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Enhancing the 60/40 Portfolio: The Value of Commodity Futures

Master Thesis

by

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Supervisor:

Paolo Giordani

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Abstract

We modify the 60/40 portfolio to include commodity futures, utilizing 146 years of monthly data from 1877, seeking to improve the traditional 60/40 portfolio in the long run. Employing a full-scale optimization in a strategic asset allocation framework, we allocate 23.5% to commodity futures, contributing to 28.5% of the total portfolio variance, with the remainder allocated to the 60/40 portfolio. The portfolio shows strong in-sample outperformance compared to the 60/40 portfolio, relying on excess returns during high inflation and positive unexpected inflation. The portfolio is optimized to be more resilient, resulting in improvements in both risk-adjusted return and the conditional correlation profile relative to a mean-variance optimal portfolio (including 48% commodity futures, with a variance contribution of 71.1%). We study the conditional correlations between stocks and bonds, the 60/40 portfolio and commodity futures, specifically during inflationary regimes and in terms of downside and upside correlations. Our findings suggest that including commodity futures enhances the 60/40 portfolio. Our extensive 146-year analysis is unprecedented in similar studies, and to our knowledge, we are the first to investigate downside and upside correlations between commodity futures and a stock-bond portfolio.

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1 | Introduction

Diversification has been said to be the only free lunch available in finance. Equity is typically the preferred asset class for investment. By diversifying across various indices, such as the S&P 500, along with those from other countries and regions, one can reduce the volatility of the equity portfolio, leading to more stable returns. The benefit of diversification depends on the degree of cross-assets correlation. While it is common to assume a constant correlation between assets, this is not necessarily the case. As Chua et al. (2009) shows, the correlations between stocks, stock indices, and other asset classes are conditional. Specifically, they observe an increase in correlation to the downside, during times of turmoil, precisely when diversification is most needed. This became evident with the increased correlation between stocks and bonds in 2022, when the 60/40 portfolio endured one of its worst years in history, with both stocks and bonds experiencing significant losses. This motivates a study on whether long-term investors should consider a more diversified allocation than the traditional 60/40 portfolio and explore the potential inclusion of commodity futures as an alternative.

The increase in conditional correlation to the downside is not as significant between asset classes as it is within them (Chua et al. (2009)). Traditionally, the most common strategy for investing across asset classes is the 60/40 portfolio, which allocates 60% to stocks and 40% to bonds. Historically, these asset classes have exhibited low correlation and have not experienced the same degree of correlation increase during times of turmoil. However, despite more favorable conditional correlations to the downside, the portfolio still undergoes prolonged periods of significant drawdowns (Hurst et al. (2017)).

There has been some research conducted considering commodity futures, specifically focusing on their statistical properties compared to stocks and bonds, and assessing the potential benefits of including them in a strategic asset allocation framework. These studies have primarily focused on diversification, with little consideration given to conditional correlations between asset classes. In general, most research concerning conditional correlation centers around models like GARCH or other modified versions, which aim to forecast conditional volatility, covariances, and correlations to engage in market timing and portfolio adjustments. However, we will not engage in any market timing or base our portfolio on forecasts of volatility, covariances, correlations, or other predictors. Instead, we focus on a strategic asset allocation approach, with the argument that long-term portfolios should be modeled similarly to airplanes, designed to withstand turbulence whenever it arises, as it is typically unpredictable. Strategic investors, such as pension plans, share a similarity with airline pilots in that their objective is not to predict the unpredictable, but rather to ensure their portfolios can weather market storms (Chua et al. (2009)).

In our research, we aim to explore the potential diversification benefits of incorporating commodity futures into a traditional 60/40 equity and bond portfolio. We also plan to assess the potential risks and drawbacks associated with this strategy, specifically by examining the conditional correlations between the asset classes and their respective properties. Notably, we study inflation regimes and their impacts on the 60/40 portfolio, and analyze how different inflationary environments influence this portfolio allocation strategy. Additionally, we investigate the potential utility of commodity futures as a tool to offset challenges posed by inflationary periods. Our overarching goal is to provide a comprehensive overview and contribute to a broader understanding of the potential benefits of including commodity futures in a 60/40 portfolio.

Our results are based on monthly returns from stocks, bonds, and commodity futures indices, extending back to 1877. The study convincingly illustrates the benefits of incorporating commodity futures into the 60/40 portfolio. We initiate our investigation by examining the relationship between income and price returns and their impact on total returns for commodity futures. This is motivated by conflicting findings in the existing literature and aims to clarify the drivers of commodity future returns and how they differ from spot returns. In essence, we find evidence to suggest that commodity futures returns are driven more by changes in commodity prices rather than merely interest rate-adjusted carry. When we extend the time horizon to five years or longer, both income and price returns explain an equal proportion of the variance in commodity futures returns. Further, we document the advantages of a commodity futures exposure to the 60/40 portfolio, and in what states, especially regarding inflation change and unexpected inflation shocks, this inclusion could prove particularly beneficial.

One problematic assumption in traditional portfolio construction is the notion of correlation being constant over time. This assumption could lead to an overestimation of diversification benefits, as evidenced by the performance of the 60/40 portfolio in 2022. After assessing the conditional correlations between the assets, we employ full-scale optimization (also known as "direct utility maximization"), motivated by Chua et al. (2009), to mitigate potential negative consequences associated with conditional correlations, as compared to mean-variance optimization. This results in an optimal allocation of 23.5% to commodity futures, with the remaining allocated to the 60/40 portfolio. This allocation to commodity futures contributes to 28.6% of the total portfolio variance. In contrast, the optimal mean-variance portfolio would allocate 48% to commodity futures, resulting in a variance contribution of 71.1% of the total portfolio variance.

Not surprisingly, compared to the 60/40 portfolio, the full-scale optimization on our in-sample data reduces the volatility while increasing both the mean return and the Sharpe ratio. As our focus is not on prediction or tactical allocation, we conduct our optimization and testing on the full sample to leverage our extensive data set. Our study stands out in its use of a lengthy 146-year period, encompassing numerous market cycles and regimes. This contributes to a more robust long-term portfolio allocation, offering an advantage over studies typically confined to 40 years of data and possibly only a single market cycle or regime.

This thesis is organized as follows: In section 2, we provide an overview of the related existing literature. In section 3, we explain the data behind our analysis. Section 4 constitutes the main portion of our research, which is conducted in four parts; 1) We examine the drivers of commodity futures returns to gain a better understanding of this asset class. 2) We compare the properties and empirical performance of the relevant asset classes; stocks, bonds, and commodity futures. 3) We analyze the assets' conditional correlations. In addition, we explore the potential enhancement of the 60/40 portfolio by including commodity futures, particularly with respect to different inflation regimes and conditional correlations. Furthermore, we strive to improve the asymmetry of the conditional correlation in our portfolio, in comparison to the mean-variance optimal portfolio, using full-scale optimization. 4) We document the results, comparing the 60/40 portfolio with our proposed portfolio. Lastly, we conclude and provide our references.

2 | Literature Review

For decades, published research has investigated the properties of commodity futures, contemplating their inclusion into traditional portfolios of equities and bonds. Most research underscores the benefit of diversification, attributing it to the low correlation between returns from commodity futures and those from equities and bonds. However, there is variation in the research approaches, leading to different explanations and recommendations concerning how, when, and why commodity futures should be incorporated. Furthermore, views on the properties of commodity futures as an asset class vary.

2.1 Commodity Futures

Commodity futures do not offer direct exposure to physical commodities, but they are derivatives with maturity claims on real assets or potential cash settlements (Gorton & Rouwenhorst (2006)). "A commodity futures contract is an agreement to buy (or sell) a specified quantity of a commodity at a future date, at a price agreed upon when entering into the contract – the future price" (Gorton & Rouwenhorst (2006)). Since there is no initial payment, the contract's value is zero at inception. Returns on the contract would be excess returns, where the cash collateral is most commonly assumed to be invested in T-bills. This provides the risk-free interest rate if the position is fully collateralized (Erb & Harvey (2016) Jensen et al. (2000)).

Just like other futures contracts, commodity futures require rolling over their contracts to extend their maturity to avoid taking physical delivery. Erb & Harvey (2016) decompose commodity futures returns into price return, roll return, and collateral return. The collateral return represents the return from holding the collateral in T-bills, as previously mentioned. Therefore, by excluding the collateral return, the remaining return, comprising the roll return and price return, constitutes the excess return. Price return refers to the change in the price of the underlying commodity of a futures contract, while roll return, also known as the carry, is associated with the gain or loss incurred when rolling the position further out on the curve. Levine et al. (2018) decompose the carry return into the convenience yield, net of storage costs, minus the risk-free rate, referred to as the interest rate-adjusted carry (ψ). This corresponds to what Erb & Harvey (2016) refer to as the "income return"; the roll return plus the collateral return (risk-free rate).

It is important to note that the two papers differ as Levine et al. (2018) uses excess spot returns and incorporates the cross-term between the income and price returns, as illustrated in Equation 4.

Futures contracts are said to be in contango if contracts further from maturity are priced higher than those closer to maturity, resulting in a negative carry. Conversely, if futures contracts with longer maturities are priced lower, this condition is referred to as backwardation, yielding a positive carry (Erb & Harvey (2016)). Futures prices are shaped by expectations regarding the future spot price, the risk-free rate, and potential risk premiums. According to the theory of normal backwardation proposed by Keynes et al. (1930) and Hicks (1939), the risk premium would, on average, accrue to the buyers of futures. This occurs predominantly because the average demand for hedging unexpected price fluctuations is higher from producers than it is from consumers. This imbalance prompts speculators to assume the opposite position, thereby demanding a risk premium (Gorton & Rouwenhorst (2006)). Both Levine et al. (2018) and Gorton & Rouwenhorst (2006) show that commodity futures returns have outperformed commodity spot returns, even without considering costs related to storage. The former study, with data from 1877 to 2015, shows that uncollateralized commodity futures had an arithmetic mean return of 4.6%, significantly more than the excess spot arithmetic mean return of 2.0%."

Erb & Harvey (2016) and Gorton & Rouwenhorst (2006) analyzed the correlations between the returns from an equal-weighted commodity spot index (price return) and an equal-weighted fully collateralized commodity futures index (total return). The latter found that commodity futures highly correlate with movements in the spot, as commodity futures benefit from unexpected increases in spot prices. The correlation was especially high during times of high volatility. Erb & Harvey (2016), on the other hand, conducted a regression on the total returns with the spot return as an explanatory variable, using rolling 10-year overlapping monthly observations and with monthly rebalancing. Contrary to the expectations based on the positive correlations found by Gorton & Rouwenhorst (2006), their results do not indicate any positive relationship between spot returns and commodity futures returns, as evidenced by an R-squared of 0.004. Instead, they found that income return has a positive coefficient of 0.51 and an R-squared of 0.54. "For an investor who believes there should be a powerful relationship between the price return of a commodity futures portfolio and its total return, this result may seem unwelcome and preposterous" Erb & Harvey (2016). Nonetheless, they noted vague clusters of returns, suggesting the existence of periods with distinct correlation between total returns and price returns. The contrasting findings between the two studies could be attributed to the difference in the time scales examined: the first study focused on monthly intervals, while the second analyzed 10-year intervals. This observation suggests that the variations in the relationship between spot commodities returns and commodity futures returns might stem from the differences in the time intervals considered.

2.2 Expected and Unexpected Inflation Regimes

Expected inflation is reflected in asset prices, therefore, investors seek to hedge unexpected inflation. Generally, commodities are inflationary, as they often serve as essential input factors for goods included in the CPI. Nevertheless, there is considerable variation within the commodity complex (Neville et al. (2021)). Most research, such as those conducted by Neville et al. (2021) and Gorton & Rouwenhorst (2006), has found a positive correlation between inflation and real commodities, with Erb & Harvey (2016) being an exception. If we assume commodities are inflationary and that there is a relationship between expected inflation and the anticipated price change in real commodities, then in this context, let's say we anticipate a specific increase in real commodities, and this expectation proves to be accurate. How would this anticipated change impact commodity futures? While commodity futures, reflecting the underlying commodity, would be expected to appreciate, they also include carry, which mirrors the expectation. Therefore, commodity futures would not increase in line with real commodities, since the carry perfectly offsets the expected rise. Consequently, the expected spot return does not contribute to futures returns. Instead, futures rise and fall with unexpected deviations from the predicted price and premiums (Gorton & Rouwenhorst (2006)).

A correlation between unexpected inflation and commodity futures is intuitive since unexpected deviations in real commodities are inflationary. This inference is supported by findings from Levine et al. (2018) and Erb & Harvey (2016).Despite this, Gorton & Rouwenhorst (2006) finds that commodity futures and inflation share no correlation at a monthly interval, yet the correlation becomes statistically significant at a one-year interval, with a correlation value of 0.29, which further increases to 0.45 at a five-year interval. Most of the research literature agrees that commodity futures serve as an effective hedge against inflation, particularly against unexpected inflation shocks. Furthermore, Levine et al. (2018) notes that arguments have been made for a risk premium associated with inflation risk in commodity futures. The potential inflation risk premium could explain why commodity futures, despite exhibiting similar volatility, have demonstrated lower expected returns compared to stocks. The negative premium might be interpreted as compensation for providing an inflation hedge.

2.3 The 60/40 Portfolio and Commodity Futures

Many research papers have been conducted to look into the opportunities of including commodity futures in portfolios consisting mainly of equities and bonds, with the 60/40 portfolio being particularly prevalent. The common argument is additional diversification benefits, which is the main reason many investors choose a 60/40 portfolio of equities and bonds. Thapar & Maloney (2021) highlights that the correlation with respect to growth sensitivity is inverse for bonds and equities, leading to the combination of these two assets being highly advantageous during both economic booms and recessions, which has made them widely popular and acknowledged.

Unfortunately, both equities and bonds are negatively affected by inflation, particularly by changes and shocks in inflation (Conover et al. (2010a) Neville et al. (2021)). Similar to their approach with growth sensitivity, Thapar & Maloney (2021) also examined the partial correlation to inflation sensitivities for equities and bonds, resulting in correlations of -0.25 and -0.45, respectively. This supports the assertion that inflation, specifically changes in inflation, negatively impacts these asset classes. During periods of high inflation, especially when inflation increases rapidly, this jointly negative correlation might impact the entire portfolio. As shown by Thapar & Maloney (2021), the correlation between stocks and bonds rises to 0.25 during inflation-dominated periods, thereby reducing the diversification benefits.

Different strategies have been proposed as potential hedging mechanisms against inflation. According to Thapar & Maloney (2021), trend following and macro momentum have performed well in periods of both negative and positive inflation shocks. Additional studies, such as Cheung & Miu (2010) and Conover et al. (2010a), have examined the utilization of monetary policy, interest rates, and inflation as indicators for tactical allocation strategies. These studies employ techniques such as dynamical conditional correlation, GARCH models, and other suitable methodologies to optimize tactical allocation, often incorporating commodity futures. Even though some of these approaches have demonstrated potential effectiveness in addressing inflation-related challenges, our goal is not to rely on active trading and tactical allocation strategies.

When considering a fully invested fixed-weight portfolio comprising 60/40 and commodity futures without any tactical allocation, previous research has suggested an allocation towards commodity futures in the range of 5% to 25%, depending on an investor's risk aversion, preferences, and familiarity with commodity futures (Erb & Harvey (2016) and Levine et al. (2018)). The optimal allocation to commodity futures in a risk parity portfolio suggests around 18% allocated in commodity futures (Bhardwaj & Janardanan (2014)). Further, Jensen et al. (2000) states the following: " In the overall study period, futures were shown to be a relatively poor stand-alone investment since they have both lower returns and higher standard deviation than stocks. In a portfolio context, however, return/risk optimization (over a range of risk levels) gave substantial weight to commodity futures over the full sample. Such allocation significantly enhanced the portfolios' returns. This evidence supports the use of futures in traditional portfolios over the last 25 years." This is further supported by Conover et al. (2010b) stating; "Since increasing commodity prices are typically one element of heightened inflation and higher interest rates, both of which tend to negatively affect equities, long positions in commodity futures are found to provide an inflation hedge for equity portfolios." That said, Thapar & Maloney (2021) remarks that commodity futures, while providing a good hedge against inflation, also suffer during periods of decreasing inflation and lower inflation than expected.

Commodity futures exhibit an intriguing statistical property whereby they tend to generate positive returns during periods of high volatility, and their skewness is generally thought of as positive. In addition, Cheung & Miu (2010) demonstrates that; "The low (high) return environment for commodity futures is associated with low (high) volatility. This positive risk-return relationship is in sharp contrast to the negative risk-return relationship for international equities. Our regime-switching analysis also reveals an important feature of the diversification benefits of commodity futures." Moreover, research by Chong & Miffre (2009) shows that correlations between the S&P500 and 11 commodities also fell in periods of above-average volatility in equity markets. This is encouraging for long-term institutional investors, who stand to benefit most from diversification during periods of high equity market volatility. Similarly, the findings suggest that incorporating commodity futures to Treasury bill portfolios can further mitigate risk in volatile interest rate environments.

However, Cheung & Miu (2010) states: "Our regime-switching analysis further reveals the manner in which commodity futures contribute to the risk-adjusted return of a portfolio. The long-run diversification benefits can be attributed to the infrequent episodes of upswings in the commodity markets. The diversification benefits are unimpressive otherwise. Unfortunately, upswings in commodity futures happen rather infrequently. Further, the common impression that the low or negative static correlations between commodities and other financial assets make commodities an ideal asset class to smooth out the bearish equity market is clearly refuted. The strength of commodities lies in their ability to enhance performance during the infrequent outbursts."

2.4 Conditional Correlations' Effect on Diversification

Diversification becomes crucial for risk mitigation during periods of market turmoil or uncertainty. As underscored by Campbell et al. (2008), it is precisely during these periods that diversification proves most necessary. Consequently, investors should be deeply concerned by any breakdown in the correlation structure. Furthermore, Chua et al. (2009) points out that exactly when they are most needed, increased downside correlations can erode the benefits of diversification. The authors demonstrate that U.S. equities, when compared to World Equities (excluding the U.S.), exhibit markedly higher downside correlation, and that correlations between different asset classes tend to display better conditional correlation profiles. This refers to the conditional correlations of both assets being jointly positive or negative theta (θ) standard deviations, as shown in Equation 4.1.

Cheung & Miu (2010) highlights that commodity futures could exhibit better performance during periods of turmoil, compared to other asset classes. This observation gains further support from Chong & Miffre (2009), who affirm: "We also observe that for more than half of our cross-section, the conditional correlations between commodity futures and global equity returns fell in periods of market turbulence. Indeed, it is precisely when stock market volatility is high that the benefits of diversification are most appreciated. This is particularly true of precious metals, which emerge as excellent risk diversifiers in periods of high volatility in equity markets, irrespectively of the data frequency."

To account for conditional correlations when constructing a portfolio, many studies utilize GARCH-models to estimate conditional volatility and correlations. For instance, Cappiello et al. (2006) employed GARCH to estimate changing correlations within equity and bond indices. However, for long-term funds, such as pension funds, mean-variance optimization has traditionally been used for portfolio construction. One of the several issues with mean-variance optimization is the assumption of constant correlations. Chua et al. (2009) points out that empirically, most financial assets demonstrate significant correlation asymmetry. This could potentially lead to flawed portfolio constructions when assuming these correlations to be constant, when in fact the correlation is higher to the downside than the upside. Therefore, correlation asymmetry presents a significant challenge in portfolio management, particularly since correlations often increase during market downturns. Paradoxically, while a high correlation during market uptrends can limit the portfolio's upside potential, an increased downside correlation during downturns can erode the diversification benefits, a scenario investors particularly depend on during adverse market conditions. Consequently, investors may face a situation where risk heightens while the potential for upside growth is simultaneously constrained (Campbell et al. (2008))

2.5 Full-Scale Optimization

When constructing a portfolio and determining fixed weights for asset allocation, it is crucial to acknowledge inherent limitations. A significant one is the inability to incorporate tactical allocation, which permits dynamic adjustments of weights in response to evolving market conditions. Factors such as economic growth, inflation, and monetary policy contribute to time-varying correlations among assets, leading to a constantly changing covariance matrix. Unfortunately, these fluctuations present a challenge when employing optimization techniques like mean-variance.

According to Adler & Kritzman (2007), mean-variance optimization assumes a normal distribution, relying solely on its expected mean and covariance matrix, or quadratic utility for investors. However, empirical evidence shows that correlations often exhibit significant asymmetry in practice, with downside correlations typically surpassing upside correlations. This asymmetry in correlation structure significantly influences the diversification benefits attained within a portfolio (Chua et al. (2009)).

To optimize a portfolio while considering conditional correlations, Chua et al. (2009) proposes an approach they refer to as 'full-scale optimization,' as originally suggested by Cremers et al. (2005) and Adler & Kritzman (2007). Full-scale optimization does not seek to exploit correlation asymmetry directly. Instead, it optimizes expected utility using a kinked log utility function, with the location of the kink and the extent of penalization determined based on investor preferences. This method implicitly takes into account all characteristics of the empirical sample, including potential skewness, kurtosis, and other unique distribution attributes such as correlation asymmetries (Chua et al. (2009)). Comparing mean-variance and full-scale optimization, Adler & Kritzman (2007) states: "Mean-variance analysis requires investors to estimate the means and variances of all assets and the covariances of all asset pairs. To the extent the out-of-sample experience of these parameters departs from the in-sample parameter values, the mean-variance approximation will be even less accurate. Full-scale optimization requires investors to estimate the entire multivariate return distribution. To the extent it varies from the in-sample distribution, full-scale optimization will also yield sub-optimal results out of sample." Chua et al. (2009) finds that portfolios optimized using the full-scale method demonstrate improved correlation asymmetry compared to mean-variance portfolios, with lower downside correlation and higher upside correlation. It is worth noting that full-scale optimization does not provide an analytical solution and requires brute-force optimization.

3 Data

We have analyzed monthly returns from 1877 to 2023 for three asset classes: commodity futures, equities, and bonds. Few, if any, studies have previously examined returns in different market regimes across all three asset classes over such an extensive period of 146 years. By including this substantial time span, we are able to generate more robust statistical measures across various market cycles and regimes.

3.1 Data Collection

One of the datasets utilized in this study originates from the work on commodities by Levine et al. (2018). Their dataset comprises monthly observations and incorporates additional variables that indicate whether the aggregate slope of the futures curve is in backwardation or contango ex-ante, as well as any unexpected inflation (change) in either direction within a given month. The commodity futures returns are derived from an equally weighted portfolio, meant to capture the average returns across commodities at any given period. The historical data spanning from 1877 to 1951 were transcribed from the annual report of the Trade and Commerce of the CBOT. Futures prices from various contracts provided by Commodities Systems Inc. were used between 1951 and 2012. Post-2012 prices were obtained from Bloomberg, as cited by Levine et al. (2018)

The annualized return on each commodity future is constructed by the following procedure done by Levine et al. (2018); "For each month-end, we calculated the return on each contract from the previous month-end. "Note that there are days with limit moves on various grains contracts, and we assumed that all contiguous limit moves were incorporated into the first move price. In addition, opening and closing prices were not recorded in the early part of the sample, so high and low values were used in the analysis—in particular, the average of these high and low values for the daily return series before closing prices were available". For each month, we held the nearest of the contracts whose delivery month was at least two months away. For example, we held an April contract through the end of February. An exception must be made for Brent crude oil, whose delivery month needs to be at least three months away, so we held the April contract for Brent crude oil through the end of January. The methodology for Brent crude oil was chosen to coincide with the procedure used for the GSCI. The returns on the held contracts were spliced together on the roll dates. Using the same rolled contract series, we constructed a rolled price series and calculated the spot returns." Levine et al. (2018) further states: "The complete list of commodities used in this study includes futures available prior to 1960 (the start of most academic studies) and those that are currently included in the S&P Goldman Sachs Commodity Index (GSCI) as a measure of liquidity. In alphabetical order, these futures are aluminum, Brent crude oil, cattle, cocoa, coffee, copper, corn, cotton, feeder cattle, gas oil, gold, heating oil, hogs, Kansas wheat, lard, lead, natural gas, nickel, oats, pork, short ribs, silver, soybeans, soybean meal, soybean oil, sugar, gasoline, wheat, WTI (West Texas intermediate) crude oil, and zinc."

Concerning questions about varying liquidity levels in different periods for the index, Levine et al. (2018) finds little evidence to suggest that the commodity futures we examine were significantly less liquid in the earlier period compared to the later one.

We derive the risk-free rate from the commodity futures dataset by subtracting the excess spot return from the spot return. Levine et al. (2018) describe their risk-free rate proxy as follows: "The US government started issuing short-term bills only after 1929. We took the short rate series prior to 1918 from Federal Reserve Economic Data provided by the St. Louis Federal Reserve Bank. The data represent New York call money rates until 1889 and the New York Times money rates until 1918. For the period after 1918, we used Global Financial Data; the data represent secondary market rates on the shortest-term US bonds available until 1931 and T-bills thereafter. A rolling one-year average of the short-term rate was used."

In order to study not just the unexpected direction of inflation provided in the Levine et al. (2018) dataset, we supplemented it with monthly data for US inflation from Global Financial Data (GFD). We used the "United States BLS Consumer Price Index Inflation Rate NSA (with GFD Extension)," a dollar-denominated US CPI index inflation rate constructed from various sources (for more details see description from GFD).

The bond index returns are derived from a dollar-denominated World Government Bond GDP-weighted return index provided by GFD. This index is constructed from various sources over time (see GFD for a detailed description). Note the significant outlier in September 1949, where the index dropped by over 20%. This dramatic decline was primarily due to a substantial currency readjustment in currencies in September 1949. The devaluation of many European currencies significantly contributed to this steep drop in the index.

The equity data is constructed in two parts. Firstly, for the period prior to 1926, we use the S&P 500 Total Return Index from GFD, which is based on the Cowles Commission's work (Common Stock Indexes, 1871-1937, Alfred Cowles) and the S&P composite index. Secondly, for the period post-1925, we employ GFD's Developed World Total Return Index. It's important to note that due to the lack of world-index data before 1926, we use US data as a proxy for world equity returns. All returns are total returns, incorporating both capital gains and reinvestments of any cash distributions, such as dividends. Interestingly, the equity index is occasionally calculated using different exchange rates than the bond index. For instance, during the Bretton Woods era (1946-1970), when exchange rates were fixed, GFD utilized Pick's black market exchange rates, as they were deemed to better represent the market value of foreign stock markets to a U.S investor than the official exchange rate (see GFD for a more detailed description). This might explain why we do not observe a similar outlier in the Equity Index in September 1949, as we did in the World Bond Index.

3.2 Variables Description

We focus on two state variables from Levine et al. (2018). The first variable is unexpected inflation (up or down), which is determined by the change in one-year inflation. This calculation is based on the inflation data used by Levine et al. (2018), which they collected as follows: "After 1913, the monthly inflation rate was calculated from the US Consumer Price Index published by the US Bureau of Labor Statistics. Prior to 1913, we used data from Shiller (2000), who used the Warren and Pearson (1935) price index."

The second variable is aggregate backwardation, which is defined as:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{F_{i,t,T1}-F_{i,t,T2}}{(T2-T1)F_{i,t,T1}}>0, \tag{3.1}$$

where $F_{\mathrm{i},t,TK}$ represents the price of a futures contract at time t for a contract expiring at time TK for commodity i, and $T2 > T1$. The value F is defined by Equation 1. This average level of backwardation theoretically reflects the level of inventories and hedging demand across commodities (Levine et al. (2018)).

3.3 Commodity Futures Breakdown

Levine et al. (2018) provides a breakdown of commodity futures, which is detailed below. Additionally, they discuss the differences between their breakdown and that given by Erb & Harvey (2006), supplementing what was presented in our section 2.1 Commodity Futures, in the literature review:

"Assuming, for simplicity, that the interest rate and net convenience yield are constant, the starting point for models of commodity futures prices is the cost-of-carry model:

$$F_{t,T} = S_t e^{(r-\psi)(T-t)} \tag{1}$$

where

$$\begin{aligned} S_t &= \text{the time } t \text{ spot price of the commodity} \\ F_{t,T} &= \text{the futures price with maturity } T \\ r \equiv \ln(1+R) &= \text{the continuously compounded riskless rate} \\ \psi \equiv \ln(1+\psi) &= \text{the convenience yield, net of storage costs} \end{aligned}$$

Defining $r^{F_T}_{t,t+1} \equiv \ln\left(1+R^{F_T}_{t,t+1}\right)$ and $r^{S}_{t,t+1} \equiv \ln\left(1+R^{S}_{t,t+1}\right)$ as, respectively, the continuously compounded return on a futures contract with maturity T and the commodity spot return produces;

$$\begin{aligned} r^{F_T}_{t,t+1} &\equiv \ln\left(\frac{F_{t+1,T}}{F_{t,T}}\right) \\ &= \underbrace{\ln\left(\frac{S_{t+1}}{S_t}\right)}_{\text{Spot return, } r^{S}_{t,t+1}} + \underbrace{(\psi - r)}_{\text{Carry}} \end{aligned} \tag{2}$$

or

$$r^{F_T}_{t,t+1} = \underbrace{\ln\left(\frac{S_{t+1}}{S_t}\right) - r}_{\text{Excess of cash spot return, } r^{eS}_{t,t+1}} + \underbrace{\psi}_{\text{Interest rate-adjusted carry}} \tag{3}$$

Equations 2 and 3 show that futures returns can be decomposed either as the sum of spot returns and carry or as the sum of the excess of cash spot returns and interest rate–adjusted carry. ... In particular, Equation 3 implies two sources of futures return premiums: excess spot return premiums, $E\left[r_{t,t+1}^{eS}\right]$, and interest rate-adjusted carry, ψ. For the interest rate-adjusted carry, the literature points to compensation for bearing inventory risk and/ or providing liquidity to hedgers, especially during periods of low inventories. For spot returns, we focused on one potential risk premium, namely commodities' hedge against inflation and exposure to the business cycle.

Erb & Harvey (2016), who labeled ψ the "income return" of the futures contract, broke down the total return on futures, $r_{t,t+1}^{F_T} + r$, into the spot return and interest rate-adjusted carry. Our decomposition differs from that of EH because we are focusing on futures returns, which are in excess of cash. ... Because a weighted sum of individual commodity returns does not aggregate in log terms (i.e., the sum of geometric returns of the individual assets does not itself equal the return on the aggregate portfolio), a convenient step is to write the equivalent of the log futures return in Equation 3 in terms of simple returns, which do aggregate well into portfolios. For a given commodity i, Equation 3 can be rewritten to incorporate simple returns as

$$R_{i,t,t+1}^{F_T} = R_{i,t,t+1}^{eS} + \psi_{i,t,t+1} + R_{i,t,t+1}^{eS}\psi_{i,t,t+1} \tag{4}$$

where $R_{i,t,t+1}^{eS} = \left(1 + R_{i,t,t+1}^{S}/1 + R_{t,t+1}\right) - 1$ is the excess of the cash spot return. From Equation 4, we see that the futures return is equal to the excess spot return plus the interest rate-adjusted carry, but this decomposition also includes a cross-term when simple returns are used. Given the well-documented negative relationship between commodity spot returns and carry, the cross-term is likely to be negative. So commodity futures returns may be overestimated by simply considering the sum of excess spot returns and interest rate-adjusted carry."

4 | Results and analysis

4.1 Commodity Futures Return Drivers

Before comparing commodity futures to bonds, equities, and the 60/40 portfolio, we aim to clarify the underlying drivers of commodity futures returns. As explained in the literature review, commodity returns can be decomposed into the sum of income returns and price returns. Figure 4.1 presents the annualized returns over a 10-year period with monthly rebalancing, providing insights into the trends observed throughout the entire sample period. Returns appear to be quite persistent, following extended periods of both strong and weak performance. The income returns seem to be more persistent than price returns, and there are instances when income returns improve as price returns decrease, and vice versa.

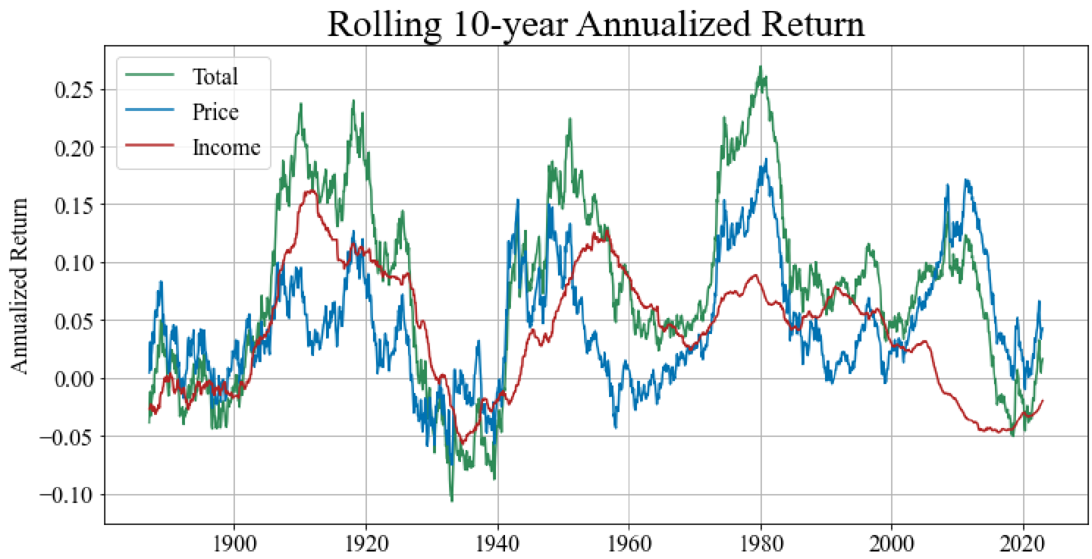


Figure 4.1: *The graph represents total return, which is the annualized commodity futures returns, fully collateralized in T-bills; price return, which represents the spot return; and income return, which refers to the interest rate-adjusted carry return. The graph is based on monthly rebalancing.*

To gain deeper insights into the underlying return drivers, we conduct separate ordinary least squares (OLS) regressions with income and price as covariates. The results, depicted in Figure 4.2, include analyses of both monthly returns and 10-year overlapping returns with monthly rebalancing. When examining the monthly income regression, we observe a high degree of noise in the data, with income exhibiting negligible explanatory power and a mere correlation of 0.04. In contrast, the monthly price regression reveals a robust relationship, with a coefficient close to 1 and an R-squared value of 0.919. This indicates that approximately 91.9% of the variance in returns can be explained by changes in the underlying spot market. Moreover, we find the correlation between price and returns to be 0.96. These findings suggest that while commodity futures encompass both the cumulative sum and the interaction between income and price returns, the majority of the monthly return variance can be attributed to fluctuations in the underlying spot market.

Furthermore, we perform the same regression using 10-year annualized overlapping returns with monthly rebalancing. A clear relationship emerges between the variables in the two respective regressions, both exhibiting similar explanatory power and correlations of 0.75. All coefficients are strongly significant, except for income in monthly returns, which is not statistically significant at the 5% level due to a t-statistic of 1.86.

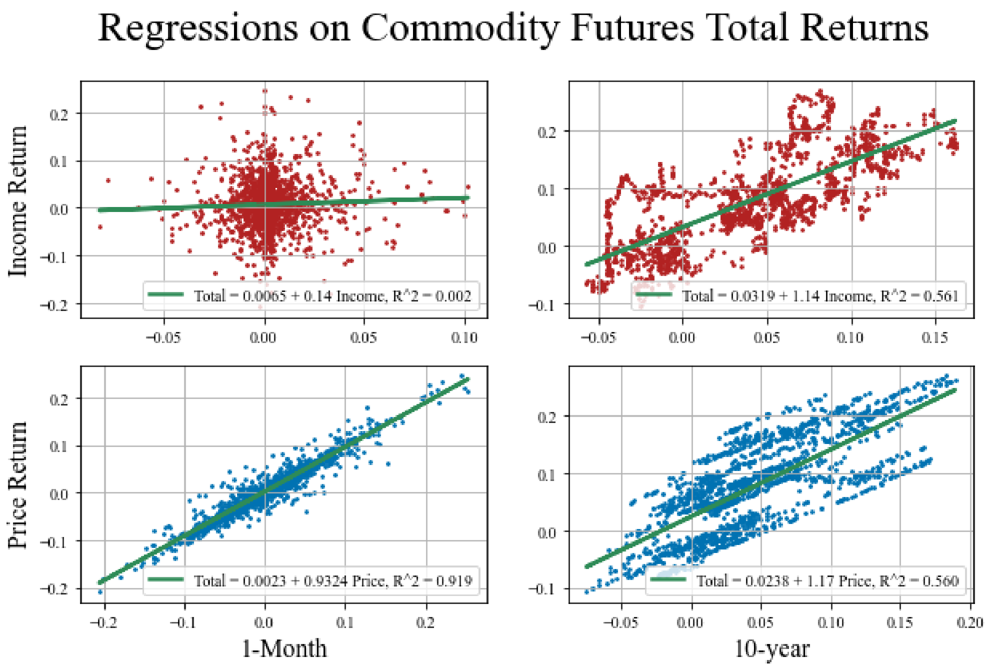


Figure 4.2: *Regressions on monthly returns (left) and 10 years overlapping annualized returns (right). Income return vs. total return (red) and price return vs. total return (blue).*

As shown above, the coefficient changes substantially for income returns when moving from a monthly interval to a 10-year interval. We present the evolution of the coefficients in Figure 4.3, with corresponding t-statistics in parentheses, as we increase the length of the interval. Notably, the coefficient for income returns grows substantially with the length of the interval, indicating a stronger and more pronounced relationship between income and total returns. The price coefficients are considerably more consistent, only increasing slightly when the interval extends beyond 5 years, where both coefficients exceed 1. Coefficients above 1 for both income and price returns are most likely attributable to the interaction term. The relationship between price return and income return is in itself interesting. This relationship can shed light on the link between the expectation of price change and the actual price change (see appendix. A.2).

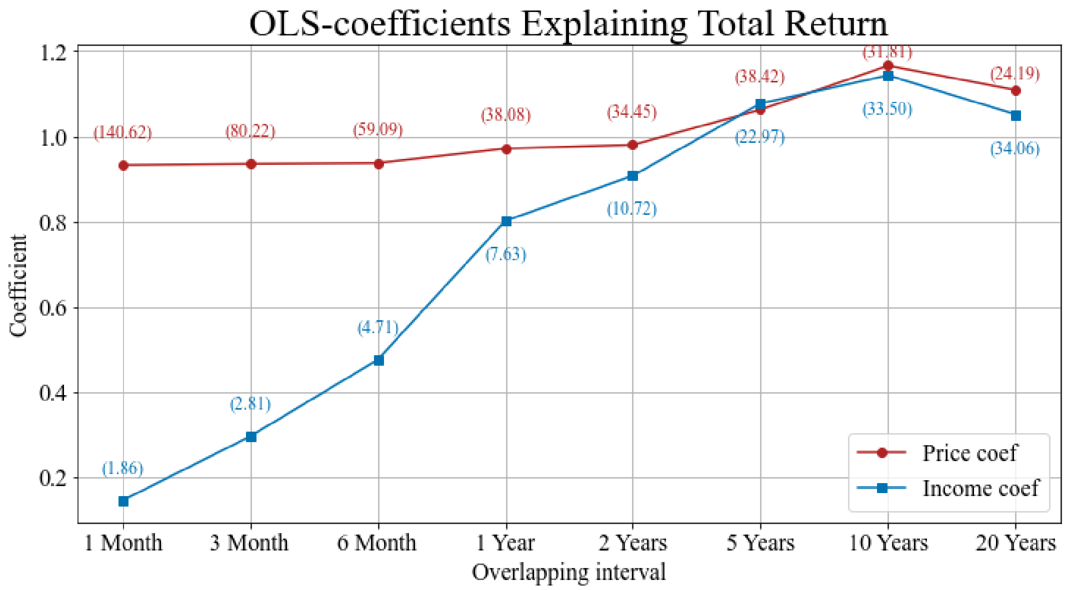
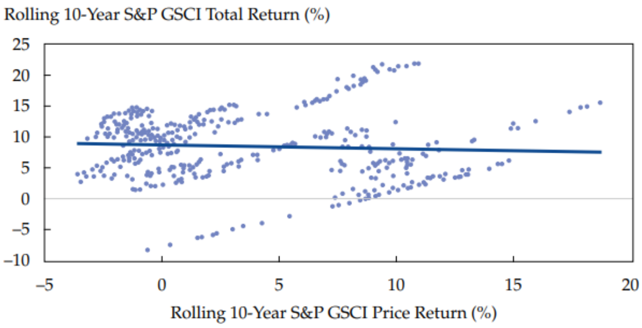


Figure 4.3: *Coefficients of price return and income return in 1-variable OLS-regressions on total return over different overlapping intervals. T-values are provided in parentheses. Newey-West robust standard errors are used for intervals over 1 month, to adjust for overlapping.*

Our results for monthly returns support the statement by Levine et al. (2018) that, "The higher volatility of the spot return suggests that commodity futures returns are fundamentally about commodity price changes, not simply carry." Yet, our findings also show that the explanatory power of income return (interest rate-adjusted carry) increases as we lengthen the interval. Both income return and price return explain equally as much of the variance as we approach intervals of 5 years and more. This supports the assertion by Bhardwaj et al. (2015) that, "Futures prices can increase during a period when spot prices decline if the spot price declines are anticipated. Unexpected fluctuations in spot prices affect spot and futures prices alike. Spot prices are different from futures prices for these reasons."

Contrary to Erb & Harvey (2016), which reported a coefficient of -0.06 and an R-squared of 0.004, Figure 4.2 demonstrates a potent relationship between price return and total return over a 10-year interval. Given the fundamental difference in these results, we carry out a regression over the same time interval as Erb & Harvey (2016) in an attempt to replicate their results. Our replicated regression, along with the original one, is displayed in Figure 4.4. Despite our efforts, their findings remain unreplicated; even though the coefficient is somewhat lower compared to the full sample regression, it retains high significance in terms of t-statistics and an R-squared of 0.488. Thus, we observe a significant relationship between price return and total return and suggest that investment in commodity futures provides exposure to commodities, contrary to the implications of the regressions by Erb & Harvey (2016). Nevertheless, as the statement from Bhardwaj et al. (2015) above clarifies, commodity futures are not synonymous with physical commodities, and their returns can diverge. Nevertheless, this relationship is attributed to the unanticipated fluctuations in spot prices, which affect both.



(a) *Total return = 0.09 – 0.06 × Price return, R-squared = 0.004*

(b) *Total return = 0.023 + 0.838 Price return, R-squared = 0.488*

Figure 4.4: *Erb & Harvey (2016) regression a) versus our attempted replication b), on the same sample size, frequency, and overlapping interval.*

4.2 Commodity Futures, Stocks and Bonds in Different Regimes

After the clarification of the roles of income and price return in driving commodity futures returns, we now turn our attention to comparing these returns with those of our other two primary asset classes: stocks and bonds. Our aim is to explore potential enhancements to the standard 60/40 portfolio by considering including commodity futures within a strategic asset allocation framework.

As depicted in Figure 4.5, stocks (red) have accumulated the highest wealth among the individual asset classes within our studied time interval. Although they have demonstrated significantly higher volatility, stocks have outperformed the 60/40 portfolio in terms of final wealth. Notably, bonds lagged behind the risk-free interest rate until approximately 1985, which also signified the peak level of interest rates. Since that point, bonds have shown strong performance relative to their relatively low volatility. Commodity futures (green) should be given attention as they have outperformed during some periods when the 60/40 portfolio (black) has performed poorly. While the performance of commodity futures has varied considerably over time, the final accumulated wealth aligns closely with that of the 60/40 portfolio. Given these observations, it may prove advantageous to consider the incorporation of commodity futures into a 60/40 portfolio, as potential diversification benefits could enhance final wealth and mitigate portfolio volatility.

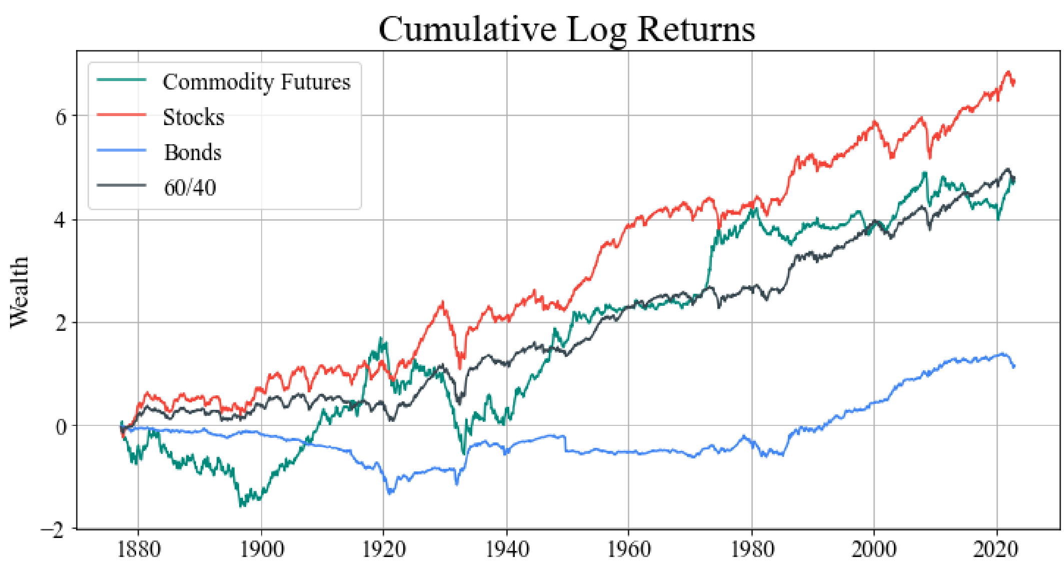


Figure 4.5: *Accumulated monthly log excess returns, rebalanced monthly. Commodity futures is an equal-weighted index. Stocks and bonds are world indices (before 1925, we used US equity data as a world-data proxy).*

	Commodity Futures	**Commodity Spot**	**60/40**	**Stocks**	**Bonds**	**Ira carry**	**ST int. Rate**
Full sample							
Arithmetic mean return	4.8%	2.4%	3.7%	5.5%	1.0%	3.5%	3.5%
Volatility	17.6%	18.1%	9.4%	13.7%	6.7%	5.5%	0.7%
Geometric mean return	3.2%	0.8%	3.3%	4.6%	0.8%	3.3%	3.5%
Skewness	0.38	0.35	-0.5	-0.51	-1.18	1.3	0.73
1877-1945							
Arithmetic mean return	3.8%	2.1%	2.7%	4.6%	-0.1%	3.2%	3.2%
Volatility	20.8%	21.5%	9.2%	13.6%	6.7%	6.8%	0.5%
Geometric mean return	1.6%	-0.1%	2.3%	3.6%	-0.4%	3.0%	3.2%
Skewness	0.39	0.39	-0.53	-0.35	-0.63	0.86	-0.5
1945-2023							
Arithmetic mean return	5.6%	2.7%	4.7%	6.4%	2.0%	3.7%	3.8%
Volatility	14.1%	14.4%	9.5%	13.9%	6.7%	3.9%	0.8%
Geometric mean return	4.6%	1.7%	4.2%	5.4%	1.8%	3.6%	3.8%
Skewness	0.35	0.19	-0.48	-0.66	-1.68	2.81	0.78
abs t-Statistics (diff-in-mean)	0.64	0.19	1.26	0.80	1.93	0.53	4.69

Table 4.1: *Mean returns and volatilities are annualized from monthly excess returns.*[1] *Commodity futures refers to an equal-weighted commodity index, and commodity spot is the spot exposure in the same portfolio. Ira carry is the interest rate adjusted carry. ST stands for short-term. t-statistics refers to the difference in mean of the two subsamples.*

Stocks exhibit the highest arithmetic mean of 5.5%, followed by commodity futures at 4.8% and bonds at 1.0%. We observe that every asset class has higher means after 1945, but these differences are not statistically significant, except for the short-term interest rate. Furthermore, it's worth mentioning the high volatility in both commodity futures and spot, especially pre-1945. Levine et al. (2018) explains this as follows: "The commodity futures index returns volatility is considerably lower after 1945 than previously, most likely because of the additional commodities included in the index as years passed, leading to a higher diversification effect for that particular index." Due to volatility drag, the high volatility results in a significantly lower geometric mean for commodity futures and spot, a trend also observed in the other assets. This effect is particularly evident pre-1945, where the arithmetic mean of commodity spot is 2.1%,

[1] Annualized returns are approximated from multiplying monthly excess returns by 12.

but due to volatility drag, the geometric mean is negative at -0.1%. Given that final wealth is determined by the geometric mean, the impact of high volatility on final wealth is considerable for standalone investments in commodity futures and spot. The presented geometric means offer valuable insights into the cumulative log returns depicted in Figure 4.5. As previously mentioned, it is observed that bonds underperformed the risk-free rate until 1985. This finding is further supported by the negative geometric mean of -0.4% prior to 1945.

The empirical skewness of monthly returns for stocks and bonds presents as negative, which contrasts with the findings of Levine et al. (2018) using US data. This is more in alignment with the common consensus regarding skewness for stocks and bonds. Furthermore, this property supports the inclusion of commodity futures in a 60/40 portfolio, given its distinctive risk attributes associated with its positive skewness. The signs of skewness are also identical across both sub-samples for all assets, except for the short-term interest rate, which may imply varying interest rate regimes over time.

The breakdown of commodity futures in terms of excess spot return and interest rate-adjusted carry is also reported. From the full sample period, it is apparent that, on average, a greater proportion of commodity futures returns are derived from the interest-adjusted carry (3.5%), as compared to the spot (2.4%). The returns from these two do not aggregate to the commodity futures returns, indicating an on average negative cross-term between interest-adjusted carry and the spot. Nonetheless, a review of the volatility underscores that the prime driver of volatility in futures returns at monthly frequencies is the spot price volatility (18.1%), as opposed to the interest rate-adjusted carry volatility (5.5%), supporting our earlier regression findings. Additionally, the historical short-term interest rate volatility is low, indicating it provides little explanation for the variation in commodity futures.

	Full Sample	Backwardation	Contango	Inflation Up	Inflation Down
#months	1751	812	939	890	860
Arithmetic mean returns					
60/40 portfolio	3.7%	1.5%	5.7%	2.6%	4.9%
Commodity futures	4.8%	7.8%	2.2%	10.0%	-0.6%
Commodity spot	2.4%	-2.8%	6.9%	6.9%	-2.2%
Interest rate adj. carry	3.5%	12.1%	-3.9%	4.3%	2.7%
Bonds	1.0%	-1.4%	3.1%	0.5%	1.5%
Stocks	5.5%	3.4%	7.4%	4.0%	7.2%
Volatility					
Commodity futures	17.6%	18.0%	17.1%	18.8%	16.1%
60/40 portfolio	9.4%	9.1%	9.6%	9.3%	9.5%
Correlations					
Commodity futures vs. bonds	0.03	0.04	0.02	0.03	0.03
Commodity futures vs. stocks	0.23	0.13	0.31	0.20	0.27
Stocks vs. bonds	0.29	0.36	0.24	0.31	0.28
Commodity futures vs. 60/40	0.21	0.13	0.28	0.19	0.24

Table 4.2: *Annualized return in different states. Inflation up and down refers to unexpected inflation (positive or negative change)*

	Backwardation/ Contango	Pre/Post 1945	Inflation Up/Down
Commodity futures	1.93	0.64	3.65
Commodity spot	3.24	0.19	3.07
Bonds	4.04	1.93	0.86
Stocks	1.77	0.80	1.40
Interest rate adj. carry	19.50	0.53	1.82
60/40	2.71	1.26	1.47
Short-term interest rate	1.87	4.69	0.27

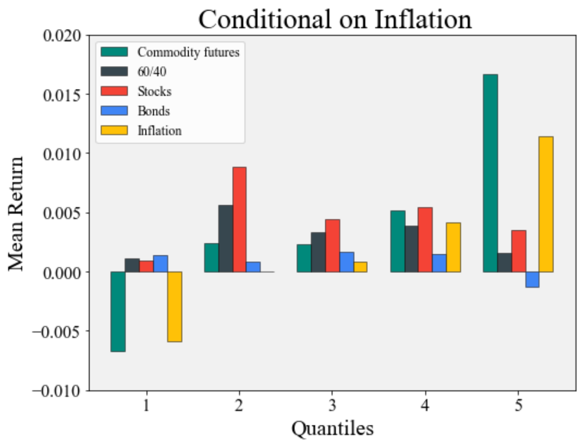
Table 4.3: *t-values of the difference in mean returns between the different states.*

In Table 4.2, we present the average returns for the different assets in states of backwardation, contango, and inflation change, which is often considered a proxy for unexpected inflation. The results indicate that commodity futures exhibit a notable positive average annualized return of 10% during periods of positive unexpected inflation. Conversely, in months of negative unexpected inflation, commodity futures report an average annualized return of -0.6%. These findings align with the conventional expectation that commodity futures tend to perform favorably during periods characterized by positive shocks to inflation while displaying poorer performance in the presence of negative shocks to inflation (Thapar & Maloney (2021)). Moreover, Table 4.2 reveals that the difference in average returns for commodity futures between the two inflation states is statistically significant, as evidenced by a t-statistic of 3.65. Hypothetically, if the market shares this expectation, it could be assumed that there are negative risk premia associated with investing in commodity futures as opposed to stocks, due to their lower average returns (4.8% vs 5.5%). The premise is that, due to this potential "inflation hedge", investors require less compensation for investing in commodity futures than other asset classes, all else being equal (Levine et al. (2018)).

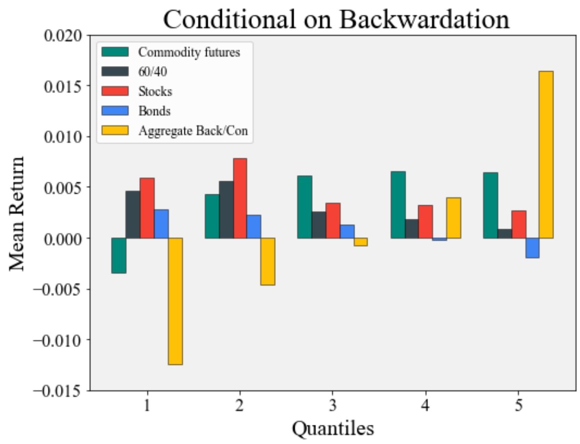
The 60/40 portfolio exhibits a contrast to commodity futures concerning the impact of inflation changes. Notably, it follows an opposite pattern, as evidenced by its conditional mean returns of 2.6% during periods of increasing inflation and 4.9% during periods of decreasing inflation. This observation aligns with previous literature indicating that portfolios of equities and bonds perform worse as inflation increases. Thus, commodity futures seem to perform well in states of the world where the 60/40 portfolio performs poorly, as we also identified in Figure 4.5. Particularly, these states seem to be when the commodity futures are in backwardation or when the economy faces unexpected higher inflation. However, commodity futures exhibit high volatility, and our study focuses solely on a single historical price path. Moreover, the "backwardation-outperformance" observed in-sample may derive from extensive periods during which a potential "mispricing" strategy to exploit this may not have been actively traded upon. It is important to note that past returns do not serve as reliable indicators of future performance.

From Table 4.2, we also confirm a well-documented result that commodity futures in backwardation outperform those in contango (7.8% versus 1.5%, with a t-statistic of 1.93). Not surprisingly, this outperformance is driven by the higher interest rate adjusted carry (12.1% versus -3.9%, with a t-statistic of 19.5) when in backwardation as opposed to contango.

Additionally, we observe that the excess spot returns display the opposite trend to the interest rate adjusted carry. This finding can be attributed to the well-established connection between the mean reversion of spot prices and the shape of the commodity futures curve, as evidenced in previous studies. When the commodity futures curve exhibits backwardation (contango), it typically signals a temporary scarcity (oversupply) situation, with the underlying expectation of spot prices eventually reverting to their equilibrium levels. This mean reversion phenomenon in spot prices elucidates the rationale behind the relatively positive average returns observed in commodity futures during contango periods, albeit to a diminished extent. In such a state, the positive excess spot returns outweigh the adverse impact of interest rate adjusted carry, thereby preserving positive returns for commodity futures (Ben-David et al. (2017)).



(a) *Monthly inflation level*



(b) *Aggregate backwardation/contango*

Figure 4.6: Monthly mean returns for given assets in quintiles of a) and b)

In Figure 4.6a, we see the same trend as in Table 4.2, but with a focus on quintiles of monthly inflation rather than unexpected inflation (change in inflation). As shown above, commodity futures perform better in months of high inflation, with increasing mean returns as the mean inflation level rises. Notably, we see particularly poor performance in the lowest inflation quintile and strong performance in the quintile with the highest inflation. This supports Thapar & Maloney (2021)'s argument that commodity futures serve as a good hedge against inflation but unfortunately underperform during periods of deflation. That said, the outperformance in inflationary periods seems to outweigh the weak returns in deflationary periods.

Furthermore, it becomes apparent that bonds offer relatively low yet consistent returns, except in the top 20% of months with the highest inflation, where returns tend to be negative. Stocks demonstrate their strongest performance, on average, in the second quintile.

They continue to perform well in higher inflationary environments while generating their lowest mean return during months with the lowest inflation. Unquestionably, the strong performance of commodity futures in high inflationary states makes them an attractive addition to the 60/40 portfolio, given that this portfolio tends to underperform during periods of high inflation. Unfortunately, incorporating commodity futures into the 60/40 portfolio would exacerbate its lower returns during the most deflationary months. Thus, the inclusion of commodity futures in this regard becomes a double-edged sword: while it enhances the portfolio's performance in high inflationary states, it simultaneously worsens it in periods of low inflation.

Figure 4.6b illustrates the mean returns of the asset across quintiles, ranging from the most contango (left) to the most backwardated (right) months of commodity futures in aggregate. Table 4.2 highlights that the commodity futures index exhibits a higher mean return when in backwardation compared to when in contango (in aggregate). One might anticipate a linear increase in commodity futures' mean returns with a higher level of backwardation, contributing to improved rollover returns. However, contrary to expectations, the observed differences in commodity futures returns among the quintiles are primarily driven by negative returns in the quintile with the highest contango, rather than a consistent increase with the level of backwardation. This suggests that the disparity in mean returns for commodity futures between states of backwardation and contango, as shown in Table 4.2, originates mainly from poor performance during the top 20% of months characterized by the highest level of contango.

Moreover, the mean returns of bonds appear to have a clear negative relationship as the level of backwardation in commodity futures increases. Furthermore, equities have the highest return in the state where commodity futures are most in contango, but a clear relationship through the quantiles is not obvious. As documented, this contributes to the enhanced performance of the 60/40 portfolio during those months where the aggregate futures curve is in contango, rather than backwardation. It is important to note that these results are measured concurrently and do not predict future returns based on states of backwardation and contango. To investigate the predictive power of the futures curve, additional analysis must be conducted by lagging the measured level of backwardation. When only considering the backwardation level metric, incorporating commodity futures would stabilize the returns of the 60/40 portfolio, as evidenced by the mean returns in each quantile.

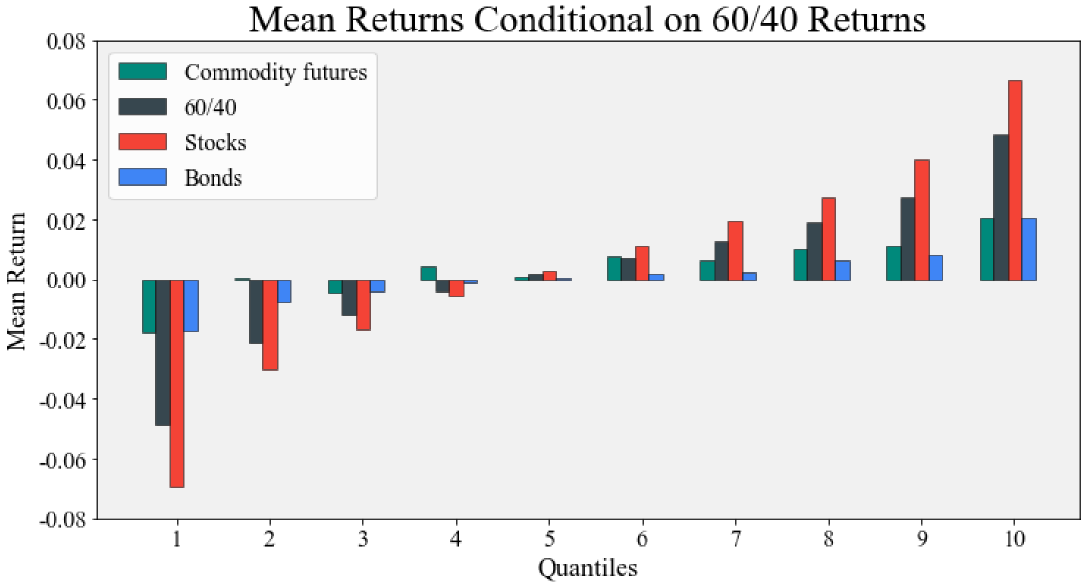


Figure 4.7: *Average monthly returns for the assets in quantiles of the 60/40 portfolio monthly returns.*

Figure 4.7 demonstrates that the mean return of commodity futures somewhat aligns with the sorted increase of the 60/40 mean returns. We observe that commodity futures have lower negative returns in the worst deciles and lower positive returns in the best decile. By incorporating some commodity futures into the 60/40 portfolio, we could diminish the negative mean in the lowest deciles while simultaneously reducing the positive mean in the highest deciles, thereby stabilizing the mean returns across deciles. Additionally, given that commodity futures exhibit a higher mean across the entire sample, it would also elevate the unconditional mean of the portfolio.

In Table 4.4, the correlations between commodity futures and equities, bonds, and commodity spot, as well as the 60/40 portfolio over increasing time intervals, are presented. Intriguingly, the correlation between commodity futures and the 60/40 portfolio remains stable at intervals ranging from monthly to one year. However, it turns negative at the 5-year interval and decreases further to -0.33 at the 10-year interval. These findings suggest that commodity futures could be effective in diversifying a stock-bond portfolio, with the benefits appearing more substantial at longer horizons.

Upon examination of Table 4.5, it is observed that commodity futures correlate positively with monthly inflation and prove statistically significant at all intervals. While the correlation does not exceed 0.15 at the monthly interval, it increases with longer intervals, peaking at 0.59 at a 5-year interval. This trend aligns with academic literature arguing that commodity futures

	Stocks	Bonds	Commodity Spot	60/40
Monthly	0.23*	0.03	0.96*	0.21*
Quarterly	0.25*	0.04	0.95*	0.24*
1-year	0.24*	0.06*	0.91*	0.23*
5-year	-0.04	-0.11*	0.77*	-0.06*
10-year	-0.21*	-0.28*	0.66*	-0.33*

Table 4.4: Correlation of commodity futures returns with stocks, bonds, commodity spot, and 60/40. Overlapping intervals of monthly returns. A "*" indicates statistical significance at the 5% level.

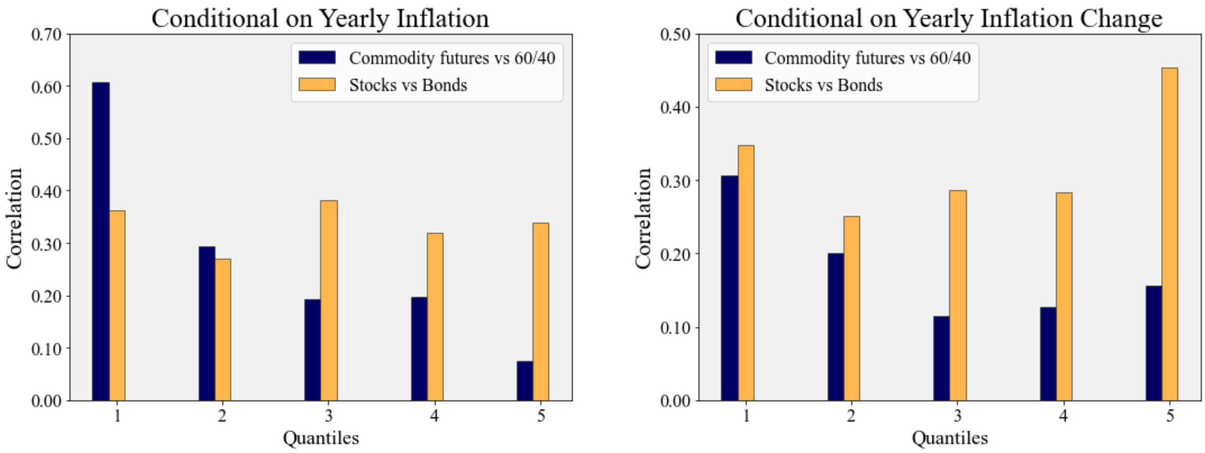
	Stocks	Bonds	Commodity futures	60/40
Monthly	0.02	-0.04	0.15*	0.00
Quarterly	0.03	-0.08*	0.32*	0.00
1-year	0.01	-0.13*	0.51*	-0.02
5-year	-0.12*	-0.26*	0.59*	-0.17*
10-year	-0.14*	-0.21*	0.58*	-0.22*

Table 4.5: Correlation of monthly inflation with stocks, bonds, commodity futures, and 60/40. Overlapping intervals of monthly returns. A "*" indicates statistical significance at the 5% level.

represent a suitable asset class for hedging inflation. Even though the correlation at monthly intervals appears less pronounced, it shows considerable improvement over longer durations. It is somewhat surprising that we do not notice any correlation between inflation and the 60/40 portfolio on monthly, quarterly, or yearly intervals. This discrepancy could stem from our focus on nominal inflation, whereas other research underscores that the negative sensitivities of stocks and bonds to inflation primarily originate from sensitivities to unexpected inflation (Gorton & Rouwenhorst (2006)). However, at 5 and 10-year intervals, we observe significant correlations of -0.17 and -0.22, respectively, between the 60/40 portfolio returns and inflation. Most notably, commodity futures exhibit an opposite exposure to inflation compared to stocks and bonds, especially over longer time horizons.

The analysis conducted herein supports the proposition that including commodity futures can enhance the performance of a 60/40 portfolio, particularly during inflationary states. Firstly, this benefit arises from exposure to commodity spot prices through investment in commodity futures, given that the spot prices are inherently inflationary. Secondly, commodity futures demonstrate strong performance during periods of high nominal inflation, as well as in periods of positive unexpected inflation (positive change). Thirdly, the low correlations observed against the 60/40 portfolio, particularly over longer horizons, present a compelling attribute for inclusion within the portfolio. Moreover, when compared to the findings of Gorton & Rouwenhorst (2006), our research reveals that the correlations between inflation and commodity futures returns were higher and exhibited statistical significance at all time horizons. Intriguingly, our findings underscore the divergent performance of commodity futures, stocks, and bonds within the same inflationary states. While commodity futures yield favorable outcomes, stocks and bonds typically under-perform during periods characterized by high inflation and high unexpected inflation, reinforcing Levine et al. (2018). This divergence underscores the unique attributes of commodity futures as an investment tool, particularly for long-term investors who prioritize portfolio resilience without relying on market timing strategies. However, it is important to recognize that unconditional correlations alone do not provide a comprehensive perspective. The evolution of conditional correlations plays a crucial role in determining the effectiveness of diversification within a given portfolio, particularly during periods when it is most needed.

The correlation between stocks and bonds, as shown in Figure 4.8a (yellow), appears quite stable across different inflation quintiles. However, in Figure 4.8b, we observe a somewhat increased correlation between stocks and bonds (yellow) when inflation levels fluctuate significantly, both in response to negative shocks and even more so towards positive shocks. Regrettably, this increase in correlation reduces the diversification effect of the 60/40 portfolio during periods of significant inflation changes. This observation carries importance as these are the states in which the 60/40 portfolio tends to underperform. Ideally, we would desire enhanced diversification during these periods, rather than the observed deterioration due to increased correlations between the assets in the 60/40 portfolio.



(a) *Yearly inflation*

(b) *Yearly inflation change, from most negative change to most positive change.*

Figure 4.8: Conditional correlations in quintiles of a) and b)

The correlation between commodity futures and the 60/40 portfolio (blue) in quintiles of inflation, shown in Figure 4.8a, exhibits a noteworthy pattern. The correlation begins at around 0.6 in the state with the lowest inflation and decreases rapidly as inflation increases, falling below 0.10 at the 20% highest levels of inflation. When considering the inclusion of commodity futures in the 60/40 portfolio, the low correlation between their returns during the highest levels of inflation contributes to enhanced diversification in these states. This enhancement contrasts with the states of lower inflation, where the diversification effect has historically been less pronounced.

Previous research suggests that hedging against unexpected inflation is the most critical aspect of inflation hedging, as inflation expectations are already embedded in asset prices. Particularly, upside inflation shocks have the potential to cause substantial losses in traditional portfolios of stocks and bonds (Conover et al. (2010b) Neville et al. (2021)). Examining the correlation between commodity futures and the 60/40 portfolio within quintiles of inflation changes, as illustrated in Figure 4.8b, provides intriguing findings. The correlation of 0.3 observed in the lowest quintile gradually decreases in the second and third quintiles, before displaying a modest increase in the fourth and fifth quintiles. The slight rise in correlation during inflation shocks (first and fifth quintiles) is somewhat unexpected, considering previous research. We might anticipate the performance of commodity futures to improve with increasing positive inflation change, paralleled by poorer performance for the 60/40 portfolio. Thus, a decreasing (if not negative) correlation would be expected, rather than the minor increase we observe. However, the correlation between commodity futures and the 60/40 portfolio remains relatively low, potentially contributing to diversification enhancement and improved returns in states of increasing inflation, as demonstrated in Table 4.2.

Below, in Figure 4.9, we illustrate the rolling correlation for the 5-year horizon, which reveals the changes in correlations over time. There are periods where the correlation between commodity futures and the 60/40 portfolio is notably low, even negative at times. Intriguingly, the correlation between stocks and bonds tends to be positive and substantial. If the 60/40 portfolio performed poorly during these periods, an allocation to commodity futures could significantly enhance diversification. However, to form a viewpoint on this, it is important to differentiate between downside and upside correlations, as we will discuss later.

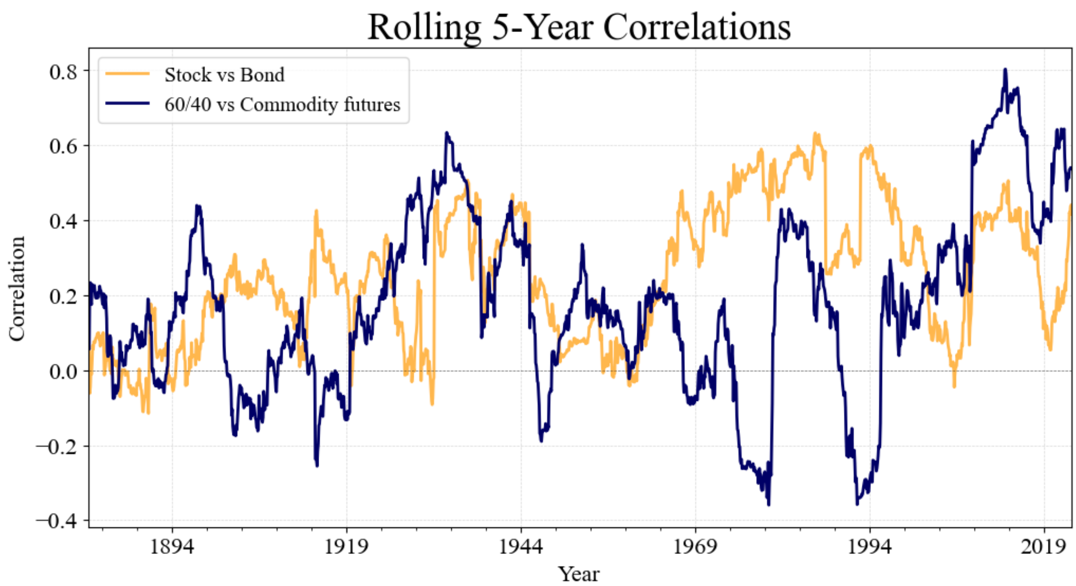


Figure 4.9: *Correlations from a 5-year rolling window of normal returns*

The yellow line in Figure 4.9 underscores an often-overlooked, yet critical facet of the 60/40 portfolio and portfolio construction in general; the correlation between stocks, bonds, and other assets can vary significantly. Empirically, the correlation between stocks and bonds ranges from 0.6 to -0.1 on a 5-year rolling window (the range expands further when the window is shortened). This introduces us to the essential concept of conditional correlations and correlation asymmetry. We desire a low correlation among our portfolio assets when they are underperforming (to the downside) when diversification benefits are most needed. Unfortunately, this issue often goes unnoticed when constant correlations and covariance matrices are assumed for portfolio optimization, thereby potentially weakening the foundation of the traditional 60/40 portfolio, as well as portfolios in general, due to overestimation of diversification benefits.

4.3 Correlation Profiles

To delve deeper into conditional correlations and asset asymmetry, we follow an approach similar to that of Chua et al. (2009). Initially, to account for differences in means and volatilities, we standardize the returns of the assets using the formula $\frac{x-\mu}{\sigma}$. Following this, we calculate the conditional correlations in instances where both assets joint exceed or fall below theta (θ) standard deviations, as demonstrated mathematically;

$$\rho(\theta) = \begin{cases} \mathrm{corr}(x, y \mid x > \theta, y > \theta) & \text{if } \theta > 0 \\ \mathrm{corr}(x, y \mid x < \theta, y < \theta) & \text{if } \theta < 0 \end{cases} \tag{4.1}$$

where the variables x and y are observed values for each asset, $\rho(\theta)$ is the conditional correlation, and θ is the threshold in terms of standard deviation exceedence.

The potential disparity between the assets' full-sample correlation (ρ) and the conditional correlations ($\rho(\theta)$), given the varying thresholds, does not necessarily imply that returns are non-normal or generated by more than a single regime. To illustrate this point, consider a hypothetical scenario with a bivariate normal distribution, denoted as x and y, which have equal means and volatilities, and an unconditional full-sample correlation of 50%. When we truncate the sample to include only observations when both x and y return positive values (or when they both return negative values), the correlation decreases from 50% to 27%. As we move towards the tails of the distribution, the correlation decreases further. For example, if we narrow our focus to observations when both x and y return values of minus (plus) one standard deviation or lower (higher), the correlation decrease to 18%. This phenomenon, referred to as conditioning bias, can potentially lead us to incorrectly conclude that diversification improves during extreme market conditions (Chua et al. (2009)).

In Table 4.6 below, the downside (upside) correlation refers to the correlation between the assets when both have negative (positive) standardized returns. For stocks and bonds, the difference between the downside and upside correlation is -0.083. This difference is advantageous for the 60/40 portfolio, as the diversification is greater to the downside than to the upside. In contrast, commodity futures and 60/40 exhibit a higher correlation to the downside compared to the upside, resulting in lower diversification to the downside than to the upside.

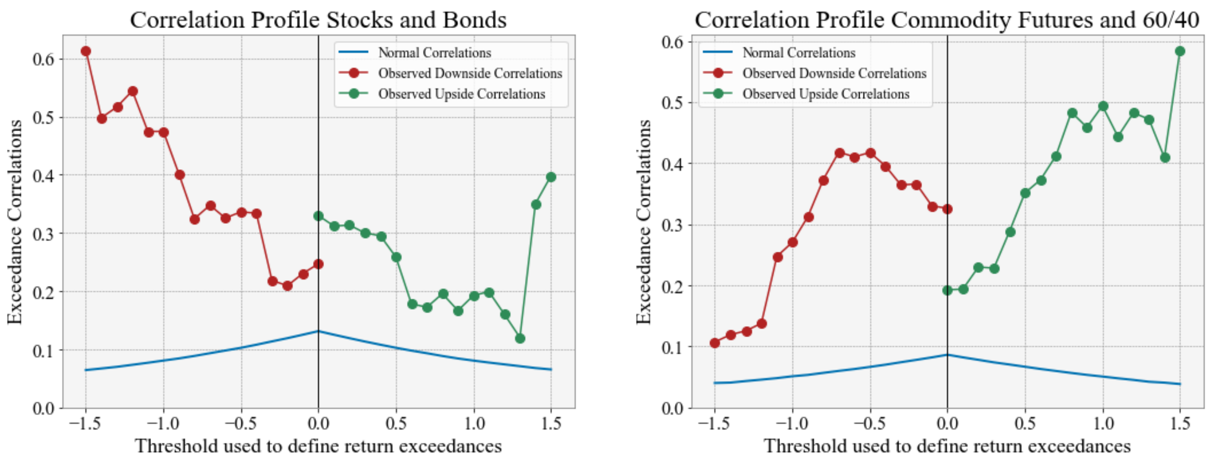
	Downside correlation	Upside Correlation	Difference	Downside frequency	Upside frequency
60/40 vs Commodity futures	0.326	0.193	0.133	27.3%	28.9%
Stock vs Bonds	0.247	0.330	-0.083	28.8%	31.1%

Table 4.6: *Downside and upside correlations for theta (θ) equal zero. Frequency refers to the number of observations that meets the two conditions in Eq. 4.1, relative to the number of observations in the full sample.*

To gain a comprehensive understanding of how the conditional correlation evolves as we move the threshold towards the tails of the distribution, we compute the different conditional correlations with a threshold (θ) within a range from -1.5 to 1.5 standard deviations, as depicted in Figure 4.10. Furthermore, to sidestep the influence of conditioning bias in our empirical analysis, we compare the observed correlation profiles with the correlation profile anticipated from two normal distributions. This comparison is achieved by randomly drawing an adequately large sample from a bivariate normal distribution with a correlation equal to the empirical full-sample correlation of the assets under investigation, as indicated by the blue line in Figure 4.10. The impact of conditioning bias becomes evident as the conditional correlations (blue line) are significantly lower than the unconditional full-sample correlation, and they decrease further as we move the threshold toward the tails of the distribution. Therefore, if the returns of stocks, bonds, and commodity futures were strictly normally distributed, the downside and upside correlation (when $\theta = 0$) would be expected to be equal to 0.09 for 60/40 vs. commodity futures, and 0.13 for stocks vs. bonds, given their unconditional full-sample empirical correlations of 0.21 and 0.29 respectively. These values are markedly lower than our observed conditional correlations, as shown in Table 4.6, and can also be observed from the points on the y-axes in Figure 4.10 below.

We observe from Figure 4.10 that the correlation profiles in both a) and b) consistently exhibit higher correlations than those anticipated from a bivariate normal distribution. Although this might initially seem peculiar, it is critical to note that only approximately 27-31% of the observed sample jointly falls below or above 0 standard deviations (as evident from Table 4.6).

Further, as we move towards the tails, the sample size decreases significantly, with only around 1-2% of the sample meeting the criteria at 1.5 standard deviations. This emphasizes an important potential bias where higher noise is associated with correlations further into the tail due to the smaller sample size. Additionally, a single outlier can considerably influence the correlations, either inflation them or deflating them.



(a) *Stocks and bonds* (b) *Commodity futures and 60/40*

Figure 4.10: *The observed correlations are conditional on the assets' standardized returns jointly exceeding or falling below theta (θ) standard deviations (Eq. 4.1). Due to the high noise resulting from the limited number of observations in the tails, we set a threshold of +/- 1.5 standard deviations. The blue line represents the expected conditional correlations given a bivariate normal distribution (with the unconditional correlation from the assets under consideration).*

Generally speaking, higher-than-expected correlations to the upside are desirable as investors favor assets that move in tandem during prosperous times, providing unification or anti-diversification. However, in challenging times, the goal should be to reduce losses. Furthermore, higher downside correlations than those expected based on the assets' unconditional correlations suggest that losses tend to move more in tandem than anticipated. This trend reduces the benefits of diversification and results in overestimated expectations of downside diversification. This phenomenon is evident in the empirical conditional correlations between stocks and bonds, as illustrated in Figure 4.10a. The observed upside trend is unfavorable, given that when both assets increase by more than 1.3 standard deviations, their correlation slightly exceeds 0.1. Conversely, when both assets decrease by the same deviation, their correlation surpasses 0.6. In essence, the diversification between stocks and bonds functions better during good times when it is not as essential and reduces in bad times. These findings indicate a potential vulnerability in the 60/40 portfolio, as the diversification benefit might be overestimated due to heightened conditional correlations in the downside tail, where diversification is most crucial. In summary, stocks and bonds exhibit an undesirable asymmetry, whereas the commodity futures and 60/40 portfolios show a more favorable conditional correlation asymmetry.

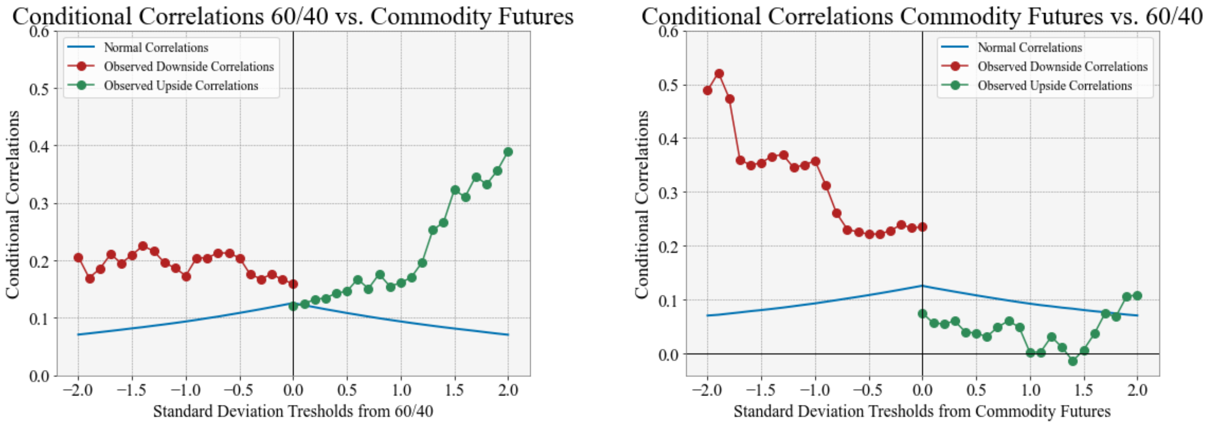
When examining the conditional correlation between the 60/40 portfolio and commodity futures, as depicted in Figure 4.10b, we notice some intriguing characteristics. Interestingly, the conditional correlation experiences a rapid increase as we move the threshold towards the right tail. Conversely, the conditional correlation initially rises before significantly dropping as we move the threshold towards the left tail. Empirically, when both commodity futures and the 60/40 portfolio are up by more than 1.5 standard deviations, their correlation is near 0.6. On the other hand, when both are down by more than 1.5 standard deviations, the correlation slightly exceeds 0.1. This suggests that the diversification between the 60/40 portfolio and commodity futures tends to work better during bad times when it is most needed. Additionally, these assets move more in sync during good times, which is preferable as one should aim to minimize the return drag from diversifiers if possible (Page (2020)).

The encouraging findings regarding the inclusion of commodity futures in the 60/40 portfolio, as depicted in Figure 4.10b, particularly in terms of the evolution of conditional correlations towards the tail, have prompted us to conduct a more in-depth investigation of their correlation relationship. To accomplish this, we will perform an analysis similar to the previous one, but this time conditioning on one asset at a time. Mathematically, this can be expressed as:

$$\rho(\theta) = \begin{cases} \operatorname{corr}(x, y \mid x > \theta) & \text{if } \theta > 0 \\ \operatorname{corr}(x, y \mid x < \theta) & \text{if } \theta < 0 \end{cases} \tag{4.2}$$

Figure 4.11 below presents the correlations between commodity futures and the 60/40 portfolio, conditioned on a) the 60/40 portfolio, and b) commodity futures, where their standardized returns exceed or fall below the threshold (θ) standard deviations. This allows us to measure the differences in tail correlations depending on which of the two assets (or markets) has sold off (or rallied). We have increased the threshold to include up to +/- 2 standard deviations, primarily because the sample size towards the tails is larger due to only one condition, as opposed to two conditions in the jointly conditional analysis.

In Figure 4.11a, which is conditioned on 60/40 returns, the downside correlations (red) are higher than the correlation profile expected (blue) under the assumption that both markets follow a normal distribution with a correlation equal to the observed full-sample empirical correlation. The downside correlation remains stable as the 60/40 returns move towards its left tail, indicating that commodity futures provide relatively good diversification when needed (i.e. when 60/40 performs poorly). However, the conditional correlation shows an increasing trend as we move toward the right tail of the 60/40 portfolio distribution. Contrary to the expectation of perfect symmetry between the upside and downside correlations under a bivariate normal distribution, with conditional correlations gradually decreasing as we move towards the tails, we observe a steady increase in the conditional correlations between the 60/40 portfolio and commodity futures returns as we move the threshold towards the 60/40 portfolio's right tail.



(a) *60/40*

(b) *Commodity Futures*

Figure 4.11: *The observed correlations are conditional on the asset a) and b) standardized returns exceeding or falling below theta (θ) standard deviations (Eq. 4.2). The blue line represents the expected conditional correlations given a bivariate normal distribution (with the unconditional correlation from the assets under consideration).*

Moreover, when conditioned on commodity futures returns, as shown in Figure 4.11b, we observe that the downside correlations are notably higher than those expected under the normality assumption. Furthermore, these downside correlations increase as we move the threshold towards the left tail of the distribution. Conversely, the observed upside correlations display the opposite trend.

We find that commodity futures offer better diversification for the 60/40 portfolio during periods of poor performance by the latter, compared to how the 60/40 portfolio diversifies commodity futures when commodity futures underperform. Also, as shown in Figure 4.11a, the correlation between the 60/40 portfolio and commodity futures increases as we move towards the right tail of the 60/40 returns. However, this desirable trend does not appear when we consider the conditional correlation moving towards the right tail of commodity futures returns in Figure 4.11b. Overall, commodity futures appear to provide desirable diversification in both tails of the 60/40 portfolio's returns.

Let us return to the jointly conditional correlations depicted in Figure 4.10. In order to compare the conditional correlation between different asset or investment pairs across a range of thresholds, we apply Equation 4.3 from Chua et al. (2009). This allows us to calculate the average excess correlations under the assumption of a normal distribution.

$$\begin{aligned}
\mu_{dn} &= \frac{1}{n}\sum_{i=1}^{n}\left[\breve{\rho}\left(\theta_i\right) - \rho\left(\theta_i\right)\right] \quad \forall\theta_i < 0 \\
\mu_{up} &= \frac{1}{n}\sum_{i=1}^{n}\left[\breve{\rho}\left(\theta_i\right) - \rho\left(\theta_i\right)\right] \quad \forall\theta_i > 0
\end{aligned} \tag{4.3}$$

Where the variables μ_{dn} and μ_{up} are the average differences for down and up markets, respectively; $\breve{\rho}\left(\theta_i\right)$ is the observed exceedance correlation at threshold θ_i; and $\rho\left(\theta_i\right)$ is the corresponding normal correlation. For up and down markets, we use n thresholds ($\theta_i \geq 0$ and $\theta_i \leq 0$, respectively) equally spaced by intervals of 0.1 standard deviations, which correspond to $n = 16$. The results are summarized in Table 4.7.

Observations are less frequent towards the tails, but these incidents have the most significant impact on a portfolio. Consequently, we pay relatively more attention to these incidents as we assign equal weight to the correlations. As shown in the results in Table 4.7, all the average excess correlations are positive. This suggests a higher correlation than expected in a normal distribution, as evident from the Figures in 4.10. Interestingly, the average excess correlation between stocks and bonds is now significantly higher on the downside compared to the upside, and the reverse is true for the correlation between the 60/40 portfolio and commodity futures. This contrast is particularly striking when compared with Table 4.6, where we observed the correlations only when θ equaled zero.

	Downside (μ_{dn})	**Upside** (μ_{up})	**Difference** (μ_{dn} - μ_{up})
60/40 vs Commodity futures	0.235	0.321	-0.086
Stocks vs Bonds	0.287	0.153	0.134

Table 4.7: *Average excess correlations versus normal distribution.*

Our results for stocks versus bonds in Table 4.7 align with the findings of Chua et al. (2009), which examined the conditional correlation between U.S. equities (Russell 3000) and U.S. Bonds. They reported an average excess downside correlation of 10.33% compared to a 6.68% average excess upside correlation, using data from 1970 to 2008. While we report higher average excess correlations and a larger asymmetry of 13.4% compared to their 3.65%, the same trend is apparent. Intriguingly, Page (2020) asserts that "During market crises, diversification across risk assets almost completely disappears. Moreover, diversification seems to work remarkably well when investors do not need it: during market rallies. This undesirable asymmetry is pervasive across markets." We observe the same trend for stocks versus bonds. However, their study did not include commodity futures, and our analysis reveals the opposite trend regarding the conditional correlation profile between commodity futures and the 60/40 portfolio.

4.4 Full-Scale Optimization

Throughout this paper, we have observed that the mean returns and the correlation between commodity futures and the 60/40 portfolio are conditional on various factors. These include nominal inflation, inflation change, backwardation (contango), and differing time intervals. Furthermore, the correlation may vary depending on the downside and upside returns. Given these findings, investors should be careful blindly using full-sample correlations for portfolio construction. Rather than optimizing portfolio weights (in our case, consisting of an allocation in the 60/40 portfolio and commodity futures) using approaches such as mean-variance, which assume a constant unconditional covariance matrix or that investors' preferences are well approximated by mean and variance, Chua et al. (2009) and Page (2020) propose the use of full-scale optimization.

Full-scale (or direct utility maximization) does not directly exploit conditional correlations or asymmetries. However, it is flexible, and it directly optimizes the empirical distribution of returns, catering to any type of investor preferences or goals. Full-scale's main advantage over mean-variance optimization is that it accounts for potentially higher moments in portfolio construction from the empirical sample. However, Harry Markovitz found that mean-variance optimization approximates utility functions and asset return distributions remarkably well (Page (2020)). Nevertheless, Page (2020) states that: "Problems with mean-variance start to occur when the utility function includes a sharp drop, a kink that represents significant aversion to loss beyond a threshold." By optimizing the in-sample utility, full-scale indirectly takes the entire in-sample distribution into account and tends to build more robust portfolios with improved correlation asymmetries compared to mean-variance (Chua et al. (2009)). But as with mean-variance optimization, full-scale would yield sub-optimal results out-of-sample if the distribution varies from the in-sample distribution (Adler & Kritzman (2007)).

Given our results and the features of full-scale optimization, we found it interesting to proceed with full-scale. We chose to use the same kinked log utility function (4.4) as Chua et al. (2009), penalizing negative returns below a certain threshold (θ).

$$U(x) = \begin{cases} \ln(1+x), & \text{if } x \geq \theta \\ v \times (x-\theta) + \ln(1+\theta), & \text{if } x < \theta \end{cases} \tag{4.4}$$

θ indicates the location of the kink, and v represents the steepness of the loss aversion slope. The parameters for the full-scale optimization should be based according to investor's preferences, where θ reflect accepted losses and v is the loss aversion beyond the kink (Chua et al. (2009)). In alignment with the method presented in Chua et al. (2009), we use a θ value of -3% and set v to 3 as our choice of parameters.

In applying full-scale optimization with our chosen parameters, we obtain an optimal allocation of 23.5% in commodity futures and a 76.5% allocation in the 60/40 portfolio. In contrast, the mean-variance portfolio, optimized with a risk aversion of 1 (which approximates the optimal growth criterion, also known as the Kelly criterion), allocates 48% to commodity futures and 52% to 60/40. The results for the full-scale, mean-variance, and 60/40 portfolio are presented in Table 4.8. It is important to remember that these portfolios are optimized solely in-sample and make use of the entire sample. Upon examining the results, we observe that both portfolios that include commodity futures enhance the mean return and accumulated wealth, compared to the 60/40 portfolio. As expected, the mean-variance portfolio exhibits the greatest increase in final wealth, given that it approximates the optimal growth portfolio.

The full-scale portfolio demonstrates reduced volatility compared to the 60/40 portfolio. In contrast, the mean-variance portfolio displays increased volatility, to the extent that its Sharpe ratio approximates that of the 60/40 portfolio. Thus, the mean-variance portfolio does not enhance risk-adjusted returns, as the increase in mean return is offset by a corresponding increase in volatility. The higher standard deviation observed in the mean-variance portfolio can largely be attributed to the high variance contribution from commodity futures. Specifically, the 48% allocation to commodity futures in the mean-variance portfolio corresponds to 71.1% of the total portfolio variance. Conversely, in the full-scale portfolio, the 23.5% allocation to commodity futures contributes to 28.6% of the total portfolio variance (refer to Table 4.8).

The full-scale portfolio increases the Sharpe-ratio compared to the 60/40 portfolio, with a higher return and a lower standard deviation. Notably, the full-scale portfolio significantly reduces the probability of losses exceeding 5% and 10% compared to both the mean-variance and 60/40 portfolio, although not surprising since this is expected by design. Overall, we observe that the full-scale portfolio provides better risk-adjusted returns compared to the other two portfolios.

Portfolios	**Commodity futures weight**	**>5% loss probability**	**>10% loss probability**	**Mean return**	**Standard deviation**	**Sharpe-Ratio**	**Accumulative log-returns**	**Variance contribution**
Full Scale	23.5%	2.74%	0.17%	3.96%	8.99%	0.441	5.18	28.6%
Mean Variance	48.0%	3.54%	0.40%	4.22%	10.57%	0.399	5.33	71.1%
60/40	0.0%	3.60%	0.23%	3.72%	9.39%	0.396	4.77	0.0%

Table 4.8: *Mean returns and standard deviations are annualized. The portfolios are rebalanced monthly. >5% and >10% loss refers to the observed percentage number of losses over 5% and 10%. Variance contribution refers to commodity futures variance contribution in the given portfolio.*

In addition, we have measured the degree of correlation asymmetry in both portfolios, optimized respectively through the mean-variance and full-scale approaches. The weighted average correlations in Table 4.9 are calculated according to:

$$\xi = \sum_i w_i \bar{\rho}_i^{dn} - \sum_i w_i \bar{\rho}_i^{up} \tag{4.5}$$

Equation 4.5 defines the degree of correlation asymmetry in a portfolio. "Where w_i is asset i's weight in the portfolio, $\bar{\rho}_i^{dn}$ is the weighted average of correlations between asset i and the other assets in the portfolio when the portfolio is down, and $\bar{\rho}_i^{up}$ is the weighted average of correlations between asset i and the other assets in the portfolio when the portfolio is up" Chua et al. (2009).

Comparing the correlation asymmetries of the full-scale and the mean-variance portfolio in Table 4.9, we notice a desirable asymmetry in the full-scale portfolio versus an undesirable asymmetry in the mean-variance portfolio (higher correlation to the downside than upside).

This indicates an improvement of 7.4% in the correlation asymmetry of the full-scale portfolio relative to the mean-variance portfolio. With a larger sample and a new asset class (commodity futures), our findings support Chua et al. (2009)'s assertion that full-scale optimization tends to improve the portfolio asymmetry compared to mean-variance optimization.

Weigthed average correlation	**Full Scale Optimal**	**Mean Variance Optimal**
Downside correlation	-0.003	-0.024
Upside correlation	0.000	-0.094
Correlation asymmetry	-0.003	0.071

Table 4.9: *Correlation asymmetries in the full-scale and the mean-variance portfolio. The asymmetry is the downside minus the upside correlation.*

4.5 Portfolio Performance

Table 4.10 presents the results of a back-test conducted on our proposed full-scale portfolio (optimized using the full-scale approach) and the traditional 60/40 portfolio, both with monthly rebalancing. The data indicates a significant improvement in the Sharpe ratio, with higher average returns and lower volatility for both sub-samples and the overall sample period. Importantly, the full-scale portfolio underperformed the 60/40 portfolio only in periods characterized by either negative unexpected inflation or when the aggregate commodity futures were in contango, which aligns with our analysis. Conversely, the full-scale portfolio significantly outperformed the 60/40 portfolio in states with positive inflation shocks or when commodity futures were in backwardation.

	Full Sample	**1877-1945**	**1945-2023**	**Inflation Up**	**Inflation Down**	**Back-wardation**	**Contango**
Mean return							
60/40	3.7%	2.7%	4.7%	2.6%	4.9%	1.5%	5.7%
Fullscale	4.0%	2.9%	4.9%	4.3%	3.6%	2.9%	4.8%
Volatility							
60/40	9.4%	9.2%	9.5%	9.3%	9.5%	9.1%	9.6%
Fullscale	9.0%	9.4%	8.6%	9.0%	8.9%	8.6%	9.3%
Sharpe ratio							
60/40	0.40	0.29	0.49	0.28	0.52	0.16	0.59
Fullscale	0.44	0.31	0.57	0.48	0.40	0.34	0.52

Table 4.10: *Annualized returns in different states from monthly normal returns. Both portfolios are rebalanced monthly.*

Beyond documenting the portfolio's performance in terms of unexpected inflation from Table 4.10, we present the mean return across deciles of inflation, as depicted in Figure 4.12. The full-scale portfolio significantly outperforms the 60/40 portfolio in the highest inflation deciles. However, in the lowest inflation decile, the inclusion of commodity futures unfortunately intensifies the already poor portfolio performance. This supports previous literature from Conover et al. (2010b), Neville et al. (2021), and Thapar & Maloney (2021). Nevertheless, the exacerbated underperformance in the lowest inflation decile is more than offset by higher and more stable mean returns in the higher inflation deciles. The cumulative benefits of the full-scale portfolio (including commodity futures) compared to the 60/40 portfolio are showcased in Figure 4.13.

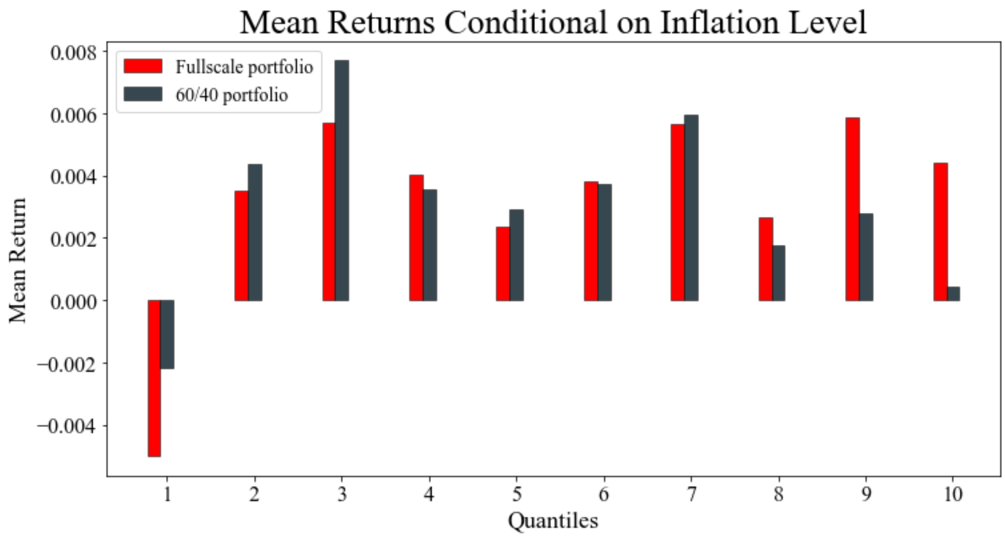


Figure 4.12: *Mean returns in quantiles of monthly inflation. Both portfolios are rebalanced monthly.*

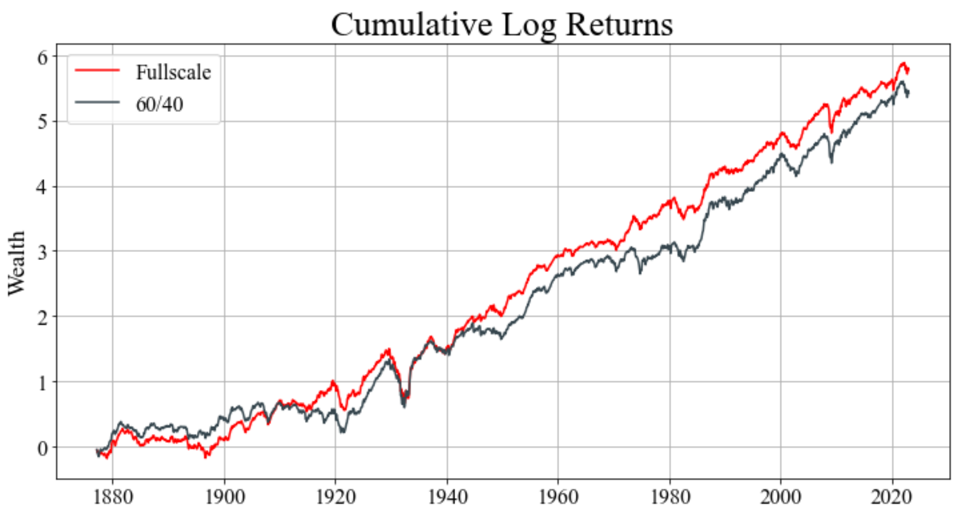


Figure 4.13: *Cumulative log returns for full-scale and 60/40 portfolio. Both portfolios are rebalanced monthly.*

It is important to acknowledge that the full-scale portfolio outperformance in the full-sample is not unexpected, as the back-test employed an in-sample methodology, whereby the portfolio optimization and evaluation were performed using the same full sample period. Nevertheless, this analysis sheds light on the historical viability of incorporating commodity futures as a favorable addition to the 60/40 portfolio as well as the potential advantages of the full-scale optimized portfolio versus the mean-variance optimization. Furthermore, given the comprehensive 146-year sample period, we studied the performance across multiple economic cycles and regimes. Therefore,it is reasonable to expect that the inclusion of commodity futures would continue to contribute positively to the portfolio's performance over the long run.

5 | Conclusion

This thesis studies commodity futures and seeks to improve the traditional 60/40 portfolio, by including commodity futures with a strategic asset allocation approach e.g. with no market timing. The study is based on 146 years of monthly data, spanning the period from 1877 to 2023. We document that the drivers of commodity returns are both income and price returns, where the price returns come from changes in the underlying spot prices, and that the volatility in commodity futures returns mostly come from the volatility in spot prices, especially at short time horizons. Empirically we find that commodity futures have different properties than stocks and bonds, complementary to the 60/40 portfolio. This seems especially evident in periods with high inflation or positive unexpected inflation, as commodity futures outperform stocks and bonds significantly in these states. Furthermore, we find these results to be even more significant at longer horizons.

We document substantial variability in the cross-correlations between the assets over time. We found undesirable asymmetry in the correlation profile towards the tails between stocks and bonds. Conveniently the correlation profile between commodity futures and the 60/40 portfolio shows better correlation asymmetry towards the tails. In addition, we find that in general, the correlation profiles tend to be higher than what we would expect from a bivariate normal distribution, potentially indicating overestimated diversification benefits. To indirectly take these conditional correlations into account we optimize our proposed portfolio using a full-scale optimization approach inspired by previous literature from Chua et al. (2009) and Adler & Kritzman (2007).

Our full-scaled optimized portfolio allocates 23.5% in commodity futures and the remaining in the 60/40 portfolio. This is significantly lower than the 48% allocated from the mean-variance optimization. The full-scale portfolio shows improved correlation asymmetry and significantly improves the Sharpe ratio (0.44) compared to the mean-variance portfolio (0.40). The higher standard deviation observed in the mean-variance portfolio can largely be explained by the high degree of variance contribution from commodity futures, as the 48% allocation to commodity futures in the mean-variance portfolio corresponds to 71.1% of the overall portfolio variance, compared to only 28.6% in the full-scale portfolio.

Further, the full-scale portfolio improves the mean return and the volatility compared to the 60/40 portfolio on the full sample and also reduces the frequency of losses exceeding 5 and 10 percent. Naturally, the new portfolio outperforms the 60/40 portfolio the most, in terms of a higher Sharpe ratio, in states of positive unexpected inflation, and when the aggregate commodity futures are in backwardation. The same holds true for both sub-samples, spanning from 1877 to 1945 and 1946 to 2023. However, the results must be considered with a grain of salt as they are in-sample optimizations. Thus, the full-scale portfolio is expected to outperform the 60/40 portfolio overall, since the 60/40 portfolio is not based on in-sample optimization. Nevertheless, our analysis shows the advantages of including commodity futures in the classic 60/40 portfolio from a large dataset including several market cycles and regimes. We show that the increased robustness is especially apparent in periods of high inflation, where the 60/40 portfolio performs poorly. We also show why considering conditional correlations matters and how assuming constant correlations could hurt portfolios and overestimate the diversification benefits.

For further research, we suggest exploring correlation predictions within a tactical asset allocation approach. Additionally, investigating the impact of different rebalancing periods on portfolio performance, in the long run, could provide valuable insights.

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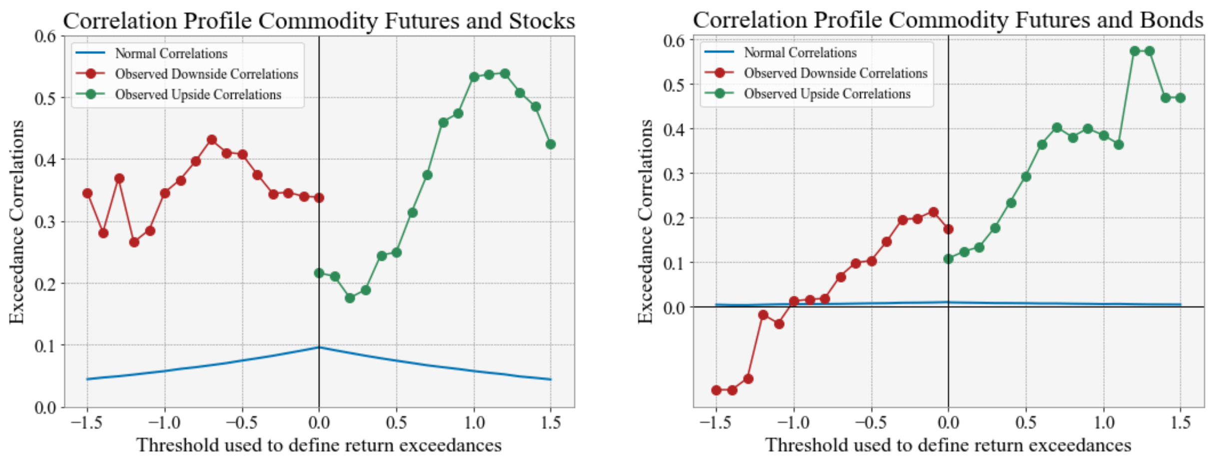
A | Appendix

A.1 Correlation Profiles



(a) *Commodity futures and stocks*   (b) *Commodity futures and bonds*

Figure A.1: *The observed correlations are conditional on the assets standardized returns jointly exceeding or falling below theta (θ) standard deviations (Eq. 4.1). Due to the high noise resulting from the limited number of observations in the tails, we set a threshold of +/- 1.5 standard deviations. The blue line represents the expected conditional correlations given a bivariate normal distribution (with the unconditional correlation from the assets under consideration).*

A.2 Price Return vs. Income Return

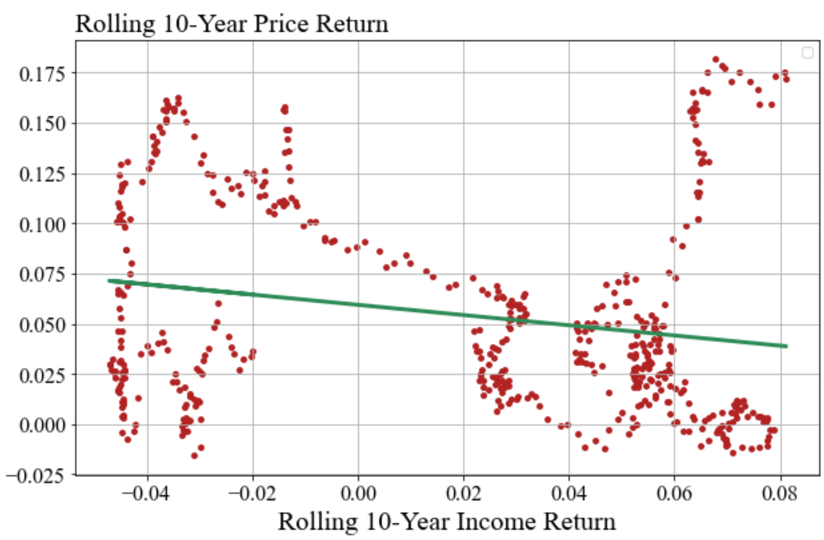
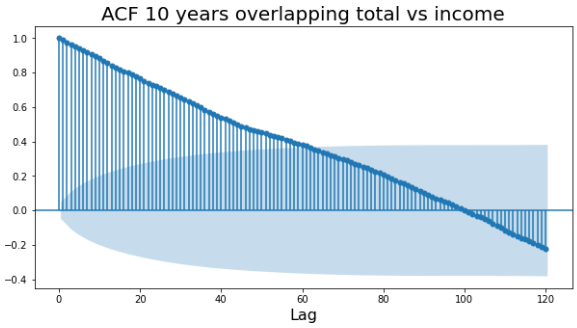
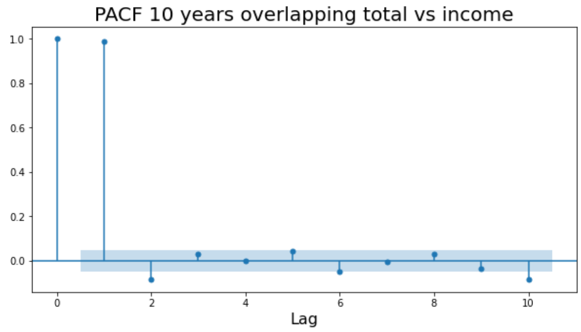


Figure A.2: *Price return = 0.059 - 0.254 Income return, R-squared = 0.046*

A.3 ACF and PACF

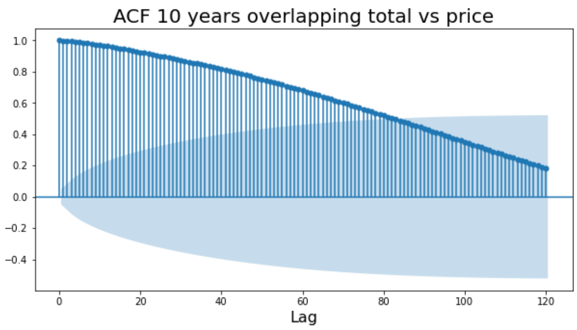


(a) *ACF*

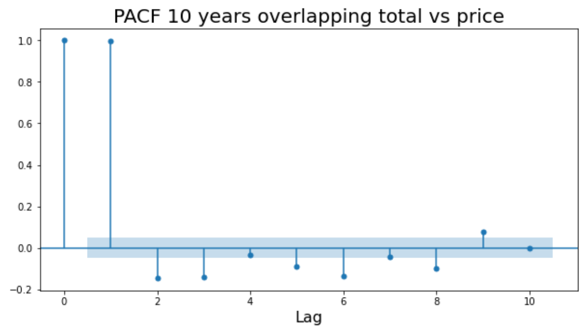


(b) *PACF*

Figure A.3: *Based on b) PACF, we include one lag in the Newwey-West robust standard errors, as this will take into account the majority of the auto-correlation from overlapping returns.*



(a) *ACF*



(b) *PACF*

Figure A.4: *Based on b) PACF, we include one lag in the Newwey-West robust standard errors, as this will take into account the majority of the auto-correlation from overlapping returns.*