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Master Thesis

- Transportation Lot Sizing in the Cement Industry -

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Preface

In our studies at BI Norwegian Business School it is required that the Master of Science students do an in-depth study in relations to their major. The research that is presented here is a study in the field of Logistics – Supply Chain and Networks.

The research that is presented in this thesis is on the topic of economical lot sizing and was initiated by a suggestion from our supervisor. The thesis presents an adaptation of the existing lot sizing theory utilized in production planning to a new transportation lot sizing model. The main contribution of this paper is that we have formulated and developed a new lot sizing model with extensions within a new area of application. The model is formulated based on an empirical case from HeidelbergCement but the emphasis in the thesis has been to develop a new transportation lot sizing model, not to come up with an optimal solution for the company

We want to thank HeidelbergCement for the cooperation and last but not least we want to thank our supervisor, Atle Nordli, for all the support, advice and inspiration during the thesis work.

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Summary

In this thesis a mathematical formulation of a new transportation lot sizing model is presented. The model is developed in three separate steps, starting with the development of a single level capacitated multi-terminal lot sizing model. This step constitutes our initial model and is referred to as Model 0. In the next step we have added the extension of period overlapping setup to our initial model. This step proved to be the most difficult to formulate in the model. The final step that concludes our final model is the implementation of the possibility for inventory shortage. Solutions to the three steps are found by the optimization tool used to solve the model – MPL with the Gurobi solver. Each step is presented separately in chapter four with a discussion of the solution together with a discussion of the challenges concerning each step.

Based on the discussion and observations made in chapter four; the stability, flexibility and functionality of the model are tested in chapter five. The tests are performed in different scenarios by changing different parameters in the model, and also by testing how the model reacts to different demand patterns. The utility value of the two extensions made in the model becomes more evident in these tests, and we also discuss the solutions and findings from each scenario.

1 Introduction

1.1 Motivation

In our first year of the Master of Science (MSc) program at BI we both chose the course Introduction to Management Science as our elective. This course was the first out of three focusing on quantitative modelling and operations research/management science within our major in Logistics – Supply Chain and Networks. The other two were Supply Chain Planning and IT Tools for Logistics Analysis. In Introduction to Management Science we were introduced to how we could use quantitative modelling for decision making in supply chain management. We spent the majority of time in this course in Excel, using the Premium Solver, but we were also briefly introduced to the optimization tool SAS/OR. Supply Chain Planning was in many ways similar to Introduction to Management Science, only focusing more on different types of theoretical models for supply chain planning, how to implement these in SAS/OR, and consequently analysing the results of such models. We also got a better understanding of why IT tools like Excel and SAS/OR can be of great help during the planning process.

As the interest for quantitative modelling and operations research grew it became more and more evident to us that this would be the focus of our master thesis. Once we had decided on this, the choice of supervisor was quite evident. Atle Nordli is the leading professor at BI within operations research. As far as we know he is also the only one with sufficient knowledge of the IT-tools we were going to use in our thesis; SAS/OR and MPL. He was also the lecturer in the three elective courses mentioned above and a professor that we knew we could cooperate well with. As we have been through several courses in our MSc-program covering operations research, we feel well prepared for using theory from this academic field in our thesis-work. Writing a master thesis is also a mandatory part of the MSc-program at BI.

We have cooperated with the Norwegian division of HeidelbergCement on our thesis. Initially the intention was to solve an aggregate planning problem for the company. But in collaboration with our supervisor we discussed several other possible problems we could look at in our thesis and found that it would be interesting to use a case from HeidelbergCement to test if we could adapt economical lot sizing theory in order to make a transportation planning model. In the Supply Chain Planning course we solved and analysed several different economic lot sizing-problems, and found this area of operations research interesting. As our supervisor wrote the dissertation for his doctorate on the topic of lot sizing, this further motivated us to pursue this research problem.

1.2 The Company

HeidelbergCement is the third largest manufacturer of cement and concrete, and the largest manufacturer of aggregates in the world. They have facilities in more than 40 countries around the world, about 53 000 employees that are situated in 2500 different locations and the company had a consolidated turnover of approximately 12 billion Euro in 2010. Their core business includes the production and distribution of cement and aggregates, which are the two essential raw materials for concrete. (*HeidelbergCement Annual Report 2010*).

The logistics division for Northern Europe is located in Oslo, Norway, and controls the logistics of Norway, Sweden, Denmark and the Baltic countries. In Norway and Sweden the company is the dominating provider of cement and concrete. The Norwegian cement-division of HeidelbergCement is Norcem AS and the Swedish cement-division is Cementa AB. Cementa produces cement to meet the demand within Sweden, but they also export cement to other countries when they have excess capacity. Our master thesis will use data obtained from one of the three production facilities in Cementa, henceforth called *the factory*. The factory has a production capacity of 1000 tons of cement each day. From the production facilities the cement is transported by vessel to eight different terminals on the coast of Sweden, where the end-customer picks up the final product.

We were so fortunate that we were invited to visit Norcem's factory in Brevik, Norway. Even though this is not the same factory that we are focusing on in this thesis, the processes and the essence of what they do are the same. We got a guided tour of the different parts of the facilities and were informed of how the process of producing, loading and transporting the cement works. This experience was educational for both of us and gave us new insight as to how the different processes of producing and shipping cement are handled. It was also interesting to hear from those close to the daily operations in the company how the supply chain planning tools affected their daily work, and what they regarded as advantages and disadvantages with such tools.

1.3 The Industry

HeidelbergCement operates in the construction and building materials industry. There are several risks that the construction and building materials industry is exposed to, and the market for their products can be quite unstable. According to HeidelbergCement's annual report (*HeidelbergCement Annual Report 2010*) the demand for building materials will fluctuate with the construction activity. If the investments in the construction industry are high, there will be an increased demand for building material and vice versa. The seasonal fluctuations in an industry like this are quite high as well. The demand for their products will most likely be dependent on the economical state in the region where they are operating as well as the weather condition and seasonal fluctuations. The construction and building material industry is described by HeidelbergCement as a cyclical industry, indicating that the demand fluctuates in certain cycles (*HeidelbergCement Annual Report 2010*). The variations will vary from country to country, and since HeidelbergCement is located in about 40 countries they are quite diversified and might be able to spread the risks that they are facing. Even though there are risks associated with operating in the construction and building materials industry, the demand for such materials will most likely always be present. Thus, one can argue that it is a somewhat stable industry in the sense that the demand will be there even though it fluctuates.

1.4 The Research Problem

The framework of this master thesis is defined by the following research problem:

"How can we adapt existing production lot sizing-models and theory to develop a transportation lot sizing model? A case from a cement producer"

The overriding objective in this thesis is to develop a new *transportation lot sizing model*. In order to do this we have to adapt existing theory and lot sizing models usually formulated for production problems. While the literature on lot sizing for production planning is vast, we are not aware of any work on the specific topic of transportation lot sizing. Hence we hope to find a new area of application for existing lot sizing models.

The new lot sizing model will be formulated in order to solve an empirical case from HeidelbergCement. Therefore the problem in this thesis is somewhat two-fold. However, while we solve this case based on the input data provided to us by the company, the data is first and foremost utilized to develop the model. We will not compare the results found from our model with the actual situation in HeidelbergCement. The main objective in this thesis is to develop a stable and functional transportation lot sizing model. Thus the model is not meant to be utilized by HeidelbergCement, but if the model seems to work well it may still be used in some capacity by the company.

The transportation in this thesis is performed by a vessel. Maritime transportation is in general an expensive and time consuming way of distributing products. One objective when formulating a transportation lot sizing model would therefore be to make the model provide a solution (transportation plan) consisting of as large lot sizes as possible for each trip. Another objective will be to formulate the model in order to make it as flexible as possible when it comes to time management. When developing a model like this, you also have to be aware that it is an iterative process. Finding a solution to one problem may shed new light over another problem. The model will be solved using the optimization tool MPL (Mathematical Programming Language) with the Gurobi solver.

1.5 Importance of topic - why develop a transportation lot sizing model?

The purpose of this thesis is first and foremost to find a new way of utilizing existing lot sizing theories through the development of a new transportation lot sizing model. We recognize that there are several other ways of solving a transportation problem like the one in our empirical case. We also recognize that some extensions have been made to integrate distribution as a result of lot sizing decisions in the production planning, but these are not made solely as transportation lot sizing models.

Up until now the focus in the lot sizing literature has been on production planning. However, some industries have low complexity in production, only one final product and a very high setup costs in production. In such industries the decision of when and how much to produce becomes irrelevant. The only true option is to produce at full capacity in all periods (except when there is downtime for maintenance). The empirical case from HeidelbergCement can be described as such an industry. In this case the decision should rather be to find an optimal transportation plan while balancing the costs of transportation and holding inventories. These are decisions that are made by a lot sizing model. From a theoretical point of view we therefore argue that it is important to expand the area of application for lot sizing models from production planning to transportation planning.

Another important aspect in lot sizing theory is that from a practical point of view the utility value of lot sizing models may seem limited to some. In order to increase the usability of lot sizing models in practice and further replicate the real-life situation it is therefore important to implement as many aspects as possible to the model. We argue that the implementation of period overlapping setups and the possibility for inventory shortage in our model increases the utility value of the transportation lot sizing model. Hence, this also underlines the (relative) importance of the work performed in this thesis.

1.6 Outline

The remainder of this master thesis is structured in the following way: In chapter two we will present the research methodology used for this paper; a multimethodology consisting of normative axiomatic research and empirical research. Chapter three contains a literature review, reviewing the different lot sizing models, theory on the model extensions of period overlapping setup and inventory shortage as well as a review on the work done on models with seasonal demand. In chapter four we start with a discussion of possible objective functions in the model before we present the mathematical formulation of the model in detail in three separate steps. The different solutions and a discussion of each step are also presented. Chapter five contains an analysis and test of the model in order to test its stability, flexibility and functionality. The final chapter summarizes the work in the thesis and discusses the practical use of the model and provides suggestions for further research.

2 Research methodology

Research in operations management usually differs somewhat from “traditional” research. It is as Meredith et al. (1989, p 297) put it:

“Due to the heritage and history of operations management, its research methodologies have been confined mainly to that of quantitative modelling and, on occasion, statistical analysis. The research methodologies in operations have largely remained stagnant”.

Meredith et al. (1989) presented a framework for research methods which consists of two dimensions;

1. Natural moving towards *artificial*
2. Rational moving towards *existential*.

Normative modeling corresponds to the ”artificial reconstruction of object reality” on the first dimension and *axiomatic research* on the second dimension. In their article, Bertrand and Fransoo (2002) classify quantitative model-based research into two distinct classes with two corresponding research types.

1. The axiomatic research approach
2. Research based on empirical findings and measurements

Both classes can be either normative or descriptive.

Bertrand and Fransoo (2002, p. 249), which base some of their article on the insights provided by Meredith et al. (1989), have defined axiomatic research as the following;

“In this class of research [axiomatic research], the primary concern of the researcher is to obtain solutions within the defined model and make sure that these solutions provide insights into the structure of the problem as defined within the model”.

This type of research is driven by an idealized model. Such a model can be interpreted as a highly simplified model of reality; for instance a lot sizing model. According to Bertrand and Fransoo (2002) almost all axiomatic research seems to be normative. Normative research mainly wants to develop policies, strategies and actions to improve the results that have been provided by the literature that already exists, and also to provide solutions to problems that have just been defined.

Mitroff, Betz, Pondy and Sagasti (1974) present a model on how to approach operational research based on quantitative modeling. We present the research model's four central phases.

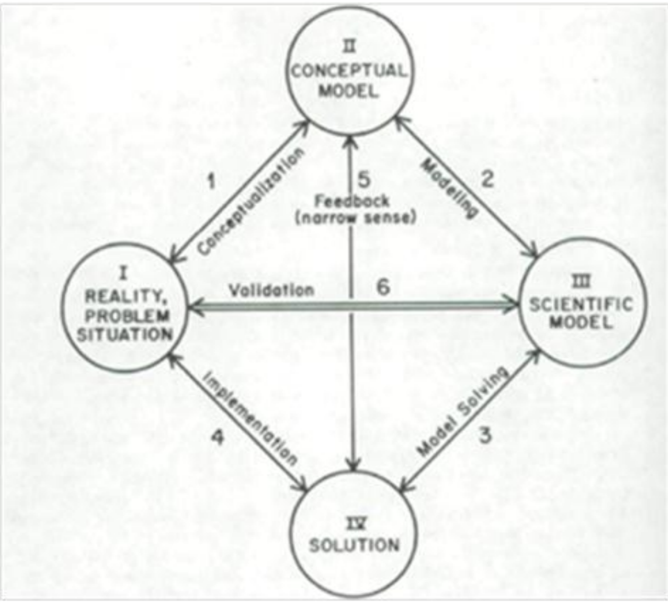

Figure 2.1: Research model (Source: Mitroff et al. (1974))

The conceptualization phase

Starting from an existing "reality", or problem, the researcher here makes decisions on which variables that need to be included and also address the scope of the problem and model. This relates to our problem where we choose theory and variables to use from the existing theory. It also relates to the empirical case as this is where the problem is narrowed down from the "real-world" to the simplified problem we will solve.

The modeling phase

Based on the conceptual model, the researcher builds the quantitative model. The causal relationships between the variables are defined. This phase is where we develop our mathematical model from the relevant theory and variables picked in the previous phase, and through parameters provided from the empirical case.

The model solving phase

This phase involves solving the quantitative model. Here we apply a computer program – MPL – to the mathematical model in order to solve the model.

The implementation phase

If the previous phase provides positive results, then the results can be implemented to the original departure point, or reality. If the model solved provides results that are possible to implement in the origin of the empirical case, this is done here. Nevertheless, we will not be implementing our model. It is not a model made for use, rather to explore the field of lot sizing. If we manage to find results that can add new insight to existing literature, this also occurs in this phase.

Mitroff et al. (1974) also include one horizontal axis, indicated by (6) in the figure, and one vertical axis, indicated by (5). The horizontal axis (6) describes the validation phase of the model, while (5) is called the feedback phase. One feature of the "framework" presented here is that it can, and perhaps should, work as an iterative process. We will probably have to adjust our problem after each iteration that is performed.

Another approach to quantitative modeling is presented by Pidd (1999). He has set up six different principles that discuss model development as a gradual process. His belief is that one should divide larger models into several smaller ones, so that it would be easier to model, and then combine them. The six different principles that he has come up with are (Pidd 1999, p 121);

1. Model simple; think complicated. It will be too time-consuming, expensive etc. to model an exact replica of the reality, but it is important to be aware that the model is just a simplification.
2. Be parsimonious; start small and add. Develop your model gradually – start with simple assumptions and add complications only as needed.
3. Divide and conquer; avoid mega models. This avoids over-general models that cannot be validated.
4. Use metaphors, analogies and similarities
5. Do not fall in love with data
6. Model building may feel like muddling through.

Given our previous experience with quantitative modeling we believe these principles to be good advice. It is always easier to start with a small and simple model and add the constraints that you need to add as you go along.

As mentioned before, the objective for our thesis can somewhat be divided into two parts. We will use quantitative modeling to test existing theory on a new area of application, while at the same time solving an empirical case. When you take existing models and tweak them a bit and try to further develop them you are using axiomatic research design. On the other hand, when you are considering a real life problem and trying to solve this you are using an empirical design. Thus one can argue that we in some ways are using a mulitmethodology; a mixture of two different designs. As Mingers and Brocklesby (1997, p 489) states:

"…in dealing with the richness of the real world, it is desirable to go beyond using a single methodology to generally combining several methodologies, in whole or in part, and possibly from different paradigms."

According to Mingers and Brocklesby (1997) one of the reasons for using multimethodology is that the real world situations are highly complex and multi-dimensional. Thus, your work might not fit into one methodology alone, as is the case for our thesis.

Based on the type of work we are going to perform, the most fitting methodology seems to be normative axiomatic research methodology. The reason why we believe this to be the case is that we want to explore a new field for the use of existing lot sizing models, and hence we can describe this as normative research (or modeling). Further we want to find solutions to our problem through the model – we want to see if, and how, the model we develop actually work and provides us with new and useful insight to our problem. This seems to correspond with axiomatic research. We will also use the two approaches proposed by Mitroff et al. and Pidd when formulating our mathematical model.

2.1 Collection of data

We have been provided with the data needed by the company. Thus, this can be considered secondary data. One of the pitfalls of secondary data is that you have not collected them yourself, and therefore it is not as easy to make sure that they are up to date, that they are not too general and that they are correct. Nevertheless, since we are obtaining realistic data through an associate (consultant) of the company, we consider the validity of the data to be secured.

The data itself should not have an impact on the quality of the mathematical model itself, but erroneous data may of course lead to wrong solutions for the company. Nevertheless, our main goal for this master thesis is not necessarily to make a decision tool for the company, rather to explore the field of lot sizing and try to further exploit the literature in order to make a transportation lot sizing model.

2.2 Reliability

The notion of reliability is that when another researcher tries to replicate your research with the same data, he or she should be able to obtain the same result. Pedhauzer (1991) defines reliability in the most general way as this: *"Reliability refers to the degree to which test scores are free from errors of measurement."*

There are three different ways of measuring the reliability; test retest, equivalent forms and internal consistency. Test retest is most likely the simplest form of testing the reliability. The concept of test retest is that you check the consistency and the repeatability of measurement (Elazar J. Pedhauzer 1991). Our model will most likely be easy to replicate by retesting the work that we have done. Along the way of making our model, we will carefully document the work, by explaining the mathematical formulation and our method of approach. The process will be documented very closely, and we believe that our results could be replicated should one want to do so.

When you are testing the reliability by equivalent forms you want to measure the same phenomenon in two different ways. According to Pedhauzer (1991), the two different methods should preferably be parallel. Nevertheless, there are several strict assumptions behind parallel measurements and this method is therefore not used as often. In our case, one way of securing reliability through equivalent forms is to implement our model in several programs. We intend to model our problem in both MPL and SAS/OR. Internal consistency is to make sure that your object of interest responds in the same manner after some time has passed. This way of securing reliability is not relevant to us and will not be elaborated further.

2.3 Methods for solving lot sizing models

According to Karimi, Fatemi Ghomi, and Wilson (2003) there are three methods of solving the lot sizing problems: (1) Exact methods, (2) Common sense or specialized heuristics and (3) Mathematical programming based heuristics.

In our thesis we will be using *exact methods*, therefore we will only be reviewing this method in more detail. Within the exact methods there are three different types of solution methods (Karimi, Fatemi Ghomi, and Wilson 2003, 369-373);

1. Implementation of a mixed integer programming formula, using branch and bound techniques to solve it
2. Cut-generation techniques
3. The variable redefinition techniques

The implementation of a mixed integer program is the most straightforward approach out of the three, and the one that we will be using in our master thesis. Nevertheless, the Gurobi solver uses a combination of these three solution methods, so one can argue that we are in fact using all three.

Branch and bound is a way of finding the best integer solution while allowing for relaxation of the variables which means that we allow them to be between 0 and 1 instead of exactly 0 or 1. Branch and bound is said to be a very effective way of solving mixed integer programs, and is therefore a good approach for us to use (Sas 2011). The model is divided into several sub problems, one for each possible outcome. If the sub problems are not integer-feasible new sub problems are defined. If a sub problem is integer-feasible it becomes the upper bound for a minimization problem or the lower bound for a maximization problem (Sas 2011).

The algorithm chooses to branch on the solution with the largest fractional value, i.e. the value closest to 1. From here the model is able to find the solution that yields the best objective value by fathoming (excluding) infeasible sub problems, sub problems where the LP solution yields binary variables for the 0-1 variables and sub problems where the objective function does not exceed the incumbent value. This is also known as the best known feasible MIP solution (Shapiro 2007). If it turns out that there is no integer solution to be found the MIP problem itself is infeasible. Nevertheless, if an integer solution is found, this is the optimal solution for the MIP program.

According to Buschkühl et al. (2008) the bounds that are found during the relaxation of lot sizing problems are quite poor. It can be beneficial to introduce inequalities in order to tighten the lower bounds, which increases the efficiency of the branch and bound method. Within the inequalities approach there are three different methods (Buschkühl et al. 2008, p 240);

1. The cutting plane method; the inequalities are generated dynamically to cut of current non-integer solutions.
2. The branch and cut; the valid inequalities are introduced in the course of the branch and bound algorithm.
3. The cut and branch procedure; the cut and branch method incorporates all the generated inequalities into the model formulation before starting the branch and bound algorithm.

Another exact method is the cut-generation technique. When you are using the cut generation technique you are adding strong inequalities. This reformulates the problem and speeds up the solution process and it will also give you what is a near to optimal solution. The reformulated problem will be solved by using branch and bound. The inequalities that are used to reformulate the models are produced by using a cutting plane procedure (Karimi, Fatemi Ghomi, and Wilson 2003, p 369).

The third exact method that is mentioned above is the variable redefinition technique. According to Martin (1987, p 821) the general idea of variable redefinition *"is to develop an alternative formulation for the special structure subproblem."* The variable redefinition can consist of a completely new set of variables, but it can also contain a subset of the variables in addition to some new auxiliary variables (Eppen and Martin 1987).

Buschkül et al (2008) also provides a thorough review of different solution approaches for lot sizing problems, and has included an additional category of solving lot sizing models; decomposition and aggregation approaches. This approach divides the model or problem into several smaller sub-problems. Each of the sub-problems are solved individually and their solutions are then coordinated. Given that we have a problem from real-life, this might be the approach that would yield the most realistic result, but not necessarily what is most optimal. Nevertheless, since our main goal is to develop a transportation lot sizing model we want to develop a model that yields the most optimal results. The best way to do this is presumably to view the problem as a whole, not dividing it into sub problems as suggested by Buschkühl et al. (2008).

2.4 Tools used

Deciding which tool to use in order to solve an optimization problem in quantitative modeling can be an important decision in regards to several aspects; how easy it is to use, how good the solver is, whether the programming language is commonly known etc. We were, as previously mentioned, introduced to SAS/OR in our course Introduction to Management Science. As we knew the program, we initially used the modeling tool SAS/OR and Enterprise Guide 4.3. Our three different steps of the model were initially implemented in SAS/OR.

SAS/OR is an optimization tool that is designed for people with a background within operations research/management science (or similar) that uses for instance mathematical programming (Sas 2011). It is a tool for constraint based programming. In our case we used the OPTMODEL-procedure which uses a solver called MILP. According to SAS (2011) *"the MILP solver implements an LP-based branch-and-bound algorithm"*.

After trying to solve the different models in SAS/OR we experienced that the solution time was extremely long. Our computers ran out of memory before the optimal solution could be found. In order to try to solve the problem more efficiently we chose to implement our models in another planning tool, MPL (Mathematical Programming Language), which is quite similar to SAS/OR but that has a better solver; GUROBI. The solution times were considerably reduced when we started using MPL. The smallest models were solved in approximately five seconds. The largest ones were cut off after two hours as we saw that the objective function were the same as when we let the program run until the PC was out of memory. The reason why the largest problems are taking so long to solve is that as more variables are added the solution time increases exponentially (Bahl, Ritzman, and Gupta 1987). We chose to terminate the program after some time because there were not really any other realistic alternatives. We could have found another solver, but his would be too time-consuming. The other option was to let the program run until the computer was out of memory, but this would not be an efficient use of our time.

According to Maximal Software, the developer of MPL, *"MPL includes an algebraic modeling language that allows the model developer to create optimization models using algebraic equations. The model is used as a basis to generate a mathematical matrix that can be relayed directly into the optimization solver. This is all done in the background so that the model developer only needs to focus on formulating the model."(Maximal Software 2011).*Since MPL is designed to handle large problems; it can be a good choice for supply chain problems since these tend to be quite large. The solver that we have chosen to use with MPL is GUROBI 4.5.1, which solves linear problems, quadratic problems, mixed integer problems (as our problem) and mixed integer quadratic problems (Maximal Software 2011). According to Gurobi; *"For MILP and MIQP models, the Gurobi Optimizer incorporates the latest methods including cutting planes and powerful solution heuristics. All models benefit from advanced presolve methods to simplify models and slash solve times" (Gurobi 2011).*

The computers that were used during the modeling and solving of our models;

- *Acer Aspire Timeline 3820T, Intel Core i3-M350 @ 2.26GHz, 4GB RAM. Windows 7, x64.*
- *Asus UL30V, Intel Core 2 Duo SU7300 @ 1.3GHz, 4GB RAM, Windows 7, x64.*

3 Literature review

In production planning and inventory management the term "*lot sizing*" refers to the determination of the optimal timing and level of production while considering the trade-offs between setup costs, production costs and inventory costs. Karimi, Fatemi Ghomi and Wilson (2003) argue that lot sizing decisions can have a large impact on a manufacturing firm's productivity and performance, and hence its ability to compete in the market. They further argue that developing and improving solution procedures for lot sizing problems therefore is very important.

The perhaps most famous lot sizing model; the *Economic Order Quantity model (EOQ)*, was originally presented by Harris in 1913. The model determines an order quantity that minimizes inventory and ordering costs for a single product, under the assumptions of no capacity constraints and a deterministic and static demand over an infinite planning horizon (Harris 1990). One can argue that these assumptions make this model highly simplified. Axsäter (1986) discusses some of the assumptions or simplifications made in lot sizing models and how valid they are in practical situations. Also for computational purposes, simplifications of the reality have to be made when formulating lot sizing models. According to Bahl, Ritzman and Gupta (1987) the computational time, or the solution time, can increase exponentially when the number of products or time periods increases. The development of computers and their processing-power allows researchers to make fewer simplifications and hence lot sizing models are able to edge closer to describing the real life problem. Then again, as the complexity increases so does the solution time. Jans and Degraeve (2008) points out that while early lot sizing models focused on the main trade-off between the production- ,inventory- and setup costs, new extensions increasingly focus on incorporating industrial concerns. They argue further that: *"The power of production planning theory comes from the ability to solve more and more complex industrial problems"*.

Wagner and Whitin (1958) introduced the extensions of dynamic demand to lot sizing decisions. Their seminal work was the beginning of what Jans and Degraeve (2008) describe in their paper as *"the dynamic lot sizing problem, with discrete time scale, deterministic dynamic demand and finite time horizon"*. This type of lot sizing problem will be the focus of this literature review. Ekşioğlu

(2009, p 93) defines the classical economic lot sizing model as follows: *"Given the demand, the unit production cost, the unit inventory holding cost for a commodity, and the set-up costs for each time period over a finite and discrete-time horizon; find a production schedule that satisfies demand at minimum cost".*

There is an extensive amount of literature on the topic of lot sizing. Jans and Degraeve (2008), Brahimi et al.(2006), Karimi, Fatemi Ghomi and Wilson (2003) and Bahl, Ritzman and Gupta (1987) have all provided some very good review papers on the topic. While almost all literature focuses on lot sizing problems in production, some have included transportation/distribution. We cannot seem to find any work on lot sizing models used specifically on transportation planning - which is the topic of our thesis. As we will try to expand and extend the use of existing lot sizing theory from production to transportation, the literature review will continue to focus on lot sizing in production.

All lot sizing models have some characteristics in common that determines their level of complexity. In their review, Karimi, Fatemi Ghomi and Wilson (2003) lists the following characteristics as decisive when modeling, classifying and determining the complexity of lot sizing models:

1. **The planning horizon:** You can have different levels of planning in accordance to how long your planning horizon is. If your schedule applies for a year or more you typically have a strategic planning tool, if it applies for 3-4 months you most likely have a tactical planning tool, and if the planning horizon is shorter than this it is typically called an operational model/planning tool. The planning horizon can either be finite or infinite.
2. **Number of levels:** A lot sizing model can either be single-level or multiple-level. You can have single/multi-level *production.* In single-level production systems the final product is a very simple one, while in multi-level systems several levels of handling exist and the demand on one level of production is dependent on its "parents'" level. You can also have single/multi-level *production and transportation*. A model is single-level if it only consists of production, while it is multi-level if transportation is included and is dependent on the production levels (its parents' level).

3. **Number of products.** You can either have a single-item model; one product for each time period only, or a multi-item model; you can produce multiple products on the same machine within the time period. Multi-item problems is said to have much higher complexity than single-item problems.
4. **Capacity or resource constraints:** When building a model there are several constraints that need to be considered. For instance capacity constraints in regards to production, vessel-size, silo capacity etc. As the number of constraints increases, the solving complexity increases.
5. **Demand:** The demand can be static; where its value *does not* change over time, or it can be dynamic; where its value *does* change over time. Most optimization models assume deterministic demand which means that demand is known in advance. Most likely the demand is also dynamic. This will increase the complexity of the modeling.
6. **Setup structure:** If the setup costs or setup times are not sequence-dependent it is called a simple setup. When it is dependent on previous periods or sequence it is called a complex setup structure. Setup carry-over or period overlapping setup is an example of a complex setup structure. Setup costs and time are generally modeled as a binary variable (0/1). This makes it harder to solve the model and also extend the solution time.
7. **Inventory shortage:** If you allow for shortage, this means that you allow for unmet demand in the current period to be met in future periods. This is known as *backlogging*. If you do not allow for inventory shortage, you allow for demand not to be satisfied at all. This is known as *lost sales.* These extensions make the lot sizing model more difficult to solve.

From here on we will concentrate on single-/multi-item lot sizing problems with a finite planning horizon, single level production, and dynamic demand since this is the most relevant to our problem. We will present one uncapacitated and one capacitated model, before we present two extensions of the model: *Period overlapping setup* (complex setup structure) and *inventory shortage*

3.1 The single-item uncapacitated lot sizing model

The simplest type of lot sizing model is the single-item uncapacitated model, which Brahimi et al. (2006, p 5) have defined as: *"a lot sizing problem where we consider a single (or aggregate) product, and the production capacity is assumed to be high enough to never bind in an optimal solution"*. Jans and Degraeve (2008) have formulated a mathematical model of the single-item uncapacitated problem. The notations used are the following:

$\boldsymbol{vc_t}$: variable cost of producing one unit.
$\boldsymbol{x_t}$: The amount of the product that is produced.
$\boldsymbol{sc_t}$: The cost of setting up the machine for production.
$\boldsymbol{y_t}$: A binary variable that is 1 if the machine is setup in period *t*, 0 otherwise.
$\boldsymbol{hc_t}$: The cost of storing one unit.
$\boldsymbol{s_t}$: The inventory in period *t*.
$\boldsymbol{d_t}$: The demand in period *t*.
$\boldsymbol{M}$: A large number
$\boldsymbol{T}$: A time period

The mathematical formulation:

$$Min \sum_{t=1}^{T} ((vc_t * x_t) + (sc_t * y_t) + (hc_t * s_t)) \tag{1}$$

subject to

$$s_{t-1} + x_t = d_t + s_t \quad \forall t \in T \tag{2}$$

$$x_t \leq (M * y_t) \quad \forall t \in T \tag{3}$$

$$x_t, s_t \geq 0; \; y_t \in [0,1] \quad \forall t \in T \tag{4}$$

The objective function is to minimize the total costs of producing, storing and setting up for production (1). The inventory balance is modeled in (2), and tells us that the demand can be covered either by the inventory from last period or the production from the current period, and any excess is carried over to the next period as inventory. Constraint (3) is a "setup logic" that symbolizes that you cannot produce the product if you have not set up the machine for production. The inventory and production variable in the model must be greater or equal to zero and the setup variable is a binary variable (4).

When working on a real life problem you are not likely to find many cases where the single-item uncapacitated lot sizing model would be realistic for the problem at hand. Every company has some kind of capacity constraints in regards to their resources. Nevertheless, Bahl, Ritzman and Gupta (1987) argue that in many cases it can be easier to approach a lot sizing problem by developing an uncapacitated model as a starting block, and then expand the model with the capacity constraints so that it eventually becomes a capacitated model. This way it can be easier to avoid the capacity infeasibility, by evolving the model step by step

3.2 The capacitated multi-item lot sizing model (CLSP)

In the capacitated model there are restrictions in regards to the capacity available, as for instance the inventory-capacity, the capacity on the chosen transportation mode or the production capacity (Karimi, Fatemi Ghomi, and Wilson 2003). You have to *calculate* what capacity will be available in each period, or use an *approximation* of an average constant value based on previous experiences. The multi-item lot sizing model is quite similar to the single-item lot sizing model. The main difference is that you have to calculate the lot-sizes for several products and not just one. As a consequence you have to change the setup-state each time you start producing a new product. In the CLSP you are going to schedule N items over a horizon of T periods while minimizing the total costs, and according to Karimi, Fatemi Ghomi, and Wilson (2003) the multi-item capacitated lot sizing model is strongly NP-hard. Strongly NP-hard problems are often solved as mixed integer programs.

Capacitated lot sizing problems can be classified into two different classes, or time buckets. The length of each period in a planning horizon is called *time buckets (Sox and Gao 1999).* There are two different types of time buckets; *small* *bucket* models and *large bucket* models. In a small bucket model you are only allowed to produce one product within one time period. The Continuous Lot Sizing Problem (CSLP) is a small bucket model. In the large bucket model you can produce several products on the same machine within one time period. The Capacitated Single-/Multi-Item Lot sizing Problem (CLSP) is a large bucket model (Jans and Degraeve 2008). Our problem is a *capacitated multi-item lot-sizing problem* and the new mathematical extensions can be formulated as follows (using the same notation as before with the addition of Cap_t, which is a capacity constraint, and vt_i is the time used to produce product *i*):

$$Min \sum_{t=1}^{T} \sum_{i=1}^{I} \left((vc_{it} * x_{it}) + (sc_{it} * y_{it}) + (hc_{it} * s_{it}) \right) \qquad (1)$$

subject to

$$s_{i,t-1} + x_{it} = d_{it} + s_{it} \quad \forall i \in I \quad \forall t \in T \qquad (2)$$

$$x_{it} \leq (M * y_{it}) \quad \forall i \in I \quad \forall t \in T \qquad (3)$$

$$\sum_{i=1}^{I} (vt_i * x_{it}) \leq Cap_t \quad \forall t \in T \qquad (4)$$

$$x_{it}, s_{it} \geq 0; \;\; y_{it} \in [0,1] \quad \forall i \in I \quad \forall t \in T \qquad (5)$$

The objective function is to minimize the total cost of producing and storing the products and the setup for the different products (1). The inventory balance (2) and the setup logic (3) now apply for each product *i* in each period *t*, and (4) is the capacity constraint. The inventory and production variable in the model must be greater or equal to zero and the setup variable is a binary variable (5).

3.3 Lot sizing models and transportation

There is a vast amount of literature on the topic of transportation. We have narrowed down our review so it only concerns literature that includes lot sizing models used in transportation planning. Molina et al. (2009) argue that while the transportation costs accounts for a substantial portion of the logistics cost for a product, the costs considered in lot sizing models are usually restricted to production, setups and inventory. Hence transportation costs in lot sizing models deserve some attention.

Speranza and Ukovich (1993) describes several strategies for finding the lowest total cost of inventory and transportation for a specific problem. One of the strategies that they describe is the *ship-when-full strategy*. Using this method you simply fill up the truck or vessel and ship it when it is completely filled. This way the transportation capacity is utilized to its fullest potential. This strategy was found to be less than optimal compared to the other strategies. Speranza and Ukovich (1993) also mention some other strategies. One of them is to weight the value of the products and then ship at a regular pace a certain quantity or the truck capacity whichever is smaller. This approach is intended for high value goods. The other strategy mentioned is to manage the different products independently. The products are shipped on different trucks and the frequency for each product is different. Hwang (2010) points out yet another strategy; in order to reduce the number of deliveries that are fairly small (less than truckload) a shipment consolidation program can be used. Here some of the units are backlogged or held back so that you can combine several small shipments into one large shipment (full truckload) or several full truckload shipments. This will lead to an overall lower shipment per unit cost.

Over the years the “traditional” lot sizing models, which only consider production and inventory, have been extended by some authors to also include transportation. Hwang (2010) presents a lot sizing model with integrated production and transportation; *the ELSP-PT*. The ELSP-PT is a model used to find optimal lot sizes when considering that the production and transportation is linked. In his paper he models the production costs as concave (economies of scale) and transportation costs as a stepwise function based on the cargo capacity (consolidation). Hwang (2010) assumes that the transportation is uncapacitated and that the demand is deterministic. Haq et al. (1991) developed mixed integer programming (MIP) model which determines the transportation lot sizes that minimizes the total transportation cost where the production, inventory and distribution are integrated. The model is built upon a multi echelon system. Neng, Lee and Tseng (2003) have also developed a MIP model which minimizes transportation costs for supply chains with discrete-period variable demand and have developed a two-phase heuristic to solve it. Cetinkaya and Lee (2000) present a model for coordinating inventory and transportation in *Vendor Managed Inventory (VMI) systems*. Kaminsky and Simchi-Levi (2003) have developed a two-stage model where items are produced at stage one, held at an inventory and then transported to stage two where additional production is completed. After this the finished products are transported to the final inventory which the customers are served from. Diaby and Martel (1993) have developed a model for planning in a multi-echelon distribution system.

According to Chen (2010) it is critical to integrate the production and transportation, and plan and schedule them jointly in a coordinated manner in order to achieve optimal operational performance in a supply chain. Chandra and Fisher (1994) investigates the value of coordinating production and transportation planning, and finds that this is cost saving compared to planning the two in separate. Another benefit of integrating the two operations, in addition to reducing costs, is that this can often lead to better customer service which is key in many industries (Chen 2010).

3.4 Period Overlapping Setup (Model extension 1)

We have done an extensive search for literature concerning period overlapping setup, but to our knowledge there are not an abundance of articles on the topic. This is also acknowledged by Suerie (2006, p 877): *"Regarding the case in which setup times overlap two (or more) periods, only a few model formulations have been proposed so far."* All the articles we have found on the topic of period overlapping setups are written in conjunction with production lot sizing (Suerie and Stadtler 2003), (Gopalakrishnan, Miller, and Schmidt 1995), (Suerie 2006), (Tempelmeier and Buschkühl 2009) and more.

Jans and Degraeve (2008) explain that setup times represent the capacity that is lost due to cleaning, machine adjustments, inspection, testing etc. when production for a new item starts. The capacitated lot sizing problem have been criticized because it does not allow a setup to be carried over from one period to the next one, even if the product that is to be produced (or in our case shipped) at the beginning of the period is the same as the one at the end of the last period (Jans and Degraeve 2008). According to Gopalakrishnan, Miller and Schmidt (1995) it is desirable to maintain and carry over a setup for a product (or trip) if it is produced (shipped) last in a period and first in the following period. In order to be able to do so it is important that the product that is produced last in period *t* and the one produced first in period *t+1* are identified. If they are not the same we cannot have a period overlapping setup.

Since the notion of period overlapping setups is a relevant problem when dealing with real-life situations there has been a development of new models which allow for such overlaps. The problem above is referred to as the capacitated lot sizing problem with *linked lot sizes*, *period overlapping setups* or *setup carryovers* (Suerie and Stadtler 2003), (Suerie 2006), (Gopalakrishnan, Miller, and Schmidt 1995), (Briskorn 2006). What is important to remember here is that maximum one setup state can be carried over from one period to the next (Suerie and Stadtler 2003).

Including a period overlapping setup might be advantageous. If the capacity is tight in the previous periods, there may not exist a feasible solution (Gopalakrishnan, Miller, and Schmidt 1995). By introducing a period overlapping setup a feasible solution might be obtained. In addition, by allowing a setup to be carried over there can be substantial cost savings since the number of setups and the inventory most likely will be reduced (Gopalakrishnan, Miller, and Schmidt 1995). An example of a practical situation where period overlapping setups is important is found in Kim et al. (2010). They argue that some industries have to produce on a “24/7 – basis” in order to avoid expensive shutdowns (as is the case with HeidelbergCement). In such situations setups can take place any time in order to make an efficient plan, and hence period overlapping setups are important. This resembles the empirical case in our thesis and makes the extension of period overlapping setup interesting.

There are several different manners in which the challenge of a period overlapping setup can be solved. One method for implementing a period overlapping setup is presented by Dirk Briskorn (2006). He established a binary variable that is 1 if the setup for a product is preserved from period *t* to period *t+1*. In addition he has a binary variable for a regular setup, as for the regular lot sizing models. The model that he has developed is presented below (Briskorn 2006, p 1045). The notations will be the same as before with the exception of the new binary variable which will be denoted by $\gamma_{i,t}$, which will be 1 if we have a period overlapping setup and 0 otherwise:

$$Min \sum_{t=1}^{T} \sum_{i=1}^{I} \left((vc_{it} * x_{it}) + (sc_{it} * y_{it}) + (hc_{it} * s_{it}) \right) \quad (1)$$

subject to

$$s_{i,t-1} + x_{it} - d_{it} = s_{it} \quad \forall i \in I \quad \forall t \in T \quad (2)$$

$$x_{it} \leq M * (y_{it} + \gamma_{it}) \quad \forall i \in I \quad \forall t \in T \quad (3)$$

$$\sum_{i=1}^{I} (vt_i * x_{it}) \leq Cap_t \quad \forall t \in T \quad (4)$$

$$\sum_{k=1}^{K} \gamma_{t,k} \leq 1 \quad \forall t \in T \quad \forall k \in K \quad (5)$$

$$\gamma_{i,t} - Y_{i,t-1} \leq 0 \quad \forall t \in T \geq 2 \quad (6)$$

$$x_{it}, s_{it} \geq 0; \quad y_{it}, \gamma_{it} \in [0,1] \quad \forall i \in I \quad \forall t \in T \quad (7)$$

Two constraints have been added and one has been changed. Constraint (2) has been altered to account for the period overlapping setup. Constraint (4) has been added in order to assure that we can have at most *one* period overlapping setup. Constraint (5) assures us that we cannot have a period overlapping setup for a product if it was not setup last period.

In addition to allowing period overlapping setups there are some models that have been able to identify the sequence of the lot-sizes, not only the first and the last but also those in between. This extension can be necessary if you have sequence-dependent setup-times or setup-cost (Gupta and Magnusson 2005),(Xiaoyan and

Wilhelm 2006), (Gopalakrishnan, Miller, and Schmidt 1995). As we have sequence *independent* setup times and costs in our model we will not explore this any further.

3.5 Inventory Shortage (Model extension 2)

Inventory shortage is usually dealt with by allowing for either lost sales or backlogging. There are several authors that have reviewed the topic of inventory shortage (Vijayan and Kumaran 2008), (Zipkin 2008), (Zipkin 2008), (Absi and Kedad-Sidhoum 2009), (Huh et al. 2009).

When you do not have enough products to supply your customers at a certain point in time you have a stock-out or inventory shortage. According to Vijayan and Kumaran (2008) there are three different ways of handling stock-outs, or inventory shortage:

1. Backlogging
2. Lost sales
3. A mixture of backlogging and lost sales

If the customers are willing to wait to get their order fulfilled in the next period instead of the current one, you have a case of backlogging. If they are not willing to wait you have a case of lost sales. Normally there will be a penalty cost for each unit that the company is unable to deliver. This penalty cost can for instance be the lost profit margin or the cost of ordering the product from another supplier. Hsu and Lowe (2001) believe that there can be costs from stock outs additional to the lost margin. If the customers have to wait a long time, you may lose future sales to these customers and your future production might decrease due to decreased demand. These costs can increase in a nonlinear manner.

The fact that all of these elements have to be included in the evaluation can make it quite difficult to estimate the real stock out cost. In many instances the cost of lost sales can be higher than the holding cost for the product (Huh et al. 2009), hence you want to be able to produce enough products in order to meet demand unless there are financial reasons or capacity restrictions that do not allow for this to happen. Nevertheless, in some situations firms *choose* to lose the sales because this will be more beneficial than taking on the orders. According to Kesen, Kanchanapiboon and Das (2010, p 182):

"Companies are increasingly convinced that when demand drops it is better to lose the sales rather than expose the supply chain to substantial inventory risk and discount pricing."

Liu et al. (2007, p. 5882) mention some of the reasons why losing sales might be beneficial; if the setup costs and production costs are too high or if the product holding and storage costs are too high. According to the authors there are two different lost sales strategies;

1. The conservation strategy where the customers' demand is not met even if the inventory is positive because there is a greater opportunity for the company in a later period.
2. The stock-out strategy where the customers' demand will be met until the inventory is empty, and from there you will have lost sales.

Kesen, Kanchanapiboon and Das (2010) have come up with the specific example of the electronics industry where the life time of a product is getting smaller and smaller and the price of the product is declining fast. Many speculate in ordering the forecasted demand and would rather lose some sales than be left with a large inventory and having to sell the products at a discounted price. To order smaller batches is the increasing trend in the retail industry.

The simplest manner of modelling inventory shortage is as follows (Absi and Kedad-Sidhoum 2009, p 1353):

$\boldsymbol{r_{i,t} \geq 0}$: A non-negative variable that accounts for the inventory shortage of product i in period t.

$\boldsymbol{R_{cost}}$: A parameter which accounts for the penalty cost incurred by lost-sales.

$$Min \sum_{t=1}^{T} \sum_{i=1}^{I} \Big((vc_{it} * x_{it}) + (sc_{it} * y_{it}) + (\, hc_{it} * s_{it}) - \big(r_{i,t} \; * R_{cost}\big) \Big)$$

Subject to:

$$x_{i,t} + r_{i,t} - s_{i,t} + s_{i,t-1} = d_{i,t} \qquad \forall i \ \forall t$$

We have only included the objective function and the constraint that is affected by the introduction of inventory shortage. The penalty cost has to be included in the objective function and the inventory shortage has to be accounted for in the inventory balance.

Vijayan and Kumaran (2008) have combined backlogging and lost sales. They allow for a mixture of backlogging and lost sales, i.e. if the customers are willing to wait they fulfil their orders, if not they lose the sale. Most articles separate the two different scenarios, but when looking at what is most realistic in real life it is most likely to have a mixture of backlogging and lost sales; some customers are willing to wait while others need the products straight away and prefer to go to a competing firm, or settle for a substitute product.

A field within the area of inventory shortage that resembles our empirical case is lateral transhipment. If a company is unable to meet a customer's demand the company might order transhipment from a store within the same chain. Then you perform an intrafirm transhipment (Wenjing, Xinxin, and Yi 2010). In order for a transhipment to be beneficial there needs to be a surplus at one of the inventories and a deficiency at another, and the surplus must be larger than the shortage. In addition the price of transferring the demand must be lower than the shortage cost.

What separates our case from lateral transhipment as described by Wenjing, Xinxin and Yi (2010) is that the product is **not sent** from one terminal to another, it is the customers that **pick up** their product from another terminal. One can argue that it is not a transhipment of products, but rather a "transhipment of demand". As one terminal loses a sale, another terminal gets an increase in demand. For the case of lateral shipments there is a second handling of the products when they are shipped to the second location, while in our case there is no re-handling of the product since the customers are sent to the "new" terminal. Most likely it will not be less expensive, but at least you do not have to re-handle the products.

3.6 Seasonal demand

Many industries are facing challenges with fluctuations in demand. There are some researchers that have developed different heuristics and models in order to cope with this challenge. Amongst them are Metters (1998) and Buxey (1993). The literature that we have found on the topic of seasonal demand has to do with production, not transportation. According to Buxey (1993) a company's capacity, when they have seasonal demand, should be about equal to the trend figure; what you will normally sell. This means that there will be an excess of products when the demand is slightly lower than what you on average produce (or ship). You can use this excess to build up your stocks and utilize it when the demand is higher than usual. On the other hand you are not able to produce (or ship) what is demanded when the demand is higher than normal unless you have stock than you can feed of or you are able to expand the capacity by using overtime or produce at a more intense level.

Some of the strategies that are well known and that are used by companies is the *"chase strategy"* and the *"level production strategy"* (Buxey 1993). When you are using the chase strategy you are constantly changing the output according to sales. When using the level strategy you are producing at a rate that is calculated by finding the monthly mean values. Based on a survey that Buxey (1993) did, he found that there was an overwhelming trend towards using the chase strategy in order to cope with the seasonal fluctuations of demand. One of the reasons why so many companies choose to follow the chase strategy is to avoid speculation and to strengthen the cash flow situation, seeing as you are not binding so much capital in the inventory (Buxey 1993). A company's capacity constraints may not allow the company to follow a chase strategy. In many cases they have to produce to stock in anticipation of a high seasonal demand.

Metters (1998, p 1397) have created five rules of thumb that could be used alongside a heuristic as a support tool; (1) minimize over-commitment risk – there are many that begin to produce at maximum capacity too early, (2) store capacity cheaply – anticipatory stock should be built taking the holding cost per unit into consideration. Build the stock with the lowest cost first. (3) Production can be seasonal regardless of demand – produce the cheaper product first. (4) Produce the sure thing first and (5) when your plan fails, produce the money-makers. These are relatively easy general rules that might provide some guidance for the planner.

4 The Model

4.1 Case description

HeidelbergCement produces the final product (cement) at the factory in Sweden before it is shipped out by a vessel to eight surrounding terminals. The production is at full capacity as the cost of a production shutdown is significant. In addition to the storage capacity at each terminal, inventory of the final product is also held at the factory. As the supply chain planning in this case is managed by a central division in the company, the inventory management is similar to the Vendor Managed Inventory (VMI). In our particular case, the company does not transport any amount of cement out to their customers from the terminals. At the terminals, each customer picks up their desired amount of cement themselves with their own trucks. There is no need for the customer to make a prior notification of their visit; hence it is important to have a good transportation planning system in place so that you are able to meet the customers' demand by shipping the vessel with a fill rate that is as high as possible. The case is illustrated in figure 4.1.

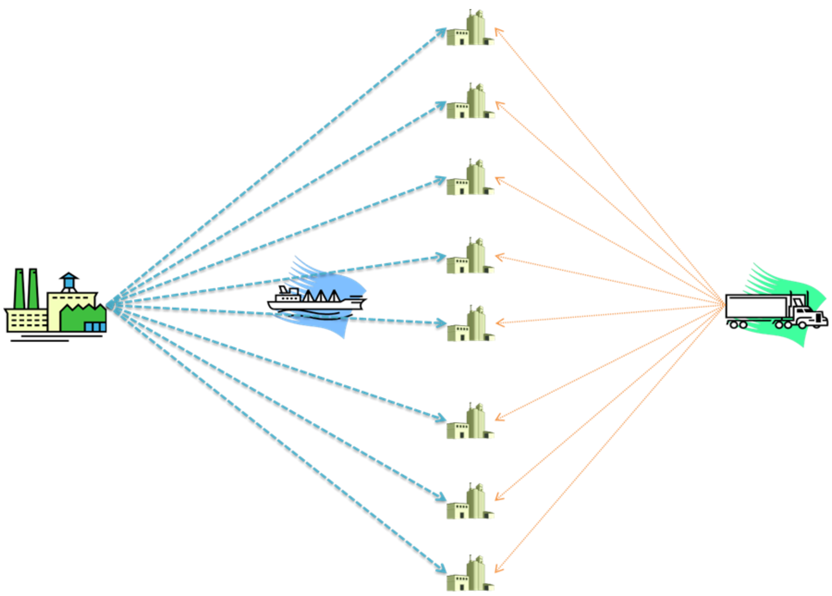

Figure 4.1: Case illustration

4.2 Model description

In order to develop a transportation lot sizing model that determines the optimal timing and level of the transportation of cement from the production facility to the surrounding terminals, we have to adapt the existing theory – usually intended for production problems – to our transportation problem.

Our model is similar to the capacitated multi-item lot sizing model. While the factory produces only one item, this does not imply that we have a single-item problem. The production quantity is assumed to be deterministic in the model – hence no decisions of when, and how much to produce are made by the model. In our model we have one vessel that is used to transport cement to *several* terminals within one period, and as a consequence each trip to each terminal requires a new setup in the model. The purpose of the model is then to decide the optimal timing of each shipment and the corresponding optimal lot size on each shipment. This is equivalent to the "traditional" multi-item problem where you can produce several products on the same machine within a time period. Hence we can say that our model is a multi-item model, or rather; a multi-terminal model. The transportation lot sizing model has one level of transportation - from the factory to the terminals, i.e. we do not transport via a distribution center or change transportation mode. This would have made the model multi-leveled. Thus we argue that our model is a *single level capacitated multi-terminal lot sizing model.*

In traditional lot sizing models the binary setup variable is connected to a setup cost and setup time that usually cover all costs/time associated with changing production from one item to another on the same machine. Hence the production resumes *after* the setup has been performed. In our model the setup cost refers to the costs associated with a specific trip. The setup time is equal to the time spent on each trip, including the loading and unloading of the vessel and the trip from the factory to the terminal and back. In other words; a setup in our model refers to a trip from the factory to terminal *k*, and when the setup is finished a new one can begin.

The number of possible setups (trips) in each period is limited by the time capacity in this period, and each period is equal to one week (or 168 hours). The vessel capacity decides how much that can be transported on each trip. The vessel capacity is 3000 tons. The factory and each of the eight terminals has an inventory capacity constraint, and the inventory capacity is 5000 tons in both the factory and the terminals. The production capacity at the factory is 7000 tons each week except from every third week when the capacity is reduced by 50 tons due to maintenance work. The holding costs are related to the cost of handling the inventory and are equal to *2 per ton* at both the factory and at the terminals. The setup cost for each trip is equal to 1000. The cost of production is irrelevant to the model and is therefore not included.

Maritime transportation is a capital intensive business. Hence supply chains that include maritime transportation usually have a significant transportation cost. When developing a planning model in such a case we argue that it should be optimal to send as few vessels as possible, with as high fill rate on each vessel as possible. Another aspect of maritime transportation is that each trip is time consuming. In our case, no trip is completed in less than 28 hours. Hence we argue that time management should be important in maritime transportation planning.

In line with the methodology presented by Pidd (1999) in chapter two and Bahl, Ritzman and Gupta (1987) in chapter three we have developed the model through several steps. We started out with a small model and gradually added variables and constraints. First we developed an initial model (henceforth called Model 0). Based on this model we have made two extensions in order to make a better fit to the empirical case, and to make the model more flexible as a planning tool. In the first extension we have added the possibility of performing a period overlapping setup in the model. In the second extension we have added the possibility of allowing for inventory shortage. After adding these two extensions to Model 0 we end up with our final model: *A single level capacitated multi-terminal transportation lot sizing model with period overlapping setups and inventory shortage.*

4.3 Underlying assumptions in the model

In quantitative modeling you have to make assumptions regarding what you include in your mathematical model. These assumptions are made to simplify the real life problem we are trying to solve in order to reduce the complexity of the model. Some of the assumptions may be more valid than others.

- The demand is dynamic and deterministic. This is a common way of treating demand in lot size modeling.
- Production is static and at full capacity, except for every third week when capacity is reduced due to maintenance work. The production costs are omitted from the model since we assume the production to be static and hence they will not affect the decision made in the model. We argue that these assumptions are valid as it is very expensive to shut down the production facilities even for a short period of time, and shut-downs are therefore avoided whenever possible.
- The vessel is available constantly throughout the planning period. This is a necessary assumption in any lot sizing model but may be a bit unrealistic in our case. In reality, factors as maintenance work, vessel breakdowns and weather conditions affect the vessel-availability for HeidelbergCement.
- There is an initial inventory level at each terminal of 2000 tons.
- The safety stock at each terminal is set to zero.
- The planning horizon is finite and set to 19 weeks. Hence the model will work as a tactical planning tool.

The following assumptions do not apply to our initial model – Model 0:

- By adding the period overlapping setup variable *C*, we assume that you are allowed to start a trip in period *t* and return in the following period *t+1*.
- All quantity shipped (*Z*) in a period overlapping setup-situation (*C*) is assumed to be delivered in period *t*, even though the vessel is returning in period *t+1*. This assumption had to be made in order to make the model work. Ideally you would know the arrival time at the terminal in order to decide which period the shipment was delivered.
- We assume that the quantity produced for period *t* is available at the beginning of the period. Hence we do not recognize the fact that production takes place "24/7". The consequence of this assumption is that you may have shipped all available production-quantity in period *t* (7000 tons), but still have more time available at the end of the period to start a new trip (setup). In reality there is a quantity of finished product available

at this point that are meant for next period (*t+1*). Ideally you should then be able to ship some of this quantity, if this is optimal.

The following assumption applies only to our final model:

- The demand at each terminal are picked up by the customer themselves. Instead of including all customers in the model we assume that the demand at terminal *k* is picked up by customer group *i*.

4.4 Objective values

The most commonly used objective value in lot sizing models is minimization of total costs. We thought however that it could be interesting to test whether we could use other objective values in our model. We tested Model 0 with three different objective functions in order to find out which one of them that would provide the most logical results, both from a theoretical and practical perspective. The tests were performed in Model 0 as they were done before we made the two extensions to the model. The three different objective functions that we have tested are:

- Maximization of tons shipped
- Maximization of time used
- Minimization of total costs

4.4.1 Maximization of tons shipped

The mathematical formulation of the objective function:

$$Max\ tons\ shipped\ \sum_{t=1}^{T}\sum_{k=1}^{K} Y_{t,k}$$

Where ***t:*** *period* and ***k:*** *terminal* are sets in the model.

$\boldsymbol{Y_{t,k}} \geq \mathbf{0}$: Variable that says how many tons the vessel ships to each terminal in each period.

When using maximization of tons shipped as the objective function, the model ships out 132 700 tons. This is equal to the produced amount over all periods. Hence the objective function works satisfactory in the manner that it is able to ship out the cement to the terminals in order to meet demand. The average utilization of the vessel capacity is not particularly good, as we see from figure 4.2. However this is not unreasonable as the objective function has no incentives to ship as few trips as possible. We also found that the vessel was on average used 143 hours a week which is the same as about 85% usage of the capacity.

# Trips	Average lot size	Average fill rate	Objective value
52	2 552	85 %	132 700

Figure 4.2: Results for maximization of tons shipped

When analyzing the data we discovered that while there are 52 trips made, there are only 49 trips that were actually transporting cement. From a practical point of view it is not realistic that a vessel would ship out to a terminal without carrying any cement. In order to avoid this, we adjusted the objective function by subtracting a number that was five times as high as the number of trips that were made. Thus the model got an incentive to make as few trips as possible. The "new" objective function:

$$Max\ tons\ shipped \sum_{t=1}^{T}\sum_{k=1}^{K} Y_{t,k} - 5 * \left(\sum_{t=1}^{T}\sum_{k=1}^{K} X_{t,k}\right)$$

Where $\boldsymbol{X_{t,k}}$ $\boldsymbol{binary}$: If the vessel ships to terminal k in period t then X = 1, otherwise 0.

When we made this adjustment the three redundant trips were eliminated and the new results are presented in figure 4.3:

# Trips	Average lot size	Average fill rate	Objective value
45	2 949	98 %	132 700

Figure 4.3: Results for maximization of tons shipped, with penalty.

The problem of empty vessels being shipped is eliminated, and the results have become more realistic. As you can see from figure 4.3 there are fewer trips made after the adjustment. This was expected since we penalized the model for each trip made. The average lot size is significantly higher, and close to optimal. The model still manage to ship out all amount produced over the period of 19 weeks. The average use of time during a week is 127 hours, which is less than before. This is due to the fact that the vessel is utilized more efficiently.

4.4.2 Maximization of time used

The mathematical formulation of the objective function:

$$Max\ time\ used\ \sum_{t=1}^{T}\sum_{k=1}^{K}(X_{t,k} * Time_k)$$

Where $\boldsymbol{Time_k}$ is the number of hours the vessel use out to each terminal *k*.

Using this objective function we find that the vessel is utilized close to its potential in terms of time-management. Over the planning period of 19 weeks there are 7 weeks where the vessel is utilized in all available hours of period *t* (168 hours) and for the remaining 12 weeks the vessel is used 166 hours a week. This gives an average time used of 167 hours per week. The number of trips is however very high, resulting in a poor utilization of the vessel capacity as seen in figure 4.4. The problem when using this objective function is the same as when we used max tons shipped; the model has no incentives to plan as few shipments as possible. Out of 63 trips the vessel only carries a load in 53 of them. Hence 10 trips are made solely for the purpose of aggregating more hours. We think it is unnecessary to extend this objective function with a penalty as we did above, since the objective function will still be to maximizing number of trips (*X*).

# Trips	Average lot size	Average fill rate	Objective value
63	2 054	68 %	3 168

Figure 4.4: Results for maximization of time used

4.4.3 *Minimization of total costs*

The mathematical formulation of the objective function:

$$Min\ total\ costs \sum_{t=1}^{T}\sum_{k=1}^{K}\left((X_{t,k} * sc) + (st_{t,k} * hct)\right) + \sum_{t=1}^{T}(sf_t * hcf)$$

Where the parameters are ***sc:*** the cost of sending a vessel, ***hct:*** the holding cost at the terminals and ***hcf:*** holding cost at the factory. The variables are $\boldsymbol{st_{t,k}} \geq \mathbf{0}$: Inventory at terminal k and $\boldsymbol{sf_t} \geq \mathbf{0}$: Inventory at the factory.

We see from figure 4.5 that the number of trips now is reduced to 43, resulting in an average fill rate of 99 % on the vessel. As this objective function gives the model an incentive to make the vessel perform as few trips as possible we have also eliminated the previous problem of shipping out empty vessels. It chooses to ship 5000 tons less than what is produced over the planning period of 19 weeks. The utilization of time is rather low with an average of 122 hours each week.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
43	2 970	99 %	127 700	1 001 524

Figure 4.5: Results for minimization of total costs

4.4.4 *Comparing results*

When the objective functions were *maximization of tons shipped* and *time used* we found that there were trips made without transporting any cement in both of the models. If we ignore the "penalty function" introduced in the first objective function there were three empty trips made when we maximized tons shipped compared to ten when we maximized time used. This gives the model with maximization of tons the benefit of larger lot sizes and thereby a larger fill-rate, compared to the model with maximization of hours. Nevertheless, from a practical point of view it is would still be wrong to ship empty vessels.

The results from the model where we used minimization of total costs as objective function were quite different from the two other models. There were no trips made without transporting cement. As mentioned before the average fill rate was close to optimal, 99%, so the utilization of the vessels capacity was better when minimizing the costs. The hours used to ship out the cement were fewer for the model with the minimization of costs. The number of hours used when maximizing tons and hours were 2717 and 3168 respectively, while when minimizing the costs only 2322 hours were used. This indicates a more efficient planning of the transportation when we use minimization of total costs as objective function.

Even though we managed to eliminate the problem of empty shipments by using a "penalty-function" when we maximized tons shipped, the results we obtained from minimizing costs were better. The output from the model also became more realistic and logical when minimizing the costs. This is most likely due to the fact that you are taking the costs of shipping a vessel into consideration and the model therefore gives us an output which uses the company's resources in the most efficient way possible still delivering all the demand that is required. From the results that we have obtained from these three tests, it is clear that the objective function that should be used in the model is to minimize total costs.

4.5 Initial model – Model 0

Model 0 is our initial model for finding the optimal timing and level of the transportation of the cement. The purpose of the model is to find out which terminals are replenished with cement in each period, and the lot size on each trip. This model does not include any extensions and the objective function is to minimize the total costs of shipments (setup costs) and the inventory costs at the terminal and factory. The following notations have been used:

Sets in the model:

t: period

k: terminal

Parameters in the model:

$\mathbf{d_{t,k}}$: Demand in period *t* from terminal *k*

$\mathbf{Cap_t}$**:** Production capacity in period *t*

$\boldsymbol{Time_k}$: The number of hours the vessel use out to terminal *k*

Limit: The maximal hours available in each period *t*

VCap: Vessel capacity

hct: Holding cost at the terminals

hcf: Holding cost at the factory

$\boldsymbol{InvCapT_k}$: Inventory capacity at each terminal *k*

$\boldsymbol{InvCapF}$: Inventory capacity at the factory

$\boldsymbol{ss_k}$: Minimum inventory at each terminal *k*

sc: Cost of sending a vessel.

Variables in the model:

$\boldsymbol{st_{t,k} \geq 0}$: Inventory at terminal *k* in period *t*.

$\boldsymbol{sf_t \geq 0}$: Inventory at the factory in period *t*.

$\boldsymbol{X_{t,k}\ binary}$: If the vessel ships to terminal *k* in period t then *X* = 1, otherwise 0.

$\boldsymbol{Y_{t,k} \geq 0}$: How much the vessel ships to each terminal *k* in each period *t*

$\boldsymbol{TimeUsed_t}$: How many hours the vessel has been utilized during period *t*

Mathematical formulation:

$$Min \sum_{t=1}^{T} \sum_{k=1}^{K} \left((X_{t,k} * sc) + (st_{t,k} * hct) \right) + \sum_{t=1}^{T} (sf_t * hcf)$$

subject to:

(1) $Y_{t,k} \leq X_{t,k} * VCap \qquad \forall t \in T \quad \forall k \in K$

(2) $st_{t,k} = X_{t,k} - d_{t,k} + st_{t-1,k} \qquad \forall t \in T \quad \forall k \in K$

(3) $st_{t,k} \leq InvCapT_k \qquad \forall t \in T \quad \forall k \in K$

(4) $st_{t,k} \geq ss_k \qquad \forall t \in T \quad \forall k \in K$

(5) $sf_t = Cap - \sum_k^K Y_{t,k} + sf_{t-1} \qquad \forall t \in T$

(6) $sf_t \leq InvCapF \qquad \forall t \in T$

(7) $TimeUsed_t = \sum_{k=1}^{K} (X_{t,k} * Time_k)$ $\quad \forall t \in T$

(8) $TimeUsed_t \leq Limit$ $\quad \forall t \in T$

(1) This constraint states that one cannot use the vessel to ship cement to a terminal *k* in time period t unless the vessel is set up for the trip of going to terminal *k* in time period *t*. We refer to this as a "setup logic".
(2) The inventory balance for the terminal needs to be defined, and this constraint also assures us that the demand for each terminal *k* in each period *t* is either met by what is shipped to the terminal or by the inventory from last period.
(3) Each terminal *k* has a maximum inventory capacity in each period *t* and this capacity cannot be exceeded.
(4) The inventory level at each terminal *k* in each period *t* cannot go below the level set as safety stock.
(5) The inventory at the factory in each period *t* is what we produced in period *t* in addition to what was at the inventory in last period *t-1* less what we have shipped out to each terminal *k* in period *t*.
(6) The inventory at the factory cannot exceed the inventory capacity.
(7) The amount of time that the vessel is used in each period *t* is calculated by summing up all trips made by the vessel in period *t*.
(8) This constraint assures us that the time used in each period *t* is not exceeding the capacity of 168 hours (1 week).

4.5.1 Results

From a theoretical perspective the results from Model 0 show that the model has found an optimal solution to the problem without violating any constraints. This is encouraging as it indicates that the model is stable and formulated in a good way (stability-tests are performed on the final model in chapter five). The solution-time was about five seconds, which is insignificant. From a practical perspective the results also seem satisfying. The highlights of the results are the same as in chapter 4.4.3 and are shown in figure 4.5. Since we want the lot size of each shipment to be as high as possible it is very positive with an average lot size of 2970 tons (out of 3000). As the holding cost at the factory and the terminals are similar, there is no cost-difference in regards to where you store the cement. Thus not everything that is produced (132 700 tons) each period has to be shipped out to the terminals. On average the vessel is used 122 hours each week which indicates a good time-utilization.

4.5.2 Challenges

Model 0 was the first step of the model. The biggest challenge when formulating this step was that we had to change our train of thoughts on how we usually solve a lot sizing problem in order to adapt the existing theories from production problems to a transportation problem. In accordance with the amount of theory on the topic, there are a lot of empirical examples from lot sizing problems in production planning and, to our knowledge, *none* from lot sizing problems in transportation planning. Hence we did not have any examples to use for comparison when developing Model 0. This made the interpretation and discussions of both theoretical and practical aspects of the model a time consuming affair.

Even though Model 0 seems to work well, we argue that the model does not reflect the real life situation from HeidelbergCement in a satisfying manner. Hence we have made two extensions to Model 0 that are presented in the two following chapters.

4.6 Model extension 1: Period overlapping setup

As the model is defined in Model 0 the vessel cannot begin a trip at week t and finish it at week $t+1$. All the trips that it makes must be completed within the same period t. In a real life scenario it is very unlikely that such a restriction would exist. In order to mend this problem, we have implemented a period overlapping setup which allows the vessel to start a trip in period t and return in the following period $t+1$. Since this extension builds upon Model 0, we will only present the parameters, variables and constraints that are changed or added by this extension.

Added parameters:

co: The cost of letting the vessel start a trip in period *t* and returning in period *t+1*

$\boldsymbol{TimeC_{t,k}}$: This is a matrix showing the time it takes to travel to terminal *k* in period *t.*

Added variables:

$\boldsymbol{Z_{t,k} \geq 0}$: The amount of tons shipped on the period overlapping setup

$\boldsymbol{C_{t,k}}$ $\boldsymbol{binary}$: If we have a period overlapping setup then *C* is equal to 1, otherwise 0

$\boldsymbol{0 \leq \alpha_{t,k} \leq 0.98}$: This identifies the share of the period overlapping setup performed in period *t+1*. It has to be less than or equal to 0.98 in order to secure that at least one hour (1 - 0.98 = 0.02) of the trip occurs in period *t*.

$\boldsymbol{0 \leq \beta_{t,k} \leq 0.98}$: This identifies the share of the period overlapping setup performed in period *t*. It has to be less than or equal to 0.98 in order to secure that at least one hour (1 - 0.98 = 0.02) of the trip occurs in period *t+1*.

Added constraints and changes in the objective function:

$$Min \sum_{t=1}^{T} \sum_{k=1}^{K} \left((X_{t,k} * sc) + (C_{t,k} * co) + (st_{t,k} * hct) \right) + \sum_{t=1}^{T} (sf_t * hcf)$$

subject to

(9) $\sum_{k=1}^{K} C_{t,k} \leq 1$ $\forall t \in T \quad \forall k \in K$

(10) $X_{t,k} + \alpha_{t,k} + \beta_{t,k} \leq 1{,}98$ $\forall t \in T \quad \forall k \in K$

(11) $Z_{t,k} \leq C_{t,k} * VCap$ $\forall t \in T \quad \forall k \in K$

(12) $\alpha_{1,k} = 0$ $\forall k \in K$

(13) $C_{19,k} = 0$ $\forall k \in K$

(14) $\alpha_{t+1,k} + \beta_{t,k} = C_{t,k}$ $\forall t \in T \ \ \forall k \in K$

(15) $TimeD_{t,k} = \left(X_{t,k} + \alpha_{t,k} + \beta_{t,k}\right) * TimeC_{t,k}$ $\forall t \in T \ \ \forall k \in K$

(16) $TimeUsed_t = \sum_{k=1}^{K} (X_{t,k} + \alpha_{t,k} + \beta_{t,k}) * Time_k$ $\forall t \in T$

Where

(9) *One* period overlapping setup is allowed in each period *t*

(10) You can only have one trip (*X*) in addition to a prospective period overlapping setup to terminal *k* in period *t*.

(11) If we have a period overlapping setup the amount of tons shipped to terminal *k* in period *t* cannot exceed the vessel capacity

(12) The vessel cannot return from terminal *k* in period *1* since this is our starting period.

(13) The vessel cannot start a period overlapping trip in week *19* without returning in the same period as this is the ending period

(14) The fraction of what is shipped *in* in period t+1 and *out* in period *t* must be equal to the binary value of the period overlapping setup-variable *C*. If we do not have a period overlapping setup, both in and out must be equal to zero

(15) This constraint helps us display the time used to each terminal *k* in each period *t*.

(16) The amount of time the vessel is used in each period *t* is calculated by summing up all trips (*X*) made by the vessel in period *t* in addition to the prospective portion of a period overlapping setup (α and/or β).

4.6.1 Results

The results from extension 1 are exactly the same as in Model 0 (see figure 4.5), with one exception – the number of hours the vessel is utilized on average per period is reduced from 122 hours to 121 hours. The transportation plan is however altered compared to Model 0. As there are many possible ways of create a transportation plan in order to end up on the same amount of total trips and total amount shipped, this is not of importance. This also explains the difference in time-utilization. As we can see in figure 4.6, the new period overlapping-variable *C* are used in 11 periods. However, it is only in period *5* that *Time Used* is equal to the maximal amount of available hours in a period (168), and this does not correspond with the start of a period overlapping trip. This indicates that the new setup variable *C* does not work as we first thought it would. We initially thought *C* would only be utilized when there was insufficient time available in period *t* to complete a trip (*X*) in this period, and thus it would choose a period overlapping trip (*C*) instead.

Period	C	β	α	Time Used	β * $Time_k$	α * $Time_k$
1				122		
2				106		
3	1	0,98		95	27	
4	1	0,10	0,02	97	6	1
5			0,90	168		53
6	1	0,98		122	67	
7			0,02	128		1
8	1	0,98		92	38	
9	1	0,02	0,02	128	1	1
10			0,98	154		27
11				113		
12	1	0,98		153	38	
13	1	0,02	0,02	128	1	1
14	1	0,98	0,98	94	67	27
15	1	0,02	0,02	152	1	1
16			0,98	140		54
17	1	0,98		126	67	
18	1	0,02	0,02	41	1	1
19			0,98	132		50

Figure 4.6: Period overlapping setups for model extension 1

What we have found is that the model uses the new variable *C* as a "substitute" for variable *X*. This can be explained by looking at period *3*. Here the model chooses to use a period overlapping setup (*C*) in order to send a vessel to terminal *T2* so that the vessel starts in period *3*, runs for 27 hours (98% of the trip) in this period, before it returns in period *4*. The scenario from period *3* is illustrated by the timeline in figure 4.7. What happens is that the vessel waits at the dock by the factory for 73 hours before it starts the trip to terminal *T2*. In real life this could have been because the vessel has to wait for the correct amount of cement to be produced. Since we assume that all production in period *t* is available at the beginning of the period, this situation will however not occur in our model.

t = 3	68 h	73 h	27 h
Terminal	T4	At the dock	T2

Figure 4.7: Illustration of period overlapping setup – timeline for period t=3

We see that in period *3* only 95 out of 168 hours are utilized. Hence there is no need for the model to choose *C* instead of *X* in this case. If the model were to be used as a planning tool in some capacity for the company, we recognize that this flaw could provide illogical results from a practical point of view.

From a theoretical perspective we argue that this does not necessarily imply that the period overlapping setup-extension does not work. The time capacity in each period is only maximized in one period. Thus, due to the nature of the empirical data the extension has not been tested in a satisfying manner. We will perform such tests in chapter five.

4.6.2 Challenges

As Model 0 appeared to be stable, there was always a risk of altering this stability by introducing new parameters, variables and constraints. When we started to implement this extension, this is also what happened. It took a while to restore the model-stability, but as it is presented here it seems stable. Although we could to a certain extent support our work on what had been done previously, we had to make some adjustments in order to be able to implement a period overlapping setup that was adapted to the transportation lot sizing problem and our empirical case.

In our model a period overlapping setup has to be linked with the setup time (time used on each trip) - i.e. we have to find the share of time used by a period overlapping trip performed in period *t* (β) and the share of time used in period *t+1* (α). Our initial thought was to connect β to the remaining time available in period *t* by saying that β had to be equal to the remaining time available for an overlapping setup in period *t*, but this made the model become non-linear. As the Gurobi-solver that we are using cannot handle such problems we had to formulate the model so that the problem became linear. We struggled for a long time to see how we could solve this before we realized that we should model period overlapping trips *independently* of the remaining time available in period t. We introduced the binary variable C that told us when a period overlapping setup was performed, and the two continuous variables β and α that represented each share of the overlapping trip in period t and $t+1$ respectively.

Initially the setup variable C was connected to the lot size variable Y, thus when a period overlapping setup occurred the lot size could be seen in the Y matrix along with those lot sizes Y connected with the setup variable X. Recall that we assume that a terminal k can be refilled by one ordinary setup X and one period overlapping setup C in each period t. When this situation occurred at a terminal k we were therefore not able to distinguish the two lot sizes from each other. Hence we introduced the lot size variable Z which is now connected to C.

Two challenges occurred that forced us to make an assumption of Z. First, we could not link Z to β and α as they were linked to the time-share used through the variable C. We argue that to distribute Z with the same distribution formula as C would be wrong since this would affect the inventory levels both at the terminals in two periods in addition to the time. We only wanted β and α to be connected to the time. Second, since we do not know the exact point of time that the shipment is unloaded from the vessel it would be too complicated to formulate this in the model and then link this to Z. Thus we chose to assume that all amount of cement in Z are delivered in period t, and not divided among period t and $t+1$. This assumption allows the model to "exploit" the period overlapping setup extension in an inappropriate way. Even if the results of the model says that only one hour of the period overlapping trip occurs in period t, it still register a (potentially) full lot size Z of 3000 tons to terminal k in period t. From a practical point of view this is wrong, and this is a challenge we have not been able to solve adequately.

4.7 Model extension 2: Inventory Shortage

Until this point we have assumed that the demand stems from each terminal k. In real life the customers pick up their own demand at their closest terminal. In order to make the model more similar to the real life case and to further increase the flexibility of the model, we extend the model with the possibility of inventory shortage. This implies that the customers have to be taken into account, and hence we declare *a set* that is customer group *i*. This allows the model to redirect customer group *i* to another terminal than their designated terminal. For simplification we only allow customer group *i* to be redirected to the two terminals closest to their designated terminal as illustrated in figure 4.8.

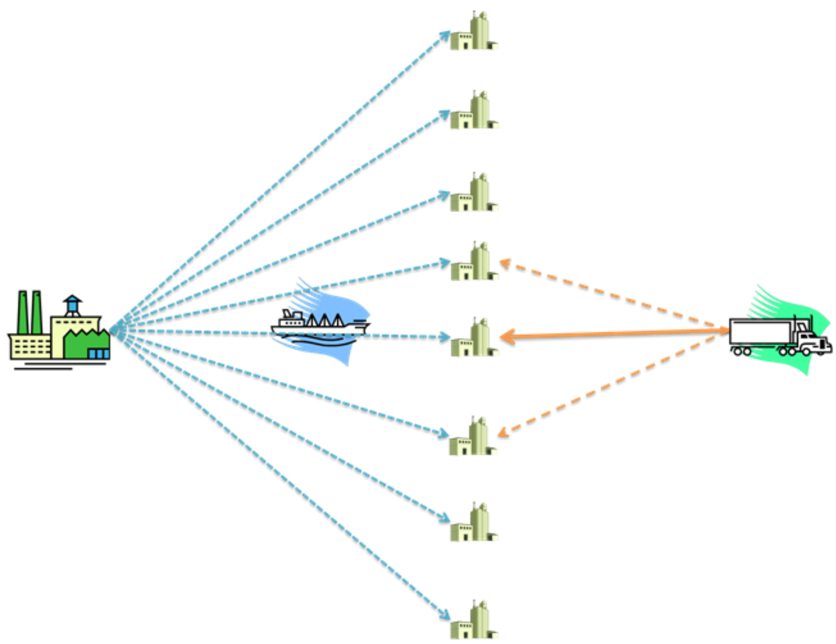

Figure 4.8: Case illustration of inventory shortage extension

We will only include the sets, parameters and variables that have been added or changed since the model builds on Model 0 with extension 1.

<u>*Added sets:*</u>

i: a customer group

<u>*Added parameters:*</u>

$\boldsymbol{d_{t,i}}$: The demand from each customer group *i* in each period *t*, which is the same as the demand used for the terminals before since it is ultimately the same.

$\boldsymbol{Dist_{k,i}}$: The distance from each customer group *i* to each terminal *k*.

Added variables:

$\boldsymbol{P_{t,k,i}}$ ***binary***: A binary variable that is 1 if customer group *i* picks up their demand from terminal *k* in period *t*, 0 otherwise.

$\boldsymbol{PDem_{t,k,i} \geq 0}$: Tells us how much cement each customer group *i* has picked up from terminal *k* in period *t*. As there could potentially be two or three customer groups *i* picking up their demand from terminal *k* in period *t* we had to introduce this variable.

Added constraints and changes in the objective function:

$$Min \sum_{t=1}^{T}\sum_{k=1}^{K}\left(\left(X_{t,k} * sc\right) + \left(C_{t,k} * co\right) + \left(st_{t,k} * hct\right)\right) + \sum_{t=1}^{T}(sf_t * hcf) + 1 * \left(\sum_{t}^{T}\sum_{k}^{K}\sum_{i}^{I}(P_{t,k,i} * Dist_{k,i})\right)$$

subject to

(17) $st_{t,k} = Y_{t,k} + Z_{t,k} - \sum_i^I PDem_{t,k,i} + st_{t-1,k}$ $\quad \forall t \in T \quad \forall k \in K$

(18) $PDem_{t,k,i} \leq \left(P_{t,k,i} * d_{t,i}\right)$ $\quad \forall t \in T \quad \forall k \in K \quad \forall i \in I$

(19) $\sum_k^K P_{t,k,i} \leq 2$ $\quad \forall t \in T \quad \forall i \in I$

(20) $d_{t,i} = \sum_k^K PDem_{t,k,i}$ $\quad \forall t \in T \quad \forall i \in I$

Where

(17) The inventory balance had to take into account that there could potentially be two or three customer groups *i* picking up their demand from terminal *k* in period *t*.

(18) In order for customer group *i* to pick up cement from terminal *k* in period *t* the trip must be set up. This is a “setup logic”.

(19) Each customer group *i* can pick up the demand in period *t* from up to two terminals; either at their own and one of their two neighboring terminals, or at the two neighboring terminals.

(20) The demand has to be covered.

4.7.1 Results

The results from adding this extension is the same as in Model 0 with extension 1 with the exception of an altered transportation plan, resulting in a slightly higher average time used per period *t* (124 hours). The objective value has also increased slightly due to the new distance cost introduced. This implies that after we included the inventory-shortage extension the model does not redirect any customers to other terminals. Since the model now is equal to the final model, the results are discussed in chapter 4.8.1. As we shall see in chapter five, you have to either increase the demand or change some of the parameters in order to see that this extension actually works and thus we refer to the discussion of the results in that chapter.

4.7.2 Challenges

In this case the literature found on the topic of inventory shortage were not very helpful during the model formulation, since the solution in these cases often is to backlog the demand or forfeit the sale (lost sales). Most similar to our problem was the literature found on lateral transhipment (Wenjing, Xinxin, and Yi 2010), but from a modelling perspective this did not help us in formulating the problem either.

The most challenging part of implementing our form of inventory shortage was to "inform" customer group *i* that they initially had to go to their designated terminal unless their demand was either higher than the current inventory, or it was beneficial from a planning perspective to pick up the demand from another terminal k. Our initial thought was to make a variable dependent on the inventory. But, as we experienced during the formulation of extension 1, this made the problem non-linear. The solution we came up with was to construct the distance matrix shown in figure 4.9. As we did not have any information of the actual distances between the terminals, the distance matrix is constructed so that the distance between the terminals is sufficiently long enough to secure that customer group i is only allowed to visit the two (one for *T1 & T8*) nearest terminals, assuming that *T2* is nearest to *T1*, *T1* and *T3* are nearest to *T2*, and so forth.

Terminal/Customer gr.	1	2	3	4	5	6	7	8
T1	1	100	999	999	999	999	999	999
T2	100	1	100	999	999	999	999	999
T3	999	100	1	100	999	999	999	999
T4	999	999	100	1	100	999	999	999
T5	999	999	999	100	1	100	999	999
T6	999	999	999	999	100	1	100	999
T7	999	999	999	999	999	100	1	100
T8	999	999	999	999	999	999	100	1

Figure 4.9: Distance Matrix

This distance matrix was then connected to the binary variable *P* so that if customer group *1* picks up the demand at their designated terminal *T1*, the distance is equal to one. If they choose to pick up the demand at terminal *T2* instead, this will result in an additional distance of 99 (+1 =100). Eventually these distances were connected to a small and insignificant cost of *one* and incorporated into the objective function of minimizing the total cost. Hence we forced the model to choose to send the customers to their own terminal, unless this was not an option due to either low inventory at terminal *t* or overriding planning objectives.

4.8 The final model

In order to give the reader a clearer picture of the final model, we sum up the different parts presented in this chapter and present the final model in its entirety. The notations and the explanations of the objective function and constraints are the same as what is explained throughout chapter four.

$$Min \sum_{t=1}^{T} \sum_{k=1}^{K} \left((X_{t,k} * sc) + (C_{t,k} * co) + (st_{t,k} * hct) \right) + \sum_{t=1}^{T} (sf_t * hcf)$$

$$+ 1 * \left(\sum_{t}^{T} \sum_{k}^{K} \sum_{i}^{I} (P_{t,k,i} * Dist_{k,i}) \right)$$

subject to

(1) $\alpha_{t+1,k} + \beta_{t,k} = C_{t,k}$ $\forall t \in T \ \ \forall k \in K$

(2) $\sum_{k=1}^{K} C_{t,k} \leq 1$ $\forall t \in T$

(3) $X_{t,k} + \alpha_{t,k} + \beta_{t,k} \leq 1{,}98$ $\forall t \in T \ \ \forall k \in K$

(4) $Y_{t,k} \leq X_{t,k} * VCap$ $\forall t \in T \ \ \forall k \in K$

(5) $Z_{t,k} \leq C_{t,k} * VCap$ $\forall t \in T \ \ \forall k \in K$

(6) $\alpha_{1,k} = 0$ $\forall k \in K$

(7) $C_{19,k} = 0$ $\forall k \in K$

(8) $\sum_{k}^{K} P_{t,k,i} \leq 2$ $\forall t \in T \ \ \forall i \in I$

(9) $PDem_{t,k,i} \leq P_{t,k,i} * d_{t,i}$ $\forall t \in T \ \ \forall k \in K \ \ \forall i \in I$

(10) $d_{t,i} = \sum_{k}^{K} PDem_{t,k,i}$ $\forall t \in T \ \ \forall i \in I$

(11) $st_{t,k} = Y_{t,k} + Z_{t,k} - \sum_{i}^{I} PDem_{t,k,i} + st_{t-1,k}$ $\forall t \in T \ \ \forall k \in K$

(12) $st_{t,k} \leq InvCapT_{k}$ $\forall t \in T \ \ \forall k \in K$

(13) $st_{t,k} \geq ss_{k}$ $\forall t \in T \ \ \forall k \in K$

(14) $sf_{t} = Cap - \sum_{k}^{K} Y_{t,k} + sf_{t-1}$ $\forall t \in T$

(15) $sf_{t} \leq InvCapF$ $\forall t \in T$

(16) $TimeUsed_{t} = \sum_{k=1}^{K} (X_{t,k} * Time_{k})$ $\forall t \in T$

(17) $TimeUsed_{t} \leq Limit$ $\forall t \in T$

(18) $TimeD_{t,k} = \left(X_{t,k} + \alpha_{t,k} + \beta_{t,k}\right) * TimeC_{t,k}$ $\forall t \in T \ \ \forall k \in K$

4.8.1 Results

The results we have found from solving the final model in MPL is exactly the same as what we found when we solve Model 0 and Model 0 with extension 1 except that the objective value has now slightly increased (see figure 4.10). The reason why the objective value has increased is because of the new distance cost introduced in extension 2. The difference (1 001 675 – 1 001 524 = 151) is equal to the distance from each customer group *i* to each terminal *k* (19 periods * 8 terminals – 1 period with no demand = 151). The transportation plan is again altered which results in an average time used per period of 124 hours.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
43	2 970	99 %	127 700	1 001 675

Figure 4.10: Results for the final model

The reason why the solution is equal to the solution found in Model 0 may be due to the nature of the input data from the empirical case. As shown in figure 4.11 the inventory levels at the terminals over the planning period of 19 periods is rather high and not once equal to zero. This indicates that the input data is such that no "inventory-bottlenecks" occur at the terminals and thus the model does not have to utilize extension 2. If the demand is increased, this may change. Some of the assumptions we have made also influence the model. Since we assume the production (and the production-cost) to be constant and the holding cost at the factory and the terminal to be equal, this does not give the model any possibilities to adjust the inventory levels by lowering the production in some periods.

Period/Terminal	T1	T2	T3	T4	T5	T6	T7	T8
1	37 %	35 %	37 %	2 %	27 %	74 %	22 %	80 %
2	35 %	31 %	35 %	31 %	25 %	58 %	9 %	66 %
3	31 %	26 %	32 %	65 %	78 %	44 %	60 %	63 %
4	87 %	23 %	29 %	21 %	66 %	63 %	43 %	53 %
5	85 %	77 %	26 %	42 %	59 %	42 %	91 %	41 %
6	79 %	71 %	86 %	2 %	54 %	12 %	75 %	87 %
7	72 %	60 %	82 %	9 %	45 %	38 %	64 %	71 %
8	69 %	46 %	78 %	37 %	99 %	71 %	55 %	58 %
9	62 %	25 %	70 %	68 %	88 %	100 %	100 %	46 %
10	60 %	72 %	66 %	85 %	67 %	65 %	89 %	28 %
11	54 %	53 %	56 %	100 %	43 %	27 %	75 %	70 %
12	46 %	30 %	50 %	55 %	80 %	54 %	62 %	52 %
13	39 %	67 %	44 %	71 %	58 %	82 %	49 %	34 %
14	33 %	48 %	39 %	87 %	99 %	56 %	39 %	19 %
15	85 %	24 %	35 %	28 %	81 %	90 %	20 %	61 %
16	78 %	66 %	29 %	36 %	67 %	70 %	9 %	49 %
17	73 %	54 %	85 %	2 %	59 %	56 %	2 %	100 %
18	70 %	48 %	83 %	41 %	51 %	48 %	58 %	95 %
19	67 %	41 %	80 %	81 %	43 %	100 %	54 %	91 %

Figure 4.11: Inventory level at the terminals

Even though the optimal solution is similar from Model 0 to the final model, two important findings are made. The first important finding is that we have found that the model produces on average a very high fill rate of 99 %, as shown in figure 4.10, with only seven out of 43 trips having a lot size that is less than maximum capacity. This means that the model is able to "understand" that it is most reasonable to ship as few vessels as possible, with as high fill rate as possible.

Total (Y+Z)	T1	T2	T3	T4	T5	T6	T7	T8
1	-	-	-	-	-	3 000	-	3 000
2	-	-	-	3 000	-	-	-	-
3	-	-	-	3 000	3 000	-	2 966	-
4	3 000	-	-	-	-	2 483	-	-
5	-	3 000	-	3 000	-	-	3 000	-
6	-	-	3 000	-	-	-	-	3 000
7	-	-	-	2 451	-	3 000	-	-
8	-	-	-	2 937	3 000	3 000	-	-
9	-	-	-	3 000	-	3 000	3 000	-
10	-	3 000	-	3 000	-	-	-	-
11	-	-	-	3 000	-	-	-	3 000
12	-	-	-	-	3 000	3 000	-	-
13	-	3 000	-	3 000	-	3 000	-	-
14	-	-	-	3 000	3 000	-	-	-
15	3 000	-	-	-	-	3 000	-	3 000
16	-	3 000	-	3 000	-	-	-	-
17	-	-	3 000	-	-	-	-	2 924
18	-	-	-	2 940	-	-	3 000	-
19	-	-	-	3 000	-	2 999	-	-

Figure 4.12: Lot sizes from the final model

The other important finding from the final model is that the period overlapping setup variable is utilized in the manner we initially intended it to. This is illustrated by the two timelines in figure 4.13. As we saw in chapter 4.6.1 this was not the case after we had implemented only extension 1. Due to the extension of period overlapping setup the model is able to set up a new trip to terminal *T4* at the end of period *9* even if it has to return in period *10*.

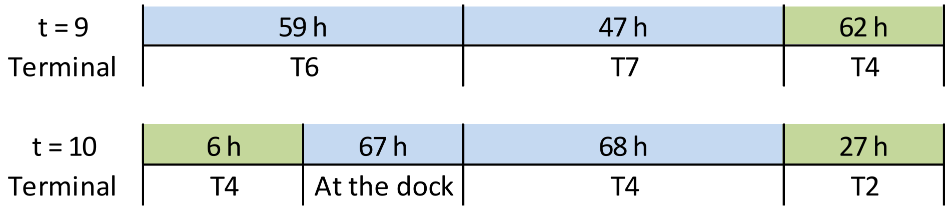

Figure 4.13: Period overlapping setup illustrated

The stability and flexibility of the model become even more evident when we adjust some of the parameters in the model and test the model for seasonal variations in demand in chapter five.

5 Model testing and analysis

5.1 The purpose of model testing

The discussion in chapter four showed that the model appeared to be stable. From a theoretical perspective it is important to make sure that the model also is stable and provides logical and acceptable results under different assumptions, parameters and input data. As we saw from chapter four the extension of period overlapping setups did not work optimally and the results for whether or not the inventory shortage worked were inconclusive. We therefore have to ensure that their functionality becomes evident under different input data. In order to ensure this, we have tested the model by changing the parameters we presume to affect the stability of the model. The industry that HeidelbergCement operates in is exposed to seasonal fluctuations in demand. Thus, from a practical perspective it is also important to test whether the model is able to handle such changes in the demand.

5.2 Model testing by changing parameters

The model-testing is divided into seven different scenarios. Except for the parameter in question in each scenario, all other parameters are held equal. Since the overriding objective in this chapter is to test the stability and functionality of the model, we do not take any investments costs that would occur from changing the parameters into consideration. Hence no evaluation of potential costs savings are made in any of the scenarios and no consideration are made in regards to the realism of changing these parameters from HeidelbergCement's perspective.

When we changed optimization tool from SAS/OR to MPL we were given the possibility of analysing the model to a larger extent than what we would have been able to do had we continued using SAS/OR. This is due to the fact that MPL has a better solver than SAS/OR does. This has been paramount in order to perform the tests in this chapter.

Scenario 1: *Inventory capacity at the terminals - increase of 2500 tons*

The inventory capacity at each terminal *k* is increased from 5000 tons to 7500 tons. By increasing the inventory at the terminals the model are given an opportunity to reallocate the inventory in the first periods of the planning horizon such that the inventory shortage may be utilized later on in the planning period and hence reduce total number of shipments.

The results from this test are unchanged and equal to those in the final model. At three of the terminals with the largest demand (*T4, T6 and T7*) the capacity-extension is utilized four times so that they are able receive two vessels in the same period, but this is not offset by fewer trips in other periods. Hence the inventory shortage-extension was not utilized the way we thought it would, in fact it was not utilized at all over the 19 periods. An interesting observation is that there are only two shipments with a lot-size lower than 3000, compared with the regular situation in the final model where there were seven. This implies that the model is working intuitively correct as it is choosing to fill up as many vessels as possible to maximal capacity when the opportunity arises.

Another interesting observation is that the average inventory for the two terminals with lowest average demand has decreased, while the average inventory at the terminal with the highest average demand has increased significantly (see figure 5.1). From a practical perspective this would be a logical solution; if you could increase the inventory-capacity you would do it where it is needed the most in order to further secure the ability to meet demand.

Terminal:	T1	T2	T3	T4	T5	T6	T7	T8
Avg. inventory scenario 1	2 105	2 520	1 954	3 422	2 811	3 146	4 178	3 017
Avg. inventory final model	3 053	2 362	2 743	2 268	3 126	3 026	2 568	3 065
Difference	***-947***	***158***	***-789***	***1 154***	***-316***	***120***	***1 609***	***-49***
Avg. demand at the terminal	246	630	210	1 910	625	1 236	593	651

Figure 5.1: Inventory levels compared

We argue that the model's reaction to the new parameter value, as highlighted in these two observations, underlines the model's ability to provide logical results and hence also its stability.

Scenario 2: *Inventory capacity at the factory - increase of 2500 tons.*

The inventory capacity at the factory is increased from 5000 tons to 7500 tons. We observed in the results from the final model that in some periods the inventory at the factory had to be built up in order to be able to produce large lot-sizes in later periods. Because of this, less than optimal (3000 tons) lot sizes were shipped out in some of those periods. When the inventory capacity at the factory is increased, the model should be able to increase the lot sizes in periods where it has to build up inventory at the factory. It should also be able to reduce the number of trips.

We see from the solutions presented in figure 5.2 that the number of trips is reduced by one and that the model has been able to further increase the average fill rate on the shipments. This indicates that the model has found results that are in accordance with what we presumed it would find, and it utilizes the capacity extension by exceeding the old capacity in five periods.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
42	2 981	99,4 %	125 200	1 000 775

Figure 5.2: Results for Scenario 2

While the reduction of one trip should mean that the reduction in objective value would be 1000, the actual reduction is only 900. This is because the inventory shortage extension is now utilized once in period nine (see figure 5.3) and the extra trip for customer group six result in an extra cost of 100. Even if it is positive that the extension is utilized, we will not draw any conclusions from it. The same scenario was tested without the extension, and found the same results. In other words; the solution in this scenario is not dependent on the extension working.

Terminal/Customer gr.	1	2	3	4	5	6	7	8
T1	340	-	-	-	-	-	-	-
T2	-	1 070	-	-	-	-	-	-
T3	-	-	443	-	-	-	-	-
T4	-	-	-	1 478	-	-	-	-
T5	-	-	-	-	581	526	-	-
T6	-	-	-	-	-	1 023	-	-
T7	-	-	-	-	-	-	744	-
T8	-	-	-	-	-	-	-	588

Figure 5.3: Illustration of the inventory shortage variable from Scenario 2

***Scenario 3:** Vessel capacity - increase of 1000 tons.*

In this scenario we want to test whether an increase in vessel capacity from 3000 tons to 4000 tons will force the model to use the inventory shortage extension. We expect that such an increase may lead to bottlenecks at some of the factories unless the inventory shortage extension is utilized. In reality, when you increase the vessel capacity the setup time will also change. A larger vessel will take longer time to load/unload. No adjustments to the setup times are made in the model, and hence the time-utilization in this scenario will be erroneous. Despite this weakness it still contributes to highlight a "logical problem" with the period overlapping setup variable *C*. As we discussed in chapter 4.6.1 the variable *C* were used as a substitute for *X* even if it did not make sense from a logical perspective. This becomes even more evident in this scenario (see figure 5.4).

Period	C	β	α	Time Used	β * $Time_k$	α * $Time_k$
1	1	0,98		53	53	
2			0,02	128		1
3	1	0,02		40	1	
4			0,98	114		46
5	1	0,98		86	58	
6			0,02	124		1
7				68		
8				106		
9	1	0,02		52	1	
10	1	0,98	0,98	159	67	53
11			0,02	60		1
12	1	0,98		95	27	
13	1	0,98	0,02	121	53	1
14	1	0,98	0,02	98	38	1
15			0,02	116		1
16	1	0,02		69	1	
17			0,98	66		27
18				113		
19				68		

Figure 5.4: Period overlapping setup, scenario 3

When the number of trips decreases the available time per week increases, and consequently the maximal time-capacity is not utilized *once* in this scenario. Still the variable *C* is used in nine periods and on all occasions either β*Time or α*Time is equal to one hour. This indicates that our formulation of the period overlapping setup seems to work in an unsatisfying manner the more slack you have in the time-capacity. This is further discussed in scenario four.

From figure 5.5 we see that number of trips made is reduced. This should come as no surprise. Even if the number of trips made is reduced, the model still manages to increase the average lot size, which is positive. The objective value is reduced by 10 602, meaning that the 11 000 saved in setup costs are reduced by an extra distance cost of 398.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
32	3 991	99,8 %	127 700	991 073

Figure 5.5: Results for Scenario 3

We find in this solution (see appendix) that the inventory shortage extension is better utilized, which is positive and what we expected. However, as we saw in scenario two, the solution is not necessary dependent on the extension. We have therefore also tested this scenario *without* the possibility of inventory shortage, and the results are shown in figure 5.6:

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
33	3 870	96,7 %	127 700	991 524

Figure 5.6: Results for Scenario 3 implemented *without* inventory shortage

It is interesting to see that this solution is not as good as when we included the inventory shortage extension. The model now only manages to reduce the number of trips to 33 and with a lower average lot size. We argue that from a planning perspective this shows the importance of including the possibility of inventory shortage. It also shows some of the flexibility of the model.

Scenario 4: *Time available - reduce the time available per period to 120 hours.*

When we implemented the period overlapping setup in the model, we experienced that the period overlapping setups were not functioning optimally, as we have explained in chapter 4.6.1 and in scenario two. Even though it functioned better in the final model there were still some "problems" where larger portions of the trip could have been performed in period *t* instead of postponing it to period *t+1*.

In order to investigate this problem, we decided to see whether or not the same issue would occur when we operated with a shorter planning period. We reduced the planning horizon per week from 7 days (168 hours) to 5 days (120 hours). When decreasing the length of the planning horizon per period we obtained the same results as for the final model with the exception of the average use of time. The average use of time is now 119 hours a week which is almost a utilization of

100%. As we can see from figure 5.7 there is a substantial use of period overlapping setups:

Period	C	β	α	Time Used	β * $Time_k$	α * $Time_k$
1	1	0,90		120	61	
2	1	0,88	0,10	120	45	7
3	1	0,22	0,12	120	12	6
4	1	0,49	0,78	120	19	42
5	1	0,78	0,51	120	53	20
6	1	0,59	0,22	120	23	15
7	1	0,66	0,41	120	45	16
8	1	0,15	0,34	120	10	23
9	1	0,91	0,85	120	62	58
10	1	0,19	0,09	120	13	6
11	1	0,10	0,81	120	6	55
12			0,90	120		53
13	1	0,68		114	46	
14	1	0,28	0,32	120	11	22
15	1	0,47	0,72	120	24	28
16			0,53	120		27
17	1	0,11		120	6	
18	1	0,22	0,89	110	15	48
19			0,78	120		53

Figure 5.7: Period overlapping setup, Scenario 4

There is a period overlapping setup performed in 16 of the 19 periods. This is because the time available during a period has been reduced since we are only operating with 5 days. Then the vessel has to take advantage of all the hours available and thereby using period overlapping setups. As you can see from figure 5.7 most of the periods where a period overlapping setup is being performed has a time utilization of 120 hours. In other words, when the time capacity is tighter, the period overlapping setup is working better.

We can also see an improvement with respect to how large the portions, α and β, of the overlapping setup are. The setup variable C does not work as a substitute for X when the planning horizon per period is reduced to five days; all the period overlapping setups that are performed are due to the fact that this is the only way the model could find and optimal solution. Hence we argue that when time-capacity is tight the period overlapping setup extension works very well and in line with what we intended it to.

For almost all the periods every customer group is picking up the demand from their own terminal. The only exception is week *16*, where customer group four is picking up the demand from terminal *T5*. This is a consequence of the fact that the demand of terminal *T4* is too high as to what the inventory level was, and terminal *T5* was supplied in week *16*.

Terminal *T4* can be said to be the “problem child” since its demand is consistently high in every period. When you have a bottleneck like this it is very helpful to have implemented the inventory shortage, so that the customers belonging to one terminal have the possibility of turning to their adjacent terminal should their own be out of stock or not have enough inventory. We can see that the implementation of inventory shortage is functioning as a planning tool.

When we reduced the planning horizon per period to five days in Model 0 and in Model 0 with extension 1, we found that the solution were infeasible. Thus, had it not been for the implementation of the period overlapping setup and the inventory shortage we could not have reduced the planning horizon per period to 120 hours. We argue that this illustrates the flexibility that the two extensions have added to the model.

Scenario 5: *Production capacity - let the production fluctuate between 5000 and 7000 tons.*

Theoretically it may not be correct to assume the production to be constant since you are then ignoring that the level of production may be adjusted in periods. Even if the machines are supposed to be running “24/7” because it is too expensive to shut them down, you may still adjust the production volumes. When you have a fluctuating production this is considered to a greater extent since you can adjust the production to the demand, or to produce the exact amount in order to achieve full vessels. Having a production capacity that is assumed to be at a constant level might be considered to be too rigid as there is no demand that is completely stable. If you are following a *level-strategy* you are producing at a constant level. Nevertheless, most companies follow the *chase-strategy* where you adjust the production to the demand. Thus, in order to test how the model would handle such a situation, we let the production capacity fluctuate between 5000 and 7000 (6950) tons in this scenario.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
39	2 564	85 %	100 009	361 917

Figure 5.8: Results for Scenario 5.

The expansion of letting the production capacity vary is highly utilized. There is only three of the periods where the production is 7000 tons (or 6950), and for 14 of the periods the production is set to the minimum; 5000 tons. As this model only takes the balance between vessel-trips (setups) and inventory holding cost into consideration, there is no setup costs connected to starting the production. Thus the model has no incentive to produce more than what is needed. By not producing more than necessary the additional cement does not have to be stored, and hence inventory costs are saved.

For the previous scenarios the production capacity has been constant, now the model can “choose”, and what is produced is actually less than what is demanded due to the fact that there is an initial inventory that the customers can feed of. This is one of the dangers of having such a short planning horizon. Since the model assumes that the world “stops” after 19 periods, it tries to save as much as possible by downsizing the inventory, not considering that this inventory might be needed to cover future demand (see figure 5.9). Nevertheless, this would happen no matter how long planning horizon you were operating with. If you are operating in an industry that has to deal with seasonal demand you would also have to build up some inventory in order to be able to cover the increased demand in future periods.

Period/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	1 832	1 733	1 850	3 006	1 353	709	1 118	977	2 081
2	1 734	1 559	1 741	1 491	1 238	2 639	428	300	4 345
3	1 548	1 323	1 580	3 190	896	1 924	3 045	3 052	424
4	1 361	1 149	1 461	3 946	288	421	2 171	2 571	2 424
5	1 233	829	1 297	2 025	2 590	2 342	1 589	1 981	1 761
6	3 254	532	1 297	0	2 383	851	807	1 265	4 481
7	2 339	532	1 110	577	1 937	2 167	2 460	1 246	800
8	2 180	2 652	921	2 070	1 634	802	2 022	588	0
9	1 840	1 582	478	592	1 053	2 253	1 278	0	2 000
10	1 725	926	316	1 473	0	484	718	1 947	1 121
11	1 431	0	2 031	0	0	384	18	1 046	121
12	1 063	0	562	765	0	0	18	145	801
13	695	926	250	126	0	0	0	0	0
14	401	0	0	0	0	0	500	0	983
15	0	0	0	0	0	0	0	0	78
16	0	0	0	0	0	0	0	0	0
17	290	0	0	72	0	0	0	0	0
18	145	0	460	1 496	0	0	435	0	0
19	0	0	0	106	0	0	0	0	4 604

Figure 5.9: Inventory levels

Lower inventories yields a lower cost, but it can be undesirable since it impacts the company's ability to deliver. As it is now the safety stock is set to zero in order to make our extension of inventory shortage as realistic as possible. From a practical perspective the company would probably like to have some sort of safety stock. The way we have developed the model you could add a value to the safety stock (or any other parameter) by changing the input-file. This makes our model more flexible to work with as you can change the parameters when needed. Since you are producing less cement there is also less cement to ship out to the terminals, thus the number of trips taken have decreased by four trips compared to the final model.

For 10 out of the 19 periods the customer groups are not picking up their demand at their designated terminal, rather going to one of the adjacent terminals. This is mainly due to the fact that it is empty at their designated terminal. In order to see how well the extension of inventory shortage is functioning in this scenario we will implement the same scenario (fluctuating production) without the extension. Doing so we obtained the following results:

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
48	2 202	73 %	105 693	412 032

Figure 5.10: Results for Scenario 5 implemented *without* inventory shortage extension

The number of trips performed now is considerably higher and the average lot-size is lower. Compared to when we implemented it in the final model, we are transporting an additional 5 684 tons of cement. Nevertheless, there are an additional *nine* trips performed. This is a quite substantial increase of trips given the small increase in the amount of cement to be shipped. This is because there is not an option to redirect the customers so that you only have to replenish a few of the terminals instead of all of them.

Here you cannot utilize the extension of inventory shortage as a planning tool. Thus, we can see the benefit of implementing the inventory shortage; the utilization of the company's resources are more efficient both in terms of the use of the vessel's capacity (higher fill rate) and in regards to the time utilized (fewer trips – more efficient use of the time). When you see that you are able to avoid nine additional trips to transport 5 684 tons of cement, you see that the inventory shortage is functioning well as a planning tool and is essential to obtain the best possible results.

Scenario 6: *Holding cost at the terminals - increase of 1.*

In this scenario we increased the holding cost at the terminals to 3, thus making it cheaper to store cement at the factory. The reason why we made this change is so that we could see whether the lot sizes would change and the number of trips would be reduced. Since the amount of cement stored at the factory is likely to increase, more cement will be available at the factory and hence the possibility for larger lot sizes should increase.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
50	2 554	85 %	127 700	1 397 687

Figure 5.11: Results for scenario 6

The number of trips that is being performed has increased compared to the final model. Thus the fill rate and the average lot size are not as good as for the final model. We did not see the desirable effect of the change in holding cost at the terminals.

The number of tons that is shipped out to the terminals is the same as before; it is the manner in which it is done that differs. The smallest lot size that is being shipped is 1000 tons, a utilization of 33%.This means that from a practical perspective the model makes inexpedient decisions in terms of the lot sizes. The model has an incentive to keep the inventory at the factory as full as possible, and thereby ships the amount of cement that makes this possible, not taking into consideration that the fill rate of the vessel is poor.

Period/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	1 832	1 733	1 850	87	1 353	2 709	1 118	977	5 000
2	1 734	1 559	2 801	1 572	1 238	1 903	428	3 240	5 000
3	1 548	1 323	2 640	2 294	3 176	1 188	2 692	3 071	5 000
4	1 361	4 149	2 521	3 050	2 568	685	1 818	2 590	5 000
5	1 233	3 829	2 357	1 129	2 207	2 241	3 551	5 000	4 050
6	974	3 532	2 357	2 104	5 000	750	2 769	4 284	5 000
7	2 722	5 000	2 170	0	4 554	1 935	2 178	3 509	5 000
8	2 563	4 320	1 981	1 493	4 251	3 549	1 740	4 822	4 050
9	2 223	3 250	1 538	3 015	3 670	5 000	996	4 234	5 000
10	2 108	2 594	1 376	5 000	2 617	3 231	3 332	3 302	5 000
11	1 814	1 668	3 879	2 739	3 367	4 331	2 632	2 401	4 050
12	1 446	511	3 567	3 504	2 245	2 717	4 984	1 500	5 000
13	1 078	2 354	3 255	3 379	1 123	1 103	4 336	2 489	5 000
14	784	1 428	3 005	4 191	2 429	2 518	3 848	1 768	4 050
15	383	3 228	2 818	1 256	1 529	1 227	2 906	3 868	5 000
16	0	4 969	2 510	0	847	3 183	2 359	3 269	5 000
17	1 699	4 376	2 308	1 290	400	2 499	5 000	2 877	4 050
18	4 554	4 033	2 191	3 300	0	2 103	4 792	2 650	5 000
19	4 409	5 000	2 074	5 000	2 600	1 707	4 584	2 423	5 000

Figure 5.12: Inventory levels

The average inventory at the factory is 4750 which is 95% of the total capacity. This means that the difference in what was stored at the factory earlier and now is the change in the inventory at the terminals. As expected the inventory at the factory is full in almost every period. Also, all customer groups are picking up their demand from their designated terminal in this scenario.

***Scenario 7:** Holding cost at the factory - increase of 1.*

From a supply chain perspective it may be beneficial to store the final product closer to the customers. One of the reasons for this is that it is easier to supply unexpected orders and that the lead time becomes better. Another reason is that you should be able to ship out fewer vessels with larger lot sizes from the factory. Since this may be more correct from a "lot sizing-point of view" we wanted to see the how the model behaved when we increased the holding cost at the factory to 3 as opposed to keeping the costs equal.

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
52	2 552	85 %	132 700	1 015 425

Figure 5.13: Results for Scenario 7

Since it is less expensive to store the cement at the terminals the model will try to ship as much of the cement as possible out to the terminals, which will increase the number of trips made. As you can see, more cement (5000 tons) are transported compared to the results in the final model, which confirms this.

As in scenario six the model makes inexpedient decisions. The lowest lot size is 950 tons for this scenario, but the reasoning is different. Now the model wants to transport as much cement out to the terminals as possible, so if there is 950 tons at the factory that there is room for at the terminals this will be shipped regardless of how poor the utilization of the vessel is as long as the cost of shipping this vessel is less than the extra holding cost of storing the product at the factory

Period/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	3 733	1 733	4 850	87	1 353	709	3 217	977	0
2	3 635	1 559	4 741	1 572	1 238	2 903	2 527	1 300	0
3	3 449	1 323	4 580	3 271	896	2 188	2 144	4 131	950
4	3 262	1 149	4 461	3 886	2 379	3 685	1 270	3 650	0
5	3 134	829	4 297	4 025	2 018	2 606	3 688	5 000	0
6	2 875	3 532	4 297	5 000	1 811	1 115	2 906	4 284	950
7	2 499	2 993	4 110	2 896	1 365	4 696	5 000	3 509	0
8	2 340	4 644	3 921	4 389	1 062	5 000	4 562	2 851	0
9	5 000	3 574	3 478	2 911	3 481	3 451	3 818	2 263	950
10	4 885	2 918	3 316	3 792	5 000	4 060	3 258	1 331	0
11	4 591	1 992	2 819	1 531	3 800	4 590	2 558	5 000	0
12	4 223	3 835	2 507	2 296	2 678	2 976	1 910	4 099	950
13	3 855	2 678	2 195	3 061	1 556	3 312	4 262	3 198	0
14	3 561	4 469	1 945	3 873	1 901	2 021	3 774	2 477	0
15	3 160	3 269	1 758	3 938	1 001	3 730	2 832	1 577	950
16	2 777	5 000	1 450	4 328	319	5 000	2 285	978	0
17	2 526	4 407	2 866	2 618	2 400	4 316	4 780	586	0
18	2 381	4 064	2 749	2 578	5 000	3 920	4 572	3 359	0
19	5 000	3 721	5 000	1 588	4 600	3 524	4 364	5 000	0

Figure 5.14: Inventory levels

The average inventory at the terminals is 3122 tons. This compares to an average utilization of the capacity of 62 %, which is quite high. This is not surprising given the change in holding cost. The average inventory at the factory has decreased significantly; from 4747 tons in the scenario six to 250 tons. The only inventory that is held is 950 tons during the periods where the production capacity is reduced by 50 tons.

5.3 Changing demand patterns

The cement industry is prone to seasonal fluctuations and variations in demand that stems from other external factors they cannot control, as for instance the recent financial crisis. In their annual report for 2010 HeidelbergCement states that *"changes in demand obviously present both opportunities and risks for HeidelbergCement"* and *"A significant risk in building material sales volumes* *results mainly from seasonal demand, especially because of the dependency on weather conditions".* In other words; it is important from both a theoretical and practical point of view to see if the model can handle fluctuations in demand and also how it reacts to the increase and decrease demand.

We first tried to reduce the demand by 15% in order to simulate a low-season, but the model could not handle such a large decrease. Since the production capacity is set and constant there was produced too much cement for the vessel and the inventories to handle, thus there was no feasible solution found to the problem. A possible solution for this is to let the production capacity fluctuate. The two different scenarios that we modelled were a low-season scenario, with a decrease in demand by 10%, and a high-season scenario, with an increase in demand by 15%. Both demand scenarios have the demand from the final model as a "starting point", which can be described as a "regular season". The results from the final model is discussed in chapter four and hence not included in this chapter.

"Low-season"

In order to simulate a low season we reduced the demand by 10%, which provided these results:

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
43	2 970	99 %	127 700	1 233 441

Figure 5.15: Results for "low-season"

The results are the same as for the final model, with the exception of the objective value. The reason why the number of shipments is still equal to the final model is that the model utilizes *the inventory shortage extension* as a mean to coordinate the shipments so that the fill rate can be as good as possible. We illustrate this with an example from the last three periods in the planning horizon. Figure 5.16 shows that the inventory levels at the terminals in these periods are very high. This is due to the low demand. This situation could also be similar to a real-life situation where the company choose to build up inventories in a "low-season". In order to handle this situation the model utilizes the inventory shortage extension (see figure 5.17). This again shows how well the implementation of the inventory shortage works as a planning tool, and the importance of implementing it to the model.

Period/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
17	2 877	5 000	5 000	3 045	4 324	4 960	5 000	2 319	3 000
18	2 438	5 000	5 000	5 000	3 607	4 960	5 000	4 926	4 000
19	4 999	5 000	4 895	5 000	5 000	4 960	4 813	4 721	5 000

Figure 5.16: Inventory level at the terminals in period 17-19

	Terminal/Customer gr.	1	2	3	4	5	6	7	8
t = 17	T1	226	-	-	-	-	-	-	-
	T2	-	534	-	-	-	-	-	-
	T3	-	-	182	424	-	-	-	-
	T4	-	-	-	1 115	402	-	-	-
	T5	-	-	-	-	-	-	-	-
	T6	-	-	-	-	-	616	323	-
	T7	-	-	-	-	-	-	-	-
	T8	-	-	-	-	-	-	-	353
t = 18	T1	131	309	-	-	-	-	-	-
	T2	-	-	-	-	-	-	-	-
	T3	-	-	-	-	-	-	-	-
	T4	-	-	105	891	-	-	-	-
	T5	-	-	-	-	360	356	-	-
	T6	-	-	-	-	-	-	-	-
	T7	-	-	-	-	-	-	-	-
	T8	-	-	-	-	-	-	187	204
t = 19	T1	131	309	-	-	-	-	-	-
	T2	-	-	-	-	-	-	-	-
	T3	-	-	105	-	-	-	-	-
	T4	-	-	-	-	-	-	-	-
	T5	-	-	-	891	360	356	-	-
	T6	-	-	-	-	-	-	-	-
	T7	-	-	-	-	-	-	187	-
	T8	-	-	-	-	-	-	-	204

Figure 5.17: Inventory shortage extension illustrated in period 17-19

As you can see from figures 5.18 and 5.19 the average inventories are also consistently higher when there is a "low-season" than for the "regular season" since the production level is the same but the demand is lower. It could be beneficial to be able to adjust the production level according to the "low-season", so that less inventory had to be stored.

Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
Average inventory	3 345	3 027	3 881	3 398	3 132	4 020	3 783	3 191	3 510
in %	67 %	61 %	78 %	68 %	63 %	80 %	76 %	64 %	70 %

Figure 5.18: Average inventory for "low-season"

Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
Average inventory	3 053	2 362	2 743	2 268	3 126	3 026	2 568	3 065	3 013
in %	61 %	47 %	55 %	45 %	63 %	61 %	51 %	61 %	60 %

Figure 5.19: Average inventory in the final model

"High-season"

For the high-season it is interesting to see whether the model is able to maintain the solution from the final model. If the model is stable it should utilize one of the extensions in order to meet the challenge of the increased demand. In order to simulate a high-season we increased the demand by 15%, giving us these results:

# Trips	Average lot size	Average fill rate	Tons shipped	Objective value
43	2 970	99 %	127 700	656 504

Figure 5.20: Results for "high-season"

The inventory at the terminals and the factory are naturally smaller than before since more cement is picked up by the customers. And as you can see from figure 5.21 the inventory at certain terminals are completely drained in some periods. This is usually not desirable since it is usual to operate with some safety stock in order to cope with extreme situations as an increase in demand or unexpected orders. Since we have allowed for inventory shortage, safety stock at the terminals becomes less important as the model has the possibility of redirecting customers in such situations. A potential problem with our formulation of the extension must however be mentioned. As you can see, the inventory levels in period 16 at terminal *T1*, *T2* and *T3* are zero. This implies that for customers designated to terminal *T1* and *T2* it is potentially not possible to pick up the demand at the adjacent terminals. They could pick up the demand at one of the other terminals, but due to the distance we have assumed to be 999 this is not probable. If the real distances were implemented in the distance matrix, this may have improved the functionality of the extension in such situation.

Period/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	1 807	1 693	1 827	2 800	1 256	515	986	824	4 000
2	1 694	1 493	1 702	4 058	1 124	2 588	192	45	5 000
3	1 480	1 221	1 517	2 562	3 730	1 766	2 752	2 851	2 950
4	1 265	1 021	1 380	2 981	3 031	3 038	1 747	2 298	3 950
5	3 861	653	1 192	3 772	2 616	1 797	1 077	4 619	2 207
6	3 563	3 312	1 192	4 443	2 378	82	178	3 796	3 157
7	3 130	2 692	976	2 024	1 865	987	2 498	2 904	4 316
8	2 947	1 910	3 759	3 290	1 517	2 417	1 995	2 148	2 316
9	2 556	3 679	3 250	1 591	848	636	4 139	1 471	3 266
10	2 424	2 925	3 063	2 154	2 638	1 602	2 857	1 038	1 266
11	2 086	1 860	2 492	2 554	1 258	2 417	2 052	2	2 266
12	1 663	530	2 133	2 984	2 967	561	1 306	1 966	216
13	1 240	3 486	444	413	412	2 969	561	930	1 260
14	902	2 421	156	897	2 333	1 485	0	101	2 260
15	440	1 041	354	107	1 298	0	1 635	1 662	898
16	0	0	0	105	514	1 697	1 006	973	2 000
17	2 029	0	2 768	1 139	0	911	593	522	0
18	1 863	2 556	2 633	0	2 540	455	354	261	1 000
19	1 696	2 161	2 499	1 862	2 080	0	114	0	5 000

Figure 5.21: Inventory levels

Nevertheless, the implementation of the inventory shortage is still working as a planning tool since the model is redirecting customers not only because their terminal is empty, but also in order to coordinate the shipments so that the fill rate will not be too poor. What this shows is that the model is able to handle increased demand due to the extension of the inventory shortage, which further underlines the importance of including this in the model.

6 Concluding remarks

6.1 Summary and conclusions

The transportation lot sizing model was developed in three separate steps. In the first step (Model 0) the optimization tool *MPL* found an optimal solution to the model within seconds. We observed that the solution showed a very good utilization of the vessel capacity with an average lot size close to maximum capacity. This indicated that the model was formulated in a way that provided logical results both from a theoretical and practical perspective. In order to increase the flexibility of the model we made two extensions to Model 0 in two separate steps; "Period overlapping setup" and "Inventory shortage". These three steps constituted our *final model*. The model did not utilize the two extensions as we initially thought it would, and hence the solution found in the final model was similar to the one found in Model 0.

In order to test the stability and flexibility of the model, and also prove that the extensions made to the model work, we changed different parameters in the model. We found that the model continued to deliver logical solutions, and we argue that this shows the stability of the model. The utility value of the two extensions also became more evident. Under tighter capacities the period overlapping setup worked very well, but less so if there was slack in the capacities. In scenario three the inventory shortage extension helped the model to find a better solution than what it did without the extension. The "model-value" of this extension became further evident when we tested the model for different demand patterns. As this is a highly relevant problem for HeidelbergCement it was important to show that the model was able to handle such changes. We argue that by including the two aforementioned extensions in the model we have improved the flexibility of the model, both from a theoretical and practical perspective.

By adapting lot sizing models and theory usually found in production planning to a transportation planning problem, we argue that we have been able to formulate a lot sizing model that can be used solely for transportation planning – *a transportation lot sizing model.* We have also been able to find a solution to the empirical case provided to us by HeidelbergCement by using the model we have developed in this thesis.

6.2 Practical use of the model

It was not an objective in our thesis to formulate a model that the company would use in its daily planning work. As the model has proven to be stable and provides what we presume to be logical solutions from HeidelbergCement's perspective, we will however briefly discuss our opinions on whether we think the model may be utilized as a planning tool for the company.

In order to formulate this model in a way that was possible to solve in MPL, we have shown that we had to make several assumptions and simplifications. While this is necessary when modelling, such simplifications will always lead to solutions that are not directly transferrable to a real life situation and this is something you always have to bear in mind when analysing the findings of these types of models. But it does not necessarily reduce the ability to use the solutions found in the model in some capacity. The planning horizon we have used in our formulation is 19 periods. According to theory the model can therefore be described as a tactical planning tool.

From a practical perspective we argue that the inventory shortage extension provides a great opportunity from a practical planning perspective to see how you can redirect customers in order to find an overall better transportation and inventory plan. The discussion in chapter five also shows that it is possible to use the model in order to see how the plan would change with different capacities. Hence the model could be utilized as a support tool in new investment analysis. The analysis of seasonal demand patterns also shows that the model can be used to simulate how the transportation and inventory plan would react under different demand scenarios. This may help the company to identify possible bottlenecks if the demand changes,

While we recognize the fact that the model should not be used on a day-to-day basis, it could still be of some practical use for the company in situations as those described above. Thus we argue that due to the stability and flexibility of the model it may be utilized for simulation studies and as a support tool in different scenarios.

6.3 Suggestions for further research

Throughout the analysis of the model we have shown that the period overlapping setup does not work entirely as we had hoped for when you have slack in the time-capacity per period. When the capacity is tight however it seems to work very well. We therefore suggest that the formulation of the period overlapping setup should be further investigated in order to find a formulation that makes more sense from a practical perspective. One possibility could be to implement some form of sequence dependent setups in order to create different "time slots" that the overlapping setup variable had to depend on. This might however be misguiding since there is no sequence dependent setup costs or setup times in the way we have formulated the model.

Regardless of the share of the period overlapping setup *C* performed in period *t*, the entire lot size *Z* has to be delivered in period *t*. This assumption leads to some of the "misbehaviour" of *C* as only one hour of the trip has to be performed in period *t* in order to deliver the whole lot size *Z* in period *t.* It would be interesting to see if it is possible to implement the point of time at each trip that the delivery is made at terminal *k.* This would give a more realistic picture of the period overlapping trip and could also help with some of the challenges with *C*.

We assumed that the production available each period *t* is constant and do not adjust concurrently with the production over the period *t*. Implicit we say that all that is produced in period *t* only becomes available at the beginning of period *t+1*. This is too simplified as production takes place 24/7 with constant replenishment of the inventory at the factory. It would be very interesting to see if it would be possible to implement a form of continuously available production in the model. This could also help to facilitate the period overlapping setup possibility as the vessel under such new production availability could be forced to stay in dock while waiting for enough cement to be produced before it could start a new trip. From a real life perspective we argue that this would certainly improve the model.

The distance matrix and cost assumptions we have made for the extension of inventory shortage is highly simplified. It would be interesting to see how this extension would work if the actual data distances and costs were analysed and implemented into the model. As this would increase the realism in the model you could also relax the assumption that one customer group can only go to two adjacent terminals. This could be particularly interesting in periods with high demand, where the customers could be willing to travel longer in order to pick up their demand. It would also benefit the company as the ability to use the model as a planning tool would increase.

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8 Table of appendices

Appendix 1: MPL source code for the final model:

```
title
	cplm


index
	period = 1..19;
	terminal = (T1, T2, T3, T4, T5, T6, T7, T8);
	customer = (C1, C2, C3, C4, C5, C6, C7, C8);


data
	D[period, customer] := EXCELRANGE("ImportFinal.xlsx","Demand!D");
	ProdCap2[period] := EXCELRANGE("Prodcap!Prodcap");
	TimeC[period, terminal] := EXCELRANGE("Timecount!Timecount");
	Time[terminal] := EXCELRANGE("Time!Time");
	Limit := EXCELRANGE("Timelimit!Timelimit");
	VCap := EXCELRANGE("Vesselcap!Vesselcap");
	HCT := EXCELRANGE("HCT!Hct");
	HCF := EXCELRANGE("HCF!Hcf");
	InvCapT[terminal] := EXCELRANGE("InvCapT!Invcapt");
	InvCapF := EXCELRANGE("InvCapF!Invcapf");
	SS[terminal] := EXCELRANGE("Safetystock!Safetystock");
	InitInvT[terminal] := EXCELRANGE("InitialInvT!Initialinv");
	InitInvF := EXCELRANGE("InitialInvF!Initialinvf");
	Setupcost[terminal] := EXCELRANGE("Setupcost!Setupcost");
	COcost := EXCELRANGE("Carryovercost!Carryovercost");
	Distance[terminal, customer] := EXCELRANGE("Distance!avstand");


variables
	InvT[period,terminal] EXPORT TO EXCELRANGE("Scenario_5.xlsx",
"InvT!Invt");
	InvF[period] EXPORT TO EXCELRANGE("InvF!Invf");
	Y[period,terminal] EXPORT TO EXCELRANGE("Y!Y");
	TimeUsed[period] EXPORT TO EXCELRANGE("TimeUsed!TimeUsed");
```

```
	C_in[period,terminal] EXPORT TO EXCELRANGE("in!in");
	C_out[period,terminal] EXPORT TO EXCELRANGE("out!out");
	TimeD[period, terminal] EXPORT TO
EXCELRANGE("CountingTime!Countingtime");
	Z[period, terminal] EXPORT TO EXCELRANGE("Z!Z");
	PDem[period,terminal,customer] EXPORT TO
EXCELRANGE("PDem!Pdem");
	ProdCap[period] EXPORT TO EXCELRANGE("ProdCap!ProdCap");

binary variables
	X[period,terminal] EXPORT TO EXCELRANGE("X!X");
	C[period,terminal] EXPORT TO EXCELRANGE("C!Cvar");
	P[period,terminal,customer] EXPORT TO EXCELRANGE("P!P");

model
	min totalcost EXPORT TO EXCELRANGE("ObjFunc.xlsx", "Scenario_5")
			= (sum(period, terminal: X[period,terminal]*Setupcost))
			+(sum(period,terminal: C[period,terminal]*COcost))
			+(sum(period: InvF[period]*HCF))
			+(sum(period,terminal: InvT[period,terminal]*HCT))
			+(sum(period,terminal,customer:
P[period,terminal,customer]*Distance[terminal,customer]*1));

subject to

conprodcap1[period]:  ProdCap[period] <= ProdCap2[period];

defineinandout[period,terminal]:  C_out[period,terminal] + (if period<19 then
C_in[period,terminal] else 0 endif) = C[period,terminal];

maxin[period,terminal]: C_in[period,terminal] <= 0.98;

maxout[period,terminal]: C_out[period,terminal] <= 0.98;
```

```
nomorethanone[period]:  sum(terminal: C[period,terminal]) <= 1;

maxoneXandC[period,terminal]:  X[period,terminal] + C_out[period,terminal] +
C_in[period,terminal] <= 1.98;

setuplogic[period, terminal]: Y[period,terminal] <= X[period,terminal]*VCap;

setuplogictwo[period,terminal]: Z[period,terminal] <= C[period,terminal]*VCap;

notin[terminal]: C_in[1,terminal] = 0;

notout[terminal]: C[19,terminal] = 0;

cangototwo[period,customer]: sum(terminal: P[period,terminal,customer]) <= 2;

setuplogiclostsales[period,terminal,customer]: PDem[period,terminal,customer]
<= P[period,terminal,customer]*D[period,customer];

mustmeetdemand[period,customer]:  D[period,customer] = sum(terminal:
PDem[period,terminal,customer]);

InventoryBalanceT[period, terminal]:  InvT[period,terminal] = Y[period,terminal]
+ Z[period,terminal] - (sum(customer: PDem[period,terminal,customer])) + (if
period>1 then InvT[period-1,terminal] else InitInvT[terminal] endif);

MaxInvT[period, terminal]: InvT[period,terminal] <= InvCapT[terminal];

MinInvT[period,terminal]: InvT[period,terminal] >= SS[terminal];

InventoryBalanceF[period]: InvF[period] = ProdCap[period] - (sum(terminal:
Y[period,terminal])) - (sum(terminal: Z[period,terminal])) + (if period>1 then
InvF[period-1] else InitInvF endif);
```

```
MaxInvF[period]: InvF[period] <= InvCapF;

TimeUsedPerWeek[period]: TimeUsed[period] = (sum(terminal:
X[period,terminal]*Time[terminal])) + (sum(terminal:
C_out[period,terminal]*Time[terminal])) + (sum(terminal:
C_in[period,terminal]*Time[terminal]));

maxtimecanuse[period]: TimeUsed[period] <= Limit;

Showtime[period,terminal]: TimeD[period,terminal] =
(X[period,terminal]*TimeC[period,terminal]) +
(C_out[period,terminal]*TimeC[period,terminal]) +
(C_in[period,terminal]*TimeC[period,terminal]);
```

Appendix 2: Results for Model 0:

Inventory levels:

Week/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	1 832	1 733	1 850	87	1 353	3 709	1 118	977	4 000
2	1 734	1 559	4 741	1 572	1 238	2 903	428	300	5 000
3	1 548	1 323	4 580	271	3 896	2 188	3 045	3 131	2 950
4	1 361	1 149	4 461	1 027	3 288	3 685	2 171	2 650	3 950
5	1 233	829	4 297	2 106	2 927	2 606	4 589	2 060	4 950
6	974	3 532	4 297	3 081	2 720	4 115	3 807	1 344	2 900
7	598	2 993	4 110	3 977	2 274	2 431	3 216	3 569	3 900
8	3 439	2 313	3 921	2 470	4 971	4 066	2 778	2 911	1 900
9	3 099	4 206	3 478	3 992	4 390	2 517	2 034	5 000	210
10	2 984	3 550	3 316	1 873	3 337	3 748	1 474	4 068	4 210
11	2 690	2 624	2 819	2 612	5 000	4 848	774	3 167	2 347
12	2 322	1 467	5 000	3 377	3 878	3 234	126	2 266	3 804
13	1 954	3 310	4 688	4 123	2 756	1 620	2 478	1 365	1 823
14	1 660	2 384	4 438	4 935	1 818	3 329	1 990	644	2 823
15	1 259	1 184	4 251	5 000	918	2 038	1 048	2 744	3 773
16	3 876	3 279	3 943	2 390	3 236	994	501	2 145	1 773
17	3 625	2 686	3 741	3 680	2 789	3 033	142	1 753	3 050
18	3 480	2 343	3 624	2 690	2 389	2 637	2 934	4 526	4 000
19	3 335	5 000	3 507	4 700	1 989	2 241	2 726	4 299	5 000

Lot sizes (Y and Z):

Week/Terminal	T1	T2	T3	T4	T5	T6	T7	T8
1	-	-	-	-	-	3 000	-	-
2	-	-	3 000	3 000	-	-	-	-
3	-	-	-	-	3 000	-	3 000	3 000
4	-	-	-	3 000	-	3 000	-	-
5	-	-	-	3 000	-	-	3 000	-
6	-	3 000	-	3 000	-	3 000	-	-
7	-	-	-	3 000	-	-	-	3 000
8	3 000	-	-	-	3 000	3 000	-	-
9	-	2 963	-	3 000	-	-	-	2 677
10	-	-	-	-	-	3 000	-	-
11	-	-	-	3 000	2 863	3 000	-	-
12	-	-	2 493	3 000	-	-	-	-
13	-	3 000	-	2 981	-	-	3 000	-
14	-	-	-	3 000	-	3 000	-	-
15	-	-	-	3 000	-	-	-	3 000
16	3 000	3 000	-	-	3 000	-	-	-
17	-	-	-	3 000	-	2 723	-	-
18	-	-	-	-	-	-	3 000	3 000
19	-	3 000	-	3 000	-	-	-	-

Time Used:

Week	
1	59
2	119
3	140
4	127
5	115
6	155
7	122
8	153
9	150
10	59
11	166
12	119
13	143
14	127
15	122
16	122
17	127
18	101
19	96

Appendix 3: Results for Model 0 with extension 1:

Inventory levels:

Week/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	1 832	1 733	4 835	3 087	1 353	709	1 118	977	1 015
2	1 734	1 559	4 726	4 572	1 238	2 386	428	2 832	-
3	1 548	4 323	4 565	3 271	896	1 671	3 045	2 663	950
4	1 361	4 149	4 446	4 027	3 288	168	2 171	2 182	1 950
5	1 233	3 829	4 282	2 106	2 927	2 089	4 589	1 592	2 950
6	974	3 532	4 282	3 081	2 720	598	3 807	3 876	3 900
7	3 598	2 993	4 095	977	5 000	1 914	3 216	3 101	2 174
8	3 439	2 313	3 906	2 470	4 697	3 549	2 778	2 443	3 174
9	3 099	1 243	3 463	992	4 116	5 000	2 034	4 855	4 124
10	2 984	3 587	3 301	1 873	3 063	3 231	4 474	3 923	2 124
11	2 690	2 661	2 804	2 612	4 863	1 331	3 774	3 022	3 124
12	2 322	4 504	2 492	3 377	3 741	2 717	3 126	2 121	1 074
13	4 928	3 347	2 180	4 142	2 619	1 103	2 478	1 220	2 100
14	4 634	2 421	1 930	1 954	4 681	2 812	1 990	499	3 100
15	4 233	1 221	1 743	2 019	3 781	1 521	1 048	2 599	4 050
16	3 850	316	1 435	2 409	3 099	3 477	501	5 000	2 050
17	3 599	2 723	4 233	3 699	2 652	2 793	142	4 608	50
18	3 454	2 380	4 116	2 709	2 252	2 397	2 934	4 381	4 000
19	3 309	2 037	3 999	4 719	4 852	2 001	2 726	4 154	5 000

Lot sizes (Y and Z):

Week/Terminal	T1	T2	T3	T4	T5	T6	T7	T8
1	-	-	2 985	3 000	-	-	-	-
2	-	-	-	3 000	-	2 483	-	2 532
3	-	3 000	-	-	-	-	3 000	-
4	-	-	-	3 000	3 000	-	-	-
5	-	-	-	-	-	3 000	3 000	-
6	-	-	-	3 000	-	-	-	3 000
7	3 000	-	-	-	2 726	3 000	-	-
8	-	-	-	3 000	-	3 000	-	-
9	-	-	-	-	-	3 000	-	3 000
10	-	3 000	-	3 000	-	-	3 000	-
11	-	-	-	3 000	3 000	-	-	-
12	-	3 000	-	3 000	-	3 000	-	-
13	2 974	-	-	3 000	-	-	-	-
14	-	-	-	-	3 000	3 000	-	-
15	-	-	-	3 000	-	-	-	3 000
16	-	-	-	3 000	-	3 000	-	3 000
17	-	3 000	3 000	3 000	-	-	-	-
18	-	-	-	-	-	-	3 000	-
19	0	0	0	3 000	3 000	0	0	0

Period Overlapping Setups:

Week	C	α	β	α*Time	β*Time
1	1	0,00	0,98	0,0	66,6
2	1	0,02	0,02	1,4	1,0
3	1	0,98	0,98	50,0	46,1
4	1	0,02	0,02	0,9	0,8
5	1	0,98	0,06	38,2	2,8
6	1	0,94	0,02	44,2	1,4
7	1	0,98	0,02	66,6	1,2
8	1	0,98	0,19	57,8	13,0
9	0	0,81	0,00	55,0	0,0
10	1	0,00	0,98	0,0	27,4
11	1	0,02	0,98	0,6	38,2
12	1	0,02	0,98	0,8	27,4
13	0	0,02	0,00	0,6	0,0
14	1	0,00	0,02	0,0	0,8
15	1	0,98	0,98	38,2	50,0
16	1	0,02	0,10	1,0	5,3
17	1	0,90	0,02	45,7	0,6
18	1	0,98	0,98	27,4	46,1
19	0	0,02	0,00	0,9	0,0

Time Used:

Week	
1	118
2	129
3	127
4	70
5	100
6	100
7	162
8	130
9	168
10	142
11	107
12	155
13	124
14	60
15	159
16	134
17	168
18	74
19	108

Appendix 4: Results for the final model:

Inventory levels:

Week/Terminal	T1	T2	T3	T4	T5	T6	T7	T8	Factory
1	1 832	1 733	1 850	87	1 353	3 709	1 118	3 977	1 000
2	1 734	1 559	1 741	1 572	1 238	2 903	428	3 300	5 000
3	1 548	1 323	1 580	3 271	3 896	2 188	3 011	3 131	2 984
4	4 361	1 149	1 461	1 027	3 288	3 168	2 137	2 650	4 501
5	4 233	3 829	1 297	2 106	2 927	2 089	4 555	2 060	2 501
6	3 974	3 532	4 297	81	2 720	598	3 773	4 344	3 451
7	3 598	2 993	4 110	428	2 274	1 914	3 182	3 569	5 000
8	3 439	2 313	3 921	1 858	4 971	3 549	2 744	2 911	3 063
9	3 099	1 243	3 478	3 380	4 390	5 000	5 000	2 323	1 013
10	2 984	3 587	3 316	4 261	3 337	3 231	4 440	1 391	2 013
11	2 690	2 661	2 819	5 000	2 137	1 331	3 740	3 490	3 013
12	2 322	1 504	2 507	2 765	4 015	2 717	3 092	2 589	3 963
13	1 954	3 347	2 195	3 530	2 893	4 103	2 444	1 688	1 963
14	1 660	2 421	1 945	4 342	4 955	2 812	1 956	967	2 963
15	4 259	1 221	1 758	1 407	4 055	4 521	1 014	3 067	913
16	3 876	3 316	1 450	1 797	3 373	3 477	467	2 468	1 913
17	3 625	2 723	4 248	87	2 926	2 793	108	5 000	2 989
18	3 480	2 380	4 131	2 037	2 526	2 397	2 900	4 773	3 999
19	3 335	2 037	4 014	4 047	2 126	5 000	2 692	4 546	5 000

Lot sizes (Y and Z):

Week/Terminal	T1	T2	T3	T4	T5	T6	T7	T8
1	-	-	-	-	-	3 000	-	3 000
2	-	-	-	3 000	-	-	-	-
3	-	-	-	3 000	3 000	-	2 966	-
4	3 000	-	-	-	-	2 483	-	-
5	-	3 000	-	3 000	-	-	3 000	-
6	-	-	3 000	-	-	-	-	3 000
7	-	-	-	2 451	-	3 000	-	-
8	-	-	-	2 937	3 000	3 000	-	-
9	-	-	-	3 000	-	3 000	3 000	-
10	-	3 000	-	3 000	-	-	-	-
11	-	-	-	3 000	-	-	-	3 000
12	-	-	-	-	3 000	3 000	-	-
13	-	3 000	-	3 000	-	3 000	-	-
14	-	-	-	3 000	3 000	-	-	-
15	3 000	-	-	-	-	3 000	-	3 000
16	-	3 000	-	3 000	-	-	-	-
17	-	-	3 000	-	-	-	-	2 924
18	-	-	-	2 940	-	-	3 000	-
19	-	-	-	3 000	-	2 999	-	-

Period Overlapping Setup:

Week	C	α	β	α*Time	β*Time
1	1	0,00	0,98	0,0	52,9
2	1	0,02	0,98	1,1	66,6
3	1	0,02	0,02	1,4	0,8
4	1	0,98	0,98	38,2	57,8
5	1	0,02	0,02	1,2	0,6
6	0	0,98	0,00	27,4	0,0
7	0	0,00	0,00	0,0	0,0
8	0	0,00	0,00	0,0	0,0
9	1	0,00	0,91	0,0	62,0
10	1	0,09	0,98	6,0	27,4
11	1	0,02	0,98	0,6	66,6
12	0	0,02	0,00	1,4	0,0
13	1	0,00	0,98	0,0	66,6
14	0	0,02	0,00	1,4	0,0
15	0	0,00	0,00	0,0	0,0
16	0	0,00	0,00	0,0	0,0
17	0	0,00	0,00	0,0	0,0
18	1	0,00	0,98	0,0	46,1
19	0	0,02	0,00	0,9	0,0

Time Used:

Week	
1	112
2	68
3	117
4	151
5	117
6	132
7	127
8	166
9	168
10	101
11	121
12	99
13	154
14	108
15	168
16	96
17	105
18	114
19	128