coefficients					
	Housing Price Index A Oslo	Housing Price Index B Oslo	Housing Price Index A Stavanger	Housing Price Index B Stavanger	OSEAX
Housing Price Index A Oslo	1				
Housing Price Index B Oslo	0.9985	1			
Housing Price Index A Stavanger	0.9792	0.9796	1		
Housing Price Index B Stavanger	0.9773	0.9767	0.9923	1	
OSEAX	0.8635	0.8615	0.8229	0.8314	1

Table B.7 Pearson Correlation Coefficients

B.8.Investment Portfolio Analysis

							Buy 4th quarter sell second quarter, In Millions. 2.5% transaction costs when buying, 2% transactioncsts when selling		Buy 4th quarter sell second quarter, In Millions. Zero transaction costs		Buy-Hold-Sell		
Quarter	Deposit interest rate(%)	Housing Price IndexA Oslo	*Price change from the fourth quarter to the second quarter the next year (two quarter price development) in Oslo*	Housing Price IndexA Stavanger	*Price change from the fourth quarter to the second quarter the next year (two quarter price development) in Stavanger*		Housing investment in Oslo	Housing investment in Stavanger	Housing investment in Oslo	Housing investment in Stavanger		Housing investment in Oslo	Housing investment in Stavanger
0						**Initial value of portfolio:**	100	100	100	101	**Initial value of portfolio:**	100	100
1	0.01419	100		100		**Keeping investment in a bank**	101.4194887	101.4194887	101.4194887	102.4336836	**Buy:**	97.5	97.5
2	0.01419	102.765		99.808			102.859127	102.859127	102.859127	103.8877182		100.1959299	97.31324137
3	0.01419	102.178		108.136			104.3192007	104.3192007	104.3192007	105.3623927		99.62343599	105.4327626
4	0.01419	97.268 -		109.754		**Buy:**	101.7112207	101.7112207	104.3192007	105.3623927		94.83622704	107.010351
5	0.00447	96.384		107.829								93.97411932	105.1332978
6	0.00447	92.845	-0.045	111.905	0.020	**Sell:**	95.1448275	101.6307341	99.57595762	107.4275683		90.52416105	109.1078277
7	0.00447	98.040		115.431		**Keeping investment in a bank**	95.57011918	102.0850174	100.0210562	107.9077631		95.58856864	112.544779
8	0.00447	103.186		118.991		**Buy:**	93.1808662	99.53289199	100.0210562	107.9077631		100.6065483	116.0167095
9	0.00323	109.019		120.050								106.2932058	117.0483606
10	0.00323	108.243	0.049	127.667	0.073	**Sell:**	95.79251352	104.6540915	104.922876	115.7753767		105.5370618	124.4755509
11	0.00323	110.889		128.593		**Keeping investment in a bank**	96.10233289	104.9925716	105.2622254	116.1498262		108.1168253	125.3785597
12	0.00323	116.417		132.436		**Buy:**	93.69977457	102.8927202	105.2622254	116.1498262		113.5067729	129.1252854
13	0.00398	120.474		137.768								117.4624453	134.3235911
14	0.00398	118.832	0.021	138.860	0.049	**Sell:**	93.73070178	105.7258303	107.4458868	121.7836382		115.8614671	135.3884681
15	0.00398	121.627		144.765		**Keeping investment in a bank**	94.10339582	106.1462196	107.8731154	122.2678769		118.5866715	141.1458324
16	0.00398	128.571		140.847		**Buy:**	91.75081092	103.4925641	107.8731154	122.2678769		125.3562568	137.3262069
17	0.00644	130.489		154.573								127.226918	150.7083931
18	0.00644	135.487	0.054	162.877	0.156	**Sell:**	94.95029417	117.9337792	113.6762072	141.3918523		132.0998635	158.8054627
19	0.00644	142.870		170.425		**Keeping investment in a bank**	95.56154328	118.6929861	114.4080056	142.3020721		139.2979561	166.1640408
20	0.00644	145.713		175.815		**Buy:**	93.17250469	116.3191264	114.4080056	142.3020721		142.0701483	171.4197982
21	0.01082	152.780		188.411								148.9605533	183.7008038
22	0.01082	155.274	0.066	193.528	0.101	**Sell:**	97.54491703	125.9457845	121.9149811	156.6384129		151.3921981	188.6896285
23	0.01082	154.039		194.417		**Keeping investment in a bank**	98.60064782	127.3088985	123.2344697	158.3337139		150.1883447	189.5563227
24	0.01082	147.179		195.470		**Buy:**	96.13563162	124.126176	123.2344697	158.3337139		143.4991107	190.583336
25	0.01131	147.148		201.797								143.4695615	196.7516942
26	0.01131	147.620	0.003	198.574	0.016	**Sell:**	94.49520065	123.5751212	123.603706	160.8477506		143.9290641	193.609435
27	0.01131	143.458		188.158		**Keeping investment in a bank**	95.5636376	124.9723585	125.0012666	162.6664216	**Hold**	139.8712866	183.4541528
28	0.01131	134.882		182.248		**Buy:**	93.17454666	122.4729113	125.0012666	162.6664216		131.5102977	177.6914835
29	0.00472	140.786		191.777								137.2663759	186.9822005
30	0.00472	145.052	0.075	203.771	0.118	**Sell:**	98.19578409	134.1984382	134.4261906	181.8776177		141.4259938	198.6771664
31	0.00472	151.914		191.156		**Keeping investment in a bank**	98.65892711	134.8313887	135.0602153	182.7354483		148.1162411	186.3774387
32	0.00472	151.662		204.158		**Buy:**	96.19245393	131.4606039	135.0602153	182.7354483		147.8707791	199.0544426
33	0.00546	155.657		211.842								151.7655321	206.5464276
34	0.00546	157.080	0.036	219.197	0.074	**Sell:**	97.63615504	138.3212502	139.8849611	196.1959371		153.1531556	213.717006
35	0.00546	159.630		216.298		**Keeping investment in a bank**	98.16877966	139.0758201	140.6480613	197.2662249		155.6391704	210.8902336
36	0.00546	165.270		224.822		**Buy:**	95.71456017	136.2943037	140.6480613	197.2662249		161.138317	219.2017104
37	0.00644	169.286		241.900								165.054019	235.8522095
38	0.00644	172.417	0.043	243.840	0.085	**Sell:**	97.85643968	144.8668577	146.730054	213.952809		168.1063623	237.7437987
39	0.00644	179.011		255.829		**Keeping investment in a bank**	98.4863973	145.7994482	147.6746389	215.330145		174.5360792	249.4328319
40	0.00644	180.696		258.960		**Buy:**	96.02423737	142.154462	147.6746389	215.330145		176.178992	252.4857112
41	0.00570	183.955		259.649								179.3564915	253.1576578
42	0.00570	192.745	0.067	260.832	0.007	**Sell:**	100.378399	140.3183626	157.5212828	216.8866244		187.926248	254.3107636
43	0.00570	201.027		287.026		**Keeping investment in a bank**	100.9506624	141.1183262	158.4193213	218.1231083		196.0013678	279.8500371
44	0.00570	197.493		275.737		**Buy:**	98.42689586	137.590368	158.4193213	218.1231083		192.5553545	268.8434334
45	0.00546	205.779		284.500								200.6347876	277.3872039
46	0.00546	205.168	0.039	275.941	0.001	**Sell:**	100.2071839	134.9383781	164.5762419	218.2845792		200.0389621	269.0424511
47	0.00546	203.360		265.780		**Keeping investment in a bank**	100.753834	135.6744938	165.474038	219.4753649		198.2762039	259.1355044
48	0.00546	190.045		254.541		**Buy:**	98.23498811	132.2826314	165.474038	219.4753649		185.2935766	248.1776817
49	0.00521	197.752		263.559								192.8079562	256.969674
50	0.00521	204.129	0.074	269.389	0.058	**Sell:**	103.4048751	137.1987005	177.7373116	232.2773573		199.0256753	262.6538795
51	0.00521	212.959		269.577			103.9435272	137.9133898	178.6631729	233.4873262		207.6345572	262.8374941
52	0.00521	213.585		255.917		**Keeping investment in a bank**	104.4849852	138.6318021	179.5938571	234.703598	**Sell:**	203.4818661	257.5807442
Value of investment							**104.4849852**	**138.6318021**	**179.5938571**	**234.703598**		**203.4818661**	**257.5807442**

				Buy-Sell-Buy		Buy-Hold-Sell	
Quarter	Deposit interest rate(%)	OSEAX	*Price development from fourth quarter to the second quarter next year*		*Invest in the OSEAX in the fourth quarter and sell in the second quarter the year after. Assume 1% transaction costs*		*Invest in the OSEAX in the first quarter and hold the investmenr untll the last period*
0				Initial value of portfolio:	100	Initial value of portfolio	100
1	0.01419	173.57			101.4194887	Buy:	99
2	0.01419	148.85		Investment in a bank	102.859127		84.9003284
3	0.01419	115.62			104.3192007		65.94676499
4	0.01419	120.33		Buy:	103.2760087		68.63323155
5	0.00447	110.6					63.08348217
6	0.00447	140.25	0.165544752	Sell:	119.1690818		79.99510284
7	0.00447	147.8		Investment in a bank	119.7017605		84.30143458
8	0.00447	178.042		Buy:	118.5047429		101.5507173
9	0.00323	201.968					115.1975111
10	0.00323	213.65	0.199997753	Sell:	140.783371		121.8606326
11	0.00323	231.747		Investment in a bank	141.2387031		132.1827101
12	0.00323	247.569		Buy:	139.8263161		141.2071844
13	0.00398	274.396					156.5086363
14	0.00398	314.169	0.269015911	Sell:	175.6674017		179.1941637
15	0.00398	373.008		Investment in a bank	176.3658942		212.7544622
16	0.00398	376.782		Buy:	174.6022352		214.9070577
17	0.00644	452.295					257.9777899
18	0.00644	433.063	0.149372847	Sell:	198.6762375		247.0083367
19	0.00644	426.275		Investment in a bank	199.9552296		243.1366308
20	0.00644	502.381		Buy:	197.9556773		286.5455954
21	0.01082	521.197					297.2777727
22	0.01082	586.858	0.168153254	Sell:	228.9301429		334.7291698
23	0.01082	575.146		Investment in a bank	231.4078589		328.048937
24	0.01082	569.971		Buy:	229.0937803		325.0972461
25	0.01131	477.398					272.2959152
26	0.01131	536.937	-0.057957335	Sell:	213.6579543	Hold	306.2554762
27	0.01131	375.617		Investment in a bank	216.0737389		214.2425707
28	0.01131	270.2		Buy:	213.9130015		154.1153425
29	0.00472	269.488					153.7092355
30	0.00472	333.082	0.232723908	Sell:	261.0587145		189.9816673
31	0.00472	370.828		Investment in a bank	262.2900049		211.5110445
32	0.00472	420.092		Buy:	259.6671048		239.6100017
33	0.00546	425.225					242.5377369
34	0.00546	379.07	-0.097650039	Sell:	231.967496		216.2120758
35	0.00546	423.43		Investment in a bank	233.2329248		241.5139137
36	0.00546	486.48		Buy:	230.9005955		277.4760615
37	0.00644	501.28					285.9176125
38	0.00644	472.22	-0.029312613	Sell:	221.8909727		269.3425131
39	0.00644	397.6		Investment in a bank	223.3194113		226.7811258
40	0.00644	442.46		Buy:	221.0862172		252.3681512
41	0.00570	481.84					274.8295212
42	0.00570	458.32	0.035845048	Sell:	226.7209526		261.4142997
43	0.00570	495.33		Investment in a bank	228.0135027		282.5238809
44	0.00570	490.52		Buy:	225.7333676		279.7803768
45	0.00546	518.7					295.8535461
46	0.00546	512.71	0.045237707	Sell:	233.5855773		292.4369995
47	0.00546	551.86		Investment in a bank	234.859833		314.7671833
48	0.00546	602.8		Buy:	232.5112347		343.8220891
49	0.00521	622.19					354.8816616
50	0.00521	690.39	0.145305242	Sell:	263.6333726		393.781241
51	0.00521	676.33		Investment in a bank	265.0066798		385.7617676
52	0.00521	619.74			266.3871408	Sell:	381.9041499
Value of investment					266.3871408		381.9041499