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Productivity Spillovers through Labor Mobility in Search Equilibrium*

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Abstract

This paper proposes an explicit model of spillovers through labor flows in a framework with search frictions. Firms can choose to innovate or to imitate by hiring a worker from a firm that has already innovated. We show that if innovating firms can commit to long-term wage contracts with their workers, productivity spillovers are fully internalized. If firms cannot commit to long-term wage contracts, there is too little innovation and too much imitation in equilibrium. Our model is tractable and allows us to analyze welfare effects of various policies in the limited commitment case. We find that subsidizing innovation and taxing imitation improves welfare. Moreover, allowing innovating firms to charge different forms of fees or rent out workers to imitating firms may also improve welfare. By contrast, non-pecuniary measures that reduce the efficiency of the search process, always reduce welfare.

Key words: Efficiency, innovation, imitation, productivity, search frictions, spillovers, worker flows.

JEL Codes: J63, J68, 031, 038.

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1 Introduction

Productivity spillovers associated with R&D are considered to be important.[1] Due to such productivity spillovers, the argument goes, R&D gives rise to positive externalities on other firms, which in turn may call for policies that spur innovation. The recent empirical literature has identified labor mobility as an important channel for such spillovers.[2] If a worker moves from a technologically advanced firm to one that is less so, she may bring valuable knowledge with her.[3] Hence worker flows create information flows.

In this paper we construct a canonical model of productivity spillovers through worker flows, and use the model to perform welfare analysis. The model has two periods, and a firm may enter as an innovating firm in period 1, or as an imitating firm in period 2. An innovating firm shares its productive idea with its worker and an imitating firm gains access to this knowledge if it hires such a worker. Between the periods, workers with knowledge do on-the-job search in a competitive search market. An innovating firm that loses a worker still possesses the required knowledge, and can therefore hire a new worker and continue production. However, due to search frictions, losing the worker is costly.

From a social planner's perspective, there is a trade-off between innovation costs on the one hand and search and waiting costs on the other. If a large fraction of the firms innovate, aggregate innovation costs are high. On the other hand, innovations come in more quickly and the planner economizes on search costs, as less job-to-job transitions are necessary in order to disseminate the knowledge to imitating firms. The optimal trade-off features both innovation and imitation. In our benchmark model, with no other frictions than the search frictions, the equilibrium allocation is efficient. If an innovating firm can commit to long-term wage contracts, it will give the employee the full match surplus of the second period. This will induce the employee to search in a way that maximizes this surplus, which the firm in turn extracts through a relatively low period-1 wage. As a result, a firm that innovates pockets the full social value of its innovation, and the decentralized equilibrium realizes the socially optimal allocation.

We then analyze the welfare properties of the equilibrium allocation with restrictions on the contracting environment for innovating firms. More specifically, we restrict the firms' ability to write long-term wage contracts. In period 2 they trade off a higher rent by lowering the wage in the second period against a lower chance of retaining the worker. This leads to a lower joint surplus in period 2, which is anticipated in period 1, implying less entry of innovating firms. On the other hand, imitation –by hiring workers from innovating firms– becomes cheaper, implying excessive entry of imitating firms. Hence, there is too little innovation and too much imitation in equilibrium compared with the social optimal levels.

It is worth noting that without search frictions, the equilibrium allocation is efficient even in the

[1]See Romer (1990), Grossman and Helpman (1993) and Aghion and Howitt (1992). Arrow (1962) first drew attention to the labor channel for spillovers. For a survey of the literature on growth and spillovers see Jones (2005).

[2]We discuss the empirical literature in more detail below.

[3]This knowledge may for instance be intangible organizational capital transferred by mangers, see e.g. Lustig, Syverson, and Van Nieuwerburgh (2011) and Eisfeldt and Papanikolaou (2013).

absence of long-term contracts, as competition for workers with knowledge protects their long-term wages. Hence, it is the combination of search frictions and limited commitment that creates the inefficiency. We conjecture that other frictions, like a lower bound on initial wages, risk averse workers, or restrictions on state-contingent contracts, may give similar effects.

We then evaluate various policy measures. This is not a trivial exercise, as the absence of long-term wage contracts reduces both the social and the private value of innovations. We find that the amount of innovation is still constrained efficient, in the sense that conditional on the amount of imitation, increasing the number of innovating firms does not lead to higher welfare. Still a subsidy to innovation will improve welfare, through general equilibrium effects, as these reduce the excessive entry of imitating firms. We find that a subsidy to innovation, together with a tax on imitation, can implement the efficient allocation.[4]

Importantly, we also study the welfare implications of firm-level measures aimed at reducing excessive turnover. This gives guidelines as to how the government and courts of law should treat firm (and industry) procedures such as covenants not to compete.[5] To what extent courts honour such contracts varies. For instance, due to different legal traditions, some states in the US enforce covenants not to compete clauses in employment contracts, whereas others are more reluctant to do so (see Gilson (1999)). The study by Saxenian (1996) suggests worker mobility as an important channel for interfirm knowledge transfers. She contrasts the high employee turnover region of Silicon Valley, where covenants not to compete are illegal, with the region of Route 128 on the East coast, where such clauses are enforced.

We model different aspects of real-world mobility restrictions to analyze the effects of each channel in isolation. We find that allowing innovating firms to charge different forms of fees or renting out workers to imitating firms may improve efficiency. By contrast, restrictions that reduce the efficiency of the search process, like restricting hirings by imitating firms or search for imitation jobs, are always detrimental. Still, firms may have an incentive to impose such restrictions in order to reduce worker turnover and extract rents from workers ex post. Hence, it follows from our analysis that courts of law should be reluctant to enforce such contracts.

Spillovers as we model it have similarities with general training. In both cases the worker acquires knowledge at one firm which can be utilized by other firms the worker moves to. The difference is that with human capital investments, the investment is lost if the worker quits. With spillovers, the investing firm still has the knowledge, and the cost associated with the worker quitting is the replacement cost of the worker. The latter is determined in search equilibrium. It is this endogenous replacement cost that is the main channel for welfare improving policies in our paper. This channel is absent in models of general human capital investments, e.g., Acemoglu (1997), Acemoglu and Pischke (1999) and Moen and Rosén (2004). This difference will be discussed in more detail below.

[4]Policies towards fostering innovation play an important role in many OECD countries. For instance, government-financed R&D in 2010, as a percentage of GDP, was 0.74 in the OECD and 0.92 in the US (OECD (2013)).

[5]According to The Economist (2013) about 90% of managerial and technical employees in the US have signed non-compete agreements, which prevents employees leaving a firm from working for a rival for a fixed period.

Related Literature. There are several strands of literature that relate to our work. First, spillovers are at the core of endogenous growth models with innovation and imitation.[6] Several papers, following the seminal work by Segerstrom (1991), also analyze optimal policy.[7] However, in these papers it is imposed by assumption that spillover effects through imitation give rise to positive externalities. In our model, similar effects are derived endogenously as a result of limited commitment and search frictions in combination. Our model thus gives a microfoundation for spillover effects in labor market equilibrium.

Spillovers through worker mobility have also been studied within the industrial organization literature. Following the seminal paper by Pakes and Nitzan (1983), this literature focuses mostly on the strategic effects that arise if competitors get access to the innovation.[8] In these papers the dissemination of ideas might be inefficient as innovating firms have an incentive to limit worker flows in order to prevent increased price competition in the product market. In our paper we abstract from product market competition and focus on the cost of information flows coming from the frictional hiring process. Such search frictions are essential, as without them equilibrium always reaches efficiency. To our knowledge, none of the papers in the industrial organization literature on imitation contains search frictions.

While our paper connects on a technical level to the literature on search with contracting under limited commitment,[9] we are not aware of any work that analyzes innovation and imitation within a labor-search environment.[10]

As noted above, our model is related to models with on-the-job investments in general human capital in the presence of search frictions. In Acemoglu (1997), there are suboptimal investments in training due to a hold-up problem. Workers and their new employer bargain over the terms of trade, and at that point in time the costs of the investments are sunk. Hence the poaching firm gets part of the gain from the investments. In our paper search is directed, and poaching firms compete for workers *ex ante*. There is no underlying hold-up problem in our model. The different effects of imitation and human capital investment on optimal policy can be seen most directly by comparing our paper with Moen and Rosén (2004), who study human capital investments with directed search and provide some policy analysis. In Moen and Rosén, the investment level in human capital is below its first best level. Still it is constrained efficient; a training subsidy would reduce welfare. In our model, by contrast, a subsidy on innovation improves welfare. Increased entry of innovating firms makes the replacement market more crowded, increases wages for workers with knowledge in innovating firms, and reduces entry of imitating firms. Interestingly, we can

[6]See Eeckhout and Jovanovic (2002) and König, Lorenz, and Zilibotti (2012) for two recent examples.

[7]In particular, see Davidson and Segerstrom (1998), Mukoyama (2003), and Segerstrom (2007).

[8]See also Cooper (2001), Fosfuri and Rønde (2004), Kim and Marschke (2005), and Combes and Duranton (2006).

[9]See Rudanko (2009) and Fernández-Blanco (2013).

[10]Silveira and Wright (2010) and Chiu, Meh, and Wright (2011) study the trade of knowledge in a framework with search frictions, but without looking at labor mobility. Akcigit, Celik, and Greenwood (2013) also analyze a frictional market for ideas, but their transmission mechanism is based on trade of patents. For a model of knowledge diffusion and worker mobility, where search is random and matches occur independent of equilibrium outcomes, see Lucas and Moll (2014). Relatedly, Marimon and Quadrini (2011) study human capital accumulation on-the-job in a setting with limited commitment, but without search frictions.

replicate the constrained-efficiency results of Moen and Rosén in our model if we assume that the innovating firm is without value if the worker quits.[11]

Empirical Motivation. There is a substantial empirical literature that provides direct and indirect evidence on spillovers through worker flows.[12] The earliest empirical studies in this regard have focused on the mobility of engineers and scientists using patent citation data. These papers find that ideas are indeed spread through the mobility of patent holders (see Jaffe, Trajtenberg, and Henderson (1993), Almeida and Kogut (1999), Kim and Marschke (2005), and Breschi and Lissoni (2009)).

More recently, Stoyanov and Zubanov (2012) study spillovers across firms through worker mobility by analyzing the productivity of the receiving firm measured as the value added per worker. Using Danish data they observe firm-to firm worker movements and that "firms that hire workers from more productive firms experienced productivity gains one year after the hiring". Greenstone, Hornbeck, and Moretti (2010) analyze productivity spillovers by comparing changes in total factor productivity of incumbent plants in a given US county stemming from the opening of new large manufacturing plants in the same county. They find that positive spillovers exist and are increasing in the worker flow between the incumbent plants' industry and the opening plants' industry.

There is also a recent strand of literature that finds evidence for labor mobility as a channel of spillovers from multinational enterprises to firms that operate only locally (see Görg and Strobl (2005), Balsvik (2011), Pesola (2011) and Poole (2013)).

Finally, Møen (2005) finds evidence that firms use wage incentives to retain workers, who have gained knowledge of the firm's innovations, by charging a discount in the beginning of the career and paying a premium later.

The paper proceeds as follows. The economy is described in section 2. Section 3 sets up the welfare function, while sections 4 and 5 analyze the equilibrium when firms can and can not commit to long-term wage contracts, respectively. Next, section 6 establishes efficiency of the equilibrium with full commitment and the inefficiency of the equilibrium with limited commitment. Then, in section 7, we discuss public policies (taxes and subsidies), while a detailed analysis of firm policies (quit fees, restrictions on mobility, and options of renting out workers) is undertaken in section 8. Section 9 provides a discussion of the differences between spillovers and human capital as well as some of our model assumptions. The last section concludes.

2 Model Environment

There are two periods, and two types of agents, workers and entrepreneurs. The number (measure) of workers is normalized to 1, while the number of entrepreneurs is determined endogenously. All agents are risk neutral and do not discount future values.

[11]See the discussion section for further details.

[12]There is also a large literature on productivity spillovers in general, see Griliches (1992) for a summary of the early literature and Bloom, Schankerman, and Van Reenen (2013) for a recent example.

Production requires an entrepreneur, a worker, and knowledge. An entrepreneur may obtain knowledge in two ways, by innovating or by hiring a worker from a firm that has already innovated. Period 1 is the innovation phase, in which an entrepreneur innovates, obtains knowledge, and posts a vacancy in the pool of available workers. During the first period, employed workers learn the innovation and become informed. Period 2 is the imitation phase. In the beginning of the period, new entrepreneurs set up imitating firms and attempt to hire a worker from an innovating firm to learn the innovation from her. In addition, there is a third market, in which innovating firms that were not matched in the first period and innovating firms that have lost their worker search to find an employee among the available workers that were not hired in period 1. We refer to this as the replacement market.[13]

We use the search and matching technology of Diamond (1982), Mortensen (1982), and Pissarides (1985), in which a matching function maps vacancies and searching workers into new matches. Our model economy has three separate matching markets; the search market in period 1 denoted by the index 1, the imitation market (I), and the replacement market (R). For each market $i \in \{1, I, R\}$, s_i and v_i are the measures of searching workers and firms with vacancies, respectively. We assume a Cobb-Douglas matching function, $m(s_i, v_i) = As_i^{\epsilon}v_i^{1-\epsilon}$, where $\epsilon \in (0,1)$ and $A \in (0,1)$ are parameters. Let $\theta_i \equiv v_i/s_i$ denote the labor market tightness in market i. The probability of finding a worker in this market is $q(\theta_i) \equiv \min\{\frac{m(s_i,v_i)}{v_i}, 1\}$, and the job finding probability is $p(\theta_i) \equiv \min\{\frac{m(s_i,v_i)}{s_i}, 1\}$. If the upper bounds do not bind, it follows that $p(\theta_i) = A\theta_i^{1-\epsilon}$, $q(\theta_i) = A\theta_i^{-\epsilon}$, and $p(\theta_i) = \theta_i q(\theta_i)$. Let $\theta^{\max}$ denote the (smallest) value of θ at which $p = 1$, and $\theta^{\min}$ the (highest) value of θ at which $q = 1$. It follows that $\theta^{\max} = A^{\frac{-1}{1-\epsilon}}$ and $\theta^{\min} = A^{\frac{1}{\epsilon}}$. In what follows we assume that the bounds on the matching function do not bind. In propositions 1 and 2 we derive conditions under which this is indeed the case. To simplify the notation, we use the shorthand $q_i \equiv q(\theta_i)$ and $p_i \equiv p(\theta_i)$ throughout the main text unless the explicit version is needed for clarity.

We employ the competitive search equilibrium framework of Moen (1997), where firms advertise vacancies with wage contracts attached to them, and where the wage contracts are observed by the workers before they make their search decisions. Firms commit to the current period wage of posted contracts, but not necessarily to future period wages. Workers can quit at any time. The key feature of the competitive search framework for our analysis is that it allows search externalities to be internalized. This makes it easier to identify the efficiency properties associated with the productivity spillovers. However, the competitive search framework is not crucial for our results. The important assumption is that the imitation and the replacement search markets are separate, so that the searching agents can direct their search towards the relevant market.
The following summarizes the timing protocol:

First Period:

1. Entrepreneurs enter and pay cost K in order to innovate and create an innovating firm.
2. Each innovating firm posts a wage contract at cost c to attract a worker.

[13] It is never optimal to innovate in period 2, as this is strictly dominated by innovation in period 1.

3. Available workers observe the posted contracts and decide which firm to apply to.
4. Matched firms produce y_1 units of output, while unmatched firms keep their innovation but stay idle. Employed workers learn the innovation.

Second Period:

1. New entrepreneurs enter and set up an imitating firm at no costs.
2. The imitating firms post a vacancy for informed workers at cost c.
3. Innovating firms that have lost their worker as well as innovating firms that remained unmatched in the first period post a vacancy for the remaining available workers at cost c_R.
4. Matched firms produce: y_2 in continuing matches; y_R in innovating firms that are being matched in period 2; y_I in imitating firms. Other firms exit.

We require that $y_2 \geq y_I$ and that $y_2 \geq y_R$. This allows for cases in which productivity spillovers are only occurring to a limited degree. We refer to the case where $y_2 = y_I = y_R$ as perfect spillovers.

3 Welfare

In this section we set up the welfare function and analyze the social planner's problem. As it is common in the literature, we measure welfare as total output net of innovation and vacancy costs. By constrained efficiency we mean that the social planner faces the same matching frictions and constraints on information flows as the agents in the market.

Since the mass of available workers is normalized to unity, aggregate output in period 1 equals $p_1 y_1 - \theta_1(c + K)$. If a worker at an innovating firm moves to an imitating firm in period 2, her contribution to output is changed by the difference between y_2 and y_I. The now vacant innovating firm will produce additional output only if it finds a new worker. It is this replacement possibility that gives rise to potential benefits of imitation. That is, replacement vacancies affect the job finding probability p_R in period 2 for workers that were not matched in period 1. Aggregate net output is then given by

$$\begin{aligned} F(\theta_1, \theta_I) = & p_1[y_1 + y_2 + p_I(y_I - y_2) - c\theta_I] \\ & + (1 - p_1)p_R y_R - (c + K)\theta_1 - c_R[p_1 p_I + \theta_1(1 - q_1)]. \end{aligned} \quad (1)$$

Note that only innovating firms that have already entered in the first period can post vacancies in the replacement market. Therefore, the market tightness θ_R is completely determined by the market tightness of the other markets. Since the mass of workers in the economy is one, we have

$$\theta_R = \frac{p_1 p_I + \theta_1(1 - q_1)}{1 - p_1}, \quad (2)$$

where the first summand of the numerator is derived from the fact that the measure of workers at innovating firms that have lost their employee at the beginning of period 2 equals the number of workers who have found a job at an imitating firm. The second summand is the number of innovating firms that remain unmatched in the first period. The denominator gives the mass of searching workers, which is equal to the mass of workers that have not found a job in the first period.

The planner chooses θ_1 and θ_I so as to maximize welfare given by (1). The first-order condition for θ_1, after some manipulation (see appendix 11.1 for details), can be written as

$$\begin{aligned}\frac{\partial F}{\partial \theta_1} =&(1-\epsilon)q_1[y_1 + y_2 + p_I(y_I - y_2) - c\theta_I - \epsilon p_R y_R - (1-p_I)((1-\epsilon)q_R y_R - c_R)] \\ &+ (1-\epsilon)q_R y_R - c_R - (c+K) = 0. \end{aligned} \tag{3}$$

Then the first-order condition with respect to θ_I can be written as

$$\frac{\partial F}{\partial \theta_I} = (1-\epsilon)q_I(y_I - y_2 + (1-\epsilon)q_R y_R - c_R) - c = 0. \tag{4}$$

The constrained efficient allocation is a pair $\{\theta_1, \theta_I\}$ that solves (3) and (4). The planner trades-off innovation costs on the one hand and search and waiting costs on the other. If a large fraction of the firms innovate, aggregate innovation costs are high. On the other hand, innovations come in more quickly and the planner economizes on search costs, as less job-to-job transitions are necessary in order to disseminate the knowledge to imitating firms. We will in the following sections show that optimal trade-off involves both innovation and imitation for parameters that are such that the imitation market is open in equilibrium.

4 Model with Full Commitment

In this section we first define the values of workers and firms and then analyze equilibrium when firms can commit to long-term wage contracts.

The value of a wage contract in an innovating firm in period 1 and 2, denoted by W_1 and W_2, are given by

$$W_1 = w_1 + W_2 \tag{5}$$
$$W_2 = p_I w_I + (1-p_I)w_2 \tag{6}$$

where w_2 is the period-2 wage offered by an innovating firm, w_I is the wage offered by an imitating firm in period 2, and p_I is the probability of finding a job at an imitating firm, which in turn depends on w_2. The income of an available worker at the beginning of period 1 and period 2 are

$$U_1 = p_1 W_1 + (1-p_1)U_2 \tag{7}$$
$$U_2 = p_R w_R, \tag{8}$$

respectively, where w_R is the wage offered in the replacement market, and p_1 and p_R are the job finding probabilities in the period-1 hiring market and the replacement market, respectively. The profit of an innovating firm in period 1 that has already hired a worker is given by

$$J_1 = y_1 - w_1 + p_I V_R + (1 - p_I)(y_2 - w_2), \tag{9}$$

where V_R is the value of a vacancy posted in the replacement market, given by

$$V_R = q_R(y_R - w_R) - c_R. \tag{10}$$

The ex-ante value of innovating and opening a vacancy in an innovating firm is

$$V_1 = q_1 J_1 + (1 - q_1)V_R - c - K, \tag{11}$$

where q_1 is the probability that the vacancy is filled. The value of a vacancy in an imitating firm is

$$V_I = q_I(y_I - w_I) - c, \tag{12}$$

where q_I is the job-filling probability.

Search is competitive as all firms have to offer an expected value of search that is no lower than the expected value workers could get elsewhere in the market. Given this market value firms then optimally trade off wages (contracts) with the probability of finding a worker. In addition to the standard assumptions regarding advertised wages and the probability of hiring workers, innovating firms also have to form expectations about the relationship between the period-2 wage w_2 they offer to the worker and the probability p_I that the worker quits. We follow here the literature on competitive on-the-job search (see Moen and Rosén (2004), Shi (2009), and Menzio and Shi (2010)). Suppose a small subset of innovating firms offer a wage w_2, which may be different from the equilibrium wage. Then a submarket opens up, and imitating firms flow into this submarket up to the point where they receive zero profits. They offer wages w_I so as to maximize profit, taking the expected market value of search of the workers in this submarket as given. It follows that the resulting values of θ_I and w_I, denoted by and $\hat{\theta}_I(w_2)$ and $\hat{w}_I(w_2)$, are given by

$$\{\hat{\theta}_I(w_2), \hat{w}_I(w_2)\} = \underset{\theta_I, w_I \ s.\ to\ V_I=0}{\arg\max}\ p_I w_I + (1 - p_I)w_2. \tag{13}$$

Note that (13) is the dual to the imitating firm's profit maximization problem subject to $W_2 \leq p_I w_I + (1 - p_I)w_2$ and the zero-profit condition, taking w_2 and W_2 as given. The assumption is that, when deciding on w_2, workers and firms alike expect that workers will quit and start in an imitating firm and receive a wage $\hat{w}_I(w_2)$ with probability $\hat{p}_I(w_2) \equiv p(\hat{\theta}_I(w_2))$. It follows that we can write

$$V_1 = q_1[y_1 - w_1 + \hat{p}_I(w_2)V_R + (1 - \hat{p}_I(w_2))(y_2 - w_2)] + (1 - q_1)V_R - c - K, \tag{14}$$

$$W_1 = w_1 + \hat{p}_I(w_2)\hat{w}_I(w_2) + (1 - \hat{p}_I(w_2))w_2. \tag{15}$$

4.1 Equilibrium

Definition 1 *An equilibrium is a vector of market tightnesses* $\{\theta_1^*, \theta_I^*, \theta_R^*\}$*, values for workers* $\{W_1^*, W_2^*, U_1^*, U_2^*\}$*, and values for firms* $\{V_1^*, V_R^*\}$*, and wages* $\{w_1^*, w_2^*\}$*,* w_I^**, and* w_R^* *satisfying the following conditions:*

1. *Optimal contract and profit maximization:*

   (a) *Given* U_1^**,* U_2^* *and* V_R^**, the contract* $\{w_1^*, w_2^*\}$ *maximizes* V_1 *given by (14) subject to (7) and (15).*

   (b) *Given* w_2^**,* $\{w_I^*, \theta_I^*\}$ *solves (13).*

   (c) *Given* U_2^**, the wage* w_R^* *maximizes* V_R *given by (10) subject to (8).*

2. *Zero-profit condition:* $V_1^* = 0$.

3. *The labor market tightness in the replacement market,* θ_R^**, is given by (2).*

4.2 Characterization of Equilibrium

We start with the period-2 decisions to solve for equilibrium. First, consider the imitating firm's problem of maximizing V_I given by (12) subject to (6). The optimal wage conditional on w_2 is given by (see 11.2 for details)

$$\hat{w}_I(w_2) = \epsilon y_I + (1-\epsilon)w_2. \tag{16}$$

This is the standard result in competitive search models: the surplus (here $y_I - w_2$) is shared between the worker and the firm according to the elasticity of the job finding probability, i.e. ϵ. Then, by using (16) to substitute out $\hat{w}_I(w_2)$ in (12), the zero-profit condition for the imitating firms implicitly determines $\hat{\theta}_I(w_2)$:

$$q(\hat{\theta}_I(w_2)) = \frac{c}{(1-\epsilon)(y_I - w_2)}. \tag{17}$$

Given the solution for $\hat{\theta}_I(w_2)$, we obtain $\hat{p}_I(w_2)$.

Next, consider the replacement market in period 2. The innovating firm sets w_R so as to maximize V_R given by (10) subject to (8), with first-order condition

$$w_R = \epsilon y_R,$$

independently of θ_R. Given θ_R, which is determined by the tightness in the other markets, this pins down V_R and U_2:

$$V_R = q_R(1-\epsilon)y_R - c_R$$
$$U_2 = p_R \epsilon y_R.$$

We now turn to the innovating firm's problem in period 1. It is instructive to divide this maximization problem into two steps:

1. *Optimal retention*: For a given W_1, find the contract $\{w_1, w_2\}$ that maximizes J_1 given the functions $\hat{p}_I(w_2)$ and $\hat{w}_I(w_2)$.
2. *Optimal recruiting*: Find the value of W_1 that maximizes V_1 subject to the constraint (7).

Let M_1 denote the joint income of the worker and the firm, which can be written

$$M_1 = y_1 + y_2 + \hat{p}(w_2)[V_R + \hat{w}_I(w_2) - y_2]. \tag{18}$$

In step 1, the firm sets w_2 so as to maximize M_1. The imitation market maximizes the gain from search of a worker, $p(\theta_I)(w_I - w_2)$, given the zero profit condition of imitation vacancies. The two maximization problems coincide if $w_2 = y_2 - V_R$. Hence the solution to the optimal retention problem is to set this wage (see appendix 11.2 for details):

$$w_2 = y_2 - V_R.$$

The wage is equal to the value of the worker to the innovating firm in period 2, i.e., the value created in period 2 net of the expected profits of the firm from hiring in the replacement market. Hence the worker is the "residual claimant" on her own search effort, and her search behavior maximizes joint income. Although the firm receives zero net profit in the second period, it can extract surplus from the worker in period 1 through w_1.

Turning to the optimal recruiting problem in step two, the firm now takes M_1 as given and maximizes $V_1 = q_1(M_1 - W_1) + (1 - q_1)V_R - c - K$ subject to (7). The first-order condition

$$W_1 = \epsilon(M_1 - V_R) + (1 - \epsilon)U_2,$$

gives that the value of the contract offered by the firm is a share of the match surplus ($M_1 - V_R - U_2$).

By substituting in equilibrium values into (11), the zero-profit condition for innovating firms can be written as:

$$V_1 = q_1(1 - \epsilon)[M_1 - V_R - U_2] + V_R - c - K. \tag{19}$$

Similarly, by substituting equilibrium values into into (12), we obtain for imitating firms:

$$V_I = q_I(1 - \epsilon)[y_I - y_2 + V_R] - c = 0. \tag{20}$$

Here we see that if the transferability of technology were limited, i.e. $y_I < y_2$, imitation would be less profitable with no entry of imitating firms in the extreme case.

To construct parameter restrictions that ensure existence of equilibrium, let $\hat{y}_I$ denote the joint income of an innovating firm-worker pair from on-the-job search when $\theta_R = 0$. In that case $V_R = y_R - c_R$, hence $\hat{y}_I = \max_{w_I, \theta_I} p_I(w_I + y_R - y_2 - c_R)$ s.t. $q_I(y_I - w_I) \geq c$. Note that $\hat{y}_I$ only depends on exogenous parameters. The exact expression for $\hat{y}_I$ is derived in appendix 11.3, where we also prove our first main result:

Proposition 1 *Suppose $y_1 + y_2 + \hat{y}_I \geq c + K$. Then an equilibrium exists. Suppose further that assumptions B1 and B2 in the appendix are satisfied. Then all three markets are open, i.e., $\theta_1, \theta_I, \theta_R$ are all greater than zero. Finally, suppose assumptions C1-C4 in the appendix are satisfied. Then there exists an interior equilibrium in which $\theta_i \in (\theta^{\min}, \theta^{\max})$ for $i = 1$, R, and I. In all cases, the equilibrium is unique.*

The first parameter restriction ensures that at least some firms find it profitable to enter and hire workers in the first period. Assumptions B1 and B2 ensure that imitating firms enter. Assumption B1 ensures that imitation is sufficiently productive, that is, y_I and y_R are sufficiently high relative to y_2, c and c_R. Note that y_R matters because it influences the wage the innovating firm sets and hence how easy it is for an imitating firm to attract an informed worker. Assumption B2 ensures that not too many innovating firms enter the market in period 1 so that replacement becomes difficult and imitation is too costly.

Finally, the parameter restrictions C1-C4 ensure that the tightness is in the interval $(\theta^{\min}, \theta^{\max})$ in all the three markets. These are only sufficient conditions. As all the markets are interlinked, and there are many parameters in the model, the sufficient conditions are somewhat involved. Numerical simulations in appendix 11.15 indicate that interior solutions are easily obtained.

Recursively along the equilibrium path, prices and the allocation unfold as follows (with internal solution): In the replacement market, the number of agents on each side of the market is predetermined by the entry of innovating and imitating firms. The wage in the replacement market is $w_R = \epsilon y_R$, and the expected income of searching workers depends positively on the tightness in the market as it influences the probability of trade. In the imitation market, the innovating firms set a wage that reflects the shadow value of a worker, $y_2 - V_R$, where V_R depends on the anticipated tightness in the replacement market. Imitation firms set a wage equal to $w_I = \epsilon y_I + (1 - \epsilon)w_2$, enter the market up to the point where they receive zero profit, and in equilibrium the searching workers' income from search is maximized given this constraint. In the first period, innovating firms anticipate the profits obtained in periods 1 and 2, enter up to the point where the zero profit constraint hits, and wages are set so that the firm gets a share $(1 - \epsilon)$ of the match surplus while the remaining share is paid to the worker.

5 Model with Limited Commitment

In the setting of the previous section firms can commit to future wages. This is arguably a strong assumption. Wage contracts which specify future wage growth are rarely seen in practice. In a world where asymmetric information make state-contingent contracts difficult to honor, binding long-term wage contracts may be costly. For instance, Boeri, Garibaldi, and Moen (2017) show that long-term wage contracts with high future wages - which cannot be made contingent on future productivity - may lead to excessive firing ex post. Firms may also want to have discretion over future wages to avoid opportunistic behavior and shirking by workers.

This section analyzes the model where firms can only commit to the wage within the current period. We call this the case of limited commitment. The model is identical to the full-commitment case except for the determination of w_2. The innovating firm posts a wage w_2 in period 2 before the worker's on-the-job search decision.[14] Hence the innovating firm takes into account that the wage it sets influences the on-the-job search decision of the worker. If the on-the-job search is unsuccessful, the worker can join the replacement market. Hence the participation constraint of the worker reads $w_2 \geq U_2$.[15]

We assume that innovating firm can match wage offers from imitating firms, similar to Postel-Vinay and Robin (2002). The wage paid by a successful imitating firm must therefore be at least $\bar{w}_I = y_2 - V_R$. Otherwise, a separation would be inefficient as the value of keeping the worker for the innovating firm is higher than searching for a new one. We refer to $\bar{w}_I$ as the lower bound on w_I.

As in the full-commitment case, the wage and the tightness in the imitation market maximizes a searching worker's pay-off given the zero-profit constraint of imitating firms, see equation (13), given the new constraint $w_I \geq \bar{w}_I$. Hence, for any wage $w_2 \geq U_2$, it follows that

$$\{\hat{\theta}_I(w_2), \hat{w}_I(w_2)\} = \underset{\substack{\theta_I, w_I \ s.\ to \\ V_I=0 \ \& \ \hat{w}_I \geq \bar{w}_I}}{\arg\max} \quad p(\theta_I)w_I + (1 - p(\theta_I)w_2. \tag{21}$$

If the bound $\bar{w}_I$ does not bind, it follows that the wage is given by equation (16): $\hat{w}_I(w_2) = \epsilon y_I + (1-\epsilon)w_2$. The associated labor market tightness $\hat{\theta}_I(w_2)$ is implicitly defined by the zero profit condition for imitating firms, $q(\hat{\theta}_I(w_2))((1-\epsilon)(y_I - w_2) - c = 0$, and the probability that the worker leaves is $\hat{p}_I(w_2) \equiv p(\hat{\theta}_I(w_2))$, as in the full-commitment case. If w_2 is set so low that $\hat{w}_I(w_2) < \bar{w}_I$, i.e., if w_2 is set lower than $\bar{w}_2$ given by $\bar{w}_2 = \frac{y_2 - \epsilon y_I - V_R}{1-\epsilon}$, then $w_I = \bar{w}_I$. If $w_2 < \bar{w}_2$, the imitating firm will still pay $\bar{w}_I$. Hence, for any $w_2 < \bar{w}_2$, it follows that $p_I = \hat{p}_I(\hat{\theta}(\bar{w}_2)) \equiv \bar{p}_I$.[16]

Denote the optimal period-2 wage of an innovating firm by w_2^{lc}. It follows that w_2^{lc} solves

$$\max_{w_2} J_2 = (1-p_I)(y_2 - w_2) + p_I V_R.$$

$$\text{s. to} \qquad p_I = \begin{cases} \hat{p}(w_2) & \text{if } w_2 \geq \bar{w}_2 \\ \bar{p}_I & \text{if } w_2 < \bar{w}_2 \end{cases}$$

$$w_2 \geq U_2, V_R \text{ given.}$$

We show in appendix 11.4 that the first-order condition for w_2, which we denote by $\tilde{w}_2$, is given by the equation

[14] In an earlier version of the paper, (see Heggedal, Moen, and Preugschat (2014)), we show that our results are robust to assuming wage bargaining instead of wage posting of w_2.

[15] Note, with full commitment this constraint never binds.

[16] Since there is free entry of imitation firms, the wages set by other innovation firms do not directly effect the firm's wage setting, only indirectly through the effect on θ_R and thereby on V_R.

$$\tilde{w}_2 = y_I - \frac{\hat{p}_I(w_2)(1-\epsilon)}{\hat{p}_I(w_2)-\epsilon}(y_I - y_2 + V_R). \tag{22}$$

In the appendix we also show that the corresponding maximization problem always has a unique solution and that the second order conditions are satisfied. Furthermore, $\tilde{w}_2$ is continuous and increasing in V_R and hence in θ_R. If $\tilde{w}_2 < \bar{w}_2$, then $w_2^{lc} = U_2$. If $\tilde{w}_2 \geq \bar{w}_2$, we have to compare the profit for $w_2 = \tilde{w}_2$ and for $w_2 = U_2$, and then pick the wage that gives the higher value.[17]

Lemma 1 *The optimal period-2 wage can be expressed as a function $w_2^{lc} = w_2^{lc}(\theta_R)$ which is strictly increasing in θ_R. For a given θ_R, the period-2 wage in innovating firms is strictly lower in the limited-commitment case than in the full-commitment case.*

Proof. See appendix 11.4. ■

Taking θ_R as given, the lemma states that the second period wage w_2 is smaller in the limited-commitment case. This is because the firm now trades off retention and rent extraction within the period. At the full-commitment wage the firm is indifferent between keeping and losing the worker. By increasing second period profits when keeping the worker through lowering w_2, the firm can now increase overall profits. Turning to period 1, innovating firms choose w_1 so as to maximize V_1 given by (14) subject to (7) and (15), with $w_2 = w_2^{lc}(\theta_R)$.

The profitability of entering the market for an innovating firm is lower than in the full-commitment case, and hence the requirements for obtaining entry of innovating firms are stricter than in the full-commitment case. In appendix 11.5 we show that a sufficient condition for profitable entry of innovating firms is that $y_1 + y_2 + \hat{y}_I^c \geq c + K$, where $\hat{y}_I^c$ is the value of on-the-job search to the worker-firm pair when θ_1 and θ_R are both zero ($\hat{y}_I^c$ may be zero).

The requirements for entry of imitating firms are identical to the requirements in the full-commitment case. This is because the wage w_I, in the limit as $\theta_I \to 0$, converges to $y_2 - V_R$ both in the full-commitment and in the limited-commitment case. Hence the equilibria are identical in that limit. Regarding requirements for an interior solution, observe from (22) that the wage $\tilde{w}_2$ converges to the full-commitment wage $w_2 = y_2 - V_R$ as $\hat{p}_I$ converges to 1. Hence, the conditions that ensure that $\theta_I \leq \theta^{\max}$ are the same in the two cases. As $\theta_I \to \theta^{\min +}$, w_2 is lower and the entry of imitating firms are higher in the limited-commitment case than in the full-commitment case. Hence the requirement that $\theta_I > \theta^{\min}$ is more lax in the limited-commitment case than in the full-commitment case. Finally, the entry conditions for innovating firms are slightly different in the two cases, and this also influences the requirements for the interior solution.

In appendix 11.5 we show the following proposition:

Proposition 2 *Suppose $y_1 + y_2 + \hat{y}_I^c \geq c + K$. Then the limited-commitment equilibrium exists. Suppose further that assumptions B1 and B2 are satisfied. Then all the three markets are open, i.e.,*

[17] A minor technical issue emerges here, as $w_2^{lc}(\theta_R)$ may be discontinuous at exactly one value of θ_R, and jump from U_2 to $\tilde{w}_2$. However, the value functions are still continuous, see appendix 11.5, particularly the proof of Lemma 6 for details.

$\theta_1, \theta_I, \theta_R$ are all greater than zero. Finally, suppose that assumptions E1-E5 are satisfied. Then there exists an interior equilibrium in which $\theta_i \in (\theta^{\min}, \theta^{\max})$ for $i = 1$, R, and I.

If $w_2^{lc} = U_2$ (lower bound), the equilibrium is unique. If $w_2^{lc} = \tilde{w}_2$ (interior wage), a sufficient (but not necessary) condition for uniqueness is that $\epsilon \leq 1/2$ (see appendix 11.5 for a proof). Uniqueness requires that V_1 defined by (19) is decreasing in θ_1. This may seem obvious. However, w_I, the wage of a worker hired by an imitating firm increases in θ_1, and this in isolation tends to increase the joint income of the worker and the firm. Numerically, we have not been able to find cases of multiple equilibria. Below we assume that the parameters are such that the equilibrium is unique.[18]

We have shown above that w_2 is lower than in the full-commitment case for any given θ_R. In fact, in appendix 11.5 we show that w_2 is lower than in the full-commitment case for any level of entry of innovating firms. This lower wage leads to a higher probability of losing the worker to an imitating firm. The total effect is that the joint income of a matched worker-firm pair in period 1 is lower, and, hence, θ_1 is also lower. This is established in the following proposition.

Proposition 3 *The limited-commitment equilibrium has a higher θ_I and a lower θ_1 than the full-commitment equilibrium.*

Proof. See appendix 11.6. ■

6 Efficiency

Comparing the first-order conditions of the planner to the zero-profit conditions in full commitment equilibrium for innovators (19) and for imitators (20), we show in appendix 11.7 that they are indeed the same. Thus, the (necessary) equilibrium conditions are identical to the necessary conditions for the interior efficient allocation.

Proposition 4 *The full-commitment equilibrium allocation is constrained efficient.*

Efficiency in the commitment case can be explained by contracting under full commitment and competitive search. The argument can be divided into several steps.

First, the on-the-job search market in period 2 maximizes the income of the searching worker given the constraint that the imitating firms must make zero profits. Hence, the worker receives the entire social gain from her knowledge about the innovation. Second, when the worker searches so as to maximize her own income in period 2, there are no externalities from her search behavior on the employer. The period-2 wage in the innovating firm is exactly equal to the opportunity cost of letting the worker move to an imitating firm, i.e. output less the value of a vacancy in the replacement market. Thus, when maximizing her own income, the worker in effect also maximizes

[18] If it is not unique, we can make the refinement that we choose the one with the highest θ_1, which will always Pareto dominate other equilibria.

joint income. Third, the firm commits to a total compensation value at the beginning of period 1. The worker therefore only cares about the total compensation and will accept a low wage in period 1. Thus the firm can extract the value of the innovation net of the total wage costs. Finally, innovating firms compete for the workers ex ante, and enter up to the point where the gain from entering is equal to the cost. Since search is competitive, this process does not create distortions, and efficiency prevails.

To sum up, the optimal decision for the firm is to give the full income to the worker in period 2, and extract income only in period 1 through w_1. Joint income maximization implies that also the worker's surplus is maximized, i.e. the worker will search optimally, which is efficient from the social planner's point of view.

We now turn to the limited commitment case. As shown in section 5 above, the limited commitment θ_I is higher and θ_1 is lower than the unique efficient allocation under full commitment. The following is immediate:

Corollary 1 *The limited-commitment allocation is not constrained efficient.*

The intuition for the inefficiency result is as follows: Since there are search frictions, the innovating firms find it in their interest to lower the period-2 wage below the efficient wage; i.e., the wage that reflects the value of the worker to the innovating firm. True, this increases the probability that the worker leaves, but this is outweighed by the gain if the worker does not leave. As a result it becomes too cheap for imitating firms to attract an informed worker, in the sense that the private cost of hiring for the imitating firm is lower than the social cost. On-the-job search creates a negative externality on the current employer, and the worker quits to often.

To be more specific, recall that the imitation market maximizes the income of the searching workers (due to competitive search and the zero-profit constraint of imitation vacancies). Since w_2 is too low, the trade-off the worker faces between a high w_I and a high probability of getting a job is tilted towards the latter relative to the trade-off that maximizes joint income. The joint income of an innovating firm and its employee is therefore lower than what it would have been if innovating firms were setting a higher wage with a corresponding lower p_I. In period 1, the innovating firm may still extract the period 2 surplus from the worker, but the joint income is smaller than in the full commitment case. As a result, fewer innovating firms enter the market, and welfare is lower. It is worth noting that it is the combination of search frictions and limited commitment that creates the inefficiency. Without search frictions the equilibrium allocation is efficient even in the absence of long-term contracts, as competition for workers with knowledge protects their long-term wages.

To gain more insight into the inefficiency result, we continue by analyzing the welfare function evaluated at the limited-commitment allocation. Recall that the aggregate output in the economy, absent any policy, is given by $F(\theta_1, \theta_I)$ defined in (1). Let θ_1^{**} and θ_I^{**} denote the limited-commitment equilibrium values of θ_1 and θ_I, respectively. Then the following holds:

Lemma 2 *The following conditions are satisfied at the limited-commitment allocation:*

$$\frac{\partial F(\theta_1^{**}, \theta_I^{**})}{\partial \theta_1} = 0$$
$$\frac{\partial F(\theta_1^{**}, \theta_I^{**})}{\partial \theta_I} < 0.$$

Proof. See appendix 11.8. ■

The excessively high equilibrium value of θ_I reduces the magnitude of θ_1 required for optimal first-period entry compared to the full commitment level. However, given θ_I^{**} the level of θ_1^{**} is welfare maximizing. In contrast, a marginal reduction in θ_I from its limited-commitment value is strictly welfare improving. This result provides a very helpful method for policy analysis. It implies that for any policy that does not alter the welfare function F itself, we know that the policy is welfare improving if and only if it reduces θ_I. Monetary transfers between the agents will not affect the structure of F. However, policies that involve real costs (like an increase of the matching friction in the on-the-job search market) will.

7 Government Policies: Taxes and Subsidies

In the equilibrium with limited commitment there is too little innovation and too much imitation compared with the full-commitment case. This inefficiency gives scope for welfare improving policies. Since our model makes the transmission mechanism of productivity spillovers explicit, our analysis not only determines the resulting welfare effects, but also illuminates the way these policies function. In this section we analyze direct policies in the form of subsidies and taxes, while in section 8 we analyze policies that extend the contracting possibilities of the firms. We assume that any net surplus or losses to the government are redistributed in a lump-sum fashion to all workers equally.

Define $\sigma \geq 0$ as a subsidy on innovation investment, i.e. a subsidy to K. The subsidy thus reduces the innovation costs from K to $K - \sigma$. We show that the welfare-maximizing stand-alone subsidy σ^* is strictly greater than zero.

Lemma 3 *The welfare-maximizing stand-alone subsidy σ^* is strictly greater than zero.*

Proof. See appendix 11.9. ■

With a slight abuse of notation, let $\theta_1(\sigma)$ and $\theta_I(\sigma)$ define the limited-commitment equilibrium values of θ_1 and θ_I as a function of a stand-alone subsidy σ. The welfare effect of a (small) subsidy on innovation is given by

$$\frac{dF(\theta_1(0), \theta_I(0))}{d\sigma} = \frac{\partial F(\theta_1(0), \theta_I(0))}{\partial \theta_1}\frac{d\theta_1(0)}{d\sigma} + \frac{\partial F(\theta_1(0), \theta_I(0))}{\partial \theta_I}\frac{d\theta_I(0)}{d\sigma}.$$

The subsidy increases θ_1. Through general equilibrium effects, an increase in θ_1 decreases θ_R , and this in turn pushes θ_I down. Hence $\frac{d\theta_I(0)}{d\sigma} > 0$. From Lemma 2 it thus follows that the introduction of a small subsidy on innovation increases welfare. Intuitively, a subsidy on innovation increases the number of innovating firms that enter the market. In itself this does not increase welfare. However, it makes the replacement market tighter. This leads to higher second period wages in innovating firms, which in turn reduces the entry of imitating firms and thereby increases welfare. This mechanism highlights the role of the general equilibrium effect coming from the replacement market: if the expected labor market tightness θ_R, and hence value V_R of replacing a worker would be unaffected by labor market conditions, a subsidy would have no positive effect on welfare.[19] If for instance an increase in U_2 induces an inflow of available workers into the period-2 market, this would make subsidization less effective as a policy tool.

In appendix 11.9 we show that $F_I(\theta_1(\sigma), \theta_I(\sigma))$ is strictly negative and we have the following result:

Corollary 2 *At the welfare-maximizing stand-alone subsidy σ^*, the level of imitation is too high, in the sense that $F_I(\theta_1(\sigma^*), \theta(\sigma^*)) < 0$.*

It follows that the stand-alone subsidy on innovation cannot attain the efficient allocation.

Consider next a tax $\tau \geq 0$ on imitation, interpreted as a tax on the creation of an imitation vacancy. In effect, the cost of opening such a vacancy thus increases from c to $c + \tau$.

Lemma 4 *The welfare-maximizing stand-alone tax τ^* is strictly greater than zero. Further, τ^* does not lead to a shut-down of the imitation market.*

Proof. See appendix 11.10. ■

A tax on imitating firms reduces θ_I, which increases welfare. Whether the tax on imitation increases or decreases θ_1 is ambiguous, as the tax redistributes surplus between the agents in the economy. However, we know from Lemma 2 that changes in θ_1 only have a second order effect on welfare. Hence the introduction of a small tax τ unambiguously increases welfare. However, the optimal tax is not so high that the imitation market shuts down, as entry of imitating firms creates gains in welfare.

Again we slightly abuse notation, and let $\theta_1(\tau)$ and $\theta_I(\tau)$ denote the limited-commitment equilibrium values of θ_1 and θ_I as a function of a stand-alone tax rate τ. In appendix 11.10 we show that $F_1(\theta_1(\tau), \theta_I(\tau))$ is strictly positive for $\tau > 0$ and we have the following result:

Corollary 3 *At the welfare-maximizing stand-alone tax τ^*, the level of innovation is too low in the sense that $F_1(\theta_1(\tau^*), \theta(\tau^*)) > 0$.*

It follows that the stand-alone tax cannot attain the efficient allocation. In appendix 11.15 we provide a numerical illustration of the tax and the subsidy policies. The example suggests that starting from the welfare-maximizing tax (subsidy), the introduction of a subsidy (tax) further increases innovation, reduces imitation and increases welfare. [20]

[19] In section 9 we discuss this issue in the context of human capital.

[20] Note that it is possible to achieve θ_I^* with a tax, however at this point entry of innovation firms has a positive fiscal externality on the government, and this leads to insufficient entry of innovation firms.

The two corollaries above indicate that a combination of a positive tax on imitation vacancies and a positive subsidy may be superior to only using one instrument. The next proposition confirms this, and states that the two instruments in combination can attain efficiency.

Proposition 5 *There exists a combination of a subsidy σ on innovation and a tax τ on imitating firms that attains the efficient allocation.*

Proof. See appendix 11.11. ■

A subsidy induces entry in period 1 so that θ_1 increases. At the optimal θ_1, we know that θ_I is too high in the limited-commitment case. Thus a tax is needed to reduce entry to imitation.

Subsidies and taxes may be linked to other variables than the innovation cost K and the vacancy cost c of creating imitation vacancies. Clearly, a subsidy of creating innovation vacancies in period 1 is identical to subsidizing innovation, and will therefore not be considered further. Instead we focus on two types of subsidies: $S1$) on innovation cost as above, and $S2$) on operating innovating firms in period 1. Further, we focus on taxes levied on: $T1$) innovating firms that lose a worker, $T2$) operating imitating firms, and $T3$) on the creation of imitation vacancies as above. By using the same logic as in the proof of Proposition 5, we can show the following:

Corollary 4 *On tax and subsidy combinations and revenue neutrality:*

i) Any combination of an optimally set subsidy on either $S1$) *innovation or* $S2$) *operating period-*1 *innovating firms, with at an optimally set tax on either* $T1$) *innovating firms that lose a worker,* $T2$) *operating imitating firms, or* $T3$) *imitation vacancies will attain efficiency.*

*ii) An optimal subsidy on operating period-*1 *innovating firms in combination with any of the tax measures at optimality is revenue neutral.*

iii) An optimal subsidy on innovation in combination with any of the tax measures at optimality. gives a net surplus to the government.

Intuitively, subsidies increase innovation and taxes reduces imitation. Start out with the efficient allocation (θ_1^*, θ_I^*). For any of the tax instrument, find the tax level that that realizes θ_I^*, taking θ_1^* as given. Then for any subsidy instrument, find the subsidy level that realizes θ_1^*, taking the optimal tax as given.

To understand the revenue neutrality result, compare the income of the various agents in the full-commitment equilibrium with the efficient tax-and subsidy equilibrium when there is a subsidy on operating innovating firms. Clearly, V_R and U_2 are the same, since θ_R is the same. Since the subsidy directly enters the match surplus, and θ_1 is the same, it follows that M_1 must be the same, and hence that U_1 is the same. Imitating firms get zero profits in both equilibria. Then, by a simple accounting exercise, it follows that the amount of net transfers to the private agents in the two equilibria must be the same. Since there are no net transfers in the full-commitment equilibrium, there cannot be any in the tax-and subsidy equilibrium when there is a subsidy on operating innovating firms either.

Revenue neutrality does not hold with subsidies on innovation investment (K). Again, it follows that U_2 and V_R are identical as in the full-commitment equilibrium. However, the subsidy is not directly included in the match surplus and a lower M_1 is now needed to induce θ_1^*. It follows directly that U_1 is lower. In aggregate the private agents are receiving less surplus in the tax-and subsidy equilibrium than in the full-commitment equilibrium, and the difference is collected by the government in the form of a budget surplus.

As noted above, a subsidy to the innovating firm in period-1 is welfare improving. A subsidy to period-2 hiring, by contrast, will *cet. par.* increase the value of entering the replacement market both for innovating firms and for available workers. The effect on imitation, and hence on welfare, depends on the response of the wage w_2. If the lower bound binds, $w_2 = U_2$, the subsidy pushes wages up and imitation down so that welfare improves. By contrast, if the lower bound does not bind, the subsidy makes it more attractive for the firm to go to the replacement market, and this leads to a lower w_2, more imitation and lower welfare. If the same subsidy is given to both period-1 and period-2 hirings, welfare improves if the lower bound on w_2 binds and is indeterminate otherwise. Similar effects arise with a general tax on vacancy creation. A tax on imitating firms reduces entry of such firms, and this improves welfare. A tax on innovation reduces entry of innovating firms. This only has a second-order effect on welfare. However, reduced entry of innovating firms in isolation leads to a lower w_2. The net effect on imitation and, hence, on welfare is therefore uncertain. Note, however, that the planner can attain the efficient solution by taxing all firms and subsidizing innovation.

8 Firm Policies

In this section we extend the contractual toolbox of the firm and of the government. We study the effects of allowing innovating firms to employ more sophisticated contracts that directly restrict turnover, and of policy instruments that the government or employer organizations can implement as a substitute if the firms are not able to implement the restrictions on their own. As taxes on imitating firms are likely to be difficult to implement in practice, firm-based measures can be relevant substitutes.

Restrictions on mobility may include covenants not to compete clauses of various forms. The milder form is that either the worker or the imitating firm has to compensate the innovating firm if the worker leaves. We refer to the former case as a quit fee and the latter case as a transfer fee. More drastic measures are restrictions that reduce the efficiency of the search process. Below we analyze the different forms of fees and restrictions on mobility separately to isolate the effects of different types of clauses in contracts. Finally, we also analyze the effects of firm options to rent out workers. Our analysis gives useful guidelines regarding the attitude the government should take towards these firm policies.

Quit Fees. In this subsection we extend the contract space by including a quit fee α paid by the worker to the innovating firm if she leaves. If the firm can commit to α at the hiring stage, efficiency will, not surprisingly, be restored. One can easily show that the innovating firm can influence the search behavior of the worker in period 2 through its choice of α. Maximizing joint income with respect to α will then be a substitute for maximizing with respect to w_2.[21]

The more interesting case is when the firm cannot commit to α in period 1, but sets α at the beginning of period 2. More precisely, the firm posts a contract $\{w_2, \alpha\}$, which the worker accepts or rejects. This reduces the value of the worker of being employed in the innovating firm while searching for a job in an imitating firm. Recall that the worker may leave the firm at will at the beginning of period 2, before searching for a job in an imitating firm. The outside option of the worker at this stage reads[22]

$$\overline{W} \equiv \max_{\theta_I, w_I \ s.\ to\ V_I=0} [p_I w_I + (1-p_I)U_2]. \tag{23}$$

The associated participation constraint,

$$W_2 \geq \bar{W}, \tag{24}$$

until now always satisfied, may bind in this setting with quit fees. We refer to this constraint, somewhat imprecisely, as the interim participation constraint. This constraint implies that the firm has to offer a contract that gives at least $\bar{W}$. Notice that for any $\alpha > 0$ the interim participation constraint implies that $w_2 > U_2$.

The expected period-2 income of the worker, if entering the period as employed by an innovating firm, is $W_2 = w_2 + p_I(w_I - \alpha - w_2)$. Analogous to (13), it follows that the values $\hat{\theta}_I$ and $\hat{w}_I$, as implicit functions of α and w_2, maximize W_2 given the zero profit constraint of imitating firms:

$$\{\hat{\theta}_I, \hat{w}_I\} = \underset{\theta_I, w_I \ s.\ to\ V_I=0}{\arg\max} [w_2 + p_I(w_I - \alpha - w_2)]. \tag{25}$$

Clearly, $\hat{w}_I$ and $\hat{p}_I$ only depend on the sum $\alpha + w_2$. The firm maximizes ex-post profits, J_2, given by

$$J_2 = p(\hat{\theta}_I(w_2+\alpha))(\alpha + V_R) + (1 - p(\hat{\theta}_I(w_2+\alpha)))(y_2 - w_2), \tag{26}$$

with respect to w_2 and α, subject to (24).

The first thing to note is that the constraint (24) always binds. If not, the innovating firm could lower w_2, and at the same time increase α by the same amount. This would not influence $p(\hat{\theta}_I)$. However, the firm's ex-post profit would increase. Substituting (24) (which is binding so that $W_2 =$

[21]It does so by influencing w_I through α. The ex-post payment to the worker when she pays a quit fee is $w_I = \epsilon y_I + (1-\epsilon)(w_2 + \alpha)$. Then, together with the fact that there is a lower bound on w_2, it is clear that by choosing α the firm can set w_I equal to the efficient level w_I^* for any given level of w_2.

[22]Notice that we assume the productivity for workers hired in the replacement market is y_R, regardless of whether they are informed.

$\overline{W}$) into the expression for J_2 from above gives

$$\begin{aligned} J_2 &= p(\hat{\theta}_I(w_2+\alpha))(\hat{w}_I+V_R) + (1-p(\hat{\theta}_I(w_2+\alpha)))y_2 - \bar{W} \\ &= M_2 - \bar{W}, \end{aligned} \tag{27}$$

where $M_2 = y_2 + p_I[V_R + w_I - y_2]$ is the period-2 joint income. This is similar to the first step of the firm's maximization problem in the full-commitment case, where the firm maximizes $M_1 - W_1$ with respect to w_2 for W_1 given. To be more precise, the problem of maximizing $M_2 - \bar{W}$ given by (27) with respect to w_2 is equivalent to the problem of maximizing M_1 given by (18) with respect to w_2 up to a constant, hence the two problems have the same solution. In both cases, the firm is the residual claimant, and thus has an incentive to maximize joint income. The firm induces optimal on-the-job search by setting $w_2 + \alpha = y_2 - V_R$.

To complete the analysis, insert the first order condition for w_I (analogous to (16), given by $w_I = \epsilon y_I + (1-\epsilon)(w_2+\alpha)$), and $w_2 + \alpha = y_2 - V_R$, into the expression for W_2 to obtain

$$\begin{aligned} W_2 &= w_2 + p(\hat{\theta}_I)(\epsilon y_I + (1-\epsilon)(y_2 - V_R) - (y_2 - V_R)) \\ &= w_2 + p(\hat{\theta}_I)\epsilon(y_I - y_2 + V_R). \end{aligned}$$

The value of w_2 then solves $W_2 = \overline{W}$. We have the following proposition:

Proposition 6 *If innovating firms can post a contract in the second period specifying a quit fee α and a wage w_2, the efficient allocation is attained. The wage w_2 is lower than in the full-commitment case.*

Efficiency is obtained because with the quit fee the firm has two instruments. This enables the firm to both extract all the rent from the worker, and in addition govern her search behavior. As a result, the trade-off between rent extraction and efficiency is defused, the firm becomes the residual claimant and implements efficiency. It follows that the government, or a court of law, should not restrict firms use of quit fees on workers that leave.[23]

Compared with the full-commitment case, the wage profile is more front-loaded with limited commitment and quit fees. Both workers and firms realize ex ante that the firms will extract rents ex post, and as a result there is fiercer competition leading to higher wages paid in period 1. It follows from our analysis that allowing the firm to charge a quit fee restores efficiency, even if it is agreed upon ex post, and hence that such arrangements should be approved by a court of law.[24]

[23] Here we assume that the worker has the option to quit and search by herself, and if she finds a job not having to pay the fee. However, this is not important for the result. Suppose the innovation firm can identify the imitation firm and require a fee even if the worker searches by her own. The relevant outside option to the worker is then that $w_2 \geq U_2$. By applying exactly the same argument, we find that this constraint will always bind in optimum, and that the contract implements the efficient solution.

[24] Note that the argument rests on the presumption that the workers ex ante anticipate that they will have to pay a quit fee if they find a new job ex post. If workers do not anticipate this, their wages will be lower than expected, and too many innovating firms will enter in period 1.

Transfer Fees Backed by Mobility Restrictions. Consider now a scenario in which the innovating firm is unable to enforce a quit fee on its worker. An alternative measure may be to charge a transfer fee f from the imitating firm ex post of a match. The innovating firm may obtain the leverage to do so if it can prevent the worker from leaving. Suppose the innovating firm can require that a worker stays on in the firm, and that a court of law will enforce the innovating firm's decision with probability $\xi \in [0, 1]$. We consider ξ as a policy measure for the government. The cost to the imitating firm of going to a court of law is $\xi(y_I - w_I)$ (the probability that the worker is denied to move times the gain from acquiring the worker). Hence, in equilibrium, the innovating firm sets a fee equal to $f = \xi(y_I - w_I)$ in which case the imitating firm accepts to pay the fee.

Anticipating the fee, the imitating firm's problem is

$$\begin{aligned} &\max_{\theta_I, w_I} \; q(\theta_I)(1-\xi)(y_I - w_I) - c \\ \text{s. to} \quad &W_2 \leq p(\theta_I)w_I + (1 - p(\theta_I))w_2, \end{aligned}$$

which gives $w_I = \epsilon y_I + (1 - \epsilon)w_2$. Thus, the fee does not enter the wage directly and is akin to a tax on imitating firms entry cost, from the imitating firms perspective. Note that as ξ approaches one, imitating firms will not cover their vacancy cost and the imitation market will shut down.

The planner wants to set ξ so as to attain the efficient allocation. To find the optimal ξ, suppose innovating firms could set ξ together with w_2 in the beginning of period 2. The firm sets (ξ, w_2) so as to maximize profit subject to the interim participation constraint of the worker. This maximization problem is almost identical to the maximization problem with quit fees. First, the participation constraint will always bind, otherwise the firm could increase profit by reducing w_2 and increasing ξ in such a way that θ_I stays constant, and hence increase its profit. Given that the interim participation constraint binds, the firm becomes the residual claimant and has an incentive to maximize the joint income M_2. Denote the solution to the innovating firm's maximization problem, evaluated at the optimal allocation $\{\theta_1^*, \theta_I^*\}$ by ξ^*. The government can then implement the first-best by setting $\xi = \xi^*$.

Corollary 5 *Suppose the planner can restrict mobility through its choice of ξ, and that innovating firms can use this to extract a fee from imitating firms. Then there exists an $\xi = \xi^*$ that attains efficiency.*

Note that if the imitating firm has to pay the fee even if the worker searches by his own outside of the innovating firm, the interim participation constraint does not bind. However, the constraint $w_2 \geq U_2$ will bind, and the result will still hold (see footnote 23).

Restrictions on Hiring. We will now analyze the effects of restrictions on the mobility of workers that are not caused by or can be undone by transfers. Such a restriction may be restrictions on workers to accept jobs if they find one, or restrictions on worker search. In our model, this can be interpreted as less efficient hiring, that is, a reduction in the number of matches for a given market tightness. More concretely, now the probability of finding a worker for imitating firms is given by

$(1-\rho)q_I$ (and the job finding probability in the imitation market by $(1-\rho)p_I$, where $\rho \in [0,1]$ is a measure of the strictness of this restriction on hiring.

To understand the welfare effects of the restrictions on mobility, let us first derive the planner's choice of ρ. More specifically, we write the matching function as $(1-\rho)m(s,v)$, and let the planner decide on ρ. For a given ρ, the equilibrium is defined as above. Note that, for a given ρ, the matching function is well defined, and the welfare function (1) becomes

$$F(\theta_1, \theta_I, \rho) =$$
$$p_1[y_1 + y_2 + (1-\rho)p_I(y_I - y_2 + q_R y_R - c_R) - c\theta_I] + \theta_1(1-q_1)[q_R y_R - c_R] - (c+K)\theta_1.$$

It is straightforward to show that, for given values of θ_I and θ_1, an increase in ρ decreases welfare. However, an increase in ρ will also change the equilibrium values of θ_1 and θ_I. For a given ρ, the matching function is well defined, and Lemma 2 holds. Hence, we know that an equilibrium response in θ_1 has no effect on welfare. However, equilibrium effects on θ_I do have welfare consequences. If θ_I increases, we know that this will reduce welfare even further. On the other hand, if θ_I decreases, this will tend to increase welfare, and the net effect is not obvious. However, in appendix 11.12 we show that welfare decreases also in this case and we have the following proposition:

Proposition 7 *Restrictions on hiring reduce welfare.*

Restrictions on hiring lower welfare even if they reduce the probability of losing a worker to imitating firms. To get more intuition, first note that since $w_I \geq y_2 - V_R$, the surplus a worker creates in an imitating firm is at least as large as that from a worker that stays in the innovating firm. Thus, the presence of imitating firms in itself is good for welfare. However, imitation would be more valuable if fewer workers would leave the innovating firm, and those who leave get a higher wage w_I. This is, in effect, what happens when the moving worker pays a quit fee to the incumbent firm, or, when entry of imitating firms is taxed. A restriction on hiring, in contrast, destroys resources, which means it reduces the matching rate without giving higher wages in return. Therefore, welfare decreases.

Next we analyze the innovating firm's incentive to implement such restrictions on mobility. Consider first a situation where firms can commit to ρ at the hiring stage in period 1. In appendix 11.13 we show that joint income, M_1, is strictly decreasing in ρ. Hence, firms find it in their interest to set $\rho = 0$.

Consider then a situation where firms set ρ at the beginning of period 2. An employee can avoid the constraint on her job search by quitting before search takes place. Hence, analogous to the scenario with a quit fee, the contract the firm offers has to satisfy the interim participation constraint of the worker. More specifically, the firm offers a contract $\{\rho, w_2\}$ that satisfies $W_2 \geq \bar{W}$, or

$$\overline{W} \leq (1-\rho)p_I w_I + (1-(1-\rho)p_I)w_2, \tag{28}$$

and, in addition, $w_2 \geq U_2$. As above, this latter inequality is always satisfied when (28) is satisfied.

An issue is how the workers' and the firms' search behavior is influenced by the restrictions on mobility. The problem is a reformulation of equation (25):

$$\{\hat{\theta}_I, \hat{w}_I\} = \underset{\theta_I, w_I \ s.\ to\ V_I=0}{\arg\max} [w_2 + (1-\rho)p_I(w_I - w_2)].$$

Suppose imitating firms cannot observe individual firms' choice of ρ. Then the constraint $V_I = 0$ is independent of a single firm's choice of ρ, and it follows that $\bar{\theta}_I$ and $\bar{w}_I$ are independent of ρ (i.e. $1-\rho$ is just a multiplier). One can show that if the imitating firms observe ρ, this has the same effect on $\{\bar{\theta}_I, \bar{w}_I\}$ as scaling up c of the imitating firm to $c/(1-\rho)$, in which case $\bar{\theta}_I$ and $\bar{w}_I$ both fall. In what follows we assume the latter scenario, although our result holds in both cases.

Suppose first that w_2^{lc} is equal to U_2, where w_2^{lc} denotes the limited-commitment wage in the absence of restrictions on hiring. In this case (28) binds, hence the firm has to compensate the worker if $\rho > 0$. Further, as in the quit fee case, when (28) binds, the objective function of the innovating firm can be written as $M_2 - \overline{W}$. Then, since M_2 is strictly decreasing in ρ, the firm sets $\rho = 0$.

Suppose next that $w_2^{lc} > U_2$. In this case the constraint (28) does not bind at $\rho = 0$, and the firm may set $\rho > 0$ without increasing w_2. Since the firm's period-2 profit J_2 is increasing in ρ (see appendix 11.13 for a formal proof) it is in the firm's interest to set $\rho > 0$. The argument applies up to the value $\bar{\rho}$ at which (28) starts to bind. Hence, the firm will set $\rho = \bar{\rho} > 0$. The intuition is that if $w_2^{lc} > U_2$, the worker receives a rent by staying on in the firm. Hence, if the firm increases ρ slightly, it can do this without compensating the worker, it only appropriates some of this rent. As a result the firm has an incentive to increase ρ up to the point at which the outside option of the worker binds.

Restrictions on Search. While the previous paragraph considered restrictions on hiring we now consider a scenario where the innovating firm may exclude some workers from searching for jobs. This may be due to covenants not to compete that fully prohibit some workers to apply to jobs in imitating firms. We model this as a fraction $\chi \in [0,1]$ of employees in innovating firms who cannot search for jobs in imitating firms.

We first derive the planner's choice of χ. The planner sets χ at the beginning of period 1. The outcome for each period-1 match is revealed at the beginning of period 2. Either the worker can search for an imitation job or she cannot. When there is only a fraction $(1-\chi)$ of employed workers searching, the welfare function given in (1) becomes

$$F(\theta_1, \theta_I, \chi) =$$
$$p_1[y_1 + y_2 + (1-\chi)\{p_I(y_I - y_2 + q_R y_R - c_R) - c\theta_I\}] + \theta_1(1-q_1)[q_R y_R - c_R] - (c+K)\theta_1.$$

As in the case of restrictions on hiring discussed above, Lemma 2 holds for a given χ, and an equilibrium response in θ_1 has no effect on welfare while a positive effect on θ_I is negative for welfare. In appendix 11.14 we show that θ_I increases with χ and that the direct effect of the policy is negative, and we have the following:

Corollary 6 *Restrictions on search reduce welfare.*

Similarly to restrictions on hiring, restrictions on search destroy resources and lower welfare.

We now turn to the innovating firm's incentives to implement restrictions on search. By the same logic as in the restrictions on hiring scenario, it is clear that the firm sets $\chi = 0$ if it can commit to this at the hiring stage in period 1.

Next consider the situation where the firm cannot set χ in period 1, but offers a contract $\{\chi, w_2\}$ at the beginning of period 2. Whether a worker that accepts the contract can search for an imitation job or not, is revealed before imitating firms posts vacancies. Since the worker can avoid this possible restriction on her search by quitting, the contract the firm offers has to satisfy the interim participation constraint of the worker. That is, the contract has to satisfy

$$\overline{W} \leq (1-\chi)p_I w_I + (1-(1-\chi)p_I)w_2,$$

in addition to $w_2 \geq U_2$.

The period-2 problem for the innovating firm is then analogous to that of the restrictions on hiring scenario. If U_2 binds for w_2^{lc}, the interim participation constraint binds, and the firm sets $\chi = 0$. By contrast, if $w_2^{lc} > U_2$, the interim participation constraint does not bind, and the firm sets $\chi > 0$ to extract rents from the worker.

Hence, as with ρ, innovating firms have an incentive to set χ strictly higher than zero in some situations. However, we know from above that restrictions on mobility, through either restricting hirings or restricting search, reduce welfare. Our analysis thus indicate that courts of law should not enforce covenants not to compete clauses that reduce the efficiency of the search process.

Renting out Workers. Finally, consider the scenario where the innovating firm has the possibility of renting out the worker to an imitating firm. In this case it is the innovating firm that does the search for a job, and it faces the same frictions as the worker does when she would search on the job. Since the firm has all the bargaining power, the interim participation constraint will again bind, and the worker receives an expected income of $\overline{W}$ as defined in (23). Denote the rental price to the imitating firm as w_I^r. The innovating firm in period 2 now maximizes:

$$\max_{\theta_I, w_I^r \text{ s. to } V_I=0} p_I(V_R + w_I^r) + (1-p_I)y_2 - \overline{W}$$

The maximand can be written as $y_2 - \overline{W} + p_I(w_I^r - (y_2 - V_R))$. Since $y_2 - V_R$ is equal to the full-commitment wage w_2, the firm's problem is equivalent to the worker's maximization in the full-commitment case up to a constant. It follows that the solution is efficient.

Proposition 8 *When innovating firms can rent out workers to imitating firms, the efficient allocation is attained.*

The intuition is straightforward. The firm is residual claimant on the value of search, and hence searches efficiently.

9 Extensions and Discussion

In this section we will discuss several of the assumptions of our model.

More on Human Capital vs Spillovers. Clearly, human capital and spillovers are related phenomena. However, when it comes to policy recommendations, there are some differences between the two that lead to different conclusions. To illustrate this, recall that the difference between spillovers and general human capital is captured by y_R, the productivity of the job if the worker leaves. With pure spillovers $y_R = y_2$, while with pure human capital y_R is low (zero).

Assume now that the productivity in the replacement market is so low that $V_R < 0$ for a tightness θ_R given by (2). Let $\bar{y}_R$ be such that, in equilibrium, $V_R(\theta_R; \bar{y}_R) = 0$, with θ_R given by (2). If y_R is below this threshold, then V_R is negative. Innovating firms with a vacancy will then randomize on whether to post the vacancy or not, and in the resulting mixed-strategy equilibrium, the tightness $\tilde{\theta}_R$ will be determined so that

$$V(\tilde{\theta}_R) = (1-\epsilon)q((\tilde{\theta}_R)y_R - c_R = 0 \tag{29}$$

Equation (29) uniquely determines $\tilde{\theta}_R$ for $y_R \in [\frac{c_R}{1-\epsilon}, \bar{y}_R]$.[25]

In addition we relax the assumption that $y_I \leq y_2$, and assume instead that $y_I > y_2 + c$. With human capital investments, this may make sense; some firms (like research firms) have an advantage in training workers, others in utilizing the skills of trained workers. This ensures that we will still have entry of imitating firms.

The limited-commitment equilibrium of the model can be defined as above, but with (2) replaced by (29). We refer to this as the training equilibrium. Furthermore, it follows from the welfare properties of competitive search equilibrium that the mixed-strategy equilibrium in the replacement market is efficient, in the sense that it maximizes aggregate output less search costs. It is straightforward to show that Proposition 3 still holds in the training equilibrium, i.e., that the equilibrium allocation has less innovation and more imitation than the optimal allocation. Hence we can show the following proposition

Proposition 9 *In the training equilibrium, a subsidy σ on training vacancies reduces welfare.*

The result is analogous to the constrained-efficiency result in Moen and Rosén (2004). The proof is straightforward: The training subsidy affects θ_1. However, the wage w_2 does not change as θ_R is uniquely determined by (29). Therefore the maximization problem of the imitating firm, is unaltered by a training subsidy, and thus also θ_I. It follows that for a given θ_1, the period-2

[25] For simplicity we assume - as in the main text - that the bounds on the matching function do not bind.

profits of innovating firms are independent of the training subsidy. Hence the training subsidy unambiguously increases θ_1. However, we know from Lemma 2 that θ_1 is constrained optimal given θ_I with $\sigma = 0$, and the proposition follows.

In comparison, when there are spillovers, i.e. $y_R > \bar{y}_R$, the equilibrium is as in the main model and the effects of a subsidy to training go through the replacement market. A subsidy increases the number of training firms entering the market, the number of imitating firms entering (for a given tightness θ_I), and the tightness of the replacement market. As a result, the value of entering the replacement market for innovating firms that have lost their worker falls, and the innovating firms therefore protect their workers more. This reduces the incentives of imitating firms to enter the market, and it is this effect that improves welfare. In the training equilibrium, this effect is defused.

Observe that Proposition 9 also holds when $y_R = 0$, in which case the replacement market shuts down. Further note that a tax on imitating firms still improves welfare, as this has a direct effect on θ_I, independently of the replacement market.

The Role of Search Frictions. It may be enlightening to analyze the Walrasian equilibrium without search frictions and vacancy costs. To simplify we assume that $y_1 = y_2 = y_R = y_I = y$ and that $y < K < 2y$, and that search costs are zero (this is not necessary for our argument). The number of matches - irrespective of whether we are within the bounds of the matching function or not - is given by $\min\{As_i^{\epsilon}v_i^{1-\epsilon}, s_i, v_i\}$. If we relax the restriction on A the number of matches in the limit becomes $\min\{s, v\}$ as A goes to infinity. In equilibrium, $s = v$ in period 1, where s is the measure of workers that do search. At this point the elasticity of q with respect to θ is not well defined. However, the competitive equilibrium can easily be derived by observing that it must satisfy the following requirements:

1. The zero-profit constraints of both innovating firms in period 1 and imitating firms in period 2.

2. Workers at the beginning of period 1 are indifferent between getting a job in period 1 or waiting to get a job in the replacement market in period 2.

3. All workers are employed in period 2.

These requirements uniquely pin down the equilibrium where: (i) a measure 1/2 of innovating firms enter in period 1 and hire half of the work force, (ii) a measure 1/2 of imitating firms enter the market in period 2 and hire all employed workers, and (iii) the innovating firms hire all the remaining available workers in the replacement market.[26]

On average, a worker works in 3/2 periods and produces y per period, and the investment cost per worker is $K/2$. The total wage income over the two periods is then $y \cdot 3/2 - K/2$. If

[26] The wage structure supporting this equilibrium is $w_1 = \frac{1}{2}y - \frac{1}{2}K$, $w_I = y$, and $w_R = \frac{3}{2}y - \frac{1}{2}K$, where w_1 denotes the period-1 wage, w_I the wage paid by imitating firms, and w_R the wage paid by innovating firms to their new hires in period 2. Note, w_1 is negative since $K > y$.

imitation was impossible, all workers would be hired in period 1, and the total wage income would be $2y-K$. Hence, the gain from imitation is $(K-y)/2>0$. It is easy to verify that the Walrasian equilibrium allocation is efficient. This allocation emerges independently of the assumptions made on commitment of innovating firms, as competition between imitating firms always increases the wage paid by imitating firms up to y. Hence, search frictions are key for our inefficiency results; without search frictions the equilibrium is efficient.

Limited Firm Size. As mentioned in the description of the model environment, an important assumption is that a single firm cannot expand indefinitely. As argued below, a maximum capacity of one worker can be thought of as a normalization, the important assumption is that firms are small relative to the market. For this reason, firms in the market earn a rent, which allows them to capitalize on their initial investments. As in many models of monopolistic competition, the scarce factor of production is labor,[27] and firms enter the market up to the point where the tightness of the labor market makes innovation just worthwhile. The most direct interpretation of limited firm size is technological, i.e., that the production function of each firm exhibits decreasing returns to scale. Limited firm size may also be interpreted as a reduced form model of product differentiation under monopolistic competition, as in the standard Dixit-Stiglitz framework. With this interpretation, each innovator creates a new product variety, and aggregate demand for each product is limited. We conjecture that our welfare results will still hold if firms are allowed to price discriminate so that the social value of opening a market is equal to the private value to the firm.

The Diamond-Mortensen-Pissarides framework models a one-good economy. We conjecture that our analysis will still hold if we extend the model to allow for many goods, with downward sloping aggregate demand curves, as long as the individual firms are price takers. The important issue is that the private value and the social value of entering the market coincide. On the other hand, our analysis abstracts from strategic considerations that may arise if firms have market power. Intuitively, one would think that if firms have market power, and this leads to a deadweight loss, imitation may lead to more firms having access to the technology and thereby erode the market power of the innovating firms. This may increase the social value of imitation. Hence, our analysis is less relevant for markets in which firms have substantial market power and where this leads to deadweight losses.

Further, we could allow for multi-worker firms as in Pissarides (2000) and Kaas and Kircher (2011), as long as the firms are small relative to the market and hence act as price takers. Suppose each innovating hires up to n workers, and that the output is proportional to the number of employees up to the capacity limit. For each position, the firm opens one vacancy, which is filled with a probability p_1. Suppose also that all workers in an innovating firm learn about the innovation. Finally, suppose that the innovation cost is nK. It is then straightforward to show that this model is isomorphic to our model, with the same equilibrium characteristics and welfare properties. In particular, the policy recommendations will still hold. Likewise, our model can also easily be

[27] See for instance Melitz (2003).

extended to allow for an expansion of innovating firms, for instance by allowing innovating firms to hire one more worker from the replacement market in period 2. This allows the innovating firms to exploit the non-rivalry of the knowledge use in-house. In all other respects, the model is as before, in particular the incumbent worker does on-the-job search. Technically, the new element of the model is that innovating firms post two vacancies in the replacement market if the incumbent worker has moved on, and one if the incumbent worker stays, instead of one and zero as in the original version. Everything else equal, this will increase the tightness in the replacement market and hence drive up w_2, both with full and limited commitment. This will tend to reduce the amount of entry by imitating firms. The effect on the amount of entry of innovating firms is not clear. On the one hand, the hiring opportunity is also a profit opportunity. This will tend to increase entry. On the other hand, the increased tightness in the replacement market will reduce period-2 profits. In addition, the outside option of available workers in period 1 (which is to enter the replacement market and cash in U_2 in period 2) will increase. The latter two effects go in the direction of a reduced entry. Hence the net effect is unclear. More important, however, is that exactly the same externalities will be present as in the original model. With full commitment, the imitation search market will maximize the profit of the incumbent worker and firm pair, without creating externalities. Hence the equilibrium will be efficient. With limited commitment, the period-2 wages paid by innovating firms to workers with knowledge will be too low to deliver efficiency, and too many imitating firms will enter the market. Hence the inefficiencies analyzed in the original model prevail. Our conjecture is that our policy results also hold with this extension.

Timing of Innovation. We assume that a firm has to hire a worker after it has innovated as opposed to the case where the firm innovates with an already hired worker. Our results, however, do not hinge on this timing assumption. We can easily adjust our model so that innovators have a worker readily available without costs, because she is already hired. The entrepreneur offers her employee a contract that satisfies the worker's participation constraint. We do not expect any qualitative changes in the outcomes from this modification. First, the search stage in period 1 in the original model is not the source of any inefficiencies. Second, the key element of our model, i.e. the search market in period 2, still remains in place.

Multiple Periods. For simplicity our model is set in two periods. We can extend the model so that within each period there are two stages (corresponding to the periods of the model in this paper), first an innovation stage and then an imitation stage. In this extended model the qualitative trade-offs for the firms are the same as in the two-period model, and it can be shown that an equilibrium with full commitment is efficient while an equilibrium with limited commitment is not.[28] Taking our framework to a to an infinite-horizon, endogenous-growth setting is on the agenda for future work. This will allow us to analyze the dynamic effects of policies and would make our framework more comparable to the related models in the growth literature.

[28] A formal model of this extension is available upon request.

10 Conclusion

In this paper we propose a model of innovation, imitation and spillovers through worker mobility, in which the worker flows are explicitly modelled by using the Diamond-Mortensen-Pissarides matching framework with wage posting. We analyze under what circumstances the decentralized equilibrium of the model gives rise to an efficient allocation of resources. We find that the equilibrium is efficient if innovating firms can commit to long-term wage contracts with their workers. In the limited-commitment case, in which such contracts are absent, there is too little innovation and a too high probability of hiring by imitating firms in equilibrium compared with the efficient allocation.

Our model allows us to analyze the effects of various policies, as well as the welfare effects of firm-level measures aimed at reducing turnover. In the limited-commitment case we find that subsidizing innovation and taxing imitation improves welfare. Moreover, allowing innovating firms to charge quit fees, transfer fees, or rent out workers to imitating firms may also improve welfare. By contrast, reducing the efficiency of the search process, through restricting hirings by imitating firms or search for imitation jobs, always reduce welfare.

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11 Appendix

11.1 Derivatives of F

Taking the derivative of the welfare function (1) with respect to θ_1, using $\theta_R = \frac{p(\theta_I)p(\theta_1)+\theta_1(1-q(\theta_1))}{1-p(\theta_1)}$ and the fact that $p'(\theta_i) = (1-\epsilon)q(\theta_i)$, we get

$$\begin{aligned}\frac{\partial F}{\partial \theta_1} &= (1-\epsilon)q(\theta_1)[y_1 + y_2 + p(\theta_I)(y_I - y_2) - c\theta_I] + \frac{d}{d\theta_1}[(1-p(\theta_1))p(\theta_R)y_R] \\ &- (c+K) - c_R[(1-\epsilon)q(\theta_1)(p(\theta_I)-1)+1].\end{aligned}$$

Note that the second summand can be written

$$\begin{aligned}&\frac{d}{d\theta_1}[(1-p(\theta_1))p(\frac{p(\theta_I)p(\theta_1)+\theta_1(1-q(\theta_1))}{1-p(\theta_1)})y_R] \\ &= -(1-\epsilon)q(\theta_1)p(\theta_R)y_R + (1-p(\theta_1))p'(\theta_R)y_R[\frac{(1-\epsilon)q(\theta_1)(p(\theta_I)p(\theta_1)+\theta_1(1-q(\theta_1)))}{(1-p(\theta_1))^2} \\ &+ \frac{(1-\epsilon)q(\theta_1)(p(\theta_I)-1)+1}{1-p(\theta_1)}] \\ &= (1-\epsilon)q(\theta_1)y_R[-\epsilon p(\theta_R) - (1-p(\theta_I))(1-\epsilon)q(\theta_R)] + (1-\epsilon)q(\theta_R)y_R.\end{aligned}$$

Using this, we can write

$$\begin{aligned}\frac{\partial F}{\partial \theta_1} &= (1-\epsilon)q(\theta_1)[y_1 + y_2 + p(\theta_I)(y_I - y_2) - c\theta_I] + (1-\epsilon)q(\theta_1)[-\epsilon p(\theta_R)y_R - (1-p(\theta_I))(1-\epsilon)q(\theta_R)y_R] \\ &+ (1-\epsilon)q(\theta_R)y_R - (c+K) - c_R[(1-\epsilon)q(\theta_1)(p(\theta_I)-1)+1] \\ &= (1-\epsilon)q(\theta_1)[y_1 + y_2 + p(\theta_I)(y_I - y_2) - \epsilon p(\theta_R)y_R - c\theta_I - (1-p(\theta_I))((1-\epsilon)q(\theta_R)y_R - c_R)] \\ &+ (1-\epsilon)q(\theta_R)y_R - c_R - (c+K).\end{aligned}$$

Next, the taking the derivative of F with respect to θ_I we get

$$\begin{aligned}\frac{\partial F}{\partial \theta_I} &= p(\theta_1)[(1-\epsilon)q(\theta_I)(y_I - y_2) - c] + (1-p(\theta_1))(1-\epsilon)q(\theta_R)\frac{d\theta_R}{d\theta_I}y_R - p(\theta_1)(1-\epsilon)q(\theta_I)c_R \\ &= p(\theta_1)[(1-\epsilon)q(\theta_I)(y_I - y_2 + (1-\epsilon)q(\theta_R)y_R - c_R) - c],\end{aligned}$$

where we have used

$$\frac{d\theta_R}{d\theta_I} = (1-\epsilon)q(\theta_I)\frac{p(\theta_1)}{1-p(\theta_1)}.$$

11.2 Optimal wage setting

We first derive w_I for given w_2. Recall that $\{\theta_I, w_I\}$ solves

$$\begin{aligned}&\max_{\theta_I, w_I} \; p(\theta_I)w_I + (1-p(\theta_I))w_2 \\ \text{s. to} \quad &q(\theta_I)(y_I - w_I) - c = 0.\end{aligned}$$

By the definition of the matching function we have $q'(\theta_I) = -\frac{\epsilon q(\theta_I)}{\theta_I}$, and the interior solution gives

$$w_I = \epsilon y_I + (1-\epsilon) w_2.$$

Next, we derive the optimal w_2. The first-order condition of the period-2 problem is:

$$\begin{aligned} \frac{dJ_1}{dw_2} &= \frac{d\hat{p}_I(w_2)}{dw_2}[V_R + w_2 - y_2] + \hat{p}_I(w_2) + \frac{d}{dw_2}[\hat{p}_I(w_2)(\hat{w}_I(w_2) - w_2)] \\ &= 0. \end{aligned} \tag{30}$$

From the envelope theorem it follows that $\frac{d}{dw_2}[\hat{p}_I(w_2)(\hat{w}_I(w_2) - w_2)] = -\hat{p}_I(w_2)$. Thus, the first-order condition with respect to w_2 reduces to

$$w_2 = y_2 - V_R.$$

Next, we establish sufficiency of the first-order condition. First we determine the derivative $\frac{d\hat{p}_I(w_2)}{dw_2}$. Using $p(\theta_I) \equiv \theta_I q(\theta_I)$ together with equation (17), the zero-profit condition of imitating firms can be written

$$\hat{p}_I(w_2) = \frac{\hat{\theta}_I(w_2) c}{(1-\epsilon)(y_I - w_2)}.$$

Totally differentiating this yields

$$d\hat{p}_I(w_2) = \frac{c}{(1-\epsilon)(y_I - w_2)} \left(\frac{\hat{\theta}_I(w_2)}{y_I - w_2} dw_2 + d\hat{\theta}_I(w_2) \right) = \frac{\hat{p}_I(w_2)}{y_I - w_2} dw_2 + q(\hat{\theta}_I(w_2)) d\hat{\theta}_I(w_2),$$

where the second equality uses equation (17) once again. Then note that $dp(\theta_I) = d(\theta_I q(\theta_I)) = q(\theta_I)(1 + \frac{dq(\theta_I)}{d\theta_I}\frac{\theta_I}{q(\theta_I)}) d\theta_I = q(\theta_I)(1-\epsilon) d\theta_I$. Therefore we can reformulate the previous expression to

$$\frac{d\hat{p}_I(w_2)}{dw_2} = -\frac{1-\epsilon}{\epsilon} \frac{\hat{p}_I(w_2)}{y_I - w_2} \leq 0, \tag{31}$$

Then, the second derivative of $\hat{p}_I(w_2)$ is

$$\frac{d^2 \hat{p}_I(w_2)}{(dw_2)^2} = -\frac{1-\epsilon}{\epsilon^2}(2\epsilon - 1) \frac{\hat{p}_I(w_2)}{(y_I - w_2)^2}. \tag{32}$$

It follows that

$$\begin{aligned} \frac{d^2 J_2}{(dw_2)^2} = \frac{d^2 \hat{p}_I(w_2)}{(dw_2)^2}[V_R + (1-\epsilon)(w_2 - y_2)] + 2\frac{d\hat{p}_I(w_2)}{dw_2} < 0 \\ \Leftrightarrow -(2\epsilon - 1)\epsilon(y_2 - w_2) - 2\epsilon(y_I - w_2) < 0, \end{aligned}$$

where we have used that at the optimum $V_R = y_2 - w_2$.

11.3 Proof of Proposition 1

Preliminaries. As the proof considers also the cases at the boundary of the matching function, we here discuss wage determination at these boundaries.

Consider the period-1 market, and suppose that $p_1 \leq 1$ binds, i.e., that $\theta_1 = \theta^{\max}$ with $q(\theta^{\max}) = A^{\frac{1}{1-\epsilon}}$. Then the wage W_1 in the market is implicitly defined by $A^{\frac{1}{1-\epsilon}}(M_1 - W_1) = c + K$, or

$$W_1 = M_1 - (c+K)A^{-\frac{1}{1-\epsilon}}. \tag{33}$$

Suppose instead that the constraint $q_1 \leq 1$ binds, i.e., that $\theta_1 = \theta^{\min}$. The zero profit condition still pins down the wage, and it simply reads that $W_1 = M_1 - c - K$.

Next consider the imitation market. Suppose that at $w_2 = y_2 - V_R$ the constraint $p_I \leq 1$ binds, i.e., that $\theta_I = \theta^{\max}$. If $p_1 = 1$, then w_I is determined by the zero profit condition $A^{\frac{1}{1-\epsilon}}(y_I - w_I) = c$. The wage w_I and M_1 are thus independent of w_2 on intervals where $p_I = 1$ binds, and w_2 is (weakly) optimal. Suppose then instead that constraint $q_I \leq 1$ binds. Then $w_I = y_I - c$. Again the wage is independent of w_2 and $w_2 = y_2 - V_R$ is (weakly) optimal.

In the replacement market there is no free entry of firms. We have

$$V_R = \begin{cases} y_R - c_R & \text{if } \theta_R < \theta^{\min} \\ \in [(1-\epsilon)y_R - c_R, y_R - c_R,] & \text{if } \theta_R = \theta^{\min} \\ q(\theta_R)(1-\epsilon)y_R - c_R & \text{if } \theta^{\min} < \theta_R < \theta^{\max} \\ \in [0, q(\theta_R)(1-\epsilon)y_R - c_R] & \text{if } \theta_R = \theta^{\max} \\ 0 & \text{if } \theta_R > \theta^{\max}. \end{cases} \tag{34}$$

For further reference we give the boundary points of θ and the matching probabilities at these points: $\theta^{\max} = A^{-\frac{1}{1-\epsilon}}$, $\theta^{\min} = A^{\frac{1}{\epsilon}}$, $q(\theta^{\max}) = A^{\frac{1}{1-\epsilon}}$, $p(\theta^{\min}) = A^{\frac{1}{\epsilon}}$. Whenever necessary for clarity we will denote the boundary point by using the index for the respective market, e.g. $q(\theta_R^{\max})$ when considering the replacement market.

Condition for entry of innovating firms. The requirement for existence reads

Requirement A1 K is less than K_A defined as $K_A = y_1 + y_2 + \hat{y}_I - c$.

We want to show that at $\theta_1 = 0$ our assumption ensures that it is profitable to enter ($\hat{y}_I$ is derived below). If $\theta_1 \leq \theta^{\min}$, then $q_1 = 1$. Hence a firm that enters attracts all the workers in the market, and hires a worker with probability 1 for an arbitrarily low expected wage above zero. If the innovating firm places a vacancy in the replacement market, it is also filled with probability 1 at an arbitrarily low wage above zero. Hence $V_R = y_R - c_R$. It follows that the value of on-the-job search is $\hat{y}_I$, which accrues to the firm through a negative period 1 wage. Hence the profit of entering the market is $y_1 + y_2 + \hat{y}_I$, which by definition is greater than the cost $c + K$. Hence $V_1 > 0$.

To complete the requirement, we calculate expressions for $\hat{y}_I$. When the bounds on θ_I do not bind, we get $\hat{y}_I = \epsilon A^{\frac{1}{\epsilon}}(y_I + y_R - y_2 - c_R)^{\frac{1}{\epsilon}}\left(\frac{1-\epsilon}{c}\right)^{\frac{1-\epsilon}{\epsilon}}$. In case the upper bound binds $\theta_I = \theta^{\max}$, and we get $\hat{y}_I = y_I + y_R - y_2 - c_R - cA^{\frac{-1}{1-\epsilon}}$. When the lower bound binds $\theta_I = \theta^{\min}$ we get $\hat{y}_I = A^{\frac{1}{\epsilon}}(y_I + y_R - y_2 - c_R - c)$. Clearly $\hat{y}_I$ is increasing in A, and $A \to 1$ gives an upper bound for $\hat{y}_I$.

Conditions for entry of imitating firms. Next, we will derive conditions for when the two other markets open up. Let $\tilde{V}_R$ denote the threshold value of V_R above which the imitation market is open. The first vacancy that enters attracts all the employed workers at a wage slightly above $y_2 - V_R$. Hence the requirement reads $y_I - y_2 + V_R \geq c$, or that

$$V_R \geq y_2 - y_I + c. \tag{35}$$

As $\theta_R \to \theta^{\min}$, $V_R \to (1-\epsilon)y_R - c_R$. For (35) to hold strictly in the limit, we assume the following

Requirement B1 $(1-\epsilon)y_R - (y_2 - y_I) > c_R + c$.

Requirement B1 is not sufficient, as the existence of leftover-vacancies from period 1 may imply that θ_R is strictly greater than $\theta^{\min}$ even without entry of imitation firms.

To ensure entry of imitating firms, define $\theta_R^b > \theta^{\min}$ as the highest value of θ_R for which an imitating firm breaks even, i.e., for which condition (35) holds with equality. Either $\theta_R^b < \theta^{\max}$ or $\theta_R^b = \theta^{\max}$. Consider the case where $\theta_R^b < \theta^{\max}$, in which case $V(\theta_R) = A\theta_R^{-\epsilon}(1-\epsilon)y_R - c_R$. Hence the value θ_R^b such that $V_R(\theta_R^b) = y_2 - y_I - c$ reads

$$\theta_R^b = \left(\frac{y_2 - y_I + c + c_R}{Ay_R(1-\epsilon)}\right)^{-\frac{1}{\epsilon}} \tag{36}$$

We want to characterize the corresponding value of θ_1. With $\theta_I = 0$, and noting that $A\theta_1^{1-\epsilon} = \theta_1 A\theta_1^{-\epsilon}$, we generally have that

$$\theta_R = \theta_1 \frac{1 - A\theta_1^{-\epsilon}}{1 - A\theta_1^{1-\epsilon}}. \tag{37}$$

Note that θ_R is strictly increasing in θ_1, is 0 at $\theta_1 = \theta^{\min}$, and goes to infinity as $\theta_1 \to \theta^{\max}$. Hence $\theta_R(\theta_1)$ defined by (37) has an inverse function, which can be written as $\theta_1 = g(\theta_R)$, where g is increasing in θ_R. Note that $g(1) = 1$. Define $\theta_1^b > \theta^{\min}$ as

$$\begin{aligned} \theta_1^b &= g(\theta_R^b) \\ &= g\left(\left(\frac{y_2 - y_I + c + c_R}{Ay_R(1-\epsilon)}\right)^{-\frac{1}{\epsilon}}\right). \end{aligned} \tag{38}$$

Since θ_R is increasing in θ_1, we know that V_R is above the break even point if firms find it strictly unprofitable to enter at θ_1^b. Since $U_2 = A\theta_R^{b1-\epsilon}\epsilon y_R$, $V_R = y_2 - y_I + c$, and no imitation, we have that

$$V_1(\theta_1^b) = A(\theta_1^b)^{-\epsilon}((1-\epsilon)[y_1 + y_2 - A(\theta_R^b)^{1-\epsilon}\epsilon y_R - y_2 + y_I - c] + y_2 - y_I + c - (c + K) < 0 \tag{39}$$

is required and get the following condition:

Requirement B2 K is strictly greater than K_B defined as

$$K_B = A(\theta_1^b)^{-\epsilon}((1-\epsilon)[y_1 + y_2 - A(\theta_R^b)^{1-\epsilon}\epsilon y_R - y_2 + y_I - c] + y_2 - y_I,$$

where $\theta_1^b = g(\theta_R^b)$ is defined by (38).

The requirement A1 ensures that the economy is sufficiently productive, so that firms enter to innovate. Requirement B2 ensures that entering as an innovating firm is not too attractive, so that the replacement market is not too tight. Clearly, there exists an interval of K-values such that both requirements are satisfied.

As an example, suppose $A = 1/2$, $\epsilon = 1/2$, and $y_2 = y_I = y_R = y$. We can construct a set of parameters as follows: Choose any value of θ_R^b, say $\theta_R^b = 1$. In order for firms to be willing to enter at $\theta_R = 1$, it follows from (36) that

$$y \geq 4(c + c_R).$$

Further, it follows that $\theta_1^b = g(1) = 1$, and hence from (39) that

$$\frac{7}{16}y < \frac{1}{2}c + K$$

which clearly can be satisfied together with with A1.

Existence and uniqueness. We want to derive the value of entering, V_1, as a continuous and strictly decreasing function of θ_1 on $[0, \theta^{\max})$. The case with $\theta_1 = \theta^{\max}$ will be treated separately.

Then, to show existence and uniqueness of the overall equilibrium define

$$M_2(\theta_1) = y_2 + \max_{\theta_I, w_I \text{ s. to } V_I = 0} \{p(\theta_I)[V_R + w_I - y_2]\} \tag{40}$$

$$\theta_R = \frac{p(\theta_1)p(\theta_I) + \theta_1(1 - q(\theta_1))}{1 - p(\theta_1)} \tag{41}$$

$$V_R = V_R(\theta_R) \text{ is defined by (34).} \tag{42}$$

Lemma 5 *The system of equations (40)-(42) has a unique solution $(\theta_I(\theta_1), \theta_R(\theta_1), M_2(\theta_1))$ for all $\theta_1 \geq 0$. Furthermore, $\theta_R(\theta_1)$ is continuous and increasing in θ_1 while $M_2(\theta_1)$ and $\theta_I(\theta_1)$ are continuous and decreasing in θ_1.*

Proof. For a given θ_1, θ_I is increasing in V_R (strictly increasing if the bounds on the matching function do not bind), θ_R is strictly increasing in θ_I, and V_R is decreasing in θ_R (strictly decreasing if the bounds do not bind). It follows that we can write $\theta_I = f(\theta_I; \theta_1)$, where f is a decreasing function in θ_I, with $f(0; \theta_1) \geq 0$. Furthermore, $f(\theta_I; \theta_1)$ is bounded. Hence the the system of equations has a solution provided that f is continuous. Furthermore, an increase in θ_1 increases θ_R for a given θ_I, and hence shifts $f(\theta_I; \theta_1)$ down. It follows that $\theta_I(\theta_1)$ is decreasing in θ_1 (strictly decreasing if the bounds do not bind). It follows directly that $\theta_R(\theta_1)$ is increasing in θ_1 (strictly increasing if the bounds do not bind), the opposite is inconsistent with $\theta_I(\theta_1)$ being decreasing in θ_1. A complicating factor is that f need not be continuous when the bounds on the matching function bind. This is what we now will deal with.

First note that if $\theta_1 > g(\theta_R^b)$ defined by (38), $\theta_I = 0$. It follows that $M_2 = y_2$ and that $\theta_R = \frac{\theta_1(1-q(\theta_1))}{1-p(\theta_1)}$. Hence the lemma trivially holds in this case. Consider therefore a case where $\theta_1 \leq g(\theta_R^b)$.

Note that θ_I is bounded from above for all $\theta_1 \geq 0$. It follows from (41) that for sufficiently small values of θ_1, $\theta_R < \theta^{\min}$. For these low values of θ_1, $V_R = y_R - c_R$, which is independent of θ_1. Denote the corresponding value of θ_I by θ_I^1. Now define $\underline{\theta}_1$ by the equation

$$\frac{p(\underline{\theta}_1)p(\theta_I^1) + \underline{\theta}_1(1-q(\underline{\theta}_1))}{1-p(\underline{\theta}_1)} = \theta^{\min}.$$

Clearly $\underline{\theta}_1$ is well defined (also if $\theta_I^1 = 0$ or if the bound on p_I binds). It follows that for $\theta_1 \in [0, \underline{\theta}_1]$, the maximization problem in (40) is well defined, and that $M_2(\theta_1)$ is constant, $\theta_I(\theta_1) = \theta_I^1$, while θ_R defined by (41) is strictly increasing and continuous. If $\theta_I^1 = 0$, the lemma holds trivially, we therefore look at the case for which $\theta_I^1 > 0$.

We know that V_R is discontinuous at the boundary points for θ_R. From (34) we know that $\lim_{\theta_R \to \theta^{\min}-} V(\theta_R) = y_R - c_R$, while $\lim_{\theta_R \to \theta^{\min}+} V(\theta_R) = (1-\epsilon)y_R - c_R$. Also at the latter point, θ_I is well defined, and denote the solution by $\theta_I^2 \geq 0$. Clearly $\theta_I^1 > \theta_I^2$. Suppose first that $\theta_I^2 > 0$. Define $\overline{\theta}_1$ as the solution to

$$\frac{p(\theta_I^2)p(\overline{\theta}_1) + \overline{\theta}_1(1-q(\overline{\theta}_1))}{1-p(\overline{\theta}_1)} = \theta^{\min}.$$

Clearly, $\overline{\theta}_1$ is well defined, and $\overline{\theta}_1 > \underline{\theta}_1$. If $\theta_I^2 = 0$, define θ_R^1 as the lowest value of θ_R for which $\theta_I = 0$, and define $\overline{\theta}_1 = g(\theta_R^1)$.

For any given $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$, $\theta_I(\theta_1)$ is defined as unique value of θ_I which in combination with θ_1 gives $\theta_R = \theta^{\min}$, i.e., the unique solution to the equation

$$\frac{p(\theta_1)p(\theta_I) + \theta_1(1-q(\theta_1))}{1-p(\theta_1)} = \theta^{\min}.$$

This is just a mechanical relationship. It follows straightforwardly that $\theta_I(\theta_1)$ is strictly decreasing and continuous. Let $V_R^n(\theta_1)$ on the interval $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$ be the unique value such that

$$\max_{\theta_I, w_I \quad s.to \ V_I=0} p(\theta_I)[V_R(\theta_1) + w_I - y_2]$$

has the solution $\theta_I(\theta_1)$. Then, as θ_I is decreasing in θ_1, and θ_I is increasing in V_R, it follows that V_R is decreasing in θ_1. From the definition of θ_I it follows that $V_R^n(\theta_1)$ is strictly continuous and strictly decreasing in $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$. Hence $V_1(\theta_1)$ is continuous and strictly decreasing for $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$. It follows that the lemma also holds in this case.

As $\theta_R \to \theta^{\max -}$, V_R converges to $A^{\frac{1}{1-\epsilon}}(1-\epsilon)y_R - c_R$. Let $\theta_I^3 \geq 0$ be the associated value of θ_I. Define $\underline{\theta}_1^h$ by the equation

$$\frac{p(\underline{\theta}_1^h)p(\theta_I^3) + \underline{\theta}_1^h(1-q(\underline{\theta}_1^h))}{1-p(\underline{\theta}_1^h)} = \theta^{\max}.$$

Suppose $\theta_1 \in (\overline{\theta}_1, \underline{\theta}_1^h)$ (and hence $\theta_R \in (\theta^{\min}, \theta^{\max})$). We will show that θ_R is strictly increasing in θ_1 on this interval. Suppose not. Then V_R and hence θ_I is non-decreasing in θ_1. But then it follows from (41) that θ_R is increasing in θ_1, a contradiction. It follows that we can write θ_R as an increasing function of θ_1, $\theta_R = \theta_R(\theta_1)$. From the envelope theorem it follows that

$$\frac{dM_2(\theta_1)}{d\theta_1} = p_I V_R'(\theta_R)\frac{d\theta_R}{d\theta_1} \leq 0$$

independently of whether the bounds on p_I binds or not, with strict inequality if $p_I > 0$. Hence the lemma holds also in this case.

If $\theta_I = 0$ at $\underline{\theta}_1^h$, the lemma holds. Consider therefore the case where $\theta_I > 0$. The analysis in this case is analogous to the analysis at $\theta^{\min}$. Let $\overline{\theta}_1^h = g(\theta^{\max})$ defined by (38). Then we know that $\theta_I(\underline{\theta}_1^h) = \theta_I^3$ and that $\theta_I(\overline{\theta}_1^h) = 0$. At the interval $[\underline{\theta}_1^h, \overline{\theta}_1^h]$, $\theta_I(\theta_1)$ is defined exactly in the same way as on the interval $[\underline{\theta}_1, \overline{\theta}_1]$. Hence $V_R(\theta_1)$ is continuous and strictly decreasing at $\theta_1 \in [\underline{\theta}_1^h, \overline{\theta}_1^h]$. It follows that the lemma also holds in this case. This completes the proof of the lemma. ■

It follows from Lemma 5 that we can write V_R and U_2 as function of θ_1. With some abuse of notation we write $V_R = V_R(\theta_1)$ and $U_2 = U_2(\theta_1)$, where V_R is decreasing and U_2 is increasing in θ_1. Suppose first that $\theta_1 < \underline{\theta}_1$. Then we know that $V_1(\theta_1) = y_1 + M_2(\theta_1) - U_2(\theta_1)$. Note that $V_1(0) = y_1 + M_2 - c - K > 0$ due to assumption A1. Up to $\min(\theta^{\min}, \underline{\theta}_1)$, $V_1(\theta_1)$ is constant and equal to $V_1(0)$, and A1 ensures that the equilibrium will not be in this interval. If $\underline{\theta}_1 < \theta^{\min}$, then $V_1(\theta_1)$ is continuous and decreasing on the interval $(\underline{\theta}_1, \theta^{\min})$. At $\theta^{\min}$, $V_1(\theta_1)$ has a discontinuity point, as

$$\lim_{\theta_1 \to \theta^{\min -}} V_1(\theta_1) = y_1 + M_2(\theta_1^{\min}) - U_2(\theta_1^{\min}) - c - K \tag{43}$$

$$\lim_{\theta_1 \to \theta^{\min +}} V_1(\theta_1) = (1-\epsilon)(y_1 + M_2(\theta_1^{\min}) - U_2(\theta_1^{\min}) - V_R(\theta_1^{\min})) + V_R(\theta_1^{\min}) - c - K \tag{44}$$

For $\theta_1 \in (\theta^{\min}, \theta^{\max})$ and for $\theta_I > 0$ we have that

$$V_1(\theta_1) = q(\theta_1)(1-\epsilon)(y_1 + M_2(\theta_1) - U_2(\theta_1) - V_R(\theta_1)) + V_R(\theta_1) - c - K$$

From Lemma 5 we see that $V_1(\theta_1)$ is continuous and decreasing in this case.

Consider then the discontinuity point at $\theta_1 = \theta^{\max}$. We have that

$$\lim_{\theta_1 \to \theta^{\max}-} V_1(\theta_1) = (1-\epsilon)q(\theta^{\max})(y_1 + M_2(\theta_1^{\max}) - U_2(\theta_1^{\max})) - c - K \tag{45}$$

$$\lim_{\theta_1 \to \theta^{\max}+} V_1(\theta_1) = -c - K \tag{46}$$

When there are no imitation vacancies around, we know that $V_R(\theta_1)$ will be discontinuous at the point where $\theta_R = \theta^{\min}$ and $\theta_R = \theta^{\max}$, and that $V_1(\theta_1)$ is discontinuous at $\theta^{\max}$. However, at all the discontinuity points V_1 jumps down. Furthermore, $V_1(0) > 0$ while $V_1(\theta) = -c + K$ for $\theta_1 > \theta^{\max}$. Hence we know that if the equilibrium exists, it is unique. We only have to show that equilibrium exists at the discontinuity points.

Consider first the discontinuity point at $\theta_1 = \theta^{\min}$. Denote the RHS of equation (43) by $V_1^-(\theta^{\min})$, and the RHS of (44) by $V_1^+(\theta^{\min})$. Suppose $V_1^+(\theta^{\min}) < 0 < V_1^-(\theta^{\min})$. Then the equilibrium of the model is obtained at $W_1 = V_1^-(\theta^{\min})$. Again the equilibrium exists and is unique.

If $\lim_{\theta_1 \to \theta^{\max}-} V_1(\theta_1) > c + K$, then $W_1 = q(\theta^{\max})(y_1 + M_2(\theta_1^{\max}) - U_2(\theta_1^{\max})) - c - K$. Again the equilibrium exists and is unique.

Finally, V_R and hence V_1 may be discontinuous at $\theta_R = \theta^{\min}$ and $\theta_R = \theta^{\max}$ if $\theta_I = 0$ at that point. Suppose $\hat{\theta}_1$ is such that $\lim_{\theta_1 \to \hat{\theta}_1^-} \theta_R(\theta_1) = \theta^{\max}$. If there are no imitation vacancies at this point, $V_R(\theta_1)$ jumps from $(1-\epsilon)q(\theta^{\max})y_R - c_R = \hat{V}$ to zero at this point. Suppose $\lim_{\theta_1 \to \hat{\theta}_1^-} V_1(\theta_1) > 0$ while $\lim_{\theta_1 \to \hat{\theta}^+} V_1(\theta_1) < 0$. In this case, w_R adjusts so that $V_1(\hat{\theta}_1) = 0$. Again the equilibrium is unique. A discontinuity at $\theta_R = \theta^{\min}$ is treated analogously. This completes the proof.

Conditions for interior solution. In addition to the requirements for existence and for entry of imitating firms, we will now give restrictions that ensure interior solution in all markets, i.e. that $\theta \in (\theta^{\min}, \theta^{\max})$ in all markets.

Suppose $\theta_R \in (\theta^{\min}, \theta^{\max})$. Below we will give conditions for when this is the case.

Above we derived sufficient conditions to ensure that $\theta_I > 0$. Now we have to derive sufficient conditions to ensure that $\theta_I \in (\theta^{\min}, \theta^{\max})$. Recall that $V_R(\theta_R) = A\theta_R^{-\epsilon}(1-\epsilon)y_R - c_R$, that $V_R(\theta_R)$ is decreasing in θ_R, and that

$$\lim_{\theta_R \to \theta^{\min}+} V_R(\theta_R) = (1-\epsilon)y_R - c_R \tag{47}$$

$$\lim_{\theta_R \to \theta^{\max}-} V_R(\theta_R) = A^{\frac{1}{1-\epsilon}}(1-\epsilon)y_R - c_R. \tag{48}$$

Note that $\theta_I < \theta^{\max}$ if

$$(1-\epsilon)A^{\frac{1}{1-\epsilon}}(y_I - y_2 + V_R) < c. \tag{49}$$

The maximum interior value of V_R is given by (47). With this value of V_R, (49) reads

$$(1-\epsilon)A^{\frac{1}{1-\epsilon}}[(1-\epsilon)y_R - c_R - (y_2 - y_I)] < c, \tag{50}$$

and we have the following requirement:

Requirement C1 $A^{\frac{1}{1-\epsilon}}[(1-\epsilon)y_R - c_R - (y_2 - y_I)] < \frac{c}{1-\epsilon}$.

Note that $\theta_I > \theta^{\min}$ if

$$(1-\epsilon)(y_I - y_2 + V_R) > c. \tag{51}$$

The maximum value of V_R is given by (47), which inserted into (51) gives that

$$\frac{c}{1-\epsilon} < (1-\epsilon)y_R - (y_2 - y_I) - c_R.$$

A necessary condition thus reads as follows:

Requirement C2 $(1-\epsilon)y_R - (y_2 - y_I) - c_R > \frac{c}{1-\epsilon}$.

Since $A^{\frac{1}{1-\epsilon}} < 1$, C2 is clearly consistent with C1.

If we insert the minimum value of V_R, given by (48), into (51), the requirement that $\theta_I > \theta^{\min}$ reads

$$(1-\epsilon)A^{\frac{1}{1-\epsilon}}y_R - (y_2 - y_I) - c_R > \frac{c}{1-\epsilon},$$

which is clearly violating C1. It follows that the equation $A(\theta_R^c)^{-\epsilon}(1-\epsilon)y_R - c_R = \frac{c}{1-\epsilon} + y_2 - y_I$ has a solution $\theta_R^c \in (\theta^{\min}, \theta^{\max})$ given by

$$\theta_R^c = (\frac{y_2 - y_I + c/(1-\epsilon) + c_R}{Ay_R(1-\epsilon)})^{-\frac{1}{\epsilon}}.$$

We want to characterize the highest value of θ_1 consistent with $\theta_R \leq \theta_R^c$. We denote this value of θ_1 by θ_1^c. To this end, recall that in the limit, as θ_R goes to θ_R^c from below, θ_I converges to $\theta^{\min}$, in which case $p(\theta_I)$ converges to $p^{\min} = A^{\frac{1}{\epsilon}}$. Hence we have that θ_1^c is given by the equation

$$\begin{aligned}\theta_R^c &= \frac{p(\theta_1^c)p^{\min} + \theta_1^c - p(\theta_1^c))}{1-p(\theta_1^c)} \\ &= \frac{A(\theta_1^c)^{1-\epsilon}A^{\frac{1}{\epsilon}} + \theta_1^c - A(\theta_1^c)^{1-\epsilon}}{1-A(\theta_1^c)^{1-\epsilon}}.\end{aligned}$$

The right-hand side is increasing in θ_1^c, is zero for $\theta_1^c = 0$ and goes to infinity as $\theta_1^c \to \theta^{\max} = A^{-\frac{1}{1-\epsilon}}$. Furthermore, if $\theta_1^c = \theta^{\min}$, then $\theta_R < \theta^{\min}$.[29] It follows that we can write $\theta_1^c = g^c(\theta_R^c)$, where $\theta_1^c \in (\theta^{\min}, \theta^{\max})$. The gross value of a vacancy, evaluated at θ_1^c, reads

$$V(\theta_1^c) = q_1(\theta_1^c)(1-\epsilon)[y_1 + y_2 + p(\theta_I)(w_I - (y_2 - V_R)) - U_2(\theta_R^c) - V_R] + V_R - c - K$$

[29] Recall that $\theta^{\min} = \theta^{\min}q(\theta^{\min}) = p(\theta^{\min})$. Suppose $\theta_1^c = \theta^{\min}$. Then $\theta_R = \frac{p(\theta^{\min})^2}{1-p(\theta^{\min})} = \frac{(\theta^{\min})^2}{1-\theta^{\min}} < \theta^{\min}$ since $\theta^{\min} < 1/2$.

At θ_1^c, $V_R = \frac{c}{1-\epsilon} + y_2 - y_I$ and $(y_I - w_I) = c$, it follows that $(w_I - (y_2 - V_R)) = \frac{\epsilon}{1-\epsilon}c$. Furthermore, $U_2 = A(\theta_R^c)^{\frac{1}{\epsilon}}(1-\epsilon)y_R$ and $p_I(\theta^{\min}) = \theta^{\min} = A^{\frac{1}{\epsilon}}$. Inserted this gives

$$V(\theta_1^c) = A(\theta_1^c)^{-\epsilon}(1-\epsilon)[y_1+y_I+A^{\frac{1}{\epsilon}}\frac{\epsilon}{1-\epsilon}c-\frac{c}{1-\epsilon}-A(\theta_R^c)^{1-\epsilon}(1-\epsilon)y_R]+\frac{c}{1-\epsilon}+y_2-y_I-c-K. \quad (52)$$

Then define K_C as the value of K that makes $V(\theta_1^c) = 0$ (clearly K_C depends on the other parameters in the model). We have the following:

Requirement C3 $K > K_C$.

Finally, define $\theta_1^{c\min} = g^c(\theta^{\min})$. It follows that $\theta_1^{c\min} > \theta^{\min}$. We want to ensure that $\theta_1 > \theta_1^{c\min}$. To this end, note that θ_R is increasing in θ_1. Hence a lower bound on $V(\theta_1^{c\min})$ is given by

$$\tilde{V}(\theta_1^{c\min}) = A(\theta_1^{c\min})^{-\epsilon}(1-\epsilon)[y_1+y_I+A^{\frac{1}{\epsilon}}\frac{\epsilon}{1-\epsilon}c-\frac{c}{1-\epsilon}-A(\theta_R^c)^{1-\epsilon}(1-\epsilon)y_R]+\frac{c}{1-\epsilon}+y_2-y_I-c-K.$$

Clearly, $\tilde{V}(\theta_1^{c\min}) > V(\theta_1^c)$. Define K_{C2} as the value of K that makes $\tilde{V}(\theta_1^{c\min}) = 0$.

Requirement C4 $K < K_{C2}$.

By construction we have shown that an interior solution exists whenever requirement C1, C2, C3, and C4 are all satisfied.

11.4 Proof of Lemma 1

The first-order condition of the firm's period-2 problem reads

$$\frac{dJ_2}{dw_2} = \hat{p}_I(w_2) - 1 + \frac{d\hat{p}_I(w_2)}{dw_2}[V_R + w_2 - y_2] = 0. \quad (53)$$

Substituting in the expression for $\frac{d\hat{p}_I(w_2)}{dw_2}$ from (31) and solving for w_2 gives

$$w_2 = y_I - \frac{\hat{p}_I(w_2)(1-\epsilon)}{\hat{p}_I(w_2) - \epsilon}\big(y_I - y_2 + V_R\big),$$

where $\hat{p}_I(w_2)$ is determined by the zero profit condition of the imitating firms:

$$q(\hat{\theta}_I(w_2)) = \frac{c}{(1-\epsilon)(y_I - w_2)}. \quad (54)$$

We can combine the two equations to:

$$q(\hat{\theta}_I(w_2)) = \frac{c}{(1-\epsilon)^2(y_I - y_2 + V_R)}[1 - \frac{\epsilon}{p(\hat{\theta}_I(w_2))}].$$

The left hand side is decreasing in θ_I, starting from 1 at $\theta_I = 0$ and approaching 0 as $\theta_I \to \infty$. The right hand is non-negative only if $p(\theta_I) \geq \epsilon$ as by assumption we must have $y_I - y_2 + V_R > 0$. For θ_I above this threshold, the right-hand side is increasing until $p(\theta_I) = 1$, where it reaches the value $c/[(1-\epsilon)(y_I - y_2 + V_R)] > 0$. Thus, a unique intersection exists. Note that the intersection point moves to the left as V_R decreases so that $\frac{p(\theta_I)(1-\epsilon)}{p(\theta_I)-\epsilon}$ decreases. Hence, w_2 increases in θ_R. It is easy to show that for given θ_R the interior wage w_2 is smaller than in the full-commitment case.

If the wage w_2 is at its lower bound U_2, it increases in θ_R since U_2 increases in θ_R. That $U_2 < y_2 - V_2$, for given θ_R, follows from the definitions of the terms together with the fact that $y_2 > \epsilon y_2 = w_R$.

Finally, we establish sufficiency at the interior solution for w_2. We need to show that

$$\frac{d^2 J_2}{(dw_2)^2} = \frac{d^2 \hat{p}_I(w_2)}{(dw_2)^2}[V_R + w_2 - y_2] + 2\frac{d\hat{p}_I(w_2)}{dw_2} < 0.$$

Using the expression for $\frac{d\hat{p}_I(w_2)}{dw_2}$ from (31) and the second derivative analogous to (32):

$$\frac{d^2 \hat{p}_I(w_2)}{(dw_2)^2} = -\frac{1-\epsilon}{\epsilon^2}(2\epsilon - 1)\frac{\hat{p}_I(w_2)}{(y_I - w_2)^2},$$

we get

$$\begin{aligned}\frac{d^2 J_2}{(dw_2)^2} &= -\frac{1-\epsilon}{\epsilon}\frac{\hat{p}_I(w_2)}{y_I - w_2}\left[(\frac{2\epsilon - 1}{\epsilon})(\frac{V_R - y_2 + w_2}{y_I - w_2} - 1) + 2\right] < 0 \\ &\Leftrightarrow (2\epsilon - 1)(V_R - y_2 + w_2) + 2\epsilon(y_I - w_2) > 0 \\ &\Leftrightarrow 2\epsilon(V_R - y_2 + y_I) > w_2 + V_R - y_2.\end{aligned}$$

When the imitation market is active, we have $y_2 - V_R < y_I$ and the wage $w_2 < y_2 - V_R$. Thus it follows that the left hand side is larger zero, and the that left hand side is less than zero. This completes the proof.

11.5 Proof of Proposition 2

Condition for entry of innovating firms. The first innovating firm that enters attracts a worker with probability 1 at an arbitrarily low wage. In the replacement market, the innovating firm also attracts a worker with probability 1 at an arbitrarily low wage. Hence $V_R = y_R - c_R$ and $U_2 = 0$. The lower bound on w_2 is thus zero, and the lower bound on w_I is $y_2 - V_R = y_2 - y_R + c_R$. Define

$$(\tilde{\theta}_I, \tilde{w}_I) = \arg\max_{\theta_I, w_I} A\theta_I^{1-\epsilon} w_I \quad \text{s.to} \quad A\theta_I^{-\epsilon}(y_I - w_I) \geq c,\ w_I \geq y_2 - y_R + c_R,\ \theta^{\min} \leq \theta_I \leq \theta^{\max}.$$

Then define $\hat{y}_I^c$ as

$$\hat{y}_I^c = A\tilde{\theta}_I^{1-\epsilon}(\tilde{w}_I - y_2 + y_R - c_R).$$

The firm finds it profitable to enter whenever $y_1 + y_2 + \hat{y}_I^c \geq c + K$.

Requirement D1 $y_1 + y_2 + \hat{y}_I^c \geq K$.

This condition may be more restrictive than condition A1 since the value of on-the-job search may be smaller without full-commitment.

Conditions for entry of imitating firms. Consider then the conditions for entry of imitating firms. Suppose there is no entry of imitating firms, and consider the first firm that enters. At $\theta_I = 0$, a firm that enters will attract infinitely many workers at wage $w_I = y_2 - V_R$. Since the innovating firm matches lower offers, imitating firms cannot hire workers at a lower wage. With full-commitment, the equilibrium imitation wage w_I is also $y_2 - V_R$ in the limit as $\theta_I \to 0$. Hence, in the limit as $\theta_I \to 0$, the models with limited and and full-commitment are isomorphic. Hence conditions D1, B1 and B2 are sufficient to ensure the existence of an equilibrium with entry of imitating firms.

Conditions for interior solution. We start out assuming that $\theta_R \in (\theta^{\min}, \theta^{\max})$, and then derive sufficient conditions under which this is true later on. For convenience we repeat the expression for the period-2 wage

$$w_2 = y_I - \frac{\hat{p}_I(w_2)(1-\epsilon)}{\hat{p}_I(w_2) - \epsilon}(y_I - y_2 + V_R).$$

First note that if $\hat{p} \to 1$, then w_2 converges to $w_2 = y_2 - V_R$, as in the full-commitment case. It follows that the equilibrium converges to the full-commitment equilibrium. Hence condition C1 ensures that $\theta_I < \theta^{\max}$. Second, as will be shown below, θ_I in the limited-commitment case is greater than in the full-commitment case. Hence conditions C2-C4 ensure that $\theta_I > \theta^{\min}$.

Although not necessary, it is convenient if $w_2 = U_2$ at $\hat{p}_I = p(\theta^{\min})$. This is surely the case if $p(\theta^{\min}) \leq \epsilon$, or that $A \leq \epsilon^{\epsilon}$. This is a mild restriction on the matching function.

Requirement E1 $A \leq \epsilon^{\epsilon}$.

A necessary condition for $\theta_I > \theta^{\min}$ is that $(1-\epsilon)(y_I - U_2(\theta_R^{\min}) > c$, giving

Requirement E2 $(1-\epsilon)(y_I - A^{\frac{1}{\epsilon}}\epsilon y_R) > c$.

Although not necessary, it is convenient to ensure that $\theta_I < \theta^{\min}$ at $\theta_R = \theta^{\max}$. This gives

Requirement E3 $(1-\epsilon)(y_I - \epsilon y_R) < c$.

Now define $\theta_R^d \in (\theta^{\min}, \theta^{\max})$ as the solution to the equation $(1-\epsilon)(y_I - p(\theta_R^d)\epsilon y_R) = c$, and define $\theta_1^d = g^c(\theta_R^d)$, where g^c is defined in the section on the interior solution of the full-commitment case. The value of entering as an innovating firm at θ_1^d is

$$V(\theta_1^d) = q_1(\theta_1^d)(1-\epsilon)[y_1 + y_2 + p(\theta_I)(w_I - (y_2 - V_R)) - U_2 - V_R] + V_R - c - K.$$

At θ_1^d, $(1-\epsilon)(y_I - U_2) = c$, hence $U_2 = y_I - \frac{c}{1-\epsilon}$. Furthermore, $(w_I - U_2) = \frac{\epsilon}{1-\epsilon}c$. Hence $w_I = U_2 + \frac{\epsilon}{1-\epsilon}c = y_I - c$. The gain from search is thus $p^{\min}(y_I - y_2 - c + V_R)$. Finally, $V_R =$

$(1-\epsilon)A(\theta_R^d)^{-\epsilon}y_R - c_R$. Recall that $p(\theta^{\min}) = \theta^{\min} = A^{1/\epsilon}$. Inserted this gives

$$\begin{aligned} V(\theta_1^d) =& A(\theta_1^d)^{-\epsilon}(1-\epsilon)[y_1 + y_2 - y_I + A^{1/\epsilon}(y_I - y_2 - c) + \frac{c}{1-\epsilon} - \\ & (1 - A^{1/\epsilon})((1-\epsilon)A(\theta_R^d)^{-\epsilon}y_R - c_R)] + (1-\epsilon)A(\theta_R^d)^{-\epsilon}y_R - c_R - c - K. \end{aligned} \tag{55}$$

Define K_D as the value of K that makes $V(\theta_1^d) = 0$

Requirement E4 $K > K_D$.

It follows that for $K > K_D$, $\theta_I > \theta^{\min}$. Still θ_R is increasing in θ_1, and we have to make sure that θ_1 is sufficiently high so that $\theta_R > \theta^{\min}$. We do this in the same way as in the full-commitment case. First, define $\tilde{\theta}_1^d = g^c(\theta^{\min})$. It follows that if $\theta_1 \geq \tilde{\theta}_1^d$ in equilibrium, then $\theta_R > \theta^{\min}$. Let $\tilde{V}(\theta_1)$ denote the value of entering at $\tilde{\theta}_1^d$. Define K_{D2} as the value of K that makes $\tilde{V}(\theta_1) = 0$. By construction, $K_{D2} > K_D$.

Requirement E5 $K < K_{D2}$.

By construction, it follows that the equilibrium has an interior solution if E1-E5 are satisfied. Note that these are sufficient, not necessary conditions.

Existence and uniqueness For a given θ_1, the period-2 equilibrium is given by equations (41)-(42), and

$$\theta_I \text{ solves} \max_{\theta_I, w_I \ s.\ to\ V_I = 0, w_I \geq y_2 - V_R} \{p(\theta_I)[w_I - w_2^{lc}]\} \tag{56}$$

$$w_2^{lc} \in \arg\max \quad (1 - p(\theta_I))(y_2 - w_2) + p(\theta_I)V_R \ \text{ S.T. } w_2 \geq U_2 \tag{57}$$

$$M_2 = y_2 + p(\theta_I)(w_I - y_2 + V_R). \tag{58}$$

We first establish the following lemma, parallel to Lemma 5:

Lemma 6 *The system of equations (41)-(42) and (56)-(58) has a unique solution* $(\theta_I(\theta_1), \theta_R(\theta_1), M_2(\theta_1))$. *Furthermore,* $\theta_R(\theta_1)$ *is continuous and increasing in* θ_1 *while* $\theta_I(\theta_1)$ *is continuous and decreasing in* θ_1.

Proof. We know that θ_I is decreasing in w_2^{lc} (strictly if the bounds on the matching function do not bind). From Lemma 1 we know that w_2^{lc} is increasing in θ_R. For a given θ_1, θ_R is strictly increasing in θ_I. Hence the solution to the system of equations can be written as $\theta_I = \tilde{f}(\theta_I; \theta_1)$. Given assumption E1-E5, the equilibrium has an interior solution, hence $\lim_{\theta_I \to \theta^{\min}+} \tilde{f}(\theta_I; \theta_1) > \theta_I$ and $\lim_{\theta_I \to \theta^{\max}-} \tilde{f}(\theta_I; \theta_1) < \theta_I$ for the relevant values of θ_1. It follows that the solution, if it exists, is unique. Furthermore, an increase in θ_1 leads to an increase in θ_R for a given θ_I, and hence to a negative shift in $\tilde{f}$. Hence $\theta_I(\theta_1)$ is decreasing in θ_1 (strictly if the bounds do not bind). It follows directly that $\theta_R(\theta_1)$ is increasing in θ_1(strictly if the bounds do not bind), the opposite is inconsistent with $\theta_I(\theta_1)$ being decreasing in θ_1.

As indicated in footnote 17 the function $w_2^{lc}(\theta_R)$ can be discontinuous at one point, at which point w_2^{lc} jumps from U_2 to the interior solution $\tilde{w}_2$. At this point, firms are indifferent between setting $w_2 = U_2$ and the interior solution $\tilde{w}_2$. In this case firms randomize between setting $w_2 = U_2$ with probability π and $w_2 = \tilde{w}_2$ with probability $1 - \pi$. To be more specific, suppose the point of discontinuity is at $\bar{\theta}_R$. Let $\underline{\theta}_1$ be defined by $\lim_{\theta_R \to \bar{\theta}_R^-} \theta_R(\theta_1) = \bar{\theta}_R$. Let $\bar{\theta}_1$ be defined by $\lim_{\theta_R \to \bar{\theta}_R^+} \theta_R(\theta_1) = \bar{\theta}_R$. On $(\underline{\theta}_1, \bar{\theta}_1)$, firms set π so that $\theta_R = \bar{\theta}_R$. Hence $V_R(\theta_1)$ is constant at this interval, while M_2 is increasing in θ_1 at this interval (since the wage is moving in the direction of the full-commitment wage).

The bounds can be treated in exactly the same case as in the full-commitment case, and the proof is therefore omitted. ■

Proof of Existence. First note that when requirements E1-E5 are satisfied, an equilibrium has an interior solution. In the following we denote $V_1^{lc}(\theta_1)$ the value of V_1 given θ_1 and given the period-two equilibrium. Using the functions from Lemma 6, the value of the innovating firm in the first period can then be written:

$$\begin{aligned} V_1^{lc}(\theta_1) &= q(\theta_1)(1-\epsilon)[y_1 + y_2 + p(\theta_I)[V_R + w_I - y_2] - V_R - U_2] + V_R - (K + c) \\ &= q(\theta_1)(1-\epsilon)[y_1 + y_2 + p(\theta_I(\theta_1))[q(\theta_R(\theta_1))(1-\epsilon)y_R - c_R + \epsilon y_I - y_2 + (1-\epsilon)(w_2^{lc}(\theta_R(\theta_1)))] \\ &\quad - q(\theta_R(\theta_1))(1-\epsilon)y_R + c_R - p(\theta_R(\theta_1))\epsilon y_R] + q(\theta_R(\theta_1))(1-\epsilon)y_R - c_R - (K + c), \end{aligned}$$

when the lower bound on w_I does not bind. We have to show that a solution to $V_1^{lc}(\theta_1) = 0$ exists. First, note even though there may be a discontinuity where $w_2^{lc}(\theta_R)$ jumps, $V_1^{lc}(\theta_1)$ is continuous since θ_R is constant (see the proof of Lemma 6 for details). The bounds on the matching function can be dealt with in exactly the same way as in the full-commitment case, and is therefore omitted. By assumption we have that $V_1^{lc}(0) \geq (1-\epsilon)(y_1 + y_2 + \hat{y}_I) - K - c > 0$. Furthermore $\lim_{\theta_1 \to \infty} V_1^{lc}(\theta_1) = -K - c < 0$. Hence, it follows from the intermediate value theorem that an equilibrium exists. Existence when the lower bound on w_I binds follows from a similar argument and is omitted.

Proof of Uniqueness. We have to show that V_1 defined by (19) is decreasing in θ_1. Taking derivatives gives

$$\frac{dV_1^{lc}}{d\theta_1} = q'(\theta_1)(1-\epsilon)[M_1 - V_R - U_2] + q(\theta_1)(1-\epsilon)\frac{d}{d\theta_1}[M_1 - U_2] + (1 - q(\theta_1)(1-\epsilon))\frac{dV_R}{d\theta_1}.$$

The first term is strictly negative. From Lemma 6 we know that θ_R is increasing in θ_1. Hence the third term is negative. We are left with the second term. More specifically, we want to show that $M_1 - U_2$ is decreasing in θ_1, or equivalently that $M_2 - U_2$ is decreasing in θ_1. Recall that M_2 can be written

$$M_2 = p_I(w_I + V_R - y_2) + y_2. \tag{59}$$

Suppose first that the lower bound on w_I binds, i.e., that $V_R = y_2 - w_I$. It follows that $M_2 = y_2$. Since U_2 is increasing in θ_R and hence in θ_1, it follows that $M_2 - U_2$ is decreasing, and hence that $dV_1/d\theta_1 < 0$.

Suppose then that the lower bound on w_2 binds, i.e., that $w_2 = U_2$, while the lower bound on w_I does not. Then $w_I = \epsilon y_I + (1-\epsilon)U_2$ and we can write

$$\begin{aligned} M_2 - U_2 &= p(\theta_I)(\epsilon y_I + (1-\epsilon)U_2 + V_R - y_2) + y_2 - U_2 \\ &= (p(\theta_I)(1-\epsilon) - 1)U_2 + p(\theta_I)(\epsilon y_I + V_R - y_2) + y_2. \end{aligned}$$

Since U_2 is increasing in θ_1, and θ_I is decreasing in θ_1, it is straightforward to show that this expression is decreasing in θ_1.

Suppose then that $w_2 = \tilde{w}_2 > U_2$. By taking derivative of M_2 in (59) it follows that

$$\frac{dM_2}{d\theta_1} = p'(\theta_I)\theta_I'(\theta_1)[w_I + V_R - y_2] + p(\theta_I)[\frac{dw_I}{d\theta_1} + \frac{dV_R}{d\theta_1}].$$

Since $w_I + V_R - y_2 > 0$, $\theta_I'(\theta_1)$ is negative, and $p'(\theta_I)$ is positive, the first term is negative. Hence a sufficient condition for M_2 to be decreasing in θ_1 is that $\frac{dw_I}{d\theta_1} + \frac{dV_R}{d\theta_1} < 0$. Since $w_I = \epsilon y_I + (1-\epsilon)w_2$, a sufficient condition for this to hold is that $\frac{dw_2}{d\theta_1} + \frac{dV_R}{d\theta_1} < 0$. We will now derive sufficient conditions for this to hold. To that end, recall from (53) that the first-order condition for w_2 reads

$$-(1 - \hat{p}_I(w_2)) + \hat{p}_I'(w_2)[w_2 + V_R - y_2] = 0.$$

Taking derivative with respect to θ_1 gives

$$\hat{p}_I'(w_2)\frac{dw_2}{d\theta_1} + \hat{p}_I''(w_2)\frac{dw_2}{d\theta_1}[w_2 + V_R - y_2] + \hat{p}_I'(w_2)\frac{d}{d\theta_1}[w_2 + V_R - y_2] = 0.$$

Hence

$$\frac{d}{d\theta_1}[w_2 + V_R] = \frac{-\hat{p}_I'(w_2) + \hat{p}_I''(w_2)[y_2 - w_2 - V_R]}{\hat{p}_I'(w_2)}\frac{dw_2}{d\theta_1}.$$

We know that $\frac{dw_2}{d\theta_1} > 0$ and that $\hat{p}_I'(w_2) < 0$. Hence the first term in the nominator is positive, while the denominator is negative. Recall that $y_2 - w_2 - V_R > 0$. Hence a sufficient condition for the right hand side to be negative is that $\hat{p}_2''(w_2) > 0$. Recall from (54) that

$$q(\theta_I) = k_1(y_I - w_2)^{-1},$$

where k_1 is a constant. Using the definition of $q(\theta_I)$, it follows that $\theta_I = k_2(y_I - w_2)^{\frac{1}{\epsilon}}$ and we can write

$$\hat{p}_2(w_2) = k_3(y_I - w_2)^{\frac{1-\epsilon}{\epsilon}},$$

where k_2 and k_3 are uninteresting constants. Then, taking derivative twice gives

$$\hat{p}_2''(w_2) = \frac{1-\epsilon}{\epsilon}\frac{1-2\epsilon}{\epsilon}k_3(y_I - w_2)^{\frac{1-3\epsilon}{\epsilon}}.$$

It follows that $\hat{p}''(w_2) > 0$ if and only if $\epsilon < 1/2$. The result thus follows.

11.6 Proof of Proposition 3

We first prove the following Lemma.

Lemma 7 *For given θ_1, θ_I is strictly higher and w_2 strictly lower than in the full-commitment case.*

Proof. In the following the arguments are based on the equilibrium outcome of the second period for a given entry of firms in period 1. We will denote equilibrium values for given θ_1 of a variable x as $x^*(\theta_1)$ and $x^{**}(\theta_1)$ for the full-commitment and limited-commitment case, respectively. Now, by contradiction suppose the opposite of the lemma is true, i.e. $\theta_I^*(\theta_1) \geq \theta_I^{**}(\theta_1)$. Then $\theta_R^*(\theta_1) \geq \theta_R^{**}(\theta_1)$ and hence $V_R^*(\theta_1) \leq V_R^{**}(\theta_1)$. Thus, $w_2^*(\theta_1) = y_2 - V_R^*(\theta_1) \geq y_2 - V_R^{**}(\theta_1) > w_2^{**}(\theta_1)$, and hence, by the zero-profit condition of the imitating firms, $\theta_I^*(\theta_1) < \theta_I^{**}(\theta_1)$, a contradiction. Further, given $\theta_I^*(\theta_1) < \theta_I^{**}(\theta_1)$, zero-profit condition implies $w_2^{**}(\theta_1) < w_2^*(\theta_1)$.

When w_I is bounded, i.e. $w_I = y_2 - V_R$, the zero-profit condition is given by

$$q(\theta_I) = \frac{c}{(y_I - y_2 + V_R)}. \tag{60}$$

To show the result in this case, again suppose the opposite is true, i.e. $\theta_I^*(\theta_1) \geq \theta_I^{**}(\theta_1)$. Then $\theta_R^*(\theta_1) \geq \theta_R^{**}(\theta_1)$ and hence $V_R^*(\theta_1) \leq V_R^{**}(\theta_1)$. Thus by (60) $\theta_I^*(\theta_1) < \theta_I^{**}(\theta_1)$, which is a contradiction. ■

To prove Proposition 3, insert for U_2 and $M_2(\theta_1) = y_1 + \mathrm{M}_2(\theta_1)$ (where $M_2(\theta_1)$ is given by (40)) in equation into (19) for the full-commitment case, to get

$$V_1^*(\theta_1) = q(\theta_1)(1-\epsilon)[y_1 + y_2 + \max_{\theta_I, w_I \ s.\ to\ V_I = 0}\{p(\theta_I)[V_R + w_I - y_2]\} - p(\theta_R)\epsilon y_R] - K - c.$$

For given θ_1 and V_R, the term within the max operator of $V_1^*(\theta_1)$ compares to the corresponding term of $V_1^{**}(\theta_1)$ in the following way:

$$\max_{\theta_I, w_I \ s.\ to\ V_I = 0}\{p(\theta_I)[V_R + w_I - y_2]\} \geq p(\theta_I^{**}(\theta_1))[V_R + w_I^{**}(\theta_1) - y_2].$$

Furthermore, we know from Lemma 7 that for given θ_1, $\theta_I^{**}(\theta_1) > \theta_I^*(\theta_1)$. Hence $\theta_R^{**}(\theta_1) > \theta_R^*(\theta_1)$ and therefore V_R ($p(\theta_R)$) is higher (lower) in the full-commitment case for given θ_1. By termwise comparison it then follows that $V_1^*(\theta_1) > V_1^{**}(\theta_1)$. Since $V_1^*(\theta_1)$ is strictly decreasing in θ_1, as established in Proposition 1, it follows that $\theta_1^* > \theta_1^{**}$.

When w_I is bounded we can write the profits of the innovating firms in equilibrium as:

$$\begin{aligned} V_1^{**}(\theta_1) &= q(\theta_1)(1-\epsilon)[y_1 + y_2 + p(\theta_I)[V_R + \check{w}_I - y_2] - p(\theta_R)\epsilon y_R] - K - c \\ &= q(\theta_1)(1-\epsilon)[y_1 + y_2 - p(\theta_R)\epsilon y_2] - K - c. \end{aligned}$$

Following a similar argument as above, showing existence is straightforward. To show Proposition 3 when $w_I = \check{w}_I$, first note that in the full-commitment case $\max_{\theta_I, w_I \ |V_I=0}\{p(\theta_I)[V_R + w_I - y_2]\} > 0$. Next it follows from Lemma 7 that for given θ_1, $p(\theta_R)$ is lower in the full-commitment case. Hence, we have the result $V_1^*(\theta_1) > V_1^{**}(\theta_1)$. By the same argument as above, we can conclude that $\theta_1^* > \theta_1^{**}$.

11.7 Proof of Proposition 4

First we compare the first-order condition of the planner to the zero profit condition $V_1 = 0$. Recall that $\frac{\partial F}{\partial \theta_1} = 0$ is

$$(1-\epsilon)q_1[y_1+y_2+p_I(y_I-y_2)-c\theta_I-\epsilon p_R-(1-p_I)((1-\epsilon)q_R y_R-c_R)]+(1-\epsilon)q_R y_R-c_R-(c+K)=0.$$

Using the free entry condition for the imitating firms, $q_I(y_I - w_I) - c = 0$, we can replace c and rewrite the condition as

$$(1-\epsilon)q_1[y_1+y_2+p_I(w_I-y_2)-\epsilon p_R-(1-p_I)((1-\epsilon)q_R y_R-c_R)]+(1-\epsilon)q_R y_R-c_R-(c+K)=0.$$

Next, using the definitions of $V_R = (1-\epsilon)q_R y_R - c_R$ and $U_2 = \epsilon p_R$, we can write

$$(1-\epsilon)q_1[y_1 + y_2 + p_I(w_I - y_2) - U_2 - (1-p_I)V_R] + V_R - (c+K) = 0. \tag{61}$$

Now we will compare this condition to the zero-profit condition for innovating firms, which can be written as

$$V_1 = (1-\epsilon)q_1[M_1 - V_R - U_2] + V_R - (c+K) = 0.$$

Substituting in the definition of M_1 as given in (18) into (61) we get the desired result.

Next, the planner's first-order condition with respect to θ_I can be written

$$(1-\epsilon)q_I(y_I - y_2 + (1-\epsilon)q_R y_R - c_R) - c = 0,$$

where we have divided $\frac{\partial F}{\partial \theta I}$ by $p_1 > 0$. Then, using the definition of V_R we get

$$(1-\epsilon)q_I(y_I - y_2 + V_R) - c = 0,$$

which is identical to the zero profit condition of imitating firms given in (20).

11.8 Proof of Lemma 2

For the first part, i.e. $\frac{\partial F(\theta_1^{**}, \theta_I^{**})}{\partial \theta_1} = 0$, we can just refer to the first part of the efficiency result (shown in appendix 11.7). We can apply that proof as the equality of the first order condition and the zero profit condition holds for any given level of θ_I. To see this note that in effect, when the firm chooses w_1 in period 1, it takes M_1 as given and maximizes $V_1 = q(\theta_1)(M_1 - W_1) + (1 - q(\theta_1))V_R - c - K$ subject to (7), which gives the first-order condition $W_1 = \epsilon(M_1 - V_R) + (1 - \epsilon)U_2$. Using this to substitute out W_1 from V_1 gives the free entry condition

$$V_1 = q(\theta_1)(1 - \epsilon)[M_1 - V_R - U_R] + V_R - (c + K) = 0,$$

which we have shown in appendix 11.7 is the same as the first-order condition for efficiency.

Next, we establish the second condition, $\frac{\partial F(\theta_1^{**}, \theta_I^{**})}{\partial \theta_I} < 0$. Recall that the derivative of the welfare function with respect to θ_I is

$$\frac{\partial F}{\partial \theta_I} = p(\theta_1)[(1 - \epsilon)q(\theta_I)(y_I - y_2 + (1 - \epsilon)q(\theta_R)y_R - c_R) - c].$$

Substituting out $c = q(\theta_I)(y_I - w_I)$, using the result $w_I = \epsilon y_I + (1 - \epsilon)w_2$ and the definition of $V_R = (1 - \epsilon)q(\theta_R)y_R - c_R$, we get

$$\frac{\partial F(\theta_1^{**}, \theta_I^{**})}{\partial \theta_I} = (1 - \epsilon)p(\theta_1)q(\theta_I)[V_R - y_2 + w_2] < 0,$$

since $w_2 < y_2 - V_R$ in the limited-commitment equilibrium (Lemma 1). Hence, at the limited commitment equilibrium allocation, the derivative of the welfare function with respect to θ_I is negative (proofs for when w_2 or w_I are bound follow exactly the same line of argument and are therefore omitted).

11.9 Proof of Lemma 3

It is immediate that a subsidy shifts V_1 up and thus increases θ_1. Furthermore,

$$\frac{dF}{d\theta_1} = \frac{\partial F}{\partial \theta_1} + \frac{\partial F}{\partial \theta_I}\frac{d\theta_I}{d\theta_1} = \frac{\partial F}{\partial \theta_I}\frac{d\theta_I}{d\theta_1} > 0,$$

where $\frac{\partial F}{\partial \theta_1} = 0$ and $\frac{\partial F}{\partial \theta_I} < 0$ from Lemma 2. Then, the inequality follows from the fact that higher θ_1 implies lower θ_I as stated in Lemma 6. It follows that $\sigma^* > 0$.

We next evaluate the derivative of F with respect to θ_I at $\{\theta_1(\sigma), \theta_I(\sigma)\}$. As in appendix 11.8 we use equilibrium results, and can write

$$\frac{\partial F(\theta_1(\sigma), \theta_I(\sigma))}{\partial \theta_I} = (1 - \epsilon)p(\theta_1)q(\theta_I)[V_R - y_2 + w_2] < 0,$$

where the inequality follows from $w_2 < y_2 - V_R$ for any θ_R (Lemma 1). Corollary 2 follows.

11.10 Proof of Lemma 4

First we show that $\frac{d\theta_I}{d\tau} < 0$. As has been established in the proof of Lemma 6, the left-hand side of the zero-profit condition for the imitating firms, $q(\theta_I) = c/(y_I - w_I)$, decreases with θ_I, whereas the right-hand side increases, regardless of whether w_I is interior or on the wage floor. Thus, an increase in c through a tax decreases θ_I for a given θ_1. The induced effect of θ_1 through the other zero-profit condition could only overturn the decrease in θ_I, if θ_1 decreases sufficiently enough. By contradiction, assume that θ_I increases with the tax. Then by the zero-profit condition of the imitating firms, w_2 must decrease. Since θ_I increases, θ_1 has to decrease sufficiently to lower θ_R in order for w_2 (as given in (22)) to go down. The zero-profit condition of the innovators can be written

$$q(\theta_1)(1-\epsilon)(M_1 - U_2) + V_R[1 - q(\theta_1)(1-\epsilon)] - K + c = 0, \tag{62}$$

where

$$\begin{aligned} M_1 =& y_1 + y_2 + p(\theta_I)[V_R + \epsilon y_I + (1-\epsilon)w_2 - y_2] \\ =& y_1 + y_2 + p(\theta_I)[\epsilon y_I - y_2 + (1-\epsilon)\frac{(1-\epsilon)p(\theta_I)y_2 - \epsilon(1-p(\theta_I))y_I}{p(\theta_I)-\epsilon} \\ &+ \left(1 - (1-\epsilon)\frac{p(\theta_I)(1-\epsilon)}{p(\theta_I)-\epsilon}\right) V_R], \end{aligned}$$

when the w_I is not bound, i.e. $w_I + V_R > y_2$. First, consider the effect of $d\theta_I$. The sign of the inner part of $\frac{dM_1}{d\theta_I}$ is then given by

$$\frac{\partial\left((1-\epsilon)\frac{(1-\epsilon)p(\theta_I)y_2 - \epsilon(1-p(\theta_I))y_I - (1-\epsilon)\frac{p(\theta_I)(1-\epsilon)}{p(\theta_I)-\epsilon}V_R}{p(\theta_I)-\epsilon}\right)}{\partial p(\theta_I)} \frac{dp(\theta_I)}{d\theta_I}.$$

Since θ_I increases, this is positive as $(1-\epsilon)\frac{\partial w_2}{\partial p(\theta_I)} = (1-\epsilon)^2 \frac{\epsilon(y_I + V_R - y_2)}{(p(\theta_I)-\epsilon)^2} > 0$ (since $y_I + V_R > y_2$) and $\frac{dp(\theta_I)}{d\theta_I} > 0$. Next, the effect on profits given by (62) of a change in θ_R reads

$$q(\theta_1)(1-\epsilon)(\frac{dM_1}{d\theta_R} - \frac{dU_2}{d\theta_R}) + \frac{\partial V_R}{\partial \theta_R}[1 - q(\theta_1)(1-\epsilon)],$$

where $\frac{dM_1}{d\theta_R} = \left(1 - (1-\epsilon)\frac{p(\theta_I)(1-\epsilon)}{p(\theta_I)-\epsilon}\right)\frac{\partial V_R}{\partial \theta_R}$. Notice that U_2 decreases as θ_R decrease. Thus, since V_R is decreasing in θ_R, to show that profits increase as θ_R decreases it is sufficient to show that

$$q(\theta_1)(1-\epsilon)\left(1 - (1-\epsilon)\frac{p(\theta_I)(1-\epsilon)}{p(\theta_I)-\epsilon}\right) + [1 - q(\theta_1)(1-\epsilon)] > 0,$$

which follows from that $1 - (1-\epsilon)\frac{p(\theta_I)(1-\epsilon)}{p(\theta_I)-\epsilon} > 0$ when $w_I + V_R > y_2$. Therefore profits increase when θ_I increases and θ_R decreases. Finally, since $M_1 - V_R - U_2 > 0$, to satisfy the zero-profit condition of innovating firms, θ_1 has to increase, a contradiction. It follows that θ_1 cannot decrease that much, hence a tax reduces θ_I. The case of $w_I = y - V_R$ can be established in a similar way.

It then follows form Lemma 2 that welfare increases for a small $\tau > 0$.

We next show that it is not welfare maximizing to shut down the imitation market. Let τ^s be a tax that exactly shuts down imitation, and consider the welfare effect of lowering the tax marginally from τ^s. This increases θ_I (from zero) and affects $M_1 = y_1 + y_2 + p(\theta_I)[V_R + w_I - y_2]$ non-negatively since separations are efficient (since $w_I \geq y_2 - V_R$). First, consider the case when θ_1 goes up. In this case θ_R clearly increases, and it follows that U_2 increases and V_R decreases. Consequently $W_1 = \epsilon(M_1 - V_R) + (1-\epsilon)U_2$ increases. It follows that $U_1 = p_1 W_1 + (1-p_1)U_2$ increases, and welfare (including the tax revenue) must increase. Second, consider the case when θ_1 goes down. Also in this case θ_R must go up. By contradiction, assume that θ_R goes down, while θ_I goes up and θ_1 goes down. Above we have established that profits given by (62) increase when θ_I increases and θ_R decreases. Then, to satisfy the zero-profit condition of innovating firms, θ_1 has to increase, a contradiction. Thus, also in case θ_1 goes down, both W_1 and U_2 increase. Next note that the dual of the optimal period-1 recruiting problem is $\max_{\{W_1, p_1\},} p_1 W_1 + (1-p_1)U_2$ subject to $V_1 = 0$, given M_1, U_2, and V_R. We can substitute out W_1 in the problem by using the constraint $V_1 = 0$, that is $W_1 = M_1 + \frac{1-q_1}{q_1} V_R - c - K$, and rewrite the problem to $\max_{p_1} p_1 W_1 + (1-p_1)U_2$, given W_1 and U_2. By the envelope theorem we then have that only changes in W_1 and U_2 affect U_1, and the desired result then follows. Since a small tax is positive, it follows by continuity that a welfare maximizing stand-alone tax lower than τ^s exists.

Last we evaluate the derivative of F with respect to θ_1 at $\{\theta_1(\tau), \theta_I(\tau)\}$. Again as in appendix 11.8 we use equilibrium results and note that with τ the imitating firms' free entry condition is $q_I(y_I - w_I) = \tau + c$. We can write

$$\begin{aligned}\frac{\partial F(\theta_1(\tau), \theta_I(\tau))}{\partial \theta_1} &= (1-\epsilon)q_1[y_1 + y_2 + p_I(w_I - y_2) - U_2 - (1-p_I)V_R] + V_R - (c+K) + (1-\epsilon)q_1\tau\theta_I \\ &= (1-\epsilon)q_1\tau\theta_I > 0,\end{aligned}$$

where we have used the fact that the free entry condition of innovating firms imply $(1-\epsilon)q_1[y_1 + y_2 + p_I(w_I - y_2) - U_2 - (1-p_I)V_R] + V_R = c + K$.

11.11 Proof of Proposition 5

To construct the efficient tax and subsidy combination $\{\tau^*, \sigma^*\}$ start out with the efficient allocation (θ_1^*, θ_I^*). Then find the tax level that that realizes θ_I^*, taking θ_1^* as given. Next, find the subsidy level that realizes θ_1^*, taking the optimal tax as given. This procedure has a solution as the the proof of proposition 2 has established that both $\theta_I(\theta_1)$ and $V^{**}(\theta_1)$ are decreasing. The optimal subsidy and taxes are therefore given by

$$\begin{aligned}V_1^{lc}(\theta_1^*, \theta_I^*) &= -\sigma^* \\ V_I^{lc}(\theta_1^*, \theta_I^*) &= \tau^*.\end{aligned}$$

11.12 Proof of Derivative of $\hat{F}$ w.r.t. ρ

First we substitute w_I back into the welfare function. Recall that in equilibrium $(1-\rho)q_I(y_I - w_I) = c$. Using this and the relationship $\theta_I q_I = p_I$, it follows that $c\theta_I = (1-\rho)p_I(y_I - w_I)$. Substituting this into the expression for F, the equilibrium welfare as a function of ρ writes

$$\hat{F}(\theta_1(\rho), \theta_I(\rho), w_I(\rho), \rho) = \tag{63}$$
$$p_1[y_1 + y_2 + (1-\rho)p_I(w_I + q_R y_R - y_2 - c_R)] + \theta_1(1-q_1)[q_R y_R - c_R] - (c+K)\theta_1.$$

Now we have

$$\frac{d\hat{F}}{d\rho} = \frac{\partial \hat{F}}{\partial \theta_1}\frac{d\theta_1}{d\rho} + \frac{\partial \hat{F}}{\partial \theta_I}\frac{d\theta_I}{d\rho} + \frac{\partial \hat{F}}{\partial w_I}\frac{dw_I}{d\rho} + \frac{\partial \hat{F}}{\partial \rho}. \tag{64}$$

We will go through each term of (64) in turn. First, from Lemma 2 we know that the first term is zero. Second,

$$\begin{aligned}
\frac{\partial \hat{F}}{\partial \theta_I} =& p(\theta_1)\frac{d}{d\theta_I}\left\{(1-\rho)p(\theta_I)[w_I + q(\frac{p(\theta_1)(1-\rho)p_I(\theta_I) + \theta_1(1-q(\theta_1))}{1-p(\theta_1)})y_R - y_2 - c_R]\right\} \\
&+ \frac{d}{d\theta_I}\left\{\theta_1(1-q(\theta_1))q(\frac{p(\theta_1)(1-\rho)p_I(\theta_I) + \theta_1(1-q(\theta_1))}{1-p(\theta_1)})\right\} \\
=& (1-\rho)p(\theta_1)[w_I + q(\theta_R)y_R(1 - \frac{\epsilon p_I(\theta_I)}{\theta_R}\frac{p(\theta_1)(1-\rho)}{1-p(\theta_1)}) - y_2 - c_R]\frac{dp(\theta_I)}{d\theta_I} \\
&- \theta_1(1-q(\theta_1))\frac{\epsilon q(\theta_R)y_R}{\theta_R}\frac{p(\theta_1)(1-\rho)}{1-p(\theta_1)}\frac{dp(\theta_I)}{d\theta_I} \\
=& (1-\rho)p(\theta_1)[w_I + V_R - y_2]\frac{dp(\theta_I)}{d\theta_I} > 0,
\end{aligned}$$

where we have used the fact that $V_R = (1-\epsilon)q(\theta_R)y_R - c_R$. The inequality follows from the wage bound on w_I. Since we are investigating the case where θ_I is strictly decreasing in ρ (if not we know welfare is falling in ρ), it follows that the second term in (64) is strictly negative.

From (63) it follows that $\partial\hat{F}/\partial w_I = p(\theta_1)(1-\rho)p(\theta_I) > 0$. In the following Lemma we show that $dw_I/d\rho < 0$.

Lemma 8 *It holds that* $\frac{dw_I}{d\rho} < 0$.

Proof. By contradiction assume $\frac{dw_I}{d\rho} \geq 0$. We consider two cases: First, assume that θ_I increases with ρ in equilibrium. Then it follows immediately from the zero-profit condition of the imitators that w_I has to fall. Second, if θ_I decreases with ρ it follows from the equilibrium value of w_2 that w_2 (and thereby w_I) can increase if and only if θ_R increases. Since θ_I decreases, θ_1 has to increase sufficiently. Recall the zero-profit condition of the innovators:

$$q(\theta_1)(1-\epsilon)(M_1 - V_R - U_2) + V_R = K + c, \tag{65}$$

where

$$M_1 = y_1 + y_2 + (1-\rho)p(\theta_I)(w_I + V_R - y_2).$$

Suppose θ_1 increases so much that θ_R increases enough so that w_2 stays constant. Then it follows that M_1 falls if $w_I + V_R > y$ as V_R is decreasing in θ_R. Furthermore, U_2 increases and, since $M_1 - V_R - U_2 > 0$, the left hand side of (65) then decreases. Given that the equilibrium is locally stable, it follows that θ_1 cannot increase that much, hence the result follows. The case of $w_I = y_2 - V_R$ can be established in a similar way. ■

Hence the third term in (64) is also negative. Finally, we have that

$$\begin{aligned}\frac{\partial \hat{F}}{\partial \rho} =& p(\theta_1)\frac{d}{d\rho}\left\{(1-\rho)p(\theta_I)[w_I + q(\frac{p(\theta_1)(1-\rho)p_I(\theta_I)+\theta_1(1-q(\theta_1))}{1-p(\theta_1)})y_R - y_2 - c_R]\right\} \\ &+ \frac{d}{d\rho}\left\{\theta_1(1-q(\theta_1))q(\frac{p(\theta_1)(1-\rho)p_I(\theta_I)+\theta_1(1-q(\theta_1))}{1-p(\theta_1)})y_R\right\} \\ =& -p(\theta_1)p(\theta_I)[w_I + V_R - y_2] < 0,\end{aligned}$$

where the steps are similar to the steps for $\partial\hat{F}/\partial\theta_I$.

11.13 Proof of $dM_1/d\rho < 0$

Given $J_2 = (1-p_I)(y_2 - w_2) + p_I V_R$ and using the definition of W_2 in (6) we can write

$$M_1 = J_2 + W_2 + y_1.$$

From the firm's perspective, θ_R is given. From the envelope theorem it follows that we have $\frac{dJ_2}{d\rho} = -p(\theta_I)[V_R + w_2 - y_2] > 0$ since $w_2 < y_2 - V_R$. Furthermore, the envelope theorem also implies that (since the imitation market maximizes the income of the searching workers) $\frac{dW_2}{d\rho} = \frac{dw_2}{d\rho}(1-(1-\rho)p(\theta_I)) - p(\theta_I)(w_I - w_2)$. Combining gives

$$\frac{dM_1}{d\rho} = -p(\theta_I)[V_R + w_I - y_2] + \frac{dw_2}{d\rho}(1-(1-\rho)p(\theta_I)),$$

where the first part is negative due to the bound $w_I \geq y_2 - V_R$. What is left to show is that $\frac{dw_2}{d\rho} \leq 0$.

Let $w_2(\rho)$ be the innovating firm's optimal period-2 wage as a function of ρ. If the wage is bound by U_2, the result is immediate as θ_R is given from the firm's perspective. For the interior period-2 wage, the first-order condition is the solution to the equation (derived analogously as in appendix 11.4):

$$w_2(\rho) = y_I - \frac{(1-\rho)\hat{p}_I(w_2(\rho))(1-\epsilon)}{(1-\rho)\hat{p}_I(w_2(\rho)) - \epsilon}[y_I - y_2 + V_R].$$

Taking derivative with respect to ρ gives

$$\frac{dw_2(\rho)}{d\rho} = -\frac{\epsilon(1-\epsilon)\hat{p}_I(w_2(\rho))[y_I - y_2 + V_R]}{(1-\rho)(\hat{p}_I(w_2(\rho)) - \epsilon)^2} + \frac{(1-\rho)(1-\epsilon)\epsilon[y_I - y_2 + V_R]}{((1-\rho)\hat{p}_I(w_2(\rho)) - \epsilon)^2} \frac{d\hat{p}_I(w_2(\rho))}{d\rho}.$$

To show the result, suppose the opposite is true, i.e. $\frac{dw_2}{d\rho} \geq 0$. Note that $y_I - y_2 + V_R > 0$ so the first term is negative. Then $\frac{dw_2}{d\rho}$ can only be positive if $\frac{\hat{p}_I(w_2(\rho))}{d\rho} > 0$. However, from the imitating firm's zero-profit condition, $(1-\rho)q(\theta_I)(1-\epsilon)(y_I - w_2) = c$, it follows that a higher ρ in tandem with a higher wage w_2 certainly means a lower θ_I and hence a lower p_I, a contradiction.

11.14 Proof of Corollary 6

First, we show that the direct effect of the policy is (weakly) negative.

$$\begin{aligned}
\frac{\partial F}{\partial \chi} &= -p_1 \{p_I(y_I - y_2 + q_R y_R - c_R) - c\theta_I\} + p_1(1-\chi)p_I y_R \frac{\partial q_R}{\partial \chi} + \theta_1(1-q_1)y_R \frac{\partial q_R}{\partial \chi} \\
&= -p_1 \{p_I(y_I - y_2 + V_R) - c\theta_I\} \\
&= -p_1 \{p_I(w_I + V_R - y_2)\} \leq 0,
\end{aligned}$$

where we in the second step have used the fact that $\theta_R = \frac{(1-\chi)p_1 p_I + \theta_1(1-q_1)}{1-p_1}$ and $\frac{\partial q_R}{\partial \chi} = \frac{\epsilon q(\theta_R)}{\theta_R}\frac{p_1 p_I}{1-p_1}$. In the last step have use the fact that $c\theta_I = p_I(y_I - w_I)$, and then the inequality follows from the lower bound on w_I ($w_I \geq y_2 - V_R$).

Second, we show that, indeed, a higher θ_I evaluated at the limited-commitment equilibrium gives lower welfare.

$$\begin{aligned}
\frac{\partial F}{\partial \theta_I} &= p_1(1-\chi)\{(1-\epsilon)q_I(y_I - y_2 + q_R y_R - c_R) - c\} + p_1(1-\chi)p_I y_R \frac{dq_R}{d\theta_R}\frac{\partial \theta_R}{\partial \theta_I} + \theta_1(1-q_1)y_R \frac{dq_R}{d\theta_R}\frac{\partial \theta_R}{\partial \theta_I} \\
&= p_1(1-\chi)\{(1-\epsilon)q_I(y_I - y_2 + V_R) - c\} \\
&= p_1(1-\chi)(1-\epsilon)q_I(w_2 - y_2 + V_R) < 0,
\end{aligned}$$

where we have used the equilibrium condition $(1-\epsilon)q_I(y_I - w_2) = c$, and then the inequality follows from $w_2 < y_2 - V_R$.

To show that restrictions on search are negative for welfare it is then sufficient to show that θ_I goes up. By contradiction, suppose that θ_I goes down. Then by the entry condition for the imitating firms $w_I = \epsilon y_I + (1-\epsilon)w_2$, and consequently θ_R, must go up. Both $(1-\chi)$ and p_I goes down, then, since $\theta_R = \frac{(1-\chi)p_1 p_I + \theta_1(1-q_1)}{1-p_1}$ goes up, it follows that θ_1 must go up. Next, recall that the zero profit condition for innovating firms is $V_1 = (1-\epsilon)q_1[M_1 - U_2] + (1-(1-\epsilon))q_1 V_R - (c+K)$. Hence, as a higher θ_R gives lower V_R and higher U_R, it is sufficient to show that M_1 decreases in χ to show that θ_1 cannot go up which implies that θ_I cannot go down. Holding θ_1 constant, the change in $M_1 = y_1 + y_2 + (1-\chi)p_I[V_R + w_I - y_2]$ can be written

$$\frac{dM_1}{d\chi} = -p_I[V_R + w_I - y_2] + (1-\chi)\frac{dp_I}{d\chi}[V_R + w_I - y_2] + (1-\chi)p_I[\frac{dV_R}{d\theta_R}\frac{d\theta_R}{d\chi} + \frac{dw_I}{d\theta_R}\frac{d\theta_R}{d\chi}].$$

The first term is negative or zero since $w_I \geq y_2 - V_R$. The second term is also negative as we are analyzing the case where θ_I goes down. For the last term, note first that χ does not directly affect the wage setting in period 2 (the firm knows whether the worker can search or not). Thus we can use that we have showed $\frac{dw_I}{d\theta_1} + \frac{dV_R}{d\theta_1} < 0$ in the proof to Proposition 2. This inequality can be rewritten $\left(\frac{dw_I}{d\theta_R} + \frac{dV_R}{d\theta_R}\right)\frac{d\theta_R}{d\theta_1} < 0$, with $\frac{d\theta_R}{d\theta_1} > 0$. Hence, as we are analyzing the case in which $\frac{d\theta_R}{d\chi} > 0$, the desired result follows.

11.15 Numerical Illustration

Numerical example for an interior allocation

In the following we provide a numerical example to confirm the existence of equilibria for the benchmark case with full commitment and the economy with limited commitment where all markets are active and illustrate the shape of the zero-profit conditions graphically. We pick the following parameter constellation:

Table 1: Parameters

Parameter	ϵ	A	y_1	y_2	y_I	y_R	c	c_R	K	θ^{min}	θ^{max}
Value	0.55	0.50	1.00	1.00	1.00	1.00	0.04	0.04	0.50	0.28	4.67

The last two columns display the minimum and maximum tightness implied by the parameters of the matching function.

The resulting equilibria for the full and limited-commitment cases, respectively, are:

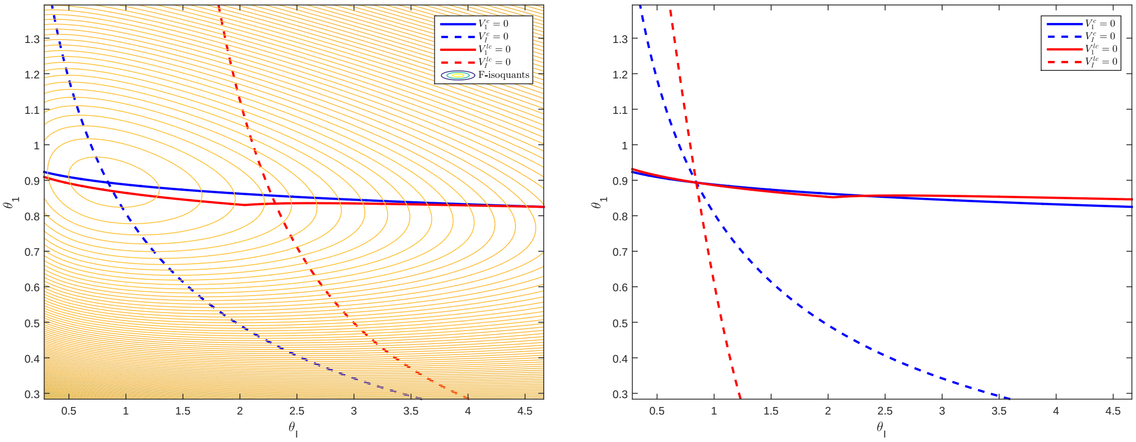
Table 2: Numerical Example of the Equilibria

	θ_1	θ_I	θ_R	U_2	w_1	w_2	w_I	$\bar{w}_I$	F
Benchmark	0.893	0.845	1.216	0.300	0.289	0.838	0.927	n.a.	0.713
Lim. Commit.	0.834	2.309	1.316	0.311	0.352	0.703	0.866	0.838	0.705

Note that we use an example with $\epsilon > .5$ to demonstrate that the condition for uniqueness following proposition 2 is only sufficient and not necessary. Welfare in the limited-commitment case is about 1.1% lower than in the benchmark. Using the given parameters we can demonstrate that for other values of K in the range $K \in [0.33, 1.00]$ further interior equilibria for both the full and limited commitment cases exist (by continuity, the set of equilibria can be extended to an neighborhood of the given parameter constellation).

Figure 2(a) depicts the equilibrium in the two cases and the isoquants of the welfare function.

Figure 1: Numerical example of the equilibrium allocations and optimal policy



(a) Equilibrium allocations and welfare (b) Optimal tax-subsidy combination

The left graph shows the zero-profit lines within the limits for the θ's and the isoquants of the welfare function.

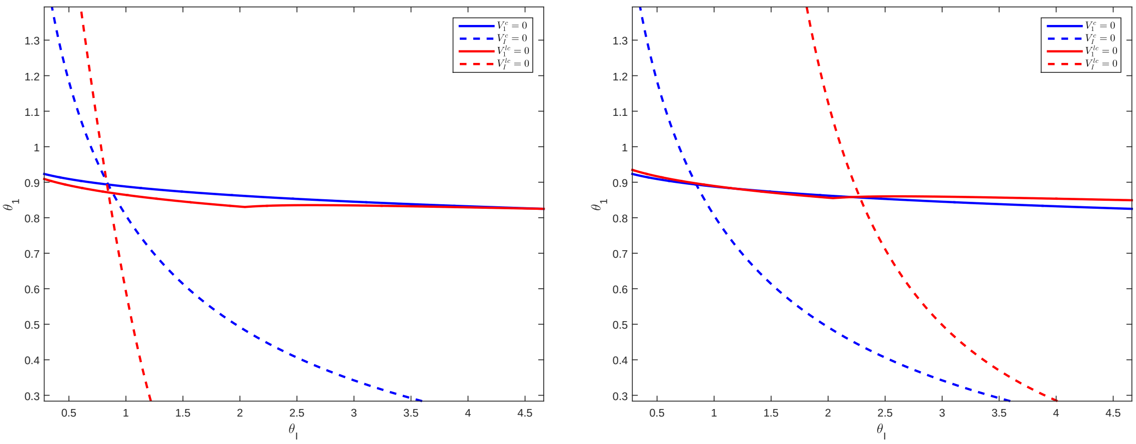
Table 3: Numerical Example of Tax Policies

Scenario	Subsidy	Tax	Lump. Tr.	F	θ_1	θ_I
Tax & Subsidy	0.0099	0.1328	0.0444	0.7131	0.8934	0.8444
Tax	0	0.1339	0.0532	0.7130	0.8712	0.8459
Subsidy	0.0113	0	-0.0097	0.7054	0.8589	2.2765

Numerical example for tax policies

Using the given parameters from table 1, we next illustrate government policies using a subsidy σ and/or a tax τ. The first scenario shows the optimal combination of innovation subsidies and taxes on imitation, which yields the efficient allocation. The first row in table 3 shows the amount of the subsidy, the tax and the implied lump sum transfer to workers, as well as the ex-ante welfare and the allocation in terms of the equilibrium market tightness. Figure 2(b) depicts this case. The next two scenarios show the cases of an optimal tax and an optimal subsidy in isolation. As figures 3(a) and 3(b) indicate, both a subsidy and a tax increase innovation and lower imitation and increase welfare. The tax increases welfare much more than the subsidy. The figures also suggest that starting from the optimal subsidy (tax), the introduction of a tax (subsidy) would further increase innovation, reduce imitation, and increase welfare.

Figure 2: Optimal stand-alone tax policies



(a) Optimal tax on imitating firms

(b) Optimal subsidy to innovating firms

Note: The graphs show the zero-profit lines with the limits for the θ's.