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- The Black-Litterman Model: An Application on the Norwegian Stock Market -

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Abstract

In this thesis we apply the Black-Litterman model on the Norwegian stock market using historical price data in the period of 2004 to 2015. A wide collection of analyst recommendations was used to determine views to feed into the model. We provide a theoretical framework of the model, and discuss the implications of some of the approaches in the literature. To best understand the functioning of the model, we compare it to several mean-variance models and pure benchmark portfolios by evaluating them based on five criteria. They asses return, both risk adjusted and not, transaction cost and predictability. With equal weights on all criteria, the Black-Litterman portfolios perform mediocre despite a positive contribution from the views. Regardless of its ranking among comparable portfolios, the model behaves intuitively and is undoubtedly an upgrade to Markowitz traditional method.

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1   Introduction

1.1   *Background*

In 1952 Harry Markowitz introduced a concept of mean-variance optimization in his article *Portfolio Selection*. The model outlined was intended to assist investors in the effort of constructing optimal portfolios based on historical financial data. His contribution would later be considered the foundation of what is today known as modern portfolio theory. Although, the approach of using mean-variance optimization in order to construct optimal portfolios were sound in theory, receiving great praise by academics, the method did not influence practitioners in the same manner. The approach had several issues that made its practical applications limited. Michaud (1989) argues why the Markowitz "optimized" portfolio might not be optimal. One of the major issues discussed by Michaud, is how the model tend to maximize the effects of errors in the input assumptions. This would, he argued, lead to unintuitive portfolios with unstable asset allocations.

Black and Litterman (1992) argued that Markowitz' model was difficult to use in practice, since the portfolios tend to behave badly. First of all, the model requires investors to assign exact expected returns for all assets. This assumption is unrealistic, since portfolio managers usually only follow a small number of securities. The resulting portfolio weights were also extremely sensitive to small changes in the inputs of expected returns. Already in 1990, Fisher Black and Robert Litterman discussed the inadequacies of the Markowitz model. This became the foundation of the Black-Litterman model; hereby referred to as the BL model. Further extensions of the model were made in 1991 and 1992, but the model has also seen a large amount of supplementary research from other authors. In Section 2, we introduce some of the most central articles and extensions related to the BL model.

The BL model builds on the same optimization approach suggested by Markowitz, but with a different set of expected returns. Instead of using historical returns as a proxy for future expected returns, Black and Litterman suggested using market equilibrium returns, with notions from the CAPM by Sharpe (1964) and Lintner (1965), as reference. The method for arriving at a set of equilibrium

returns is further based on the works of the Global CAPM by Black (1989). The improvements include a new way to easily let investors adjust their portfolio based on a set of views. The original model that consisted only of fixed income assets and currencies were expanded, in Black and Litterman (1991) and (1992), to include equities as well. The optimal portfolio suggested would, simply put, be a combination of the market capitalization weighted portfolio and the investors own views. Hence, an investor with neutral views would hold the equilibrium (market capitalization weighted) portfolio. However, when possessing additional views, the model would tilt the optimal portfolio weights according to these views.

1.2   *Research Question*

In this thesis, we explore the BL Model and apply it to the Norwegian stock market. The process involves setting specific views for individual assets; this is done using historical analyst recommendations. We seek to clarify on the implementation process and explore the challenges and difficulties associated with the BL model. We investigate the implications of model buildup and reasons for different outputs. This will help shed light on various aspects of the model behavior, as well as other issues related to the process. Even though the idea and the end result of the model might be intuitive, the implementation process can appear quite complicated. We further discuss the improvements the BL model offers to the traditional mean-variance approach. In addition, we evaluate the performance, based on several criteria, of BL portfolios, Markowitz portfolios, in combination with an equally weighted and market weighted portfolio.

1.3   *Outline*

This thesis will consist of two main parts. Firstly, in Section 2 we will present a review of existing literature and Section 3 will consist of a clarification of the theoretical framework. Section 4 will in short introduce the data used in the thesis. Secondly, Section 5 presents the methodology, while we in Section 6 present the results of the empirical analysis. The optimal portfolios constructed using the BL model, both with and without short-constraints, will be compared against an equally weighted portfolio (1/N), a market weighted portfolio (Mkt_weights), and several mean-variance portfolios. These portfolios will be evaluated using the

performance criteria; cumulative return, Sharpe ratio, portfolio turnover, certainty equivalent return (CEQ), and tracking error.

1.4   Key Findings

Measured by Sharpe ratio we did not find a significant difference between the BL portfolios and the market or the equally weighted portfolio. However, the traditional Markowitz portfolio significantly underperformed the two best performers in our sample, which are a Markowitz portfolio with constraints on shorting and a Markowitz portfolio where the variance is minimized. This was due to poor performance by the traditional Markowitz portfolio resulting from excessive trading. In terms of CEQ, the Markowitz portfolio significantly underperformed all other portfolios and benchmarks, again due to its poor performance. Among the other portfolios, no significant differences were detected.

The impact of the analyst recommendations leads the BL portfolios to outperform the market in terms of cumulative returns. However, high levels of trading can be observed for the BL portfolios, thus, incurring substantial transaction costs. Based on all performance criteria, the equally weighted portfolio is the best performer, and the traditional Markowitz portfolio is the poorest.

2   Literature Review

A starting point for Markowitz was John Burr Williams' *Theory of Investment Value (1938).* Here, Williams claimed that the value of a security should be the same as the net present value of its future dividends. With future dividends being unknown, Markowitz claimed that the expected future returns could serve as a proxy for future dividends, and hence the value of a security. Together with the expected future return, Markowitz (1952) argued that one has to take into consideration the variance, i.e. the risk, associated with investing in a portfolio. Since dealing with a portfolio of more than one asset, co-movements between assets had to be dealt with, which is represented by the covariance of returns.

Markowitz' mean-variance portfolio model serves the two basic objectives of investing; namely *maximizing expected return* and *minimizing the risk*. His framework has stood the test of time and is still considered academically sound. However, multiple practical issues have strongly prevented the model's impact in the professional investment management world.

Many problems with the use of Markowitz' mean-variance portfolio model has been advocated. Black and Litterman (1992, 1) highlighted that *"A good part of the problem is that such models are difficult to use and tend to result in portfolios that are badly behaved"*. They elaborate by saying that without constraints the model results in large short positions in one or several assets. With no shorting of assets, the model frequently assigns zero weights to many assets, i.e. "corner" solutions. Also, the model does not take into consideration the market capitalization of assets, ending in large positions in assets with low market capitalization.

These unintuitive and unreasonable results stem, according to Black and Litterman (1992), from two known problems. First, the Markowitz formulation requires investment managers to specify expected returns for all assets included in the model. This seems laborious, knowing that investors tend to focus only on particular segments of the investment universe. In addition to being time consuming, expected returns are hard to estimate, and the historical returns that are often used is, according to Black and Litterman (1992), a poor proxy for future

returns. Second, the weights assigned to each asset in the mean-variance portfolio are extremely sensitive to the vector of expected returns. Together, compounding each other, these problems produce highly unstable portfolios. As Best and Grauer (1991) demonstrated, a small increase in one individual asset's expected return can drive half of the assets from the portfolio (with constraints on shorting).

In Michaud (1989) several problems were discussed. One critique states that the Markowitz' optimizers maximize estimation errors. The estimates of expected returns, variances, and covariance are subject to estimation error. The basis for the critique lies in the fact that Markowitz optimization overweighs those assets that have large expected returns, negative covariance and small variance. His argument is that *"these securities are, of course, the ones most likely to have large estimation errors"* (1989, 34). He further claims that using sample means from historical data as expected returns contributes to the maximization of estimation errors. Also, he touches upon the issues that the model does not account for asset's market capitalization weights, and the instability of results with respect to the expected return input in particular.

The BL model successfully closed some gaps that Markowitz' left behind with his mean-variance optimization model. It creates stable, mean-variance efficient portfolios, and according to Lee (2000), there is no longer real issues caused by estimation error-maximization. The first significant contribution to asset allocation by Black and Litterman is that it provides an intuitive and neutral starting point (prior), namely the equilibrium market portfolio, building on the work of Black (1989). The second contribution made is that investors can express their own views, either relative or absolute, and these views are combined with the equilibrium market portfolio, resulting in stable and intuitive portfolios. The weight of an asset increases if the investor becomes bullish toward the asset, holding everything else equal. The weight also increases if the investor becomes more confident about the bullish view. These features serve as two new dimensions to portfolio management, which together with the neutral starting point of the equilibrium market portfolio makes the model stable, intuitive, and valuable to practitioners. As Black and Litterman (1992) writes, they have combined two established theories of modern portfolio theory – the mean-

variance optimization framework of Markowitz and the CAPM of Sharpe and Lintner.

In today's myriad of versions of the BL model, Jay Walters (2014) has sorted the models into three distinct reference models, based on two central dimensions. The first dimension separates Bayesian from non-Bayesian models. The original BL (1991a, 1991b, 1992), together with Bevan and Winkelmann (1998) and He and Litterman (1999) uses a Bayesian approach. Walters calls these models, Canonical Reference Models (CRM). Non-Bayesian models are further split in two parts, models including the parameter τ, often referred to as the 'weight on views', called Hybrid, while models excluding τ, which are pure mixing models, are called Alternative.

The original articles by Black and Litterman (1991b, 1992, 1991a) and He and Litterman (1999) focused on the features and overview, rather than on the derivation of the formulas behind the model. This made it hard to reproduce, and even hard to obtain full understanding of the models build-up. Bevan and Winkelmann (1998) provided insight on the internal process of working with the BL model within Goldman Sachs. Full derivation of the model is still not presented, but an explanation of how they set target risk levels, focusing on tracking error and Market Exposure, presented by Litterman and Winkelmann (1996), contributes to the development on how to use the model.

In Satchell and Scowcroft (2000), their main objective was to give a mathematical depiction of the model. However, their contribution, in retrospect, is the introduction of a new non-Bayesian (Hybrid) model. It uses point estimates instead of distributions, which leads τ and Ω to affect shrinkage of the views, rather than the 'weight on views' and uncertainty in the views respectively. Fusai and Meucci (2003), and Meucci (2005), focused on a non-Bayesian model, which excludes the parameter τ, as Ω alone was considered sufficient in shrinking the influence of the views. Using Jay Walters' analogy, Meucci's model is the Alternative Reference Model (ARM). Meucci (2005) himself coined the phrase, "Beyond BL" referring to his model. Looking at the last decade, the most influential models, according to Walters, has been the CRM and "Beyond BL".

Idzorek (2002) introduced "user-specified confidence levels". The method of using confidence levels let the investor establish a confidence to each view, instead of calculating the less intuitive variance of each view. According to Idzorek, this new method should increase the usability of the model. Even if the model Idzorek uses is a Hybrid Reference Model it can also be applied to the CRM.

3   Theory

In this part we will introduce some of the theoretical framework the BL model relies on. Black and Litterman presented their approach as an upgrade to the traditional mean-variance approach. We will therefore start off by explaining some of the most essential aspects of Markowitz (1952). Thereafter, we will present the BL model and its framework, to understand how it functions from a mathematical point of view.

3.1   The Markowitz Mean-Variance Model

The main concept of the mean-variance method is that an investor can, theoretically, significantly reduce the risk of a portfolio, while at the same time keep a certain level of expected return, or the investor can maximize the expected return, given a level of risk. This can be achieved by combining assets that have low or negative correlation with one another. Markowitz defined risk as the variance of the portfolio, which is determined by the variances of individual asset returns, as well as their covariance. The Markowitz model is a normative model, rather than a descriptive, attempting to explain how one should select a portfolio, and not how people usually go about doing it (Sharpe 1967). Diversification as a technique, to lower risk and not influencing the expected return, was well known before 1952, but Markowitz' mean-variance optimization would prove to form a comprehensive and convincing argument to validate diversification for decades. The investor is assumed to be risk averse, and is only interested in expected return and risk.

In order to form traditional mean-variance portfolios an investor needs a proxy for expected return, commonly used is the historical returns of all assets, as well as the variance and covariance of returns between them. By convention, excess returns are used to calculate the covariance matrix and to state the expected returns.

Attainable portfolios are reached by solving the following problems

$$\min_{w} w^T \Sigma w \tag{1}$$

st.

$$w^T \bar{r} = \bar{r}_p \tag{2}$$

or

$$\max_{w} w^T \bar{r} \tag{3}$$

st.

$$w^T \Sigma w = \sigma_p^2 \tag{4}$$

Where

w   is the vector of portfolio weights
w*   is the optimal portfolio
σ_p^2 is the variance of the portfolio
$\bar{r}$ is the expected returns of the portfolio
$\bar{r}_p$ is the required returns of the portfolio
μ   is the vector of expected excess returns
Σ   is the covariance matrix
δ is the risk aversion coefficient that states the tradeoff between risk and return

However, often the following problem is solved, and its result is referred to as the Markowitz' optimal portfolio (w*).

$$\max_{w} w^T \mu - \frac{\delta}{2} w^T \Sigma w \tag{5}$$

$$w^* = (\delta \Sigma)^{-1} \mu \tag{6}$$

3.2   Causes of the Mean-Variance Methods' Limited Practical Use

As previously stated, the Markowitz model has received praise in the academic world, but its practical impact through implementation amongst practitioners has been limited. Several reasons for this have been presented over the years. From a theoretical standpoint two assumptions must hold for the mean-variance model to provide the optimal risk-return tradeoff. Firstly, returns have to be jointly normal; secondly, one must have perfect knowledge of Σ and μ.

As stated by Michaud (1989) and Black and Litterman (1992), maximization of errors is one of the most important problems, at least fundamentally. The reason for the problem is that all inputs need to be estimated, i.e. expected returns, variances and covariance. These factors are estimates, and will be affected by estimation error. The argument made is that assets that have high expected return, low variance, or negative covariance tend to be subject to large estimation errors relative to their counterparts. Such assets receive overweight due to their attractive features, which results in portfolios that 'maximize' errors.

Portfolio weights have been reported to be extremely sensitive to changes in inputs, and expected returns in particular, by, among others Best and Grauer (1991). Michaud (1989) also comment upon this and attributes this feature of oversensitivity to ill-conditioned covariance matrices. Greater samples of historical data to estimate the covariance matrix are the only remedy proposed. In DeMiguel, Garlappi and Uppal (2005), they found that 3 000 months were needed in the estimation window (of a 25 asset portfolio) for the sample-based mean-variance strategy to outperform the equally weighted portfolio, based on several criteria. By this they aim to highlight the severity of estimation error, which also cause oversensitive portfolios.

A sample mean of historical returns is often used as expected returns in the mean-variance model. According to Michaud (1989) this causes portfolios to maximize errors and to be overly sensitive.

The Markowitz' model does not account for differences in market capitalization between assets. Therefore, it can suggest large long positions in companies with low market capitalization. This can impose a direct problem in the implementation, but also, in some cases, there will be a strong price effect that is hard to anticipate.

The problems mentioned above has a technical nature, however, the main issue is highly practical. The final weights suggested by the Markowitz' model are typically extreme, meaning unreasonably large short and long positions to exploit the in-sample-based features of the portfolio. The gearing proposed is generally not feasible, and implementation would be costly. Also, most practitioners are constrained from shorting, implying that the theoretically appealing features of optimal diversification is not relevant to start with. If constraints on shorting were to be imposed, the suggested portfolio would be 'corner solutions', meaning zero weight in many stocks and large weight in few stocks. With little diversification, these corner solutions are risky and undesirable to investors. This fact combined with the argument of estimation error maximization and portfolio sensitivity makes the applicability of the model limited.

3.2.1   Markowitz' Comment

Many of the problems connected to the mean-variance method of Markowitz are in great extent dealt with by the BL model. More stable portfolios, less corner solutions proposed, market capitalization accounted for, and maximization of errors reduced due to less extreme portfolios. Thus, the BL model is valuable in practice. The only difference however, is the calculation of expected returns. Instead of using mean historical returns, which has been the conventional method associated with Markowitz optimization, BL expected returns are used. Markowitz *(1959, 14)* comments upon the difficulty of forming inputs to the model.

> *"Portfolio selection should be based on reasonable beliefs about future rather than past performances per se. Choice based on past performances alone assume, in effect, that average returns of the past are good estimates of the "likely" return in the future; and variability of return in the past is a good measure of the uncertainty of return in the future."*

Further, he states that he created a model for portfolio optimization and that it is the job of security analysts to figure out what input to feed into the model.

3.3   The Black-Litterman Model

When the BL model was first introduced in 1990, it represented a solution to some the problems with the theoretical Markowitz framework. The new approach to the portfolio selection problem builds on the same maximization of risk and return tradeoff. The main difference, however, lies in the estimation of expected returns. As a starting point one often uses the market capitalization weighted portfolio, which is later tilted towards the views of the portfolio manager. These views are subjective in nature, but since most portfolio managers have certain beliefs about how the market will develop; the model serves as a tool for practical use. In addition to using the market equilibrium portfolio as a starting point, Meucci (2009) illustrates that an investors' current portfolio or an index can be applied. In reaching the BL expected return vector a Bayesian approach is used to combine the implied equilibrium returns with views. Optimal weights are then determined by mean-variance optimization.

We will now present the BL master formula and its input. It shows how the BL returns are calculated, which is the cornerstone of the model.

$$\hat{\mu} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q] \qquad (7)$$

Where

$\hat{\mu}$	is the vector of mean expected excess returns
τ	is a scalar that represents the 'weight on views'
Σ	is the covariance matrix of historical excess returns
P	is the matrix linking assets to views
Ω	is a diagonal matrix comprised of the uncertainty of each view represented by its error terms
Π	is the vector of implied equilibrium expected excess returns
Q	is a vector consisting of the investor views

These BL expected returns are then used to solve problem (3) above, and optimal weights are obtained.

When applying the BL model using subjective views about specific industries, companies, markets or asset classes, one assumes that the efficient market hypothesis in its semi-strong form do not hold. Therefore, to benefit from own conceptions about the future market development, mispricing must at least prevail occasionally.

3.3.1   The Canonical Reference Model

Again, using the analogy of Jay Walters (2013), the CRM refers to the original BL. We will here define the reference model for returns, which is fundamental in defining the model. It is here the various versions of the BL model differ. This presentation will contribute to the understanding of the original model, as well as its main difference compared to another model we will deploy later, namely the ARM. The reference model clarifies which variables that are random and which parameters that are modeled, unlike the variables that are not random and parameters that are not modeled.

We first assume that the expected returns are normally distributed

$$r \sim N\,(\mu\,,\Sigma) \qquad (8)$$

These are the expected returns we eventually want to model. These two moments are the only needed inputs to solve a Markowitz optimization, however, some more modeling is needed.

Now, we define the expected return, i.e. the mean return, as a random variable

$$\mu \sim N\left(\pi, \Sigma_\pi\right) \tag{9}$$

where π represents the estimate of the mean, μ, and Σ_π is the variance of the estimate of the true mean, μ. A linear relationship as shown below is obtained.

$$\mu = \pi + \varepsilon \tag{10}$$

The interpretation is that the expected returns are normally distributed around our estimate π, with a disturbance term ε. Further, the disturbance term has a mean of zero and a variance of Σ_π. The disturbance term is also considered to be uncorrelated with the prior, μ. As a final step we define Σ_r as the variance of returns, r, about the estimate, π. This leads to the following equation.

$$\Sigma_r = \Sigma + \Sigma_\pi \tag{11}$$

Now, we can define the CRM for returns as

$$r \sim N\left(\pi, \Sigma_r\right) \tag{12}$$

Equation (8) is the reference model for the ARM. The difference is that Equation (8) uses a point estimate of μ, rather than a distribution. Knowing the difference between the reference models is important to understand how parameters influence the outcome. In our implementation of the BL model we will apply both the CRM and the ARM. A presentation of the latter model will therefore follow in Section 3.3.6.

3.3.2   *Reverse Optimization and the Risk Aversion Parameter*

The implied equilibrium returns are estimated using a method called reverse optimization. First of all, the BL model utilizes General Equilibrium theory, which says that each sub-portfolio must be in equilibrium if the aggregate portfolio is. We will work with a quadratic utility function and assume that there is a risk free asset available to investors. The equilibrium returns are then CAPM

(Capital Asset Pricing Model) returns, and the portfolio without views is equal to the CAPM Market portfolio. CAPM returns are not hard to calculate for assets individually, however, since the CAPM market portfolio consist of all investable assets it becomes demanding. The technique of reverse optimization solves this problem. In its derivation we start with the quadratic utility function

$$U = w^T \Pi - \left(\frac{\delta}{2}\right) w^T \Sigma w \tag{13}$$

U   The objective function that represent investors utility
w   Row vector of weights
Π   Row vector of equilibrium excess returns
δ   Risk aversion
Σ   Covariance matrix of excess returns

Maximizing utility with respect to weights will give us the solution. So, we take the first derivative of (13) w.r.t. w and then we solve for Π and obtain

$$\Pi = \delta \Sigma w \tag{14}$$

The covariance matrix is estimated using historical data, and the market weights can easily be found in the market (at least for listed companies etc.). The risk aversion parameter however, is not yet accounted for. Multiplying (14) above with w^T and changing to scalar terms, and then solving for δ, we obtain the following expression for the risk aversion parameter

$$\delta = (r - r_f)/\sigma^2 \tag{15}$$

where r is the total market return ($w^T \Pi + r_f$), r_f is the risk free rate, and σ^2 is the variance of the market ($w^T \Sigma w$). Once δ is calculated, one can plug it in to (14), together with the covariance matrix (Σ) and the market weights (w), to find the implied equilibrium returns (also called the prior).

Now we can develop the prior distribution using an assumption made by Black and Litterman; the covariance of the estimate is proportional to the covariance of returns ($\Sigma_\pi = \tau \Sigma$). Thus, τ is a parameter of this proportionality. Making the prior distribution for the BL model, P(A), equal to

$$P(A) \sim N(\Pi, \tau\Sigma), r_A \sim N(P(A), \Sigma) \qquad (16)$$

The mean is an estimate and the variance is set proportionally. τ is often appointed low values. Black and Litterman (1992) for instance assumed τ equal to 0.025, because the variance of the estimated mean is assumed to be smaller than the variance of returns in itself. Using the CRM, (12), the prior distribution, can be written as

$$r_A \sim N(\Pi, (1 + \tau)\Sigma) \qquad (17)$$

From this, a curiosity of the models behavior can be explained. It is often stated that a BL model without views will be equal to the market portfolio, but this is only true if one constrains the model to invest 100% in the portfolio. Without this budget constraint $\tau/(1 + \tau)$ will be invested at a risk free rate. Below is the mathematical proof from Walters (2014), using what we already know.

$$\Pi = \delta\Sigma w$$
$$w = (\delta\Sigma)^{-1}\Pi$$
$$\widehat{w} = ((1 + \tau)\delta\Sigma)^{-1}\Pi$$
$$\widehat{w} = \left(\frac{1}{1 + \tau}\right)(\delta\Sigma)^{-1}\Pi$$
$$\widehat{w} = \left(\frac{1}{1 + \tau}\right)w \qquad (18)$$

3.3.3 Building the Inputs

Merging the implied equilibrium returns with views requires the user to specify the view vector (Q), the uncertainty in views (Ω), and the link matrix (P). The view vector is a $k \times 1$ column vector, where k is the number of views. It discloses what return an investor believes an asset will reap (for absolute views), or the difference in return between assets (for relative views).

Since there is uncertainty related to the views, each view has an error term (ε_i). It can be presented like this

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

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The error terms do not enter the BL formula, and they are non-observable at the time of investment. Without the error terms the investor would be 100% confident, due to absolute certainty. To incorporate these errors, estimated variances of the views are included in the model through Ω. It is, in most cases, a diagonal covariance matrix consisting of the variance of the error terms (ω). The off-diagonal positions are hence zero, meaning that the views are considered to be uncorrelated. According to Walters (2014) the problem is constrained in this way to improve stability and increase simplicity of the problem. Mankert (2006) argues that such an assumption is inconsistent, due to lack of uncorrelated error terms in Σ; meaning that the returns of the assets are correlated and so should the views.

Since Ω is a set of variances of the error terms connected to each view it represent the uncertainty of views. Entering the model is Ω^{-1}, which denote the confidence, or precision, related to the views.

There are multiple ways to calculate Ω. We will here present the original method that assumes proportionality between the variance of returns and the variance of the views. He and Litterman (1999) defined the variance of views as follows

$$\Omega = diag(P(\tau\Sigma)P^T) \tag{19}$$

This is the method most frequently used, although the alternative presented by Meucci (2005) has gained a foothold as well. The greatest difference in his alternative is that the matrix is not diagonal, implying that views are allowed to have other than zero covariance. He sets

$$\Omega = \left(\frac{1}{c}\right)P\Sigma P^T \tag{20}$$

where c > 1 and often set equal to τ^{-1}.

This leads to the following Ω matrices

$$\begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix} \quad or \quad \begin{bmatrix} \omega_{1,1} & \cdots & \omega_{1,k} \\ \vdots & \ddots & \vdots \\ \omega_{k,1} & \cdots & \omega_{k,k} \end{bmatrix}$$

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To connect the view vector (Q) to assets, one must specify a link matrix (P). It is a $k \times n$ matrix, where there are k views on n assets. In the implementation that later will follow, the P matrix will simply be, by construction, the identity matrix. This is due to the fact that only absolute views will be applied, and that k will equal n at all times. We will elaborate more in Section (5).

Determining a distribution for the views is not always possible due to the fact that the P matrix does not have to be of full rank (Walters 2014), meaning that incomplete or relative views may create a non-invertible variance. Even if it is hard to interpret the conditional distribution we find it important to elaborate in order to better grasp the model. Following Walters (2014), the conditional distribution in view and asset space is given respectively

$$P(B|A) \sim N(Q, \Omega) \tag{21}$$

$$P(B|A) \sim N(P^{-1}Q, [P^T \Omega^{-1} P]^{-1}) \tag{22}$$

3.3.4 τ; Function and Reason

(i) What is meant by a precision weighted average: The expected returns calculated using the BL model is often referred to as a precision weighted average. From how the model usually is presented that is hard to tell. However, with some small modifications of the 'master formula' it becomes more apparent.

$$\hat{\mu} = [(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q][(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$$

$$= \frac{(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q}{(\tau\Sigma)^{-1} + P^T\Omega^{-1}P} = \frac{(\tau\Sigma)^{-1}}{(\tau\Sigma)^{-1} + P^T\Omega^{-1}P}\Pi + \frac{P^T\Omega^{-1}P}{(\tau\Sigma)^{-1} + P^T\Omega^{-1}P}P^{-1}Q$$

$\tau\Sigma$ and Ω are referred to as the uncertainty of the prior and the views respectively. Thus, the inverse of uncertainty is called the precision, and it is evident that the fractions above form a precision weighted average. Also, from this it is clear that the posterior distribution will have a higher precision than either the prior or the conditional distribution.

(ii) BL expected returns when Ω is proportional to the variance of asset returns: According to Walters (2014) this is the most common method in specifying Ω. By construction, the variance of the prior is set proportionally to the variance of

assets ($\Sigma_\pi = \tau \Sigma$). If the variance of the views (Ω) is specified by the same proportionality (τ), then the weight of the prior distribution and conditional distribution will be equal in forming the posterior distribution. This is best understood if looking at the equation above, where the precision of the implied equilibrium returns (Π) and the returns of the views ($P^{-1}Q$) is the same. The reason is that both are set proportionately to the variance of asset returns (Σ). A proof will follow with two initial assumptions.

1. $\Omega = P(\tau\Sigma)P^T$
2. $P^T = P = I$

Where I is the identity matrix. Further, we define $V = P^{-1}Q$.

$$\hat{\mu} = [(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q][(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$$
$$= [(\tau\Sigma)^{-1}\Pi + P^T(P(\tau\Sigma)P^T)^{-1}Q][(\tau\Sigma)^{-1} + P^T(P(\tau\Sigma)P^T)^{-1}P]^{-1}$$
$$= [(\tau\Sigma)^{-1}\Pi + (\tau\Sigma)^{-1}V][(\tau\Sigma)^{-1} + (\tau\Sigma)^{-1}]^{-1}$$
$$= \frac{(\tau\Sigma)^{-1}(\Pi + V)}{(\tau\Sigma)^{-1} + (\tau\Sigma)^{-1}} = \frac{(\tau\Sigma)^{-1}(\Pi + V)}{2(\tau\Sigma)^{-1}} = \frac{\Pi + V}{2}$$

Hence, defining Ω in this way and having a link (P) matrix equal to the identity matrix, τ will be irrelevant for the outcome of the model. As shown above, BL expected returns would be a simple average of the implied equilibrium returns and the absolute returns stated in the views.

3.3.5 Using Bayes Theorem for the Estimation Model

Estimating the BL returns require a blending of the implied equilibrium returns and the views. Both of them are normally distributed and by applying Bayesian theory we can model the posterior distribution, which is the blending of the prior (17) and conditional (22) distribution.

The posterior distribution becomes a precision weighted average of the prior and conditional distribution. Applying Bayes Theorem, the posterior distribution can be constructed, also called the BL master formula (Walters 2014).

$$P(A|B) \sim N([(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi \qquad (23)$$
$$+ P^T\Omega^{-1}Q], [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1})$$

It is, however, worthwhile to present the mean return in the following manner for intuitive purposes

$$\hat{\mu} = \Pi + \tau\Sigma P^T[(P\tau\Sigma P^T) + \Omega]^{-1}[Q - P\Pi] \tag{24}$$

To provide further intuition of this equation, two extreme scenarios are presented. First, we let Ω be zero, meaning that there is 100% certainty in the views. Commonly Ω is estimated as $diag(P(\tau\Sigma)P^T)$, but this is just one possible method of obtaining Ω. Instead, we set all variances (and covariance) of the views ($\omega_{k,k}$) equal to zero, so (24) transforms to the following

$$\hat{\mu} = \Pi + \Sigma P^T[P\Sigma P^T]^{-1}[Q - P\Pi] \tag{25}$$

Ω is simply left out and τ cancels since there is only one term left in the brackets which is put in inverse. If a view is specified for every asset, then P will be invertible and further simplification can easily be performed. We obtain

$$\hat{\mu} = P^{-1}Q \tag{26}$$

It makes sense that the views solely set expected returns if there is no uncertainty related to them. On the other hand, if the practitioner of the model is not certain about the views ($\Omega = \infty$), then the expected returns are equal to the implied returns. To see this, the second term on the right hand side of (24) is divided by an infinitely large number, and thus converges toward zero.

3.3.6   The Alternative Reference Model

This is the most commonly used extension or alternative to the original BL model. It is used in Satchell and Scowcroft (2000) and in Meucci (2005). The ARM is as follows

$$E(r) \sim N(\mu, \Sigma) \tag{27}$$

Here, $E(r)$ is normally distributed with a variance of Σ. We do not model μ as a random variable in this case; it is rather a point estimate. This is frequently referred to as setting $\tau = 1$, but τ is actually eliminated altogether. Black and Litterman assumed in their original model that the covariance of the estimate was proportional to the covariance of returns. This is where τ entered, forming the assumption: $\Sigma_\pi = \tau\Sigma$. When eliminating τ from the model, Ω is the only

parameter that controls the merging of the implied equilibrium returns and the views. Rewriting (24) results in the following expression for the BL expected returns.

$$\widehat{\Pi} = \Pi + \Sigma P^T \big((P \Sigma P^T) + \Omega \big)^{-1} (\Omega - P\Pi) \tag{28}$$

With these expected returns, unlike with Bayesian returns, the outcome without views would be equal to the market weights, since we previously had $\widehat{w} = \left(\frac{1}{1+\tau} \right) w$.

4   Data

In this Section we describe the data used, and the sources it is collected from.

In order to form a good basis for the construction of an optimal and diversified portfolio we begin by specifying an investment universe consisting of Norwegian stocks listed on Oslo Stock Exchange (OSE). For it to be diversified, the relative weight of each stock needed to be non-negligible. On those grounds, we selected the 20 largest companies that were listed before 2004, see Appendix, Table A1. We collected weekly prices for these companies from Bloomberg, starting from the beginning of 2004 to the end of 2015. In order to reflect the total return to investors, prices with adjustments for dividends were used (Total Return Net of Dividends). Further, logarithmic (ln) returns were calculated to meet the normality assumption. We decided to use weekly return data as this will provide us with what we believe is a sufficient number of observations.

In calculating the covariance matrix, we used excess returns, which is the conventional method. To arrive at the excess returns, we proceeded to collect a proxy for the risk-free rate. As the investment period is 10 years, we decided to use 10-year Norwegian Government bonds in order to match the risk-free alternative attainable over a similar time horizon. The data was collected from Norges Bank for the period of 2004-2015. Additionally, in order to calculate the implied equilibrium returns, market weights were needed on every rebalancing date, i.e. the last Friday every month from 2006 to 2015. The current market capitalization was used as a proxy for each firms' size and further used to arrive at the company's specific weight at each week and month.

Data on analysts' recommendations were collected from the Bloomberg database for the investment period 2006-2015. For each company, the total number of buy, hold and sell recommendations on the last Friday of every month, i.e. the rebalance date, were retrieved. Based on these sentiment samples, practices were developed to determine whether the BL portfolio would underweight or overweight a stock, and to what extent.

5   Methodology

In this Section we clarify the different empirical approaches used in this thesis and the motivation for the choices made. The methods of choice are based on the original work by Black and Litterman (1990, 1991 and 1992), as well as the later publications by Meucci (2005, 2010).

As mentioned earlier, the BL model can be constructed using different reference models. These reference models differ in framework and applicability. We will employ both what is referred to as the CRM and the ARM (Walters 2014).

The BL models are applied to the Norwegian Stock Market using weekly data from 2004-2015, where the first two years are used solely as input for the covariance matrix, i.e. a rolling window consisting of 104 observations. The investment period starts on January 1st 2006 and lasts for 10 years with rebalancing occurring on the last Friday of every month. We define our investment universe as the 20 largest stocks, measured by market capitalization. The portfolios constructed using the BL approach will be compared against a traditional Markowitz maximum Sharpe portfolio and three other benchmark portfolios. These benchmark portfolios are; market capitalization weighted, equally weighted, and minimum variance. The performance of all portfolios will be evaluated by the following five performance criteria: (i) cumulative returns; (ii) the out-of-sample Sharpe ratio; (iii) portfolio turnover (trading volume), (iv) CEQ, and (v) tracking error.

We will construct portfolios both with and without constraints on short selling. There are two main reasons for this; first, mutual funds are not allowed to short sell stocks. This means that many potential users of the BL model and mean-variance methods are restricted. The second reason is that the mean-variance approach has a tendency to suggest unreasonably large short positions in one or several assets.

In Section *3.3.4 (ii)* we proved that if one defines Ω as $P(\tau\Sigma)P^T$, and at the same time use a P matrix equal to the identity matrix, the BL expected returns will be a

simple average of the implied equilibrium returns (Π) and the returns of the views ($P^{-1}Q$). This definition of Ω appears to be most common, which is somewhat peculiar in our understanding, as it renders the model unaffected by τ. Few authors pinpoint this fact in their work even though it obviously is important in the understanding of the model. Knowing this we will still apply the original model (CRM) with Ω as outlined in (19) and the ARM will be applied with Ω as outlined in (20). A discussion of how to determine the value of τ will follow, however, its value will be irrelevant in our CRM and work as a scalar to set the uncertainty related to views in the ARM ($1/c = \tau$). We hope that this will shed light on the different aspects of the models, both in terms of their behavior and end results.

5.1   Constructing a New Set of Expected Returns

This Section will clarify how our model has been build, with remark to some specific details. The BL expected returns relies heavily on how the parameters are set and defined, therefore we devote this Section to describe just that.

5.1.1   Estimating the Covariance Matrix

In the BL model, together with many other quantitative financial models, the returns are assumed to be normally distributed. We assume the return data to be log normally distributed, thus the covariance matrix is estimated with ln returns in order to achieve normality. In addition, excess returns are used, meaning asset returns above the weekly risk free rate. The covariance matrix is updated for every rebalancing, i.e. estimation of a new portfolio, with a rolling window of two years or 104 weekly returns.

5.1.2   Setting a Value for τ

How to set τ has been a subject of great discussion. Some authors set small values for τ, while some ignore it. Meucci (2005) sets τ to 1 and in effect eliminate τ using the ARM, while Black and Litterman (1992), He and Litterman (1999), and Idzorek (2005) choose a value of τ between 0.025 and 0.05. Satchell and Scowcroft (2000) discuss the use of τ "around" 1, while Walters (2014, 20) state that this has *no intuitive connection to the data*. Taking into account the convincing argument that τ is closer to 0 than 1, because the uncertainty in the mean is lower than the uncertainty in returns, the statement from Walters in his

2014 article makes sense. To see why, from a statistical standpoint, consider the following. Let's say one bootstraps a distribution of means using m samples of size n (with replacement). As the number of samples (m) goes to infinity, the central limit theory tells us that Σ_π converges toward Σ/n (Walters 2013). Knowing that $\Sigma_\pi = \tau\Sigma$, we thus have

$$\Sigma_\pi = \tau\Sigma = \Sigma/n \;\rightarrow \tau = 1/n$$

The most intuitive definition of n in this particular case is to set n equal to the number of observations used to determine the covariance matrix, which is two years of weekly observations, i.e. n = 104. Thus, the value of τ in our models will be 0.00962 (1/104).

5.1.3   Risk Aversion

The risk-return tradeoff for the investor is determined by the risk aversion parameter δ. The value of δ is generally calculated as the rolling average of market return divided by the market volatility; see (15). The parameter directly affects the value of the implied equilibrium return (Π) vector. We found the value of δ, when using the general calculation method, to be volatile, and in some cases negative. This created some problems with parts of the calculation. In He and Litterman (1999), the parameter of δ is set as a constant value of δ = 2.5. This is assumed to be a representation of the world average risk tolerance. The above reasoning leads us to apply a constant δ of 2.5 across the entire investment period.

5.1.4   Investor Views

The specified views by the portfolio managers have a large impact on the final weights suggested by the BL optimization problem. A view can either be defined as an absolute view or a relative view. An absolute view is set on an individual asset basis, while a relative view is set for more than one asset. The latter would be useful in the event where an investor believe one or more assets will outperform other assets. In the BL model, a view is assigned a value of 1, 0, or -1 to represent the nature of the view. A bullish view is assigned a value of 1, while a bearish view is assigned a value of -1, and a neutral view is given a value of 0. The nature of the view determines in which direction the portfolio will be tilted. A bullish view will increase the weights for a given stock keeping everything else equal, while a bearish view will decrease the weights for the stock.

The P matrix regulates which assets are related to the different views. In our case, as the specified views are absolute and all assets have a view, by construction, at all times, the P matrix is the identity matrix. Since the portfolio is estimated 120 times it is easier not to adjust the size of the P matrix continuously. When a neutral view (0) is assigned to a stock, the return in the Q matrix will be equal to the corresponding implied equilibrium return. The Q vector contains the actual quantified investor views in the form of expected returns.

In order to arrive at these expected returns in the Q vector we utilize the sentiment of analyst recommendations. If more than 70% of analysts recommend a buy, the implied equilibrium returns are scaled upwards by 30%. If the buy ratio is between 60 and 70% the tilt is 20%, and with a buy ratio of between 50 and 60% the tilt is 10%. The same goes for sell signals, only that the scaling is respectively 30, 20, and 10 % downward. The view is set as neutral when less than half of the analysts possess neither a bullish or bearish view. In essence, three levels for each sentiment are used, i.e. strong buy, buy and weak buy, or strong sell, sell and weak sell.

The model input of investor views is of a highly subjective nature. In order to mitigate this issue, we have formed our views using a compilation of analyst recommendations. Hence, we collected buy, hold and sell recommendations from a wide selection of financial institutions. In so doing, we will form some sort of analyst consensus, and also, the extent to which analysts agree will serve as an extra dimension in the information gathering. From these views we will estimate new portfolios on a monthly basis (120 times).

In our CRM Ω is defined as $diag(P(\tau\Sigma)P^T)$, but remembering the proof under Section 3.3.4, Ω has no impact on the end result. In our ARM however, Ω is defined as $(1/c)P\Sigma P^T$. Using the same arguments as for τ, 1/c is set equal to 1/104. This level of uncertainty is used as a starting point.

5.1.5   Combining All Input into BL Expected Returns

The process of constructing a new set of expected returns is the most central part of the BL model. To arrive at the new set of expected returns, we apply the BL master formula (23). The formula is restated here

$$\hat{\mu} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q] \tag{29}$$

The implied equilibrium returns (Π) are the starting point, and is reached by performing reverse optimization as described in Section 3.3.2. Following this, the investor views are stated through the P matrix and Q matrix. The Ω matrix captures the variance of the specified views and the covariance between the different views (in the ARM) acting as a measure of uncertainty in the views. We proceed to separately calculate the first (30) and the second term (31) of the BL master formula, before combining them to arrive at the final set of new expected returns.

$$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1} \tag{30}$$

$$[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q] \tag{31}$$

5.2   Estimating the portfolios

The method of estimating the portfolio weights is performed using the traditional Markowitz optimization method. All of the required automation processes were possible to achieve using the Macro function in Microsoft Excel, and simple use of VBA coding. For every rebalancing a solver-computation is run for each portfolio. Because the starting weights matter for the end result using solver, each portfolio is assigned the current market weights before executed. A constraint forcing the aggregate portfolio weights equal to one is included; hence, investing in a risk free asset is not an option. The initial weights in January 2006 for all portfolios, including the market capitalization weighted portfolio, are displayed in the Appendix, Table 2.

The optimal portfolios are obtained by maximizing the Sharpe ratio using the Markowitz framework

$$\max_p SR = \frac{R_p - R_f}{\sigma_p^2} \tag{32}$$

s.t $$\sum_{i=1}^{N} w_i = 1, where\ N = 20$$

and the constrained portfolios, s.t

$$w_i \geq 0$$

5.3   Benchmark Portfolios

5.3.1   Markowitz Maximum Sharpe Ratio Portfolio

This is the traditional Markowitz optimal portfolio that optimizes the Sharpe ratio, as shown in (32). Both the input in the covariance matrix and the calculations for the mean historical return uses an estimation window of 2 years, or 104 observations. The return data used in the calculations are ln values. Because of unfeasible weights occurring for a large portion of the investment period we chose to make a constrained version in order to have a more sensible basis for comparison. This version constrains the weights of individual stocks to be at most, long or short, 100% of the portfolio. Although constraining the portfolio will prevent a theoretically optimal model to unfold, we find it necessary in order to implement the portfolio, without its features being lost for comparison purposes. The aforementioned will be referred to as the MV portfolio; in addition, we include a version with constraints on short selling, hereby referred to the MV_ns.

5.3.2   Market Capitalization Weighted Portfolio

The benchmark will act as a proxy for the market portfolio in our study, and will be referred to as Mkt_weights. It contains the 20 stocks that comprise our investment universe. Weights are calculated based on market capitalizations in 2006. Once invested in these weights in January 2006, no rebalancing is needed, thus the turnover of this portfolio is zero.

5.3.3   Equally Weighted Portfolio

This benchmark is commonly referred to as the naive portfolio where an equal amount is invested in each of the 20 stocks, hereby denoted 1/N. Each stock will be given a portfolio weight of 5 percent and will be rebalanced every month ($w_i = 1/N$).

5.3.4   Minimum Variance Portfolio

This portfolio follows the techniques of Markowitz and is constructed by minimizing the portfolio variance, from this time on referred to as Min_var. Hence, the portfolio is not influenced by the estimation error related to expected returns.

$$\min_{p} \sigma_p^2 \tag{33}$$

s.t $\sum_{i=1}^{N} w_i = 1, \, where \, N = 20.$

5.4   Portfolio Performance Criteria

In order to properly evaluate our BL portfolios we compare them to the Markowitz maximum Sharpe portfolios and the benchmark portfolios along five different performance criteria. To analyze if the strategies are statistically distinguishable, we performed tests on the out-of-sample Sharpe ratio and the CEQ, for all strategy pairs.

5.4.1   Cumulative Return

The cumulative return obtained out-of-sample for each portfolio is calculated throughout the investment period. The model estimation is based on ln returns, however when estimating the resulting change in wealth, raw returns are used. Also, keep in mind that returns are reported in excess of the risk free rate, i.e. above the yield of 10-year Norwegian Government bonds.

5.4.2   Out-of-Sample Sharpe Ratio

We gather the in- and out-of-sample Sharpe ratio at each rebalance date. The in-sample Sharpe ratio compared to the out-of-sample Sharpe ratio gauge the effect of estimation error, or, in other words, the effect of a non-clairvoyant investor. Strategy k's out-of-sample Sharpe is defined as the sample mean of out-of-sample excess returns over their standard deviation $(\hat{\mu}_k / \hat{\sigma}_k)$[1]. The in-sample Sharpe ratio using Markowitz optimization is the theoretically optimal solution, while the out-of-sample Sharpe ratio is calculated using these optimal portfolio weights and post-rebalance returns. Markowitz in-sample Sharpe will be used as a benchmark representing the maximum Sharpe ratio throughout the analysis. To tell whether

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[1] We have used ln values of out-of-sample excess returns to obtain a better estimate of the mean, see Hudson and Gregoriou (2010).

or not the Sharpe ratios (out-of sample) are statistically distinguishable we performed the test proposed by Jobson and Korkie (1981), with the correction of Memmel (2003).[2]

5.4.3   Portfolio Turnover

The portfolio turnover, or the trading volume, is recorded for each portfolio at every rebalance date. The turnover of the portfolio represents the average portion of the total portfolio that is reallocated each month. Hence, a high level of turnover translates into higher transaction costs for the portfolio. We define turnover as in the article *1/N* (DeMiguel, Garlappi, Uppal 2005).[3]

5.4.4   Certainty Equivalent Return

The CEQ is the return an investor, with a given risk aversion level, would choose with certainty over a higher but uncertain portfolio return. Assuming that investors are mean-variance optimizers and have a quadratic utility, the CEQ of portfolio k is

$$\widehat{CEQ}_k = \hat{\mu}_k - \frac{\delta}{2}\hat{\sigma}_k^2, \tag{34}$$

where $\hat{\mu}_k$ and $\hat{\sigma}_k^2$ is the sample mean and variance of excess returns and δ is still the risk aversion parameter (set to 2.5). Other authors apply a δ of 1 when calculating CEQ, see for example 1/N (DeMiguel, Garlappi, Uppal).

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[2] With two portfolios i and n, the test for difference in Sharpe ratios, $H_0: \hat{\mu}_i/\hat{\sigma}_i - \hat{\mu}_n/\hat{\sigma}_n = 0$, has the following test statistic $\hat{z}_{JK}$, which has a standard normal distribution:

$$\hat{z}_{JK} = \frac{\hat{\sigma}_n\hat{\mu}_i - \hat{\sigma}_i\hat{\mu}_n}{\sqrt{\hat{\vartheta}}}$$

and

$$\hat{\vartheta} = \frac{1}{T-M}\left(2\hat{\sigma}_i^2\hat{\sigma}_n^2 - 2\hat{\sigma}_i\hat{\sigma}_n\hat{\sigma}_{i,n} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_n^2 + \frac{1}{2}\hat{\mu}_n^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_n}{\hat{\sigma}_i\hat{\sigma}_n}\hat{\sigma}_{i,n}^2\right)$$

where the means, variances, and covariance are estimated over the sample size T – M.

[3] Turnover is defined as the average absolute value traded per rebalancing:
$Turnover = \frac{1}{T-M}\sum_{t=1}^{T-M}\sum_{j=1}^{N}\left(\left|\hat{w}_{k,j,t+1} - \hat{w}_{k,j,t}\right|\right)$, where T is the length of the total dataset and M is the length of the estimation window. Further, subscript k, j, and t represent strategies, assets, and time respectively. Note that for the market weighted portfolio the turnover is zero, but for the equally weighted portfolio it is not due to fluctuations in market prices.

The test used to quantify the statistical difference between portfolios builds on the work by Greene (1997).[4]

5.4.5   *Tracking Error*

The tracking error is assessed for the BL portfolios and the Markowitz portfolios (including Min_var) against the market weighted portfolio and the equally weighted portfolio. Tracking error is measured as the standard deviation of the difference in return between a portfolio and a benchmark. The rationale for including this criterion is that large deviations from a benchmark can reveal unpredictable behavior.   Hence, measuring how different an investor can anticipate the portfolio to evolve in relation to the benchmark.   The market weighted portfolio and the equally weighted portfolio are stable portfolios that serve as intuitive benchmarks for this matter. Market weighted portfolios are often imitated by ETFs (exchange traded funds) and serves as easily available, highly diversified investments. The equally weighted portfolio is known as the naive portfolio, investing the same amount in every security included in a portfolio. Because of their availability and widespread use in simple diversification we employ these portfolios as benchmarks.

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[4] If $f(v) = \left(\mu_i - \frac{\delta}{2}\sigma_i^2\right) - (\hat{\mu}_n - \frac{\delta}{2}\hat{\sigma}_n^2)$, where i and n represent two different portfolios, the asymptotic distribution of $f(v)$ is according to Greene (1997): $\sqrt{T}\big(f(\hat{v}) - f(v)\big) \rightarrow$ $N\left(0, \frac{\delta f}{\delta v}^T \Theta \frac{\delta f}{\delta v}\right)$, where

$$\Theta = \begin{pmatrix} \sigma_i^2 & \sigma_{i,n} & 0 & 0 \\ \sigma_{i,n} & \sigma_n^2 & 0 & 0 \\ 0 & 0 & 2\sigma_i^4 & 2\sigma_{i,n}^2 \\ 0 & 0 & 2\sigma_{i,n}^2 & 2\sigma_i^4 \end{pmatrix}$$

6   Empirical Results and Analysis

In this chapter we present the results of our empirical study and elaborate on our findings. We will discuss the performance of our optimal BL and Markowitz portfolios and the accompanied benchmarks in the period of 2006-2015. Hopefully, this will enlighten various aspects of the portfolios and reveal their main differences.

Empirical results will be presented for a total of nine different strategies or portfolios; the three benchmarks (Mkt_weights, 1/N, and Min_var), MV with and without constraints on shorting, together with 4 different BL portfolios. As previously mentioned we have employed two versions of the BL model. First is the CRM (BL (Can.) and BL_ns (Can.)) where, due to how Ω is defined, the BL expected returns are a simple average of the prior and conditional distribution (shown in Section 3.3.4). Therefore, the often mentioned weights-on-views will play no role. Our ARM (BL (Alt.) and BL_ns (Alt.)) will contrast this since the uncertainty in views is scaled by 1/c. These two models are common despite that they are fundamentally different from a theoretical standpoint and despite that only one of them offers a tool to influence the end result based on the confidence related to views.

6.1   Performance Criteria

The portfolios will be evaluated and assigned a rank according to each performance criteria as well as an overall ranking. Assigning ranks to each portfolio will act as a way to organize the evaluation process. When looking at the performance it is important to keep in mind the fact that the BL portfolios are, of course, heavily influenced by the investor views. However, assessing the turnover and tracking error will highlight the behavior of the BL model rather than the effects of views in particular.

6.1.1   Cumulative Return

Graph 1: Cumulative Returns 2006-2015
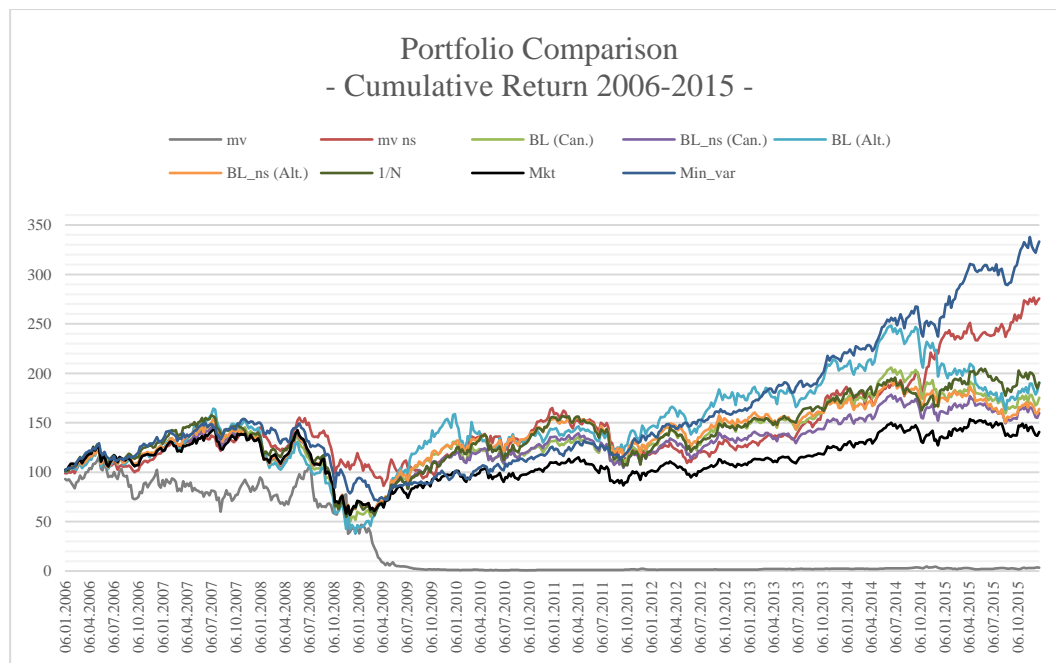
Graph 1 shows the cumulative returns over the investment period for all portfolios. The highest performing portfolio in terms of cumulative return over the period is the Min_var portfolio, with an excess return of 233%, or an average annual return of 12.8% (see Table 1, Panel B). All the portfolios constructed using the BL approach outperform the market weighted portfolio; however, only the portfolios constructed using the ARM outperform the equally weighted portfolio. This is due to the fact that the ARM assigns a higher confidence in views; hence, the portfolios deviate from the market to a higher degree. Further, the market weights in this sample are dominated by a small number of companies with relatively large market capitalization compared to the average company included. Hence, the portfolios constructed using the BL approach will be more sensitive to share price fluctuations in the largest companies in our sample. The 1/N portfolio overweight small companies relative to large ones, thus implying that with our sample over the period 2006-2015 smaller than average companies outperform larger than average companies.

Even though the end results between the two BL portfolios are quite similar, the ARMs deviate more from the market portfolio than the CRM. This can be seen studying the graphs of cumulative returns.

Graph 2: Comparison of Yearly Returns
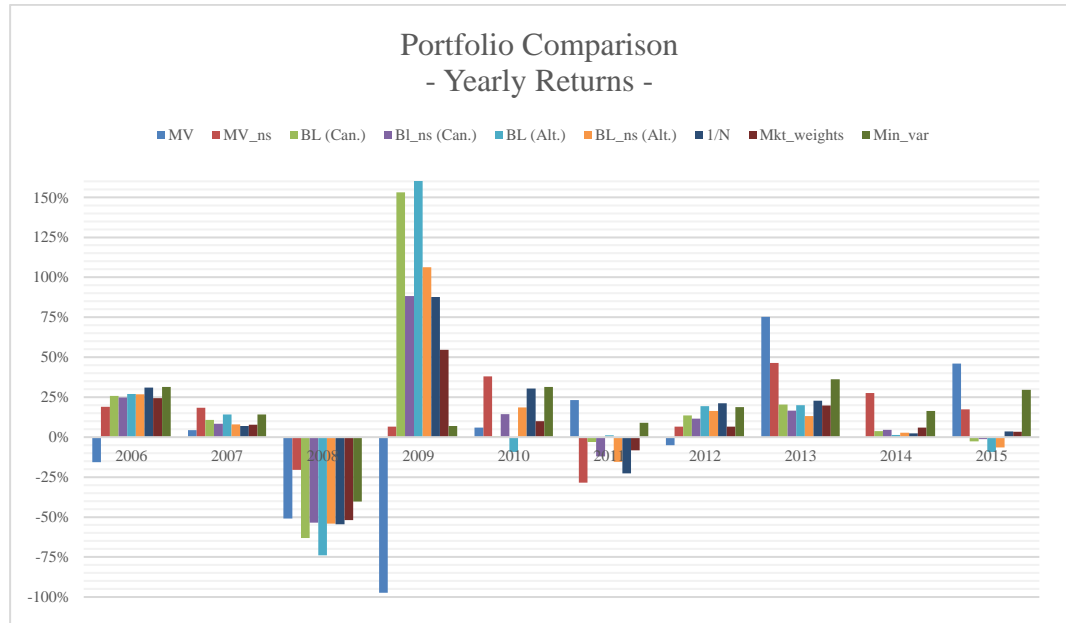
Graph 2 indicates a clear correlation of yearly portfolio returns. The co-movement is particularly strong the first four years. Correlations of raw returns are presented in Table 4. All of the portfolios are highly correlated with the market, except for the MV portfolio.

Table 1, Panel A shows the development for each portfolio return year by year. Again, the correlation in returns is evident. However, some are still varying more than others, even though they tend to move in the same direction. All portfolios experience steep declines in 2008, but most show a strong recovery in 2009, recuperating from the financial crisis the year before. The MV portfolios and the Min_var portfolio, however, do not recover in 2009. The MV portfolio plummets badly in the end of 2008 and the beginning of 2009, with the worst week showing negative returns in excess of 50%, and never recovers from the resulting low portfolio value. The MV_ns and Min_var portfolios do not recover as strongly in 2009 and their correlation with the market is not as high as for the other portfolios (0.624 and 0.564 respectively). The two portfolios reap high profits the last four years, ending up being the best performing portfolios (looking at compounding raw returns).

Table 1: Yearly Returns for All Portfolios

The table lists the raw returns for all portfolios during the investment period. Panel A displays the returns on a yearly basis. Panel B shows the average compounded return of each portfolio across the 10-year period. The investments period spans from the first month of 2006 to the last month of 2015. The return data is calculated using weekly excess returns, i.e. the return above 10-year Norwegian government bonds.

Strategy	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt weights	Min. var
Panel A: Yearly Returns for All Portfolios									
2006	-15.56 %	19.04 %	25.82 %	24.80 %	26.96 %	26.72 %	31.01 %	24.46 %	31.37 %
2007	4.26 %	18.32 %	10.79 %	8.45 %	14.20 %	8.03 %	6.96 %	7.74 %	14.11 %
2008	-50.89 %	-20.39 %	-63.02 %	-53.59 %	-73.98 %	-54.18 %	-54.51 %	-51.87 %	-40.39 %
2009	-97.43 %	6.55 %	153.2 %	88.28 %	318 %	106.2 %	87.70 %	54.54 %	6.97 %
2010	5.97 %	38.00 %	0.49 %	14.42 %	-9.19 %	18.56 %	30.33 %	9.90 %	31.40 %
2011	23.11 %	-28.54 %	-3.12 %	-12.09 %	1.10 %	-15.37 %	-22.65 %	-8.25 %	8.96 %
2012	-5.15 %	6.50 %	13.63 %	11.58 %	19.32 %	16.34 %	21.10 %	6.61 %	18.71 %
2013	75.33 %	46.44 %	20.42 %	16.51 %	20.00 %	13.07 %	22.85 %	19.70 %	36.23 %
2014	-0.14 %	27.67 %	3.74 %	4.57 %	1.31 %	2.76 %	2.41 %	6.02 %	16.28 %
2015	46.08 %	17.47 %	-2.74 %	-1.20 %	-9.31 %	-6.48 %	3.65 %	3.37 %	29.54 %
Panel B: Average Compounded Returns									
2006-2015	-28.44 %	10.67 %	5.78 %	4.80 %	6.65 %	5.07 %	6.65 %	3.47 %	12.80 %

Table A3 displays a high correlation between the BL portfolios. Due to the low uncertainty in the views for the ARM, the returns display a higher volatility and are less correlated with the market. This is intuitive as lower uncertainty assigned to investor views will cause larger weight to be placed on views rather than the market portfolio. The average compounding return of the BL portfolios show that the ARM outperforms the CRM. The reason is that the views contribute to outperforming the market, so that the more weight being put on them the better. If the views contributed negatively, the ARM would underperform.

If we ignore the MV portfolio, the worst performer overall is the market portfolio. The BL portfolios outperform the market because of the investor views. MV_ns is not as easy to interpret, but resemble a momentum strategy when historical returns are used as a proxy for expected returns. The 1/N portfolio overweight small stocks compared to the market, indicating that, in our sample, small stocks outperform large stocks.

6.1.2   Sharpe Ratio

Table 2: Comparison of In-Sample and Out-Of-Sample Sharpe Ratios

The table displays the weekly Sharpe ratios using logarithmic values. Panel A lists the In-Sample Sharpe ratios as calculated by the solver function in Microsoft Excel. Panel B lists the out-of-sample Sharpe ratios representing the actual Sharpe ratios achieved calculated using ln values of returns. The reason for using ln values of returns is to more accurately estimate an average level of out-of-sample excess return (and the accompanying standard deviation) (Hudson and Gregoriou 2010). Also, since working with ln returns in-sample, the two are comparable.

Strategy	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt weights	Min var
Panel A: In-Sample									
Expected Portfolio Return	0.0232	0.0061	0.0041	0.0037	0.0054	0.0042	0.0033	0.0033	0.0021
Portfolio Standard Deviation	0.0672	0.0353	0.0400	0.0374	0.0463	0.0387	0.0379	0.0365	0.0207
Sharpe Ratio	0.4114	0.2161	0.0958	0.0942	0.1100	0.1033	0.0812	0.0866	0.1185
Panel B: Out-of-Sample									
Portfolio Returns	-0.0064	0.0019	0.0011	0.0009	0.0012	0.0009	0.0012	0.0007	0.0023
Portfolio Standard Deviation	0.0978	0.0309	0.0388	0.0345	0.0488	0.0357	0.0360	0.0336	0.0261
Sharpe Ratio	-0.0657	0.0629	0.0278	0.0261	0.0253	0.0266	0.0344	0.0195	0.0884

In Table 2, Panel A and B, the in- and out-of-sample weekly Sharpe ratios are presented. The difference between them demonstrates the degree of estimation error and reliability associated with the model estimates. As expected, the portfolios constructed using the traditional MV approach, the MV and MV_ns, show high Sharpe ratios in-sample, 0.411 and 0.216 respectively. The Sharpe ratio of the MV portfolio represents an optimal diversification strategy without estimation error. However, these levels are far off the Sharpe ratios achieved out-of-sample, with levels of -0.066 for the MV and 0.063 for the MV_ns. Deviations can also be observed for the BL portfolios although to a lower degree than for the MV portfolios. The BL portfolios perform close to identically in terms of Sharpe ratios out-of-sample, ranging from 0.025 (BL (Alt.)) to 0.028 (BL (Can.)). In-sample, a minor distinction is evident between the CRM and the ARM, with 0.096 and 0.110, 0.094 (ns) and 0.103 (ns) respectively. It becomes evident that the model overestimates the portfolios expected return for all the portfolios, as well as the benchmarks, with one exception, namely the Min_var. The highest performing portfolio in terms of out-of-sample Sharpe ratio is the Min_var portfolio. It also has the lowest deviation between in- and out-of-sample Sharpe ratios.

Table 3: Annualized Sharpe Ratios

The table lists the annualized Sharpe Ratios for all portfolios on a yearly basis. The annualized Sharpe ratio is calculated using the yearly returns from Table 3, and the annualized standard deviation values from Table 7. The Sharpe ratios displayed are derived from raw excess return data, i.e. the return above the 10-year Norwegian government bonds.

Year	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt_weights	Min_var
2006	-0.3757	0.9392	1.1705	1.1607	1.1149	1.2096	1.5134	1.1770	1.9703
2007	0.0893	1.0088	0.5706	0.4750	0.6760	0.4593	0.4253	0.4186	0.9898
2008	-0.6950	-0.7212	-1.1537	-1.1744	-1.0327	-1.1465	-1.1756	-1.1701	-1.1471
2009	-0.9045	0.2206	4.1750	2.7186	6.5677	3.0930	2.6252	1.7912	0.3058
2010	0.0969	1.5188	0.0204	0.6821	-0.2960	0.8510	1.3314	0.4746	1.8237
2011	0.3593	-1.0543	-0.1428	-0.5175	0.0492	-0.6464	-0.8237	-0.3517	0.5497
2012	-0.1297	0.4222	0.8571	0.7880	0.9884	1.0264	1.2913	0.5005	1.8376
2013	2.1496	3.4653	1.6667	1.4501	1.2247	0.9993	2.0065	1.9680	3.2588
2014	-0.0020	1.2751	0.1855	0.2583	0.0548	0.1560	0.1296	0.3410	0.9611
2015	0.5281	1.3209	-0.1556	-0.0736	-0.4567	-0.3981	0.2001	0.1930	2.2318
Average	0.1116	0.8395	0.7194	0.5767	0.8890	0.5604	0.7523	0.5342	1.2781

Table 3 shows the annualized (out-of-sample) Sharpe ratios for each year included in the investment period for each individual portfolio. The annualized Sharpe ratios are calculated using the yearly compounding returns and annualized standard deviations. To arrive at the annualized standard deviation, we apply the technique explained in, among others, Kritzman (1991), where the standard deviation of returns used are scaled to yearly values by multiplying with the square root of the number of observations in a full year, i.e. 52 for weekly data, see Table A4. The results indicate that on average the short-restricted BL portfolios, both constructed using the CRM and ARM, perform close to the market capitalization weighted portfolio in terms of risk-adjusted returns. The portfolio constructed by minimizing the portfolio variance shows the highest risk adjusted returns, in terms of Sharpe ratio, followed by the BL portfolio constructed using the ARM.

Table A5 presents the p-values for differences in Sharpe ratios. The Min_var portfolio is statistically distinguishable from the MV, Mkt_weights, and BL_ns (Can.) portfolios at a 10% level. Min_var is the best performer in the sample, and the other three are ranked ninth, eighth, and sixth respectively. None of the pairs including two BL portfolios are close to being statistically distinguishable. Up to this point they have been reported to yield about the same return, both risk adjusted and not, therefore this result is not considered surprising. Other

performance criteria may measure their differences more clearly, such as turnover and tracking error.

6.1.3   Portfolio Turnover

Graph 3: Comparison of Portfolio Turnover
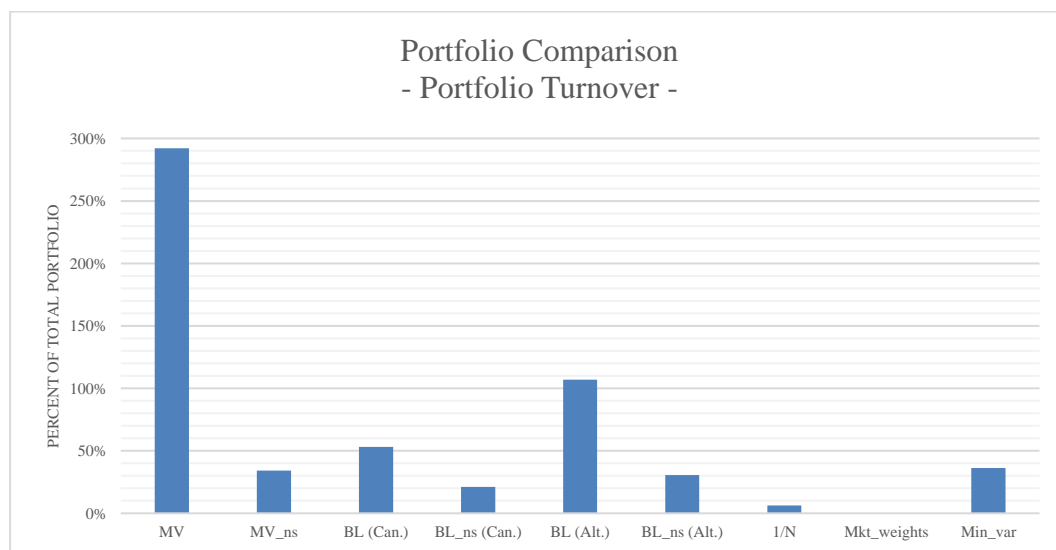
Table A6 shows the level of turnover inherent in each portfolio, the same results are displayed graphically above. The turnover criterion represents the portion of a portfolio that on average is being reallocated at each rebalance date, i.e. every month. Thus, this criterion is used to measure the transaction cost associated with the different portfolio. As expected, the MV portfolio with no short constraints holds the highest level of turnover. On average, just below 300% of the portfolio is reallocated every month. This is due to the excessively large long and short positions suggested in several stocks, and the instability of these positions.

Comparing the BL portfolios constructed using the CRM and the ARM, the latter has twice the turnover without constraints (1.07 vs 0.53) and close to 50% higher with constraints on shorting (0.31 vs 0.21). This result translates into greater implied transaction costs in the implementation of the ARM. The greater fluctuation in suggested weights, the greater the turnover or transaction cost will end up being. Because the BL portfolios apply the market (with zero turnover by construction) as a prior or starting point, the ARM ends up with a higher turnover due to its stronger confidence in views. Naturally, portfolios with constraints on shorting have lower turnover, holding everything else equal.

6.1.4   Certainty Equivalent Return

Table 10, Panel A, shows the obtained CEQ for all portfolios and Table 10, Panel B, presents the p-values of the differences in CEQ. Again, the results are calculated with a risk aversion parameter of 2.5. Due to uncertainty related to how to determine the risk aversion parameter, we want to emphasize that δ only affects the estimation of the BL portfolios. Meaning that one can easily assess, for all portfolios except the BL portfolios, the impact of δ on CEQ. To determine the new CEQ values for the BL portfolios one have to calculate new out-of-sample excess returns with the new risk aversion parameter.

Table 4: Certainty Equivalent Return and Statistical
Test of Difference in CEQ

The certainty equivalent return for each portfolio is displayed in Panel A below. A statistical test of the differences in CEQ is shown in Panel B. The certainty equivalent return is the return an investor would accept with certainty instead of investing in a given risky investment. The CEQ is calculated by subtracting from the mean return, the parameter δ divided by two and multiplied by the variance of return. In this calculation the ln returns are used, and the CEQ is reported in weekly returns. The statistical test of the difference in CEQ follows the methodology by Greene (1997) described in Section 4.4.4. It shows whether or not two portfolios have a significant different CEQ, and to which extent.

Strategy	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt_weights	Min_var
Panel A: Certainty Equivalent Return for All Portfolios									
CEQ	-1.84 %	0.08 %	-0.08 %	-0.06 %	-0.17 %	-0.06 %	-0.04 %	-0.08 %	0.15 %
Panel B: P-values for differences in CEQ									
MV		0.0001	0.0001	0.0001	0.0004	0.0001	0.0000	0.0001	0.0000
MV_ns			0.5672	0.6067	0.4148	0.5991	0.6717	0.5550	0.7522
BL (Can.)				0.9445	0.8074	0.9590	0.8935	0.9864	0.3603
BL_ns (Can.)					0.7448	0.9855	0.9467	0.9547	0.3843
BL (Alt.)						0.7606	0.6985	0.7758	0.2514
BL_ns (Alt.)							0.9333	0.9700	0.3783
1/N								0.9013	0.4452
Mkt weighted									0.3423
Min_var									

The CEQ of the MV portfolio is -1.84%, which means that an investor would be willing to accept a weekly loss of 1.84% with certainty over investing in the MV portfolio. The CEQ of the MV portfolio stands out from the rest with its poor performance; however, six out of the other eight is also negative. BL (Alt.) has the second lowest CEQ with a value of -0.17%, which in yearly terms translates into a loss of 9.2%. The CEQ of the market changes from -0.08% (-4.0% yearly) to 0.009% (0.4% yearly) when changing δ from 2.5 to 1. This highlights the

somewhat arbitrary value of the CEQ, caused by the seemingly arbitrary choice of δ. Even though the level of the values can be hard to interpret, the ranking of the portfolios stay the same.

Analyzing Table 4, Panel B, the differences in CEQ between the portfolios are not statistically significant for all portfolios except the MV portfolio. All pairs including the MV portfolio are statistically significant at the 1% level. Thus, the MV portfolio can be separated from the other portfolios, but among the remaining portfolios the CEQ is not statistically distinguishable.

6.1.5   Tracking Error

Table 5 shows the weekly tracking error for all portfolios related to two of the benchmarks. The tracking error for the MV portfolio exceeds 10% for both benchmarks. The weekly deviations are extremely high and underpin the unpredictability inherent in the MV portfolio. The MV_ns and Min_var portfolios are at reasonable levels (2.78%/2.77% and 2.83/3.06% respectively) even though they do not follow the market as closely as the BL portfolios.

Table 5: Tracking Error

The tracking error of the optimized portfolios is displayed in the table below. The first row shows the tracking error of the Markowitz portfolios and the BL portfolios against the market capitalization weighted portfolio. The second row shows the tracking error for the same portfolios as row 1 but in relation to the equally weighted portfolio. Lastly, the tracking error is reported in weekly returns.

Benchmark	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	Min_var
Mkt_weights	10.13 %	2.78 %	1.41 %	0.61 %	2.91 %	1.01 %	2.83 %
1/N	10.55 %	2.77 %	1.90 %	1.24 %	3.16 %	1.27 %	3.06 %

With the market portfolio as a benchmark the tracking error of the BL models constructed using the ARM is about twice as large as the CRM counterparts (2.91% vs. 1.41% and 1.01% vs. 0.61%). Again, the reason is that the ARM is tilted to a greater extent towards its views. The difference in tracking error is not as large using the 1/N portfolio as a benchmark since tweaked versions of the market are all somewhat different from the 1/N portfolio.

When interpreting the level of tracking error for the BL portfolios it is evident that what contributes to the level of deviation is the certainty to the views, rather than the views itself. If a given investor view makes a portfolio over- or underperform the market by 5%, the tracking error would be the same. However, if more views

are added, or the confidence to the views is changed, the tracking error could change. The only difference between the CRM and the ARM in our implementation is the confidence to the views, thus being the only parameter influencing the difference in tracking error.

6.2   Total Ranking

Table 12 displays the overall ranking, based on all the previously mentioned performance criteria. The equally weighted (1/N) portfolio receives the lowest overall score, and is hence ranked as the best performing portfolio. The 1/N portfolio outperforms the market in all criteria related to returns (cumulative return, Sharpe ratio, and CEQ). This was attributed to the fact that smaller than average stocks outperform larger than average stocks. Also, since the BL portfolios are heavily related to the market, the 1/N portfolio beats all the BL portfolios on all return related criteria. If one excludes the MV portfolio from the analysis, due to overly poor performance, the 1/N portfolio underperforms based on returns relative to the remaining MV_ns and Min_var. These are the best performers based on returns, but after accounting for turnover and tracking error, 1/N comes out on top. The 1/N portfolio is by construction the best performer (together with Mkt_weights) by tracking error, and its turnover is obviously low. Hence, an evaluation on these grounds looks promising for the 1/N portfolio once it outperforms the market.

The BL portfolios that are constrained from shorting consistently outperform the market by returns, both risk adjusted and not. This is not the case for the unconstrained BL portfolios, since they underperform the market based on CEQ. Looking at cumulative returns, the BL (Alt.) is the best performing BL portfolio. We find this intuitive, due to successful views and the fact that the BL (Alt.) portfolio is the one that is tilted the most towards the views. Second best performer, based on cumulative returns, is the BL (Can.), followed by the BL_ns (Alt.) and BL_ns (Can.). Thus, constraints on shorting suppresses the cumulative return, however, it boosts the risk adjusted return. In terms of Sharpe ratio, the BL (Can.) performs the best (out of the BL portfolios), but the BL_ns (Alt.) and BL_ns (Can.) surpasses the BL (Alt.) portfolio. By CEQ, both portfolios with constraints on shorting (and the market) have outperformed the unrestricted

portfolios. This indicates that the CEQ (with a δ of 2.5) compensates more for risk than the Sharpe ratio.

By all appearances, the MV portfolio is the worst performer of the nine. The reason lies in its excessive trading and poor returns. Min_var and MV_ns are the best performers by return, but are surpassed by 1/N due to higher turnover and tracking error. Pinpointing the causes for Min_var and MV_ns' good performance is hard, but in this case it has been lucrative to use the historical average mean or just to minimize the variance of returns.

Table 12: Total Ranking Based on All Performance Criteria

This table reports the ranking of all nine portfolios based on all five criteria. The overall score is just the sum of ranks, therefore a low score is favorable. From the overall score, an overall rank is calculated. We want to emphasize that this ranking is not absolute in telling which strategy is better. It is rather a tool used to analyze the portfolios and their different characteristics.

Rank based on:	Cumulative return	Sharpe ratio	Turnover	CEQ	Tracking error	Overall score	Overall rank
MV	9	9	9	9	9	45	9
MV_ns	2	2	5	2	6	17	3
BL (Can.)	5	4	7	7	5	28	7
BL_ns (Can.)	7	6	3	4	3	23	4
BL (Alt.)	4	7	8	8	8	35	8
BL_ns (Alt.)	6	5	4	5	4	24	5
1/N	3	3	2	3	1	12	1
Mkt_weights	8	8	1	6	1	24	6
Min_var	1	1	6	1	7	16	2

7   Conclusion

In this thesis we applied the BL model to data from the Norwegian stock market. We aimed to clarify the implementation process as well as discuss various ambiguities inherent in the BL model. Further, we wanted to determine whether the model offered any improvements to the traditional Markowitz paradigm.

The most surprising discovery from a theoretical perspective was the fact that the CRM, with its rather complicated buildup, simplifies to a simple average of the prior and conditional distribution. Discussing the parameter τ, which has gained great attention, will then be futile. The only rationale approach, in our opinion, is to define the variance of views (Ω) before setting a value for τ.

In the empirical study we measured the performance of the BL model relative to the Markowitz model and other benchmarks (1/N, Mkt_weights, and Min_var). What follows are subsections were we first discuss the significance of the results, then the BL portfolios are compared to the Markowitz maximum Sharpe portfolios and the two benchmarks (Mkt_weights and 1/N), and finally we evaluate the BL portfolios internally.

When assessing the performance in relation to risk, using the Sharpe ratio, we did not find significant differences between the BL portfolios and the market weighted portfolio or the naive portfolio. The MV portfolio had a significantly different Sharpe ratio at the 5 and 10% level against the MV_ns and Min_var respectively. However, this result was due to extreme deviations in the MV portfolio in terms of portfolio weightings and the proceeding returns. In terms of CEQ, we find it to be statistically different for the MV portfolio against all other portfolios. Similar reasons as aforementioned also apply in this case.

BL vs. Markowitz: The unrestricted MV portfolio is the worst performer in all criteria, it is not implementable in practice due to excessive trading, and its tracking error is extremely high (in excess of 10% per week). It may be theoretically sound, but practically, the MV portfolio using historical means is useless. The restricted MV portfolio performs well when assessing returns, and

risk adjusted returns, and even the turnover is not too bad. The strategy performs well in our sample, but the frequent use of corner solutions, and hence lack of diversification, may scare investors off.

BL vs. mkt_weights and 1/N: As previously mentioned the BL portfolios beat the market on returns, due to successful views. Adjusting for risk, the BL portfolios still outperform the market in Sharpe ratio, but only the restricted BL portfolios perform better than the market in CEQ. For the BL portfolios to be a better alternative investment than the market, it simply needs to outperform based on returns. Holding a market portfolio requires no portfolio turnover, therefore when tilting the portfolio, it is imperative to the BL portfolio that it is on average more correct than not. Thus, gaining abnormal return, and not only incur excessive transaction costs. However, from a practical point of view, the option to incorporate views into a portfolio selection model has value in itself.

Many of the same arguments apply to the 1/N portfolio, with its low turnover and tracking error. In our sample the 1/N portfolio overweight smaller than average stocks and underweight larger than average stocks. One practical issue is that the strategy does not take into account the market capitalization of each stock. This could induce strong price effects negatively affecting the portfolio return. However, the bet on small stocks reap good returns in our sample, and with a low turnover and tracking error the 1/N portfolio is ranked best in class. The pros of 1/N are its simplicity, and its low transaction costs, and a con is the fact that it ignores market capitalization.

Internal BL evaluation: There are two clear distinctions along two dimensions when evaluating the BL portfolios. They are both intuitive, thus verifying the logic of the model. First, the unrestricted portfolios yield higher returns than the restricted; however, the latter perform better when adjusting for risk. Further, the unrestricted portfolios demand higher turnovers and obtain higher tracking errors. Second, the ARM yield higher return, demand higher turnover, and obtain higher tracking error than the CRM. For our dataset, it can be summarized in the following; the more tilted the better the return, higher the risk, higher transaction cost, and higher uncertainty.

8   Further Research

In this thesis we have focused on the implementation and application of the BL model using a set of Norwegian Stocks during the last 10 years. As a suggestion for further research it could be of interest to consider a wider scope of stocks and a longer time period. This could possibly unveil other interesting aspects of the model behavior that might have been lost due to limited geographical location of assets and the time period of the study. Further, one could also include other asset classes in order to broaden the investment universe and this way more closely mirror the possibilities of an actual investor. This could include using a broad international index of stocks, fixed income assets and real estate investment trusts or a combination of these.

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Appendix

Table A1: Company Names and Tickers

The table lists the stocks included in our sample, and consists of the 20 largest stocks, measured by market capitalization, listed on Oslo Stock Exchange in the period of 2004-2015.

Company Name	Ticker
Atea	ATEA
DNB	DNB
DNO International	DNO
Fred. Olsen Energy	FOE
Frontline	FRO
Kongsberg Gruppen	KOG
Lerøy Seafood Group	LSG
Norsk Hydro	NHY
Orkla	ORK
Petroleum Geo-Services	PGS
Prosafe	PRS
Royal Caribbean Cruises	RCL
Schibsted ser. A	SCHA
Storebrand	STB
Statoil	STL
Subsea 7	SUBC
Telenor	TEL
TGS-NOPEC Geophysical Company	TGS
Tomra Systems	TOM
Veidekke	VEI

Table A2: Starting Portfolio Weights 01.01.2006

The table displays the portfolio weights suggested by the portfolio optimization at the start of the investment period. Optimal weights for all the BL portfolios and the MV maximum Sharpe portfolios, with and without constraint on short selling, are shown.

	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt_weights	Min_var
ATEA	-0.2745	0.0000	-0.0149	0.0000	-0.0315	0.0000	0.0500	0.0009	-0.0570
DNB	0.1444	0.1113	0.0340	0.0486	-0.0321	0.0053	0.0500	0.0968	0.3553
DNO	0.2854	0.1696	0.0370	0.0274	0.0610	0.0325	0.0500	0.0142	-0.0127
FOE	0.4359	0.1173	0.0839	0.0651	0.1567	0.0954	0.0500	0.0149	-0.0197
FRO	0.1408	0.0534	0.0209	0.0168	0.0225	0.0023	0.0500	0.0194	-0.0609
KOG	-0.2476	0.0000	0.0014	0.0000	-0.0010	0.0000	0.0500	0.0037	0.0828
LSG	0.1198	0.0422	0.0249	0.0023	0.0481	0.0016	0.0500	0.0029	0.0724
NHY	-0.0408	0.0000	0.0351	0.0376	-0.1182	0.0000	0.0500	0.1804	0.0966
ORK	0.3067	0.0258	0.0956	0.0999	0.1347	0.1385	0.0500	0.0585	0.0569
PGS	-0.1341	0.0000	0.0513	0.0316	0.0923	0.0415	0.0500	0.0125	0.0155
PRS	-0.1044	0.0000	-0.0567	0.0000	-0.1266	0.0000	0.0500	0.0098	0.0890
RCL	0.0551	0.0000	0.1191	0.1127	0.1770	0.1377	0.0500	0.0641	0.0306
SCHA	-0.1727	0.0000	0.0129	0.0000	0.0118	0.0000	0.0500	0.0140	0.0523
STB	-0.1017	0.0000	-0.0079	0.0000	-0.0322	0.0000	0.0500	0.0151	0.0959
STL	-0.3504	0.0000	0.4170	0.3917	0.4967	0.3740	0.0500	0.3413	-0.0159
SUBC	-0.1311	0.0000	0.0529	0.0405	0.0927	0.0549	0.0500	0.0151	-0.0945
TEL	0.3072	0.0964	0.1536	0.1170	0.1957	0.1114	0.0500	0.1137	0.1712
TGS	0.0871	0.0000	-0.0312	0.0000	-0.0729	0.0000	0.0500	0.0083	-0.0013
TOM	0.0690	0.0000	0.0071	0.0088	0.0054	0.0048	0.0500	0.0087	-0.0040
VEI	0.6058	0.3841	-0.0361	0.0000	-0.0799	0.0000	0.0500	0.0055	0.1474
SUM	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A3: Correlation Matrix of LN Returns

The table below displays the level of correlation between the different portfolios included in the study. The highest correlations can be seen between the BL_ns constructed using the CRM against the BL_ns constructed using the ARM, and also for the market weighted portfolio against the short restricted BL portfolios.

Strategy	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1N	Mkt_weights	Min_var
MV	1.0000	0.2876	-0.0239	-0.0347	-0.0252	-0.0308	-0.1449	-0.0435	0.2551
MV_ns		1.0000	0.5780	0.6387	0.4945	0.6418	0.6589	0.6237	0.5950
BL (Can.)			1.0000	0.9621	0.9659	0.9576	0.8712	0.9336	0.4710
BL_ns (Can.)				1.0000	0.8676	0.9929	0.9365	0.9837	0.5404
BL (Alt.)					1.0000	0.8776	0.7609	0.8110	0.3534
BL_ns (Alt.)						1.0000	0.9356	0.9585	0.5191
1N							1.0000	0.9300	0.5402
Mkt_weights								1.0000	0.5640
Min_var									1.0000

Table A4: Annualized Standard Deviation of Return for Empirical Data

The table lists the annualized standard deviations of all portfolios for every year. The annualized standard deviations are calculated as the standard deviation of weekly excess return for a given year multiplied by the square root of 52.

Year	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt_weights	Min_var
2006	0.4143	0.2027	0.2206	0.2137	0.2418	0.2209	0.2049	0.2078	0.1592
2007	0.4774	0.1816	0.1891	0.1779	0.2101	0.1748	0.1636	0.1849	0.1426
2008	0.7323	0.2828	0.5463	0.4563	0.7164	0.4726	0.4637	0.4433	0.3521
2009	1.0771	0.2970	0.3669	0.3247	0.4842	0.3433	0.3341	0.3045	0.2280
2010	0.6159	0.2502	0.2386	0.2114	0.3106	0.2181	0.2278	0.2085	0.1722
2011	0.6432	0.2707	0.2184	0.2337	0.2235	0.2378	0.2750	0.2346	0.1630
2012	0.3969	0.1539	0.1591	0.1470	0.1955	0.1592	0.1634	0.1321	0.1018
2013	0.3504	0.1340	0.1225	0.1139	0.1633	0.1308	0.1139	0.1001	0.1112
2014	0.7127	0.2170	0.2018	0.1768	0.2383	0.1768	0.1861	0.1766	0.1694
2015	0.8726	0.1323	0.1763	0.1635	0.2040	0.1629	0.1824	0.1745	0.1323
Average	0.6293	0.2122	0.2440	0.2219	0.2988	0.2297	0.2315	0.2167	0.1732

Table A5: Statistical Test for the Difference in Sharpe Ratios

This table shows the p-values from the statistical test performed for the difference in Sharpe ratios using the framework suggested in Jobson and Korkie (1981), and corrected by Memmel (2003) (as described in Section 4.4.2).

Strategy	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt_weights	Min_var
MV		0.0156	0.1410	0.1494	0.1518	0.1464	0.1340	0.1821	0.0039
MV_ns			0.3787	0.3187	0.3900	0.3236	0.4253	0.2493	0.5173
BL (Can.)				0.8874	0.8306	0.9276	0.7663	0.5977	0.1696
BL_ns (Can.)					0.9020	0.8069	0.6336	0.2329	0.0979
BL (Alt.)						0.9785	0.8842	0.8472	0.3170
BL_ns (Alt.)							0.6479	0.4690	0.1180
1/N								0.3609	0.1901
Mkt_weights									0.0870
Min_var									

Table A6: Portfolio Turnover

The portfolio turnover for each of the portfolios is displayed in the table below. The level of turnover represents how much of the portfolio is reallocated on average each month. A value of 1 would mean that 100% of the portfolio is reallocated each month. The turnover of 1/N is different from zero due to fluctuations in market prices. This price effect is accounted for each month when calculating the turnover for every portfolio.

	MV	MV_ns	BL (Can.)	BL_ns (Can.)	BL (Alt.)	BL_ns (Alt.)	1/N	Mkt_weights	Min_var
Turnover	2.9227	0.3420	0.5303	0.2108	1.0704	0.3054	0.0630	0	0.3625

Preliminary Thesis Report

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Introduction

Harry Markowitz introduced the first asset allocation model in his 1952 article *Portfolio Selection.* The model he presented was a mean-variance optimization technique designed to generate optimal portfolios, using historical data of return and predicted expected returns. His contribution received praise in the academic world and is considered to represent the beginning of modern portfolio theory. The article proposed a mean-variance optimization technique. Despite gaining appraisal in the academic world, portfolio managers were not equally excited. The approach had several issues that made its practical application limited. It was difficult to use and had a tendency to suggest portfolios that behaved badly (Black and Litterman 1992). First of all, the model required investors to assign exact expected returns for all assets. Also, the resulting portfolio weights were extremely sensitive to small changes in the expected returns, resulting in the portfolio weights changing drastically. Michaud (1989) states that the major problem with the technique is the way it would tend to maximize the effects of errors in the input assumptions.

As a solution to some of the inadequacies of the Markowitz models, Fisher Black and Robert Litterman put forth a new approach in 1990, further expanded in 1991 and 1992, which will be the focus of our thesis. By combining the mean-variance optimization techniques introduced by Markowitz with the equilibrium concepts of the CAPM (Capital Asset Pricing Model) by Sharpe (1964) and Lintner (1965b, 1965a), surfaces an approach to portfolio choice that came to be known as the Black-Litterman model, hereby referred to as the B-L model. Black and Litterman writes in their article *Global Portfolio Optimization* that "Quantitative asset allocation models have not played the important role they should in global portfolio management" (1992, 1). As it would turn out, more than two decades after their initial contribution, the use of quantitative models in portfolio choice have gained a strong position, and is to this day used by many agents as a viable tool in the investment industry.

Black and Litterman proposed a number of new features to the standard asset allocation model. Among the most important was the way it utilizes the equilibrium returns generated by a Global CAPM suggested in Black (1989) as a starting point, and how it lets investors easily adjust their portfolio based on a set

of views. The original model that consisted only of fixed income assets and currencies were expanded the year after, in Black and Litterman (1991b, 1992), to include equities as well. The optimal portfolio suggested by the model would in essence be a combination of the market portfolio and the investors own views. Hence, an investor with neutral views would hold the equilibrium (market capitalization weighted) portfolio. However, when possessing additional views, the model would tilt the optimal portfolio weights according to these views.

The importance of quantitative asset allocation models in today's investment world is perhaps greater than ever. On a daily basis portfolio managers attempt to make sense out of a vast stream of financial data and macroeconomic news. Quantitative asset allocation models are designed as a tool that can guide portfolio managers in their attempt to make more balanced and optimal portfolios. The B-L approach has been shown to be effective also in real markets, see among others Bevan and Winkelmann (1998) and Black and Litterman (1992), and we believe that it would be interesting to explore and test its applicability in practice further.

The aim of our research will be to add to the relatively limited empirical evidence of the performance of the model in real markets. We will do this by applying he B-L model on the Norwegian Stock Market. In addition to testing the model we seek to clarify on the implementation process of the model. The latter will help shed light on various aspects of the model behavior, as well as other issues related to the process. Even though the idea and the end result of the model might be intuitive, the implementation process can be quite complicated.

Literature review

The inception of modern portfolio theory is by most academics and practitioners considered to be the article *Portfolio Selection,* published in 1952 by Harry Markowitz. A starting point for Markowitz was John Burr Williams' *Theory of Investment Value (1938).* Here, Williams claimed that the value of a security should be the same as the net present value of its future dividends. With future dividends being unknown, Markowitz claimed that the expected future returns could serve as a proxy for future dividends, and hence the value of a security. Together with the expected future return, Markowitz argued that one have to take into consideration the variance, i.e. the risk, associated with investing in a portfolio. Since dealing with a portfolio of more than one asset, the co-movements between assets needed to be dealt with, represented by the covariances of returns.

Markowitz' mean-variance portfolio model serves the two basic objectives of investing; namely *maximizing expected return* and *minimizing the risk*. His framework has stood the test of time and is still considered academically sound. However, multiple practical issues have strongly prevented the model's impact in the professional investment management world.

Many problems with the use of Markowitz' mean-variance portfolio model has been advocated. Black and Litterman (1992, 1) highlighted that "A good part of the problem is that such models are difficult to use and tend to result in portfolios that are badly behaved". They elaborate by saying that without constraints the model results in large short positions in one or several assets. With no shorting of assets, the model frequently assigns zero weights to many assets, i.e. "corner" solutions. Also, the model does not take into consideration the market capitalization of assets, ending in large positions in assets with low market capitalization.

These unintuitive and unreasonable results stem, according to Black and Litterman (1992), from two known problems. First, the Markowitz formulation requires investment managers to specify expected returns for all assets included in the model. This seems laborious, knowing that investors tend to focus only on particular segments of the investment universe. In addition to being time consuming, expected returns are hard to estimate, and the historical returns that are often used is, according to Black and Litterman (1992), a poor proxy for future

returns. Second, the weights assigned to each asset in the mean-variance portfolio are extremely sensitive to the vector of expected returns. Together, compounding each other, these problems produce highly unstable portfolios. As Best and Grauer (1991) demonstrated, a small increase in one individual asset's expected return can drive half of the assets from the portfolio (with constraints on shorting).

Michaud wrote the article *The Markowitz Optimization Enigma: Is 'Optimized' Optimal?* in 1989 where several problems were discussed. One critique in Michaud (1989) states that the Markowitz' optimizers maximize estimation errors. The estimates of expected returns, variances, and covariances are subject to estimation error. The basis for the critique lies in the fact that Markowitz optimization overweighs those assets that have large expected returns, negative covariance and small variance. His argument is that "these securities are, of course, the ones most likely to have large estimation errors" (1989, 34). He further claims that using sample means from historical data as expected returns contributes to the maximization of estimation errors. Also, he touches upon the issues that the model does not account for asset's market capitalization weights, and the instability of results with respect to the expected return input in particular.

Fischer Black and Robert Litterman published in 1990 the article *Asset Allocation: Combining Investor Views With Market Equilibrium*, which proved to be the introduction of the B-L model. They extended the model in 1991 and 1992, and it was soon established as the B-L model. From this point on, many extensions and versions of the B-L model has been published.

The B-L model successfully filled many of the gaps that Markowitz' left behind with his mean-variance optimization model. It creates stable, mean-variance efficient portfolios, and according to Lee (2000), there is no longer real issues caused by estimation error-maximization. The first significant contribution to asset allocation by Black and Litterman is that it provides an intuitive and neutral starting point (prior), namely the equilibrium market portfolio, building on the work of Black (1989). The second contribution made is that investors can express their own views, either relative or absolute, and these views are combined with the equilibrium market portfolio, resulting in stable and intuitive portfolios. The weight of an asset increases if the investor becomes bullish toward the asset, all other equal. The weight also increases if the investor becomes more confident

about the bullish view. These features serve as two new dimensions to portfolio management, which together with the neutral starting point of the equilibrium market portfolio makes the model stable, intuitive, and valuable to practitioners. As Black and Litterman (1992) writes, they have combined two established theories of modern portfolio theory – the mean-variance optimization framework of Markowitz and the CAPM of Sharpe and Lintner.

In today's myriad of versions of the B-L model, Jay Walters (2014) has sorted the models into three distinct reference models, based on two central dimensions. The first dimension separates Bayesian from non-Bayesian models. The original B-L (1991a, 1991b, 1992), together with Bevan and Winkelmann (1998) and He and Litterman (1999) uses a Bayesian approach. Walters call these models, *canonical* B-L models. Non-Bayesian models are further split in two, models including the parameter τ (often referred to as the 'weight on views') are called *hybrid*, and models excluding τ and becomes pure mixing models are called *alternative*.

The original articles by Black and Litterman (1991a, 1991b, 1992) and He and Litterman (1999) focused on the features and overview, rather than on the derivation of the formulas behind the model. This made it hard to reproduce, and even hard to obtain full understanding of the models build-up. Bevan and Winkelmann (1998) provided insight on the internal process of working with the B-L model within Goldman Sachs. Full derivation of the model is still not presented, but an explanation on how they set target risk levels, focusing on tracking error and Market Exposure, presented by Litterman and Winkelmann (1996), contributes to the development on how to use the model.

Satchell and Scowcroft wrote the article *A demystification of the Black–Litterman model* in year 2000 where their main objective was to give a mathematical depiction of the model. However, their contribution in retrospect is the introduction of a new non-Bayesian (*hybrid*) model. It uses point estimates instead of distributions, leading τ and Ω to affect shrinkage of the views, rather than the original interpretations which is the 'weight on views' and uncertainty in the views respectively. Fusai and Meucci (2003), and later only Meucci (2005), replaced the model of Satchell and Scowcroft. It is a non-Bayesian model, which in addition excludes the parameter τ since Ω alone was considered sufficient in shrinking the influence of the views. Using Jay Walters' analogy, Meucci's model

is an Alternative Reference Model. Meucci himself coined the phrase, "Beyond B-L" referring to his model. Looking at the last decade, the most influential models, according to Walters, has been a mixture of the *canonical* and "Beyond B-L".

In 2002 Thomas M. Idzorek wrote *A Step-By-Step Guide To The Black-Litterman Model* where he introduced "user-specified confidence levels". The method of using confidence levels let the investor establish a confidence to each view, instead of calculating the less intuitive variance of each view. According to Idzorek, this new method should increase the usability of the model. Even if the model Idzorek uses is a Hybrid Reference Model it can also be applied to the *canonical* B-L.

Theory

In this part we will present some of the theoretical framework the B-L builds on. Black and Litterman present their approach as an upgrade to the traditional mean-variance approach. We will here start off by briefly presenting some of the most essential aspects of the Markowitz model presented in his paper *Portfolio Selection* from 1952. The main concept of the mean-variance method is that an investor can significantly reduce the risk of a portfolio, while at the same time keeping a certain level of expected return, or the investor can maximize the expected return, given a level of risk. This can be achieved by combining assets that have a low or negative correlation. The investor is assumed to be risk averse, and is only interested in expected return and risk.

Attainable portfolios are reached by solving the following problems:

$$\min_{\mathbf{w}} \mathbf{w}^T \Sigma \mathbf{w} \qquad\qquad (1)$$
$$\mathbf{w}^T \bar{r} = \bar{r}_p$$

or

$$\max_{\mathbf{w}} \mathbf{w}^T \bar{r} \qquad\qquad (2)$$
$$\mathbf{w}^T \Sigma \mathbf{w} = \sigma_p^2$$

Where

$\mathbf{w}$	is the vector of portfolio weights
$\mathbf{w}^*$	is the optimal portfolio
σ_p^2	is the variance of the portfolio
$\bar{r}_p$	is the required returns of the portfolio
$\bar{r}$	is the expected returns of the portfolio

μ   is the vector of expected excess returns
Σ   is the covariance matrix
δ   is the risk aversion coefficient

However, often the following problem is solved, and its result is referred to as the Markowitz' optimal portfolio (**w***).

$$\max_{\mathbf{w}} \mathbf{w}^T \mu - \frac{\delta}{2} \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \tag{3}$$

$$\mathbf{w}^* = (\delta \mathbf{\Sigma})^{-1} \mu$$

The expected return, using Markowitz' mean-variance approach, is often the mean historical return. However, in the B-L model, the expected returns are a combination of the equilibrium market portfolio and investors views. After having found the expected (excess) returns, the optimization process is similar for the B-L method as for the Markowitz' method. The following equation is often referred to as the B-L master formula.

$$\bar{\mu} = \left[(\tau \textstyle\sum)^{-1} + P^T \Omega^{-1} P \right]^{-1} \left[(\tau \textstyle\sum)^{-1} \Pi + P^T \Omega^{-1} Q \right] \tag{4}$$

Where

$\bar{\mu}$ is the vector of mean expected excess returns
τ is a scalar that represents the 'weight on views'
Σ is the covariance matrix of historical excess returns
P   is the matrix consisting of the assets involved in the views
Ω is a diagonal matrix comprised of the uncertainty of each view
             represented by its error terms
Π is a vector of equilibrium expected excess returns
Q   is a vector consisting of the investor views

These B-L expected returns are then used to solve problem (3) above, and optimal weights are obtained.

When applying the B-L model using subjective views about specific industries, companies, markets or asset classes one assume that the efficient market hypothesis in its semi-strong form do not hold. Therefore, to benefit from own conceptions about the future market development, mispricing must at least prevail occasionally.

Methodology

In this section we clarify the empirical approach we intend to use and the motivation for this choice. The method we will use are based on the original approach by Black and Litterman (1991a, 1991b, 1992).

Our initial plan is to apply the B-L model on the Norwegian Stock Market. We will do this by comparing the performance of a portfolio constructed using the B-L approach against a traditional mean-variance method and a benchmark. The benchmark we intend to use is the OBX-index, consisting of the 25 most traded stocks on the Oslo Stock Exchange. In order to form a good basis for comparison we will define these 25 stocks as our investment universe. Our test will be conducted on weekly data in the period of 2010-2015. The performance of the model will be evaluated by assessing the cumulative returns, as well as the risk-adjusted returns in order to separate differences in return associated with excessive risk-taking.

We will construct optimal portfolios both with and without constraints on short selling. There are two main reasons for this. First, mutual funds are not allowed to short sell stocks. This means that many potential users of the B-L model and mean-variance methods are restricted. Second, the mean-variance optimization method has a tendency to suggest unreasonably large short positions one or several assets. Our hope is that this will enlighten different aspects of the models, both in terms of their behavior and their results.

Without investor views the optimal portfolio will be identical to the market portfolio, here the benchmark index (OBX). However, views are highly subjective in nature. In order to mitigate this issue, we intend to base our views on a sum of aggregate analysts' recommendations. We will hence collect "buy", "hold" and "sell" recommendations from various financial institutions, in order to form our views. By having a large pool of recommendations we hope to reduce subjectivity and the risk of bias. Based on these views we will rebalance the portfolio on a yearly basis.

In the B-L model, a view is assigned a value of 1, 0, or -1 to represent the nature of a view. A bullish view will be assigned a value of 1, while a bearish view will be assigned a value of -1, and a neutral view with a value of 0. The nature of the

view will then tilt the optimal portfolio, by increasing the weights of stocks with bullish views, or decreasing the weights of stocks for which views are bearish.

The confidence in each view will be based on the collective sentiment of analysts. An important aspect of the model is the option investors have to specify their certainty in views. We will use a slightly modified approach by utilizing the extension suggested by Idzorek (2002), where confidence levels of 0-100 are specified. This will provide us with an intuitive tool in determining the confidence corresponding to each view. In addition, the target prices of individual stocks will be used to determine expected returns associated with the views.

Data

We will collect weekly data from the Norwegian Stock Market in the period of 2009-2015. The investment period starts in January 2010, thus the first year of data will be used solely as input in order to construct the starting portfolio. The data will consist of returns on the 25 most traded stocks on OSE (Oslo Stock Exchange), as well as the OBX-index. We decided to use weekly return data as this will provide us with what we believe is a sufficient number of observations. To perform some of the calculations we also need the risk-free rate. As a proxy for the risk-free rate we will use data on 10-year Norwegian government bonds, collected from Norges Bank. In addition, we will gather and analyze data on analysts' recommendations in order to form our inputs for the views, and the corresponding expected return and confidence levels. In cases where no recommendation can be found for a given stock, no view will be assigned. The stock data we use will be adjusted for dividends, as well as splits. For this reason, we will have to adjust target prices accordingly. The data on stock returns and analysts' recommendations will be collected from Bloomberg.

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